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# An energy-efficient reliable path finding algorithm for stochastic road networks with electric vehicles

Liang Shen[a], Hu Shao[a,c,*], Ting Wu[b], William H. K. Lam[c], Emily C. Zhu[d]

*[a]School of Mathematics, China University of Mining and Technology, Xuzhou, Jiangsu, 221116, China.*
*[b]Department of Mathematics, Nanjing University, Nanjing, Jiangsu, 210093, China*
*[c] Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China.*
*[d] Desautels Faculty of Management, McGill University, 1001 Rue Sherbrooke West, Montreal, Quebec H3A 1G5, Canada*

## Abstract

In this paper, we develop a novel reliable path finding algorithm for a stochastic road network with uncertainty in travel times while both electric vehicle energy and efficiency are simultaneously taken into account. We first propose a bi-objective optimization model to maximize (1) the on-time arrival reliability and (2) energy-efficiency for battery electric vehicles (BEVs) in a path finding problem. The former objective requires finding the reliable shortest path (RSP), which is the path with the minimal effective travel time measured by the sum of the mean travel time and a travel time safety margin for any given origin-destination (OD) pair. Then, we refer to energy-efficiency as the minimum of the electric energy consumption. We discuss the non-additive property of the RSP problem since we also consider the link travel time correlations, whereas the latter objective satisfies the additive criterion. To this end, we illustrate the existence of non-dominated solutions that satisfy both of the two objectives. Furthermore, it is shown that the intersection of two candidate sets – one for the RSPs and the other for paths with minimal energy-consumption - actually contains the optimal solution for the bi-objective optimization problem. The upper and lower bounds of the effective travel time are mathematically deduced and can be used to generate the candidate path set of this bi-objective problem via the K-shortest algorithm. Our proposed algorithm overcomes the infeasibility of traditional path finding algorithms (e.g., the Dijkstra algorithm) for RSPs. Moreover, using two numerical examples, we verify the effectiveness and efficiency of the proposed algorithm. We numerically demonstrate promising potential applications of the proposed algorithm in real-life road traffic networks.

**Keywords:** Reliable path finding algorithm; Link travel time correlation; Energy-efficient path; Non-dominated solution.

Declarations of interest: none

# 1. Introduction

During the recent decade, battery electric vehicles (BEVs) have gained more and more popularity among travelers and governments due to their environment-friendly features. However, the supporting infrastructures for BEVs are still in the cradle stage and far more than mature, exposing BEV users to operational challenges and inconveniences. For instance, by connecting to an outlet or charging device, it generally takes several hours to fully recharge a BEV (Johnson et al., 2013) in the sharp contrast to the refueling process for fuel vehicles. As the charging process for BEVs is more time consuming than fuel ones, BEV travelers have to ponder energy efficiency when making path choice decisions besides saving travel time. To address this issue, this paper proposes a path finding algorithm that also explicitly considers the energy efficiency for travelers in BEVs.

Travelers are always keen on finding the best path (Yu et al., 2009, 2011; Wu and Nie, 2011; Nie et al., 2012; Xu et al., 2017), although the criteria of an optimal path may vary over time and from person to person (Nazemi and Omidi, 2013; Faro and Giordano, 2016). Traditionally, transportation researchers have focused on finding the most reliable paths for travelers who aim to avoid being late when traveling in a transportation network with uncertainties (Xu et al., 2011). In fact, traffic uncertainties can be caused by many factors, such as adverse weather conditions, traffic accidents, vehicle malfunctions, signal failures, big events, and demand variations (Asakura and Kashiwadani, 1991; Clark and Watling, 2005; Walting, 2006; Shao et al., 2006; Gao et al., 2009). Recently, due to the growing awareness of eco-sustainability as well as the constantly rising fuel prices, transportation studies have started to study eco-routing, which involves the path-finding problem and considers carbon emissions reductions (Sun and Liu, 2015; Zeng et al., 2016; Androutsopoulos and Zografos, 2017). This study aims to find a path that simultaneously takes account of on-time arrival reliability and energy-efficiency.

The most reliable path, which is the path that maximizes the on-time arrival probability for a given travel time budget, was first introduced by Frank (1969). Similar to Asakura and Kashiwadani (1991), here, we refer to travel time reliability as the probability that travelers can arrive at their destination node on time. This concept of travel time reliability suggests that the path choice criterion consists of two factors: the travel time and reliability of the path. Then, Hall (1983) noted that travelers tend to reserve a safety margin to hedge against variations of travel times. The sum of the mean travel time and the safety margin is called effective travel time or travel time budget. Actually, the safety margin is the product of the standard deviation of travel time. In the specific case where travel times are normally distributed, the reliability measured by the effective travel time (Wu and Nie, 2011), becomes the sum of the mean travel time and a path-specific safety margin; and the safety margin, the extra time buffer, improves the likelihood of arriving on time (Helander et al., 1997; Xing et al., 2011; Samaranayake et al., 2012; Xiao and Lo, 2013; Srinivasan et al., 2014; Yao et al., 2014; Doan et al., 2015). Indeed, Rakha et al. (2010) noted that the key parameters for estimating path travel time reliability included not only the mean travel time but also the travel time variance. Hence, the reliable path is actually the path with the minimal effective travel time.

Methods of finding the reliable path have attracted the attention of researchers over the last few decades. Mirchandani (1976) introduced a recursive algorithm that enumerated all paths and all travel time possibilities to solve the discrete version of Frank's problem (Frank, 1969). Recently, there have been two major streams –

finding (1) the shortest paths and (2) the alpha-shortest paths. The former stream targets the most reliable path given a certain travel time budget. Specifically, the reliable path finding problem can be formulated as a multi-objective path finding problem with the constraint of variance (Sivakumar and Batta, 1994). Shao et al. (2004) proposed a heuristic method for solving this problem that is based on the relationship between the mean and standard deviation of the link travel times. Nie and Wu (2009) proposed a label-correcting algorithm to find the most reliable path by generating all of the non-dominated paths under first-order stochastic dominance. Chen at al. (2014) investigated the time-dependent reliable shortest path problem (TD-RSPP), which is commonly encountered in congested urban road networks.

On the other hand, the alpha-shortest path, which was first proposed by Chen and Ji (2005), refers to the path that minimizes the required travel time budget given an on-time arrival probability threshold. Meanwhile, many experts have proposed algorithms to solve the reliable shortest path problem (RSPP) in transportation networks over the past few decades. Chen et al. (2013a) addressed the problem of finding the path with the shortest expected time in stochastic time-dependent (STD) road networks by proposing an efficient multicriteria $A^*$ algorithm to exactly determine the least-expected-time path in STD networks. Chen et al. (2013b) presented the multicriteria label-setting and $A^*$ algorithm to find the alpha-reliable path when link travel times followed normal distributions. Yang et al. (2014) considered the non-anticipativity constraint associated with the a priori path in STD road networks and proposed a number of reformulations to establish linear inequalities that could be easily dualized by a Lagrangian relaxation. Although most works considered either the shortest or alpha-shortest path, Nikolova (2006) developed a parametric approach to determine both the alpha-reliable and most reliable paths.

Our work also relates to the stream of energy-efficiency optimization. Specifically, Jiang et al. (2013) extended a network equilibrium model to consider mixed gasoline and electric vehicular flows. Frank et al. (2013) presented a novel eco-driving application that informs the driver about his energy efficiency. Levin et al. (2014) demonstrated the impact of the road grade on network wide vehicle energy consumption by integrating energy consumption equations based on road load equations. It is clear that different types of formulas can be used to describe electric energy consumption (EEC). Due to concerns about climate change, the advancement of battery technologies and expeditiously rising prices of crude oil, BEVs have gained attention in recent years (Riemann et al., 2015; He et al., 2015; He et al., 2016; Chen et al., 2016; Chen et al., 2017a, 2017b; Lee and Madanat, 2017; He et al., 2018; Yi et al., 2018; Lacobucci et al., 2019; Pan et al., 2019; Wang et al., 2019; Sun and Yin, 2019). Researchers have proposed a large number of energy consumption formulas, as energy consumption is influenced by many factors, including speed (Yang et al., 2014; Zhang and Yao, 2015; Xu et al., 2018), distance and travel time (Wang et al., 2013; Kluge et al., 2013; He et al., 2014; Yang et al., 2015). He et al. (2014) formulated three mathematical models to describe the resulting network equilibrium flow distribution on regional or metropolitan road networks considering varied flow dependencies on the energy consumption of BEVs and their recharging time. Yang et al. (2014) proposed an electricity consumption model to study the effects of the road's slope on the BEV's electricity consumption (Zhang and Yao, 2015). Without a loss of generality, we use the formulas presented in He et al. (2014) and Zhang and Yao (2015), respectively. The formula adopted in He et al. (2014) assumes that the energy consumption of vehicles is determined according to distance and speed and is independent of traffic congestion. Meanwhile, the energy consumption of vehicles proposed in Zhang and Yao (2015) is affected by BEV's mass (kg), the road gradient and the vehicle speed (m/s). Hence, the energy consumption of BEVs satisfies the property of additivity; consequentially, we use the traditional K-shortest algorithm to solve the proposed energy-efficient model.

Although the above studies address either the reliable path finding or energy-efficient problem, they fail to consider the correlations of travel times between different links. On the other hand, Ji et al. (2011) extended the alpha-reliable path finding model into a multi-objective reliable path finding model by simultaneously considering travelers' multiple confidence requirements for travel time reliability and the link travel time correlations specified by a given covariance matrix. Chen et al. (2012) studied the problem of finding the reliable shortest path in a stochastic network with spatial correlated link travel times, where the link travel time is assumed to only be correlated to the neighboring links within a local 'impact area'. Later, Wang et al. (2016) presented the assumption of spatial correlations restricted to adjacent links and investigated the constrained shortest path problem in a transportation network in which the link travel times are random variables that follow certain joint probability mass functions. However, their work did not consider the reliability in a traffic network. By contrast, Zeng et al. (2015) investigated the important problem of determining a reliable path in a stochastic network with correlated link travel times. The Lagrangian relaxation (LR) approach is applied to solve the non-linear and non-additive problem. Zhang et al. (2017) proposed a novel LR approach based on a new convex problem reformulation to update the Lagrangian multipliers and handle the negative cycles of the resulting shortest path problems. In this work, we model the spatial correlations of link travel times using a variance-covariance matrix. Differently from previous studies, we propose an innovative method of the inequality technique to solve the reliable path finding problem in a transportation network with uncertainty. Specifically, we contribute to the literature by designing a heuristic algorithm to address the non-linear and non-additive properties of the effective travel time, for which traditional path-finding algorithms (e.g., the Dijkstra algorithm) are inapplicable.

We summarize the classification of existing studies in Table 1. This paper focuses on the above two model frameworks, namely, the optimal reliable path model and energy-efficient path model. Our proposed algorithm is based on a rigorous mathematical proof of the upper- and lower- bounds of the effective travel time. Moreover, the optimal reliable path problem takes the travel time correlations among different links into account and models the link travel time correlations by a variance-covariance matrix. These upper- and lower- bounds are further used to generate the candidate path set for energy-efficient reliable path finding using the conventional K-shortest algorithm (Chen & Feng, 2000). Succeeding in avoiding enumeration, our proposed algorithm is thus efficient for medium- to large-scale transportation networks.

Table 1 Categories of existing studies

| | Reliability | | | | Energy Consumption |
|---|---|---|---|---|---|
| | Deterministic | Stochastic | | | |
| | | Independent | Correlation | | |
| | | | Adjacent Links | All Links | |
| Huang & Lam, 2002; | √ | × | × | × | × |
| Shao et al. 2004; Chen & Ji, 2005; Nie & Wu, 2009; Chen et al. 2014. | × | √ | × | × | × |
| Chen et al. 2012. | × | × | √ | × | × |
| Ji et al. 2011; Zeng et al. 2015; Zhang et al. 2017 | × | × | × | √ | × |
| This paper | × | × | × | √ | √ |

Specifically, this paper bridges a gap in the literature and makes the following contributions.

(1) The proposed bi-objective model simultaneously considers the optimal path reliability and BEV energy-efficiency.

(2) A heuristic algorithm based on the inequality technique is proposed that avoids path enumeration and results in polynomial computational complexity. Hence, this algorithm can potentially be applied in large-scale networks.

(3) Several theorems for the optimal reliable path finding problem are established to demonstrate the existence, effectiveness and efficiency of the proposed algorithm.

The rest of this paper is organized as follows. In section 2, the model formulation of the bi-objective model is presented. Then, after mathematically demonstrating the foundations, a heuristic solution algorithm is proposed in Section 3. We numerically show the effectiveness and efficiency of the algorithm in two extensive case studies in Section 4. Finally, conclusions and further studies are discussed in Section 5.

## 2. Model formulation

### *2.1. Notations and assumptions*

*2.1.1. The notations used throughout the paper are listed as follows, unless otherwise specified.*

**Nomenclature**

***Sets:***

| | |
|---|---|
| **A** | Set of links in the network. |
| **E** | Set of links between OD pairs. |

| | |
|---|---|
| $\mathbf{R^n}$ | Set of feasible solutions. |
| $\mathbf{G} = (\mathbf{N}, \mathbf{A})$ | A road network, with $\mathbf{N}$ being the set of nodes and $\mathbf{A}$ being the set of links. |
| $\mathbf{E_k}$ | Kth subset of $\mathbf{E}$. |
| $\mathbf{M}$ | Set of energy-efficient paths. |
| $\mathbf{X}$ | Subset of $\mathbf{R^n}$. |
| $\mathbf{Q}$ | Candidate path set |
| ***Variables:*** | |
| $\mu_a$ | Mean of the link travel time. |
| $\sigma_a$ | Standard deviation of the link travel time. |
| $T_a$ | Travel time on link $a$. |
| $T_k^{rs}$ | Travel time on path $k$ between OD pair $rs$. |
| $\mu_k^{rs}$ | Mean of the travel time on path $k$ between OD pair $rs$. |
| $\sigma_k^{rs}$ | Standard deviation of the travel time on path $k$ between OD pair $rs$. |
| $\rho_{ab}$ | Correlation coefficient of the link travel time between links $a$ and $b$. |
| $t_{k_i}^{rs}$ | Effective travel time on path $k_i$. |
| $\varepsilon_k^{rs}$ | The safety margin. |
| $\Phi^{-1}(\cdot)$ | Inverse function of the standard normal cumulative distribution function. |
| $k^*$ | The reliable path. |
| $k_r^*$ | The optimal reliable path between OD pair $rs$. |
| $c_{k_i}^{rs}$ | Energy consumption of the BEVs on path $k_i$ between OD pair $rs$. |
| $d_a$ | The distance of link $a$. |
| $k_e^*$ | The optimal energy-efficient path between OD pair $rs$. |
| $f$ | A vector valued objective function. |
| $\hat{t}_{k_i,\min}^{rs}$ | The lower boundary of path $k_i$ between OD pair $rs$. |
| $\hat{t}_{k_i,\max}^{rs}$ | The upper boundary of path $k_i$ between OD pair $rs$. |
| $\hat{u}$ | The minimum of the upper bounds. |
| $t_{k^*}^{rs}$ | The minimum effective travel time. |
| $\rho_{\mathrm{M}}$ | Correlation coefficient matrix |
| ***Parameters:*** | |
| $\theta$ | The confidence level for the on-time arrival probability. |
| K | Number of $K$-shortest paths |
| $\delta_{a,k}^{rs}$ | Element of the link-path incidence matrix $\Delta$. |
| $\lambda$ | The multiplier for the correlation coefficient matrix ($0 \le \lambda \le 1$) |

### *2.1.2. Basic assumption*

**A1.** It is assumed that the link travel times are non-negative and follow a multivariate normal distribution (Chen et al., 2012). Further, the path travel time (sum of link travel times) distribution is approximated by normal distribution, which is more computationally tractable and has an acceptable compromise on accuracy (Zeng et al., 2015; Zhang et al., 2017).

**A2.** The formula of energy consumption presented in He et al. (2014) is not affected by traffic congestion. That is, consumption is flow-independent.

**A3.** In this paper, it is assumed that the on-time arrival probability of choosing a path is greater than or equal to 50%, i.e., $\theta \geq 50\%$, which represents the travelers' risk-neutral and risk-averse path choice behaviors.

*2.2. Concept of the optimal reliable path under the condition of correlation*

Consider a directed network $\mathbf{G = (N, A)}$ consisting of a set of nodes $\mathbf{N}$ and a set of links $\mathbf{A}$. The path travel time, which is denoted as $T_k^{rs}$, is the sum of the related link travel times along the path as

$$T_k^{rs} = \sum_{a=1}^{m} \delta_{a,k}^{rs} T_a \tag{1}$$

where $m$ is the number of links. The indicator $\delta_{a,k}^{rs} = 1$ means that link $a$ is on path $k$, and $\delta_{a,k}^{rs} = 0$ otherwise. Similar to conventional studies, the path travel time $T_k^{rs}$ is the travel time distribution of all of the links along the path and is also a random variable with its mean and standard deviation denoted as $\mu_k^{rs}$ and $\sigma_k^{rs}$, respectively.

In fact, the distribution of link travel times has been extensively investigated in the last few decades, because it is an important issue for modeling travel time reliability. So far, the typical random distributions usually considered in literature include the normal distribution (Watling, 2006; Chen et al., 2013b), the lognormal distribution (Srinivasan et al., 2014), and the gamma distribution (Nie et al., 2012). Due to the skewness of the link travel time distribution, many studies recommend the lognormal distribution for link travel time modeling. Recently, Zeng et al. (2015), used the normal distribution to approximate the distribution of path travel times to make the computation of the $\alpha$ -reliable paths mathematically tractable. As they showed that if link travel times follow the log-normal distribution, the path travel time distribution can only be derived numerically by computing the sum of link travel time distribution, due to the unavailable closed form PDF or CDF for the sum of the truncated lognormal distributed variables.. The assumption is also justified in the simulation study shown in the Appendix of Zeng et al. (2015). Thus, we assume the path travel time follows a normal distribution (Chen et al., 2012; Zeng et al., 2015) in this study, which means that $T_a \sim N(\mu_a, (\sigma_a)^2)$. In this article, the mean and variance of the path travel time between OD pair $rs$ can be calculated as follows:

$$\mu_k^{rs} = \sum_{a \in \mathbf{E}} \delta_{a,k}^{rs} \mu_a \tag{2}$$

$$\sigma_k^{rs} = \sqrt{\sum_{a \in \mathbf{E}} \delta_{a,k}^{rs} (\sigma_a)^2 + 2 \sum_{a,b \in \mathbf{E}} \delta_{a,k}^{rs} \delta_{b,k}^{rs} \rho_{ab} \sigma_a \sigma_b} \tag{3}$$

where $\mu_k^{rs}$ and $\sigma_k^{rs}$ are the mean and standard deviation of the path travel time between OD pair $rs$, respectively. The variable $\rho_{ab}$ is the correlation coefficient between links $a$ and $b$, and $\mu_a$ and $\sigma_a$ are the mean and standard deviation of link travel time, respectively. The notion $\mathbf{E}$ is the set of links between OD pair $rs$, and $\delta_{a,k}^{rs}$ is the decision variable regarding the link-path incidence relationship, where $\delta_{a,k}^{rs} = 1$ means that the link $a$ is on path $k$, and $\delta_{a,k}^{rs} = 0$ otherwise.

The detailed description of the effective travel time or travel time budget can be found in Lo et al. (2006) and Shao et al. (2006). The corresponding effective travel time $t_k^{rs}$ on path $k$ between OD pair $rs$ is depicted as the sum of the mean time $\mu_k^{rs}$ and safety margin $\varepsilon_k^{rs}$ (Lo et al., 2006; Shao et al., 2006).

$$t_k^{rs} = \mu_k^{rs} + \varepsilon_k^{rs} \tag{4}$$

where the safety margin can be determined by the following chance constraint model:

$$\begin{aligned} &\min_{\varepsilon_k^{rs}} t_k^{rs} = \mu_k^{rs} + \varepsilon_k^{rs} \\ &s.t. \Pr[T_k^{rs} \le t_k^{rs}] \ge \theta \end{aligned} \tag{5}$$

where $\theta$ is a given confidence level. By a simple manipulation, the effective travel time in equation (5) can be expressed as follows:

$$t_k^{rs} = \mu_k^{rs} + \Phi^{-1}(\theta)\sigma_k^{rs} \tag{6}$$

where $\Phi^{-1}(\theta)$ is the inverse cumulative distribution function (CDF) of the standard normal distribution at the $\theta$ confidence level. That is, $\Phi^{-1}(\theta)$ is the probability of the travel time not exceeding the effective travel time that is not less than $\theta$. The definition of the effective travel time ensures the reliability of the travel time to be greater than or equal to $\theta$ while minimizing the travel time. Then, the optimal reliable path can be defined as follows.

**Definition1.** Among all of the feasible paths between OD pair $rs$, the optimal reliable path is defined as the path with the minimum effective travel time. Mathematically, path $k_r^*$ is the optimal reliable path between OD pair $rs$ if and only if

$$k_r^* = \arg\min t_k^{rs} \tag{7}$$

Due to the non-linear and non-additive properties of Eq. (6), the conventional shortest path finding algorithms cannot be adopted to find the optimal reliable path.

*2.3. Concept of an energy-efficient path*

On account of concerns regarding climate change and rising prices of crude oil, electric bicycles and vehicles have gained attention in recent years. Thus, energy consumption of BEVs is taken into consideration in this paper. Actually, researchers have proposed a large number of energy consumption formulas. In this paper, we consider two different types of energy consumption formulae. The first one is the same as in He et al. (2014), which reflect the fact that drivers' speeds are not affected by traffic congestion. Thus, the consumption is flow-independent. Under this circumstance, a BEV does not always speed up and speed down when it traverses a link. This formula is typically applied to driving on the highways without traffic signals. The second type of formula refers to the ones presented in Yang et al. (2014) and Zhang and Yao (2015), which incorporate the fact that BEVs' speeds are affected by many factors, such as the BEV' s mass, the vehicle speed and the road gradient. In Fig.1, we describe the detailed speeding process of BEVs in link $a$. In this case, a BEV has to speed up at the beginning of a link and speed down at the end of a link. This type formula applies for driving in urban transportation networks consisting of arterial roads with signalized intersections.

*Case A. The first type formula presented in He et al. (2014)*

The energy consumption of BEVs $c_k^{rs}$ on path $k$ between OD pair $rs$ can be calculated as follows:

$$c_k^{rs} = 0.174 \times \sum_{a \in \mathbf{E}} \delta_{a,k}^{rs} d_a + 0.116 \times \sum_{a \in \mathbf{E}} \delta_{a,k}^{rs} \mu_a \quad (8a)$$

where $d_a$ is the distance of link $a$ and $\mu_a$ is the mean travel time of link $a$. Eq. (8) is as proposed by He et al. (2014).

*Case B. The second type formula presented in Yang et al. (2014) and Zhang and Yao (2015)*

According to the discussions in Yang et al. (2014) and Zhang and Yao (2015), the output power of the BEV is calculated as follows:

$$P_m = v(emg\cos\alpha + mg\sin\alpha + \delta m\, dv/dt + 0.5C_D H \chi v^2)/(\eta_c \eta_m) + P_{accessory} \quad (8b)$$

where $m$ is the BEV's mass (kg); $e$ is rolling resistance coefficient; $g$ is the gravitational constant; $\alpha$ is the road gradient; $\delta$ is the coefficient related to the BEV's mass; $v$ is vehicle speed (m/s); $C_D$ is the aerodynamic drag coefficient; $H$ is the BEV's frontal area (m$^2$); $\chi$ is the air density (kg/m$^3$); $\eta_c$ stands for the controller efficiency; $\eta_m$ represents the motor efficiency. $P_{accessory}$ denotes the electricity consumed by other accessories (e.g. the electric power steering).

Considering the braking process, the BEV's regenerative braking power can be formulated as follows:

$$P_r = w\eta_c\eta_m(emg\cos\alpha + mg\sin\alpha + \delta m\, dv/dt + 0.5C_D H \chi v^2)v + P_{accessory} \quad (8c)$$

where $w$ ( $0 < w < 1$ ) is the regenerative braking factor, which indicates the percentage of the total braking energy that can be recovered by the motor. Moreover, we define the parameter $w$ as follows:

$$w = \begin{cases} 0.5 \times \dfrac{v}{5} & v < 5\,m/s \\ 0.5 + 0.3 \times \dfrac{v-5}{20} & v \geq 5\,m/s \end{cases} \quad (8d)$$

In the light of the relationship between the power and energy, the BEV's energy consumption can be calculated as follows:

$$c_k^{rs} = \sum_{a \in \mathbf{E}} P_a \times \delta_{a,k}^{rs} t_a \quad (8e)$$

where $P_a$ is the total power of the BEV's battery of link $a$, which is the sum of $P_m$ and $P_r$. $t_a$ is mean travel time of link $a$.

In urban transportation networks, the drivers should decelerate at the end of a link due to traffic signals or pedestrian crossing at the intersections. For safety reasons, most drivers comply with this rule in practice. For convenience of calculation, the BEV's energy consumption can normally be divided into the following three stages shown in Figure 1 ( Yang et al. 2014):

- The first stage: the BEV's energy consumption sharply increases because its speed increases to start a trip. The energy consumption of this stage can be calculated by Eq. (8b) .
- The second stage: the BEV's energy consumption is constant since its speed has been stable during a trip. The energy consumption of this stage can be calculated by Eq. (8b).

- The third stage: the BEV's energy consumption is decreasing due to the BEV regenerates electricity during the process at the end of a trip. The energy consumption of this stage can be calculated by Eq. (8c).

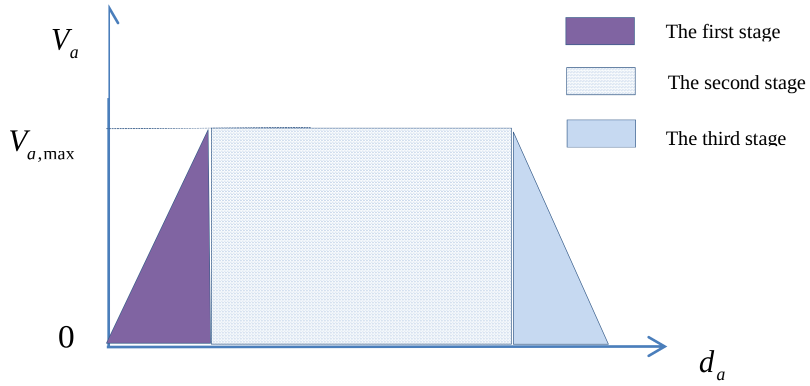


Fig.1 The detailed speeding process of a BEV in link $a$

The notations $V_a$ and $d_a$ represent speed and distance of link $a$, respectively. It is well known that the BEV's mass is a random variable in networks.

**Definition2.** Among all of the feasible paths between OD pair $rs$, the energy-efficient path between OD pair $rs$ is the path with the minimum energy consumption of BEVs. Mathematically, path $k_e^*$ is the energy-efficient path between OD pair $rs$ if and only if

$$k_e^* = \arg\min c_k^{rs} \tag{9}$$

From Eqs. (8a-8e), it is obvious that the conventional algorithm (e.g., the K-shortest algorithm) can be adopted to solve the energy-efficient path problem due to the additive property.

### *2.4. Bi-objective path finding model*

In this paper, according to assumption A1, the link travel times are non-negative and follow a normal distribution; all of the link travel times are correlated. These assumptions are commonly used in studies of stochastic shortest path problems (Chen et al., 2012). Based on recent empirical studies, it is also found that a normal distribution appears to reflect the reality of most link travel times, and the normality assumption can be sufficient from a practical standpoint given its computational simplicity (Rakha et al. 2006). Under the normality assumption, the bi-objective model can be described as follows:

Case A: The energy consumption formula of BEVs presented in He et al. (2014).

$$\begin{cases} \min t_k^{rs} = \sum_{a\in\mathbf{E}} \delta_{a,k}^{rs}\mu_a + \Phi^{-1}(\theta)\sqrt{\sum_{a\in\mathbf{E}} \delta_{a,k}^{rs}(\sigma_a)^2 + 2\sum_{a,b\in\mathbf{E}} \delta_{a,k}^{rs}\delta_{b,k}^{rs}\rho_{ab}\sigma_a\sigma_b} \\ \min c_k^{rs} = 0.174\times\sum_{a\in\mathbf{E}} \delta_{a,k}^{rs} d_a + 0.116\times\sum_{a\in\mathbf{E}} \delta_{a,k}^{rs}\mu_a \end{cases} \tag{10a}$$

Case B. The energy consumption formula of BEVs presented in Zhang and Yao (2015)

$$\begin{cases} \min t_k^{rs} = \sum_{a\in\mathbf{E}} \delta_{a,k}^{rs}\mu_a + \Phi^{-1}(\theta)\sqrt{\sum_{a\in\mathbf{E}} \delta_{a,k}^{rs}(\sigma_a)^2 + 2\sum_{a,b\in\mathbf{E}} \delta_{a,k}^{rs}\delta_{b,k}^{rs}\rho_{ab}\sigma_a\sigma_b} \\ \min c_k^{rs} = \sum_{a\in\mathbf{E}} P_a \times \delta_{a,k}^{rs} t_a \end{cases} \tag{10b}$$

Subject to

$$\sum_{a\in\mathbf{E}}\delta_{a,k}^{rs}-\sum_{b\in\mathbf{E}}\delta_{b,k}^{rs}=\begin{cases}-1 & \forall a\notin k, b\in k\\ 0 & \forall a,b\in k;\forall a,b\notin k\\ 1 & \forall a\in k, b\notin k\end{cases} \tag{11}$$

$$\delta_{a,k}^{rs}\in\{0,1\},\quad \forall a\in\mathbf{E} \tag{12}$$

Eqs. (10a) and (10b) are the objective functions that travelers want to minimize. Eq. (11) ensures that the reliable shortest path is feasible. Eq. (12) is concerned with the link-path incidence variables, which should be binary in nature. Due to the non-additive property of the objective function, the conventional algorithm cannot be adopted to solve the model. Thus, a heuristic algorithm will be proposed in next section.

## 3. Solution algorithm for solving the proposed bi-objective model

### *3.1. Concept of the Pareto efficient solution*

The multi-objective optimization problem (MOP) can be defined as follows:

$$\min\quad F(\mathbf{x})=(f_1(\mathbf{x}),\cdots f_p(\mathbf{x}))^{\mathrm{T}} \tag{13}$$

$$\text{s.t.}\quad \mathbf{x}\in\Omega \tag{14}$$

where $\mathbf{x}=(x_1,\cdots,x_n)\in\mathbf{R}^n$ is the vector of design variables, $\Omega$ is the feasible search domain, and $f_i(x)$ is the $i$ th objective function.

Two basic definitions are given as follows.

- Let $\mathbf{x}_1$ and $\mathbf{x}_2\in\Omega$ be two solutions of an MOP. $\mathbf{x}_1$ is said to dominate $\mathbf{x}_2$ if and only if $f_i(\mathbf{x}_1)\le f_i(\mathbf{x}_2), \forall i\in\{1,\cdots,p\}$.
- Let $\mathbf{x}^*\in\Omega$ be called Pareto optimal if there is no other solution in $\Omega$ that dominates $\mathbf{x}^*$.

### *3.2. Theoretical basis for the first objective function*

It is well-known that the conventional path finding algorithm (e.g., the Dijkstra algorithm) is inapplicable in the event of a non-additive effective travel time. The optimal reliable path and energy-efficient path can be easily found by calculating the effective travel time and energy consumption on all paths with the method of enumeration. Meanwhile, the non-dominated efficient solution of the proposed bi-objective model can also be obtained through the enumeration method. However, path enumeration is time consuming and almost infeasible for large-scale networks. Therefore, the enumeration method is usually not acceptable in real-world applications. To overcome this difficulty, this paper proposes a heuristic algorithm for the bi-objective model that avoids path enumeration. The algorithm principles are described as follows.

The first principle of the algorithm is to determine the estimated upper- and lower-bounds of the effective travel time. How to establish a proper estimation of the upper- and lower- bound $(\hat{t}_{k_i,\min}^{rs},\hat{t}_{k_i,\max}^{rs})$ for the effective travel time of the path $k_i$ is crucial in the proposed algorithm. We address this issue in the following theorem.

**Theorem 1** The effective travel time $t_{k_i}^{rs}$ satisfies the following inequality considering the link travel time correlations ( $\theta \geq 50\%$ ):

$$\sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \mu_a \leq t_{k_i}^{rs} \leq \sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} t_a^{rs}, \forall rs \in \mathbf{N}, \forall k \in \mathbf{E} \quad (15)$$

where $t_a^{rs} = \mu_a + \Phi^{-1}(\theta)\sigma_a$ .

Proof: According to Eq. (2), the left hand-side of the above inequality (15) can be written as follows:

$$\mu_{k_i}^{rs} = \sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \mu_a, \forall rs \in \mathbf{N}, \forall k_i \in \mathbf{E} \quad (16)$$

Because $\theta \geq 50\%$, it follows that $\Phi^{-1}(\theta) \geq 0$ and $\sigma_{k_i}^{rs} \geq 0$. Then, the following follows Eqs. (6) and (16):

$$t_{k_i}^{rs} = \mu_{k_i}^{rs} + \Phi^{-1}(\theta)\sigma_{k_i}^{rs} \geq \mu_{k_i}^{rs} = \sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \mu_a, \forall rs \in \mathbf{N}, \forall k_i \in \mathbf{E}. \quad (17)$$

We then prove the right hand-side of inequality (15). According to Eq. (3), $\sigma_a \geq 0$ and $|\rho_{ab}| \leq 1$, we have:

$$\begin{aligned} \sigma_{k_i}^{rs} &= \sqrt{\sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} (\sigma_a)^2 + 2\sum_{a,b\in\mathbf{E}} \delta_{a,k_i}^{rs} \delta_{b,k_i}^{rs} \rho_{ab} \sigma_a \sigma_b} \\ &\leq \sqrt{\sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} (\sigma_a)^2 + 2\sum_{a,b\in\mathbf{E}} \delta_{a,k_i}^{rs} \delta_{b,k_i}^{rs} \sigma_a \sigma_b} \\ &= \sqrt{(\sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \sigma_a)^2}, \forall rs \in \mathbf{N}, \forall k_i \in \mathbf{E}. \end{aligned} \quad (18)$$

Plugging inequality (18) into Eq. (6), we have

$$\begin{aligned} t_{k_i}^{rs} &= \mu_{k_i}^{rs} + \Phi^{-1}(\theta)\sigma_{k_i}^{rs} \\ &= \mu_{k_i}^{rs} + \Phi^{-1}(\theta)\sqrt{\sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} (\sigma_a)^2 + 2\sum_{a,b\in\mathbf{E}} \delta_{a,k_i}^{rs} \delta_{b,k_i}^{rs} \rho_{ab} \sigma_a \sigma_b} \\ &\leq \mu_{k_i}^{rs} + \Phi^{-1}(\theta)\sqrt{(\sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \sigma_a)^2} \\ &= \mu_{k_i}^{rs} + \Phi^{-1}(\theta)\sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \sigma_a, \ \forall rs \in \mathbf{N}, \forall k_i \in \mathbf{E} \end{aligned} \quad (19)$$

Then, substituting Eq. (16) into inequality (19), we have

$$\begin{aligned} t_{k_i}^{rs} &\leq \mu_{k_i}^{rs} + \Phi^{-1}(\theta)\sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \sigma_a \\ &= \sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \mu_a + \Phi^{-1}(\theta)\sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \sigma_a, \ \forall rs \in \mathbf{N}, \forall k_i \in \mathbf{E} \\ &= \sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} t_a^{rs}, \ \forall rs \in \mathbf{N}, \forall k_i \in \mathbf{E} \end{aligned} \quad (20)$$

Therefore, theorem 1 has been proved according to inequalities (17) and (20). □

According to theorem 1, the lower- and the upper-bounds of the effective travel times are given as below:

Lower bound:

$$\hat{t}_{k_i,\min}^{rs} = \sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} \mu_a, \forall rs \in \mathbf{N}, \forall k_i \in \mathbf{E} \quad (21)$$

Upper bound:

$$\hat{t}_{k_i,\max}^{rs} = \sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} t_a^{rs} = \sum_{a\in\mathbf{E}} \delta_{a,k_i}^{rs} (\mu_a + \Phi^{-1}(\theta)\sigma_a), \ \forall rs \in \mathbf{N}, \forall k_i \in \mathbf{E} \quad . \quad (22)$$

Eq. (22) shows that the lower- and upper-bounds of the effective travel times are additive with respect to each link. Therefore, the K-shortest algorithm can be applied to find the $i$-shortest path $k_i$ between OD pair $rs$ using the lower bounds of the effective travel times (see Eq. (21)). Then, the correlated stochastic path finding problem can be transformed into a deterministic path finding problem.

The second principle in the algorithm is to determine the candidate set of reliable paths. We assume that there are $l$ paths between OD pair $rs$, and we denote these paths as $k_1, k_2, \cdots k_l \in \mathbf{E}$, the path set $\mathbf{E}$ is defined as a set of all feasible paths between OD pair $rs$. The corresponding effective travel times are denoted as $t_{k_1}^{rs}, t_{k_2}^{rs}, \cdots t_{k_l}^{rs}$. For each path $k_i$, we use Eqs. (21) and (22) to estimate upper- and lower- bounds of the effective travel times $\left(\hat{t}_{k_i,\min}^{rs}, \hat{t}_{k_i,\max}^{rs}\right)$. The estimated upper bounds of those paths are denoted as $\hat{t}_{k_1,\max}^{rs}, \hat{t}_{k_2,\max}^{rs}, \cdots \hat{t}_{k_l,\max}^{rs}$, and the minimal upper bound of the paths is set as

$$\hat{u} = \min\{\hat{t}_{k_1,\max}^{rs}, \hat{t}_{k_2,\max}^{rs}, \cdots \hat{t}_{k_l,\max}^{rs}\} \quad . \tag{23}$$

It is clear that $t_{k^*}^{rs}$ is the minimal effective travel time, therefore

$$t_{k^*}^{rs} \le t_{k_i}^{rs}, \forall k_i \in \mathbf{E} \tag{24}$$

The effective travel time of path $k_i$ satisfies

$$t_{k_i}^{rs} \le \hat{t}_{k_i,\max}^{rs} \tag{25}$$

Then, we incorporate inequality (16) with (17):

$$t_{k^*}^{rs} \le \hat{t}_{k_i,\max}^{rs}, \forall k_i \in \mathbf{E} \tag{26}$$

Hence, path $k_i$ is not the reliable path between OD pair $rs$ if path $k_i \in \mathbf{E}$ and it satisfies the inequalition $\hat{t}_{k_i,\min}^{rs} > \hat{u}$. Otherwise, path $k_i$ may be the reliable path.

**Theorem 2** Let path set $\mathbf{Q}$ be the set that contains the alternative reliable paths. The reliable path $k^*$ must be included in the candidate path set $\mathbf{Q}$ if path $k_i$ satisfies the following:

(i) if $k_i \in \mathbf{Q}$, then $\hat{t}_{k_i,\min}^{rs} \le \hat{u}$, and

(ii) if $k_i \notin \mathbf{Q}$, then $\hat{t}_{k_i,\min}^{rs} > \hat{u}$.

Proof: This theorem is proved via reduction to absurdity. Assume that $k^*$ is the optimal path with the minimal effective travel time and path $k^*$ does not belong to $\mathbf{Q}$ (i.e., $k^* \notin \mathbf{Q}$). Hence, if the optimal reliable path $k^*$ does not belong to $\mathbf{Q}$, it satisfies the following:

$$\hat{t}_{k^*,\min}^{rs} > \hat{u} \tag{27}$$

Based on Eq. (23), the minimal upper bound of the paths between OD pair $rs$ satisfies the following inequality:

$$\hat{t}_{k_i,\max}^{rs} \le \hat{u},\ \exists k_i \in \mathbf{E} \tag{28}$$

Following Eqs. (27) and (28), an inequality can be obtained as follows:

$$\hat{t}_{k_i,\max}^{rs} \le \hat{u} < \hat{t}_{k^*,\min}^{rs},\ \exists k_i \in \mathbf{E} \tag{29}$$

Apparently, path $k_i$ and path $k^*$ satisfy the following inequalities:

$$t_{k_i}^{rs} \le \hat{t}_{k_i,\max}^{rs} \tag{30}$$

$$t_{k^*}^{rs} \ge \hat{t}_{k^*,\min}^{rs} \tag{31}$$

Then, the new inequality can be obtained while incorporating inequalities (29) and (30) into (31), as below:

$$t_{k_i}^{rs} \le \hat{t}_{k_i,\max}^{rs} \le \hat{u} < \hat{t}_{k^*,\min}^{rs} \le t_{k^*}^{rs},\ \exists k_i \in \mathbf{E}, k_i \ne k^* \quad (32)$$

Inequality (32) shows that the effective travel time on path $k_i$ is less than that on path $k^*$. Therefore, path $k^*$ is not the optimal reliable path, which contradicts the assumption that path $k^*$ is the optimal reliable path with the minimal effective travel time. To this end, path $k^*$ must be an alternative reliable path (i.e., $k^* \notin \mathbf{Q}$). □

Therefore, we use the K-shortest algorithm in the proposed algorithm by using the lower bound criterion (see Eq. (21)). As the value of parameter 'K' increases, the number of candidate paths will increase. When the parameter 'K' equals the total number of paths between the OD pair, the optimal path for the RSPP must be included in the candidate path set. In other words, if the parameter 'K' in the K-shortest algorithm is sufficiently large, the optimal path in the RSPP must be in the candidate path set.

**Remark:**

In practice, the parameter 'K' can be set as a positive integer. For example, if 'K=10', the number of paths in the candidate path set is less than or equal to 10. As parameter 'K' increases, the optimal path in the RSPP becomes more and more likely to be included in the candidate path set. However, a larger value of parameter 'K' means a longer computational time. Thus, we recommend to set a reasonable 'K' to balance the efficiency and feasibility of the proposed algorithm, though the techniques to determine an optimal K is out of the scope of this study.

Based on Eq. (21), we can find out that the lower-bound of the effective travel time is not affected by on-time arrival probability $\theta$. On the contrary, the upper-bound of the effective travel time is determined by on-time arrival probability $\theta$, as shown in Eq. (22). Using the on-time arrival probability, the travelers' risk attitudes toward travel time uncertainty can be identified by the following three types (Yin and Ieda, 2001; Chen et al., 2012):

- Risk-averse: if on-time arrival probability $\theta > 50\%$;
- Risk-neutral: if on-time arrival probability $\theta = 50\%$;
- Risk-seeking: if on-time arrival probability $\theta < 50\%$.

Therefore, BEV users can be classified into three types in terms of their risk-taking attitudes under travel time uncertainty. In this paper, we only consider the non-trivial case of risk-averse behaviors; and hence, the associated on-time arrival probability $\theta$ is greater than 50%. We also assume that the on-time arrival probability $\theta$ is the same for all travelers in the network.

### *3.3. Theoretical basis for the second objective function*

Recent researches have proposed a large number of energy consumption formulas; and in this paper we adopt two different types of energy consumption formulas--- the one that is proposed by He et al. (2014) (see Eq. (8a)) and the other proposed by Yang et al. (2014) and Zhang and Yao (2015) (see Eq. (8b)). Apparently, both types of the formulas satisfy the additive property (He et al. (2014), Yang et al. (2014) and Zhang and Yao (2015)). Thus, many conventional algorithms can be used to solve the second objective. In this paper, we aim to find some feasible paths, whose energy consumptions are less than or equal to the specified value $EEC_{\max}$. Therefore, we use the K-shortest algorithm (Yang and Chen, 2006) to find the accurate solution and then to address the energy consumption issue.

### *3.3. Solution algorithm*

Based on the above theorems, the detailed procedures of our presented heuristic algorithm are described as below.

**Inputs:**

OD pair $(r,s)$ and on-time arrival probability $\theta$,

The mean and variance of the link travel time $\mu_a$ and $\sigma_a$,

Link distance $d_a$,

Correlation coefficient of the link travel times $\rho_{ab}$, and

A pre-given size of the candidate set $K_{\max} > 0$, which is a sufficiently large integer.

$EEC_{\max}$ denotes the upper bound of acceptance range for travelers' energy consumption.

$ETT_{\max}$ denotes the upper bound of acceptance range for travelers' effective travel time.

**Outputs:**

The non-dominated efficient solutions to the optimal reliable and energy-efficient path finding problems.

**Step 1:** Initialization: set $i = 1$, $\mathbf{Q} = \varphi$.

**Step 2:** Carry out the K-shortest ($K = K_{\max}$) path algorithm to calculate the energy consumption results with Eqs. (8a) and (8e). The K-shortest path finding algorithm will stop if the energy consumption of path $k_n$ (Case A) or $k_m$ (Case B) is greater than the specified value $EEC_{\max}$. Then, put the results into sets $\mathbf{M}_1$ and $\mathbf{M}_2$.

**Step 3:** Find the corresponding paths among the energy consumption results and denote them as $k_i(i = 1, 2, \cdots n) \in \mathbf{M}_1$ and $k_j(i = 1, 2, \cdots m) \in \mathbf{M}_2$ with Eq. (9).

**Step 4:** Use the Dijkstra algorithm (Dijkstra, 1959) with respect to the upper bound of the effective travel time (Eq. (22)) to find a path that is denoted as the current optimal reliable path. Then, put it into the candidate path set $\mathbf{Q}$. The corresponding upper bound of this path is denoted as $\hat{u}$, as defined in Eqs. (22) and (23).

**Step 5:** Find the $i$-shortest path $k_i$ between OD pair $rs$ using the $K$-shortest algorithm with the lower bounds of the effective travel times $\hat{t}^{rs}_{k_i,\min}$ (see Eq. (21)).

**Step 6:** If $\hat{t}^{rs}_{k_i,\min} \le \hat{u}$

If $i < K_{\max}$

If $\hat{t}^{rs}_{k_i,\min} \le ETT_{\max}$

Put the path $k_i$ into candidate path set $\mathbf{Q}$ and $i = i + 1$.

Then, go to step 5.
Else
Go to step 7.
Else
Go to step 7.
End
Else
Go to step 7.
End

**Step 7:** Calculate the effective travel time of path $k_i (i = 1, 2, \cdots q)$ in set $\mathbf{Q}$. If the effective travel time of path $k_i$ is greater than $ETT_{\max}$, remove the path $k_i$ from set $\mathbf{Q}$. Then, denote the number of paths in path set $\mathbf{Q}$ is $\tilde{q}$.

**Step 8:** Compare the energy-efficient paths ($\mathbf{M}_1$ and $\mathbf{M}_2$) in step 3 with the reliable paths ($\mathbf{Q}$) in step 6. The path $k_p^*$ is a non-dominated efficient solution and satisfies the following criterion:

There is not a path $k$ that satisfies the inequalities $c_k^{rs} \le c_{k_p^*}^{rs}$ and $t_k^{rs} \le t_{k_p^*}^{rs}$, and there is at least one path $k_i$ that satisfies the inequality $c_{k_i}^{rs} < c_{k_p^*}^{rs}$ or $t_{k_i}^{rs} < t_{k_p^*}^{rs}$.

*Remark:* Because in this algorithm we use the K-shortest algorithm with the polynomial computational complexity (Yang and Chen, 2006), the complexity of our proposed algorithm is polynomial.

In order to demonstrate the accuracy of the obtained, we present the following theorem to show the small error between the obtained heuristic solution and the optimal one.

**Theorem 3** The solution of the reliable path using the proposed algorithm is denoted as $t_k^{rs}$. According to Eqs. (21) and (22), the upper and lower bounds of effective travel time $t_k^{rs}$ can be calculated and denoted as $t_{k,\min}^{rs}$ and $t_{k,\max}^{rs}$. The maximal error between the solution $t_k^{rs}$ and the optimal solution $t_{m,optimal}^{rs}$ is $t_{k,\max}^{rs} - t_{k,\min}^{rs}$.

**Proof:** We assumed that the solution $t_k^{rs}$ is not the optimal solution. Then, there must exist an optimal solution $t_{m,optimal}^{rs}$ ($m > k$). According to Eqs. (21) and (22), the upper and lower bounds of effective travel time $t_k^{rs}$ can be calculated and denoted as $t_{k,\max}^{rs}$ and $t_{k,\min}^{rs}$. Similarly, we can also obtained the upper and lower bounds of optimal solution $t_{m,optimal}^{rs}$ and denoted as $t_{m,\max}^{rs}$ and $t_{m,\min}^{rs}$. Based on Step 5 in the algorithm, the lower bound of the effective travel time can be obtained with the method of K-shortest algorithm. Therefore, the lower bound of the optimal solution $t_{m,\min}^{rs}$ is equal or greater than the lower bound of the solution $t_{k,\min}^{rs}$ ($t_{k,\min}^{rs} \le t_{m,\min}^{rs}$) provided $m > k$. The upper bound of $t_k^{rs}$ is $t_{k,\max}^{rs}$ and the lower bound of optimal solution $t_{m,optimal}^{rs}$ is $t_{m,\min}^{rs}$. Thus, they satisfy the following inequalities:

$$t_k^{rs} \le t_{k,\max}^{rs} \tag{33}$$

$$-t_{m,optimal}^{rs} \le -t_{m,\min}^{rs} \tag{34}$$

The error between the solution $t_k^{rs}$ and the optimal solution $t_{m,optimal}^{rs}$ can be expressed as:

$$\left|t_k^{rs} - t_{m,optimal}^{rs}\right| = \left|t_k^{rs} + (-t_{m,optimal}^{rs})\right| \le \left|t_{k,\max}^{rs} + (-t_{m,\min}^{rs})\right| \tag{35}$$

Because $m > k$, an inequality can be obtained as follows:

$$t_{k,\min}^{rs} \le t_{m,\min}^{rs} \Rightarrow -t_{m,\min}^{rs} \le -t_{k,\min}^{rs} \tag{36}$$

Then, substituting inequality (36) into inequality (35), we have

$$\begin{aligned} \left|t_k^{rs} - t_{m,optimal}^{rs}\right| &= \left|t_k^{rs} + (-t_{m,optimal}^{rs})\right| \\ &\le \left|t_{k,\max}^{rs} + (-t_{m,\min}^{rs})\right| \\ &\le \left|t_{k,\max}^{rs} + (-t_{k,\min}^{rs})\right| \\ &= \left|t_{k,\max}^{rs} - t_{k,\min}^{rs})\right| \end{aligned} \tag{37}$$

Therefore, theorem 3 has been proved according to inequality (37).□

*3.4. An illustrative simple network*

To this end, we apply this reliable energy-efficient path finding algorithm to a simple network (Figure 2) with BEVs to demonstrate the reliability effectiveness of our proposed algorithm. In Figure 2, the mean and standard deviation of the link travel times (in minutes) are marked near each link with the format "mean/standard deviation". Table 2 presents the link travel time correlation coefficient matrix of this simple network and we set the value $ETT_{\max}$ equals to 50 minutes. We add some input data for the second objective function. The link distances are assumed to be the average speed (30km/h) multiplied by the mean link travel times. The BEV's mass is assumed to be 2000kg. The values of other BEV's related parameters are shown in Table 6.

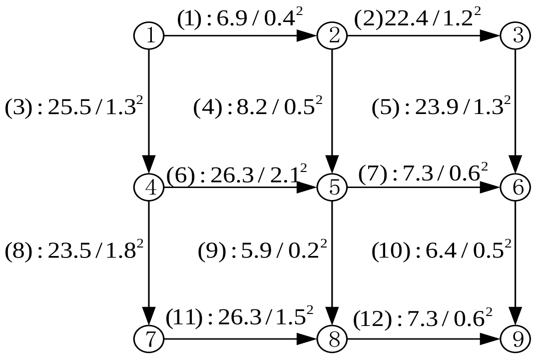


Fig. 2. A simple illustrative network

Table 2 Link travel time correlation coefficient matrix of the simple network

| Link | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | 1 | 0.53 | -0.26 | 0.69 | -0.13 | 0.25 | 0.32 | -0.15 | 0.23 | 0.15 | -0.34 | 0.21 |
| (2) | 0.53 | 1 | 0.63 | 0.15 | 0.59 | -0.21 | 0.34 | 0.15 | 0.16 | 0.26 | 0.33 | 0.28 |
| (3) | -0.26 | 0.63 | 1 | 0.23 | -0.34 | 0.69 | 0.17 | -0.34 | 0.22 | 0.31 | 0.15 | -0.26 |
| (4) | 0.69 | 0.15 | 0.23 | 1 | 0.65 | -0.23 | 0.76 | 0.32 | -0.14 | 0.23 | -0.25 | 0.16 |
| (5) | -0.13 | 0.59 | -0.34 | 0.65 | 1 | 0.73 | 0.21 | -0.69 | 0.32 | 0.16 | 0.24 | 0.26 |
| (6) | 0.25 | -0.21 | 0.69 | -0.23 | 0.73 | 1 | 0.33 | 0.24 | 0.81 | -0.35 | 0.16 | -0.32 |
| (7) | 0.32 | 0.34 | 0.17 | 0.76 | 0.21 | 0.33 | 1 | 0.76 | 0.23 | 0.14 | -0.32 | 0.31 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (8) | -0.15 | 0.15 | -0.34 | 0.32 | -0.69 | 0.24 | 0.76 | 1 | 0.73 | -0.21 | 0.25 | 0.36 |
| (9) | 0.23 | 0.16 | 0.22 | -0.14 | 0.32 | 0.81 | 0.23 | 0.73 | 1 | 0.35 | -0.43 | 0.36 |
| (10) | 0.15 | 0.26 | 0.31 | 0.23 | 0.16 | -0.35 | 0.14 | -0.21 | 0.35 | 1 | 0.56 | 0.25 |
| (11) | -0.34 | 0.33 | 0.15 | -0.25 | 0.24 | 0.16 | -0.32 | 0.25 | -0.43 | 0.56 | 1 | 0.39 |
| (12) | 0.21 | 0.28 | -0.26 | 0.16 | 0.26 | -0.32 | 0.31 | 0.36 | 0.36 | 0.25 | 0.39 | 1 |

Using this network, we illustrate the steps of our proposed algorithms below.

(1) Set the on-time arrival probability as $\theta = 80\%$, and consequently, $\Phi^{-1}(0.8) = 0.84$.

(2) Denote the six paths from node 1 to node 9 as $k_1, k_2, \cdots, k_6$ and their corresponding effective travel time of the paths as $t_{k_1}^{rs}, t_{k_2}^{rs}, \cdots, t_{k_6}^{rs}$. According to step 4 in Section 3.3, the minimal upper bound of the effective travel times (Eqs. (22) and (23)) is $\hat{u} = 29.73$.

(3) Estimate the lower bound using Eq. (21).

$\hat{t}_{k_1,\min}^{rs} = \sum_{a \in \mathbf{E}} \delta_{a,k_1}^{rs} \mu_a = 28.30$; $path_{k_1} = [1,4,9,12]$ (link sequence of path)

$\hat{t}_{k_2,\min}^{rs} = \sum_{a \in \mathbf{E}} \delta_{a,k_2}^{rs} \mu_a = 28.80$; $path_{k_2} = [1,4,7,10]$ (link sequence of path)

$\hat{t}_{k_3,\min}^{rs} = \sum_{a \in \mathbf{E}} \delta_{a,k_3}^{rs} \mu_a = 59.60$; $path_{k_3} = [1,2,5,10]$ (link sequence of path)

(4) Calculate $\hat{t}_{k_3,\min}^{rs} = 59.60 > \hat{u} = 29.73$. This indicates that path $k_3$ is not a reliable path. As a result, the iteration stops and the resulting candidate set of reliable paths is $\mathbf{Q} = \{k_1, k_2\}$.

(5) Calculate the effective travel times of paths in ($\mathbf{Q} = \{k_1, k_2\}$) using Eq. (6).

$t_{k_1}^{rs} = 29.29$; $path_{k_1} = [1,4,9,12]$ (link sequence of path)

$t_{k_2}^{rs} = 30.04$; $path_{k_2} = [1,4,7,10]$ (link sequence of path)

We can conclude that path $k_1$ is the optimal reliable path from origin node 1 to destination node 9 with an effective travel time of $t_{k_1}^{rs} = 29.29$ minutes. To verify the proposed algorithm, we enumerate the effective travel times of all 6 paths in Table 3. The optimal energy efficient path from origin node 1 to destination node 9 is path $k_1$ ([1, 4, 9, 12] ) and the corresponding energy consumptions are 5.47 and 4.77 for cases A and B. The results of the enumeration method (in Table 3) illustrate that the proposed algorithm can find the energy-efficient reliable path ($k_1$). Here we set K =2 in Step 6 of the K-shortest path algorithm.

Table 3 Results of a simple network with different algorithms

| The results of the proposed algorithm | | | | | The results of the enumeration algorithm | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Path $k_i$ | Effective travel Time $t_{k_i}^{rs}$ (mins) | Energy consumption | | Link sequence | Path $k_i$ | Effective travel time $t_{k_i}^{rs}$ (mins) | Energy consumption | | Link sequence |
| | | A | B | | | | A | B | |
| $k_1$ | **29.29** | **5.74** | **4.77** | [1,4,9,12] | $k_1$ | **29.29** | **5.74** | **4.77** | [1,4,9,12] |
| $k_2$ | 30.04 | 5.85 | 4.79 | [1,4,7,10] | $k_2$ | 30.04 | 5.85 | 4.79 | [1,4,7,10] |
| | | | | | $k_3$ | 61.71 | 12.1 | 5.43 | [1,2,5,10] |
| | | | | | $k_4$ | 67.65 | 13.1 | 5.55 | [3,6,9,12] |
| | | | | | $k_5$ | 68.33 | 13.3 | 5.56 | [3,6,7,10] |
| | | | | | $k_6$ | 85.11 | 16.7 | 5.92 | [3,8,11,12] |

After comparing the results in Table 3, our proposed algorithm results in the identical solution as the enumeration, which requires the calculation of the effective travel time of 6 paths. We thus verify the effectiveness and efficiency of our proposed energy-efficient reliable path finding algorithm.

## 4. Numerical Examples

In this section, we apply our proposed energy-efficient reliable path algorithm to 2 real-life transportation networks and compare the algorithm with alternative approaches if applicable. Our numerical experiments are conducted using Matlab 7.0 on the Windows 10 platform running on a PC with an Intel Core(TM) i7-6500U 2.5 GHz CPU and 4 GB of memory.

*4.1. Example 1: Effectiveness of finding the non-dominated efficient solutions to the bi-objective model*

The Tuen Mun Road Corridor Network of Hong Kong (Lam et al., 2001; Lam et al., 2002; Shao et al., 2018), shown in Figure 3, consists of 6 OD pairs, 4 nodes, 10 links, and 20 paths. The mean travel time was observed over 50 identical independent workday time periods. The variance-covariance matrix of the link travel times was generated using 50 simulations from the standard stochastic user equilibrium (SUE) model (Shao et al., 2018). The resulting mean and variance-covariance matrix of the link travel times are as shown in Tables 4 and 5, respectively. The related parameters' values of Eqs. (8b-8e) are shown in Table 6 (Yang et al., 2014). The variables $ETT_{\max}$ and $EEC_{\max}$ are set as 60 minutes and 2.5 kwh, respectively.

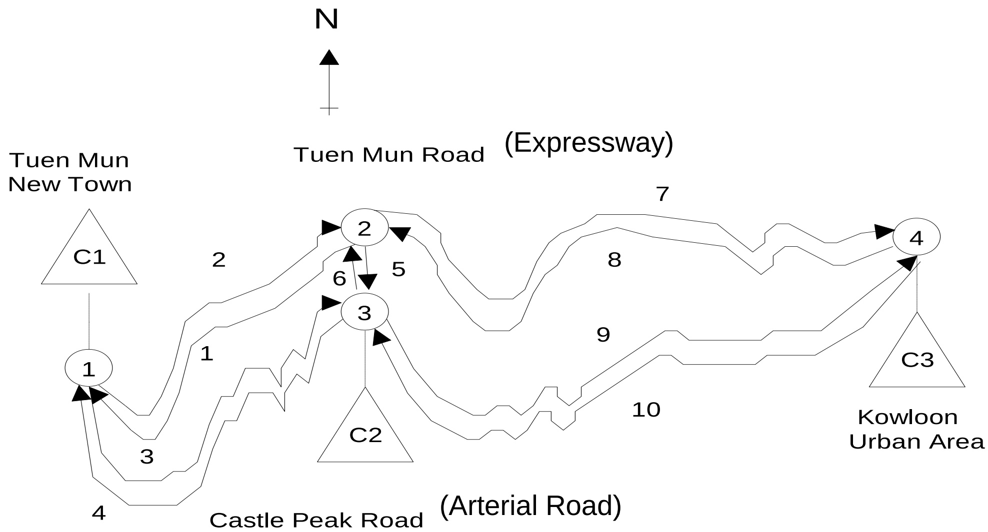


Fig. 3. Tuen Mun Road Corridor Network in Hong Kong

Table 4. Mean and Variance of the Link Travel Times

| Link no. | Mean link travel time (mins) | Variance link travel time (mins $^2$) | Distance (km) |
|---|---|---|---|
| 1 | 8.514 | 0.07860 | 5.89 |
| 2 | 17.592 | 0.87510 | 5.89 |
| 3 | 18.618 | 0.85944 | 5.51 |
| 4 | 9.102 | 0.07230 | 5.51 |
| 5 | 0.798 | 0.00096 | 0.28 |
| 6 | 0.498 | 0.00042 | 0.28 |
| 7 | 8.868 | 0.26616 | 2.68 |
| 8 | 3.600 | 0.01464 | 2.68 |

| | | | |
|---|---|---|---|
| 9 | 9.606 | 0.24036 | 4.6 |
| 10 | 5.022 | 0.00642 | 4.6 |

Table 5. Link Travel Time Covariance Matrix (mins $^2$ )

| Link no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.07860 | -0.00834 | -0.00624 | 0.07464 | 0.00330 | 0.00012 | 0.00522 | 0.01164 | 0.00258 | 0.01056 |
| 2 | -0.00834 | 0.87510 | 0.86664 | -0.01434 | 0.00504 | 0.00378 | 0.20754 | -0.00138 | 0.20562 | -0.00264 |
| 3 | -0.00624 | 0.86664 | 0.85944 | -0.01314 | 0.00582 | 0.00318 | 0.21324 | -0.00294 | 0.21012 | -0.00318 |
| 4 | 0.07464 | -0.01434 | -0.01314 | 0.07230 | 0.00216 | 0.00060 | -0.00312 | 0.01398 | -0.00462 | 0.01152 |
| 5 | 0.00330 | 0.00504 | 0.00582 | 0.00216 | 0.00096 | -0.00036 | 0.00852 | -0.00108 | 0.00744 | -0.00030 |
| 6 | 0.00012 | 0.00378 | 0.00318 | 0.00060 | -0.00036 | 0.00042 | -0.00156 | 0.00168 | -0.00096 | 0.00084 |
| 7 | 0.00522 | 0.20754 | 0.21324 | -0.00312 | 0.00852 | -0.00156 | 0.26616 | -0.00432 | 0.25236 | -0.00108 |
| 8 | 0.01164 | -0.00138 | -0.00294 | 0.01398 | -0.00108 | 0.00168 | -0.00432 | 0.01464 | -0.00300 | 0.00924 |
| 9 | 0.00258 | 0.20562 | 0.21012 | -0.00462 | 0.00744 | -0.00096 | 0.25236 | -0.00300 | 0.24036 | -0.00078 |
| 10 | 0.01056 | -0.00264 | -0.00318 | 0.01152 | -0.00030 | 0.00084 | -0.00108 | 0.00924 | -0.00078 | 0.00642 |

Table 6 The values of the BEV's related parameters

| | |
|---|---|
| $e$ | 0.015 |
| $g$ | $6.67\times 10^{-11}$ |
| $\alpha$ | $1^{o}$ |
| $\delta$ | 1.1 |
| $C_D$ | 0.3 |
| $H$ | 1.8 |
| $\chi$ | 1.2 |
| $\eta_c$ | 0.85 |
| $\eta_m$ | 0.85 |
| $P_{accessory}$ | 1000 |

*4.1.1. Spatial correlation analysis*

To normalize the link travel time covariance in the whole network, we first calculate the correlation coefficient for every pair, $l_{ab}$ and $l_{cd}$, using the following formula:

$$\rho_{ab}^{cd} = \frac{\mathrm{cov}(l_{ab}, l_{cd})}{\sigma_{ab}\sigma_{cd}}$$

(38)

The resulting correlation coefficient matrix is presented in Table 7. The value of $\rho_{ab}^{cd}$ is between -1 and +1, with $\rho_{ab}^{cd}$ =+1 indicating a positive correlation and $\rho_{ab}^{cd}$ =-1 indicating a negative correlation. To facilitate the sensitivity analysis in the following subsection, we introduce a multiplier $\lambda$ to vary the values of the correlation covariance matrix.

Table 7. Correlation Coefficient Matrix

| Link no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | -0.0318 | -0.0240 | 0.9901 | 0.3799 | 0.0209 | 0.0361 | 0.3431 | 0.0188 | 0.4701 |
| 2 | -0.0318 | 1 | 0.9993 | -0.0570 | 0.1739 | 0.1972 | 0.4300 | -0.0122 | 0.4483 | -0.0352 |
| 3 | -0.0240 | 0.9993 | 1 | -0.0527 | 0.2026 | 0.1674 | 0.4459 | -0.0262 | 0.4623 | -0.0428 |
| 4 | 0.9901 | -0.0570 | -0.0527 | 1 | 0.2593 | 0.1089 | -0.0225 | 0.4297 | -0.0350 | 0.5347 |
| 5 | 0.3799 | 0.1739 | 0.2026 | 0.2593 | 1 | -0.5669 | 0.5330 | -0.2881 | 0.4898 | -0.1208 |
| 6 | 0.0209 | 0.1972 | 0.1674 | 0.1089 | -0.5669 | 1 | -0.1475 | 0.6775 | -0.0955 | 0.5115 |
| 7 | 0.0361 | 0.4300 | 0.4459 | -0.0225 | 0.5330 | -0.1475 | 1 | -0.0692 | 0.9977 | -0.0261 |
| 8 | 0.3431 | -0.0122 | -0.0262 | 0.4297 | -0.2881 | 0.6775 | -0.0692 | 1 | -0.0506 | 0.9531 |
| 9 | 0.0188 | 0.4483 | 0.4623 | -0.0350 | 0.4898 | -0.0955 | 0.9977 | -0.0506 | 1 | -0.0199 |
| 10 | 0.4701 | -0.0352 | -0.0428 | 0.5347 | -0.1208 | 0.5115 | -0.0261 | 0.9531 | -0.0199 | 1 |

*4.1.2 Numerical solutions analysis*

To illustrate how the variables of the on-time arrival probability and correlation coefficients impact the optimal paths, we consider the three scenarios shown in Table 8. To facilitate the sensitivity analysis in the following subsection, we introduce a multiplier $\lambda$ to vary the values of the correlation covariance matrix. As mentioned previously, the energy consumption function of BEVs satisfies the property of additivity. In other words, parameters $\theta$ and $\lambda$ do not influence the results of the proposed energy-efficient path model. To this end, we focus on the impacts of parameters $\theta$ and $\lambda$ on the reliable shortest path problem.

Table 8. Three cases illustrating the impacts of $\theta$ and $\lambda$ on reliable paths

| Scenario | Variable $\theta$ | Variable $\lambda$ | Goal |
|---|---|---|---|
| A | √(varying) | ×(fix) | Account for parameter $\theta$'s impact on the reliable path only. |
| B | ×(fix) | √(varying) | Account for parameter $\lambda$'s impact on the reliable path only. |
| C | √(varying) | √(varying) | Account for the joint effects of parameters $\theta$'s and $\lambda$'s impacts on the reliable path. |

Scenario A shows how the on-time arrival probability $\theta$ impacts the reliable path while keeping $\lambda$=1. There are four paths from origin node 1 to destination node 4 (Figure 3). Path 1 (2, 7) (link sequence) is the optimal reliable path, and the effective travel time is 0.4628 hours, with an on-time arrival probability of $\theta$=90%. Figure 4 illustrates that the paths' effective travel time increases while the on-time arrival probability $\theta$ increases. For instance, the effective travel times on path 1 (2, 7) (link sequence) with on-time arrival probabilities of 50%, 75%, and 90% are 0.441, 0.4525 and 0.4628 hour, respectively.

In scenario B, we exhibit the impacts of the multiplier of the correlation coefficient matrix $\lambda$ on the reliable paths while keeping an on-time arrival probability of $\theta$=90%. Figure 5 depicts the effective travel times of all 4 paths under varied amounts of the multiplier $\lambda$. It is observed that the path effective travel time increases as the multiplier $\lambda$ increases. For instance, the effective travel times on path 1 with multipliers of 25%, 50%, and 75% are 0.4518, 0.4563 and 0.4598 hours, respectively. Interestingly, Figure 5 also shows an extraordinarily small gap in the effective travel time between paths 2 and 3.

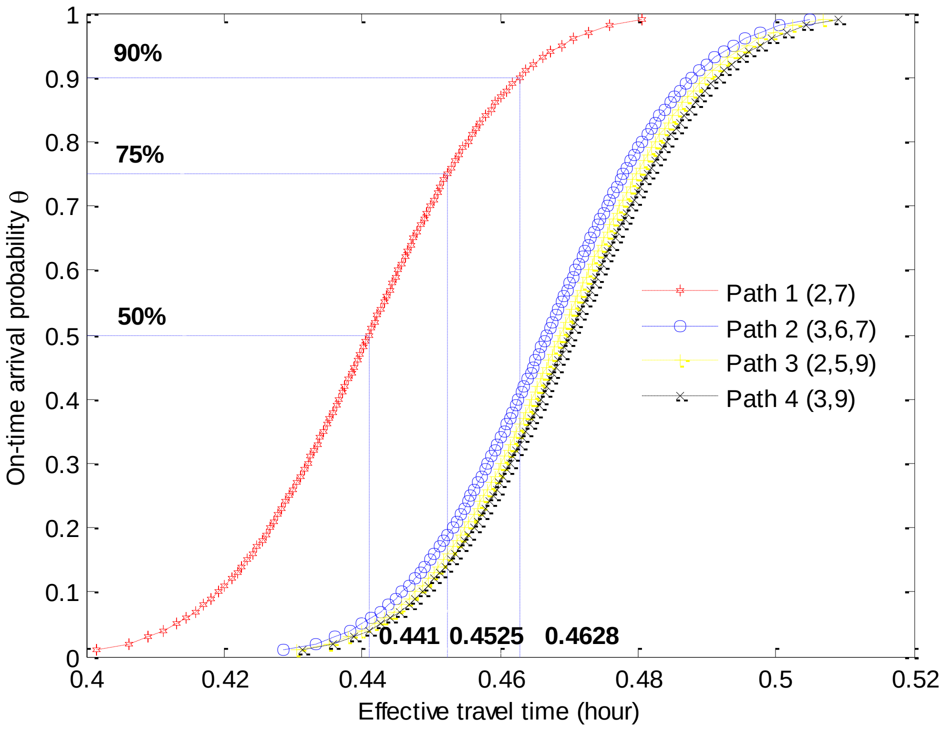


Fig. 4. Scenario A: Effective travel times of paths under different values of on-time arrival probability $\theta$

In scenario C, we demonstrate the joint effects of the on-time arrival probability $\theta$ and the multiplier of the correlation coefficient matrix $\lambda$ on the optimal reliable path (path 1). The results are shown in Figure 6. For comparison purposes, we also depict the plane of the on-time arrival probability $\theta$=50% as the benchmark plane. The effective travel time of path 1 increases from 0.441 hours to 0.481 hours as the multiplier $\lambda$ increases with the on-time arrival probability $\theta$, ranging from 50% to 100%. By contrast, the effective travel time of path 1 decreases (from 0.441 hours to 0.402 hours) as the multiplier $\lambda$ increases if the on-time arrival probability $\theta$ falls below 50%. This effect is due to the increasing monotonicity of the inverse cumulative density function of the normal distribution when $\theta$>0.5, and vice versa for $\theta$<0.5. Therefore, it suggests that ignoring the on-time arrival probability $\theta$ and correlation coefficient matrix $\lambda$ may lead to the over- or under-estimation the path's effective travel time. For better illustration purposes, we present the precise amounts of the effective travel times for all of the reliable paths in Table 9.

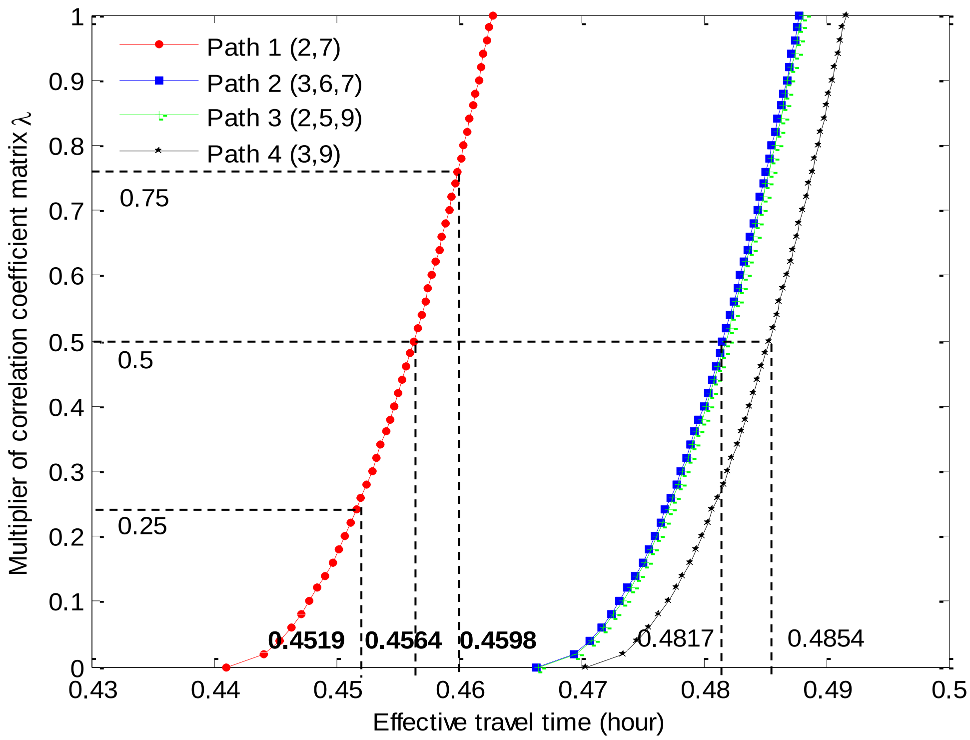


Fig. 5. Scenario B: Effective travel times of paths under different values of the multiplier of the correlation coefficient matrix $\lambda$

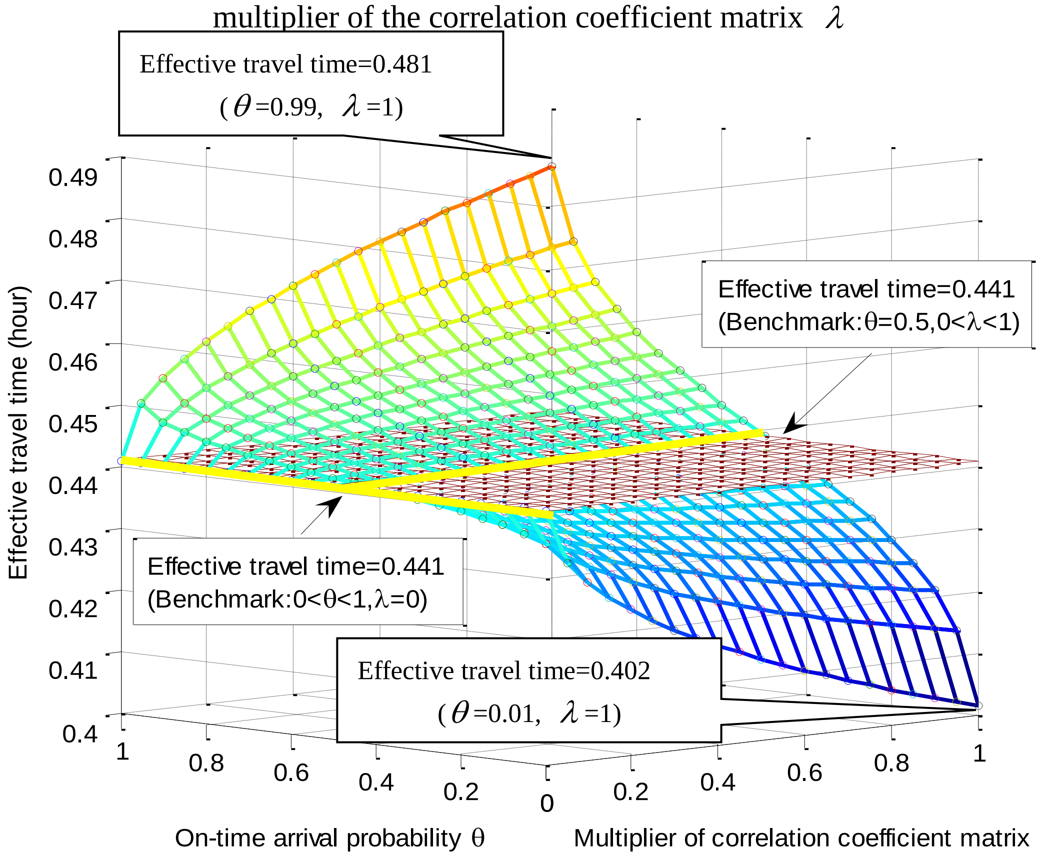


Fig. 6. Scenario C: Effective travel time under different values of the on-time arrival probability $\theta$ and the multiplier of correlation coefficient matrix $\lambda$ $(0<\theta<1, 0\leq\lambda\leq1)$

We observe that the effective travel time of optimal reliable path 1 increases as the multiplier of the correlation coefficient matrix $\lambda$ increases if the on-time arrival probability is $\theta$=90%. For instance, the effective travel time on path 1 with multipliers of the correlation coefficient of $\lambda$=0.25, 0.5, 0.75, and 1 are 0.4518, 0.4563, 0.4598 and 0.4628 hours, respectively. On the other hand, the effective travel time of optimal reliable path 1 decreases as the multiplier of the correlation coefficient matrix $\lambda$ increases given an on-time arrival probability of $\theta$=10%. That is, the effective travel time on path 1 with multipliers of the correlation coefficient of $\lambda$=0.25, 0.5, 0.75, and 1 are 0.4302, 0.4257, 0.4222 and 0.4193 hours, respectively. Due to the zero value of the inverse cumulative density function, the path effective travel times are independent of the multiplier of the correlation coefficient matrix $\lambda$ when the on-time arrival probability is $\theta$=50%. To this end, we show that the parameters of the on-time arrival probability $\theta$ and correlation coefficient matrix $\lambda$ play important roles in our proposed bi-objective model.

Table 9. Results in the Tune Mun Road Network under different values of multipliers of the correlation coefficient matrix $\lambda$ and on-time arrival probability $\theta$ among OD pairs 1-4

| $\theta$ | Paths (Link sequence) | Effective travel time (hour) | | | |
|---|---|---|---|---|---|
| | | $\lambda$=1 | $\lambda$=0.75 | $\lambda$=0.5 | $\lambda$=0.25 |
| $\theta$=90% | Path 1 (2, 7) | **0.4628** | **0.4598** | **0.4564** | **0.4519** |
| | Path 2 (3, 6, 7) | 0.4879 | 0.4850 | 0.4816 | 0.4772 |
| | Path 3 (2, 5, 9) | 0.4881 | 0.4852 | 0.4818 | 0.4773 |
| | Path 4 (3, 9) | 0.4916 | 0.4888 | 0.4854 | 0.4810 |
| $\theta$=50% | Path 1 (2, 7) | 0.4410 | | | |
| | Path 2 (3, 6, 7) | 0.4664 | | | |
| | Path 3 (2, 5, 9) | 0.4666 | | | |
| | Path 4 (3, 9) | 0.4704 | | | |
| $\theta$=10% | Path 1 (2, 7) | **0.4193** | **0.4222** | **0.4256** | **0.4301** |
| | Path 2 (3, 6, 7) | 0.4449 | 0.4478 | 0.4512 | 0.4556 |
| | Path 3 (2, 5, 9) | 0.4451 | 0.4480 | 0.4514 | 0.4559 |
| | Path 4 (3, 9) | 0.4492 | 0.4520 | 0.4554 | 0.4598 |

### *4.1.3 Effects of BEV's mass on the energy consumption in case B*

In this section, we study the effects of BEV's mass on their energy consumption. The categories of BEVs considered are: Compact, Medium-sized, Luxury, Transporter. The mass of the four reference vehicles are assumed to be 1500kg, 2000kg, 2500kg and 3000kg, respectively. Then, the detailed results are shown in Figure 7.

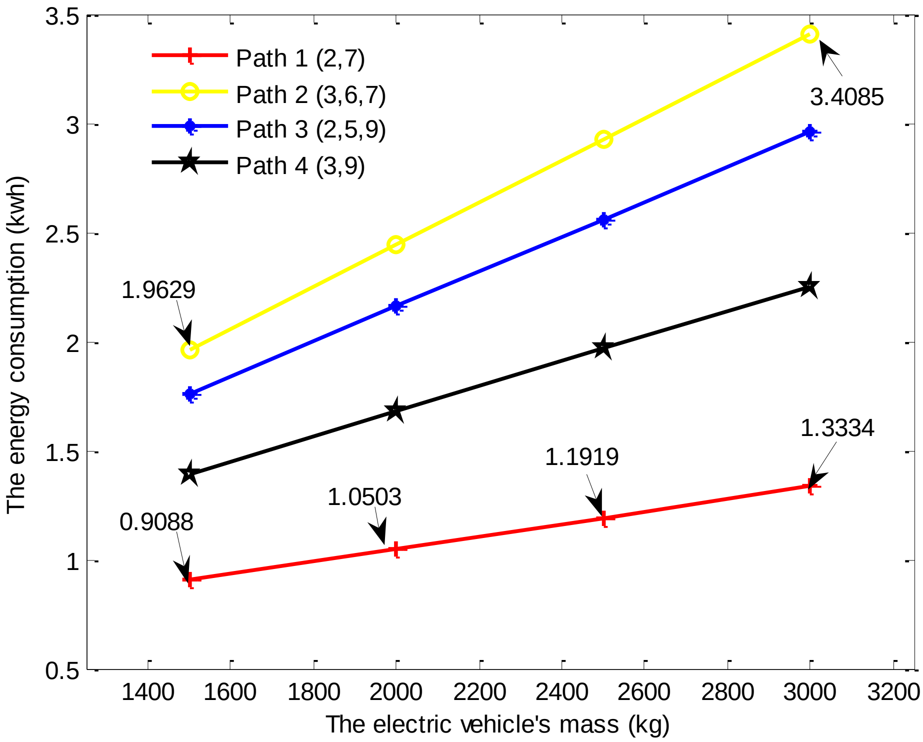


Fig. 7. Case B: Energy consumption of BEVs with different masses

Apparently, the energy consumption of BEVs increases as their mass increases. For instance, the energy consumption on path 1 with the mass of $m = 1500$, 2000, 2500 and 3000 kg are 0.9088, 1.0503, 1.1919 and 1.3334 kwh, respectively. In figure 7, the energy consumption of path 2 is 1.9629 kwh and much larger than path 1. This is because there are three links in path 2 and only two links in path 1. In each link, the traveler need to speed up in the first stage and the acceleration process (the first stage) will consume the most energy along the link, echoing the finding in Yang et al., (2014). As the BEV's mass increases, the difference between the energy consumption of path 1 and path 2 also increases. For instance, the difference between the EEC of path 1 and path 2 is $1.9629 - 0.9088 = 1.0541$ kwh with the BEV's mass of 1500 kg. Whereas, the difference becomes $3.4085 - 1.3334 = 2.0751$ kwh when the BEV's mass is 3000 kg. This indicates that the BEV's mass plays an influential role on energy consumption.

*4.1.4 Effects of speed limit on the proposed bi-objective model*

The increased traffic uncertainty triggered by speed limit may impact the non-dominated efficient solutions to our bi-objective problem. Mathematically, we use the mean and variance of the link travel times to depict the uncertainty of the speed of BEV (the average speed is defined as divide the distance of link $a$ by the mean of the link travel time). Actually, we adopted two different formulas to calculate the energy consumption of BEVs. The first formula presented by He et al. (2014) is easy to calculate with K-shortest algorithm (Yang and Chen, 2006). Following the results of the second formula proposed by Yang et al. (2014), the time consuming of first stage (acceleration process) is about 6 seconds and the third stage (deceleration process) will cost about 4 seconds. Based on the above discussion, the energy consumption formula proposed by Yang et al. (2014) and Zhang and Yao. (2015) can also be divided into three parts. The BEV's mass is assumed to be 2000kg. Meanwhile, the value of maximum speed of BEV is assumed to be double of the average speed of link $a$. We compare the original results of the proposed bi-objective model with those of the doubled mean and variance of the link travel times presented in Table 10.

Specifically, both the effective travel time and energy consumption grow with the uncertainty of the link travel time in case A. For example, when the values of the mean and variance are doubled given an on-time arrival probability of $\theta = 90\%$, the effective travel time of path 1 increases from 0.4628 to 0.9233 hours and the energy consumption of path 2 (3, 6, 7) increases from 1.53 to 1.58 kwh. However, the energy consumption decreases when the values of the mean and variance double in case B. This is because when the values of the mean and variance become double, the average speed and the maximum speed are reduced to half. Under this circumstance, the energy consumption of the first stage (acceleration process) must decrease as the maximum speed is reduced to half. For example, when the values of the mean and variance become double given an on-time arrival probability of $\theta = 90\%$, the energy consumption of path 1 (2, 7) decreases from 1.05 to 0.98 kwh. These results indicate that the speed of BEV may influence the effective travel time and the energy consumption. That is, the speed of BEV has a significant impact on the optimum solution to the bi-objective model.

In Table 10, we also present the non-dominated efficient solutions (path 1 and path 2) in case A in this network and suggest that the optimum solution of this bi-level problem, which depends on the travelers' priority if they prioritize on-time arrival reliability over energy-efficiency, is Path 1. By contrast, if energy consumption is taken as a priority over on-time arrival reliability, Path 2 becomes the optimum solution. However, the path 1 (2,7) is the optimal solution with the minimum effective travel time 0.4628 hour and the minimum energy consumption 1.0503 kwh in case B.

Table 10. Results in the Tune Mun Road Corridor Network under different values of the mean link travel time and variance link travel time among OD pairs 1-4 ($\lambda = 1$)

| | Paths | Effective travel time (hour) | | | EEC (kwh) | |
|---|---|---|---|---|---|---|
| | (Link sequence) | $\theta = 90\%$ | $\theta = 50\%$ | $\theta = 10\%$ | Case A | Case B |
| Mean link travel time ($\mu_a$) Variance link travel time ($\sigma_a^2$) | Path1 (2, 7) | **0.4628** | 0.4410 | 0.4193 | 1.5423 | **1.0503** |
| | Path 2 (3, 6, 7) | 0.4879 | 0.4664 | 0.4449 | **1.5279** | 2.4448 |
| | Path 3 (2, 5, 9) | 0.4881 | 0.4666 | 0.4451 | 1.9281 | 2.1588 |
| | Path 4 (3, 9) | 0.4916 | 0.4704 | 0.4492 | 1.8137 | 1.6798 |
| Mean link travel time ($2\mu_a$) Variance link travel time ($2\sigma_a^2$) | Path1 (2, 7) | **0.9233** | 0.8820 | 0.8406 | 1.5935 | **0.9756** |
| | Path 2 (3, 6, 7) | 0.9735 | 0.9328 | 0.8920 | **1.5820** | 1.2139 |
| | Path 3 (2, 5, 9) | 0.9741 | 0.9332 | 0.8922 | 1.9822 | 1.1822 |
| | Path 4 (3, 9) | 0.9811 | 0.9408 | 0.9004 | 1.8683 | 1.1187 |

### *4.2. Example 2: Efficiency of the proposed algorithm*

The Sioux Falls network consists of 24 nodes and 76 links. The paths are initially generated for the 96 OD pairs (Origin nodes: 1, 4, 2 and 5, Destination nodes: 13, 20, 21 and 24; Origin nodes: 13, 20, 21 and 24, Destination nodes: 1, 2, 4 and 5; Origin nodes: 6, 7, 8 and 18, Destination nodes: 3, 12, 14 and 23; Origin nodes: 3, 12, 14 and 23, Destination nodes: 6, 7, 8 and 18; Origin nodes: 9, 10, 11 and 16, Destination nodes: 15, 17, 19 and 22; and Origin nodes: 15, 17, 19 and 22, Destination nodes: 9, 10, 11 and 16).

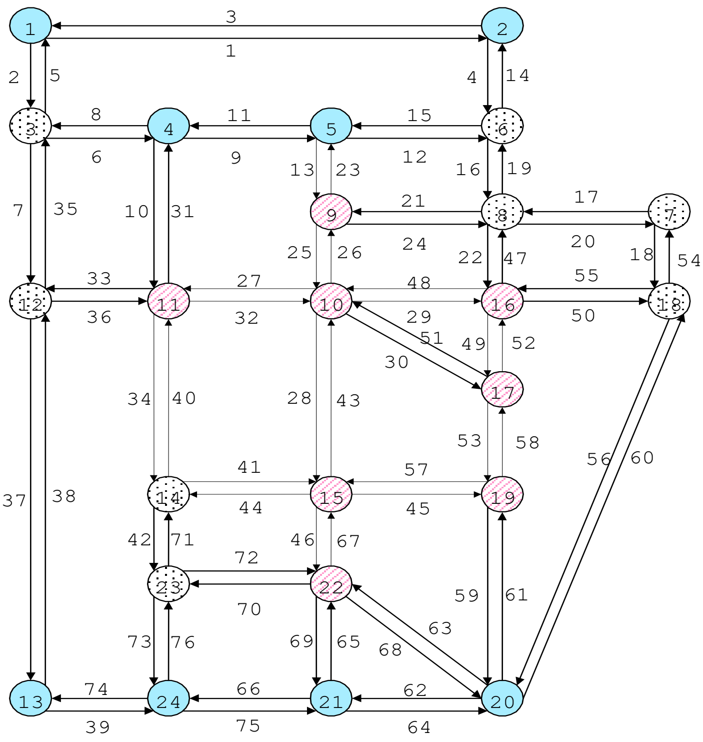


Fig. 8. Sioux Falls network for numerical example 2

Similarly to Shao et al., (2018), we test our proposed algorithm with the mean link travel time observed over 50 homogeneous, independent workday time periods and the variance-covariance matrix of link travel time generated from 50 simulations with the standard SUE model. The link distances are assumed to be 2.5 times greater than those of the link free-flow travel times and the BEV's mass is assumed to be 2000kg. The values of other BEV's related parameters are shown in Table 6. The variables $ETT_{\max}$ and $EEC_{\max}$ are set as 120 minutes and 15 kwh, respectively.

As we verify the efficiency of the algorithm in this network, the detailed results are omitted. The average computing time between OD pair 1-20 under different values of the on-time arrival probability $\theta$ are shown in Table 11. To be specific, the average running time increases as the on-time arrival probability $\theta$ increases because a higher arrival probability requires more paths to be selected in the candidate path set. The average running time is almost 3 seconds for this medium size traffic network with 90% on-time reliability, thus suggesting the promising potential of efficiency in a large-scale network.

Table 11. Computational time of the proposed algorithm

| On-time arrival probability $\theta$ (%) | 60 | 70 | 80 | 90 |
|---|---|---|---|---|
| Maximum number of paths $K_{\max}$ | 4 | 21 | 62 | 95 |

| | | | | |
|---|---|---|---|---|
| Average computational time (second) | 0.28 | 0.51 | 1.63 | 2.97 |

### *4.3. Example 3: The non-dominated solutions of the proposed model in large-scale networks*

In this section, we conduct the experiments using the large-scale networks existing in most urban areas. The grid-based network as shown in Figure 9 is adopted in this example. The test network consists of 220 nodes and 409 links as shown in Figure 10. The mean and variance of link travel times are randomly generated using a normal distribution, as shown in Table 12. The covariance matrix of the link travel time is randomly generated to be a positive definite matrix, consistent with the variances of generated link travel times. The BEV's mass is assumed to be 2000kg. The values of other BEV's related parameters are shown in Table 6. The variables $ETT_{\max}$ and $EEC_{\max}$ are set as 220 minutes and 35 kwh, respectively.

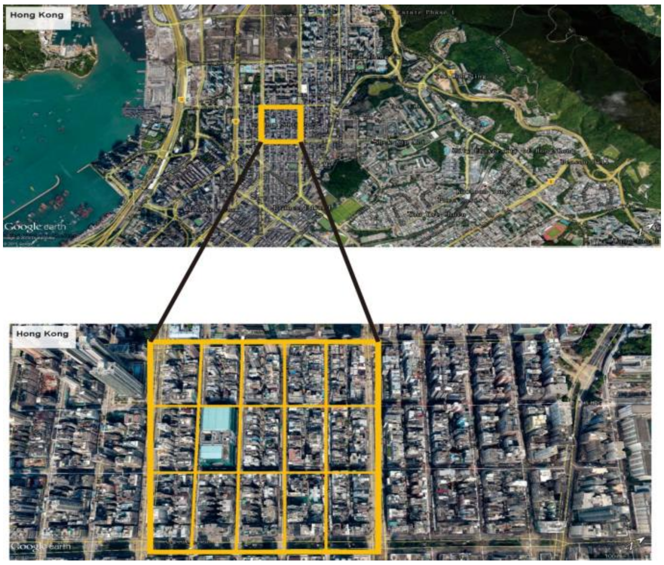


**Fig. 9.** Grid-based network in Hong Kong

**Table 12.** Bounded constraints for randomly generating link travel time

| | |
|---|---|
| Mean of link travel time (minute) | [3 16] |
| Variance of link travel time (minute $^2$) | [0.1 0.2] |
| On-time arrival probability | 90% |
| Distance of each link (meter) | [1000 3000] |

According to the proposed model and the heuristic algorithm in sections 2 and 3, the detailed results between OD pair 1 and 220 are shown in Figures 11 and 12. The results in Figure 10 are obtained with the energy consumption formula proposed by He et al. (2014) (Case A). Obviously, there are three non-dominated solutions in the test network. Meanwhile, the results in Figure 11 are calculated with the energy consumption formula proposed by Yang et al. (2014) (Case B). Apparently, there are seven non-dominated solutions in the test network. The energy consumptions with different formulas are different. Thus, different kinds of energy consumption formulas may play a significant role in the travelers' choice. The computational time of a desktop computer with Core(TM) i7-6500U 2.5 GHz CPU, a 4 GB memory and a Windows 10 operation system is 702 s. It is evidenced from this example that proposed heuristic solution algorithm can be applied to a large-scale network.

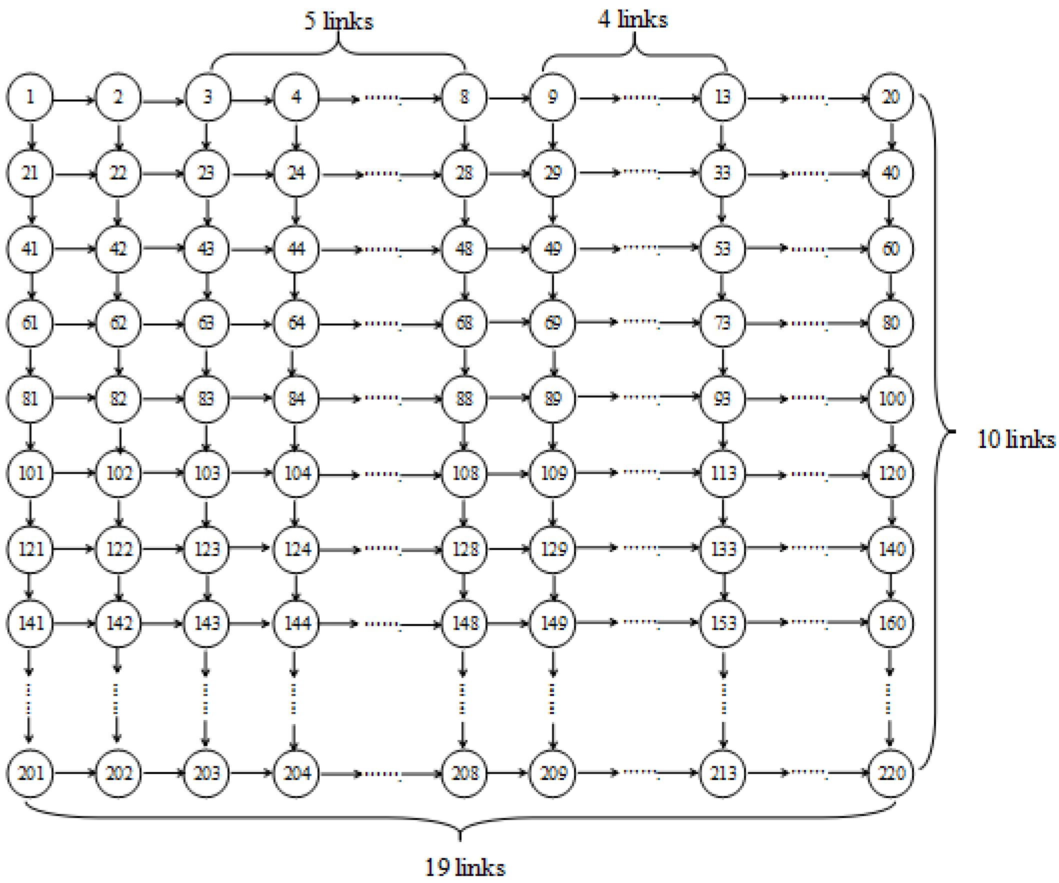


**Fig. 10.** Test network for example 3

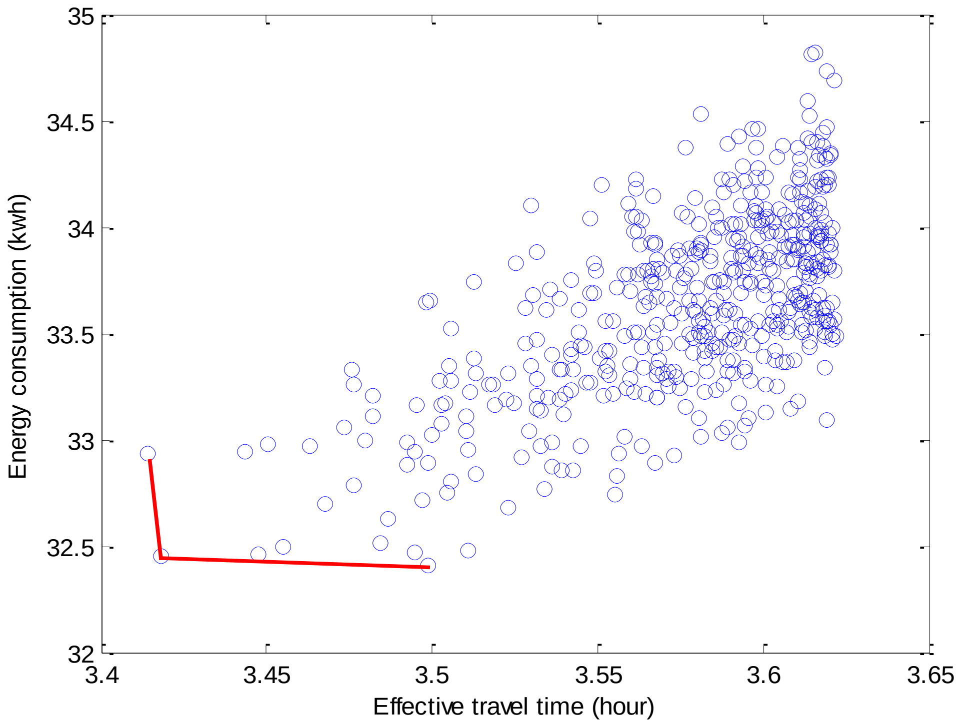


**Fig. 11.** Case A: The non-dominated solutions between OD pair 1 and 220

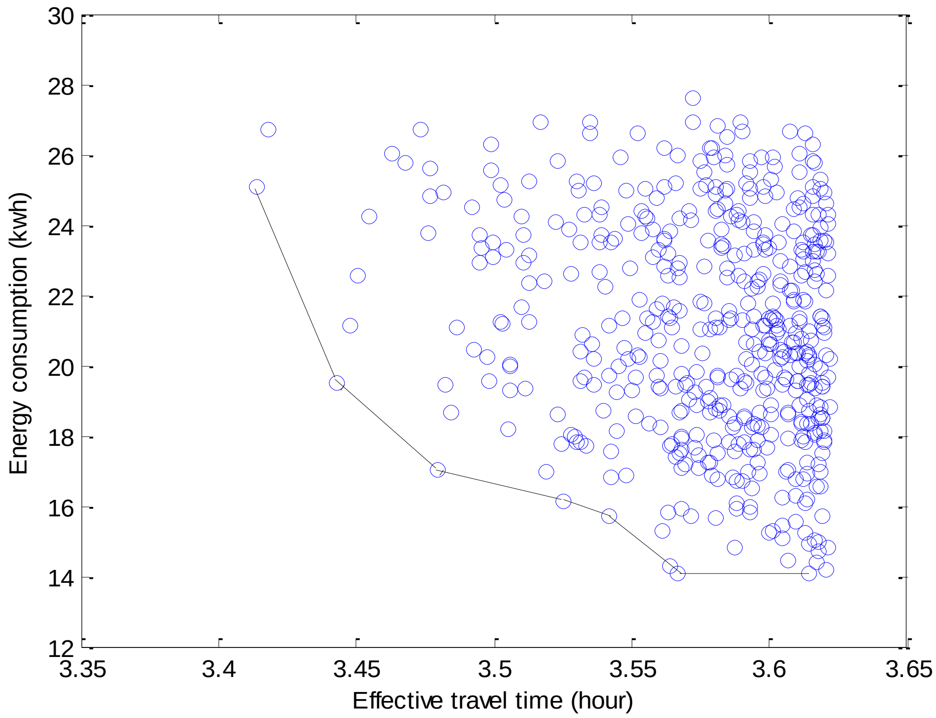


**Fig. 12.** Case B: The non-dominated solutions between OD pair 1 and 220

In order to show the impacts of the on-time arrival probability $\theta$ on the non-dominated paths, we present the non-dominated solutions under various values of $\theta$ in Figure 13.

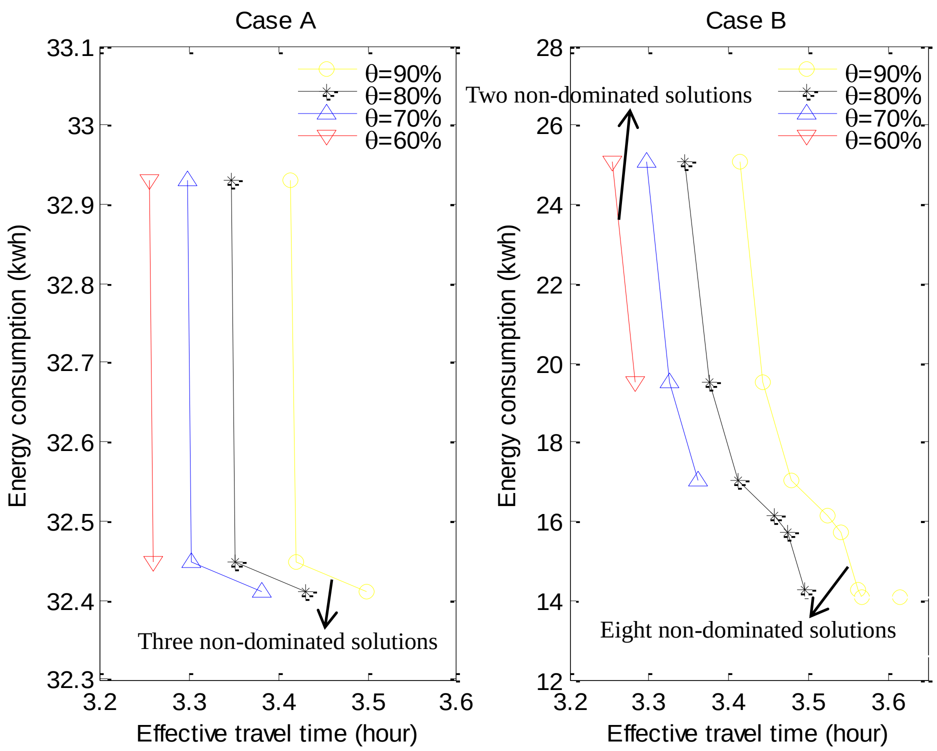


**Fig. 13.** The non-dominated solutions between OD pair 1 and 220 under different values of on-time arrival probability $\theta$

Figure 13 shows the influences of on-time arrival probability $\theta$ on the non-dominated solutions. For example, in Case B, the numbers of non-dominated solutions are 2, 3, 6, 8 for on-time arrival probabilities $\theta$=60%, 70%, 80% and 90%, respectively. The results indicate that a higher on-time arrival probability can lead to a larger number of non-dominated solutions. This is because the effective travel times of non-dominated paths increase as on-time arrival probability $\theta$ increases. This example also shows that energy consumption issue could impact on the non-dominated solutions. For instance, there are three non-dominated solutions in Case A with an on-time arrival probability $\theta$ of 90%. By contrast, the number of non-dominated solutions is 8 in Case B even with the same $\theta$. This is because Cases A and B utilize the energy consumption formulae Eqs. (8a) and (8e) respectively. As a result, different amounts of non-dominated solutions are found in Cases A and B. This also indicates that the number of non-dominated paths for the arterial roads (Case B) is greater than that of expressways (Case A), all the other parameters remaining the same.

## 5. Conclusions and further studies

This paper proposed a bi-objective model for finding an optimal reliable energy-efficient path in stochastic traffic networks with BEVs. To be specific, the proposed model accounts for both the on-time arrival probability and correlated link travel times in a stochastic network. Our proposed algorithm takes advantage of the inequality technique to address the non-linear and non-additive issue embedded in the on-time arrival reliability objective and the features in the polynomial computational complexity. With the use of the illustrative example studied in this paper, we demonstrated the following:

(i) The effective travel times of the reliable energy-efficient paths depend on the on-time arrival probability and correlation coefficient matrix as well as the mean and variance of the link travel times in uncertain road networks,

(ii) The optimal path to this bi-objective problem is dependent on the travelers' personal preferences, and

(iii) The effectiveness and efficiency of our proposed algorithm for applications.

Our proposed bi-objective model and its solution algorithm can significantly help travelers budget their travel time with a given on-time arrival probability (Yu et al., 2011) and fixed energy consumption, even in a large-scale transportation network with uncertainties. However, this paper has several limitations, and thus, we suggest the following future research works.

(1) The proposed reliability-based (or reliable) path finding model is a static model that calculates the effective travel times in stochastic road networks. Extending the reliable path finding model to a time-varying stochastic network in a dynamic setting (Xu, et al., 2011b; Sever et al., 2018) is promising for further research.

(2) Our proposed reliable path finding model only considers the spatial link travel time correlations. The temporal correlations should also be considered in the future.

(3) Different values of on-time arrival probability $\theta$ to represent various travelers' risk-taking attitudes can be worthwhile for further study.

(4) To verify the application of our proposed algorithm, a large-scale transportation network is worth being constructed in future research (Yu et al., 2012).

(5) The proposed algorithm can be further extended for solving the RSPP for risk-

seeking ($\theta < 50\%$) path choice behaviors. Further research could be carried out to design more robust path finding algorithms for multiple risk-taking path choice behaviors under network uncertainty.

(6) The proposed method depends on the assumption that the path travel time follows multivariate normal distribution. Actually, the path travel time distribution should be calibrated using real data, which may not follow normal distribution with symmetric density function. How to overcome this limitation by considering other distribution of path travel time in reliable path finding problem deserves further extension of our study.

(7) There are many different electric vehicle classes in transportation network. The proposed model only considers the effective travel time and energy consumption of private electric vehicles. Further studies could be conducted to consider the reliable path finding problem for electric buses (Chen et al., 2018).

(8) With the developments of big data and machine learning technologies, how to use the methods of deep learning and reinforcement learning (Yao and Moawad, 2019; Qi et al., 2019) to accurately estimate the effective travel time and energy consumption deserves further study.

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