https://dx.doi.org/10.1016/j.trb.2019.07.018

PARADOX LINKS CAN IMPROVE SYSTEM EFFICIENCY: AN ILLUSTRATION IN TRAFFIC ASSIGNMENT PROBLEM

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ABSTRACT

This paper demonstrates a counter-intuitive phenomenon that "paradox links" (i.e. marginally improving or adding these links will increase a system's cost) can sometimes decrease a system's cost. It can be expressed that simultaneously improving the paradox link to a certain threshold (rather than only marginal improvement) or adding more paradox links may counter-intuitively avoid the paradox. Here we refer this phenomenon as the "non-monotonicity" of the paradox with regard to the degree of link improvement and the number of additional paradox links. Firstly, a formal definition of "non-monotonicity" property of paradox in a rigorous mathematical manner is proposed. Then this non-monotonicity property is demonstrated to widely exist in the user equilibrium (UE), the stochastic assignment, and the stochastic user equilibrium (SUE) models by two simple networks, where the underlying reasons for this phenomenon in different scenarios are analyzed and compared. Finally, the non-monotonicity of the traffic paradox is corroborated in a road sub-network of Harbin. The conclusions of this study provide new insights into features of traffic paradoxes and new ideas to eliminate them.

Keywords: traffic paradox; non-monotonicity; user equilibrium; stochastic assignment; stochastic user equilibrium

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1. Introduction

A common traffic paradox is evaluated by determining whether improving an existing link or adding a new link increases the total/average travel costs. The famous Braess paradox has drawn great attention since being proposed in 1968 (Braess, 1968; Braess et al., 2005), and it has been widely investigated from both analytical and empirical aspects. First, the conditions for the occurrence of Braess' paradox were explored in some general networks (Frank, 1981; Steinberg and Zangwill, 1983; Pas and Principio, 1997; Prashker and Bekhor, 2000). Then, studies regarding how to eliminate Braess' paradox and improve a system's efficiency were further conducted (Bazzan and Klvgl, 2005; Bagloee and Ceder, 2014; Sun et al., 2015). During recent years, the characteristics of Braess' paradox under special cases have also been extensively studied (Xia and Hill, 2013; Zverovich and Avineri, 2015; Di and He, 2014; Jansuwan and Chen, 2015).

As the paradoxical phenomenon was receiving increasingly more attention, other types of traffic paradoxes were proposed, such as stochastic assignment paradox (Sheffi and Daganzo,1978; Sheffi,Y., 1985; Yao and Chen, 2014; Zhao et al., 2014; Yao et al., 2018), capacity paradox (Yang and Bell, 1998; Jiang and Szeto, 2016), emissions paradox (Nagurey, 2000; Szeto et al. ,2008), reliability paradox (Yin and Ieda, 2002; Szeto, 2011), transit assignment paradox (Szeto and Jiang, 2014), informational Braess' paradox (Acemoglu et al., 2017) and exclusive bus lanes' setting paradox (Yao et al., 2015). From a practical perspective, several studies have investigated how to detect paradox link(s) in a real network (Bagloee and Ceder, 2014; Sun et al., 2015).

Most of the aforementioned traffic paradoxes are caused by the discrepancy between the user equilibrium (UE) and the system optimum (SO), which only exist in a flow-dependent (congested) network. Although travelers unilaterally improve their own travel time, this does not guarantee the system will acquire the minimal travel costs. Unlike the other paradoxes, the stochastic assignment paradox proposed by Sheffi and Daganzo (1978) can occur in a flow-independent network. The cause of the stochastic paradox is a travelers' perceived error as the objective of the stochastic traffic assignment is to minimize travelers' perceived travel costs rather than the total actual travel costs.

There has been a plethora of studies showing that congestion and perceived error could result in traffic paradox (i.e. improving/adding a link in such cases could rather increase the total travel costs). Based on these studies, this study further demonstrates the "non-monotonicity" of the paradox with regard to the amount of link improvement and the number of additional paradox links under the UE, the stochastic assignment, and the SUE principles, respectively. For one thing, the paradox caused by improving a link could disappear when continuously improving that link to a certain threshold. For another, if there are several "paradox links" that will incur a paradox when independently added to a network, simultaneously adding these "paradox links" to the network may counter-intuitively avoid the paradox. In this paper, we adopted the Multinomial Logit (MNL) model to represent travelers' stochastic route choice behavior. Due to the identical independent distribution (IID) assumption of the MNL model, we further used the Multinomial Probit (MNP) model to address the inability of the MNL model to consider route overlapping in the real road network.

The contribution of this study is twofold. From a theoretical perspective, to the best of our knowledge, it is the first time in the literature to note the "non-monotonicity" of a traffic paradox, and provide the some insights about the reason for the non-monotonic property in different scenarios. From a practical perspective, the common method to address existing paradox links is to close them (Bagloee and Ceder, 2014; Sun et al., 2015); the finding of this study provides an alternative solution—properly adding new links—to transfer the inefficiency of the existing links to efficiency.

The main body of this paper starts with a formal definition of the "non-monotonicity" property of paradox. Then in section 3, the "non-monotonicity" of the paradox is demonstrated in UE, stochastic assignment and SUE based on two designed networks. Section 4 further corroborates this phenomenon in a real road sub-network of Harbin. Conclusions and further analyses are summarized in Section 5.

2. Definition of the "non-monotonicity" of paradox

In the paper, the total travel cost of the whole road network is used to measure the traffic assignment paradox, if the road network is improved in terms of improving an existing link or adding new links, the total travel cost is increased, then the "so called" paradox emerges; otherwise, there will be no paradox.

Before introducing the definition of paradox's "non-monotonicity", some notations should be clarified. Considering a road transportation network $N(V, A)$, where V is the link set, A is the node set, and let the number of links and nodes be $\|V\|$ and $\|A\|$, respectively. Let c_i represent the free flow travel cost of link i, and $\boldsymbol{c} = [c_1, c_2, \dots, c_n\ ]^T$ be the cost vector.

- We define that cost vector $\boldsymbol{c}^1 \leq \boldsymbol{c}^2$ if and only if $c_i^1 \leq c_i^2$ for all $i \in \{1, \dots, n\}$.
- We define that $V^1 \leq V^2$ if and only if $V^1 \subseteq V^2$.

Define $TC(N(V, A), Q, M)$ to be the total travel cost of network N under the O-D demand Q and the traffic assignment model M.

- If for all $\boldsymbol{c}^1 \leq \boldsymbol{c}^2$, there is always $TC^1 \leq TC^2$, then the total network travel cost is monotonically increasing to the travel cost of links.
- If for all $V^1 \leq V^2$, there is always $TC^1 \geq TC^2$, then the total network travel cost is monotonically decreasing to the number of links.

Obviously, the appearance of paradox can ascribe the non-monotonically increasing to the link's travel cost or the non-monotonically decreasing to the link numbers. In other words, $\exists i \in \{1, \dots, n\}$ satisfies $c_i^1 \leq c_i^2$, but there is not always $TC^1 \leq TC^2$; or $\exists V^1 \leq V^2$, but there is not always $TC^1 \geq TC^2$. The focus of this paper is on the non-monotonicity feature of the above paradox, which implies that the paradox will not always exit but only can appear in some local processes when the road network is improved, and the definition of "non-monotonicity" property of paradox can be expressed as follows:

Proposition 1: The paradox may not always occur with the increase of link improvement when this link is paradoxical for marginal improvement, that is $\exists i \in \{1, \ldots, n\}$ satisfies $c_i^1 > c_i^2 > c_i^3$, , there is $TC^1 < TC^2$, but there is not $TC^1 < TC^3$.

Proposition 2: The paradox may not always occur with the increase in the number of paradox links, that is existing such additional paradox links, when added any one individually, there will be $V^1 < V^2$, and $TC^1 < TC^2$; when added more simultaneous, there will be $V^1 < V^2 < V^3$ but there is not always $TC^1 < TC^3$.

According to the above propositions, the "non-monotonicity" of paradox should be well understood. Next, we will conduct the demonstration of the paradox's non-monotonicity in three classical traffic assignment principles: (1) the UE case, (2) the stochastic assignment case, and (3) the SUE case. Further, the causes of this phenomenon under different scenarios should be different, therefore the underlying reasons behind this non-monotonic phenomenon are also analyzed and compared in each subsection.

3. Demonstrations for the non-monotonicity of traffic paradoxes

To illustrate this "non-monotonicity" widely exists in traffic assignment paradox, this section provides three different cases (based on UE, stochastic assignment, and SUE respectively) to show the counter-intuitive phenomenon—increasing the improvement amount of paradox link or simultaneously adding more "paradox links" to a network can sometimes improve a system's efficiency. In each case, the demonstrations of the "non-monotonicity" will be conducted from two perspectives: 1) improving an existing link; 2) adding new links.

3.1 Analysis in a user equilibrium

As one of the most classical assignment principle, the user equilibrium should be first applied to illustrate the non-monotonicity of paradox. Note that only the congestion effect is considered during the process. The UE assignment results can be readily obtained by using the F-W algorithm (LeBlanc et al., 1975), and the total travel cost of the whole network can also be accordingly calculated.

3.1.1 Illustration by improving an existing link

The classical Braess network shown in Fig.1 is used to show the paradox's non-monotonicity with regard to the amount of link improvement. Let c_k be the travel cost of link k, then $c_1 = 0.01x_1$, $c_2 = 15$, $c_3 = 15$, $c_4 = 0.01x_4$, $c_5 = 10$. Here Link 5 is taken as the improved link[1] to demonstrate the non-monotonicity feature of paradox.

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[1] Here improving the link means decreasing the link's travel cost/free flow travel cost.

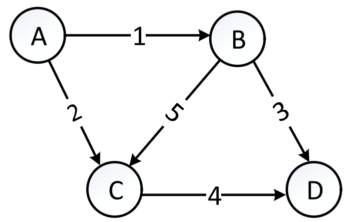
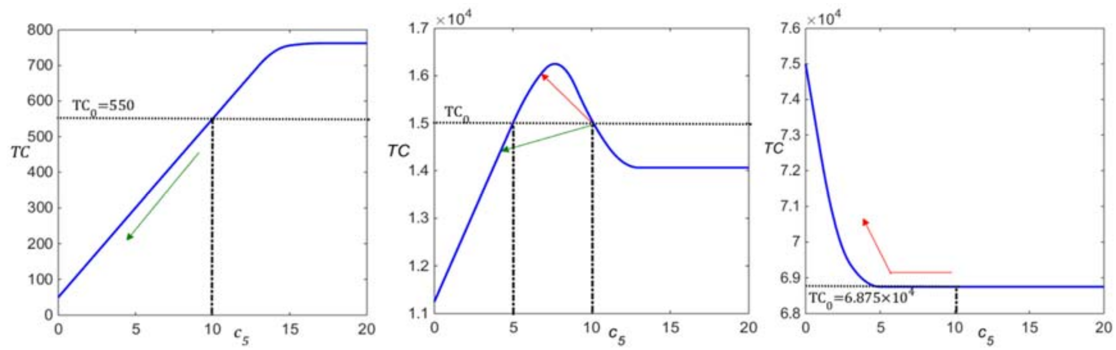


Fig. 1 The Braess network

To illustrate the non-monotonicity of paradox when improving an existing link, we should get that marginally improving a link can result in paradox, but continuously increasing the improvement degree can eliminate the paradox. Therefore, we need get the change of the total travel cost (TC) when decreasing the travel cost of link 5 (c_5). In the UE case, once the total travel demand is fixed, how the total travel cost (TC) changes with respect to $c_5$can be depicted, as shown in Fig.2.



(a) Q=50 (b) Q=750 (c) Q=2500

Fig. 2 Evaluation of TC with respect to c_5 under different travel demand level

According to Fig.2, it's clear that in a certain road network, the congestion level has a great influence on the paradox and its non-monotonicity. When Q=50, as shown in Fig.2 (a), improving link 5 will never cause the occurrence of paradox (with the decreasing of c_5, the total travel cost TC is also decreased), thus no non-monotonicity feature of paradox exists. When Q=2500, shown in Fig.2 (c), with the decreasing of c_5, TC will never lower than that of the original network (TC_0), which means improving link 5 always causes paradox, and the non-monotonicity of paradox does not exist either. But when Q=750, shown as Fig.2 (b), TC will increase first and then decrease to a value lower than TC_0. It implies marginally (or slightly) improving link 5 can bring paradox, while further increasing the improvement degree will eliminate the paradox and improve system's efficiency, which can be called the non-monotonicity of paradox with regard to the amount of link improvement.

In fact, the appearance of the above results is related to the nature of the UE principle. At a lower demand level, the less impact of congestion often results in the positive correlation between the system's total travel cost and link's travel cost, and improving an existing link cannot cause paradox. But as the level of congestion increases, due to the selfish behavior of travelers, the total travel cost of the system is no longer positively related to the link's travel cost, so decreasing link's travel cost may increase the total travel cost and result in the paradox. Especially, when some certain conditions (e.g., demand levels, the network structure) are met, the paradox can be eliminated by increasing the improvement degree. Moreover, the above analyses also indicate that in the UE assignment, sometimes marginally improving link can also result in paradox, similar as the stochastic assignment paradox (Sheffi and Daganzo, 1978).

3.1.2 Illustration by adding new links

For the illustrations of paradox's non-monotonicity with respect to the additions of paradox links, what we should do is to illustrate that in the UE assignment, simultaneously adding multiple similar paradox links (all of which can bring paradox when separately added to the network) can counter-intuitively avoid the paradox. The networks shown in Fig.3 will be applied to demonstrate it.

As shown in Fig. 3, the original road network has two O-D pairs (A, D) and (B, E), and the link cost functions are as follows: $c_1 = 0.1x_1$, $c_2 = 0.05x_2$, $c_3 = 50$, $c_4 = 5 + 0.1x_4$, $c_5 = 25$, $c_6 = 0.05x_6$ and $c_a = 0.1x_a$, $c_b = 0.01x_b$. Networks I and II are obtained by adding link a and link b, respectively; Network III is obtained by simultaneously adding the two links. Then under the fixed travel demands, the total travel cost of each network ($C_i, i \in \{0,1,2,3\}$) will be calculated.

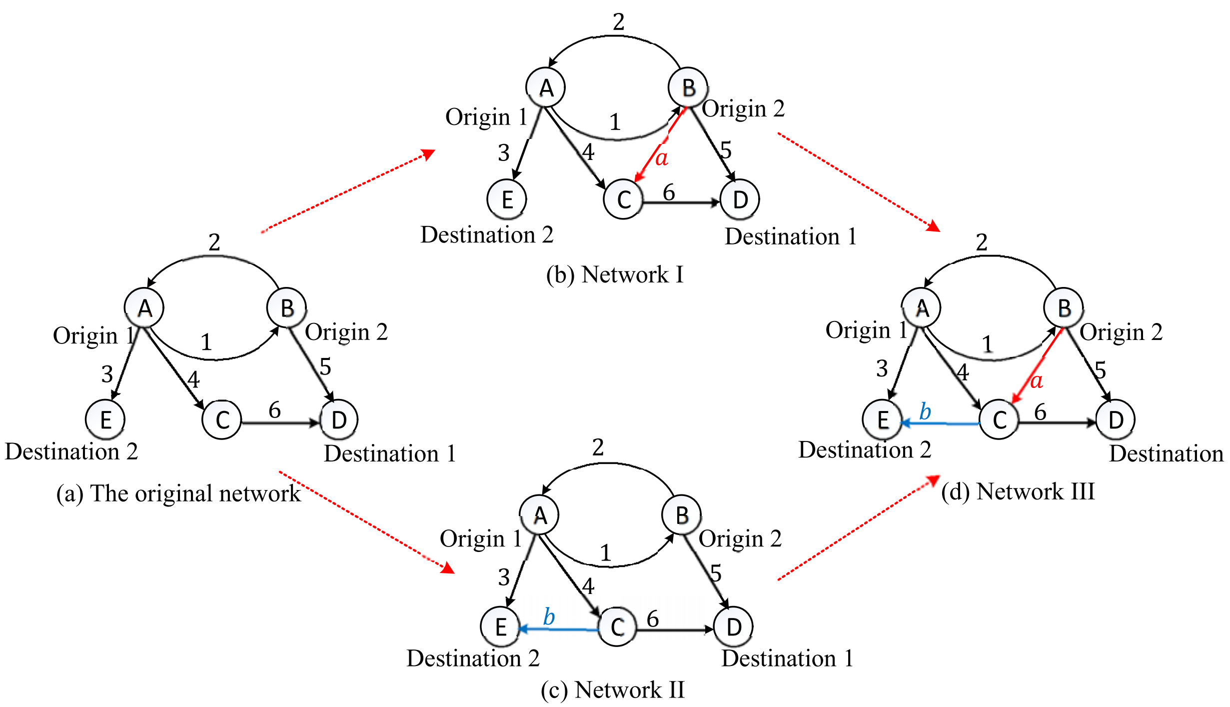
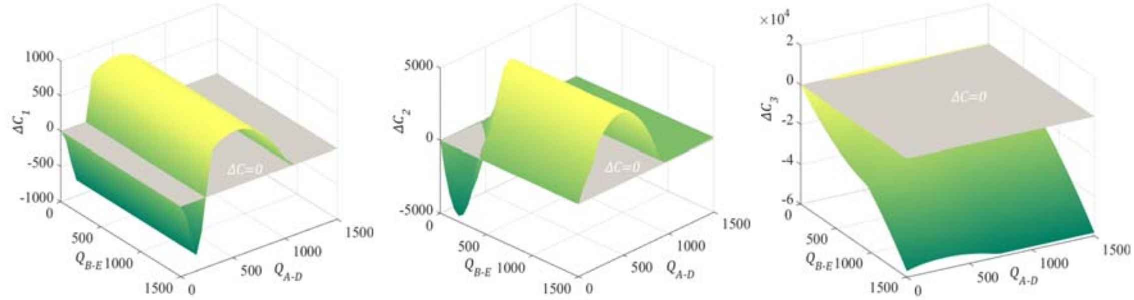
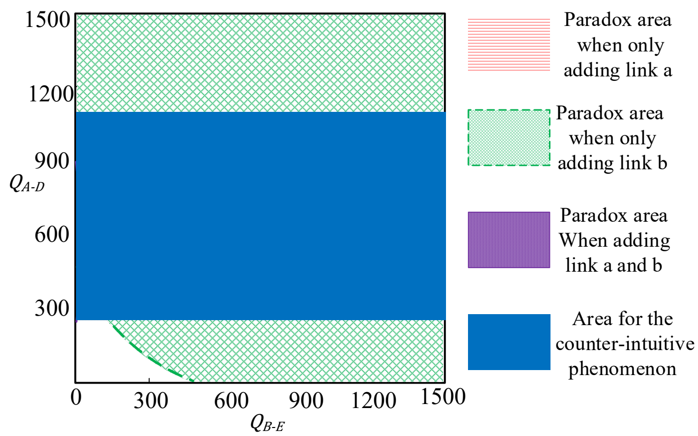


Fig. 3 Demonstration of adding new links to the original network with two O-D pairs

Denote $\Delta C_i = C_i - C_0$ $i \in \{1,2,3\}$ to be the difference between the total travel costs of Network i and the original network, where $\Delta C_i > 0$ indicates that adding new link/links causes a paradox, otherwise no paradox. How ΔC_i changes with the different total travel demands of the two O-D pairs is shown in Fig. 4 (a). To more intuitively show the differences in the paradox areas, Fig. 4 (b) depicts the demand ranges in which paradoxes occur under different conditions.



(a) ΔC_i with the different travel demands of two O-D pairs



(b) paradox area for different conditions

Fig. 4 Paradox of different conditions under the UE principle

As shown in Fig.4, it is conspicuous that there exist certain ranges of travel demands under which $\Delta C_1 > 0$ and $\Delta C_2 > 0$, but $\Delta C_3 < 0$. This implies that adding either link a or link b results in the classic Braess paradox, but the paradox disappears when simultaneously adding them. Therefore, it can be said that, under the UE assignment, the traditional Braess paradox is non-monotonic with respect to the number of paradox links.

In this case of adding new links, it is still the discrepancy between the individuals' unilateral optimum and the system's optimum that causes the paradox and its "non-monotonicity". Because of the uncooperative selfish route choice, the total travel cost of UE assignment can be higher than that of the system optimum, and they are not monotonic with regard to the number of link additions, which results in the traffic paradox and its "non-monotonicity" feature. However, it should be noted that the total travel cost under a system optimal assignment will never increase with an increase in the link number. Therefore, there is no paradox in the system optimal assignment, not to mention its "non-monotonicity."

3.2 Analysis in the stochastic assignment

As we all know, under the UE principle, the paradox and its non-monotonicity feature will be affected significantly by the congestion effect. In this section, to further explore under other assignment principles, whether the paradox still has the similar non-monotonic feature, the stochastic assignment principle is applied to do some related demonstrations. Unlike the Braess paradox, the stochastic assignment paradox can occur in uncongested networks. Therefore, here the demonstrations will be conducted in a fixed cost case.

3.2.1 Illustration by improving an existing link

In this case, we still take the network shown in Fig.1 as an example network. Since the congestion effect is not considered, we reset links' travel costs: $c_1 = c_4 = C_1$, $c_2 = c_3 = C_2$, and $c_5 = C_x$; Here the MNL model is used to calculate the route choice probability, and link 5 is still as an improved link to do illustration.

For the network shown in Fig1, the choice probability of route i can be expressed as follows:

$$P_i = \frac{e^{-\theta c_i}}{\sum_{i \in I} e^{-\theta c_i}}. \quad (1)$$

where c_i is the travel cost of route i, θ is the positive dispersion parameter related to the perception variance, and I is the route set. If $Q = 1$, then the total travel cost is as follows:

$$TC = Q \frac{c_i e^{-\theta c_i}}{\sum_{i \in I} e^{-\theta c_i}} = \frac{2(C_1+C_2)e^{-\theta(C_1+C_2)}+(2C_1+C_x)e^{-\theta(2C_1+C_x)}}{2e^{-\theta(C_1+C_2)}+e^{-\theta(2C_1+C_x)}}. \quad (2)$$

To evaluate how the change in the cost of link 5 affects the total travel costs, the partial derivative of TC with respect to C_x is derived as follows:

$$\frac{\partial TC}{\partial C_x} = \frac{e^{-\theta(2C_1+C_x)}[(1-\theta C_1+\theta C_2-\theta C_x)e^{-\theta(C_1+C_2)}+e^{-\theta(2C_1+C_x)}]}{\left(2e^{-\theta(C_1+C_2)}+e^{-\theta(2C_1+C_x)}\right)^2}. \quad (3)$$

The sign of Eq. (3) represents whether the total travel cost will increase ($\frac{\partial TC}{\partial C_x} < 0$) or decrease ($\frac{\partial TC}{\partial C_x} > 0$) when marginally improving link 5. To evaluate the sign of Eq. (3), let$f(C_x) = 1 - \theta C_1 + \theta C_2 - \theta C_x)e^{-\theta(C_1+C_2)} + e^{-\theta(2C_1+C_x)}$. Note that

$$\frac{\partial f(C_x)}{\partial C_x} = -\theta\left(e^{-\theta(C_1+C_2)} + e^{-\theta(2C_1+C_x)}\right) < 0. \quad (4)$$

Therefore, $f(C_x)$ is a decreasing function. It is not difficult to find that when $C_x \to 0$, if $f(C_x) < 0$, then $\frac{\partial TC}{\partial C_x} < 0$ will always hold, and TC should be an decreasing function with respect to C_x, therefore, there will never be the non-monotonicity feature of paradox. However, when $C_x \to 0$, if $f(C_x) > 0$ (this condition can be satisfied as long as $C_2 \geq C_1$), due to $C_x \to +\infty, f(C_x) < 0$, there must exists C_{x0} to satisfy $f(C_{x0})$=0. Therefore, we can get that when $C_x < C_{x0}$, then $f(C_x) > 0 \Longrightarrow \frac{\partial TC}{\partial C_x} > 0$; when $C_x > C_{x0}$, then $f(C_x) < 0 \Longrightarrow \frac{\partial TC}{\partial C_x} < 0$. Based on the aforementioned analysis, under the condition that $C_2 \geq C_1$, how TC changes with C_x is shown in

Fig. 5.

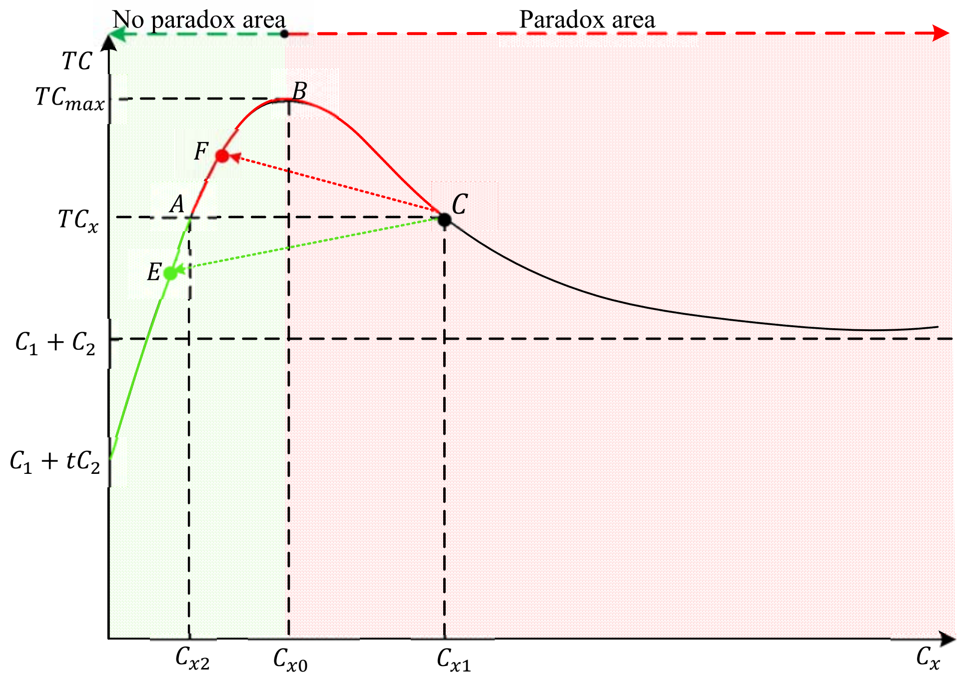


Fig. 5 Evaluation of TC with respect to C_x, $t = \frac{2e^{-\theta(C_1+C_2)}+\frac{C_1}{C_2}e^{-2\theta C_1}}{2e^{-\theta(C_1+C_2)}+e^{-2\theta C_1}} < 1$

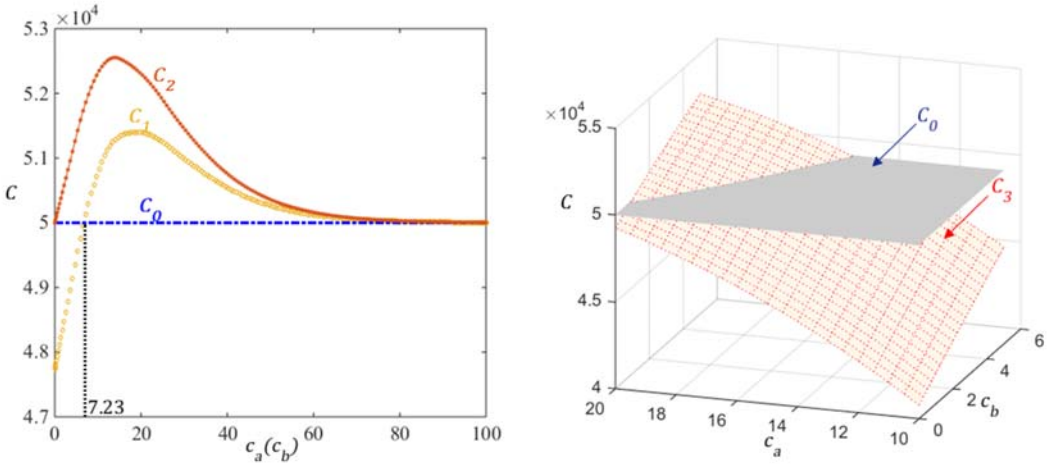
Similar as Fig.2 (b) of the UE case, TC is also non-monotonic with respect to C_x. When $C_x > C_{x0}$, marginally improving link 5 will increase the total travel cost because $\partial TC/\partial C_x < 0$. However, if enlarging the improvement degree (e.g., decreasing C_x from point C to point E, $\Delta C > C_{x1} - C_{x2}$), TC can be less than the original cost TC_x. Moreover, when continuously improving C_x, TC could reach a value that is less than $C_1 + C_2$, which could never be reached if the decision to improve link 5 was denied by the paradox evaluation of marginally improving link 5.

Clearly, in an uncongested network, the non-monotonicity of paradox under stochastic assignment still exists, and the reason for the above phenomenon can be attributed to the existence of stochastic perceived error. Because of the stochastic perceived error, marginally improving an inferior route can result in that more travelers shift to the improved route (it is still an inferior route), and accordingly cause the paradox (Sheffi and Daganzo, 1978). However, when increasing the improvement degree, the inferior route should change into a superior one, and now more travelers will use the superior route, then the total travel cost will decrease and the paradox will naturally disappear.

3.2.2 Illustration by adding new links

For the stochastic assignment case, the non-monotonicity of the stochastic paradox with regard to the number of additional paradox links is illustrated by the networks shown in Fig.3. Because of no congestion effect, the links' costs should be fixed, which is set as: $c_1 = 10$, $c_2 = 10$, $c_3 = 15$, $c_4 = 15$, $c_5 = 15$, $c_6 = 10$, and the travel costs of additional links a and b are set as c_a and

c_b respectively. Here we set the travel demand of each OD pair is $Q_{A-D} = 1000$, and $Q_{B-E} = 1000$, $\theta = 0.1$, then how the total travel cost of each network ($C_i, i \in \{0,1,2,3\}$) changes with c_a and c_b is given in Fig.6.



(a) adding link a and b separately (b) adding link a and b simultaneously

Fig.6 The total travel cost of each network

Fig. 6 (a) shows how the total travel costs of network I (C_1) and network II (C_2) change with respect to the cost of their new links (a and b). Then we can get that when $c_a > 7.23$, there is $C_1 > C_0$, and now the additional link a is a paradox link. Similarly, when adding link b, there is always $C_2 > C_0$, and the new link b is also a paradox link. Then, how the total travel cost of network III changes when adding both paradox links simultaneously is depicted in Fig.6 (b). Note that these two additional links should be both paradox links now, because only in this way, the impact of adding two paradox links simultaneously on system's efficiency can be accurately reflected. From Fig. 6 (b), it can be found that in the area of the black part, $C_3 < C_0$, which means that paradox cannot occur when adding two paradox links simultaneously.

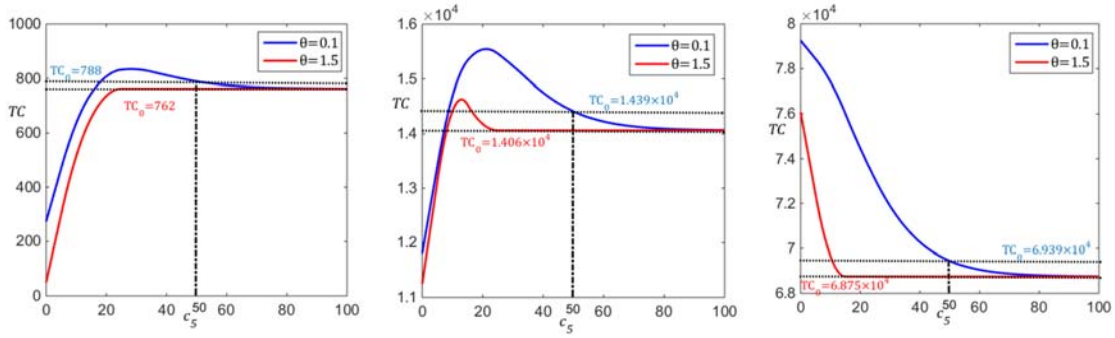
In fact, the "non-monotonicity" of the paradox can also be explained by the perceived error. When adding a new link to the original network brings inferior alternative route(s), there will always be a portion of travelers who choose the inferior route(s) because of the perceived error, which results in the increase in the total travel cost. After adding more links (these links result in a paradox when separately added), some superior routes are formed, which do not exist when adding theses links independently. When the travel-cost decreased in the superior routes outweighs the travel-costs increased in the inferior routes, the total travel cost will be decreased and the paradox should disappear.

3.3 Analysis in a stochastic user equilibrium

The above analyses separately consider the congestion effect and the stochastic effect. To generalize the aforementioned findings, this section demonstrates the non-monotonicity of paradox based on the SUE model, in which both stochastic perceived error and congestion effects are considered.

3.3.1 Illustration by improving an existing link

Similar as section 3.1.1, the same network shown in Fig.1 is still applied to conduct the illustrations for the non-monotonicity of paradox when improving link 5, where $c_1 = 0.01x_1$, $c_2 = 15$, $c_3 = 15$, $c_4 = 0.01x_4$, $c_5 = 50$. Once the travel demand Q and θ are fixed, the result of logit-based SUE assignment can be obtained by the method of successive average (MSA), then the total travel cost of the whole network can be calculated. Under different travel demand levels and different θ, how the total travel cost changes with respect to c_5 are given in Fig. 7



(a) Q=50 (b) Q=750 (c) Q=2500

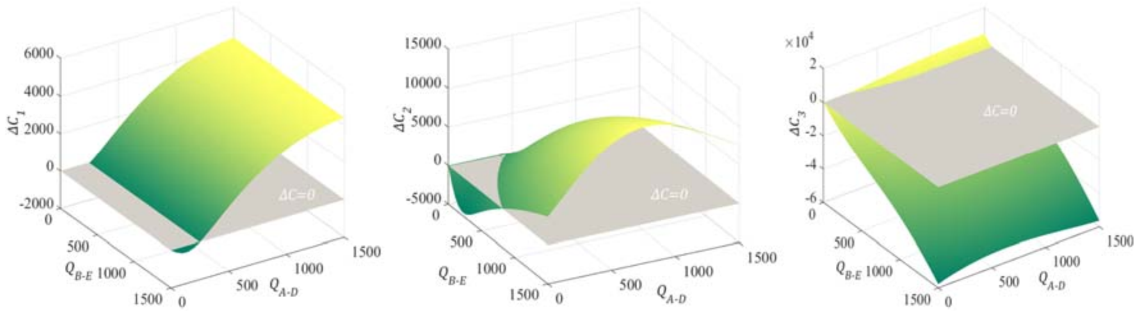
Fig. 7 Evaluation of TC with respect to c_5 under different conditions

For a lower θ (θ=0.1), when the demand level is relatively low, shown as Fig.7 (a) and (b), improving link 5 can result in the paradox and its non-monotonicity, because of less impact of congestion effect but more impact of stochastic effect, the reason is similar as that of the above stochastic assignment case without considering congestion. However, under the higher demand level, as shown in Fig.7 (c), even the travel cost of link 5 closes to zero, the route passing that link is still an inferior one, and improving this link certainly cannot eliminate the paradox, it is why only paradox occurs but no non-monotonicity feature during the decreasing of c_5. As we all know, the larger the value of θ, the weaker the randomness, the results will be closer to that under the UE case. Therefore, by comparing Fig. 7 with Fig.2, it can be found that the total travel cost of the SUE for a larger θ can be more similar to that of the UE.

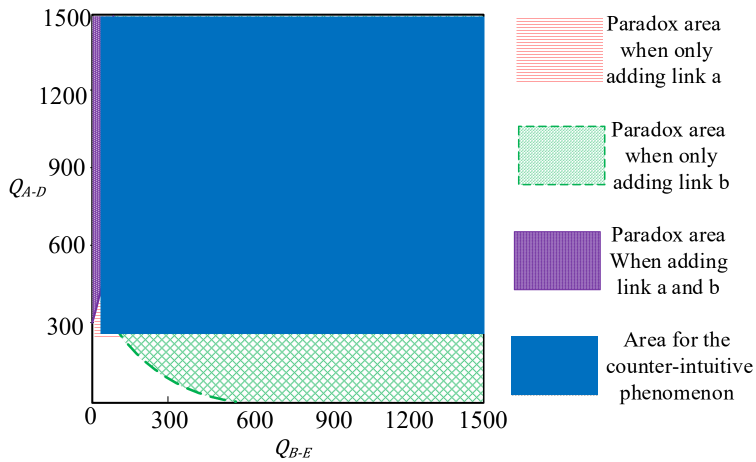
According to analyses above, it can be clearly to find that under the SUE case, sometimes the occurrence of paradox can still be non-monotonic with respect to link's improvement degree. Both the travel demand level and the value of θ have great impacts on the occurrence of paradox and its non-monotonicity,

3.3.2 Illustration by adding new links

For the analyses of paradox's non-monotonicity with respect to the addition of paradox links, here we still use the networks shown in Fig. 3 and the MNL model to do demonstrations. Let $\theta = 0.1$, similar analysis process as the UE case, the changes in ΔC_i with different total travel demands of the two O-D pairs are depicted in Fig. 8. It is obvious that under certain ranges of travel demands $\Delta C_1 > 0$ and $\Delta C_2 > 0$, but $\Delta C_3 < 0$. That is to say, the non-monotonicity of the paradox with regard to the number of additional links still exists when considering both the stochastic and the congestion factors.



(a) ΔC_i with different travel demands of the two OD pairs

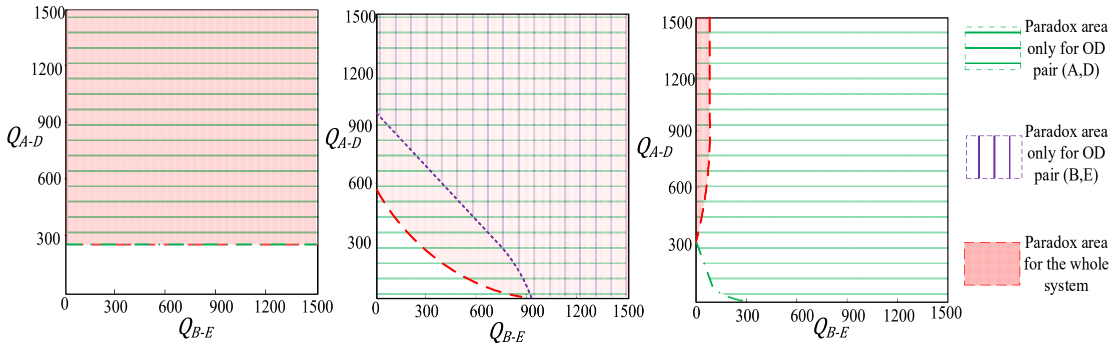


(b) paradox area for different conditions

Fig. 8 Paradox of different conditions under the SUE principle when θ=0.1

In summary, the non-monotonicity of the traffic paradox can not only exist in UE assignment and the stochastic assignment, it can also occur in SUE assignment. However, the underlying reasons for this phenomenon in the three assignment models are different. As mentioned in section 3.1 and 3.2, the factors that cause this phenomenon in UE assignment is uncooperative selfish route choice; for stochastic assignment, it lies in the perceived error; while for the SUE assignment, this phenomenon can be explained as the mixture of perceived error and uncooperative selfish route choice.

Furthermore, with the same networks shown in Fig.3, we will analyze paradox links' different effects on the travel costs of the two O-D pairs. As shown in Fig. 9, we compare the paradox areas of each O-D pair under the three situations (only adding link a, only adding link b, and adding both links). It is obvious that the addition of link a only affects the travel cost of the O-D pair (A, D), but adding link b can affect that of both O-D pairs. When simultaneously adding the two links , as shown in Fig. 9 (c), there is no paradox area for O-D pair (B, E), and the paradox area of the whole system is much smaller than that of the O-D pair (A, D). This can be explained by the fact that simultaneously adding the two links will increase the travel cost of O-D pair (A, D) but will decrease that of O-D pair (B, E).



(a) Only adding link a (b) Only adding link b (c)Adding link a and b

Fig.9 Paradox area under different travel demands of the two O-D pairs

It is understandable that the newly added link has different effects on different O-D pairs. The overall change in total travel cost is the combination of the cost-change in each O-D pair, such as the traffic paradox. Inspired by this, we can design a differentiated link usage strategy to avoid the negative impact of a link to certain O-D pairs. For example, simultaneously adding link a and link b decreases the travel cost of O-D pair (B, E) but increases the travel cost of O-D pair (A, D). We can forbid travelers of O-D pair (A, D) to use link a and link b to avoid the increase in the travel cost of O-D pair (A, D) and the new links can only be opened for travelers between O-D pair (B, E). In this particular example, we have tested that this differentiated link usage strategy can reduce a system's total travel cost. However, note that the flow re-distribution of one O-D pair can affect the flow distribution of other O-D pairs in a congested network; whether a link should be closed for some O-D pairs should be integrally evaluated with other O-D pairs.

4. Application in a generalized traffic network

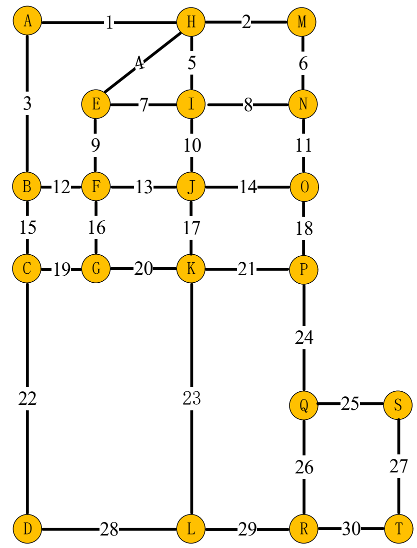
As described in this section, to verify the previous arguments, the "non-monotonicity" of the traffic paradox was demonstrated in a real road sub-network of Harbin (the capital city of Heilongjiang Province in China) under the SUE assignment. For the SUE principle, the route set was obtained using the link penalty method (De La Barra et al., 1993).[2] The MNL model was still used in the route choice process. Considering the unrealistic IID assumption in the MNL model, we further used the MNP model to verify our findings. To solve the SUE assignment, we used a route-based algorithm and the method of successive average (MSA).

As shown in Fig. 10, the road sub-network of Harbin consists of 20 intersections and 30 road segments. Each segment is bidirectional and has the same cost function in both directions. Here

[2] Note that the choice of route set can affect the traffic distribution results to a certain extent, and accordingly may have impact on the judgment of traffic assignment paradox, especially in a generalized larger road network.

we applied BPR function $c_a = c_{0a}(1 + 0.15 \times \left(\frac{x_a}{1000}\right)^4)$ as the links' travel cost function, where $c_{0a}$is the travel cost with free flow, x_a is the flow volume of the segment a. We assumed this road network had five O-D pairs: (A, T), (B, T), (C, S), (D, M), and (E, S). The travel demand of each O-D pair is set as 100η, and η can be used to measure the congestion level. Then we still conducted the analyses from the following two aspects: (1) improving an existing link; (2) adding paradox links.

(a)



(b)

Fig. 10 Road sub-network of Harbin

4.1 Improving an existing link

Here we chose link F-J as a target link to illustrate that the traffic paradox is not monotonic with regard to the amount of link improvement, and the free flow travel cost of each link c_{0a} is shown in Table.1. Then set $\theta = 0.1$, $\eta = 5$, when improving link F-J (decreasing c_{0FJ}), how the total system cost TC changes is shown in Fig. 11.

Tab.1 Free flow travel costs of links (c_{0a})

Link	c_{0a}(s)	Link	c_{0a} (s)	Link	c_{0a} (s)
1	50	11	17	21	41
2	39	12	25	22	60
3	50	13	c_{0FJ}	23	60
4	55	14	39	24	30
5	20	15	20	25	27
6	20	16	40	26	30
7	30	17	18	27	30
8	40	18	18	28	55
9	18	19	24	29	42
10	17	20	32	30	30

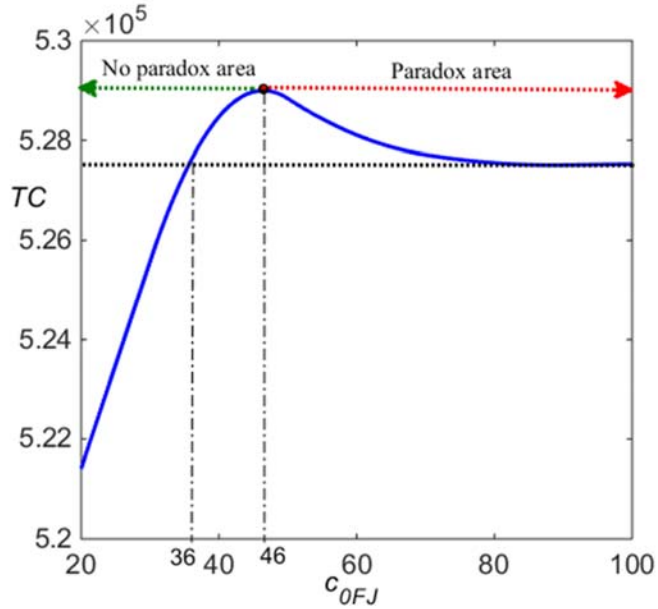
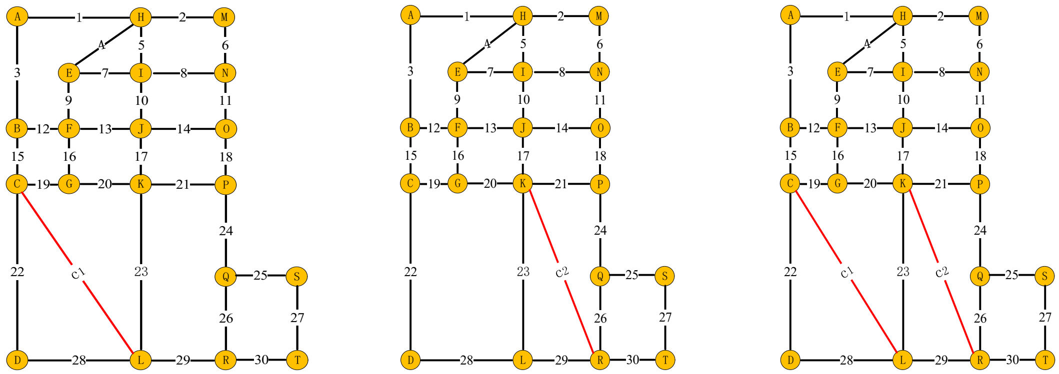


Fig. 11 Evaluation of *TC* with respect to c_{0FJ}

Fig.11 shows that, TC is not monotonic with regard to c_{0FJ}, with the increasing of c_{0FJ}, the total travel cost TC increases first and then decreases. When $c_{0FJ} > 46$, marginally decreasing c_{0FJ} can result in the increasing of TC and cause paradox; while if we enlarge the improvement degree, TC can be reduced to a value less than the original one before improving the link. That is to say, in the real road network, the marginal-improvement paradox does not mean that a sufficient amount of improvement also can result in a paradox, and sometimes the occurrence of paradox is still non- monotonic with respect to the degree of link improvement.

4.2 Adding new links

This section further illustrates that the impact of the traffic paradox is not monotonic with regard to the number of additional paradox links. Assuming we plan to add new link(s) to the network shown in Fig 10. Three link-addition plans are shown in Fig. 12. The travel cost functions of the existing links are remained unchanged, where $c_{0FJ} = 30$, and $c_{FJ} = 30(1 + 0.15(\frac{x_{FJ}}{1000})^4)$. Then we set the travel cost functions of the new additional links as: $c_{CL} = 10(1 + 0.15(\frac{x_{CL}}{125})^4)$and $c_{KR} = 10(1 + 0.15(\frac{x_{KR}}{100})^4)$, and θ is still fixed to 0.1 in the following analyses.



(a) Network I (b) Network II (c) Network III

Fig. 12 Demonstration of adding new links to the original road network

Fig. 13 shows how ΔTC_i (the difference in the total travel costs of Network i and the original network) changes with respect to η. It is clear that when $\eta \in (3.6, 4.1)$ (approximately), there are $\Delta TC_1 > 0$ and $\Delta TC_2 > 0$, but $\Delta TC_3 < 0$. Under this condition, new links C-L and Q-R are both paradox links, and adding any of them will increase the total travel cost, but simultaneously adding the two links can reduce the total travel cost. Therefore, the occurrence of the paradox is non-monotonic with respect to the increase in the number of paradox links.

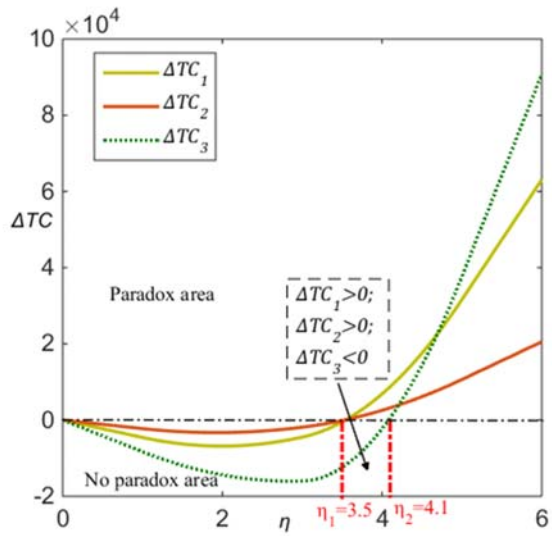


Fig. 13 Evaluation of ΔTC with respect to η

4.3 Verification using the MNP model

Route overlapping widely exists in real networks and it can still affect travelers' route choice. Because of the IID assumption, the MNL model cannot address the route overlapping effect. To compensate for this, we further verified the existence of the aforementioned phenomenon when considering the route overlapping effect. Although there are numerous closed-form alternatives (such as C-Logit, Path-Size Logit, etc.) for the MNL model that can address overlapping, most have some other defects. Therefore, here only the MNP model was used as a standard to further verify the previous findings. For a link with travel cost c, the MNP model assumes the perceived cost of the link satisfies normal distribution $N(c,(c \times \sigma)^2)$, where σ is a factor that controls the magnitude of the perceived error, and the MNP model was conducted using a Monte Carlo simulation (Sheffi,Y., 1985). Both improving a link and adding a new link(s) were evaluated.

For the link improvement, we re-performed the case described in section 4.1 by the MNP model, and here σ was set to 0.2. How the total travel cost of the network changes with respect to c_{0FJ} is depicted in Fig.14. Apparently, according to the MNP model, we can obtain the same conclusions as using the MNL model, and the "non-monotonicity" of the traffic paradox with respect to link improvement still exists in this network when considering the overlapping route effect.

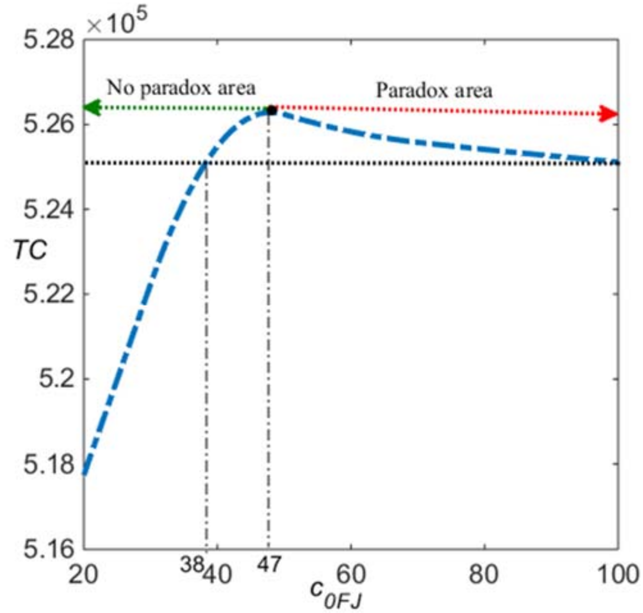


Fig. 14 Evaluation of *TC* with respect to c_{0FJ}

For the link addition, we apply the case described in section 4.2 using the MNP model. As analyses above, under the MNL model, when $\eta \in (3.6, 4.1)$, the non-monotonicity feature can exist. Therefore, the traffic demand level η is fixed to 4, and we evaluated total travel cost under different perceived error σ. Fig. 15 shows how the total travel costs of the different link-addition plans change with the different σ. It is found that when $\sigma \in (0, \sigma_1)$, $TC_1 > TC_0$, $TC_2 > TC_0$, but $TC_3 < TC_0$, thus even considering the route overlapping effect, sometimes adding more paradox links counter-intuitively can eliminate the paradox.

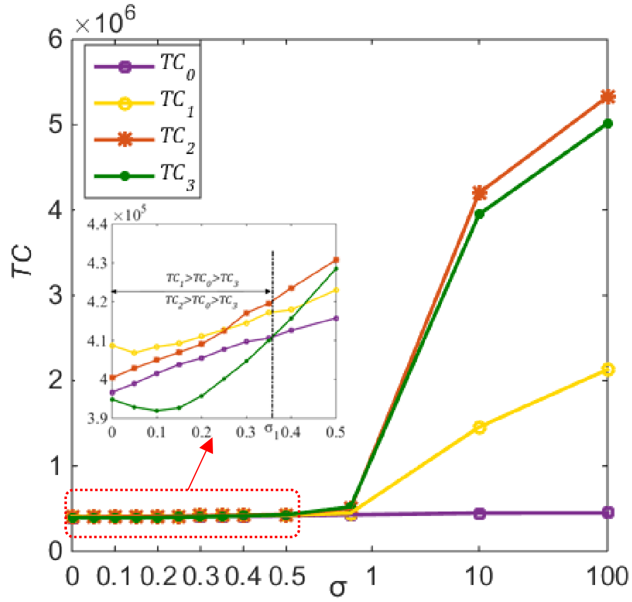


Fig. 15 Evaluation of *TC* in different networks with respect to σ

Fig. 15 further shows that the variance in the perceived error can also affect this feature. The perceived error increases with the increase in σ, and travelers randomly choose routes when $\sigma \rightarrow +\infty$. In this particular case, it is found that the non-monotonicity phenomenon disappears when σ increases. When $\sigma = 0$, the MNP model is equivalent to UE, and Fig. 15 shows that during the process of adding new links, the non-monotonicity of the paradox also exists in the UE assignment.

5. Conclusions

This paper illustrated the "non-monotonicity" of the traffic assignment paradox and presented a counter-institutive phenomenon that paradox links can sometimes improve system efficiency. Based on the UE, stochastic assignment and SUE principles, we demonstrated that (1) a paradox caused by marginally improving a link can disappear when continuously improving that link to a certain threshold and (2) simultaneously adding several paradox links (which results in a paradox when separately added to a network) to a network may counter-intuitively avoid the paradox. Certainly, the reasons for the non-monotonicity of the traffic paradox under different scenarios were also explained. Because of the perceived error and uncooperative selfish route choice, the total travel costs can't be monotonic with regard to the degree of link improvement and the number of link additions, which usually results in the traffic paradox and its "non-monotonicity" feature. In addition, we further conducted the relevant demonstrations in a real road network basing on both the MNL model and the MNP model, and the results show that under some certain conditions, the "non-monotonicity" property of paradox can indeed appear in some real road networks.

Our research has important implications in transportation planning and operations. On the one hand, our findings reflect some limitations of the current sensitivity analysis in designing traffic infrastructure. Sensitivity analysis is usually conducted by evaluating marginal improvement, but this may result in a confined judgment and a local optimum in network optimization and evaluation. On the other hand, our findings provide new visions for the design of traffic networks. In current practice, link construction plans are often independently evaluated one-by-one. Our results show that the functions of different links in a network are related (adding several paradox links can counter-intuitively eliminate the paradox), and the combination of several paradox links can also bring benefits to the network. In addition, section 3.2 shows that creative policies (e.g., differentiated restrictions for travelers from different O-D pairs) for some paradox links may also help these links play a positive role in the network.

Further researches can be conducted from the following aspects. First, the findings of this study can further be verified and compared to those of other route choice models such as the C-logit (Cascetta et al.,1996; Zhou et al., 2012), path-size Logit (PSL) (Ben-Akiva and Bierlaire,1999; Bovy et al., 2009; Chen et al., 2012), Paired Combinatorial Logit (PCL) (Chen et al., 2014), cross nested Logit (Bekhor et al., 2007), and generalized nested Logit (GNL) models (Bekhor and Prashker, 2001). Second, it is interesting to evaluate whether a similar phenomenon exists in other equilibrium models, for instance, bi-objective user equilibrium (Wang et al., 2013), boundedly rational user equilibrium (Mahmassani and Chang, 1987; Di and He, 2014), dynamic user equilibrium (Friesz et al., 1993 ; Lu et al. 2008), traffic assignment with capacity constraints (Correa et al., 2004), or whether other paradoxes can also have the similar non-monotonicity feature, such as the capacity paradox (Yang and Bell, 1998; Jiang and Szeto, 2016), emission paradox (Nagurey, 2000; Szeto et al., 2008), and noise paradox (Wang and Szeto, 2017). Lastly, this study only demonstrates the effect of combining two paradox links. More general researches should be conducted to evaluate the combination of more paradox links, and algorithms to identify the related paradox links in real road networks are worth exploring.

Acknowledgments

The authors are grateful to three anonymous referees and Professor Hai Yang for their constructive comments and suggestions to improve the quality and clarity of the paper. This research was partially supported by the National Natural Science Foundation of China (71501053, 71871075, 91846301), China Postdoctoral Science Foundation (2015M570297), International Postdoctoral Exchange Fellowship (20160076) of China Postdoctoral Council, Fundamental Research Funds for the Central Universities (HIT.NSRIF.201661), Research Grants Council of the Hong Kong Special Administrative Region (15212217), and CCF-DiDi Big Data Joint Lab.

References

Acemoglu, D., Makhdoumi, A., Malekian, A., Ozdaglar, A., 2017. Informational Braess' Paradox: The effect of information on traffic congestion. Computer Science and Game Theory. arXiv:1601.02039v2

Bagloee, S.A., Ceder, A., Tavana, M., Bozic, C., 2014. A heuristic methodology to tackle the Braess Paradox detecting problem tailored for real road networks. Transportmetrica A 10 (5), 437- 456.

Bazzan, A.L.C., Klvgl, F., 2005. Case studies on the Braess Paradox: Simulating route recommendation and learning in abstract and microscopic models. Transportation Research Part C 13 (4), 299-319.

Bekhor, S., Prashker, J.N., 2001. Stochastic user equilibrium formulation for generalized nested logit model. Transportation Research Record 1752 (1), 84-90.

Bekhor, S. Reznikova, L., Toledo, T., 2007. Application of cross-nested logit rout choice model in stochastic user equilibrium traffic assignment. Transportation Research Record 2003, 41-49.

Ben-Akiva, M., Bierlaire, M., 1999. Discrete choice methods and their applications to short term travel decisions. In, Hall, R.W. (Ed.), Handbook of Transportation Science, Springer US, Boston, MA, pp. 5-33.

Bovy, P.H.L., Bekhor, S., Prato C.G, 2008. The factor of the revisited path size: Alternative derivation. Transportation Research Record 2076 (2076), 132-140.

Braess, D., 1968. Über ein paradox in der verkehrsplanung. Unternehmensforschung 12 (1), 258-268.

Braess, D., Nagurney, A., Wakolbinger, T. 2005. On a paradox of traffic planning. Transportation Science 39 (4), 446-450.

Cascetta, E., Nuzzolo, A., Russo, F., Vitetta, A., 1996. A modified Logit route choice model overcoming path overlapping problems: Specification and some calibration results. The 13th International Symposium on Transportation and Traffic Theory, Lyon, France, 24-26 July 1996.

Chen, A., Pravinvongvuth, S., Xu, X., Ryu, S., Chootinan, P., 2012. Examining the scaling effect and overlapping problem in logit-based stochastic user equilibrium models. Transportation Research Part A 46 (8), 1343-1358.

Chen, A., Ryu, S., Xu, X., Choi, K., 2014. Computation and application of the paired combinatorial logit stochastic traffic equilibrium problem. Computers and Operations Research 43(1), 68-77.

Correa, J. R., Schulz, A. S., Stier-Moses, N.E., 2004. Selfish Routing in Capacitated Networks. Mathematics of Operations Research, 29(4):961-976.

De La Barra, T., Perez, B. and Anez, J., 1993. Multidimensional path search and assignment. In 21st PTRC Summer Annual Meeting, University of Manchester, United Kingdom, 307-319.

Di, X., He, X., Guo, X., Liu, H.X., 2014. Braess paradox under the boundedly rational user equilibria. Transportation Research Part B 67 (3), 86-108.

Friesz, T.L., Bernstein, D., Smith, T.E., Tobin, R.L., Wie, B.W., 1993. A variational inequality formulation of the dynamic network user equilibrium problem. Operations Research 41(1) 179–191.

Frank, M., 1981. The Braess paradox. Mathematical Programming 20 (1), 283-302.

Jansuwan, S., Chen, A., 2015. Considering perception errors in network efficiency measure: an application to bridge importance ranking in degradable transportation networks.

Transportmetrica A: Transport Science 11(9), 793-818.
Jiang, Y., Szeto, W.Y., 2016. Reliability-based stochastic transit assignment: Formulations and capacity paradox. Transportation Research Part B 93,181-206.
LeBlanc, L.J., Morlok, E.K., Pierskalla, W.P., 1975. An efficient approach to solving the road network equilibrium traffic assignment problem. Transportation Research 9, 309–318.
Lu, C., Mahmassani, H. S., Zhou, X., 2008. A bi-criterion dynamic user equilibrium traffic assignment model and solution algorithm for evaluating dynamic road pricing strategies. ransportation Research Part C, 16(4), 371-389.
Mahmassani, H. S., Chang, G., 1987. On boundedly rational user equilibrium in transportation systems. 21(2), 89-99.
Nagurney, A., 2000. Congested urban transportation networks and emission paradoxes. Transportation Research Part D 5 (2), 145-151.
Pas, E., Principio, S., 1997. Braess' paradox: Some new insights. Transportation Research Part B 31 (3), 265-276.
Prashker, J., Bekhor, S., 2000. Some observations on stochastic user equilibrium and system optimal of traffic assignment. Transportation Research Part B: Methodological 34(4), 277-291.
Sheffi, Y., 1985. Urban Transportation Networks. Prentice-Hall, Englewood Cliffs, NJ.
Sheffi, Y., Daganzo, C., 1978. Another paradox of traffic flow. Transportation Research 12 (1), 43-46.
Steinberg, S., Zangwill, W., 1983. Prevalence of Braess' Paradox. Transportation Science 17 (3), 301-318.
Sun, L., Liu, L., Xu, Z., Jie, Y., Wei, D., Wang, P., 2015. Locating inefficient links in a large-scale transportation network. Physica A 419, 537-545.
Szeto, W.Y., 2011. Cooperative game approaches to measuring network reliability considering paradoxes. Transportation Research Part C 19 (2), 229-241.
Szeto, W.Y., Jiang, Y., 2014. Transit assignment: Approach-based formulation, extragradient method, and paradox. Transportation Research Part B 62 (4), 51-76.
Szeto, W.Y., Li, X., O'Mahony, M., 2008. Simultaneous Occurrence of the Braess and Emission Paradoxes. Proceedings of the Conference on Traffic and Transportation Studies, ICTTS 322 : 625-634.
Wang, J. Y.T., Ehrgott, M., Chen, A., 2013. A bi-objective user equilibrium model of travel time reliability in a road network. Transportation Research Part B 66,4-15.
Wang, Y., Szeto, W.Y., 2017. Excessive noise paradoxes in urban transportation networks. Transportmetrica A: Transport Science 13(3), 195-221.
Xia, Y., Hill, D.J., 2013. Dynamic Braess's Paradox in complex communication networks. IEEE Transactions on Circuits and Systems II: Express Briefs 60 (3), 172-176.
Yang, H., Bell, M.G.H., 1998. A capacity paradox in network design and how to avoid it. Transportation Research Part A 32 (7), 539-545
Yao, J., Chen, A., 2014. An analysis of logit and weibit route choices in stochastic assignment paradox. Transportation Research Part B 69, 31-49.
Yao, J., Cheng, Z., Dai, J., Chen, A., An, S., 2019. Traffic assignment paradox incorporating congestion and stochastic perceived error simultaneously. Transportmetrica A: Transport Science 15(2), 307-325.
Yao, J., Shi, F., An, S., Wang, J., 2015. Evaluation of exclusive bus lanes in a bi-modal

degradable road network. Transportation Research Part C 60, 36-51.
Yin, Y., Ieda, H., 2002. Optimum improvement scheme for network reliability. Transportation Research Record 1783 (1), 1-6.
Zhao, C., Fu, B., Wang, T., 2014. Braess paradox and robustness of traffic networks under stochastic user equilibrium. Transportation Research Part E 61, 135-141.
Zhou, Z., Chen, A., Bekhor, S., 2012. C-logit stochastic user equilibrium model: formulations and solution algorithm. Transportmetrica 8(1), 17-41.
Zverovich, V., Avineri, E., 2015. Braess' paradox in a generalised traffic network. Journal of Advanced Transportation 49(1), 114-138.