Min-max regret model for liner shipping fleet deployment with uncertain demand

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Abstract

This paper proposes a minimax regret model for liner shipping fleet deployment with uncertain demand. The minimax regret model does not need the probability distribution function of the demand, and it is consistent with how network planners are evaluated. However, the size of the model is large due to the incorporation of all possible demand scenarios. A dynamic scenario inclusion method is proposed to efficiently solve the minimax model by utilizing only a small subset of the demand scenarios. A case study based on an Asia-Europe-Oceania liner shipping network demonstrates the applicability of the proposed model and method.

Key Words: Liner shipping fleet deployment; maritime container transportation; uncertain demand; minimax regret

1 Introduction

Maritime transportation is the backbone of world trade: around 80 per cent of global trade by volume and over 70 per cent by value is carried by sea (*1*). Among all the seaborne commodities, 52 per cent by value are containerized (*1*). Containers are usually transported by liner shipping services with fixed sequences of ports of call and regular (usually weekly) frequency, which are published by liner shipping companies in advance to attract more cargoes of shippers. Shippers or freight forwarders can pick up and deliver their cargoes at the desired ports. A single shipper usually has far less than a full shipload of cargo but containerships have to keep to their published departure dates even when a full payload is not available.

Containerships are becoming larger and larger because liner shipping companies seek to take advantage of the economies of scale in ship size to gain competitive edge. The high crude oil price has driven up the bunker fuel prices in recent years. For these two reasons, the daily operating cost of a containership is very high. Once a containership is deployed on a liner service route, it usually serves that route for a period of three to six months. Therefore, it is important for liner shipping companies to determine the suitable ships to deploy on their shipping services.

To make fleet deployment decisions, a liner shipping company first predicts the future container shipment demand between ports in the network. Based on the predicted demand, the company determines which type of ship to assign to each ship route. After that, the company informs potential customers of the shipping services so that customers could book ship slots for their cargo. Since larger ships have higher chartering and voyage costs, deploying ships with too large capacity may lose money if the future demand is not high enough. On the other hand, if too small ships are deployed and the future demand is high, then the company could not earn as much as it should. Consequently, liner shipping companies try to assign ships to services in a manner to balance the trade-off. This tactical decision problem is referred to as liner shipping fleet deployment (LSFD).

A major challenge for solving the LSFD is that the future demand cannot be predicted accurately. The future container shipment demand depends on the export and import volume of each country, which is reliant on the economic growth and economic policy. As a result, there are always errors associated with the predicted demand, and the errors can sometimes be dramatic. The focus of this study is to address the LSFD with uncertain demand.

1.1 Literature review

There are a number of efforts on port operations (*2–11*) and shipping operations (*12–14*). Studies on shipping operations include for example network design (*15–17*), ship routing (*18–21*), fleet deployment (*22*), ship capacity utilization (*23*), container routing (*24*). Most of the studies on LSFD have modeled deterministic fixed demand. Although the meaning of the fixed demand is not mentioned, it can be a predicted value, or the average of several predicted values, or the most likely value of several predicted ones. Optimization models with fixed demand have assumed that the future container shipment demand is deterministic, i.e., identical to the fixed demand (*25–33*).

There are also a few studies related to LSFD with uncertain demand. Meng and Wang (*34*) developed a chance constrained programming model for LSFD with uncertain container shipment demand. They assumed that the future container shipment demand for each origin-to-destination (OD) pair is a random variable with known probability distribution. A certain level of service, defined as the probability that all container shipment demands on a ship route are fulfilled, was imposed. The objective is to minimize the total cost subject to the above level of service constraints (chance constraints). The chance constraints could then be transformed to deterministic linear constraints.

Meng et al. (*35*) also modeled the uncertain demand as a random variable with known probability distribution. They maximized the expected total profit. A two-stage stochastic integer programming model was formulated. A solution algorithm, integrating the sample average approximation (SAA) with a dual decomposition and Lagrangian relaxation approach, was then proposed.

Wang et al. (*36*) extended the model of Meng et al. (*35*) by considering not only the expected profit, but also the variance of the profit in the objective function. High variance of the profit, or more exactly, the possibility of very low profit or even loss of money, is undesirable to liner shipping companies if they are risk-averse. A weighted sum of expected profit and variability of the profit is taken as the objective function.

In all the above three studies with uncertain demand, the joint probability distribution of container shipment demands of all origin-to-destination (OD) pairs must be known. This is an intimidating task, considering that a liner shipping network may have several hundred OD pairs. In reality, a liner shipping company may only be capable of predicting a few scenarios. For example, if there is an election in a country, and two parties have different economic policies, then there might be two demand scenarios associated with the export/import volume of ports in the country, depending on which party holds power. Among the predicted scenarios, only one scenario will happen. It should be noted that it may be half a year later when the fleet deployment decisions are made again, and hence the demand scenarios at that time will be considerably different. In this sense, maximizing the expected profit (or minimizing the expected cost) may not be very practical: it is impossible for a liner shipping company to evaluate whether its network planners have maximized the expected profit as only one scenario can be observed, and therefore network planners have no motivation to maximize the expected profit.

1.2 Objectives and contributions

The objective of this study is to develop a practical LSFD model for network planners. In view of the uncertain demand, and in view of the fact that exactly one demand scenario will happen, the liner shipping company may evaluate the network planners as follows. First, it implements the LSFD decision suggested by network planners. Then, in the process of operating the liner shipping services, the company observes the real demand and obtains profit. It also observes the lost demand (if the ship capacity is not sufficient) or the wasted ship slots (if the ship capacity is too large). The volume of the lost demand or the wasted ship slots could be used as an indicator to assess the network planners. In mathematical words, the evaluation indicator is the difference between the maximum possible profit (if the future demand could be exactly known) and the profit obtained based on the LSFD decision made by network planners. The former is not smaller than the latter, and hence the difference is always nonnegative. The difference depends on the LSFD decision and on the demand scenario that occurs. In this research, we assume that the network planners are pessimistic in that they aim to minimize the difference if the most undesirable scenario occurs. In other words, they make the LSFD decision that minimizes the maximum difference, i.e., the maximum regret. To reflect such a decision, we develop a minimax regret LSFD model with uncertain demand. The model also incorporates container transhipment and empty container repositioning.

Considering all the demand scenarios in the minimax regret model will inevitably increase the size of the model, especially if the number of scenarios is large. This is because the container routing decisions have to be copied for each demand scenario. However, we find that it is possible to identify the optimal LSFD decision using only a small number of scenarios. Based on this observation, we develop a dynamic scenario inclusion method that can effectively reduce the size of the problem and increase the computational efficiency.

1.3 An illustrative example

Before describing the minimax LSFD model for a general liner shipping network, we illustrate the concept using a simple example. Consider a network with only two ports p_1 and p_2, and one ship route connecting the two ports. A total of five types of ships are available: 2000-TEU (twenty-foot equivalent unit), 5000-TEU, 8000-TEU, 9000-TEU, 10000-TEU, as shown in the first row of Table 1. The deployment cost (sum of time-charter cost and voyage cost) of each type of ship (million USD/week) is shown in Row 1. The revenue for shipping one container from p_1 to p_2 is 1000 USD. There are two demand scenarios from p_1 to p_2: 2000 TEUs/week and 10000 TEUs/week. The profit (million USD/week) for each of the two scenarios with each of the five types of ships is shown in Row 2 ad Row 3 of Table 1, which is calculated by deducting the deployment cost from the revenue. For instance, under scenario 1, if 8000-TEU ships are deployed, the revenue is 2000 TEUs/week×1000 USD/TEU, the deployment cost is 2.8 million USD/week, and therefore the profit is –0.8 million USD/week. Therefore, if we knew that scenario 1 would happen, we would deploy 2000-TEU ships, and the profit would be 0.6; if we knew that scenario 2 would happen, we would deploy 10000-

TEU ships, and the profit would be 6.9.

The maximum regret for deploying each type of ship is shown in Row 4 of Table 1. Suppose that we deploy 2000-TEU ships. (i) If the demand turns out to be 2000 TEUs/week, we can gain a profit of 0.6. If we knew the demand would be 2000 TEUs/week, we would still deploy 2000-TEU ships and gain a profit of 0.6. Therefore, the regret in scenario 1 is 0. (ii) If the demand turns out to be 10,000 TEUs/week, we can gain a profit of 0.6. If we knew the demand would be 10,000 TEUs/week, we would deploy 10,000-TEU ships and gain a profit of 6.9. Therefore, the regret in scenario 2 is 6.9−0.6=6.3. Hence, the maximum regret for deploying 2000-TEU ships is max(0, 6.3)=6.3. Similarly, the maximum regrets for deploying other types of ships are all calculated and shown in Row 4 of Table 1. If we minimize the maximum regret under the two demand scenarios, we will deploy 9000-TEU ships, and the maximum regret is 1.6 (when scenario 1 happens).

<insert Table 1 here>

It should be noted that given a few demand scenarios, one might be attempted to use their average value (by assuming that the probability of each scenario is the same) as input for the LSFD. Using such a fixed demand value may not minimize the maximum regret. In the example of Table 1, the average demand is 6000 TEUs/week, and the optimal ship type under the average demand is 8000-TEU ship, as shown in Row 5, which is different from the 9000-TEU type that minimizes the maximum regret shown in Row 4.

If there are three scenarios shown in Table 2, two of which are the same as the two scenarios in Table 1 and the third of which is 6000 TEUs/week, then the optimal ship type that minimizes the maximum regret is still 9000-TEU ship, which is the same as Table 1 with only two scenarios. In other words, whether the demand scenario 6000 TEUs/week is included in modeling does not affect the optimal solution. Therefore, it is advantageous to consider only those scenarios that affect the optimal solution. However, one could not know a priori which scenarios affect the optimal solution. Hence, dynamically generating the potential scenarios that affect the optimal solution is a viable approach.

<insert Table 2 here>

This simple example shows the key idea of minimizing the maximum regret. It also demonstrates that using the average demand as the input may not minimize the maximum regret. Moreover, the example indicates that it is possible to use only a few scenarios rather than all scenarios to obtain the optimal LSFD decision.

2 Problem statement

Consider a set $\mathcal{R}$ of ship routes, regularly serving a group of ports denoted by the set $\mathcal{P}$. Ship route $r \in \mathcal{R}$ can be expressed as:

$$p_{r1} \to p_{r2} \to \cdots \to p_{rN_r} \to p_{r1} \tag{1}$$

where N_r is the number of ports of call and p_{ri} is the i th port of call, $i = 1, 2, \cdots, N_r$. Define $\mathcal{I}_r := \{1, 2, \cdots, N_r\}$. The voyage from port of call i to port of call $i+1$ is called leg i and leg N_r is the voyage from port of call N_r to the first port of call. In Fig. 1 three ship routes are shown: ship route 1 has three legs, ship route 2 has five legs, and ship route 3 has three legs. Each ship route has a weekly service frequency, which means that each port of call is visited on the same day every week.

<insert Figure 1 here>

2.1 Ship fleet

Let $\mathcal{V}$ be the set of ship types. Ships in the same type are homogeneous in terms of capacity, deployment cost, and other ship specific characteristics. The capacity of a ship of type $v \in \mathcal{V}$ is denoted by E_v (TEUs). The deployment cost of ship route $r \in \mathcal{R}$ is C_{rv} if ships in type $v \in \mathcal{V}$ are deployed on it. Exactly one type of ship is deployed on each ship route to maintain a weekly frequency.

2.2 Uncertain demand

Represent by $\mathcal{W}$ the set of OD port pairs. To simplify the notation, we assume that $\mathcal{W} = \mathcal{P} \times \mathcal{P}$: if there is no demand between a port pair, we simply set the demand at 0. To make the fleet deployment decisions, the liner shipping company predicts a set of possible demand scenarios Θ. For instance, $\Theta = \{\text{high demand, medium demand, low demand}\}$. Under scenario $\omega \in \Theta$, the demand for OD pair $(o,d) \in \mathcal{W}$ is denoted by q_{ω}^{od} (TEUs/week). Note that the uncertain demand here is different from the seasonality of the demand. For example, the demand a few weeks before Christmas is usually high. Such a pattern is known and can be dealt with by deploying extra capacity before Christmas. The uncertain demand here means the unpredictability of the demand: the LSFD decision must be made before knowing which demand scenario will occur. Uncertain demand is common in logistics studies (*37*–*39*).

The revenue for shipping a container in OD pair (o,d) is g^{od} (USD/TEU). Containers can be transshipped at any port from origins to destinations. The load, transshipment, discharge cost (USD/TEU) at port $p \in \mathcal{P}$ are denoted by $\hat{c}_p$, $\bar{c}_p$ and $\tilde{c}_p$, respectively. In reality we have

$$\bar{c}_p < \hat{c}_p + \tilde{c}_p \tag{2}$$

because liner shipping companies can choose the transshipment ports more freely than the export and import ports. The liner shipping company can fulfill any proportion of the demand.

2.3 Empty container repositioning

The liner shipping company transports not only laden containers, but also empty containers, due to the imbalance of trade. For example, if there are 2000 TEUs/week of laden container exported from Rotterdam, and 2500 TEUs/week of laden containers imported to Rotterdam, then 500 TEUs/week of empty containers are accumulated at Rotterdam, and must be repositioned to ports that are short of empty containers. We let $\hat{c}_p^E$, $\bar{c}_p^E$, and $\tilde{c}_p^E$ represent the load, transshipment, discharge cost (USD/TEU) of empty containers, respectively. Similar to laden containers, we have

$$\bar{c}_p^E < \hat{c}_p^E + \tilde{c}_p^E \tag{3}$$

The purpose of the liner shipping company is to maximize the profit, which is the revenue for shipping laden containers, minus the deployment cost of ship routes and container handling costs, while considering container transshipment and empty container repositioning in the liner shipping network.

3 Modeling and solution approach

3.1 LSFD with deterministic demand

3.1.1 Mixed-integer linear programming model

We first consider only one demand scenario $\omega \in \Theta$. Assuming that the company knows that this scenario will occur, we develop a model that obtains the LSFD decision that maximizes the profit. The purpose of knowing the profit is to calculate the regret.

The decision variables are as follows.

x_{rv}: A binary decision variable which equals 1 if ships of type $v \in \mathcal{V}$ are deployed on ship route $r \in \mathcal{R}$ and 0 otherwise;

y_{ω}^{od}: Fulfilled demand (TEUs/week) for $(o,d) \in \mathcal{W}$;

$f_{ri\omega}^{o}$: Total volume of laden containers (TEUs/week) with origin port $o \in \mathcal{P}$ and any destination that are transported on leg i of ship route $r \in \mathcal{R}$;

$\hat{z}_{ri\omega}^{o}$: Total volume of laden containers (TEUs/week) with origin port $o \in \mathcal{P}$ and any destination that are loaded onto a ship when the ship visits port of call i on ship route $r \in \mathcal{R}$;

$\tilde{z}_{ri\omega}^{o}$: Total volume of laden containers (TEUs/week) with origin port $o \in \mathcal{P}$ and any destination that are discharged from a ship when the ship visits port of call i on ship route $r \in \mathcal{R}$;

$\hat{z}_{p\omega}$: Total volume of laden containers (TEUs/week) that are exported at port $p \in \mathcal{P}$;

$\tilde{z}_{p\omega}$: Total volume of laden containers (TEUs/week) that are imported at port $p \in \mathcal{P}$;

$\overline{z}_{p\omega}$: Total volume of laden containers (TEUs/week) that are transshipped at port $p \in \mathcal{P}$;

$f^E_{ri\omega}$: Total volume of empty containers (TEUs/week) that are transported on leg i of ship route $r \in \mathcal{R}$;

$\hat{z}^E_{ri\omega}$: Total volume of empty containers (TEUs/week) that are loaded onto a ship when the ship visits port of call i on ship route $r \in \mathcal{R}$;

$\tilde{z}^E_{ri\omega}$: Total volume of empty containers (TEUs/week) that are discharged from a ship when the ship visits port of call i on ship route $r \in \mathcal{R}$;

$\hat{z}^E_{p\omega}$: Total volume of empty containers (TEUs/week) that are repositioned from port $p \in \mathcal{P}$;

$\tilde{z}^E_{p\omega}$: Total volume of empty containers (TEUs/week) that are repositioned to port $p \in \mathcal{P}$;

$\overline{z}^E_{p\omega}$: Total volume of empty containers (TEUs/week) that are transshipped at port $p \in \mathcal{P}$;

The LSFD with a given demand scenario $\omega \in \Theta$ can be formulated as:

[LSFD-ω]
$$\begin{aligned}\max \sum_{(o,d)\in\mathcal{W}} g^{od} y^{od}_{\omega} - \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}} C_{rv}x_{rv} - \sum_{p\in\mathcal{P}}\left(\hat{c}_p\hat{z}_{p\omega} + \overline{c}_p\overline{z}_{p\omega} + \tilde{c}_p\tilde{z}_{p\omega}\right) \\ - \sum_{p\in\mathcal{P}}\left(\hat{c}^E_p\hat{z}^E_{p\omega} + \overline{c}^E_p\overline{z}^E_{p\omega} + \tilde{c}^E_p\tilde{z}^E_{p\omega}\right)\end{aligned} \tag{4}$$

subject to:

$$\sum_{v\in\mathcal{V}} x_{rv} = 1, \forall r \in \mathcal{R} \tag{5}$$

$$f^o_{r,i-1,\omega} + \hat{z}^o_{ri\omega} = f^o_{ri\omega} + \tilde{z}^o_{ri\omega}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{6}$$

$$f^E_{r,i-1,\omega} + \hat{z}^E_{ri\omega} = f^E_{ri\omega} + \tilde{z}^E_{ri\omega}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{7}$$

$$\hat{z}_{p\omega} = \sum_{(p,d)\in\mathcal{W}} y^{pd}_{\omega}, \forall p \in \mathcal{P} \tag{8}$$

$$\tilde{z}_{p\omega} = \sum_{(o,p)\in\mathcal{W}} y^{op}_{\omega}, \forall p \in \mathcal{P} \tag{9}$$

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r, p_{ri}=p}\sum_{o\in\mathcal{P}} \hat{z}^o_{ri\omega} = \overline{z}_{p\omega} + \hat{z}_{p\omega}, \forall p \in \mathcal{P} \tag{10}$$

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r, p_{ri}=p}\sum_{o\in\mathcal{P}} \tilde{z}^o_{ri\omega} = \overline{z}_{p\omega} + \tilde{z}_{p\omega}, \forall p \in \mathcal{P} \tag{11}$$

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r, p_{ri}=p}\left(\hat{z}^E_{ri\omega} - \tilde{z}^E_{ri\omega}\right) = \sum_{(o,p)\in\mathcal{W}} y^{op}_{\omega} - \sum_{(p,d)\in\mathcal{W}} y^{pd}_{\omega}, \forall p \in \mathcal{P} \tag{12}$$

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r, p_{ri}=p} \hat{z}^E_{ri\omega} = \overline{z}^E_{p\omega} + \hat{z}^E_{p\omega}, \forall p \in \mathcal{P} \tag{13}$$

$$\sum_{r\in\mathcal{R}}\sum_{i\in I_r,p_{ri}=p}\tilde{z}^E_{ri\omega}=\overline{z}^E_{p\omega}+\tilde{z}^E_{p\omega},\forall p\in\mathcal{P} \tag{14}$$

$$\sum_{r\in\mathcal{R}}\sum_{i\in I_r,p_{ri}=p}\left(\hat{z}^o_{ri\omega}-\tilde{z}^o_{ri\omega}\right)=\begin{cases}\sum_{(o,d)\in\mathcal{W}}y^{od}_\omega, p=o\\ -y^{op}_\omega,\text{otherwise}\end{cases},\forall o\in\mathcal{P},\forall p\in\mathcal{P} \tag{15}$$

$$\sum_{o\in\mathcal{P}}f^o_{ri\omega}+f^E_{ri\omega}\le\sum_{v\in\mathcal{V}}E_v x_{rv},\forall r\in\mathcal{R},\forall i\in I_r \tag{16}$$

$$y^{od}_\omega\le q^{od}_\omega,\forall(o,d)\in\mathcal{W} \tag{17}$$

$$x_{rv}\in\{0,1\},\forall r\in\mathcal{R},\forall v\in\mathcal{V} \tag{18}$$

$$\hat{z}^o_{ri\omega}\ge 0,\tilde{z}^o_{ri\omega}\ge 0,f^o_{ri\omega}\ge 0,\forall r\in\mathcal{R},\forall i\in I_r,\forall o\in\mathcal{P} \tag{19}$$

$$\hat{z}^E_{ri\omega}\ge 0,\tilde{z}^E_{ri\omega}\ge 0,f^E_{ri\omega}\ge 0,\forall r\in\mathcal{R},\forall i\in I_r \tag{20}$$

$$\hat{z}^E_{p\omega}\ge 0,\overline{z}^E_{p\omega}\ge 0,\tilde{z}^E_{p\omega}\ge 0,\forall p\in\mathcal{P} \tag{21}$$

$$y^{od}_\omega\ge 0,\forall(o,d)\in\mathcal{W} \tag{22}$$

The objective function (4) maximizes the total profit under the demand scenario: the first term is the total revenue, the second term is the deployment cost of ship routes, the third term is the total laden container handling cost, and the fourth term is the total empty container handling cost. Constraint (5)–(22) are straightforward.

3.1.2 Formulation of empty container repositioning

In this sub-section we prove that constraints (12)–(14) correctly capture the total volume of loaded, discharged, and transshipped empty containers at each port. At a port $p\in\mathcal{P}$, the total volume of laden containers exported is $\sum_{(p,d)\in\mathcal{W}}y^{pd}$, and the total volume of laden containers imported is $\sum_{(o,p)\in\mathcal{W}}y^{op}$. If the former is larger, port p has a shortage of $\sum_{(p,d)\in\mathcal{W}}y^{pd}-\sum_{(o,p)\in\mathcal{W}}y^{op}$ empty containers; if the latter is larger, port p has a surplus of $\sum_{(o,p)\in\mathcal{W}}y^{op}-\sum_{(p,d)\in\mathcal{W}}y^{pd}$ empty containers. Consequently, the volumes of empty containers repositioned to and from a port should be computed as follows:

$$\begin{cases} \begin{Bmatrix} \hat{z}^E_{p\omega} = 0 \\ \tilde{z}^E_{p\omega} = -\sum_{(o,p)\in\mathcal{W}} y^{op} + \sum_{(p,d)\in\mathcal{W}} y^{pd} \end{Bmatrix}, \text{ if } \sum_{(o,p)\in\mathcal{W}} y^{op} - \sum_{(p,d)\in\mathcal{W}} y^{pd} < 0 \\ \begin{Bmatrix} \hat{z}^E_{p\omega} = \sum_{(o,p)\in\mathcal{W}} y^{op} - \sum_{(p,d)\in\mathcal{W}} y^{pd} \\ \tilde{z}^E_{p\omega} = 0 \end{Bmatrix}, \text{ if } \sum_{(o,p)\in\mathcal{W}} y^{op} - \sum_{(p,d)\in\mathcal{W}} y^{pd} \geq 0 \end{cases}, \forall p \in \mathcal{P} \tag{23}$$

At a port $p \in \mathcal{P}$, the total number of empty container loading operations is $\sum_{r\in\mathcal{R}}\sum_{i\in I_r, p_{ri}=p} \hat{z}^E_{ri\omega}$, and the total number of empty container discharging operations is $\sum_{r\in\mathcal{R}}\sum_{i\in I_r, p_{ri}=p} \tilde{z}^E_{ri\omega}$. If the former is larger, the port has a total of $\sum_{r\in\mathcal{R}}\sum_{i\in I_r, p_{ri}=p}\left(\hat{z}^E_{ri\omega} - \tilde{z}^E_{ri\omega}\right)$ surplus empty containers, and the total number of transshipment empty containers is $\sum_{r\in\mathcal{R}}\sum_{i\in I_r, p_{ri}=p} \tilde{z}^E_{ri\omega}$. If the latter is larger, the port has a shortage of $\sum_{r\in\mathcal{R}}\sum_{i\in I_r, p_{ri}=p}\left(\tilde{z}^E_{ri\omega} - \hat{z}^E_{ri\omega}\right)$ empty containers, and the total number of transshipment empty containers is $\sum_{r\in\mathcal{R}}\sum_{i\in I_r, p_{ri}=p} \hat{z}^E_{ri\omega}$. Therefore, the total number of transshipment empty containers is

$$\bar{z}^E_{p\omega} = \min\left\{\sum_{r\in\mathcal{R}}\sum_{i\in I_r, p_{ri}=p} \hat{z}^E_{ri\omega}, \sum_{r\in\mathcal{R}}\sum_{i\in I_r, p_{ri}=p} \tilde{z}^E_{ri\omega}\right\}, \forall p \in \mathcal{P} \tag{24}$$

Eqs. (23)–(24) are nonlinear due to "if" and "min", and it is computationally expensive to linearize them using the big-M method. Hence, we do not use them in model [LSFD-ω]. The correctness of the mixed-integer linear programming model [LSFD-ω] lies in the following proposition, which can be easily proved:

Proposition 1: Constraints (12)–(14) and the nonnegativity constraint (21) correctly capture the total volume of surplus, deficit, and transshipped empty containers at each port.

In sum, constraints (12)–(14) linearizes Eqs. (23)–(24) without introducing any integer variable. Such a nice result is derived from the practical liner shipping characteristic Eq. (3). This property enables us to develop the simple model [LSFD-ω] and the subsequent minimax regret model.

3.2 Minimax regret model

Now we examine how to minimize the maximum regret in view of the demand scenarios Θ. Let vector $\mathbf{x} := (x_{rv}, r \in \mathcal{R}, v \in \mathcal{V})$ be the fleet deployment decision. The domain of $\mathbf{x}$ is

$$X = \left\{ \mathbf{x} \mid \sum_{v \in \mathcal{V}} x_{rv} = 1, \forall r \in \mathcal{R}; x_{rv} \in \{0,1\}, \forall r \in \mathcal{R}, \forall v \in \mathcal{V} \right\} \tag{25}$$

Let $C(\mathbf{x}, \omega)$ be the maximum profit under demand scenario ω with fleet deployment decision $\mathbf{x}$, $\mathbf{x} \in X$, $\omega \in \Theta$. In other words, $C(\mathbf{x}, \omega)$ is the optimal objective value of model [LSFD-ω] with fixed fleet deployment decision $\mathbf{x}$. Define vector $\mathbf{x}(\omega)$ as the optimal fleet deployment decision under demand scenario ω. That is,

$$\mathbf{x}(\omega) \in \arg\max_{\mathbf{x} \in X} C(\mathbf{x}, \omega), \omega \in \Theta \tag{26}$$

We further let $C(\omega)$ be the maximum profit under demand scenario ω. That is,

$$C(\omega) = \max_{\mathbf{x} \in X} C(\mathbf{x}, \omega), \omega \in \Theta \tag{27}$$

In other words, $\mathbf{x}(\omega)$ is the optimal fleet deployment solution to [LSFD-ω], and $C(\omega)$ is the optimal objective value of [LSFD-ω].

To minimize the maximum regret, we formulate the following model:

[LSFD-Θ1]

$$\min_{\mathbf{x} \in X} \max_{\omega \in \Theta} \left[C(\omega) - C(\mathbf{x}, \omega) \right] \tag{28}$$

where the term $\max_{\omega \in \Theta} \left[C(\omega) - C(\mathbf{x}, \omega) \right]$ is the maximum regret under all scenarios with a given fleet deployment decision $\mathbf{x}$.

[LSFD-Θ1] can be linearized. Define decision variable U as the maximum regret, model [LSFD-Θ1] is equivalent to

[LSFD-Θ2]

$$\min_{\mathbf{x} \in X} U \tag{29}$$

subject to:

$$U \geq C(\omega) - C(\mathbf{x}, \omega), \forall \omega \in \Theta \tag{30}$$

The value of $C(\omega)$ can be obtained by solving [FD-ω]. Hence, [LSFD-Θ2] is a mixed-integer linear programming model, and it can be written as:

[LSFD-Θ]

$$\min U \tag{31}$$

subject to:

$$U \geq C(\omega) - \left[\begin{array}{c} \sum_{(o,d) \in \mathcal{W}} g^{od} y_{\omega}^{od} - \sum_{r \in \mathcal{R}} \sum_{v \in \mathcal{V}} C_{rv} x_{rv} - \sum_{p \in \mathcal{P}} \left(\hat{c}_p \hat{z}_{p\omega} + \overline{c}_p \overline{z}_{p\omega} + \tilde{c}_p \tilde{z}_{p\omega} \right) \\ - \sum_{p \in \mathcal{P}} \left(\hat{c}_p^E \hat{z}_{p\omega}^E + \overline{c}_p^E \overline{z}_{p\omega}^E + \tilde{c}_p^E \tilde{z}_{p\omega}^E \right) \end{array} \right], \forall \omega \in \Theta \tag{32}$$

$$\sum_{v \in \mathcal{V}} x_{rv} = 1, \forall r \in \mathcal{R} \tag{33}$$

$$f^{o}_{r,i-1,\omega} + \hat{z}^{o}_{ri\omega} = f^{o}_{ri\omega} + \tilde{z}^{o}_{ri\omega}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P}, \forall \omega \in \Theta \tag{34}$$

$$f^{E}_{r,i-1,\omega} + \hat{z}^{E}_{ri\omega} = f^{E}_{ri\omega} + \tilde{z}^{E}_{ri\omega}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall \omega \in \Theta \tag{35}$$

$$\hat{z}_{p\omega} = \sum_{(p,d) \in \mathcal{W}} y^{pd}_{\omega}, \forall p \in \mathcal{P}, \forall \omega \in \Theta \tag{36}$$

$$\tilde{z}_{p\omega} = \sum_{(o,p) \in \mathcal{W}} y^{op}_{\omega}, \forall p \in \mathcal{P}, \forall \omega \in \Theta \tag{37}$$

$$\overline{z}_{p\omega} = \sum_{r \in \mathcal{R}} \sum_{i \in I_r, p_{ri} = p} \sum_{o \in \mathcal{P}} \hat{z}^{o}_{ri\omega} - \hat{z}_{p\omega}, \forall p \in \mathcal{P}, \forall \omega \in \Theta \tag{38}$$

$$\sum_{r \in \mathcal{R}} \sum_{i \in I_r, p_{ri} = p} \left(\hat{z}^{E}_{ri\omega} - \tilde{z}^{E}_{ri\omega} \right) = \sum_{(o,p) \in \mathcal{W}} y^{op} - \sum_{(p,d) \in \mathcal{W}} y^{pd}, \forall p \in \mathcal{P}, \forall \omega \in \Theta \tag{39}$$

$$\sum_{r \in \mathcal{R}} \sum_{i \in I_r, p_{ri} = p} \hat{z}^{E}_{ri\omega} = \overline{z}^{E}_{p\omega} + \hat{z}^{E}_{p\omega}, \forall p \in \mathcal{P}, \forall \omega \in \Theta \tag{40}$$

$$\sum_{r \in \mathcal{R}} \sum_{i \in I_r, p_{ri} = p} \tilde{z}^{E}_{ri\omega} = \overline{z}^{E}_{p\omega} + \tilde{z}^{E}_{p\omega}, \forall p \in \mathcal{P}, \forall \omega \in \Theta \tag{41}$$

$$\sum_{r \in \mathcal{R}} \sum_{i \in I_r, p_{ri} = p} \left(\hat{z}^{o}_{ri} - \tilde{z}^{o}_{ri} \right) = \begin{cases} \sum_{(o,d) \in \mathcal{W}} y^{od}, p = o \\ -y^{op}, \text{otherwise} \end{cases}, \forall o \in \mathcal{P}, \forall p \in \mathcal{P}, \forall \omega \in \Theta \tag{42}$$

$$\sum_{o \in \mathcal{P}} f^{o}_{ri\omega} + f^{E}_{ri\omega} \le \sum_{v \in \mathcal{V}} E_v x_{rv}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall \omega \in \Theta \tag{43}$$

$$y^{od}_{\omega} \le q^{od}_{\omega}, \forall (o,d) \in \mathcal{W}, \forall \omega \in \Theta \tag{44}$$

$$x_{rv} \in \{0,1\}, \forall r \in \mathcal{R}, \forall v \in \mathcal{V} \tag{45}$$

$$\hat{z}^{o}_{ri\omega} \ge 0, \tilde{z}^{o}_{ri\omega} \ge 0, f^{o}_{ri\omega} \ge 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P}, \forall \omega \in \Theta \tag{46}$$

$$\hat{z}^{E}_{ri\omega} \ge 0, \tilde{z}^{E}_{ri\omega} \ge 0, f^{E}_{ri\omega} \ge 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall \omega \in \Theta \tag{47}$$

$$\hat{z}^{E}_{p\omega} \ge 0, \overline{z}^{E}_{p\omega} \ge 0, \tilde{z}^{E}_{p\omega} \ge 0, \forall p \in \mathcal{P}, \forall \omega \in \Theta \tag{48}$$

$$y^{od}_{\omega} \ge 0, \forall (o,d) \in \mathcal{W}, \forall \omega \in \Theta \tag{49}$$

The solution method for the LSFD with uncertain demand can be summarized as follows:

Step 1. For each demand scenario $\omega \in \Theta$, solve [LSFD- ω] and let $C(\omega)$ represent the optimal objective value (maximum possible profit).

Step 2. Using the $C(\omega)$ obtained in Step 1, solve model [LSFD-Θ]. Its optimal objective value is the maximum regret. Its optimal fleet deployment decision is the fleet deployment decision that could minimize the maximum regret.□

3.3 Dynamic scenario inclusion based method

If the number of scenarios in Θ is too large, model [LSFD-Θ] may have too many variables and hence be difficult to solve (*40–43*). To overcome this difficulty, we find that not all scenarios will lead to the maximum regret. In other words, some scenarios can be dropped without affecting the optimal fleet deployment decision.

However, we do not know a priori which demand scenarios do not affect the optimal fleet deployment decision. A possible approach is to use a few scenarios and obtain the fleet deployment decision. Under such a fleet deployment decision, if there exists a scenario that is not included in the model while leading to a larger regret, then this scenario should also be used. This process is repeated until no scenario that is not used by the model would lead to a larger regret. Formally, such a dynamic scenario inclusion based method is stated below:

Dynamic scenario inclusion based method:

Step 0. For each demand scenario $\omega \in \Theta$, solve [LSFD-ω] and let $C(\omega)$ represent the optimal objective value.

Step 1. Define a set of scenarios to be used denoted by $\bar{\Theta}$, $\bar{\Theta} \subseteq \Theta$. Arbitrarily choose one element from Θ, denoted by ω_1, and set $\bar{\Theta} := \{\omega_1\}$.

Step 2. Solve model [LSFD-$\bar{\Theta}$]. Note that model [LSFD-$\bar{\Theta}$] is obtained by replacing Θ in model [LSFD-Θ] by $\bar{\Theta}$. Hence, the number of scenarios to be considered in [LSFD-$\bar{\Theta}$] is $|\bar{\Theta}|$. The optimal fleet deployment decision to model [LSFD-$\bar{\Theta}$] is denoted by $\mathbf{x}^*$, and the optimal objective value is denoted by U^*.

Step 3. Compute the value of $C(\mathbf{x}^*, \omega)$ for each of the scenarios $\omega \in \Theta \setminus \bar{\Theta}$ that are not included in the model [LSFD-$\bar{\Theta}$]. $C(\mathbf{x}^*, \omega)$ is the optimal objective value of model [LSFD-ω] when $\mathbf{x}$ is fixed at $\mathbf{x}^*$. Define

$$\omega^* \in \arg\max_{\omega \in \Theta \setminus \bar{\Theta}} \left[C(\omega) - C(\mathbf{x}^*, \omega) \right] \tag{50}$$

If

$$U^* < C(\omega^*) - C(\mathbf{x}^*, \omega^*) \tag{51}$$

then the regret in scenario ω^* with the fleet deployment decision $\mathbf{x}^*$ is larger than that of any scenario in $\bar{\Theta}$, and therefore we let $\bar{\Theta} := \bar{\Theta} \bigcup \{\omega^*\}$ and go to Step 2. Otherwise $\mathbf{x}^*$ is the optimal solution to [LSFD-Θ], U^* is the smallest maximum regret of all scenarios in Θ, and stop.□

Since in each iteration one scenario is added to set $\bar{\Theta}$, U^* is non-decreasing throughout the algorithm. Moreover, since Θ is finite, the dynamic scenario inclusion based method terminates at most after $|\Theta|$ iterations. The novelty of the method is that the cardinality of the set $\bar{\Theta}$ is usually much smaller than that of Θ. Therefore, model [LSFD-$\bar{\Theta}$] could be much more efficiently solved than model [LSFD-Θ]. Although model [LSFD-$\bar{\Theta}$] needs to be solved several times (model [LSFD-ω] with fixed $\mathbf{x}$ is a linear program and hence the computation time of model (50) can be ignored), the overall computation time of the dynamic scenario inclusion based method may still be much shorter than directly solving model [LSFD-Θ].

4 Case study

To test the validity and applicability of the proposed models and solution methods, we carry out numerical experiments based on an Asia-Europe-Oceania liner shipping network provided by a global liner shipping company. The mixed-integer linear programming models [LSFD-ω], [LSFD-Θ], and [LSFD-$\bar{\Theta}$] are solved by CPLEX-12.2, which is called by matlab, running on a 2.8 GHz 4 Core PC with 8 GB of RAM and 64-bit Windows 7 Professional operating system.

4.1 Description of the network

The Asia-Europe-Oceania liner shipping network considered consists of 46 ports and these port are the same as Wang and Meng (*15*). Eleven ship routes are operated in the network. We consider a total of three types of ships: 3000-TEU, 5000-TEU and 10000-TEU, whose deployment costs are 76,900, 115,400, and 173,100 USD/week, respectively. The container handling costs at all ports are the same. The loading, unloading, and transshipment costs of laden containers are 100, 100, and 150 USD/TEU, respectively; the loading, unloading, and transshipment costs of empty containers are 80, 80, and 100 USD/TEU, respectively. There are a total of 652 OD pairs in the network.

4.2 Case study with three demand scenarios

We first randomly generate three demand scenarios. The total volumes of container shipment demand in the three scenarios are shown in the second row of Table 3. We solve [LSFD- ω] for each demand scenario $\omega \in \Theta$. The results in Table 3 show that a significant volume of laden and empty containers are transshipped. This demonstrates the importance of incorporating container transshipment in the model. Moreover, we can see that the volume of empty containers to be transported is about half as large as laden containers, which indicates that LSFD should incorporate empty container repositioning in the model. The bottom part of Table 3 shows the optimal ship type to deploy on each ship route under each demand scenario.

We then solve model [LSFD- Θ], and the resulting minimax regret is 0.73 million USD/week. The optimal fleet deployment is the same as the last column of Table 3. Hence, there is no regret if demand scenario 3 happens. If demand scenario 1 happens, the profit will be 25.80, which leads to the regret of 0.73. If demand scenario 2 happens, the profit will be 38.53[1], resulting in a regret of 0.34. Therefore, model [LSFD- Θ] have correctly obtained the fleet deployment decision that minimizes the maximum regret.

<insert Table 3 here>

[1] It should be noted that this value is not the same as the term in the square bracket of Eq. (32) at the optimal solution to [LSFD- Θ]. This is because scenario 2 does not lead to the maximum regret at the optimal fleet deployment decision $\mathbf{x}^*$, and hence, as long as the solver finds a solution to scenario 2 such that the regret is smaller than the regret of the optimal solution to scenario 1, i.e., $C(\omega_1) - C(\mathbf{x}^*, \omega_1)$, the computation will stop. In other words, the container routing decisions obtained by [LSFD- Θ] are guaranteed to be optimal for the scenario with maximum regret (scenario 1 in this example), but may not be optimal for other scenarios. The value 38.53 is obtained by solving model [LSFD- ω_2] while fixing $\mathbf{x}$ at $\mathbf{x}^*$.

4.3 Computation time of the dynamic scenario inclusion method

We conduct more numerical experiments to test the efficiency of the dynamic scenario inclusion method. The shipping company uses many different demand forecasting tools or hires many consulting teams. Each method/team provides 652 estimates for the 652 OD pairs, the overall of which corresponds to one demand scenario. We thus randomly generate 3, 5, 10, 15, 20, and 50 demand scenarios. The computational time is shown in Table 4. We can see that the dynamic scenario inclusion method can handle a maximum of 20 scenarios. This is sufficient for practical purposes as it is hardly possible for a liner shipping company to hire more than 20 consulting teams to predict the demand for each of the 652 OD pairs.

<insert Table 4 here>

We discuss some details about the case with 10 demand scenarios. We first arbitrarily include one demand scenario in $\bar{\Theta}$, and solve [LSFD-$\bar{\Theta}$] (we will have $U^* = 0$). Based on the obtained fleet deployment decision, we find the scenario with the maximum regret (13.82), greater than U^*. Hence, we incorporate this scenario in $\bar{\Theta}$, and solve [LSFD-$\bar{\Theta}$] again. We obtain $U^* = 0.85$. Based on the obtained fleet deployment decision, no scenario in $\Theta \backslash \bar{\Theta}$ could lead to a larger regret than U^*. Hence, we have obtained the optimal LSFD solution.

5 Conclusions and future work

We have presented a minimax regret model for liner shipping fleet deployment (LSFD) with uncertain demand. The advantage of the minimax regret model is that it does not need the probability distribution function of the demand, and it is consistent with how network planners are evaluated. As a consequence, the model is practical for network planners to deal with uncertain demand.

The idea of the minimax regret model could also be used in other planning problems in liner shipping. For example, almost all existing studies on liner shipping network design use fixed demand as input except Lo et al. (*44*) on ferry network design. If the uncertain demand scenarios are to be incorporated, a minimax regret network design problem would be of interest for liner shipping companies. Another potential venue is container flow optimization for analyzing world trade and port capacity (*45*). Using minimax regret model could help port owners to determine the expansion of capacity under uncertainty.

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List of Tables and Figures

Table 1 The profit under each case in the illustrative example with two demand scenarios

	Ship type	2000-TEU	5000-TEU	8000-TEU	9000-TEU	10,000-TEU	Optimal ship
Row 1	Deployment cost	1.4	2.2	2.8	3.0	3.1	
Row 2	Scenario 1: 2000 TEUs/week	0.6	−0.2	−0.8	−1.0	−1.1	2000-TEU
Row 3	Scenario 2: 10,000 TEUs/week	0.6	2.8	5.2	6.0	6.9	10000-TEU
Row 4	Maximum regret with scenarios 1 & 2	6.3	4.1	1.7	1.6	1.7	9000-TEU
Row 5	Average demand: 6000 TEUs/week	0.6	2.8	3.2	3.0	2.9	8000-TEU

Table 2 The profit under each case in the illustrative example with three demand scenarios

	Ship type	2000-TEU	5000-TEU	8000-TEU	9000-TEU	10000-TEU	Optimal ship
Row 1	Deployment cost	1.4	2.2	2.8	3.0	3.1	
Row 2	Scenario 1: 2000 TEUs/week	0.6	−0.2	−0.8	−1.0	−1.1	2000-TEU
Row 3	Scenario 2: 10000 TEUs/week	0.6	2.8	5.2	6.0	6.9	10000-TEU
Row 4	Scenario 3: 6000 TEUs/week	0.6	2.8	3.2	3.0	2.9	8000-TEU
Row 5	Maximum regret with scenarios 1 & 2 & 3	6.3	4.1	1.7	1.6	1.7	9000-TEU

Table 3 Optimal solution to models [LSFD- ω] under each demand scenario

Scenario ω	1	2	3
Total demand $\sum q_{\omega}^{od}$	15,974	22,942	29,489
Maximum profit $C(\omega)$ (million USD/week)	26.53	38.87	50.73
Fulfilled demand $\sum y_{\omega}^{od}$	15,974	22,942	29,432
Laden transshipment $\sum \bar{z}_{p\omega}$	16,546	21,540	31,962
Empty volume $\sum_{p\in\mathcal{P}} \left| \sum_{(o,p)\in\mathcal{W}} y^{op} - \sum_{(p,d)\in\mathcal{W}} y^{pd} \right|$	7,703	10,685	13,865
Empty transshipment $\sum \bar{z}_{p\omega}^{E}$	4,292	5,605	9,013
Ship route 1	3000-TEU	3000-TEU	3000-TEU
Ship route 2	5000-TEU	3000-TEU	5000-TEU
Ship route 3	3000-TEU	3000-TEU	3000-TEU
Ship route 4	3000-TEU	3000-TEU	3000-TEU
Ship route 5	3000-TEU	3000-TEU	3000-TEU
Ship route 6	3000-TEU	3000-TEU	3000-TEU
Ship route 7	3000-TEU	3000-TEU	3000-TEU
Ship route 8	3000-TEU	3000-TEU	3000-TEU
Ship route 9	3000-TEU	3000-TEU	3000-TEU
Ship route 10	3000-TEU	5000-TEU	10000-TEU
Ship route 11	5000-TEU	10000-TEU	10000-TEU

Table 4 Computation time for different numbers of demand scenarios

Number of scenarios	3	5	10	15	20	50
Computation time (s)	42	87	215	1120	13,034	>36,000

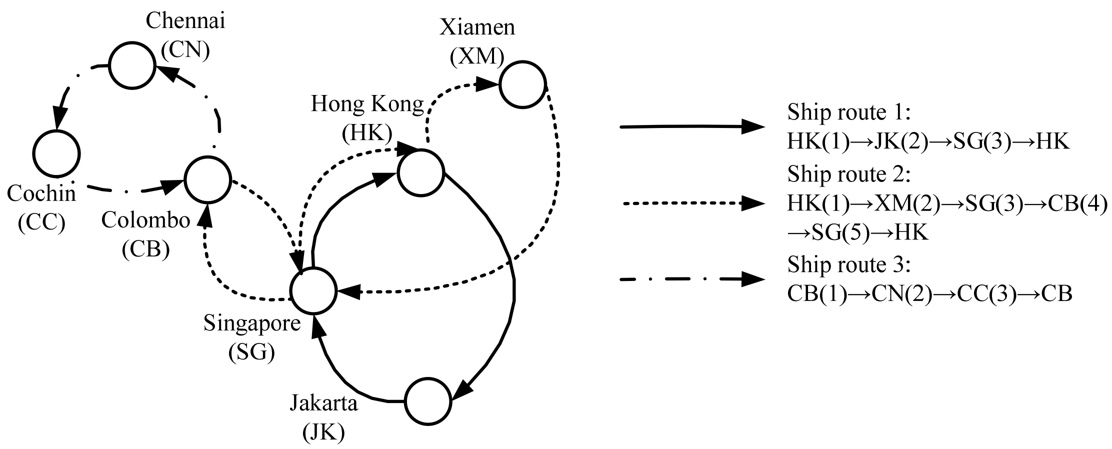

Fig. 1 An illustrative liner shipping network (*22*)