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Joint Optimization of Bus Scheduling and Targeted Bus Exterior Advertising

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Abstract

Bus exterior advertising provides a powerful way to establish brand awareness since it can reach a mass of audiences with a high frequency. For a certain advertisement category, the advertising effectiveness is largely depended upon its exposure times to the target audience who takes interests of the advertisement, which is termed as targeted advertising. Given that the distribution of target audiences over a city varies among different advertisement categories, a practical way of enhancing overall advertising effectiveness is to deploy the bus with certain advertisement category to the bus line that best fits it “target area”. This gives rise to a decision-making problem of targeted bus exterior advertising and bus scheduling. In this paper, the problem is formulated as a bi-objective optimization model with objectives of maximizing the quantified advertising effectiveness and minimizing the number of bus fleet size to cover all trips. The advertising effectiveness is quantified using the audience demographic data. The deadheading of buses is also enabled in the scheduling process to facilitate both objectives. The NSGA-II-LNS algorithm is developed to solve the bi-objective problem with the incorporation of large neighborhood search operators into the framework of the NSGA-II to improve

solution quality. Various experiments are set up to verify the proposed model and solution algorithm.

Keywords: Targeted bus exterior advertising; bus scheduling; bus deadheading; bi-objective optimization; NSGA-II-LNS.

Introduction

Transit advertising is a form of out-of-home (OOH) media that displays advertisements on the public transportation vehicles or in any related public transportation areas. For its high visibility and frequent exposure to the audiences, transit advertising is still a competitive and powerful way to win their attention despite the fast emergence of many new media, e.g., online media (Huang et al. 2022a). According to the American Public Transportation Association (APTA), the transit advertising can reach up to 83% of audiences on weekdays and 69% on weekends (APTA 2019). Out of all sorts of transit advertising, the bus exterior advertising is of particular advantage because it can cover a broad range of city areas through the mobility of bus and impact not only the passenger but anyone who can see the advertisement along the bus line (Roux 2014). Hence, more emphasis should be placed on the bus exterior advertising in launching a transit advertising campaign.

When considering bus exterior advertising, it is very important to assess the demographic that the bus line system can reach. Specifically, some audiences might not be interested in the content of certain advertisements even though they are constantly exposed. There also exists a variance in the number of interested audiences across different city areas. For example, in the tech hub of a city, people are more likely to prefer the advertisements related with high-tech products. Therefore, the concept of "targeted advertising" is proposed with the intention of maximizing the effectiveness of advertising (Johnson 2013; Tucker 2014; Wang et al. 2019), in which the core task is to expose the advertisement as many times as possible to its real "target audience". Furthermore, massive researches on transportation big data in recent years have enabled to capture the number and distribution of target audience across the city (Wang et al. 2022; Huang et al. 2021b; Rajput et al. 2022). As for the targeted bus exterior advertising, the exposure times directly determined by the bus scheduling plan, which gives space for optimization. For example, for an advertisement category which takes one certain region as the target market, the total exposure times can be increased by deploying more buses that are applied with this advertisement to serve the line passing this region.

The conventional bus scheduling assumes that each bus can serve only one bus line, in which sense the effectiveness of bus exterior advertising is relatively consistent (Huang et al. 2022b; Teng et al. 2020; Jiang and Zhang 2022). While in the paper, a more practical situation is considered that the buses are allowed to shifted the service from one line to another (interlining). As deadheading trips (a bus departs empty from a dispatching terminal stop to a designated stop) are usually demanded for accomplishing the line change, we name this scenario as “bus scheduling under deadheading scheme”. The original intention of introducing deadheading was to reduce the bus fleet size that can cover all the trips, in that the vehicle resources could be arranged in a more flexible and efficient way (Ceder 2016; Huang et al. 2021a; Zhang et al. 2021). Meanwhile, as the deadheading scheme enables line change, it is possible for the buses to reach a wider range of city areas and thus absorb more target audiences.

Hence, it is natural to raise a decision-making question based on the discussion above: under the deadheading scheme, how to properly schedule the buses on a service timetable and select the advertisement category applied on each bus, so as to maximize the overall effectiveness of bus exterior advertising as well as maintain a small bus fleet size. It has to be clarified that the improvement of advertising effectiveness is only an added value, rather than the purpose of bus scheduling. The solution does not violate any basic constraints of normal bus scheduling, but provides an alternative option from the perspective of advertising. In this paper, a bi-objective optimization program is proposed with the objectives of maximizing advertising effectiveness and minimizing the bus fleet size.

Literature review

Transportation economics is an emerging intersectional research field that has attracted attention from both public transportation research and marketing and advertising research (Wexler and Fan 2022). For the topic of transit advertising, existed studies mainly focused either on the quantitative evaluation of target audiences and potential effects from transit advertising, or the maximization of advertising effectiveness through different methods. Zhang et al. (2017a) captured the patterns of passengers and bus stations to quantitatively measure the advertising effectiveness from the Smart Card Transaction (SCT) data, geographic data and point of interests (POIs) data. A bus route recommendation model was then proposed to maximize the advertising effectiveness. A similar work was done by Zhang et al. (2017b), where the motion patterns and user interests were learnt by a probabilistic data model, and the top-k retrieval problem for advertisement recommendation was solved to support real-time decision making. Faroqi et al. (2019) used the smart card data to model the passengers' travel behavior and then proposed two behavioral advertising models regarding different optimization targets. Both models are formulated as linear programming models. They extended their work by clustering passengers with similar activities as a targeted group, and then developed an optimization model to allocate advertisements to the activity-trip groups (Faroqi et al. 2021). Huang et al. (2022a) estimated the distribution of target audiences by mobile phone data and land use data. Then, two distinct bus selection model were created to maximize the advertising effectiveness. For well-established brands, the goal was to expand the coverage of audiences, while for new brands, the goal was to acquire a high level of exposure.

Deadheading has been viewed and studied as an operational strategy in bus vehicle scheduling by the authors (Mahdavi Moghaddam et al. 2019; Wang et al. 2020; Huang and Wang 2022c; Liu et al. 2022; He et al. 2022). Ceder and Stern (1981) first introduced the concept of deadheading trips. They constructed a deficit function as the graphical interactive interface and considered to insert an empty trip between two terminals aiming at reduce the bus fleet size with respect to the original departures from the terminals. Furth (1985) applied the concept on a single bus line with a directional imbalance in passenger demand. For the direction with lower demand, some buses were selected to return empty (skip the whole trip), while the others returned in service. In this way, they found that the number of vehicles needed was reduced, as well as the waiting time of passengers. Yu et al. (2012) presented a two-phase partway deadheading strategy to improve the bus service on peak directions. The first phase assessed whether a partway deadheading was necessary based on the service reliability, and the second phase determined the beginning stop for the service of a deadheading vehicle. Liu et al. (2013) developed a bus stop-skipping scheme and considered the deadheading problem as a special case. The stop-skipping (deadheading) was formulated as an optimization model with objectives of minimizing costs of both passengers and bus operators. Tang et al. (2019) developed a model based on the deficit function which combined deadheading with other trip adjustment strategies, including limited stop and short turning, to reduce the required number of vehicles for a single line.

A research gap is identified from the previous studies. Although an increasing awareness of the potential market value in bus exterior advertising has led to a growing body of literature on this domain, and some studies have already sought to improve the effectiveness of advertising through proper bus scheduling or route selection, there is still no study that puts this problem under a deadheading scheme. According to the literature, the deadheading can successfully reduce the fleet size, and it is speculated that it can help to boost bus exterior advertising due to a wider coverage on the city area. Besides, in view of practical meaning, the deadheading scheme provides the most direct and cost-effective way to enhance the transit system given that the bus resources are always limited. Consequently, it is worthwhile to study this untouched problem.

Objectives and contributions

This study has two main contributions. First, to remedy the gap in the existing literature, it proposes a joint bus scheduling and advertisement selection problem under the deadheading scheme. The problem is then formulated as a bi-objective optimization model with aims of (1) maximizing the advertising effectiveness of bus exterior advertisements, and (2) minimizing the bus fleet size to cover all the trips. Second, due to the NP-hardness of the proposed bi-objective model, it is difficult to find an exact method. So, a heuristic-based method named NSGA-II-LNS is designed as the solution algorithm. This method embeds the large neighborhood search operator into the framework of NSGA-II to refine individual solutions. Also, a piecewise linear approximation method is adopted to solve the advertisement selection subproblem which determines the optimal advertising plan for a given scheduling solution.

The remainder of this paper is structured as follows. Section “Problem description” describes the basic concepts in the problem, including the quantitative measurement of advertising effectiveness and the bus scheduling under deadheading scheme. Section “Model formulation” provides the bi-objective formulation of joint bus scheduling and advertisement selection problem, with elaboration on the model constraints. Section “Solution algorithm” develops a solution algorithm for the bi-objective problem based on the NSGA-II. Section “Numerical example” presents the numerical example to verify the proposed model and the solution algorithm. Finally, we conclude this paper in section “Conclusions” and point out some directions for future research.

Problem description

Advertising effectiveness measurement

The effectiveness of bus exterior advertising relates to many factors such as the target audience distribution, the exposure frequency, and the advertising format. Consequently, it is complicated to measure the overall advertising effectiveness (AE) quantitatively (Huang et al. 2022a).

For the purpose of modeling AE in a practical manner, the study area is first divided into several zones and each of them has a unique bus stop inside. The shape and size of divided zones are determined by the configuration of streets and blocks in the city. Then, the bus stops are looked as the centroids of their corresponding zones, aggregating the people who live in this region and may become the potential audience of bus exterior advertisements. Fig. 1 provides a simple illustration of the zone division strategy. The number of zones is identical to the number of bus stops. Thus, the term bus stop is used to refer to a zone hereafter. For a given bus stop p in the bus stop set P and advertisement category set A, the target audience profile is denoted as follows,

$$\left\{\left(a_1,\tau_{a_1p}\right),\left(a_2,\tau_{a_2p}\right),\ldots,\left(a,\tau_{ap}\right),\ldots\right\}, \tag{1}$$

where a is an advertisement category, $a \in A$, and τ_{ap} represents the number of target audiences of a around the stop p. Existing studies have verified that the target audience profile has a close relationship to the land use type of that location (Zhang et al. 2017a; Sun et al. 2020).

[Insert Fig. 1 here]

Then, the AE of a particular advertisement category a at bus stop p is measured by the accumulative exposure times within the time period (a target audience view the advertisement once is counted as one exposure), which is denoted as,

$$\beta_{ap} = \tau_{ap} \sum_{k \in K^a} n_{kp}, \tag{2}$$

where β_{ap} is the value of AE for advertisement category a at bus stop p. n_{kp} is the number of times the bus applied with advertisement category a pass by the bus stop p. K^a is a subset of the bus set K. Note that those values are in essence determined by the transit route structure and the bus scheduling plans. Then, by accumulating the AEs at all stops, we acquire the global effectiveness of advertising of category a, which is denoted as,

$$\beta_a = \sum_{p \in P} \beta_{ap}. \tag{3}$$

Further, this study considers a non-linear extension of Eq. (2). Wells (2014) claimed that multiple exposures to an advertisement increase audience awareness of the advertising message and facilitate consumer processing of the included information. However, the audience's attitude towards a brand does not increase linearly with the times getting exposed to the advertisement. Instead, the attitude displays a diminishing marginal utility or even excessive exposure times can cause a side effect on the advertisement. Schmidt and Eisend (2015) modelled the effect of advertising repetition as a nonlinear quadratic course of effect which shaped as an inverted U curve. Following this principle, the expression of AE (Eq. (2)) is reconstructed as follows,

$$\beta_{ap} = \tau_{ap}\varphi\left(n_{ap}\right), \tag{4}$$

$$\varphi\left(n_{ap}\right) = \begin{cases} -\dfrac{\varphi_0}{n_0^2} n_{ap}{}^2 + \dfrac{2\varphi_0}{n_0} n_{ap} & 0 \le n_{ap} < n_0 \\ \varphi_0 & n_{ap} \ge n_0 \end{cases}, \tag{5}$$

where $\varphi(\cdot)$ is a non-decreasing piecewise function which maps the actual exposure times to the "effective exposure times" to better describe the practical influence of bus exterior advertisements. Fig. 2 illustrates the function curve. The effective exposure times grows gradually before leveling off at a constant value φ_0 when the actual exposure times reach the upper limit n_0, implying that no more advertising benefit can be made from more bus visiting.

[Insert Fig. 2 here]

Bus scheduling under deadheading scheme

The classic vehicle scheduling problem (VSP) in public transportation is defined as establishing the daily working schedules (rotations) for a fleet of buses, to cover a coordinated timetable. Each trip in the timetable with specified departure time, arrival time, start stop, and terminal stop is covered by exactly one rotation (Kliewer et al. 2006). Many previous studies on VSP make the preliminary assumption that the bus vehicle are tied with the bus route, while in this study we tackle with a more practical situation where buses can be dispatched across the lines. Since additional deadheading trips are inserted into the rotation plan when the terminal stop of the former trip and the start stop of the subsequent trip are different, this problem is named as VSP under deadheading scheme. The minimum required fleet size of bus can be reduced as long as the deadheading trips are properly arranged within the scheduling plan, despite the increased total working load (Ceder 2016). Another important reason to consider deadheading trips is from the perspective of advertisement spreading. As it breaks the bindings between bus and route, there is higher chance for the bus exterior advertisement to reach a wider range of city areas and to reach the actual target audiences.

Consider a graph representation of the VSP under deadheading scheme (see Fig. 3). The scheduling network is denoted as $G=(N,E)$. Each node $i \in N$ represents a trip (or depot) and is associated with a beginning time bt_i, an ending time et_i, and the bus line it operates on. The set of arcs E contains the possible connections between nodes. Two types of arcs are involved: for the case where a pair of nodes (i,j) are set up on the same line, as long as their time periods are not overlapped, there will be an *in-line arc* to connect them; for the other case where (i,j) are on different lines, there will be a *deadheading arc* if their time interval is longer than the minimum required deadheading time δ_{ij} (i.e., $bt_j - et_i \geq \delta_{ij}$). Note that the deadheading can take place either within one single bus stop (implying that there is no extra time duration for line changing) or between two distances stops (the bus needs to take an actual deadheading trip for line changing). A daily rotation of a bus is then modelled as a path starting from the depot, passing by a sequence of nodes, and returning to the depot eventually.

[Insert Fig. 3 here]

Model formulation

In this paper, optimization of the bus scheduling and the bus exterior advertisement selection are considered jointly. The three-index binary decision variables x_{ij}^k are introduced as the bus scheduling variable that equals to 1 if bus k serves trip j after i, and 0 otherwise. The two-index binary decision variables y_a^k reflect the advertising strategy that equal to 1 if bus k is applied with advertisement category a, and 0 otherwise.

Two objectives are addressed in this model. First, from the side of bus advertising, the objective is to maximize the total AE of bus exterior advertisements. Second, from the side of bus scheduling, we seek to minimize the bus fleet size because it is a direct reflection of the efficiency of bus utilization. With smaller number of buses needed to run all the trips, it means the bus scheduling plan is executed under a compact and smart timetable. Since the optimization of both objectives are facilitated by the introduction of deadheading trips as discussed in section "advertising effectiveness measurement", it is assumed that the increased operating costs (driver and vehicle travel cost) for the transit company to add deadheading trips into the timetable is far less than the capital cost of saving a bus. Furthermore, as those two objectives are speculated to contradict in nature (e.g., improving the AE generally needs a larger bus fleet size), rather than addressing them via the weighted sum method, it is more reasonable to formulate the problem as a Bi-objective Optimization Problem (BOP) and then solve it using pareto-based algorithms.

The mathematical formulation of joint bus scheduling and advertisement selection problem **[P1]** is provides as follows.

[P1]

$$\max z_1 = \sum_{a \in A} \sum_{p \in P} \beta_{ap} \tag{6}$$

$$\min z_2 = \sum_{k \in K} \sum_{i \in N} x_{0i}^k \tag{7}$$

s.t.

$$\beta_{ap} = \tau_{ap} \varphi \left(\sum_{k \in K} \sum_{i \in N} \sum_{j \in N \cup \{0\}} \lambda_{ip} x_{ij}^k y_a^k \right), \ \forall a \in A, p \in P, \tag{8}$$

$$\sum_{k \in K} \sum_{j \in N \cup \{0\}} x_{ij}^k = 1, \ \forall i \in N, \tag{9}$$

$$\sum_{j \in N \cup \{0\}} x_{ij}^k - \sum_{j \in N \cup \{0\}} x_{ji}^k = 0, \ \forall i \in N, k \in K, \tag{10}$$

$$\sum_{i \in N} x_{0i}^k \le 1, \ \forall k \in K, \tag{11}$$

$$\sum_{i\in N}\sum_{j\in N} c_{ij}x_{ij}^k \le MaxCr,\ \forall k\in K\ , \tag{12}$$

$$et_i - bt_j + \delta_{ij} + Mx_{ij}^k \le M,\ \forall i,j\in N\bigcup\{0\}, k\in K\ , \tag{13}$$

$$\sum_{a\in A} y_a^k = 1,\ \forall k\in K\ , \tag{14}$$

$$LB \le \sum_{k\in K}\sum_{i\in N} x_{0i}^k y_a^k \le UB,\ \forall a\in A\ , \tag{15}$$

$$x_{ij}^k \in \{0,1\},\ \forall i,j\in N, k\in K\ , \tag{16}$$

$$y_a^k \in \{0,1\},\ \forall a\in A, k\in K\ . \tag{17}$$

Eq. (6) is the first objective function which is to maximize the AE of bus exterior advertisements. Eq. (7) is the second objective function which is to minimize the bus fleet size to cover the trips. Note that the expression x_{0i}^k stands for the arc from the virtual depot node to the rest trip nodes in the network. Constraint (8) specifies the expression of the AE as elaborated earlier in Eqs. (4) and (5). λ_{ip} is a binary parameter which equals to 1 if trip i contains stop p, and equals to 0 otherwise. Constraint (9) enforces the rotations to cover every trip. Each trip must be served by exact one bus. Constraints (10) and (11) define the trip chain of a bus. A valid trip chain must start from the depot node, pass by a string of nodes one by one (or none), and return to the depot node again finally. Constraint (12) restricts the upper limit of deadheading trips for a bus within the study time period as too much deadheading maybe impractical even though the extra costs are ignored. c_{ij} is a binary parameter which equals to 1 if serving trip j after trip i incurs a deadheading trip, and equals to 0 otherwise. $MaxCr$ denotes the maximum number of deadheading trips for each bus within the time period. Constraint (13) ensures the layover time between two consecutive trips must be no less than the minimum required deadheading time. Note that this constraint together with the former constraints also forbids subtours in the solution. Constraint (14) stipulates that each bus is only allowed to be applied with one advertisement category during the planned period. Constraint (15) ensures the total number of buses that are applied with the same advertisement category should be within a determined range. This constraint indicates the fairness consideration among all the advertisement categories. If there wasn't such a constraint, most or even all of the buses would be applied with one advertisement category, which in general has more target audience compared to other categories to maximize the total AE. Constraints (16) and (17) define the binary decision variables.

Solution algorithm

Since constraint (8) is non-linear and contains the multiplication of decision variables, the bi-objective programming model is a non-linear integer BOP. It has been noted that a single-objective bus vehicle scheduling problem is already NP-hard (Kliewer et al. 2006; Liu et al. 2013; Bie et al. 2021). The proposed problem also includes the selection of bus exterior advertisement and a bi-objective structure that is more complicated to address than the general single-level VSP. Thus, the joint bus scheduling and advertisement selection problem is also NP-hard.

Given the extreme difficulty of applying an exact algorithm for the NP-hard problem, a new heuristic-based solution algorithm named NSGA-II-LNS is proposed to solve the bi-objective problem. The main idea lies on the integration of the group evolution mechanism and the neighborhood search operator into one algorithm. The non-dominated sorting genetic algorithm with the elitist strategy (NSGA-II) is a well-developed multi-objective optimization algorithm and has been used widely for its strong global search ability and robustness (Deb et al. 2002). To further refine the solutions that are produced by standard NSGA-II during each iteration, a large neighborhood search (LNS) operator is incorporated into the framework to search for new elite individuals. The algorithm also includes a piecewise linear approximation method for the nonlinear convex objective to make use of the mixed-integer linear programming solvers to solve the advertisement selection subproblem. Details of the algorithm are described in the rest of this section.

Overview of NSGA-II-LNS

First, some preliminary knowledges are provided on the multi-objective problem and the Pareto optimal solution. Consider a multi-objective problem,

$$\min F(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_t(\mathbf{x})\right) \tag{18}$$

s.t.

$$\mathbf{x} \in \Omega, \tag{19}$$

where $\mathbf{x}$ is the decision variable vector, t is the number of objective functions, and Ω is the feasible space. The dominance rule between solutions is defined as follows. Let two solution vectors $\mathbf{u}, \mathbf{v} \in \Omega$, $\mathbf{u}$ is said to dominate $\mathbf{v}$ if and only if $f_i(\mathbf{u}) \le f_i(\mathbf{v})$ for every objective function index $i \in \{1, 2, ..., t\}$ and $f_j(\mathbf{u}) < f_j(\mathbf{v})$ for at least one objective function index $j \in \{1, 2, ..., t\}$. $\mathbf{u}$ and $\mathbf{v}$ are said to be non-dominated if neither of solution dominates the other. A solution vector $\mathbf{x}^*$ is a Pareto optimal solution if there exists no other solution in the decision space that can dominate $\mathbf{x}^*$. Since typical multi-objective problems involve competing objectives, and no solution may make all objectives optimal simultaneously, we can obtain a set of non-dominated Pareto optimal solutions which is termed as the Pareto Front (PF) of the problem.

The NSGA-II operates and evolves on a population of solutions towards the better approximation of the PF. The core advantage of NSGA-II is its elitist strategy within the population. In particular, the fast non-dominated sorting procedure and the crowding distance calculation are applied in this algorithm. The fast non-dominated sorting ranks the solutions based on a hierarchical order with multiple levels. Solutions within the same level are non-dominated but dominate at least one solution in the lower levels. Further, on each non-dominated level, the non-dominated solutions are ranked by the crowding distance according to the descending order. The crowding distance is expressed as,

$$D_r = \sum_{i=1}^{t} \left(F_i'(r+1) - F_i'(r-1)\right), \tag{20}$$

where D_r is the crowding distance of solution r, $F_i'(r+1)$ and $F_i'(r-1)$ represent the normalized value of its next and previous solutions at objective function i, assuming that the solutions are sorted accordingly. The crowding distance of all marginal solutions are set as a very high value.

The steps of the NSGA-II-LNS algorithm can be described as follows.

Step 0: (Initialize input parameters) Set the parameters, including the population size N_P, the value of crossover probability ρ_c, the value of mutation probability ρ_m, and the maximum number of iterations $I_{\max}$. Set the iteration counter $I = 0$.

Step 1: (Initialize population) Generate the solutions as the number of population size N_P via the subroutine described in section "Solution generation subroutine", and form the initial population (the first parent population). A checking procedure is then carried out to avoid duplicated solutions in the population.

Step 2: (Large Neighborhood Search) Perform the LNS for each solution in the current parent population. Evaluate the fitness of the newly-generated neighbor solutions. If the neighbor solution is not dominated by its original solution, it is added to the population as a new individual.

Step 3: (Genetic operators) Select two candidate solutions μ and ν from the parent population based on the tournament strategy. Set a uniformly distributed random number γ_c between [0,1]. If $\gamma_c < \rho_c$, we conduct the crossover operation as follows. For each trip in the offspring solution, randomly pick one assigned bus from μ and ν conditioned that at least one of them is feasible, otherwise a third feasible bus is used. Similarly, if another random number $\gamma_m < \rho_m$, we conduct the mutation operation which is identical to the LNS in step 2 on the offspring solution. After that, the offspring solution is added to the child population. The former procedures are done for multiple times until a complete child population is generated.

Step 4: (Generate new parent population) Combine the parent population and the child population, and conduct the fast non-dominated sorting and crowding distance calculation as aforementioned. The best N_P solutions in the combined population are retained and formed as the new parent population.

Step 5: (Stopping criterion) If the iteration counter $I = I_{\max}$, terminate the algorithm and output the final PF; otherwise, let $I = I + 1$ and return to Step 2.

Solution generation subroutine

In the NSGA-II-LNS algorithm, new solutions need to be generated in the phase of population initialization. As described in section "Model formulation", each solution is comprised of two elements, namely, the bus scheduling variables (i.e., x_{ij}^k) and the advertisement selection variables (i.e., y_a^k). The following procedures are adopted to generate values for both of them.

Generation of bus scheduling solution

As for the bus scheduling variables, the mission of establishing the rotation for a fleet of buses is equivalent to assigning an available bus for each of the trip to be served. First, sort all the trips by the trip beginning time in an ascending order. Then, for each trip i in the sorted trip list, a candidate bus set is created to contain all the available bus that can be currently assigned to trip i, complying to the trip chain constraints (10) and (11), and the deadheading constraints (12) and (13). Hence, to ensure the feasibility of the bus scheduling, a bus k is randomly chosen from the candidate set for this trip. When there is no available bus for this trip, it has to be served by a dummy bus, implying the solution is infeasible. Besides, when choosing, priorities are given to the bus which has been already deployed for some trips, which is likely to leave more buses unused after finishing all trips and thus reduce the fleet size of bus to cover the trip.

Approximation of advertisement selection solution

A notable feature of the bi-objective model **[P1]** is that, once fixing the value of bus scheduling variables, the bus fleet size (the second objective function) is then determined as a constant, and **[P1]** is therefore reduced to a single-objective optimization problem with only decisions on the advertisement selection variable to maximize the AE, which should be much easier to solve. While on the other hand, fixing the value of advertisement selection variables will not decrease the complexity of the model as much. Even though each bus has a designated advertisement category, the problem still remains an VSP with two objective functions. Given the inherent asymmetry of the solution elements, we can determine the value of y_a^k after the generation of x_{ij}^k in the way of mathematical programming.

Consider a reduced problem **[P2]** of **[P1]**, which is denoted as,

[P2]

$$\max \sum_{a \in A} \sum_{p \in P} \beta_{ap} = \sum_{a \in A} \sum_{p \in P} \tau_{ap} \varphi \left(\sum_{k \in K} \tilde{n}_{kp} y_a^k \right) \tag{21}$$

s.t.

$$\sum_{a \in A} y_a^k = 1, \ \forall k \in K, \tag{22}$$

$$LB \le \sum_{k \in K} \sum_{i \in N} \tilde{x}_{0i}^k y_a^k \le UB, \ \forall a \in A, \tag{23}$$

$$y_a^k \in \{0,1\}, \ \forall a \in A, k \in K. \tag{24}$$

In **[P2]**, $\tilde{n}_{kp}$ is the number of times that the bus k passes by the bus stop p. With the known values of decision variable $\tilde{\mathbf{x}}$, it is also fixed as $\tilde{n}_{kp} = \sum_{i \in N} \sum_{j \in N \cup \{0\}} \lambda_{ip} \tilde{x}_{ij}^k$. So, the only decision variable contained in this problem is y_a^k. It is also noticed from Fig. 3 that the function $\varphi(\cdot)$ shown in Eq. (5) is concave, therefore it is not difficult to prove that the maximization problem **[P2]** is an integer programming problem with linear constraints and a convex objective function. In order to efficiently solve this problem in practice, an approximation of the convex objective function is made by a piecewise linear function. The commercial solvers, such as CPLEX, is then applied to solve the approximation problem and output the optimal solution value of the bus advertising variable as $\tilde{\mathbf{y}}$. Together with the bus scheduling variable, it forms a complete initial solution, denoted as $(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})$.

Large Neighborhood Search

It is acknowledged that the Large Neighborhood Search (LNS), which basically perform destroy and repair operators repetitively on a single solution, has several advantages on enhancing the solution quality. As long as the operators are properly designed, the searching space can contain a large variety of neighborhoods with better solutions inside. Since the problem is over-constrained, to ensure the neighbor solution is still feasible, A tailored swap operator is designed to generate new bus scheduling solution and also use the method described in section "Approximation of advertisement selection solution" to determine the corresponding advertisement selection solution. Detailed steps are as follows.

Step 1: (Swap operator) For a solution $\left(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}\right)$ in the population, define a set Φ that consists of all the trip pairs that can swap their locations in the bus scheduling solution $\tilde{\mathbf{x}}$, meaning that the new solution after swapping is still feasible. Specifically, there are arcs in the graph to connect the corresponding nodes after swapping, and the deadheading constraint (12) and the layover constraint (13) are still satisfied.

Step 2: (Generate neighbor solution) For the set Φ, a greedy strategy is applied by sorting all the trip pairs based on the increment in the total AE (the first objective function value) once the trips are swapped in a descending order. In this way, the trip pair with the maximum potential to improve the solution is preferred for swapping. Based on the new bus scheduling solution, update the bus advertising solution via the approximation method as aforementioned. The new solution is denoted as $\left(\hat{\mathbf{x}}, \hat{\mathbf{y}}\right)$.

Step 3: (Check acceptance rule) Calculate the value of the two objectives for the solution $\left(\hat{\mathbf{x}}, \hat{\mathbf{y}}\right)$ according to Eqs. (6) and (7). Subsequently, check the dominance relationship between the two solutions. If $\left(\hat{\mathbf{x}}, \hat{\mathbf{y}}\right)$ is not dominated by $\left(\hat{\mathbf{x}}, \hat{\mathbf{y}}\right)$, add $\left(\hat{\mathbf{x}}, \hat{\mathbf{y}}\right)$ into the current population; otherwise give up this neighbor solution.

Numerical example

Data settings

The proposed joint bus scheduling and advertisement selection problem and the NSGA-II-LNS algorithm are numerically verified in this section. Since the proposed problem is relatively new and there is no existing benchmark instances, we construct a test transit system on the Sioux-Falls network, which has been widely used in transportation studies (Meng and Yang 2002; Wang et al. 2013). The network has 24 bus stops and 38 undirected links as depicted in Fig. 4. The link travel time is also labeled next to each link.

Five bus lines are manually designed within the transit system. Table 1 shows the route structures and headway settings of them. Once the route structure is decided, the one-way trip time of a line is also determined by summing up the time of each link it travels by. As the lines are bi-directional, a total of 10 distinct trips (i.e., both inbound and outbound trips) are involved. It should be noted that only the stops 1, 2, 13, 20 are taken as the beginning/terminal stop. Those four bus stops are indicated by red and dashed line in the figure. To establish the bus service timetable under the network, the study time period is set as $T=[0,720]$ and it is assumed that each line starts the first (inbound and outbound) trip at time 0, arrange the following trips sequentially according to the headway shown in Table 2, and the ending time of the last trip must not exceed the upper time limit. In this way, a unique timetable is generated. There are 144 trips in total encoded from 1 to 144. As for the connection between trips, both in-line arcs and deadheading arcs are created according to the definitions in problem description. And in order to avoid long-distance deadheading, the deadheading trips are only allowed in the following three situations (let tp_i and bp_j be the terminal stop of the former trip and the beginning stop of the subsequent trip respectively): (1) $tp_i = bp_j$, and trip i and j belong to different lines; (2) tp_i and bp_j are in the bus stop pair (1, 2); (3) tp_i and bp_j are in the bus stop pair $(13,20)$. The value set of minimum deadheading time between trip i and j is therefore determined as $\delta_{ij} \in \{0, 6\min, 13\min\}$.

Regarding the input data related to bus advertising, consider a total of three bus advertisement categories (i.e., $|A| = 3$). It is assumed that the number of target audiences of a category around the bus stop follows a normal distribution, namely, $\tau_{ap} \sim N(\mu_a, \sigma_a^2)$, where μ_a and σ_a are the mean and standard deviation specific to the advertisement category a, respectively. μ_a is uniformly generated between $[80, 120]$, and σ_a also follows a uniform distribution between $[10, 20]$. Hence, a synthetic dataset on target audience distribution is formed.

[Insert Fig. 4 here]

[Insert Table 1 here]

Optimization results for the bi-objective problem

We use the NSGA-II-LNS algorithm to solve the proposed problem. Some parameters inside the algorithm are set as follows. The population size and the maximum number of iterations are $N_P = 100$ and $I_{\max} = 50$. The value of crossover probability and mutation probability are $\rho_c = 0.8$ and $\rho_m = 0.05$. Also, the lower bound and upper bound shown in constraint (15) are set as $LB = 5$ and $UB = 10$ respectively based on the rough estimation of the solution value. The parameters that affect the shape of AE function (seen in Eq. (5)) are set as $n_0 = 20$ and $\varphi_0 = 10$. The maximum number of total deadheading trips allowed for each bus is set as $MaxCr = 5$. The algorithm is coded in Python and implemented on a personal computer with AMD Ryzen 7-5800HS @ 3.20 GHz and 16 GB RAM. The approximation problem is solved by the commercial solver CPLEX 20.1.0, invoking its functions on the piecewise linear optimization.

When the algorithm terminates, it returns a Pareto Front with 5 non-dominated solutions inside, which possess different bus scheduling plans and exterior advertisement selections. Table 3 presents the value of both objectives for them as shown in the second and third column respectively. The overall result confirms the hypothesis that the two objectives optimized in the model are contradictive in nature. The solution with larger value of AE also has a larger bus fleet size (see Eqs. (6) and (7) for the expressions of objective functions). The inherent causes of this phenomenon could be attributed to two aspects: First, increasing the fleet size enlarges the decision space of both the bus scheduling variable and the advertisement selection variable, implying that there is higher chance to assign a proper advertisement category on the bus, thus to improve the total AE achieved from bus advertising. Second, given that the effective exposure time function, as shown in Eq. (5), displays a diminishing marginal utility with the number of actual exposure times, increasing the fleet size may decrease the average times of each advertisement category being exposed at the bus stops. Therefore, the exterior bus advertisements are promoted in a more efficient way.

Table 2 also provides the statistics on the trips, as shown in the right three columns. It is observed that the number of deadheading trips increases in general with the bus fleet size. Recall that the total number of trips is the same for each solution, this result may indicate that with more deadheading trips inserted into the scheduling plan, the effectiveness of advertising can be boosted. It is also noted that there is no significant difference on the average number of deadheading trips per bus of each solution. However, nearly all of them are close to the upper limit of the deadheading trips ($MaxCr = 5$).

[Insert Table 2 here]

Sensitivity analysis

Effect of deadheading scheme

To further investigate the impact of the deadheading scheme on the optimization result, a group of comparative tests are carried out by setting the maximum number of deadheading trips for each bus at different values ($MaxCr = 2,3,4,5,\infty$). Still, the NSGA-II-LNS algorithm is adopted as solution algorithm. Fig. 5 presents the optimal Pareto Front of each test group. It shows apparently that the results are better in the group with higher value of $MaxCr$. The variance in total AE for each group is quite large when the bus fleet size is small, while it gets narrower with increasing number of bus fleet size. This is because the maximum deadheading trips constraint is easier to be satisfied when the bus serves fewer trips on average. Besides, since the total number of trips is fixed, we cannot improve the optimization results by infinitely raising the value $MaxCr$. This explains why the Pareto Front of $MaxCr = 5$ almost coincide with that of the group with infinite value of $MaxCr$ (constraint (12) is said to be removed from the formulation).

Fig. 6 shows the number of deadheading trips and bus fleet size for each test group. As expected, the group with larger value of $MaxCr$ generally takes more deadheading trips in the bus scheduling plan, which means that the deadheading trip is preferable for its potential in reducing the bus fleet size and improving advertising effectiveness as long as the number does not exceed the upper limit. However, it does not necessarily mean more deadheading is always better in practice. If so, the bus drivers have to get familiar with more bus line and the whole scheduling plan gets more difficult to operate.

[Insert Fig. 5 here]

[Insert Fig. 6 here]

Effect of approximation method

In the NSGA-II-LNS algorithm, the approximation method acts as the main component for strengthening the solution quality. To investigate its performance, another comparative test is carried out in this subsection. As aforementioned, when the bus scheduling variable is given, the approximation method is then invoked to determine the advertisement selection variable by solving the piecewise linear approximation of the convex subproblem **[P2]**. A random generation method is designed to replace this procedure for the comparison. The bus advertisement category for each bus is randomly assigned and the overall advertising still needs to satisfy constraints (22) and (23). The objective values and the detailed advertising plan of the Pareto Front provided by each method are presented in Table 3. Clearly, the result from approximation method is superior to that from random generation method with higher values of total AE under the same bus fleet size. This indicates that the performance of a heuristic solution algorithm can be greatly improved by incorporating the mathematical programming approach.

[Insert Table 3 here]

Conclusions

In this paper, the joint bus scheduling and advertisement selection problem under deadheading scheme is proposed. The problem is formulated as a bi-objective optimization problem with the objectives of maximizing the advertising effectiveness of bus exterior advertisements and minimizing the bus fleet size. Both the trip chain and the advertisement category of each bus are taken into consideration as decision variables. The NSGA-II-LNS algorithm is applied to solve the proposed problem by incorporating a large neighborhood search operator into the standard framework of the NSGA-II. Also, a piecewise linear approximation method is used to solve the advertisement selection subproblem determining the optimal advertising plan for a given scheduling solution. To test the performance of the NSGA-II-LNS algorithm, A synthetic bus line system is built on the Sioux-Falls network and randomly generate the target audience profile following the normal distribution. A Pareto Front with 5 non-dominated solutions is obtained from the experiment, indicating that better advertising effectiveness needs a larger bus fleet size to realize. Moreover, the sensitivity test on the deadheading scheme shows that adding more deadheading trips can produce solutions with better values at both objectives.

It should be acknowledged that this study still has limitations. Due to the unavailability of field data, the experiments are conducted on an artificial network and the size of it is relatively small. Future works may seek to verify the model on a real-world transit network with more bus stops and lines. The mobile phone location data and land use data could be gathered to estimate the potential target audience distribution. The scheduling problem considered in this study is quite trivial, and more complicated scenario and more operational constraints should be considered. In addition, traffic dynamics (Cheng et al. 2021, 2022; Zhou et al. 2022) can be taken into consideration for the bus scheduling problem in future studies.

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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Fig. 1. An illustration of the concept of zone division.

Fig. 2. An illustration of nonlinear advertising effectiveness function.

Fig. 3. An illustration of bus scheduling network with deadheading trips.

Fig. 4. Sioux-Falls network.

Fig. 5. Optimal Pareto Front resulted from different values of maximum deadheading trips for each bus.

Fig. 6. Number of deadheading trips and bus fleet size resulted from different values of maximum deadheading trips for each bus.

Table 1. Bus line information of the transit network

Line id	Stop sequence	Trip time (min)	Headway (min)
1	1-3-12-11-10-16-17-19-15-14-23-24-13	45	10
2	1-3-4-5-6-8-9-10-17-19-15-22-21-24-13	54	15
3	1-3-4-11-14-23-22-20	31	10
4	2-6-8-7-18-16-17-19-20	23	15
5	2-6-5-9-10-15-22-23-24-13	36	20

Table 2. Solutions inside the optimal Pareto Front

Solution id	TAE	BFS	Trips/bus	Deadheading trips	Deadheading trips/bus
1	78705.2	10	14.4	37	3.7
2	78800.8	11	13.1	40	3.6
3	78943.8	12	12.0	47	3.9
4	78952.2	13	11.1	58	4.5
5	78967.2	14	10.3	56	4.0

Note: TAE = total advertising effectiveness; and BFS = bus fleet size.

Table 3. Comparisons of optimization results between approximation method and random generation method

Approximation method			Random generation method		
TAE	BFS	Advertisement plan	TAE	BFS	Advertisement plan
78705.2	10	0,0,1,2,0,0,2,2,1,1	78636.8	10	0,1,2,2,2,1,1,0,0,0
78800.8	11	2,0,1,2,0,2,0,1,2,1,1	78679.6	11	2,0,2,2,1,1,0,0,1,2,1
78943.8	12	2,2,2,0,1,1,0,0,0,2,1,1	78879.9	12	1,2,0,1,1,2,1,0,1,0,2,0
78952.2	13	0,2,1,1,0,0,2,1,1,0,2,1,2	78932.4	13	0,0,1,2,2,0,1,1,2,2,2,1,0
78967.2	14	1,2,1,1,2,1,2,0,0,0,2,0,1,0			

Note: TAE = total advertising effectiveness; and BFS = bus fleet size.