

One-Dimensional Nonlinear Finite Strain Analysis of Self-Weight Consolidation of Soft Clay Considering Creep

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Abstract:

Creep as an intrinsic property of clay is non-negligible in predicting ground settlements in land reclamation construction and foundations on reclaimed land and soft soils, but it is unfortunately difficult to characterize in current self-weight consolidation analysis. This paper develops a one-dimensional (1D) finite strain consolidation model to take into account the creep of soft clay, particularly during the early stages of reclamation construction, mainly in a self-weight consolidation process. In this model, the Yin-Graham 1D Elastic Visco-Plastic (EVP) model is first extended for modelling self-weight finite strain consolidation of soft clays to describe the creep of soil skeleton under extremely high water content. The Darcian and non-Darcian flow, nonlinear compressibility and permeability of soft clays with a huge variety of water content are also considered. Governing partial differential equations using the EVP model are derived. These nonlinear partial differential equations are solved using the Crank-Nicholson finite difference method. Three case studies involving a wide range of initial void ratio values are simulated, which show that the present model, with more realistic consideration of creep feature of clays than previous ones, can capture the self-weight consolidation process well when compared with physical model test results. The influences of the initial height and void ratio on the self-weight consolidation process are analyzed to provide some recommendations for engineering practice.

Keywords: finite strain; creep; soft clay; self-weight consolidation; viscoplasticity; permeability

Introduction

To compensate for the short supply of sand in coastal cities, local marine deposits are widely used to fill portions of reclaimed areas using the dredging and pumping method. During the reclamation, the local marine deposits that are in a slurry state usually undergo settling, self-weight consolidation, and accelerated consolidation with vertical drains and pre-loading. One-dimensional (1D) self-weight consolidation is highly significant during the early stages of reclamation that use marine deposits as fill material. Vrakas and Anagnostou (2015) pointed that the small strain theory is not suitable once the strain exceeds 10%. In addition, the finite strain theory is more appropriate when the soil has a large initial water content higher than liquid limit generally resulting in large settlement. During the self-weight consolidation process, dredged soft soil settles significantly relative to the thickness of the soil layer. Moreover, in cases featuring obvious changes in void ratio, compressibility and permeability can be found to have a nonlinear relationship with the void ratio. Accordingly, the self-weight consolidation of slurry is a finite strain problem instead of a small strain problem. Furthermore, soft soil exhibits evident creep that inevitably causes inaccurate assessment of settlement and stress distribution if not taken into account (Bjerrum, 1967, 1973) in the consolidation theory. The creep of soil is mainly due to interparticle sliding, the expulsion of water from double layers of clay particles, and the rearrangement of adsorbed water molecules and cations into different positions that are time-dependent. Sills (1995) presented several experimental investigations that the settlement induced by creep was comparable with that due to the effective stress, and creep settlement is theoretically important in cases of self-weight

consolidation since creep is an intrinsic property of clay. Therefore, an accurate prediction for self-weight consolidation process considering creep of soft clay is needed for engineering design, construction control and safety assessment of structures on or in reclaimed land.

Developing a consolidation prediction method has historically been highly challenging. Terzaghi (1943) proposed a 1D consolidation theory based on a series of simplifying assumptions, including small strain assumption, which are approximately applicable in practice. Richard (1957) and McNabb (1960) were among the first to consider finite strain consolidation problems in soil, although the details of 1D finite strain consolidation governing equations that treat void ratio as the main variable were proposed and obtained by Gibson et al. (1967, 1981). Many governing equations for 1D primary consolidation of saturated thick soil layers that have been advocated from time to time are either equivalent to or special cases of the finite strain consolidation governing equations proposed by Gibson et al. (1967, 1981) (e.g., Schiffman, 1980; Cargill, 1986; McVay et al., 1986). However, the soil skeleton in the foregoing consolidation analysis were considered elastic.

Carter et al. (1979) presented a numerical method for analysing finite elasto-plastic consolidation, and Borja et al. (1994), Borja and Alarcon (1995) and Borja et al. (1998) successfully developed a mathematical framework and finite element method for finite strain consolidation analysis using an elasto-plastic model with some case studies. Fox and colleagues (Fox and Berles, 1997; Fox, 1999; Fox et al., 2014) and Pu et al. (2020) developed dimensionless piecewise-linear numerical models, named CS2 and CS3, for 1D finite strain consolidation analysis. Both CS2 and CS3 accounted for large strain, self-weight of soil, elastic

or elasto-plastic constitutive modelling, the relative velocities of the fluid and solid phases and variable hydraulic conductivity and compressibility during the consolidation process. Although Hawlader et al. (2003) developed a 1D consolidation model considering viscosity effects of clay based on a strain rate dependent constitutive model. However, from then few works investigated or even considered the time-dependent nonlinear creep of clayey soils (an intrinsic property of clay) in the context of self-weight finite strain consolidation problems starting from a high initial water content.

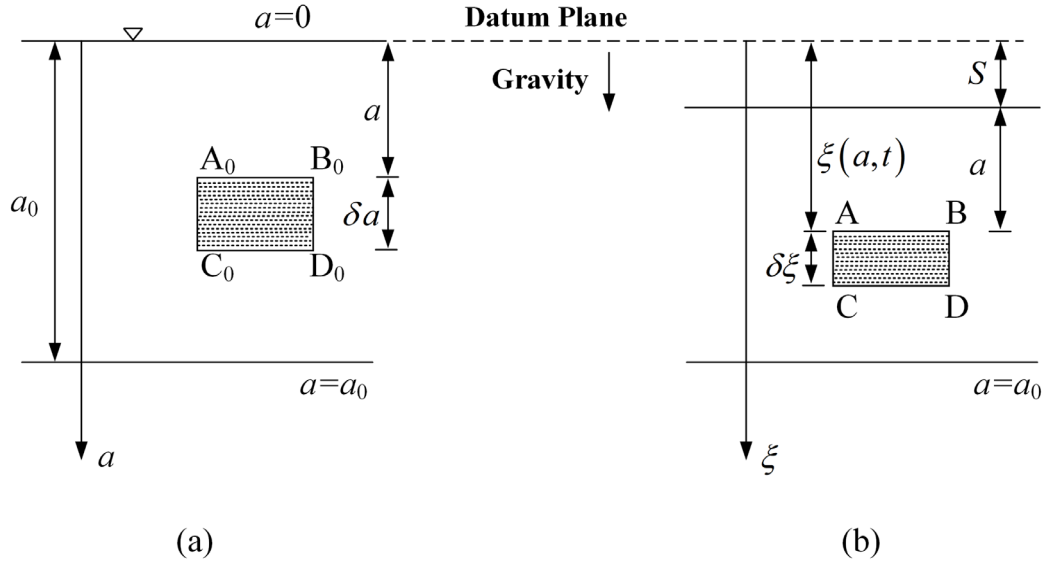
In this paper, a 1D finite strain consolidation theory is developed and verified that takes into account creep for self-weight consolidation of soft clay. Firstly, the 1D Yin-Graham EVP model is extended for the finite strain case by modifications of two logarithmic functions of instant and reference time lines for the creep equation. Secondly, the existing 1D finite strain consolidation theory is extended to consider creep, self-weight, changing compressibility and hydraulic conductivity of slurry by incorporating the extended 1D EVP model. Nonlinear partial differential equations are then solved using the Crank-Nicholson finite difference method. The extended 1D finite strain consolidation model is applied to simulate three case studies. Further, some parametric studies of the proposed consolidation model are conducted.

1D finite strain consolidation theory accounting for creep

Coordinate systems

Figure 1 shows a saturated soil layer in Lagrangian coordinates and convective coordinates: (a) initial configuration at $t = 0$ with vertical Lagrangian coordinate a , $a = 0$ at the

84 soil surface and $a = a_0$ at the soil bottom; (b) configuration at time t with vertical convective
85 coordinate ξ .



86
87 Figure 1—Lagrangian coordinates for finite strain consolidation theory: (a) initial configuration at $t = 0$ and
88 (b) configuration at time t (also called convective coordinate)

89 The coordinate system generally used in geotechnical engineering is the Eulerian system,
90 in which material deformation is related to planes fixed in space. A real measuring system is
91 one that connects with the material particles, especially under finite strain. Small strain
92 consolidation theory assumes that a clay layer's deformation is small compared with its
93 thickness, removing the need to distinguish among Eulerian systems, Lagrangian systems and
94 convective coordinates. For finite strain problems in geotechnical engineering, however,
95 changes in surface height are large compared with the thickness of the soil layer, so that the
96 Eulerian system is no longer applicable. Furthermore, there are also several obvious advantages
97 of using Lagrangian coordinates instead of convective coordinates. Firstly, there is no need to

consider the coordinate change of the soil surface with the consolidation process in Lagrangian coordinates, because the soil surface is always at $a = 0$.

Secondly, Lagrangian coordinate a is an independent variable, whereas convective coordinate ξ is dependent on a and t : $\xi = \xi(a, t)$. These advantages of the Lagrangian coordinate system will aid mathematical analysis of finite strain consolidation. Converting variables expressed in Lagrangian coordinates into the convective system is straightforward using the relationship between a and ξ given by Gibson et al. (1967, 1981),

$$\frac{\partial \xi}{\partial a} = \frac{1+e}{1+e_0} \quad (1)$$

where e is the void ratio of the clay layer at (a, t) and e_0 the initial void ratio of the clay layer.

It is also evident that convective coordinate ξ and Lagrangian coordinate a are related to the settlement S of the soil layer from Figure 1:

$$\xi = a + S \quad (2)$$

1D finite strain consolidation theory

Firstly, we introduce some general equations in convective coordinates. Considering a current infinitesimal element of ABCD at t in Figure 1(b), changed from the initial element $A_0B_0C_0D_0$ at $t = 0$ in Figure 1(a) and the vertical force equilibrium of a soil particle, we obtain (Gibson et al., 1967; Yin and Zhu, 2020)

$$\frac{\partial \sigma}{\partial \xi} = \gamma_m = \frac{G_s + e}{1+e} \gamma_w \quad (3)$$

where σ is the total vertical stress, γ_m the current unit weight of saturated soil, G_s the specific gravity of the soil particle and γ_w the unit weight of water.

Based on the equilibrium of the pore water, we obtain

$$u_w = u_e + u_h \quad (4)$$

$$\frac{\partial u_w}{\partial \xi} = \frac{\partial u_e}{\partial \xi} + \gamma_w \quad (5)$$

where u_w is the total pore water pressure, u_e the excess pore water pressure and u_h the hydrostatic pressure.

Darcian law was initially observed from the seepage tests of sand, which have large interparticle voids. However, the Darcian law was not always applicable, particularly in fine grain soils (Miller and Low, 1963; Kutilek, 1964; Kutilek, 1972; Olsen, 1985). In fine soils, Non-Darcy's flow may occur due to the altered viscosity of pore water due to the influence of the soil's solid surface (Kutilek, 1964), the electrical streaming potential effect, osmosis because of chemical reactions such as those involved in aging effects (Olsen, 1985), or the change of the geometric arrangement of particles inside of the soil sample. Using a simplified non-Darcian flow relationship proposed by Indraratna et al. (2017) and taking into account the relative velocity of the fluid and solid phases in soil, we obtain

$$v = -ki^\beta \quad (6)$$

$$v = \frac{e}{1+e}(v_w - v_s) \quad (7)$$

$$\frac{e}{1+e}(v_w - v_s) = -k \left(\frac{1}{\gamma_w} \frac{\partial u_e}{\partial \xi} \right)^\beta \quad (8)$$

where v is the exit velocity, k the hydraulic conductivity, i the hydraulic gradient, β is a flow parameter, it should be noted that when $\beta=0$, Eq.(6) can degenerate into Darcy's law, and v_w and v_s the velocities of fluid and solid phases. Notably, the hydraulic conductivity k is a function of current void ratio e : $k = k(e)$.

As already mentioned, Lagrangian coordinates can simplify the solving of finite strain consolidation equations. Accordingly, general equations in Lagrangian coordinates will be developed.

The following formula has been shown to be satisfied in Lagrangian coordinates (Imai, 1995):

$$-\frac{1}{1+e_0} \frac{\partial e}{\partial t} = \frac{\partial v}{\partial a} \quad (9)$$

Incorporating Eqs.(6)–(8), Eq.(9) can be rewritten as

$$\frac{\partial}{\partial a} \left[k \left(\frac{1}{\gamma_w} \frac{\partial u_e}{\partial \xi} \right)^\beta \right] = \frac{1}{1+e_0} \frac{\partial e}{\partial t} \quad (10)$$

Based on Eqs.(1) and (5), Eq.(10) can be absolutely expressed in Lagrangian coordinates as

$$\frac{\partial}{\partial a} \left[\frac{k}{\gamma_w^\beta} \left(\frac{1+e_0}{1+e} \frac{\partial u_w}{\partial a} - \gamma_w \right)^\beta \right] = \frac{1}{1+e_0} \frac{\partial e}{\partial t} \quad (11)$$

where t is the elapsed time.

Extension of Yin–Graham EVP model for finite strain

As already noted, few 1D finite strain consolidation theories has accounted for the creep of clayey soil slurry. This section introduces the 1D Yin–Graham EVP model and its modification for finite strain consolidation modelling. Bjerrum (1967) was among the first to investigate nonlinear time–dependent stress–strain in 1D oedometer conditions. Based on his work, Yin and Graham (1989, 1994) proposed and verified the 1D Yin–Graham EVP model for time–dependent behaviour of clays,

$$\dot{\varepsilon} = \frac{\kappa}{V} \frac{\dot{\sigma}'}{\sigma'} + \frac{\psi}{V t_0} \exp \left[- \left(\varepsilon - \varepsilon_p^{ep} \right) \frac{V}{\psi} \right] \left(\frac{\sigma'}{\sigma'_p} \right)^{\lambda/\psi} \quad (12)$$

where ε and $\dot{\varepsilon}$ are the vertical strain and strain rate (compressive strain/stress is positive), σ' and $\dot{\sigma}'$ the vertical effective stress and stress rate, κ and λ the elastic and elastic–plastic compression index in Cam–clay models, t_0 and ψ the creep parameters, ε_p^{ep} the strain related to σ'_p and V the specific volume of the clay, with σ'_p having a role somewhat similar to that of pre–consolidation pressure p_c .

Eq.(12) is equivalent to

$$\dot{e} = -\kappa \frac{\dot{\sigma}'}{\sigma'} - \frac{\psi}{t_0} \exp \left(\frac{e - e_p^{ep}}{\psi} \right) \left(\frac{\sigma'}{\sigma'_p} \right)^{\lambda/\psi} \quad (13)$$

where e_p^{ep} is the void ratio related to σ'_p . σ'_p and e_p^{ep} together make a reference state point on the reference time line.

This study selects the 1D Yin–Graham EVP model, which is a representative creep model and equivalent to other kind of EVP models (Kutter and Sathialingam, 1992; Vermeer and

Neher, 1999; Yin et al., 2010). The 1D Yin–Graham EVP model can describe 1D stress or strain responses under general conditions, including multistage loading with creep straining, continuous loading and unloading or reloading. More importantly, the 1D Yin–Graham EVP model also belongs to Hypothesis B, assuming that creep contribution should be included throughout the consolidation and compression process. In addition, the model was widely adopted and verified by Yin and Graham (1996), Nash (2001), Nash and Ryde (2001), and Indraratna et al. (2018) to analyze the consolidation of soft soils with creep. However, for finite strain consolidation problems such as the self-weight consolidation of clayey slurry, the initial effective stress is small, even zero, and thus Eq.(12) is invalid because the strain will be infinite. In addition, as the deformation is negligible compared with soil thickness in finite strain problems, the relationship between vertical strain ε and current void ratio e exhibits two patterns: the first in convective coordinates,

$$\varepsilon_n = \ln \frac{1+e_0}{1+e} \quad (14)$$

where ε_n is called natural strain, and the second in Lagrangian coordinates,

$$\varepsilon_e = \frac{e_0 - e}{1 + e_0} \quad (15)$$

where ε_e is called engineering strain. To avoid these two problems, the 1D Yin–Graham EVP model must be extended for the finite strain case, as elaborated in the follows.

Firstly, two logarithmic functions, for instant time line and reference time line, together with the creep equation used in the 1D Yin–Graham EVP model, are modified by introducing a non-zero small reference value σ'_{ref} , ranged from 0 to 1 kPa in the extended 1D EVP model.

For very soft soils with high water content, the initial value of σ' is close to 0 kPa. As shown in Eqs. (16)-(17), adding σ'_{ref} to the 1D Yin–Graham EVP model can keep strain from becoming too large or even infinite, which is particularly important for very soft clay in a soil ground with zero initial effective stress at the top of the surface. Secondly, the current 1D Yin–Graham EVP model is used mainly for small strain problems, meaning that there is no difference between using strain or void ratio to express the Yin–Graham model. For finite strain consolidation, the 1D Yin–Graham EVP model is better expressed using void ratio e as a variable instead of strain, to avoid inconsistency of strain expression. According to test data from Znidarcic et al. (1984, 1986) ($3 < e < 15$ for Florida Phosphatic Clay); Fox (1996) ($2 < e < 5$ for Southern Panther Creek Clay Slurry); Abu–Hejleh et al. (1996) ($3 < e < 13$ for Florida Phosphatic Clay); Sills (1998) ($1 < e < 6$ for Ketelmeer Mud); Stark et al. (2005) ($2 < e < 10$ for Georgia Inorganic Clay); and Pu et al. (2021) ($1 < e < 7$ for Stamford Harbor Sandy Clay), we know that void ratio e and logarithm of effective stress σ' show a nearly linear relation within a large range of void ratio e (see Figure 2), confirming the reasonableness of extending the 1D Yin–Graham EVP model to finite strain analyses.

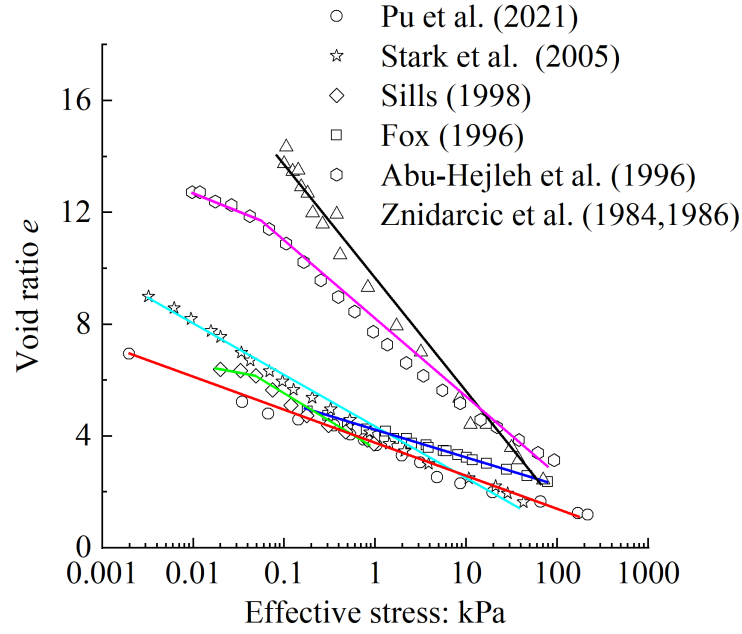


Figure 2–Void ratio–effective stress relationships of soil in high water content

Therefore, the modifications are expressed as

$$\Delta e^e = -\kappa \ln \left(\frac{\sigma'_{ref} + \sigma'}{\sigma'_{ref}} \right) \quad (16)$$

$$\Delta e^{ep} = -\lambda \ln \frac{\sigma'_{ref} + \sigma'}{\sigma'_{ref} + \sigma'_p} \quad (17)$$

$$\Delta e_{creep} = -\psi \ln \frac{t_0 + t_e}{t_0} \quad (18)$$

where σ'_{ref} is a reference non-zero small value, which can be taken as 0 to 1 kPa or checked by fitting test data at very small vertical effective stress, t_e the equivalent time in the 1D Yin–Graham EVP model.

Based on the above modifications, the 1D Yin–Graham EVP model is revised as the extended 1D EVP model for finite strain, as shown in Figure 3 and Eq.(19):

216

$$\dot{\epsilon} = -\kappa \frac{\dot{\epsilon}}{\sigma'_{ref} + \sigma'} - \frac{\psi}{t_0} \exp\left(\frac{e - e_p^{ep}}{\psi}\right) \left(\frac{\sigma'_{ref} + \sigma'}{\sigma'_{ref} + \sigma'_p}\right)^{\lambda/\psi} \quad (19)$$

217

The effective stress principle is expressed as

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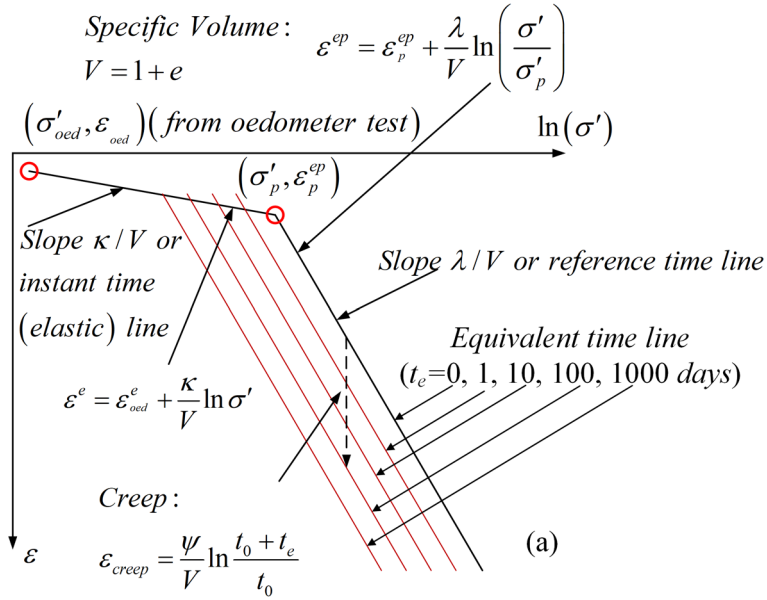
$$\sigma = \sigma' + u_w \quad (20)$$

219

Incorporating the effective stress principle, Eq.(19) could also be rewritten as

220

$$\frac{\partial e}{\partial t} = -\kappa \frac{\partial(\sigma - u_w)/\partial t}{\sigma'_{ref} + (\sigma - u_w)} - \frac{\psi}{t_0} \exp\left(\frac{e - e_p^{ep}}{\psi}\right) \left(\frac{\sigma'_{ref} + (\sigma - u_w)}{\sigma'_{ref} + \sigma'_p}\right)^{\lambda/\psi} \quad (21)$$



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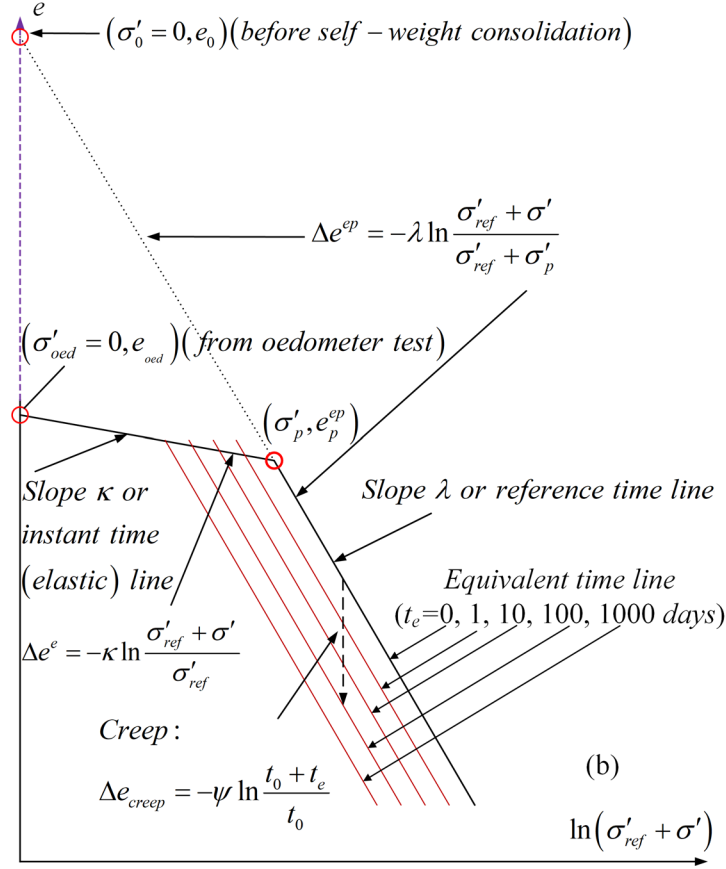


Figure 3–Existing 1D Yin–Graham EVP model and its extension for finite strain conditions: (a) Existing 1D Yin–Graham EVP model; (b) Extended 1D Yin–Graham EVP model for finite strain conditions

Extension of 1D finite strain consolidation considering creep

Equations (3), (11), and (21) can be used to solve for the three unknowns e , u_w and σ . Vertical strain ε , vertical settlement S , excess pore pressure u_e and vertical effective stress σ' can be readily calculated from the values of e , u_w and σ as rewritten in Eqs.(22)-(24) in Lagrangian coordinates

$$\frac{\partial \sigma}{\partial a} = \frac{G_s + e}{1 + e_0} \gamma_w \quad (22)$$

$$\frac{\partial}{\partial a} \left[\frac{k}{\gamma_w^\beta} \left(\frac{1 + e_0}{1 + e} \frac{\partial u_w}{\partial a} - \gamma_w \right)^\beta \right] = -m_v \frac{\partial (\sigma - u_w)}{\partial t} - g(u_w, e, \sigma) \quad (23)$$

$$\frac{1}{1+e_0} \frac{\partial e}{\partial t} = -m_v \frac{\partial (\sigma - u_w)}{\partial t} - g(u_w, e, \sigma) \quad (24)$$

where

$$m_v = \frac{\kappa / (1+e_0)}{\sigma'_{ref} + (\sigma - u_w)} \quad (25)$$

$$g(u, e, \sigma) = \frac{\psi}{(1+e_0)t_0} \exp\left(\frac{e - e_p^{ep}}{\psi}\right) \left[\frac{\sigma'_{ref} + (\sigma - u_w)}{\sigma'_{ref} + \sigma'_p} \right]^{\lambda/\psi} \quad (26)$$

Observing that $(1+e)$ occurs in the present finite strain consolidation equations, we choose an empirical power function equation used by Zeng et al. (2020), Wang et al. (2020), and Winterwerp (1999) for quantitatively describing k , expressed as

$$k = k_0 (1+e)^n \quad (27)$$

where k_0 and n are empirical coefficients. This expression is not only the same in nature as the expression of $k=C(e)^D$, which has well modelled the permeability for clay with both high and low water content (Pane and Schiffman, 1997; Berilgen et al., 2006; Dolinar, 2009), but also inherits the advantages of the expression of $k = k_0(1+e)$ used by Been and Sills (1981), making it helpful for simplifying the consolidation equations by eliminating separate $(1+e)$ terms.

We usually want to obtain the excess pore water pressure u_e directly, so here we introduce

$$\sigma = \sigma_{ah} + u_h \quad (28)$$

where σ_{ah} is the total stress above hydrostatic pressure.

Also, combining Eqs.(4), (20), and (28), we can express effective stress by

$$\sigma' = \sigma_{ah} - u_e \quad (29)$$

Using u_e , σ_{ah} instead of u_w , σ respectively, Eqs.(22)–(26) could be rewritten as

$$\frac{\partial \sigma_{ah}}{\partial a} = \frac{G_s - 1}{1 + e_0} \gamma_w \quad (30)$$

$$\frac{\partial}{\partial a} \left[\frac{k}{\gamma_w^\beta} \left(\frac{1 + e_0}{1 + e} \frac{\partial u_e}{\partial a} \right)^\beta \right] = -m_v \frac{\partial (\sigma_{ah} - u_e)}{\partial t} - g(u_e, e, \sigma_{ah}) \quad (31)$$

$$\frac{1}{1 + e_0} \frac{\partial e}{\partial t} = m_v \frac{\partial u_e}{\partial t} - g(u_e, e, \sigma_{ah}) \quad (32)$$

where

$$m_v = \frac{\kappa / (1 + e_0)}{\sigma'_{ref} + (\sigma_{ah} - u_e)} \quad (33)$$

$$g(u_e, e, \sigma_{ah}) = \frac{\psi}{(1 + e_0) t_0} \exp \left(\frac{e - e_p^{ep}}{\psi} \right) \left(\frac{\sigma'_{ref} + (\sigma_{ah} - u_e)}{\sigma'_{ref} + \sigma'_p} \right)^{\lambda/\psi} \quad (34)$$

which is to say that Eqs.(22)–(26) or Eqs.(30)–(34) formulate a 1D finite strain consolidation theory that takes into account soil creep. It should be noted that permeability k is expressed by Eq.(27).

Note that the above modifications and derivations are also applicable to other elastic visco-plastic models of soft clays (e.g. Stolle et al., 1999; Kim and Leroueil, 2001; Yin and Wang, 2012). Imai (1981) summarized that the slurry in a settling column experiences three stages: flocculation stage, settling stage, and consolidation stage. The proposed consolidation model of this study is to capture the settling of soil particles already with inter-particle contact forces and self-weight consolidation behaviour in the last two stages since the effective stress begins to develop. In addition, it also should be noted that the proposed finite strain

consolidation model can capture the creep characteristics of clayey soil once the consolidation process started no matter how long or how short the process of self-weight consolidation.

Finite difference method–based solution

The proposed 1D finite strain consolidation model with consideration of creep for clayey soil consists of a set of highly nonlinear partial differential equations. Yin and Graham (1996) and Zhu and Yin (1999) used the finite difference method and finite element method to solve a set of nonlinear partial differential equations. In this section, the Crank–Nicholson finite difference program, much as in Yin and Graham (1996), is formulated to solve Eqs.(30)–(34) due to its good stability and accuracy. The Crank–Nicolson finite difference method combines the forward Euler and the backward Euler methods. The central difference is used in space to reduce the calculation error.

Performing the difference in time and space, as shown in Figure 4, for Eqs.(30)–(32), we obtain

$$(\sigma_{ah})_{i+1,j} = (G_s - 1) / (1 + e_0) \gamma_w \Delta a + (\sigma_{ah})_{i,j} \quad (35)$$

$$A_{i,j} u_{i-1,j+1} - (2A_{i,j} + 1) u_{i,j+1} + A_{i,j} u_{i+1,j+1} = (2A_{i,j} - 1) u_{i,j} - (A_{i,j} + B_{i,j}) u_{i+1,j} + (B_{i,j} - A_{i,j}) u_{i-1,j} - C_{i,j} \quad (36)$$

$$e_{i,j+1} = \left\{ (m_v)_{i,j} \times (u_{i,j+1} - u_{i,j}) - \Delta t \times [g(u_e, e, \sigma_{ah})]_{i,j} \right\} \times (1 + e_0) + e_{i,j} \quad (37)$$

where the subscripts i (0, 1, ..., N) represent variation in depth, described by the a coordinate, and the subscripts j (0, 1, ..., M) represent variation in the time t coordinate, with

$$A_{i,j} = r_{i,j} \beta (1 + e)_{i,j}, \quad B_{i,j} = r_{i,j} (n - \beta) (e_{i+1,j} - e_{i,j}), \quad C_{i,j} = \Delta t [g(u_e, e, \sigma_{ah}) / m_v]_{i,j},$$

287 $r_{i,j} = (C_v)_{i,j} \frac{\Delta t}{2(\Delta a)^2}$, and $C_v = \frac{k_0(1+e_0)^\beta (1+e)^{n-\beta-1}}{\gamma_w^\beta} \left(\frac{\partial u_e}{\partial a} \right)^{\beta-1}$. The depth increment

288 $\Delta a = a_{i+1} - a_i$ has been kept constant, but the time increment $\Delta t = t_{j+1} - t_j$ has been allowed to
 289 increase with the development of consolidation for good convergence in the early stage of the
 290 consolidation, as well as quicker calculation in the later stage.

291 Figure 4 also shows the boundaries in Lagrangian coordinates. The initial thickness of the
 292 soil layer is a_0 . Assuming that the top of the soil layer ($a = 0$) is freely draining, the pore water
 293 pressure u_w will be constant and the excess pore water pressure u_e will be zero. At the bottom
 294 of the soil, we consider an undrained condition that can be expressed by $\partial u_e / \partial a = 0$. The
 295 complex drainage boundaries (e.g., Mei et al., 2022; Feng et al., 2020; Feng et al., 2019; Mei et
 296 al., 2014) are not considered here. Because the effective stress is zero at the top of the soil layer
 297 during the process, the initial void ratio at $a = 0$ is $e = e_0$. These boundaries can be expressed
 298 in a finite difference form:

299
$$(u_e)_{0,j} = 0 \text{ (where } j=0, 1, 2, 3, \dots, M)$$
 (38)

300
$$(u_e)_{n,j} - (u_e)_{n-1,j} = 0 \text{ (where } j=0, 1, 2, 3, \dots, M)$$
 (39)

301
$$(e)_{0,j} = e_0 \text{ (where } j=0, 1, 2, 3, \dots, M)$$
 (40)

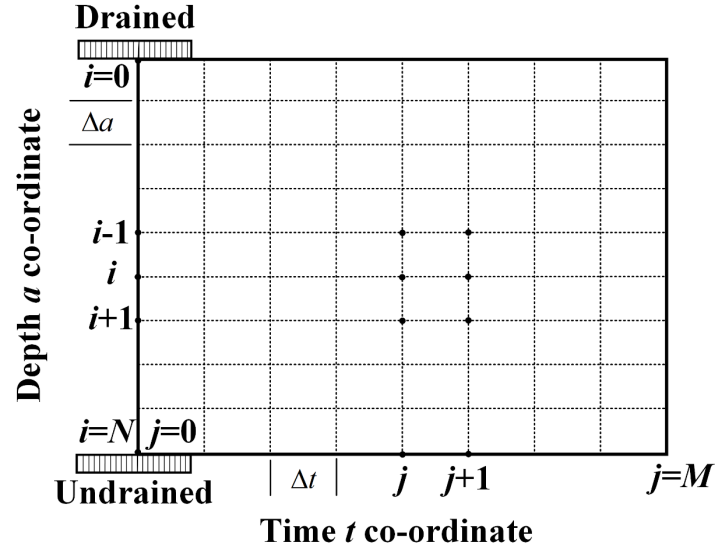


Figure 4–Finite difference grid and boundaries

Settlement can be calculated as

$$S = \int_0^a \left(\frac{e - e_0}{1 + e_0} \right) da \quad (41)$$

An approximate numerical solution of Eq.(41) can be written

$$S_j = \left[0.5 \left(\frac{e - e_0}{1 + e_0} \right)_{0,j} + 0.5 \left(\frac{e - e_0}{1 + e_0} \right)_{n,j} + \sum_{i=1}^{n-1} \left(\frac{e - e_0}{1 + e_0} \right)_{i,j} \right] \Delta a \quad (42)$$

For the case of self-weight consolidation, initial consolidation excess pore water distributes linearly along the height of soil column and the effective stress is zero. By combining equations (29) and (30), the initial excess pore water pressure is obtained, which distributes linearly along the height of soil column with the constant slope of $(G_s - 1)\gamma_w / (1 + e_0)$. The distribution can be expressed as a function of depth as follow:

$$u_e(a, 0) = \frac{(G_s - 1)}{(1 + e_0)} \gamma_w a \quad (43)$$

The proposed consolidation model was rigorously derived. Therefore, it is suitable for simulating the consolidation process with surcharge or fill. The surcharge can be time-dependent or constant, which can be applied through adjusting the total stress expression. Further, the numerical algorithm follows the Crank–Nicholson finite difference method which is flexible in setting the boundary and initial conditions. The proposed model can also be used to describe the case of double-sided drainage by adjusting the boundary conditions (e.g., Eqs. (38) and (39)). It is convenient to set the initial conditions by giving different void ratios or initial stress to simulate the case with a non-uniform void or stress distribution.

Application in self-weight consolidation case studies

Many settling columns have been used to research settling and consolidation behaviour. Been and Sills (1981) was the first to set a series of laboratory model tests of the development and consolidation of soft soil in settling columns, with measurement of density (using an accurate, non-destructive X-ray technique), total stress, pore pressure and surface settlement. A workshop “Sidere” was held in Oxford in 2000, in which four self-weight tests were carried out and the four of the test results were presented to participants for them to predict the results of the fifth experiment for which only the initial condition was known (Barhtolomeeusen et al., 2002). Alexis et al. (2004) provided a more accurate follow-up on densities and interstitial excess pore pressures. Bonin et al. (2014) presented a detailed examination of thickened tailings undergoing self-weight consolidation and a slurry consolidometer obtaining the compressibility relationship under low vertical effective stresses.

In this paper, the Experiment 15 reported by Been and Sills (1981) and Sidc 1 and Sidc 3 reported by Barhtolomeeusen et al. (2002), here called Case 1 and Case 2, respectively, were

chosen to verify proposed 1D finite strain theory for self-weight consolidation with creep. Case 1 used a larger initial void ratio than Case 2. In addition, Case 1 consisted of only Class C prediction, while Case 2 included both Class C and Class A predictions, with Class A prediction defined as the prediction made before construction based entirely on data available at that time, and Class C made after the occurrence of the event being predicted (Lambe, 1973). For simplicity, it is assumed that the flow of water in all case studies follows Darcy's law, i.e. $\beta = 1$.

Case Study 1

Experiment 15 reported by Been and Sills (1981) was chosen as Case 1 to verify the proposed 1D finite strain theory for two reasons: (1) more detailed data were provided than in other consolidation experiments, and (2) the setting column in this experiment underwent less settling suspension and self-weight consolidation from a state with high water content and a large void ratio. In Case 1, a natural soil from the River Parrett estuary was used that has a liquid limit (w_L) of 53%, plastic limit (w_P) of 25%, plasticity index (I_P) of 28%, and specific gravity of 2.66. The clay content (CI) of the soil is approximately 30%. The initial height and unit weight of the soil column are 0.643 m and 11.2 kN/m³, respectively. Further details of Case 1 are presented and discussed in Been (1980) and Been and Sills (1981).

All parameters in current 1D finite strain theory should have been determined by using the detailed procedure proposed by Yin and Graham (1994), according to the oedometer test data. For the extremely soft soil with high initial water content, Hong et al. (2010) modified the conventional oedometer apparatus using a light loading cap and two weight hangers system which can be used to perform tests on the soil with initial water content that is double of liquid limit allowing a small load (e.g. 0.5 kPa). Furthermore, Xu et al. (2015) improved the

conventional oedometer apparatus by a leverage for the soil sample with high initial water content to 4.4 times of the liquid limit allowing a small load (e.g. 0.1 kPa). For clay with water content higher than 4.4 times of its liquid limit, the settling column test is an effective method to investigate the compression behaviour with very high void ratio and very small effective stress. However, due to the absence of oedometer test data for the soil used in Been and Sills (1981), a simple approximation method for determining all parameters of the 1D finite strain theory is proposed.

Firstly, the reference stress σ'_{ref} can be obtained from stress–strain curves of Experiment 15 (Fig. 13, Been and Sills, 1981). Herein, by fitting the test data at the range of small effective stress (Been and Sills, 1981; Znidarcic et al. 1984, 1986; Fox, 1996; Barhtolomeeusen et al., 2002; Abu–Hejleh et al., 1996; Sills, 1998; Stark et al., 2005, Pu et al., 2021), an exponential relationship between the initial void ratio e_0 and the reference stress σ'_{ref} is assumed:

$$\sigma'_{ref} = 0.0193 \exp(-0.3e_0) \quad (44)$$

The Eq.(44) shows higher void ratio gives smaller reference effective stress, which is reasonable.

The compression index C_c can then be obtained by drawing a best–fit line through the stress–strain data of Experiment 15 (Fig. 13, Been and Sills, 1981). The ratio between unloading–reloading compression index C_r and compression index C_c is approximately $1/5 \sim 1/10$ by many researchers, and $C_r = C_c/6$ is adopted in this paper. The most straightforward method of obtaining the creep index is by laboratory tests. However, we usually

did not have detailed test data to get it. Therefore, the practical methods of determining the creep index are summarized to provide helpful reference for the corresponding application. One approach is using the empirical formula for the creep coefficient based on physical parameters of soils (Zhang et al., 2020; Jin et al., 2019; Yin, 1999). Another simple method is to calculate the creep index C_α by a typical ratio of creep index C_α to the compression index C_c based on soil classification and components (Mesri and Castro, 1987; Mitchell and Soga, 2005). The creep index C_α is obtained using the recently proposed correlation (Jin et al., 2019)

$$\ln(C_\alpha) = \left(0.3114 \frac{I_p^2}{CI} - 0.1229 \frac{1}{I_p^2} + 0.6455 \frac{1}{I_p} \right) e - 5.1308 \quad (45)$$

where I_p is the plasticity index and CI the clay content.

According to Yin and Zhu (2020), we can calculate current 1D finite consolidation model parameters κ , λ , ψ and t_0 using the approximate relationships

$$\begin{cases} \kappa \approx \frac{C_r}{\ln(10)} & \lambda \approx \frac{C_c}{\ln(10)} & \psi \approx \frac{C_\alpha}{\ln(10)} \\ t_0 = 1440 \text{ min} \end{cases} \quad (46)$$

The parameter t_0 is chosen as 1400 min or 1 day corresponding to a loading duration of conventional oedometer test, which is also widely adopted in other creep models (e.g. Kutter and Sathialingam, 1992; Leoni et al., 2008; Yin et al., 2010). It is a relatively simple and practically useful way. Using Eq.(27) to draw a best-fit line through the permeability profiles of Experiment 15 (Fig. 15, Been and Sills, 1981), the relationship between permeability and void ratio can be expressed by $k = 1.38 \times 10^{-10} (1 + e)^{4.03}$. Finally, $e_{oed} = 1.5$ is assumed, where the e_{oed} is the initial void ratio of the soil sample, which is used to obtain the point (σ'_p, e_p^{ep}) in

the reference time line in Figure 3(b). All the parameters of current 1D finite strain theory are obtained from Experiment 15 (Been and Sills, 1981) as shown in Table 1.

Table 1 Compression and consolidation parameters used in the case studies

Parameters	Case Study 1	Case Study 2	
	(Been and Sills, 1981)	(Barhtolomeeusen et al., 2002)	
	Experiment 15,	Sidc 1	Sidc 3
e_0	9.87	2.48	2.09
λ	1.02	0.2	0.15
I_P	28%	11%	11%
CI	30%	10%	10%
t_0 / min	1440	1440	1440
σ'_p / kPa	8.70	0.99	1.65
$k_0 / (\text{m} / \text{s})$	1.38×10^{-10}	8.85×10^{-12}	5.59×10^{-12}
n	4.0	9.7	10.6

As proposed in Carrier et al. (1983), Sills (1998) and Stark et al. (2005), consolidation occurs only when a continuous soil structure is developed. The corresponding void ratio and density are called initial void ratio and structural density, respectively. According to Carrier et al. (1983), the initial void ratio used to distinguish the end of sedimentation and the beginning of consolidation can be obtained using the correlation $e_0 = 0.07 G_s (100 w_L)$. The initial void ratio of Experiment 15 (Been and Sills, 1981) determined by the preceding correlation is 9.87 (7.5 times the void ratio at the liquid limit), and the corresponding structural density of soil is 11.3 kN/m^3 , which is higher than the initial density, 11.2 kN/m^3 , of soil in Experiment 15 (Been and Sills, 1981). Thus soil consolidation did not start at the beginning of the experiment, and a little settling suspension was seen.

In Experiment 15 (Been and Sills, 1981), the top of the soil column ($a = 0$) is permeable and the bottom of soil column ($a = a_0$) impermeable. Accordingly, the boundary conditions can be simply stated mathematically with respect to excess pore water pressure u_e as

$$u_e(0, t) = 0 \quad (47)$$

$$\frac{\partial u_e}{\partial a}(a_0, t) = 0 \quad (48)$$

At the beginning of self-weight consolidation, the initial excess pore water pressure distribution can be obtained from Eq.(43):

$$u_e(a, 0) = 1.497a \quad (49)$$

To verify the accuracy of current 1D finite strain theory, Case 1 is reproduced using the Crank–Nicholson finite difference program already proposed. The predicted surface settlement, total stress, excess pore water pressure, void ratio and density profiles are obtained.

Figure 5 shows the predicted and measured surface settlement of Case 1 at different periods. It can be seen that the predicted settlement under current 1D finite strain theory captures the progress of self-weight consolidation in the experiment and the surface settlement profile quite well.

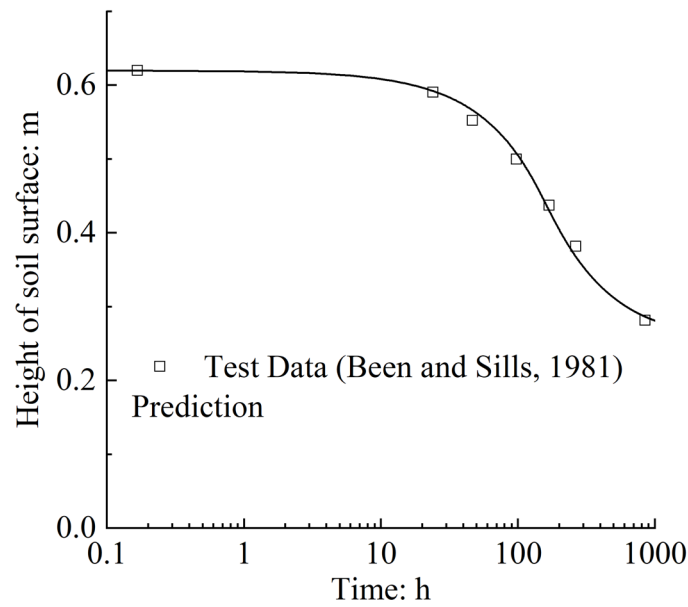
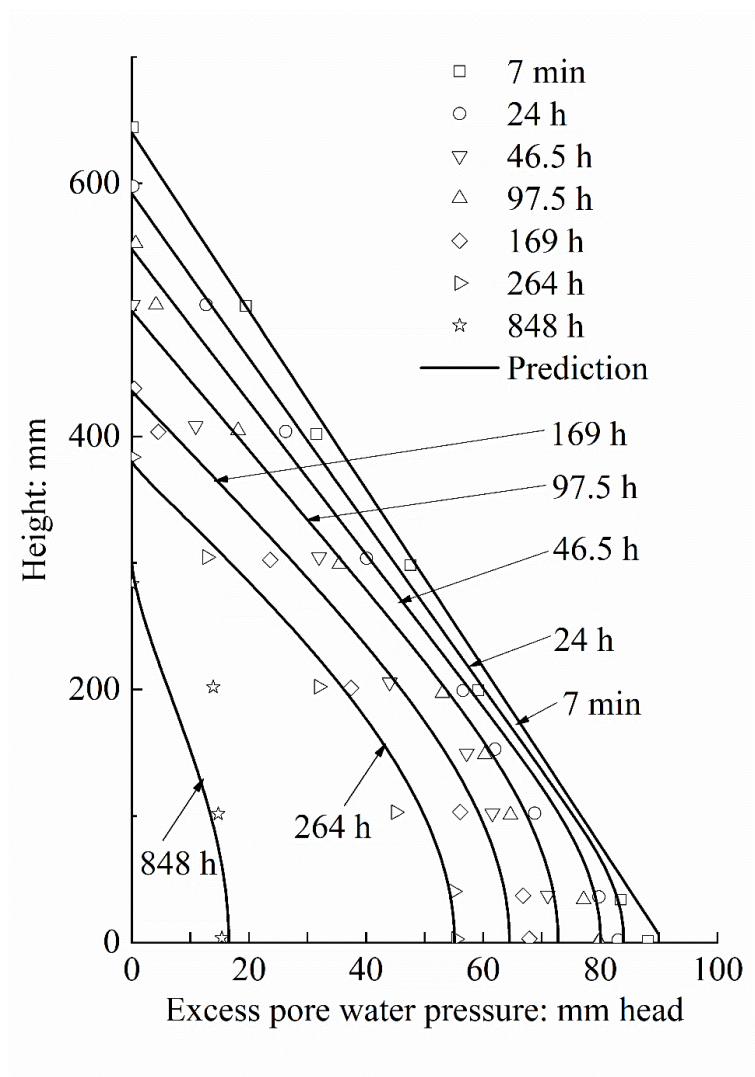


Figure 5—Predicted and measured settlement, experiment 15 (Been and Sills, 1981)

The settlement profile is highly significant in reclamation projects, but the density profile and pore pressure dissipation process are also of great interest in a riverbed design and resuspension of mine tailings, because the strength of a soil bed depends on the effective stress and void ratio – that is, successful modelling of the self-weight consolidation process must include prediction of not only surface settlement but also excess pore pressure and void ratio profile. Figure 6 and Figure 7 show the excess pore water pressure and total stress profiles determined from the finite strain modelling and observed in Case 1. According to Figure 6, the excess pore water pressure distributed linearly along the height of the soil column at the beginning of the consolidation process and the slope is $(G_s - 1)\gamma_w / (1 + e_0)$. With time, the excess pore water pressure gradually dissipates upwards from the bottom of the soil column, a phenomenon also observed by Bartholomeeusen et al. (2002) and Bonin et al. (2014). The excess pore water pressure distributes linearly along the depth. The maximum excess pore

440 water pressure is in the bottom of the soil column. The predictions concerning the excess pore
 441 water pressure dissipation process by the finite strain model agree well with the measured data,
 442 confirming the reliability of the finite strain theory. Figure 7 also verifies that the current 1D
 443 finite strain theory simulates the total stress on the soil column base in the consolidation process
 444 well. Based on Figure 7, the total stress of the column base is unchanged during the self-weight
 445 consolidation process, with no water or soil discharged during the whole experiment, and thus
 446 agrees well with the phenomenon observed in the experiment.



447

448 Figure 6–Predicted and measured excess pore water pressure, experiment 15 (Been and Sills, 1981)

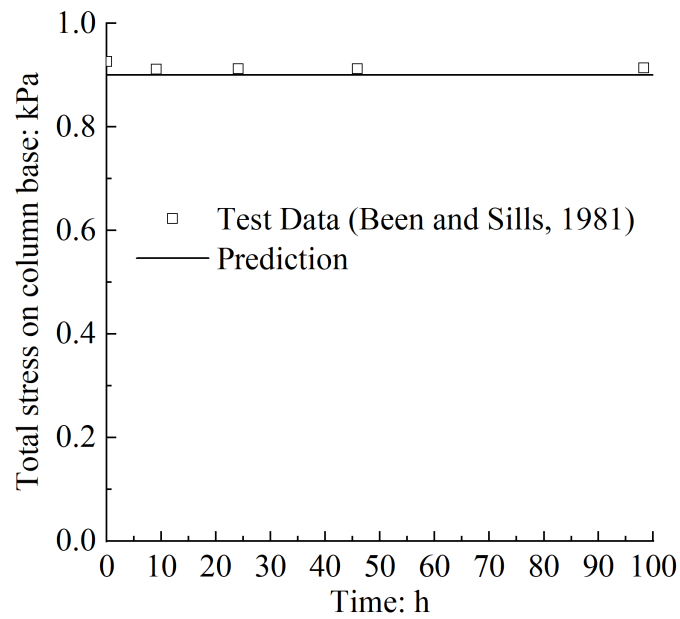


Figure 7—Predicted and experimental total stress on column base, experiment 15 (Been and Sills, 1981)

In addition, Figure 8 shows a comparison between the measured and predicted void ratio distribution with the consolidation process, which is equivalent to the density profile (Figure 9). Based on the void ratio distribution time and space, self-weight consolidation can cause redistribution of the void ratio along the soil column height. In the self-weight consolidation process, the void ratio of the soil column base decreases firstly and the other portion stays at the initial void ratio unless the self-weight consolidation begins. Theoretically speaking, this phenomenon is reasonable. The above evolution of void ratio in the self-weight consolidation process is consistent with the results reported by Pu et al. (2018) and Bonin et al. (2014). There is discrepancy that measured void ratios at different positions in the soil column did not keep constant at the early consolidation stage as predicted, which can be possibly explained as follows: (1) the existing inhomogeneity of the dredged mud in the experiment was ignored in the simulation where the initial void ratio of the whole soil column was assumed homogenous,

as reported by Been and Sills (1981); (2) the experimental stress and strain curves provided by Been and Sills (1981) are dispersal, leading to inaccurate model parameters. Under a high initial void ratio, a small change of effective stress may correspond to a big change of void ratio, which may also cause the deviation between the predicted and experimental values. It should be noted, however, that the predicted density file is somewhat different from that observed during the early stages of self-consolidation, especially near the bottom of the soil column. However, as self-consolidation develops, the predicted and measured densities show better agreement, possibly because the soil sample used in the self-weight experiment was taken from natural soil. During the early stages of self-consolidation, the density distribution is less uniform compared with its state when the structure of the soil layer increases. Overall, all prediction profiles confirm the reliability of the finite strain theory proposed in this paper for simulating self-consolidation of soft soil.

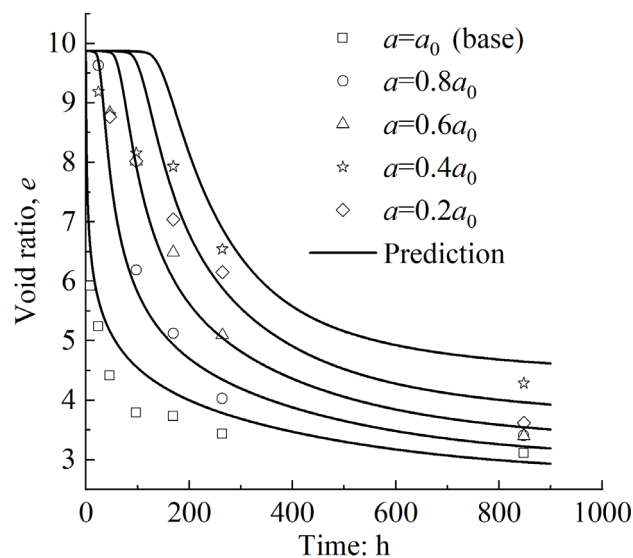


Figure 8—Comparison between predicted and experimental void ratio, experiment 15 (Been and Sills, 1981)

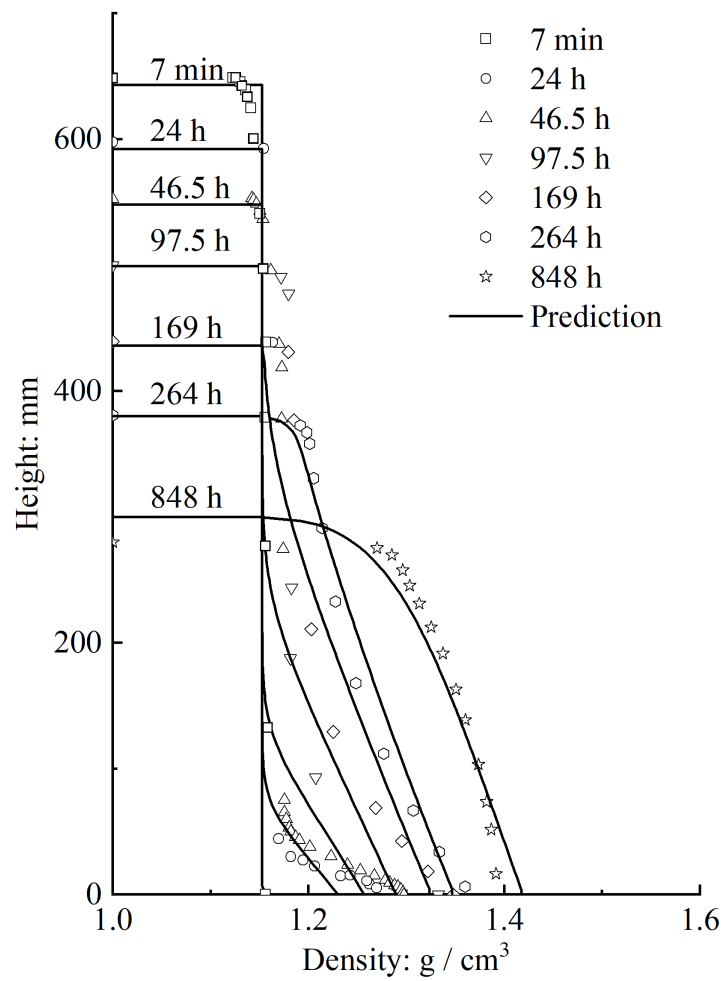


Figure 9–Predicted and measured density, experiment 15 (Been and Sills, 1981)

Furthermore, the comparison between the measured and predicted settlement and excess pore water pressure with and without considering creep was also conducted. From Figure 10, it can be seen that the predicted settlement with creep is almost consistent with that without considering creep within the first 200 h. However, the deviation of settlement curve with and without creep begins to appear after 200 h. The difference becomes larger with time going by. It also should be noted that the simulation with creep better captures the development trend of settlement in the monitoring period than that without considering creep. The predicted

settlement without considering creep is kept stable after about 1000 h, which underestimates the settlement. Figure 11 shows the excess pore water pressures predicted using the proposed model with and without the creep. The dissipation rate of excess pore water pressure is found overestimated if the creep of clay is not considered which is consistent with the analysis reported by Liu et al. (2018) and Indraratna et al. (2018). This phenomenon may be due to the deformation caused by soil creep, which reduces the dissipation of pore water when the same settlement occurs.

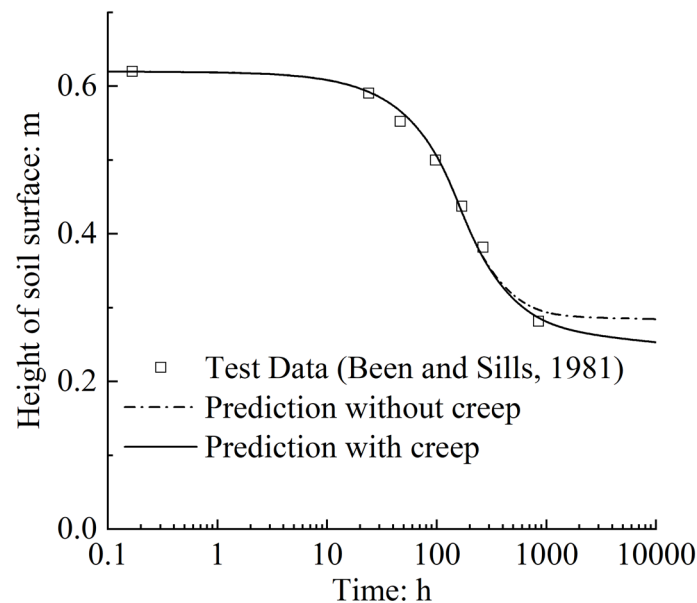


Figure 10–Comparison between measured and calculated settlements with and without considering creep for the experiment 15 of Been and Sills (1981)

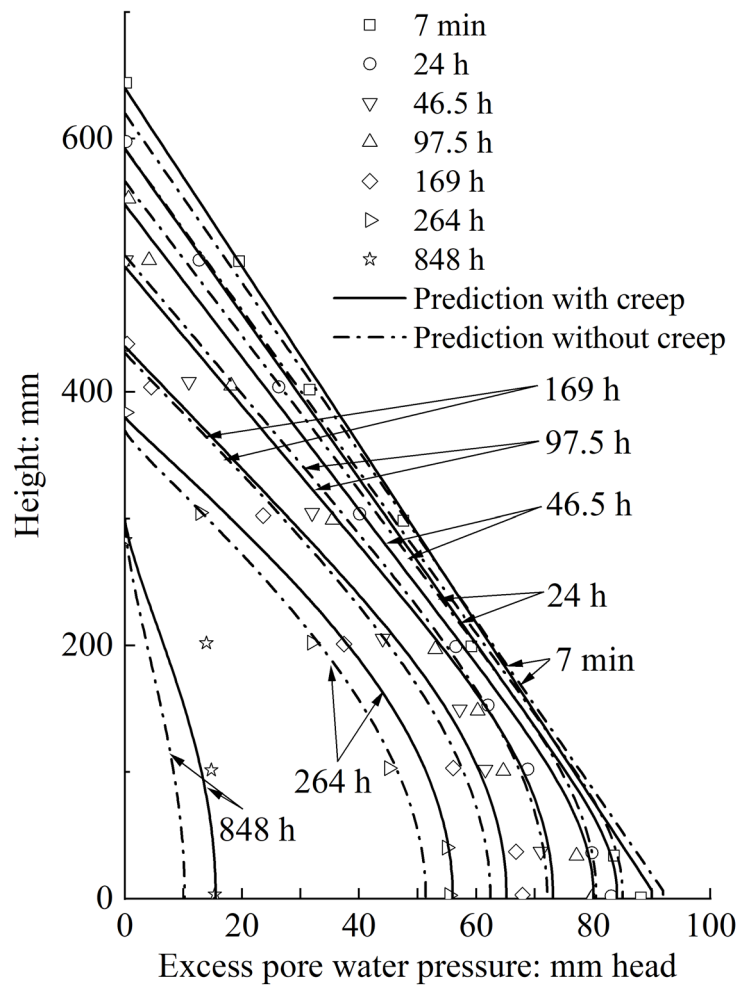


Figure 11–Comparison between measured and calculated excess pore water pressures with and without considering creep for the experiment 15 of Been and Sills (1981)

Case Study 2

Sidc1 and Side3, reported by Bartholomeeusen et al. (2002), were chosen as Case 2 to verify the reliability when the initial void ratio (2 times void ratio at the liquid limit) is relatively smaller than in Case 1 (7.5 times void ratio at the liquid limit). In addition, Case 2 consisted of not only Class A but also Class C prediction and has been simulated by many researchers, such as Hawlader et al. (2008), whose prediction results are compared with those by other researchers. In Case 2, the soil was collected from the river Schelde in Antwerpen, which has

a liquid limit of 39%, plastic limit of 28%, plasticity index of 11%, and specific gravity of 2.72. The clay content of the soil is less than 10%. The boundary conditions of Case 2 are the same as in Case 1. Further details of Case 2 are presented and discussed in Bartholomeeusen et al. (2002). The method used to determine the parameters in current 1D finite strain theory is the same as in Case Study 1 and the parameters are shown in Table 1.

Hawlder et al. (2008) developed a state-dependent model of the compressibility behaviour of soft clay sediments at a low effective stress level, which can capture the feature whereby stress-strain relationships are not unique but rather depend on the state of the element concerning its location (Been and Sills, 1981; Bartholomeeusen et al., 2002). However, the constitutive model proposed by Hawlder et al. (2008) is a complete empirical formula and is not related to the conventional compressibility indexes. Because the time required to attain the same effective stress through pore water pressure dissipation differs for soil elements along the depth, Been and Sills (1981) and Sills (1995) have attributed differences in the stress-strain relationships of different locations to creep of clayey soil. Accordingly, Case 2 is used to verify that present model can also simulate the self-weight consolidation well, even better than described in Hawlder et al. (2008), by considering the creep of clayey soil.

Figure 12 compares the predicted surface settlement by the proposed model and Hawlder et al. (2008) and measured surface settlement for Case 2, showing that the predicted settlement under both current 1D finite strain theory and Hawlder can capture the surface settlement profile of Side 3 in Case 2 quite well. However, for the Class C prediction, Side 1, Hawlder et al. (2008) clearly overestimated the settlement, predicting that the majority of the settlement would be almost completed after 7 days. But the present model prediction results indicate that

settlement of Side 1 is still on-going at 7 days, as the experiment showed. Thus the present model's prediction of settlement during the self-weight consolidation process (small effective stress range) equals or exceeds Hawlader et al. (2008)'s results.

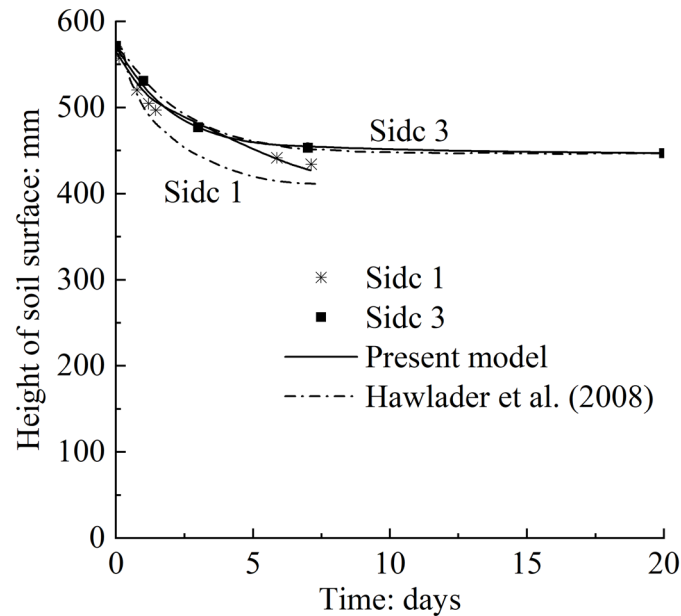


Figure 12–Predicted and measured settlement, Side 1 and Side 3 (Barhtolomeeussen et al., 2002; Hawlader et al., 2008)

Figure 13 compares the measured and predicted excess pore water pressure distribution during the consolidation process, showing that the settlement predicted by both current 1D finite strain theory and Hawlader et al. (2008) reflects the surface settlement profile of Side 3 in Case 2 quite well during early stages 0–1 d. However, as self-weight consolidation time lengthened, Hawlader et al. (2008) predicted quicker excess pore water pressure dissipation process than seen in experimental observations. Current 1D finite strain theory thus simulated the excess pore water pressure dissipation process better.

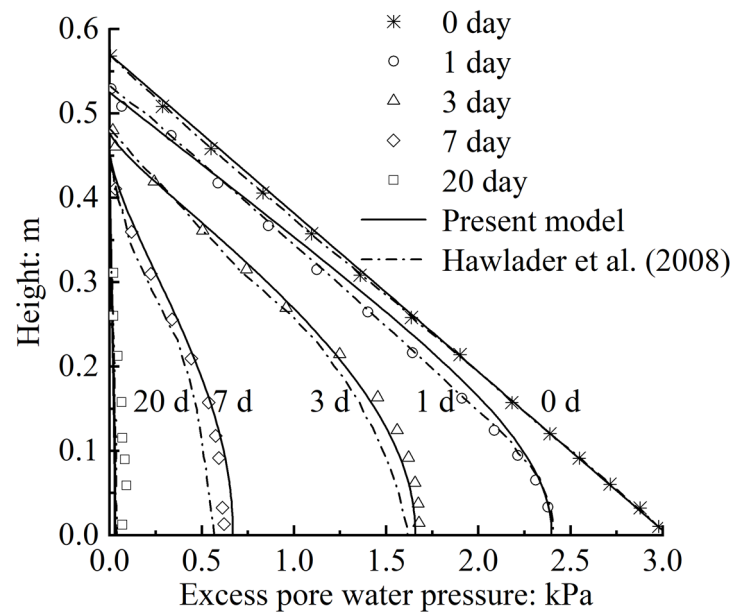


Figure 13—Predicted and measured excess pore water pressure, Side 3 (Barhtolomeeussen et al., 2002; Hawlader et al., 2008)

Figure 14 compares the measured and predicted density distribution during the consolidation process, indicating that the density distribution predicted by the present model was better than the prediction made by Hawlader et al. (2008) in the bottom zone but worse in the top zone, where the effective stress is relatively small. This phenomenon occurs because the state-dependent constitutive model proposed by Hawlader et al. (2008) can describe the stress-strain relationship better than the current model in the extreme effective stress range.

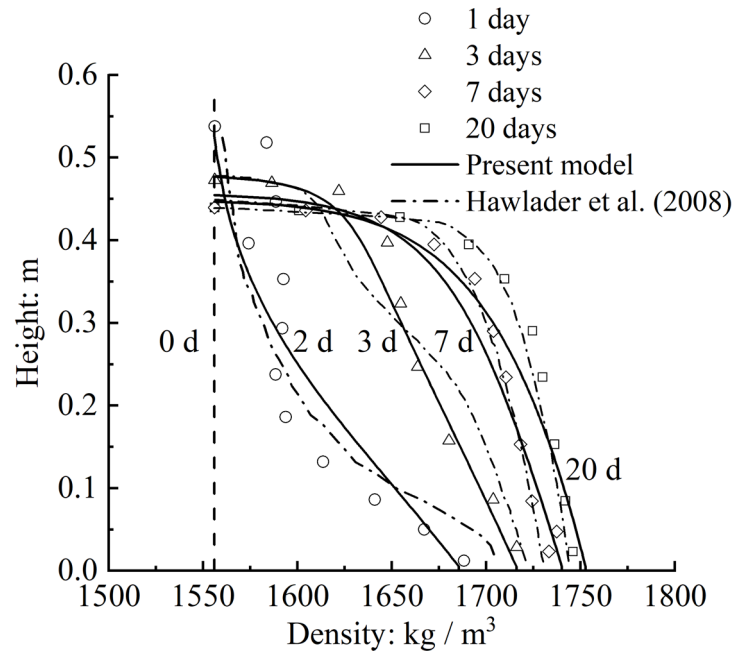


Figure 14—Predicted and measured density, Side 3 (Barhtolomeeusen et al., 2002; Hawladar et al., 2008)

The preceding numerical modelling case studies mainly reflect Class A prediction, which might be not as convincing as Class C. Accordingly, Side 1 of Case Study 2 is also simulated to verify the reliability of current 1D finite strain theory that takes into account creep for Class C predictions. Figure 15 and Figure 16 show the settlement and predicted density distribution of Side 1. The predicted results were compared with the test data of Side 1 and the best numerical simulation results obtained by Znidarcic and Winterwerp at the Sidere held in Oxford in 2000. The better agreement with test data than in Znidarcic and Winterwerp's results further illustrates the reliability of present 1D finite strain theory for Class C prediction.

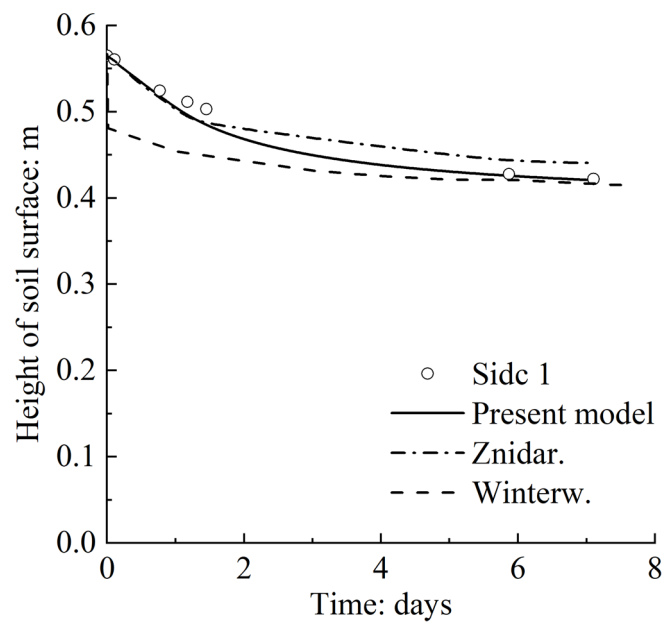


Figure 15–Predicted and measured settlement, Side 1 (Barhtolomeeusen et al., 2002)

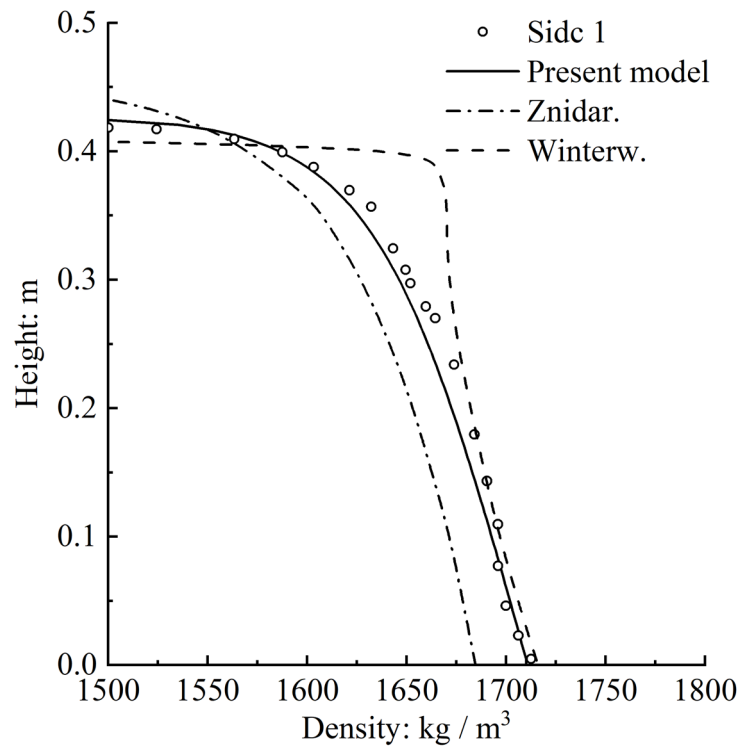
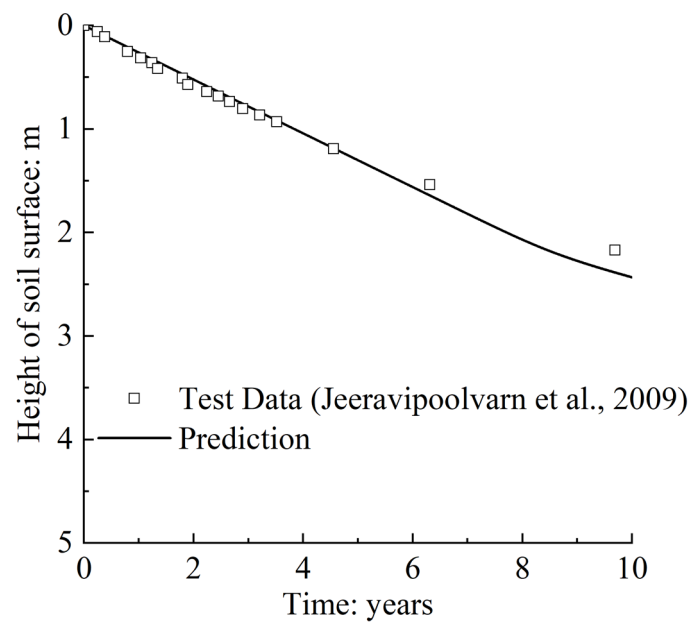


Figure 16–Predicted and measured density, Side 1 (Barhtolomeeusen et al., 2002)

Case Study 3

In this section, a 10 m standpipe test reported by Jeeravipoolvarn et al. (2009) was selected as an application example to further demonstrate the applicability of the proposed model. The tailings properties used in the standpipe test 1 were found from Jeeravipoolvarn et al. (2009). The typical value $t_0 = 1$ day was simply taken. The $\psi=0.041$ was obtained by fitting. From the standpipe test 1, the Figure (18) reported by Jeeravipoolvarn et al. (2009) was used to determine the values of κ and λ as 0.142 and 1.030, respectively. The point (σ'_p, e_p^{ep}) was determined as (0.33 kPa, 4.93). In addition, using Eq.(27) to draw a best-fit line through the permeability profiles of the standpipe test 1, the relationship between the permeability coefficient and void ratio was determined by $k=4.32 \times 10^{-7}(1+e)^{4.93}$ m/day. All parameters were used to predict the self-weight consolidation. Figure 17 and Figure 18 present the comparison between the simulated and observed settlement and excess pore water pressure respectively. The total consolidation time is about 10 years. Good agreement between measurements and predictions for both settlement and excess pore water pressure was obtained.



579

580 Figure 17–Comparison between measured and predicted settlement for standpipe 1 of Jeeravipoolvarn et

581 al. (2009)

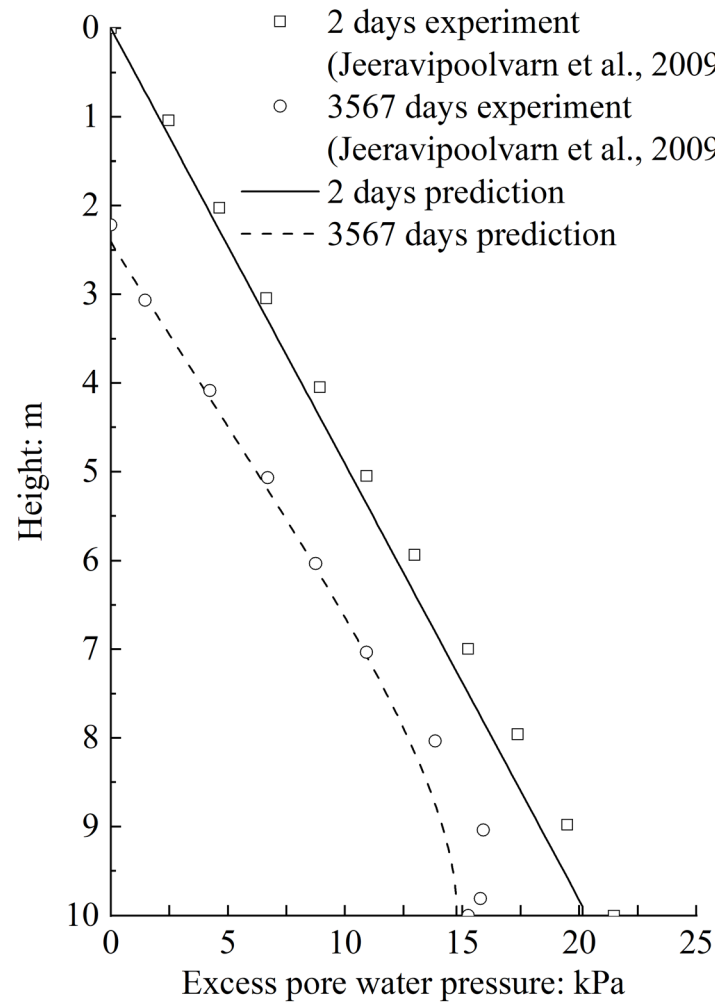


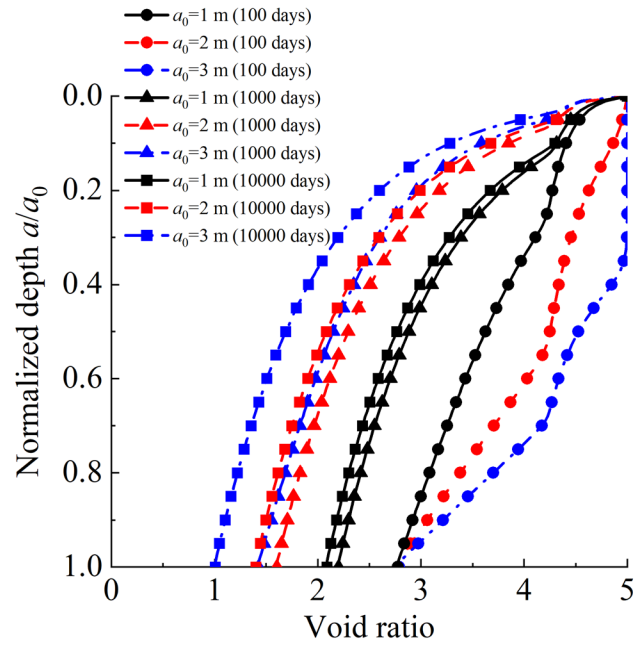
Figure 18–Comparison between measured and predicted excess pore water pressure for standpipe 1 of Jeeravipoolvarn et al. (2009)

Parametric studies

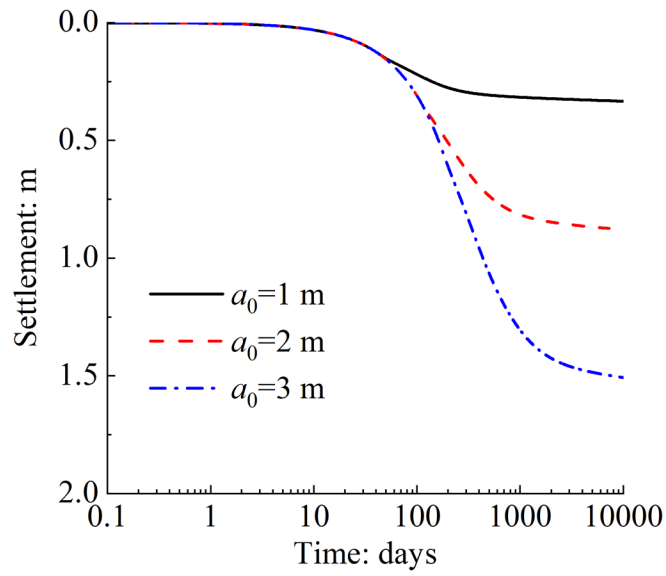
The initial height and initial void ratio are generally recognized as state variables that influence the self-weight consolidation of soil (Been and Sills, 1981; Bonin et al., 2014; Zhang et al., 2022). They are also critical design parameters in an engineering project, and thus are chosen for parametric study with other parameters kept constant. The values of κ , λ , and ψ are

0.2, 1, and 0.04, respectively. The typical value $t_0 = 1$ day was simply taken. The point (σ'_p, e_p^{ep}) was taken as (0.2 kPa, 5). The permeability coefficient function was $k=8.64 \times 10^{-6}(1+e)^4$ m/day.

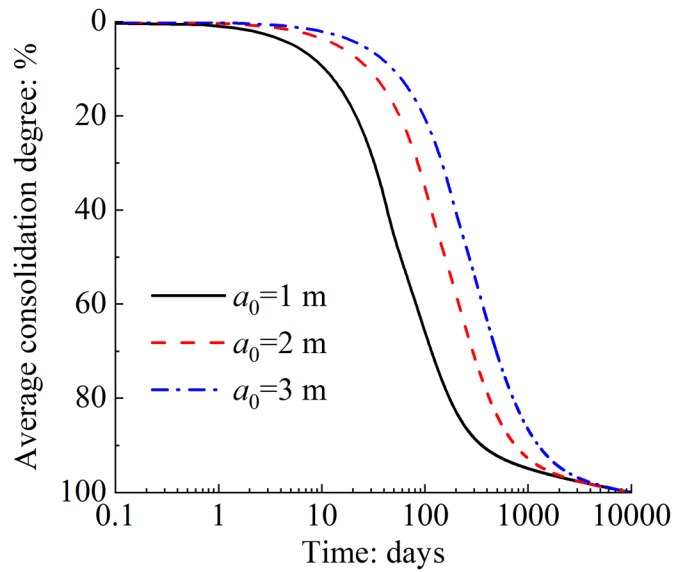
Figures 19(a)-(c) show the influence of the soil's initial height (i.e. 1 m, 2 m and 3 m, all with initial void ratio of 5) on the self-weight consolidation process. Interestingly, the soil column with the smallest initial height has the smallest void ratio at the same relative depth on the 100th day of self-weight consolidation. This is because the consolidation is faster when the initial height is smaller, as shown in Figure 19(c). The average consolidation degrees of the soil columns with three initial heights were 65.9%, 35.2%, and 20.6%, respectively. However, As the consolidation continues, such as on the 1000th and 10000th day, the soil column with higher initial height has smaller void ratio at the same relative depth. This is because, the soil column with bigger initial height has bigger effective stress due to more soil quantity and higher initial excess pore water pressure. Higher excess pore pressure dissipated would cause bigger effective stress and result in smaller void ratio. The total settlements on the 10000th day of the three cases are 0.333 m, 0.875 m, and 1.508 m respectively with the corresponding vertical strains of 33.3%, 43.8% and 50.3% (Figure 19(b)). Overall, the effect of initial height on the consolidation process is significant.



(a)



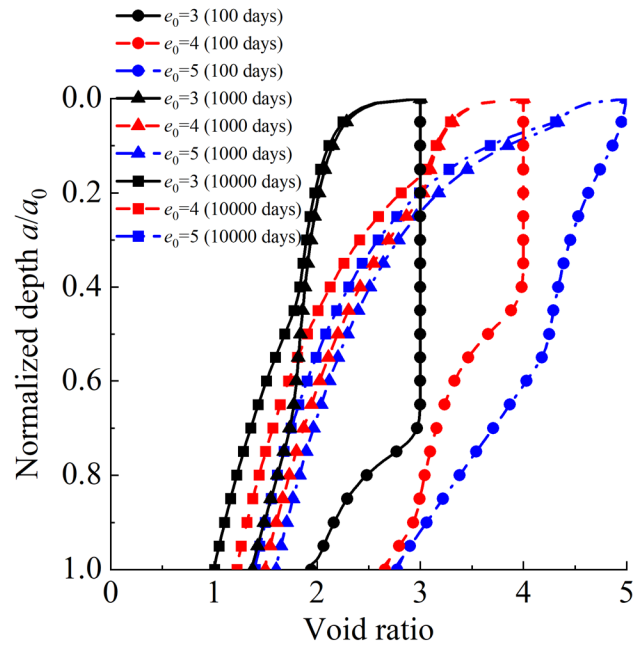
(b)



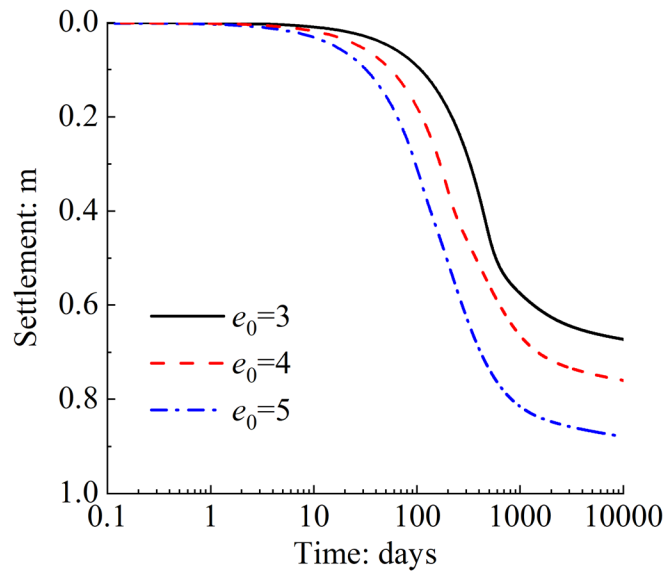
(c)

Figure 19– Consolidation process of slurry with different initial heights for evolutions of: (a) void ratio distribution; (b) settlement; and (c) average consolidation degree

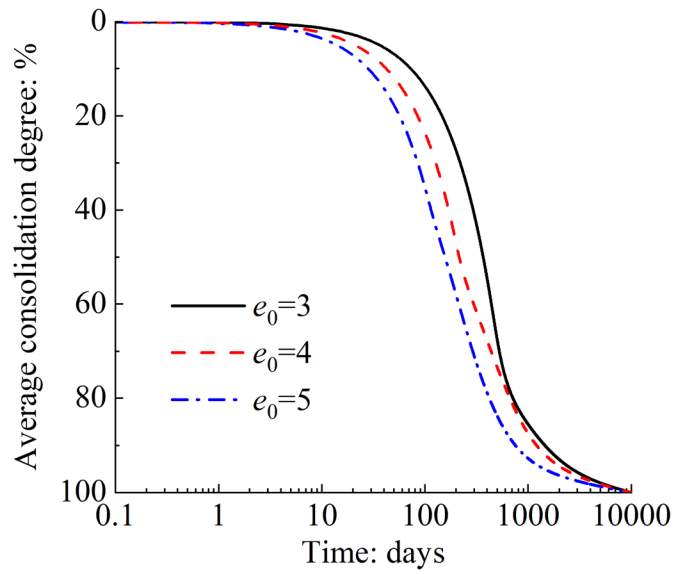
Figures 20(a)-(c) show how the initial void ratio affects the self-weight consolidation process of slurry. In these simulations, the initial void ratios are 3, 4, and 5 with an identical initial height of 2 m. The total settlement of soil on the 10000th day is 0.673 m, 0.760 m, and 0.875 m respectively (Figure 20(b)). The time required to complete 90% of the consolidation is 1452 days, 1208 days, and 752 days respectively (Figure 20(c)). In general, the soil column with bigger void ratio has bigger settlement but still bigger void ratio during consolidation. The effect of initial void ratio on the consolidation speed is significant. The consolidation speed of the soil column increases with the initial void ratio.



(a)



(b)



(c)

Figure 20— Consolidation process of slurry with different initial void ratios for evolution of: (a) void ratio distribution; (b) settlement; and (c) average consolidation degree

Conclusions

Yin-Graham one-dimensional Elastic Visco-Plastic (1D EVP) model has firstly been extended for finite strain modelling of self-weight consolidation of very soft clays. The extended EVP model inherits all the advantages of the original model, which can describe 1D stress or strain responses in soft soils exhibiting creep under general conditions including multistage loading, continuous loading, and unloading or reloading while overcoming the original model's limitation of being meaningless when the initial effective stress is very low or even zero.

Partial differential equations for 1D finite strain consolidation of very soft clays using the 1D EVP model have all been derived, in which nonlinear compressibility and permeability of

very soft soil skeleton are also taken into account. The Crank–Nicholson finite difference method has been used to solve nonlinear partial differential equations set. Finally, the present 1D finite strain consolidation model using the extended 1D EVP constitutive model has been validated through three case studies, which covers large initial void ratio of 5.17 and 9.87 and smaller initial void ratios of 2.094 and 2.475. The influences of some important parameters such as initial height and void ratio on the self-weight consolidation process are also analyzed. Comparing the prediction results with test data and numerical results of other researchers, the following conclusions can be drawn:

(a) There is a good agreement between predicted and measured results, confirming that the proposed 1D finite strain consolidation model can simulate changes in surface settlement, excess pore water pressure, total stress, void ratio and density distribution with time and space with good accuracy;

(b) The proposed model is reliable for finite strain consolidation analysis of very soft soils with a wide range of initial void ratios, and

(c) More importantly, the proposed model well captures the features of finite strain consolidation of very soft soils that the stress–strain relationships are not unique but rather are highly dependent on the locations or heights, surpassing which by taking into account the creep of these soft soils.

There are several deficiencies that need to be further studied. For example, in the proposed finite strain consolidation model, the influence of soil structure was not considered. Further

work can be conducted to extend the consolidation model for simulating the collapse of structured soil by relating the creep characteristics to the structure change of clayey soil with flocs. In addition, since there are few creep test data of clayey soil at very low-stress level which is significant for expanding our understanding of the properties of soils, further experimental studies should be conducted to investigate the creep behaviour of extremely soft soil with high water content by improving the compressive testing apparatus. The proposed finite strain consolidation model will be further extended to two- and three-dimensional to assess the anisotropy of slurry treated using prefabricated vertical drains (PVD), prefabricated horizontal drains (PHD), or other methods in practice.

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