A joint optimization model for liner container cargo assignment problem using state-augmented shipping network framework

Abstract

This paper proposes a state-augmented shipping (SAS) network framework to explicitly integrate various activities in the liner container shipping chain, including container loading/unloading, transshipment, dwelling at visited ports, in-transit waiting (transshipment waiting and delivery waiting at origin port) and in-sea transport process together. Based on the SAS network framework, we develop a chance-constrained optimization model for the joint cargo assignment problem. The model attempts to maximize the carrier's profit by simultaneously determining the optimal ship fleet capacity setting, ship route schedules and cargo allocation scheme. With a few disparities from previous studies, we take into account two differentiated container demands: deterministic contracted basis demand received from large manufacturers and uncertain individual demand collected from the spot market. The economies of scale of ship size are incorporated to examine the scaling effect of ship capacity setting in the cargo assignment problem. Meanwhile, the schedule coordination strategy is introduced to precisely measure the in-transit waiting time and resultant inventory cost. Through numerical examples, it is demonstrated that the proposed chance-constrained joint optimization model can well characterize the impact of carrier's risk preference on decisions of the container cargo allocation. Moreover, considering the scaling effect of large ships can in certain degree alleviate the concern of cargo overload rejection and consequently help carriers make more promising ship deployment schemes.

Keywords: state-augmented shipping network, cargo assignment, economies of scale of ship size, uncertain demand, schedule coordination

1. Introduction

In past decades, we have witnessed a booming development of global trade and economy, and already realized that maritime freight transportation turns to be more and more important for promoting the world trade. Every year, more than million-ton productions are containerized and delivered among worldwide consumptive markets. According to the statistical report of maritime transportation review (UNCTAD, 2014), among all the seaborne cargoes, more than 50% in dollar terms are transported by the container shipping service, and the global containerized trade grew by 4.6% in 2013 taking total volumes to 160 million Twenty-foot Equivalent Units (TEUs), up from 153 million TEUs in 2012. Allured by a prosperous prospect of the growing shipping market, shipping companies would like to quickly expand related container shipping business, and consequently bring about intense competitions in this active market. In order to enhance their competitiveness, the shipping companies (carriers) desire to design more effective and efficient cargo allocation schemes to maximize their profits, particularly combining with other joint management strategies.

The cargo assignment/allocation problem, also called classical multi-commodity flow (MCF) problem, is not a new emerging research topic. Since the recent half century, a large number of researchers have put their efforts on this problem, especially in the urban and airline freight transportation systems. Without exception, the container cargo allocation problem has attracted much attention as well in the maritime studies, for example, Ronen (1983), Christiansen et al. (2004), Hsu and Hsieh (2007), Zeng et al. (2010), Brouer et al. (2011), Bell et al. (2011), Song and Dong (2012), Christiansen et al. (2013), Bell et al. (2013), Lin and Tsai (2014),Wang et al. (2014b), and Karsten et al. (2015), to name but a few. For instance, Hsu and Hsieh (2007) formulated a bi-objective decision-making model for a hub-and-spoke container network by optimizing liner routing, ship size and service frequency. Brouer et al. (2011) investigated the cargo allocation problem subject to the availability of empty containers and put forward both arc-flow and path-flow formulations. Bell et al. (2011) proposed a frequency-based liner container assignment model incorporating empty container repositioning with an objective to minimize the total transport time of all container shipments. Subsequently, Bell et al. (2013) developed a cost-based liner container assignment model that minimizes the total shipping cost including container handling costs, container rental and inventory costs. Song and Dong (2012) discussed a joint optimization problem of cargo allocation and empty container repositioning in the operational level planning for a shipping network constituting with multi-route, multiple vessel types and multi-voyage. Lin and Tsai (2014) studied the ship routing and freight assignment problem in liner shipping along the Pacific Rim. Wang et al. (2014b) proposed a simultaneous cargo allocation and ship schedule coordination model in order to evaluate an in-transit inventory cost resulted from extra delivery waiting time. In addition to the above deterministic cargo assignment models, a few researchers also paid attention to developing the related models under demand uncertainty, e.g., the robust resource allocation model for a pendulum shipping line network (Zeng et al., 2010). Furthermore, empty container repositioning problem, as an endogenous allocation subproblem of specific empty containers, has also been emphasized in the literature (Li et al., 2007; Lam et al., 2007; Dong and Song, 2009; Brouer et al., 2011; Song and Dong, 2012) [Hans: check Brouer et al., 2011, it is the wrong paper!]. For instance, Brouer et al. (2011) formulated a dynamic stochastic empty container repositioning model and solved it by a simulation based approach in a small two–ports and two–voyages network [Hans: I am sure this is the wrong paper!]. To account for the inter-balance of empty container repositioning demand, Lam et al. (2007) investigated the cargo allocation problem by using an approximate dynamic programming method.

In previous studies, quite a few researchers have pointed out that the container cargo assignment problem refers to a series of container shipment activities, including container loading/unloading, inter-transshipment among different ship routes and/or intra-transshipment of the same ship route at revisiting, dwelling at the visited port, in-transit waiting (transshipment waiting and delivery waiting at the origin port) and in-sea transport etc. For example, Bell et al. (2013) considered the container handling costs, rental and inventory costs as well as in-sea shipping costs in the design objective function of the cargo assignment problem. Song and Dong (2012) integrated the customer demand backlog costs, in-transit inventory costs at the transshipment ports as vital cost components when making cargo allocation decisions. Wang et al. (2014b) pointed out the importance of optimizing ship route schedules so as to address extra container inventory costs of waiting for delivery. Recently, Brouer et al.

(2014) recommended a conceptual framework of building dummy links to describe container transshipment operations [Hans: Are you sure it is this paper? Or is it the attached paper "The time constrained multi-commodity network flow problem"?]. However, to the best of our knowledge, no existing network establishment approach is provided to make an explicit integration of various operations of cargo shipments from the viewpoint of container shipping activity chain. An integrated shipping network would help decision-makers precisely estimate the total cost of container cargo shipments and facilitate various applications of shipping network optimization, e.g., shipping network design and shipping revenue management. To bridge this gap, this study aims to propose a state-augmented shipping (SAS) network framework to build the integrated liner container shipping network.

Typically, the cargo allocation problem is categorized into the operational-level decision-making and can be further considered as a subproblem in many tactical-level decision problems, for example, in the network design problem (e.g., Shintani et al., 2007; Agarwal and Ergun, 2008; Imai et al., 2009; Meng and Wang, 2011; Chen et al., 2014; Brouer et al., 2014), ship route schedule design (e.g., Wang and Meng, 2012) and fleet deployment optimization problem (e.g., Fagerholt, 1999). Although we have rough definitions of the decision-makings in different planning levels, much of the literature has indicated that the decisions made in different planning levels always intertwine together (e.g., Christiansen et al., 2004; Agarwal and Ergun, 2008). For example, a tactical-level setting of ship route schedules invariably affects the results of operational-level cargo allocation. In this study, we still focus on discussing the cargo assignment problem in the tactical-level by simultaneously considering other joint optimization strategies in the shipping service.

Recently, more and more evidence shows that interest in the research area of cargo allocation or related ship routing/scheduling problems is dramatically increasing. Rather than revisiting the literature once again, we recommend a few seminal reviews, including Ronen (1983) and Ronen (1993) for early developments, and Christiansen et al. (2004), Christiansen et al. (2013), Meng et al. (2014) for the advances in the recent decade. Our aim here focuses on discussing several important and practical issues that may not receive sufficient attention in the literature. We start with the current situation of container cargo demand. Almost all previous studies assumed that the container cargoes were collected from a homogeneous shipping market. We can consider that a carrier receives fixed container cargoes subject to long-term contracts from large manufacturers, and meanwhile collects individual container cargoes from the spot market. It is not surprising that two categories of demands would exhibit quite diverse features. The contracted container cargoes are generally shipped with low freight rates; whereas shippers in the spot market need to pay high service charges. Moreover, different from the former deterministic contracted basis demand, the daily individual demand collected from the spot market may inherently have high uncertainty. With the above disparities, we need to distinguish these two kinds of cargo demands in the cargo assignment problem, and should notice that the arising uncertain individual demand gives rise to new challenges in model formulation and impact evaluation of the carrier's risk preference.

Another important issue is the economies of scale of large ships. The value of the scaling effect of ship capacity setting has not been sufficiently investigated at the current stage. The first quantitative assessment on the scaling effect of container ship size was carried out by Cullinane and Khanna (1999). In their work, some useful aggregated shipping cost functions by taking into account the economies of scale of large container ships were derived based on the analysis of ship log historical data. Subsequently, Imai et al. (2006) discussed the container mega-ship viability by a comparison analysis between two scenarios: hub-and-spoke network for mega-ship and multi-port calling network for conventional ship size. Song and Dong (2012) took into account multi-vessel in the cargo allocation problem, but the economies of scale of ship size were not addressed yet. Wang et al. (2014a) stressed the importance of addressing economies of scale of large container ships in decision-making and considered the optimization of ship capacity setting as one of crucial game strategies in the marketing competition analysis. Evidently, the scaling effect of ship capacity setting is vital to the tactical-level cargo allocation problem. Considering the scaling effect of different ship fleet capacity settings in the cargo allocation scheme is rewarding to the carrier in the following two aspects. First, the differentiated shipping costs with respect to different ship fleet capacity settings generate a more precise estimation of the carrier's profit, which can assist the carrier to make tangible operational schemes and reasonable management policies.

Second, a joint optimization combining ship fleet capacity setting provides a tractable method to resolve the cargo overload rejection problem existing in the cargo allocation decision. The cargo overload rejection issue has not attracted sufficient attention yet in developing cargo assignment models. Wang et al. (2014b) found that some profitable containers were not shipped due to setting hard capacity constraints. Related literature indicated that almost all existing studies preferred setting hard capacity constraint that does not allow for container overload, for example, in the shipping network design (e.g., Shintani et al., 2007; Agarwal and Ergun, 2008; Meng and Wang, 2011), cargo routing and allocation problem (e.g., Lam et al., 2007; Zeng et al., 2010; Song and Dong, 2012) and ship scheduling optimization (e.g., Wang and Meng, 2012). In reality, the carrier desires to reduce redundant containers in both tactical-level and operational-level decisions. To make a favorable decision, the carrier naturally expects to optimize the ship fleet capacity provision by choosing suitable container ships. Therefore, a joint optimization of ship capacity setting and cargo allocation scheme would be better to appropriately meet potential demand in the complex shipping market.

One more issue is how to precisely quantify the in-transit waiting time, including both initial delivery waiting time and transshipment waiting time, and resultant inventory costs. Rather limited number of studies focus on examining the in-transit waiting time. In what follows, we review some representative works regarding the in-transit waiting time. Bell et al. (2013) and Song and Dong (2012) incorporated the in-transit inventory costs into the design objective function of cargo assignment problem. However, both studies assumed fixed in-transit inventory costs under given ship route schedules. Wang et al. (2014b) proposed a ship schedule coordination scheme to incorporate the in-transit inventory costs, but the in-transit waiting time of transshipment containers was not considered. Because the in-transit waiting time heavily depends on the ship route schedules as well as cargo collection date, it is also necessary to determine an optimal ship route schedule coordination scheme coupled with the cargo allocation.

To address the above mentioned issues, we make attempt to fulfill two tasks in this study. We firstly propose a framework to build the SAS shipping network. This tractable network establishment framework is capable to integrate various container shipment activities existing in the liner container shipping chain together. Based on the SAS network framework, we then focus on the joint optimization of cargo assignment problem, and extend the previous works by considering differentiated container demands (deterministic contracted demand and uncertain individual demand), the scaling effect of ship capacity setting and ship route schedule coordination strategy.

The contributions of this paper are multifold. First, we propose a SAS network framework to describe the integration of the activities of container shipments. This fundamental shipping network establishment method would help stakeholders make more reasonable decisions in the tactical-level planning. Second, in the joint optimization model, we consider two differentiated container demands: contracted basis demand and uncertain individual demand. This tailored consideration can account for the diversity of freight rates for different customers in a real market, and can well capture the impact of decision-maker's risk preference under the environment of uncertain individual demand. Third, the joint liner container cargo assignment model takes into account the scaling effect of ship capacity setting and ship route schedule coordination management. The carrier's planning decision can be properly characterized by the flexible joint optimization model, since the carrier is more likely to enjoy more profit from the simultaneous optimization strategy. Moreover, the incorporation of ship capacity setting in certain extent alleviates the concern of cargo overload rejection issue.

The remainder of this paper is organized as follows. Section 2 presents a detailed problem description. The SAS network framework is explicitly introduced in the Section 3. Section 4 formulates a chance-constrained optimization model for the joint cargo assignment problem. The numerical examples are carried out in Section 5 to demonstrate the applicability of the developed model. In the last section, conclusions and future works are presented. The notation used in this paper is explicitly defined in *Appendix I*.

2. Problem description

Consider a liner container shipping company (i.e., carrier) who provides several ship routes, denoted by a set R, and each element $r \in R$ represents a particular ship route with service capacity C_r. All available ship routes are assumed to provide regular weekly shipping services. Fig. 1 shows an illustrative shipping network comprising two ship routes. Take ship route 1 as an example. We can freely choose one port of call in the ship route 1 as the first port of call. A leg (also called a link) is a voyage from one port of call to the next one. For example, if Shanghai is chosen as the first port of call, then the second port of call is Hong Kong, and the third port of call is Singapore. In such case, the first leg is the voyage from Shanghai to Hong Kong, and the second one is the voyage from Hong Kong to Singapore. In a liner container shipping network, each ship route visits a number of ports in order to load and/or discharge container cargoes. The visited ports are called origins and/or destinations. Each origin-destination (OD) pair is represented by w, and the set of OD pairs is denoted by W.

In the shipping market, the carrier always provides multiple ship routes such that each of them ensures weekly shipping service. The carrier collects daily deliveries of container cargoes from two types of customers, namely regular large manufacturers and individual shippers in the spot market. Two differentiated container cargo demands are thus considered. One is the daily contracted basis demand collected from large manufacturers (e.g., Nike and Lenovo). Such shipment demands are often subject to long-term shipping contracts. The other one is the daily individual delivery demand collected from the spot market. It is easy to understand that the daily basis delivery demand subject to binding contracts is fixed and known in a given planning period. However, the daily individual demand would show certain uncertainty

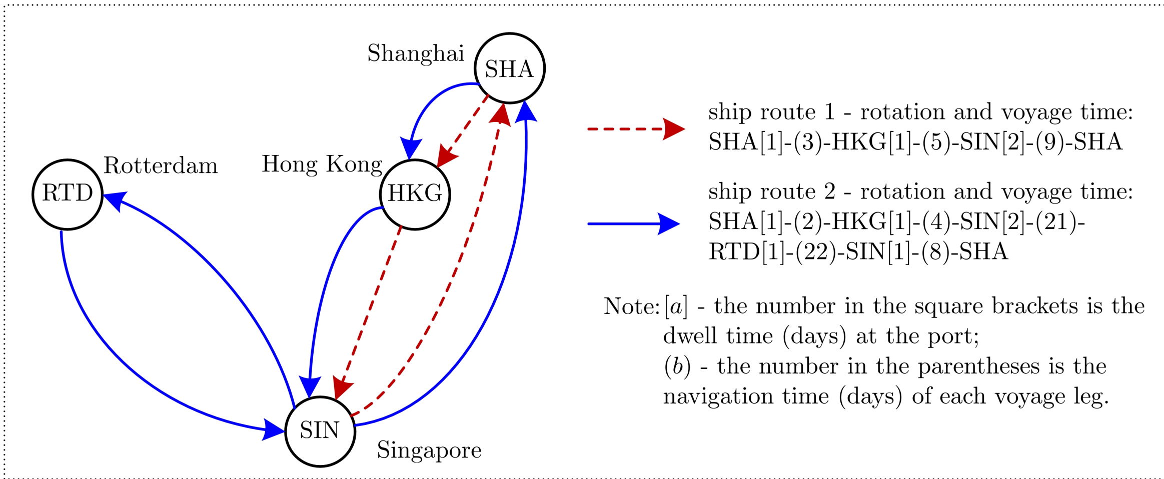

Fig. 1 An illustrative shipping network with two ship routes

due to potential fluctuations of business activities and some stochastic impacts of market factors (e.g., pricing floating, competitions from newly launched shipping lines and imperfect marketing information). Hence, we should take into account uncertain individual demands in the liner container cargo assignment problem.

In order to maximize his profit, the carrier will make a joint optimization scheme of container cargo assignment in the tactical-level planning. The joint optimization scheme comprises three major strategies: cargo allocation scheme, optimization of ship capacity setting and ship route schedule coordination. The cargo allocation scheme is to assign the container cargoes of each OD pair to desirable ship routes. The ship capacity setting is to determine optimal ship sizes of the ship fleet deployment. The shipping schedule coordination is to optimize the schedule for each ship route. In addition, the joint optimization scheme helps the carrier determine whether and which containers would be rejected for shipments.

To secure a tangible joint cargo assignment scheme, the carrier also pays attention to several practical issues in the real container shipping market. One inevitable issue is the transshipment problem. As we know, the carrier operates a liner container shipping network that comprises multiple ship routes. On one hand, the bulk of container cargoes to be delivered from origin port $\mathcal{O}$ to destination port $\mathcal{D}$ can be directly transported by one individual ship route. On the other hand, the rest containers cannot be delivered by only one accessible ship route but fulfilled via transshipments among multiple ship routes. The following two questions consequently arise: (i) how many containers will be transshipped and (ii) which transshipment routes will be used. In the modeling side, the occurrence of transshipments leads to a path-based cargo assignment formulation.

One more practical issue is the potential container overload rejection. This implies that a few containers may not be transported due to insufficient ship capacity provision. Rather than setting hard capacity constraint, the carrier expects to adopt more flexible countermeasures. When making an overload rejection design, the carrier should pour attention to two aspects. On one hand, if it is profitable to ship so-called redundant containers, the carrier could decide to use large ships. Therefore, the ship capacity setting should be simultaneously considered as one of design variables so as to make a flexible and optimal overload rejection decision. On the other hand, it is necessary to differentiate between the contracted basis demand and individual demand if the overload rejection occurs. Since the contracted basis demands are subject to long-term contracts with large manufacturers, they should be successfully accommodated[1]. For the individual demand, the carrier needs to determine how many expected daily individual demand should be collected from the spot market according to capacity constraints. In other words, for some OD pairs, their individual demands might be rejected without compensations. The above design is more flexible and reasonable to handle the overload rejection problem in real world.

The empty container repositioning problem is also important. It inherently exists in the shipping business due to inevitable trade imbalance. An increasing number of researchers have emphasized the importance of considering empty container repositioning problem (Li et al., 2007; Lam et al., 2007; Shintani et al., 2007; Song and Carter, 2009; Brouer et al., 2011; Meng and Wang, 2011; Song and Dong, 2012). In this study, we also address the empty container repositioning problem.

In summary, this paper aims to investigate a joint optimization problem of liner container cargo assignment by taking into account a series of practical issues in the real market. A SAS network framework is proposed to precisely describe various activities that happen in the shipping process, including cargo loading/unloading, transshipment, dwelling, in-transit waiting (both delivery waiting at the origin port and transshipment waiting) and in-sea transport. Based on the SAS network framework, we develop a joint optimization model of cargo assignment problem which takes into account demand uncertainty, cargo transshipment and empty container repositioning. In details, two kinds of differentiated container cargo demands are considered: the daily contracted basis demand from large manufacturers and uncertain individual demand from the spot market. The economies of scale of ship size are used to capture the scaling effect of ship capacity setting and to resolve the concern of cargo overload rejection issue. Integrating the above factors, the developed model makes an attempt to maximize the carrier's profit by simultaneously determining optimal ship capacity setting, ship route schedule and cargo allocation scheme. Eventually, the carrier's risk preference towards the uncertain individual demand in the spot market can be hopefully characterized by the proposed model.

3. The state-augmented shipping (SAS) network framework

Before giving the optimization model, we introduce a state-augmented shipping (SAS) network to describe the container delivery states in the shipping network. The concept of the state-augmented network is first proposed by Bertsekas (1995). Subsequently, Lo et al. (2003) applied it for a multi-modal transportation network in order to illustrate the mode transfer problem and non-linear fare structures (i.e., non-additive path costs). In a liner container shipping network, there exist a number of cargo operations, including cargo

[1]Generally, if the contracted basis containers are rejected to transport, the carrier needs to compensate shippers according to delivery agreements. A penalty item can be accordingly incorporated into the design objective function to address the compensation for possible overload rejection. This study assumes that the contracted basis demands will be fully transported. The carrier can freely determine optimal ship fleet capacity according to signed long-term contracts at hand. In turn, the carrier can determine his expected contracted basis demands provided a selected ship capacity supply.

loading/discharging, dwelling at port for next voyage shipment, cargo stored at port to wait for initial shipment and possible transshipment. We here introduce a SAS network to analyze various delivery states of the cargo shipments.

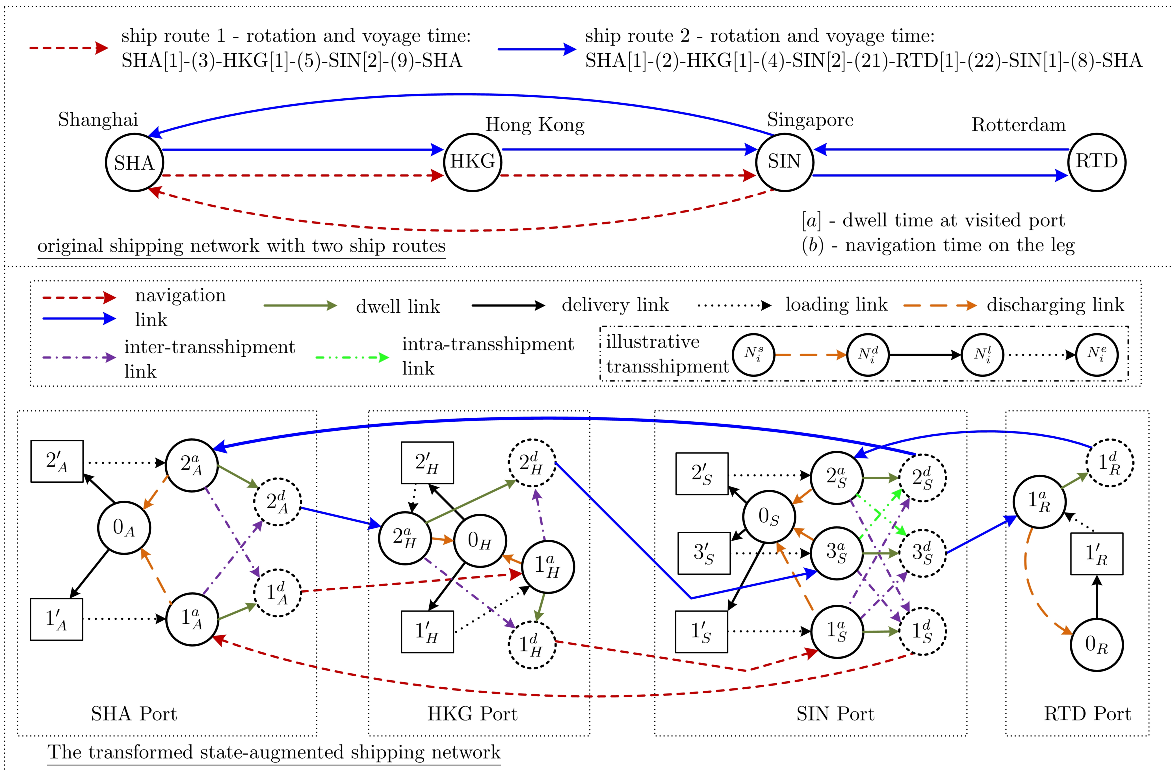

Fig. 2 The transformed state-augmented shipping network

Consider a shipping network $G = (U, V)$ (see Fig. 2 as an example, colored in the electronic version), where U, V, respectively, are the sets of physical nodes (i.e., ports) and directed links (i.e., voyage legs). Motivated by the seminal works (Bertsekas, 1995; Lo et al., 2003; Brouer et al., 2014)[Hans: here the paper Brouer 2014 should also be the paper I attached], through a state augmentation approach, we can transform the physical shipping network $G = (U, V)$ into an extended SAS network $G' = (N, A)$, where N is a set of nodes and A is a set of directed links. In the shipping network, each original physical node $i \in N_O$ (each origin or destination port) could be replicated to three types of dummy nodes by distinguishing the accessible operation states (the original physical node i still stays in the SAS network). The three types of replicated dummy nodes are defined as waiting-delivery node $i \in N_W$, arrival node $i \in N_A$ and departure node $i \in N_D$. When shippers have already transferred their cargoes to the carrier, we use the dummy waiting-delivery node to describe the operation of cargoes stored at origin port waiting for being loaded to expected ship routes. The arrival node indicates a ship arrives at the focal port, which is defined to depict the incoming/outgoing containers at the visited port, including new loading containers, possible transshipment and discharged arrival containers. The departure node marks a ship departure from the visited port. Such node is connectted from the arrival node in order to represent the dwell time at the visited port. We eventually have the node set $N = N_O \cup N_W \cup N_A \cup N_D$.

By taking HKG port shown in Fig. 2 as an example, we briefly explain the building of node set in the SAS network. The original HKG Port denoted by 0_H is replicated to three categories of dummy nodes. Each replicated node category contains two dummy nodes. The two dummy waiting-delivery nodes for HKG Port are denoted by $1'_H$ and $2'_H$. For each of them, the numeric symbol 1 and 2 means the dummy node numbers for corresponding ship routes 1 and 2 respectively; the superscript $'$ defines a label of replicated waiting-delivery nodes; and the subscript H identifies that the two dummy nodes are particularly replicated for the HKG port. Node $1'_H$ represents the arrival of cargoes to HKG Port that are to be exported on ship route 1. These cargoes may need to wait at the yard because of the periodicity of ship arrivals. Similarly, it is not difficult to generate the replicated arrival nodes (see 1^a_H and 2^a_H in Fig. 2) and departure nodes (see 1^d_H and 2^d_H in Fig. 2) for HKG port.

Next, let us define the links in the SAS network. Actually, a core problem for the SAS network establishment is to properly declare and create corresponding links via distinguishing cargo operation states at each port. According to the above cargo operation states, we eventually define 7 types of links for the SAS network. As a result, the set of links is denoted by $A = \cup_k A_k, k = 1, 2, 3, 4, 5, 6, 7$, where A_k is the subset of k^{th} type of links. The 7 types of links are navigation link, dwell link, delivery link, loading link, discharging link, inter-transshipment link and intra-transshipment link. Each of them is illustrated explicitly as follows:

(i) *navigation link* $a \in A_1$: The navigation link indicates a real leg in the waters, which is used to connect a departure node at the previous port and an arrival node at the focal port (see the red dash line or blue line shown in Fig. 2). We are concerning with two link properties, namely navigation time $t_{n,a}$ and its service capacity $C_{n,a}$. A longer navigation time usually means larger bunker consumption and higher ship usage cost (capital and voyage cost). Since the navigation link $a \in A_1$ is a component of ship route r, it has an identical capacity of the corresponding ship route r.

(ii) *dwell link* $a \in A_2$: The dwell link connects an arrival node and a departure node at the visited port. It is used to address the dwell time of ships staying at port, $t_{g,a}$, during the arrival time and departure time at each visited port. It is assumed that all carried containers could be successfully loaded and/or discharged within given dwell time at port. That is, the dwell link has sufficiently large container handling capacity.

(iii) *delivery link* $a \in A_3$: The delivery link connecting the origin port to a waiting-delivery is used to record the extra in-transit time $t_{v,a}$ of delivered containers that wait for shipping services at the origin port (i.e., the container cargo has arrived at the port but the ship that will transport the cargo has not come yet). It is a hypothetical link. An in-transit waiting time on the delivery link results in extra inventory cost of cargo shipment. The delivery link is also assumed to have sufficient large capacity.

(iv) *loading link* $a \in A_4$: The loading link is a virtual link to connect a waiting-delivery node and an arrival node at the port. It is used to describe the container loading operation and represent the handling cost of loading a container from the origin port to the vessel, denoted by $c_{l,a}$. We assume that there is no capacity constraint for the loading link.

(v) *discharging link* $a \in A_5$: The discharging link is a virtual link to connect an arrival node and a destination node at the port. It is used to describe the unloading operation. The handling cost of discharging the container from the vessel to the land is denoted by $c_{u,a}$. As well, each discharging link has a sufficiently large capacity.

(vi) *inter-transshipment link* $a \in A_6$: The inter-transshipment link is defined to describe the transshipment operation between two different ship routes at port. For example, as shown in Fig. 2, the containers delivered from SHA port to RTD port could be transshipped at SIN port from ship route 1 to ship route 2. A complete transshipment comprises the operations of cargo discharging from the coming ship route, cargo storing at the port for transshipment delivery and cargo loading to the second ship route. Therefore, each inter-transshipment link aggregates three component links: discharging link, delivery link, and loading link. We can further define the property of an inter-transshipment link by a triplet $\langle c_{u,a}, t_{v,a}, c_{l,a}\rangle$[2]. The first item in the triplet is the unloading cost on the discharging link of unloading cargoes from the first ship route at the transshipment port. The second item recorded by delivery link is to describe the waiting time of the transshipped cargoes for reloading to the next ship route. The last item is the loading cost on the component loading link of reloading the transshipped cargoes to the second ship route. The costs of three component links are additive in monetary value by introducing the parameter of value of time. As a virtual link, the inter-transshipment link is also assumed to have sufficient large capacity.

(vii) *intra-transshipment link* $a \in A_7$: The intra-transshipment link is used to illustrate the transshipment operation in the same ship route due to multi-visit at the intra-transshipment port. It has almost the same link structure and property as the inter-transshipment link. The only difference is that the intra-transshipment delivery waiting time is fixed and independent of the ship route schedule. This will be explained later.

For the sake of presentation, a unified link notation is given. Each link $a \in A$ is denoted by a nonuplet, $\langle i, j, r, t_{n,a}, t_{g,a}, t_{v,a}, c_{l,a}, c_{u,a}, C_a\rangle$, in which i, j respectively represent the tail node i and head node j of link a; r denotes the ship route r with service capacity C_r; $t_{n,a}$ is the navigation time of the physical link a (i.e., voyage leg); $t_{g,a}$ is the dwell time at the visited port; $t_{v,a}$ is the in-transit waiting time (either delivery waiting time or transshipment waiting time); $c_{l,a}$ and $c_{u,a}$ are, respectively, the unit handling cost of loading a laden container and that of discharging a laden container (differentiated costs between non-transshipment and transshipment laden containers); C_a is the link capacity. As mentioned above, only physical navigation links are subject to service capacity constraints. Therefore, each navigation link has its own capacity $C_a = C_r,$ for all $\delta_{a,r} = 1, a \in A_1$. For the virtual links $a \in A \setminus A_1$, there is no need to consider capacity constraints since they are all assumed to have sufficiently large service capacities.

We further discuss how to determine the attribute values for the link $a \in A$. For given ship routes, the link attributes: voyage navigation time $t_{n,a}$, dwell time $t_{g,a}$, unit loading cost $c_{l,a}$ and unit discharging cost $c_{u,a}$ are all known and fixed. We now pay our attention to the attribute variable $t_{v,a}$. We need to examine in-transit waiting times for three types of links, namely delivery link, inter-transshipment link and intra-transshipment link. Assume that the containerized cargoes will arrive at the focal port at least one day ahead of the ship departure in order to guarantee adequate handling time at the port. Let $D = \{1, 2, 3, 4, 5, 6, 0\}$ be a set of weekly dates. For each particular weekly date, $d \in D$, $d = 1, 2, 3, 4, 5, 6, 0$ represents Monday, Tuesday through to Sunday, respectively. Let D_c and D_s denote the sets of collection weekly

[2]To encourage the transshipments, the unit handling costs of loading/discharging transshipped container would generally be set to be lower than the non-transshipment container. Meanwhile, the waiting times for transshipment delivery and initial delivery at origin port are different.

dates of daily contracted basis cargoes and daily individual cargoes respectively, $D_c \subseteq D$ and $D_s \subseteq D$.

We know that the in-transit waiting times on the three links depend on the arrival and departure times of the involved ship routes at the visited port. Let us take the departure time as the base reference. The departure time for each visited port by ship route r can be easily determined according to the date of 1^{st} port of call, the fixed navigation time on each leg and the fixed dwell time at the port. Suppose that ship route r departs from the first port of call on weekly date $m_{1,r} \in D$. Then, the departure time of the k^{th} port of call by ship route r, denoted by weekly date $m_{k,r}$, can be calculated by

$$m_{k,r} \equiv (m_{1,r} + t_{1-k,r}) (\text{mod } 7), \tag{1}$$

where $m_{k,r}$ and $(m_{1,r} + t_{1-k,r})(\text{mod } 7)$ are called congruent modulo 7, and $t_{1-k,r}$ is the transport time from the 1^{st} port of call to the k^{th} port of call (including both the in-sea navigation time and dwell time at ports). Consequently, we would have $m_{k,r} \in D$. Similarly, we can determine the arrival time of ship route r at each visited port.

Once the arrival times and departure times of the ports of call of all involved ship routes are determined, the extra in-transit waiting time $t_{v,a}$ at each port can then be calculated. Let us start with the delivery link. The in-transit waiting time of delivery link a at the k^{th} port of call is simultaneously influenced by the departure time of the k^{th} port of call and the cargo collection date $d \in D$. It is further defined as $t_{v,a}(d, m_{k,r})$ and can be calculated by

$$t_{v,a}(d, m_{k,r}) = \begin{cases} m_{k,r} - d, & \text{if } m_{k,r} > d \\ m_{k,r} + 7 - d, & \text{if } m_{k,r} \leq d \end{cases}.$$

Therefore, we can obtain a timetable for the in-transit waiting time of delivery link a (see Table 1).

Table: 1 In-transit waiting time of delivery link $a \in A_3$

in-transit waiting time (days)		departure time of the k^{th} port of call (date $m_{k,r} \in D$)						
		Mon	Tue	Wed	Thu	Fri	Sat	Sun
cargo collection date ($d \in D_s \cup D_c$)	Mon	7	1	2	3	4	5	6
	Tue	6	7	1	2	3	4	5
	Wed	5	6	7	1	2	3	4
	Thu	4	5	6	7	1	2	3
	Fri	3	4	5	6	7	1	2
	Sat	2	3	4	5	6	7	1
	Sun	1	2	3	4	5	6	7

We then focus on the inter-transshipment link. The in-transit waiting time of the inter-transshipment link (from ship route r to ship route r', $r \neq r'$) depends on the arrival time of ship route r at the k^{th} port of call and departure time of ship route r' at the $k'^{,\text{th}}$ port of call (i.e., it relies on the departure times of the 1^{st} ports of call of two involved ship routes, $m_{1,r}$ and $m_{1,r'}$). Notice that ship route r at the k^{th} port of call and ship route r' at the $k'^{,\text{th}}$ port of call share the same transshipment port. The in-transit waiting time of the inter-transshipment link is redefined as $t_{v,a}(m_{k,r}, m_{k',r'})$, and can be calculated by

$$t_{v,a}(m_{k,r}, m_{k',r'}) = \begin{cases} m_{k,r} - m_{k',r'}, & \text{if } m_{k,r} > m_{k',r'} \\ m_{k,r} + 7 - m_{k',r'}, & \text{if } m_{k,r} \leq m_{k',r'} \end{cases},$$

where $m_{k,r}$ and $m_{k',r'}$ are, respectively, the arrival time of ship route r at the k^{th} port of call and the departure time of ship route r' at the $k'^{,\text{th}}$ port of call. Then, it is rather easy to obtain a similar timetable for the in-transit waiting time of the inter-transshipment link.

At last, we take a look at the intra-transshipment link. Since the transport time between the first-visit, denoted by the k^{th} port of call, and the revisit, denoted by the $k'^{,\text{th}}$ port of call, at the transshipment port is fixed for a given ship route r, we have a constant in-transit waiting time on the intra-transshipment link that is denoted by $t_{v,a} = \bar{t}_{v,a}$, for all m. The fixed in-transit waiting time $\bar{t}_{v,a}$ can be calculated by

$$\bar{t}_{v,a} = \begin{cases} t_{k-k',r} (\text{mod } 7), & \text{if transships from first-visit voyage to revisit voyage} \\ t_{k'-k,r} (\text{mod } 7), & \text{if transships from revisit voyage to first-visit voyage} \end{cases},$$

where $t_{k-k',r}$ is the traversing transport time during first-visit and revisit at the k^{th} intra-transshipment port, and $t_{k'-k,r}$ is the traversing transport time during revisit and first-visit.

So far, we have discussed the determination of the in-transit waiting times $t_{v,a}$ of delivery link and two types of transshipment links, and have indicated that $t_{v,a}$ inevitably depends on the ship schedule coordination scheme.

4. Model formulation for the joint cargo assignment problem

Based on the introduced SAS network framework, we now present the model formulations for the joint cargo assignment problem. A path-based chance-constrained optimization model is developed in this section.

4.1. Model assumptions

To facilitate the model formulation, a few essential assumptions are made. We firstly make two assumptions for the container demand. The contracted basis demand for each OD pair $w, w \in W$ on weekly date $d, d \in D$, denoted by $q^c_{d,w}$, is assumed to be fixed and known. The daily individual demand for each OD pair $w, w \in W$ collected on weekly date $d, d \in D$ from the spot market, $Q^s_{d,w}$, is assumed to follow a known probability distribution over the entire planning period. Specifically, the uncertain daily individual demand $Q^s_{k,w}$ is assumed to follow a given *Normal* distribution that $Q^s_{d,w} \sim N(\mu_w, \sigma^2_w), d \in D, w \in W$. Our model to be developed is applicable to all types of demand distributions.

It is also assumed that every ship route provides weekly service and deploys a ship fleet with the same ship capacity (i.e., the container ships with identical size are deployed for each ship route). In addition, for the consideration of real application, no shipped containers are allowed to be transshipped more than a given maximal permitted number of transshipment times (e.g., two times used in Song and Dong, 2012)[3].

4.2. Decision variables

In the SAS network, the container cargo shipments for each OD pair $w, w \in W$ can be fulfilled by corresponding shipping paths (either non-transshipment path or transshipment

[3]An alternative way to reasonably model the transshipment is to set proper time constraint (e.g., maximal acceptable transport time) for each OD pair.

path). In the container cargo assignment problem, the carrier makes the following decisions: (i) allocate the container cargoes to favorable shipping paths (including empty container repositioning), (ii) coordinate the schedules of ship routes, and (iii) determine an optimal ship capacity setting for each ship route.

For the container allocation, three decision variables are specifically considered. The first one is the quantity of contracted laden containers collected on weekly date d that will be loaded on path $p \in P_w, w \in W$ for shipment, denoted by $x^c_{d,p,w}$. The second one is the expected quantity of individual laden containers collected on weekly date d that will be shipped by path $p \in P_w, w \in W$, which is denoted by $x^s_{d,p,w}$. This helps the carrier decide how many daily individual demands they should collect in the spot market. The third one is the expected quantity of empty containers that will be transported by path $p \in P_w, w \in W$, denoted by $x^e_{p,w}$.

Regarding the ship schedule coordination, a set of binary decision variables are introduced, denoted by $Y_{r,m}$. The variable $Y_{r,m}$ is an indicator which equals 1 if and only if the ship of ship route r departs from the 1st port of call on weekly date $m, m \in D$. We then have $m_{1,r} = \sum_{m\in D} mY_{r,m}$. Notice that, once $m_{1,r}$ is determined, the arrival and departure days of all ports of call on ship route r can be calculated by Eqn. (1).

We simultaneously introduce a set of binary variables, $Z_{r,l}$, for determining the optimal ship capacity setting of each ship route r. Let L be a set of ship types with different ship capacities, $L = \{1, 2, \cdots, |L|\}$. The variable $Z_{r,l}$ is an indicator which equals 1 if and only if the ship fleet of ship route r deploys a number of vessels with identical ship capacity $\mathcal{C}_l$, where $\mathcal{C}_l \in \{\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_{|L|}\}$ and $l \in L$. We thus have

$$C_r = \sum\nolimits_{l\in L} Z_{r,l} \times \mathcal{C}_l, \forall r \in R.$$

4.3. Design objective function and related cost components

The carrier expects to obtain as much profit as possible, but as low as possible operating costs, including in-transit inventory cost, container handling cost and shipping transport cost (mainly including bunker cost and ship capital cost). The gross revenue of container cargo shipments, F_{rev}, can be expressed by

$$F_{\text{rev}} = \sum\nolimits_{w\in W} \sum\nolimits_{p\in P_w} \left(\sum\nolimits_{d\in D_c} \tau^c_w \times x^c_{d,p,w} + \sum\nolimits_{d\in D_s} \tau^s_w \times x^s_{d,p,w} \right),$$

where τ^c_w and τ^s_w are, respectively, the freight rates of shipping one contracted container and one individual container between OD pair w.

The involved operating costs are elaborated as follows. We firstly discuss the operating costs related to the two types of laden containers. Let α be the daily inventory cost of storing laden containers at the port (\$/(TEU·day)). For the laden containers of OD pair $w \in W$ transported by shipping path $p \in P_w$, we can determine the extra in-transit inventory cost $c_{v,d,p,w}$ by

$$c_{v,d,p,w} = \sum\nolimits_{a\in A} \delta_{a,p,w} \times c_{v,d,a}$$

where $\delta_{a,p,w}$ is the link-path indicator, $\delta_{a,p,w} = 1$ if link a is on the shipping path $p \in P_w$, otherwise, $\delta_{a,p,w} = 0$; and the unit link inventory cost $c_{v,d,a}$ can be expressed as

$$c_{v,d,a} = \begin{cases} \sum_{m \in D} \max\{\alpha Y_{r,m}(t_{v,a}(d, m_{k,r}) - \Delta_i), 0\}, \text{ if } a \in A_3 \\ \sum_{m \in D} \sum_{m' \in D} \max\{\alpha Y_{r,m} Y_{r'm'}(t_{v,a}(m_{k,r}, m_{k',r'}) - \Delta'_i), 0\}, \text{ if } a \in A_6 \\ \max\{\alpha(\bar{t}_{v,a} - \Delta'_i), 0\}, \text{ if } a \in A_7 \end{cases}, \tag{2}$$

where Δ_i is the free-usage time (days) for storing exporting cargoes at the port i and Δ'_i is the free storage time (days) at the transshipment port i.

Then the total extra in-transit inventory cost is

$$F_{\text{inv}} = \sum_{w \in W} \sum_{p \in P_w} \left(\sum_{d \in D_c} c_{v,d,p,w} x^c_{d,p,w} + \sum_{d \in D_s} c_{v,d,p,w} x^s_{d,p,w} \right).$$

Meanwhile, it is fairly easy to address the unit container handling cost $c_{h,p,w}$ of transporting one laden container by shipping path $p \in P_w, w \in W$:

$$c_{h,p,w} = \sum_{a \in A} \delta_{a,p,w} \times (c_{l,a} + c_{u,a}).$$

Therefore, we can obtain the total handling cost F_{han} by

$$F_{\text{han}} = \sum_{w \in W} \sum_{p \in P_w} c_{h,p,w} \left(\sum_{d \in D_c} x^c_{d,p,w} + \sum_{d \in D_s} x^s_{d,p,w} \right).$$

Next, we focus on the shipping cost that occurs at the in-sea transport. As suggested by Cullinane and Khanna (1999, 2000), Imai et al. (2006) and Wang et al. (2014a), it is important to take into account the economies of scale of ship size in estimating the shipping cost. We know that the shipping cost is generally positively related to the navigation time (or equivalent voyage distance) but negatively related to the ship size (i.e., the ship capacity setting). The cost that occurs during the time a ship spends at port and the cost associated with in-sea navigation can be aggregated to generate the shipping cost (excluding cargo handling charges and ship capital costs). The cost of port-related services includes port dues, pilotage, towage, wharfage, dockage fee, light/tonnage/buoy/anchorage dues etc. (Cullinane and Khanna, 1999). The cost incurred in the in-sea navigation mainly includes the bunker consumption and container usage cost during ocean navigation.

For simplicity, the total shipping cost for the laden containers is assumed to be a linear function with respect to shipping capacity setting, and can be calculated by [4]

$$F_{\text{nav}} = \sum_{p \in P_w} \sum_{w \in W} \sum_{a \in A} \delta_{a,p,w} t_{n,a} \pi_n(C_a) \left(\sum_{d \in D_c} x^c_{d,p,w} + \sum_{d \in D_s} x^s_{d,p,w} \right), \tag{3}$$

[4]The linear function (3) is applicable for the case that the container ship a full-loaded or near full-loaded. In this study, we take use of this function for two considerations. First, the carrier can freely optimize the ship capacity setting. To reduce the shipping cost, the carrier naturally expects to choose an appropriate ship capacity to closely meet the demand. Second, the empty container repositioning problem is taken into account, which in some degree makes the ship capacity achieve full-utilization. In practice, if the ship is not fully loaded, we can use the following function to approximate the total shipping cost:

$$F_{\text{nav}} = \sum_{r \in R} \pi'_n(C_r) \times t_r \times C_r.$$

where t_r is the transport time of a round trip of ship route r; $\pi'_n(C_r)$ is the modified unit shipping cost in terms of ship capacity provision. The above function assumes that the marginal effect of adding one container is negligible, compared to the huge bunker cost of driving a container ship with heavy deadweight.

where $\pi_n(C_a)$ is the unit shipping cost of transporting one container per day (or per nautical mile) by the container ship fleet with service capacity C_a. Notice that, the parameter $\pi_n(C_a)$ should be calibrated by real market data. Some empirical values of $\pi_n(C_a)$ can be found in Cullinane and Khanna (1999, 2000).

To sum up, the total operating cost (excluding the ship capital cost) for transporting one laden container collected on weekly date d can be estimated by

$$\pi_{d,p,w} = \alpha t_{v,d,p,w} + c_{h,p,w} + \sum\nolimits_{a\in A} \delta_{a,p,w} t_{n,a} \pi_n(C_a),$$

[Hans: $t_{v,d,p,w}$ is not defined.]where $\pi_n(C_a)$ can be equivalently replaced by

$$\pi_n(C_a) = \pi_n(C_r) = \sum\nolimits_{l\in L} Z_{r,l} \pi_b(\mathcal{C}_l),$$

where $\pi_b(\mathcal{C}_l)$ is pre-determined unit shipping cost for a given ship capacity parameter $\mathcal{C}_l$ (see Cullinane and Khanna, 1999). The notations $\pi_n(C_a)$ and $\pi_n(C_r)$ are used interchangeably in this paper since C_a and C_r are automatically related according to the link definition.

Now, we analyze the operating cost of empty container repositioning. The operating cost of empty container repositioning contains the container handling cost and the shipping cost, which can be calculated by

$$F_{\text{emp}} = \sum\nolimits_{w\in W} \sum\nolimits_{p\in P_w} \pi^e_{p,w} x^e_{p,w},$$

where $\pi^e_{p,w}$ is the unit operating cost (excluding the ship capital cost) to transport one empty container by path $p \in P_w$ between OD pair $w \in W$. It is the summation of the unit handling cost for empty containers and unit shipping cost, as shown below:

$$\pi^e_{p,w} = \sum\nolimits_{a\in A} \delta_{a,p,w} \left(c^e_{l,a} + c^e_{u,a} + t_{n,a}\pi_n(C_a)\right),$$

where $c^e_{l,a}$ and $c^e_{u,a}$ are, respectively, the unit loading cost and unit unloading cost for empty containers.[Hans: You did not define $c^e_{l,a}$ or $c^e_{u,a}$ whe you were defining nonuplet $\langle i, j, r, t_{n,a}, t_{g,a}, t_{v,a}, c_{l,a}, c_{u,a}, C_a\rangle$]

Another component cost is the ship capital cost. We firstly examine the daily ship capital cost. When the purchase prices for vessels with different capacities are known and given, we can roughly estimate the daily capital cost for each kind of vessel. The capital cost can be annualized over an estimated useful life (e.g., 20-year) at a certain percent interest rate (e.g., 6%). Dividing the annual value by 365 days gives a daily capital cost. The total ship capital cost can be approximately estimated by (Wang et al., 2014a)

$$F_{\text{ship}} = \sum\nolimits_{r\in R} \pi_{\text{ship}}(C_r) \times t_r. \tag{4}$$

where $\pi_{\text{ship}}(C_r)$ is the daily capital cost for the container ship with capacity of C_r (e.g., 4000 TEUs per ship). The round-trip shipping transport time t_r is calculated by

$$t_r = \sum\nolimits_{a\in A_1\cup A_2} \delta_{a,r}(t_{n,a} + t_{g,a}).$$

Notice that, t_r guarantees a round shipping trip of cargo shipments so that there is no need to address the ship fleet size. The expression (4) then can be rewritten as

$$F_{\text{ship}} = \sum\nolimits_{r\in R} \sum\nolimits_{l\in L} Z_{r,l} \pi_{\text{cap}}(\mathcal{C}_l) \times t_r.$$

where $\pi_{\text{cap}}(\mathcal{C}_l)$ is pre-determined daily ship cost for a given ship capacity parameter $\mathcal{C}_l$.

So far, we have introduced the cost components of the total operating cost.

4.4. A chance-constrained optimization model

Based on the above analyses in Subsections 4.2 and 4.3, we propose the optimization model for the joint cargo assignment problem. As a profit-maximizer, the carrier aims to maximize his net profit by jointly determining the optimal cargo allocation scheme, ship route schedules and ship capacity setting. The proposed joint cargo assignment problem (JCAP) is then formulated as multi-commodity flow model taking into account a set of bundle constraints as below:

[JCAP Model]

$$\begin{aligned}\max \quad & F(\boldsymbol{x}_c, \boldsymbol{x}_s, \boldsymbol{x}_e, \boldsymbol{C}, \boldsymbol{Y}_m, \boldsymbol{Z}_l) = F_{\text{rev}} - F_{\text{han}} - F_{\text{nav}} - F_{\text{emp}} - F_{\text{ship}} \\ &= \sum_{w \in W} \sum_{p \in P_w} \sum_{d \in D_c} (\tau_w^c - \pi_{d,p,w}) x_{d,p,w}^c + \\ &\quad \sum_{w \in W} \sum_{p \in P_w} \sum_{d \in D_s} (\tau_w^s - \pi_{d,p,w}) x_{d,p,w}^s - \\ &\quad \sum_{w \in W} \sum_{p \in P_w} \pi_{p,w}^e x_{p,w}^e - \sum_{r \in R} \pi_{\text{ship}}(C_r) \times t_r \end{aligned} \tag{5}$$

subject to

$$\begin{aligned} & \sum_{w \in W} \sum_{p \in P_w} \left(\sum_{d \in D_c} x_{d,p,w}^c \delta_{a,p,w} + \sum_{d \in D_s} x_{d,p,w}^s \delta_{a,p,w} \right) + \\ & \sum_{w \in W} \sum_{p \in P_w} x_{p,w}^e \delta_{a,p,w} \le C_a, \forall a \in A_1 \cup A_2 \end{aligned} \tag{6}$$

$$\sum_{p \in P_w} x_{d,p,w}^c = q_{d,w}^c, \forall d, w \tag{7}$$

$$\sum_{p \in P_w} x_{d,p,w}^s \le Q_{d,w}^s, \forall d, w \tag{8}$$

$$\begin{aligned} & \sum_{w \in W} \sum_{p \in P_w} \sum_{d \in D_c} \sum_{j \in N} (\delta_{\langle i,j \rangle,p,w} - \delta_{\langle j,i \rangle,p,w}) x_{d,p,w}^c + \\ & \sum_{w \in W} \sum_{p \in P_w} \sum_{d \in D_s} \sum_{j \in N} (\delta_{\langle i,j \rangle,p,w} - \delta_{\langle j,i \rangle,p,w}) x_{d,p,w}^s + \\ & \sum_{w \in W} \sum_{p \in P_w} \left(\delta_{\langle i,j \rangle,p,w} - \delta_{\langle j,i \rangle,p,w} \right) x_{p,w}^e = 0, \forall i \in N_o \end{aligned} \tag{9}$$

$$x_{d,p,w}^c, x_{d,p,w}^s, x_{p,w}^e \in \mathbb{Z}^+, \forall d, p, w \tag{10}$$

$$Y_{r,m}, Z_{r,l} \in \{0, 1\}, \forall r, m, l \tag{11}$$

$$\sum_{m \in D} Y_{r,m} = 1, \forall r \tag{12}$$

$$\sum_{l \in L} Z_{r,l} = 1, \forall r \tag{13}$$

$$C_r = \sum_{l \in L} Z_{r,l} \times \mathcal{C}_l, \forall r \tag{14}$$

In the above developed model, the design objective function is to maximize the total net profit, of which cost components have already been discussed and defined in Section 4.3. The side constraints (6)–(14) are elaborated as follows. Constraint (6) implies that, for each voyage leg, the total containers shipped should not exceed its service capacity. Remember that the variables C_a and C_r totally depend on the decision variable $Z_{r,l}$. That is, whether and how many container cargoes will be rejected are related to the decision variable $Z_{r,l}$. The flow conservation constraint (7) requires that, for each OD pair w, the daily contracted demand must be successfully accommodated. The flow conservation condition for the daily individual demand is described by constraint (8). Such an inequality constraint is able to model the possible case of redundant containers of individual container demand. Bundle constraints (9) are the inter-balancing constraints which explain the empty container repositioning problem. Constraints (10) ensure nonnegative and integral path flows. Constraints (11) give definitional conditions for design variables, $Y_{r,m}$ and $Z_{r,l}$. Constraint (12) indicates that each shipping route provides weekly service, and can and only can depart from the 1^{st} port of call on one specific weekly date. Constraints (13) show that, for each ship route, the ship fleet with the same ship capacity setting is deployed. The last constraints (14) are used to define the capacities of ship routes in terms of the decision variable, $Z_{r,l}$.

In the developed JCAP model, the constraints (8) need some more interpretations. It incorporates a random variable, namely the daily individual demand $Q^s_{d,w}$. To handle such constraint with probabilistic uncertainty, we formulate a chance constraint:

$$\Pr\left\{Q^s_{d,w} - \sum\nolimits_{p\in P_w} x^s_{d,p,w} \geq 0\right\} \geq \rho, \forall d, w,$$

where ρ is a given confidence level, which can be interpreted as the risk perference level of the carrier. The larger ρ is, the more risk averse the shipping line is.

Recall that, the individual container demand collected from the spot market for each OD pair is assumed to follow *Normal* distribution, namely $Q^s_{d,w} \sim N(\mu_w, \sigma^2_w)$. The constraint (8) can be further rewritten as the following equivalent deterministic form:

$$\sum\nolimits_{p\in P_w} x^s_{d,p,w} \leq \mu_w + \Phi^{-1}(\rho)\sigma_w, \forall d, w, \tag{15}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of cumulative distribution function for the standard *Normal* distribution.

A few comments are made regarding the solution method for the above model. In the spirit of Wang et al. (2013), we use a two-stage method to solve the JCAP model. At the first stage, the candidate shipping path set will be generated. Theoretically, there may be an infinite number of shipping paths for each OD pair. However, in practice, the number of shipping paths is quite limited because of operational constraints and some business considerations. In this study, two practical requirements are considered in the path set generation. Firstly, it is required that, for each OD pair $w \in W$, the path transport time $t_{p,w}$ cannot exceed the permitted maximal transport time $\bar{t}_w$. This constraint is formulated as

$$t_{p,w} \leq \bar{t}_w, \forall p, w,$$

where the path transport time, $t_{p,w}$ can be determined by

$$t_{p,w} = \sum\nolimits_{a\in A_1\cup A_2\cup A_3\cup A_6\cup A_7} \delta_{a,p,w}(t_{n,a} + t_{g,a} + t_{v,a}).$$

Secondly, the restraint of maximal transshipment times $\bar{n}_w$ is set to guarantee an effective cargo allocation scheme in real world, which is expressed by

$$n_{p,w} \leq \bar{n}_w, \forall p, w.$$

For more details of the shipping path generation, interested readers could refer to Wang et al. (2013). In the second stage, the nonlinear JCAP optimization model can be transformed into an equivalent mixed-integer linear program (MILP). For the MILP transformation, we only need to replace the combinatorial items (hidden nonlinear parts in the objective function, e.g, $Y_{r,m}x^c_{d,p,w}$ and $Z_{r,l}x^c_{d,p,w}$) by corresponding auxiliary variables. To save the space, we do not attempt to make a lot of discussions on the transformation (a specific transformation process can be found in Wang et al. (2014b)). Given the generated shipping path set, the transformed MILP can be efficiently solved by state-of-the-art MILP solvers (e.g., CPLEX and GUROBI).

5. Numerical examples

Numerical examples are carried out to test the applicability of the proposed JCAP model and understand the importance of considering uncertain individual demand in the spot market. The tested shipping network is shown in Fig. 2. Four ports are considered, namely Shanghai (SHA), Hong Kong (HKG), Singapore (SIN) and Rotterdam (RTD). More detailed parameter setting is given below.

5.1. Preliminary parameter setting

Two ship routes are taken into account. The rotation of each ship route, in-sea navigation time of each voyage leg and dwell time at port of each ship route are already given in Fig. 2. Let Shanghai be the 1^{st} port of call for both ship routes. Each ship route can depart from Shanghai on any weekly date $m, m \in D$. For both contracted cargo demand and individual cargo demand, 12 port-to-port OD pairs are considered. Both kinds of cargoes are supposed to be collected from Monday to Friday in every week so that we have cargo collection date $d \in D_c = \{1, 2, \cdots, 5\}, D_s = D_c$. The daily contracted basis demand for each OD pair is given in Table 2. The probability distribution of the daily individual cargo demand between each OD pair is provided in Table 3. The shipping service freight rates for two kinds of OD demands are listed in Table 4.

Table: 2 Containerized cargo demand for each O-D pair (daily contracted demand $q_{c,d,w}$)

cargo demand (TEUs/day)		destination port			
		SHA	HKG	SIN	RTD
origin port	SHA	-	120/220/220/260/140	180/230/280/320/230	200/240/220/300/250
	HKG	140/170/160/190/150	-	120/145/220/210/190	150/100/250/300/200
	SIN	145/160/160/170/150	150/150/150/175/145	-	100/180/200/250/170
	RTD	170/200/260/190/150	200/180/220/190/150	140/160/200/175/150	-

Note: (a/b/c/d/e) mean the cargo demands produced in (Mon/Tue/Wed/Thu/Fri).

In this case study, we assume that there is no maximal transport time restraint for any OD pair and the number of maximal transshipment times is set as $\bar{n}_w = 2$. There is no free storage time at each port, $\Delta_i = 0, \Delta'_i = 0$. The confidence level ρ is set to change from 50% to 95% that each incrumental step is 5%. We consider 8 types of containerships with different service capacities, $\mathcal{C}_l = 1000 \times l + 2000, l = 1, 2, 3, 4, 5, 6, 7, 8$. The parameter setting

Table: 3 Containerized cargo demand for each O-D pair (daily individual demand $Q_{s,d,w}$)

cargo demand (TEUs/day)		destination port SHA	HKG	SIN	RTD
origin port	SHA	-	$(290, 80^2)$	$(340, 120^2)$	$(320, 110^2)$
	HKG	$(180, 60^2)$	-	$(260, 80^2)$	$(200, 50^2)$
	SIN	$(170, 50^2)$	$(180, 50^2)$	-	$(220, 60^2)$
	RTD	$(300, 100^2)$	$(120, 40^2)$	$(150, 50^2)$	-

Note: numerics (a, b^2) are the mean cargo demand and its variance, namely $Nor\ (\mu_w, \sigma_w^2)$.

Table: 4 Freight rate of shipping service for each O-D pair

freight rate ($/TEU)		destination port SHA	HKG	SIN	RTD
origin port	SHA	-	550/440	1300/1040	3000/2400
	HKG	700/560	-	900/720	2500/2000
	SIN	950/760	1400/1120	-	1500/1200
	RTD	1800/1440	2000/1600	1100/880	-

Note: (a/b) are the freight rates for individual demand and contracted demand, (τ_w^s/τ_w^c).

Table: 5 Other parameters used in the tests

parameter		value	unit
daily unit inventory cost of storing laden containers at the origin port (α) for both non-transshipment and transshipment laden containers		10.0	\$/(TEU·day)
unit loading/unloading cost ($c_{l,a}$,$c_{u,a}$)	for non-transshipment laden containers	69.0	\$/TEU
	for transshipment laden containers	32.5	\$/TEU
unit loading/unloading cost ($c^e_{l,a}$,$c^e_{u,a}$)	for non-transshipment empty containers	48.0	\$/TEU
	for transshipment laden containers	22.5	\$/TEU
unit shipping cost ($\pi_n(C_r)$)	for $C_r = 3000$	21.40	\$/(TEU·day)
	for $C_r = 4000$	19.45	\$/(TEU·day)
	for $C_r = 5000$	18.30	\$/(TEU·day)
	for $C_r = 6000$	17.44	\$/(TEU·day)
	for $C_r = 7000$	16.79	\$/(TEU·day)
	for $C_r = 8000$	16.36	\$/(TEU·day)
	for $C_r = 9000$	16.16	\$/(TEU·day)
	for $C_r = 10000$	16.05	\$/(TEU·day)
daily capital cost ($\pi_{\text{ship}}(C_r)$) note: $\pi_{\text{ship}}(C_a) = \pi_{\text{ship}}(C_r)$	for $C_r = 3000$	12767.56	\$/day
	for $C_r = 4000$	15883.14	\$/day
	for $C_r = 5000$	18814.43	\$/day
	for $C_r = 6000$	21606.76	\$/day
	for $C_r = 7000$	24288.59	\$/day
	for $C_r = 8000$	26879.31	\$/day
	for $C_r = 9000$	29392.93	\$/day
	for $C_r = 10000$	31839.99	\$/day

for other given and known variables is summarized in Table 5. The attractive shipping path set is provided in *Appendix II*.

The programming code is compiled by Visual Studio 2010 and ILOG CPLEX 12.6, and runs on Windows 7 system with the following attributes: Intel Core i5-2400 3.1GHz×2 and 4GB RAM. All the tests can be completed within 10 minutes.

5.2. Optimization effect of the JCAP model

In this test, the confidence level (carrier's risk preference level) is set as $\rho = 50\%$. The optimization outcomes for the proposed JCAP model are summarized in Table 6.

Table: 6 Optimal solution for the SCAP model ($\rho = 50\%$)

cargo allocation outcomes:			
path no.	path flow of contracted containers ($x^c_{d,p,w}$)	path flow of expected individual containers ($x^s_{d,p,w}$)	path flow of empty containers ($x^e_{p,w}$)
1	0/0/220/260/140	0/0/290/290/290	0
2	120/220/0/0/0	290/290/0/0/0	0
3	0/0/280/320/230	0/0/340/340/340	0
5	180/230/0/0/0	340/340/0/0/0	0
10	200/240/220/300/250	320/320/320/320/320	0
11	140/170/0/0/0	180/180/160/0/180	0
14	0/0/160/190/150	0/0/20/180/0	0
15	120/145/0/0/0	260/260/0/0/0	0
16	0/0/220/210/190	0/0/260/260/260	0
18	150/100/250/300/200	200/200/200/200/200	0
19	145/160/0/0/0	170/170/0/0/0	630
20	0/0/160/170/150	0/0/170/170/170	765
21	150/150/0/0/0	180/180/0/0/0	0
23	0/0/150/175/145	0/0/180/180/180	0
25	100/180/200/250/170	220/220/220/220/220	0
27	170/200/260/190/150	300/300/300/300/300	950
30	200/180/220/190/150	120/120/120/120/120	275
32	140/160/200/175/150	150/150/150/150/150	0
departure time from the 1st port of call for each ship route $\boldsymbol{Y}_{r,m}$:			
	ship route 1	$\mathbf{Y}_{1,m} = \{0, 0, 0, 0, 0, 1, 0\}$,	departs on Sat.
	ship route 2	$\mathbf{Y}_{2,m} = \{0, 0, 1, 0, 0, 0, 0\}$,	departs on Wed.
ship capacity indicator $\boldsymbol{Y}_{r,l}$:			
	ship route 1	$\mathbf{Y}_{1,l} = \{0, 0, 1, 0, 0, 0, 0, 0\}$,	$C_1 = 5000$
	ship route 2	$\mathbf{Y}_{2,l} = \{0, 0, 0, 0, 0, 1, 0, 0\}$,	$C_2 = 8000$
design objective value (profit):			\$20.154 million

Note: the paths 4, 6-9, 12, 13,17, 22, 24, 26, 28, 29 and 31 with zero flows are not used.

As shown in the table, ship route 1 will depart from the 1st port of call at SHA on Saturday and ship route 2 will leave the 1st port of call at SHA on Wednesday. 5000-TEUs containerships are deployed for the fleet of ship route 1 and 8000-TEUs containerships are taken by ship route 2. The contracted and individual container cargoes collected on each weekly date can be properly allocated to 18 shipping paths, either non-transshipment or transshipment paths. The rest 14 unfavorable shipping paths: 4, 6–9, 12, 13, 17, 22, 24, 26, 28, 29 and 31 are not considered for any OD pairs. The empty container repositioning will be fulfilled by the shipping paths 19, 20, 27 and 30.

It is not difficult to examine that all containerized cargoes (including both total contracted demand and total expected individual demand) can be fully accommodated by the fleet deployments of two ship routes. The concern of possible redundant containers revealed in Wang et al. (2014b) is eliminated here due to introducing the flexible decision-making of ship capacity setting in the planning level. In a commercial market, an attractive service capacity expansion means the increasing accommodated demand can lead to sufficient profit to cover the cost of capacity expansion. In other words, the issue of potential redundant containers can be alleviated by a profitable capacity expansion. In this test, it is demonstrated that the JCAP model can help carriers determine appropriate ship capacity setting scheme.

Eventually, by the joint optimal schedule coordination, ship fleet capacity setting and cargo allocation scheme, the carrier will obtain profit of \$20.154 million. Overall, the proposed model is capable to help carriers make reasonable joint cargo assignment scheme of simultaneously optimizing shipping schedules, cargo routing and fleet capacity setting.

5.3. Impact analysis of carrier's risk preference

Recall that the confidence level in the chance constraint (15) can be interpreted as the carrier's risk preference towards the uncertain individual demand when making marketing decision of joint cargo assignment scheme. A small confidence level of ρ implies that the carrier shows an aggressive attitude (risk seeking) to the market with uncertain demand; on the contrary, a high confidence level of ρ means that the carrier behaves a conservative attitude (risk averse) to the uncertain market environment. In this subsection, the impact analysis of carrier's risk preference is explored by changing the confidence level ρ from 50% to 95% with each incremental step of 5%. Other parameters are the same ones introduced in Subsection 5.1.

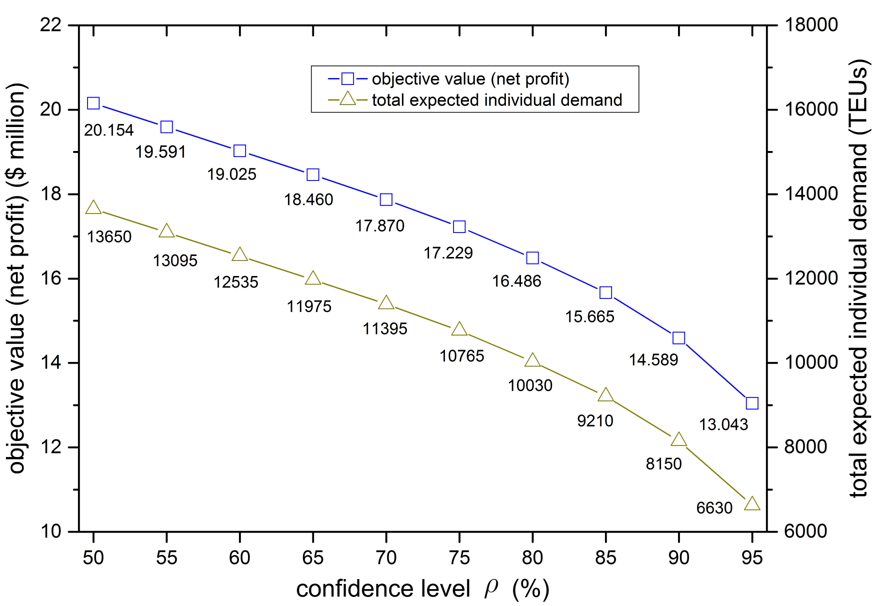

Fig. 3 Optimization effects with different confidence levels

Table: 7 Optimal solutions with different confidence levels

confidence level ρ	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%
ship capacity C_1	**5000**	**5000**	**5000**	**4000**	**4000**	**4000**	**3000**	**3000**	**3000**	**3000**
ship capacity C_2	8000	8000	8000	8000	8000	8000	8000	8000	8000	8000
ship schedule $Y_{1,m}$	Sat	Sat	Sat	Sat	Sat	Sat	Sat	Sat	Sat	**Fri**
ship schedule $Y_{2,m}$	Wed	Wed	Wed	Wed	Wed	Wed	Wed	Wed	Wed	Wed

Fig. 3 displays the results of the impact analysis. It can be seen that the design objective value monotonically decreases with the increase of confidence level ρ. In details, the objective value (profit) reduces from \$ 20.154 million to \$13.043 million when the carrier changes his risk neutral attitude to extreme risk averse. This fact could be easily explained. Once the decision-maker is inclined to be risk averse, he would have a more conservative estimation of the total expected individual containers collected from a fluctuated spot market. Therefore, as shown in Fig. 3, the total expected individual demand gradually decreases from 13650 TEUs to 6630 TEUs when the confidence level ρ changes form 50% (risk neutral) to 95% (risk averse).

Table: 8 Comparison of shipping path flows: $\rho = 50\%$ vs. $\rho = 95\%$

OD pair	path	shipping path flows			
	no.	path flows $x^c_{d,p,w}$		path flows $x^e_{p,w}$	
		$\rho = 50\%$	$\rho = 95\%$	$\rho = 50\%$	$\rho = 95\%$
SHA-HKG	1	0/0/220/260/**140**	0/0/220/260/**0**	0	0
	2	120/220/0/0/**0**	120/220/0/0/**140**	0	0
SHA-SIN	3	0/0/**280/320/230**	0/0/0/0/0	0	0
	5	**180/230**/0/0/0	**180/230/280/320/230**	0	0
SHA-RTD	10	200/240/220/300/250	200/240/220/300/250	0	0
HKG-SHA	11	**140/170**/0/0/0	0/0/0/0/0	0	0
	12	0/0/0/0/0	0/**170/160/190/150**	0	0
	13	0/0/0/0/0	**140**/0/0/0/0	0	0
	14	0/0/**160/190/150**	0/0/0/0/0	0	0
HKG-SIN	15	**120/145**/0/0/0	0/0/0/0/0	0	0
	16	0/0/**220/210/190**	**120/145/220/210/190**	0	0
HKG-RTD	18	150/100/250/300/200	150/100/250/300/200	0	0
SIN-SHA	19	**145/160**/0/0/0	**93**/0/0/0/0	**630**	**0**
	20	0/0/160/170/150	**52/160**/160/170/150	**765**	**655**
SIN-HKG	21	**150/150**/0/0/0	0/0/0/0/0	0	0
	23	0/0/150/175/145	**150/150**/150/175/145	0	0
SIN-RTD	25	100/180/200/250/170	100/180/200/250/170	0	0
RTD-SHA	27	170/200/260/190/150	170/200/260/190/150	**950**	**870**
RTD-HKG	30	200/180/220/190/150	200/180/220/190/150	**275**	**110**
RTD-SIN	32	140/160/200/175/150	140/160/200/175/150	0	0
sum		11295	11295	**2620**	**1635**

As an intuitive countermeasure to the decreasing expectation of individual demand collection in the spot market, the carrier determines to adjust his ship fleet deployment by using small containerships. In Table 7, it can be seen that, when the total expected individual cargo demand goes down, the carrier decides to redeploy the ship fleet of ship route 1. Specifically, when the confidence level ρ is less than 60%, the optimal ship capacity setting for the fleet of ship route 1 is 5000 TEUs; it changes to 4000 TEUs when confidence level ρ falls to the interval $[65, 75]$; and C_1 eventually reduces to 3000 TEUs when the confidence level ρ is set to be larger than 75%. The ship fleet capacity adjustment gives a convincing answer that it is important to consider a simultaneous optimization of ship capacity setting and cargo allocation scheme, especially for the case with uncertain demand in the market. Meanwhile, we can see that the effect of schedule coordination seems less significant than the adjustment of ship fleet capacity setting. The carrier will change the schedule of ship route 1 to Friday only when he behaves an extreme risk averse attitude, i.e., $\rho = 95\%$.

We also examine the impact of carrier's risk preference on the cargo allocation scheme, namely shipping path flow pattern. With no doubt, the shipping path flow pattern for the expected individual containers $x^s_{d,p,w}$ will be influenced by the carrier's risk preference as the total expected individual container demand shown in Fig. 3 decreases with the confidence level ρ. We now take a look at the shipping path flow patterns of the contracted containers and empty containers. Table 8 presents a comparison of shipping path flows of $x^c_{d,p,w}$ and $x^e_{p,w}$ between the cases with $\rho = 50\%$ and $\rho = 95\%$.

Let us firstly discuss the shipping path flow pattern of the contracted containers. It is found in the impact analysis tests that the total contracted cargo demand does not change no matter what confidence level is set. This is due to the compulsory conservation constraints (7) set for the contract basis cargo demand between each OD pair. Nevertheless, the related shipping path flow pattern is still remarkably influenced by the confidence level ρ. As shown in Table 8, 18 shipping paths are utilized for the contracted container transportation in the case with $\rho = 50\%$. However, 15 shipping paths are taken for the contracted container shipments in the case with $\rho = 95\%$. In details, shipping paths 12 and 13 are not attractive in the scenario that $\rho = 50\%$, but they are favorable in the scenario with $\rho = 95\%$. In turn, paths 3, 11, 14, 15 and 21 are not considered when the carrier shows high risk averse manner ($\rho = 95\%$), but they are all taken in the case when the carrier behaves risk neutral ($\rho = 50\%$). Evidently, as highlighted by the bold numerics in the table, the risk preference exerts explicit impact on the shipping path flow pattern of the contracted containers, although the total contracted demand keeps unchanged.

We then focus on the variation of path flows regarding empty container repositioning. It is not difficult to understand the shrink of the expected repositioned empty container demand when the carrier's risk behavior turns to be risk averse (larger value of ρ). Because the total expected individual demand decreases with the increase of confidence level ρ, the imbalance of OD demand distribution accordingly goes down. Consequently, we see that in Table **??** the path flows of expected repositioned empty containers when $\rho = 95\%$ are all less than or equal to those in the case with $\rho = 50\%$.

In summary, the impact analysis tells us that the optimal cargo allocation scheme is largely influenced by the carrier's risk preference. Moreover, if the carrier's risk preference is overlooked in a market environment with uncertain demand, an optimistic market demand estimation in the spot market would lead to overestimation of profit gain and make the shipping company be subject to certain risk of service capacity underutilization.

6. Conclusions and future works

In this paper, we studied a new and practical tactical-level decision problem in terms of a joint optimization of ship route scheduling, ship fleet capacity setting and cargo allocation scheme for the liner container shipping industry. For the container cargo assignment problem, we take into account a number of practical shipment issues in order to help the carriers make tangible tactical-level planning decisions. These important issues include the constitute of the liner container demand market, scaling effect of ship capacity setting, in-transit waiting time (extra delivery waiting time and transshipment waiting time), transshipment and empty container repositioning problems, differentiated charges and handling costs for non-transshipment and transshipment shipments, restraints of transshipment times and cargo permitted transport time etc. Two types of cargo demands in the liner container market are considered, namely the contracted basis demand received from large manufacturers and the individual demand collected from the spot market. For each OD pair, given the fluctuation of individual demand in the spot market, it is assumed to be a stochastic variable that follows a known probability distribution. The economies of scale of ship size is incorporated to measure the scaling effect of ship fleet capacity setting in the cargo assignment problem. The in-transit waiting time inherently depends on the ship route schedules and cargo collection weekly dates.

In order to properly describe various activities in the shipping transportation chain, we introduce a SAS network framework to integrate the processes of cargo loading/unloading, container transshipment, in-transit waiting or dwelling at the port, delivery waiting at the origin port and in-sea transport together. Based on this framework, we develop a chance-constrained optimization model for the joint cargo assignment problem. The model aims to maximize the total net profit for the container carrier by jointly determining the optimal ship route schedules, ship fleet capacity setting and the cargo allocation scheme. Finally, the numerical examples are carried out to test the optimization effect of the developed model and to analyze the impact of carrier's risk preference in decision-making. It is found that the differentiation of two cargo demands and the related chance-constrained model help the carrier realize the potential investment/operation risk in the liner container market. Considering schedule-based in-transit waiting time and scaling effect of ship size generates a more precise estimation of the profit for the liner container shipping service. More importantly, taking the scaling effect of ship size into the cargo assignment model in large degree alleviates the concern of cargo overload problem, since a more appropriate ship deployment scheme can be determined (although there probably still exists non-profitable cargo rejection).

This research makes an effort on filling the gap in container shipping planning by developing an optimization model for the joint cargo assignment problem by simultaneously optimizing the ship route schedules, ship capacity setting and cargo allocation scheme. We can further make some interesting extensions. First, a price-sensitive demand function can be considered so that a more practical planning including service charge optimization could be designed. Second, this study carries out numerical tests in a small shipping network in order to make an exact impact analysis of the carrier's risk preference. How to design a more efficient solution algorithm for the joint cargo assignment problem in a large-size shipping network is a worthwhile yet challenging task.

Appendix I

Table: 9 Notation used the paper

sets:	
N	a set of nodes in the liner shipping network;
N_k	a subset of k^{th} kind of nodes, $k \in \{O, W, A, D\}$, which denotes original nodes (physical ports), waiting-delivery nodes, arrival nodes and departure nodes accordingly;
A	a set of links in the liner shipping network;
A_k	a subset of k^{th} kind of links, $k = 1, 2, 3, 4, 5, 6, 7$;
W	a set of OD pairs (all port pairs) in the liner shipping network;
R	a set of ship routes in the liner shipping network;
P_w	a set of shipping paths for OD pair $w \in W$ in the SAS network;
D	a set of weekly dates, $D = \{1, 2, 3, 4, 5, 6, 0\}$;
D_c	a set of collection weekly dates of daily contracted containerized cargoes, $D_c \subseteq D$;
D_s	a set of collection weekly dates of daily individual containerized cargoes, $D_s \subseteq D$;
L	a set of ship types with different ship capacities, $L = \{1, 2, \cdots, \lvert L\rvert\}$;
indices:	
r	a particular ship route, $r \in R$;
w	a particular OD pair, $w \in W$;
i, j	particular nodes, $i \in N, j \in N$;
a	a particular link, $a \in A$, and another link form is $\langle i, j\rangle$;
p	a particular shipping path, $p \in P_w$;
m	a particular weekly date for the ship route departs from the 1st port of call, $m \in D$;
$m_{k,r}$	the departure time of ship route r at the k^{th} port of call, $m_{k,r} \in D$;
d	a particular cargo collection weekly date, $d \in D_c$ or $d \in D_s$;
design variables:	
$Y_{r,m}$	the indicator which equals 1 if and only if a ship departs from the 1st port of call of ship route r on weekly date $m \in D$; and 0 otherwise;
$Z_{r,l}$	the indicator which equals 1 if and only if the ships deployed for ship route r have equal ship capacities $\mathcal{C}_l$, where $C_l \in \{\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_{\lvert L\rvert}\}$ and $l \in L = 1, 2, \cdots, \lvert L\rvert$; and 0 otherwise;
C_r	the service capacity of ship route r, $r \in R$, $C_r = \sum_l Y_r^l \times \mathcal{C}_l, \forall l \in L$;
$x^c_{d,p,w}$	the quantity of contracted laden containers collected on weekly date d that will be loaded on path $p \in P_w, w \in W$ for shipment;
$x^s_{d,p,w}$	the expected quantity of individual laden containers collected on weekly date d that will be loaded on path $p \in P_w, w \in W$ for shipment;
$x^e_{p,w}$	the expected quantity of empty containers that will be transported by path $p \in P_w, w \in W$;
$\boldsymbol{Y}_m, \boldsymbol{Z}_l$	vectors for the design variables $Y_{r,m}$ and $Z_{r,l}$;
$\boldsymbol{x}_c, \boldsymbol{x}_s, \boldsymbol{x}_e$	vectors for the variables $x^c_{d,p,w}$, $x^s_{d,p,w}$ and $x^e_{p,w}$ respectively;
variables to be determined:	
$t_{v,a}$	the in-transit waiting time (delivery waiting time or transshipment waiting time) for link $a \in A_3 \cup A_6 \cup A_7$;
$c_{v,d,p,w}$	the extra in-transit inventory cost of shipping path p for the cargoes collected on weekly date d between OD pair w;
C_a	the link capacity, $C_a = C_r, \forall \delta_{a,r} = 1, a \in A_1$;
$\pi_n(C_r)$	the unit shipping cost of transporting one container per day (or per nautical mile) by the container ship fleet with service capacity C_r;

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$\pi_n(C_a)$	the interchangeable form of $\pi_n(C_r)$;
$\pi_{d,p,w}$	the shipping cost (excluding the ship capital cost) of transporting one laden cargo container collected on date d by shipping path $p \in P_w$;
$\pi^e_{p,w}$	the shipping cost (excluding the ship capital cost) of transporting one empty container by shipping path $p \in P_w$ for OD pair $w \in W$;
$\pi_{\text{ship}}(C_r)$	the daily capital cost for the container ship with capacity of C_r;
F_k	the cost components where $k \in \{\text{rev,inv,han,nav,emp}\}$, which are gross revenue, in-transit inventory cost, handling cost, shipping cost and empty container operating cost;
$t_{p,w}$	the transport time of shipping path $p \in P_w$ for OD pair w;
$n_{p,w}$	the number of transshipment times by using shipping path $p \in P_w$;
parameters given:	
$q^c_{d,w}$	the daily received containerized cargoes (TEUs) binded by long-term contacts with the large manufacturers for OD pair $w \in W$;
$Q^s_{d,w}$	the daily individual containerized cargoes (TEUs) collected from the spot market for OD pair $w \in W$, which follows a *Normal* distribution with mean of μ_w and standard variance of σ_w over the planning horizon T;
$\delta_{a,r}$	the link-route indicator, 1 if link a belongs to ship route $r \in R$, 0 otherwise;
$\delta_{a,p,w}$	the link-path indicator, 1 if the shipping path $p \in P_w$ uses link a, 0 otherwise;
$\delta_{\langle i,j\rangle,p,w}$	another form of the link-path indicator $\delta_{a,p,w}$;
τ^c_w	the freight rate for shipping one contracted container $q_{c,d,w}$ of OD pair w by ship route r;
τ^s_w	the freight rate for shipping one individual container $q_{s,d,w}$ of OD pair w by ship route r;
1 $t_{n,a}$	the navigation time of the physical link $a \in A_1$;
$t_{g,a}$	the dwell time of dwell link $a \in A_2$ at the visited port;
$c_{l,a}$	the unit handling cost of loading a laden container (differentiating non-transshipment and transshipment containers);
$c_{u,a}$	the unit handling cost of discharging a laden container (differentiating non-transshipment and transshipment containers);
$c^e_{l,a}$	the unit loading cost for empty containers;
$c^e_{u,a}$	the unit discharging cost for empty containers;
$c_{h,p,w}$	the total handling cost for one unit laden container on shipping path $p \in P_w$;
$t_{1-k,r}$	the transport time from the 1$^{\text{st}}$ port of call to the k^{th} port of call by ship route r;
$t_{k-k',r}$	the transport time during first-visit and revisit at the k^{th} port of call by ship route r;
$t_{k'-k,r}$	the transport time during revisit and first-visit at the k^{th} port of call by ship route r;
α	the daily inventory cost of storing laden containers at the port;
Δ_i	the free-usage time (days) for storing exporting cargoes at the port i;
Δ'_i	the free demurrage time (days) at the transshipment port i;
$t_{n,p,w}$	in-sea navigation time for the containers transported by shipping path $p \in P_w, w \in W$;
t_r	the transport time (including navigation time at sea and dwell time at port) of a round trip of ship route r;
$\pi_b(\mathcal{C}_l)$	the pre-determined unit shipping cost for a given ship capacity parameter $\mathcal{C}_l$;
$\pi_{\text{cap}}(\mathcal{C}_l)$	pre-determined daily ship cost for a given ship capacity parameter $\mathcal{C}_l$;
$\bar{t}_w$	the permitted maximal transport time for OD pair w;
$\bar{n}_w$	the permitted maximal transshipment times for OD pair w;
ρ	a given confidence level to measure the carrier's risk preference;

Appendix II

Table: 10 Shipping paths used for the numerical examples

OD pair	path No.	elemental links for each shipping path	ship routes used
SHA $\to$ HKG	1	$0_A \to 1'_A \to 1^a_A \to 1^d_A \to 1^a_H \to 0_H$	route ①
	2	$0_A \to 2'_A \to 2^a_A \to 2^d_A \to 2^a_H \to 0_H$	route ②
SHA $\to$ SIN	3	$0_A \to 1'_A \to 1^a_A \to 1^d_A \to 1^a_H \to 1^d_H \to 1^a_S \to 0_S$	route ①
	4	$0_A \to 1'_A \to 1^a_A \to 1^d_A \to 1^a_H \dashrightarrow 2^d_H \to 3^a_S \to 0_S$	routes ① $\dashrightarrow$ ②
	5	$0_A \to 2'_A \to 2^a_A \to 2^d_A \to 2^a_H \to 2^d_H \to 3^a_S \to 0_S$	route ②
	6	$0_A \to 2'_A \to 2^a_A \to 2^d_A \to 2^a_H \dashrightarrow 1^d_H \to 1^a_S \to 0_S$	routes ② $\dashrightarrow$ ①
SHA $\to$ RTD	7	$0_A \to 1'_A \to 1^a_A \to 1^d_A \to 1^a_H \to 1^d_H \to 1^a_S \to 3^d_S \to 1^a_R \to 0_R$	routes ① $\dashrightarrow$ ②
	8	$0_A \to 1'_A \to 1^a_A \to 1^d_A \to 1^a_H \dashrightarrow 2^d_H \to 3^a_S \to 3^d_S \to 1^a_R \to 0_R$	routes ① $\dashrightarrow$ ②
	9	$0_A \to 2'_A \to 2^a_A \to 2^d_A \to 2^a_H \dashrightarrow 1^d_H \to 1^a_S \to 3^d_S \to 1^a_R \to 0_R$	routes ② $\dashrightarrow$ ① $\dashrightarrow$ ②
	10	$0_A \to 2'_A \to 2^a_A \to 2^d_A \to 2^a_H \to 2^d_H \to 3^a_S \to 3^d_S \to 1^a_R \to 0_R$	route ②
HKG $\to$ SHA†	11	$0_H \to 1'_H \to 1^a_H \to 1^d_H \to 1^a_S \to 1^d_S \to 1^a_A \to 0_A$	route ①
	12	$0_H \to 2'_H \to 2^a_H \to 2^d_H \to 3^a_S \dashrightarrow 2^d_S \to 2^a_A \to 0_A$	routes ② $\dashrightarrow$ ②
	13	$0_H \to 1'_H \to 1^a_H \to 1^d_H \to 1^a_S \dashrightarrow 2^d_S \to 2^a_A \to 0_A$	routes ① $\dashrightarrow$ ②
	14	$0_H \to 2'_H \to 2^a_H \to 2^d_H \to 3^a_S \dashrightarrow 1^d_S \to 1^a_A \to 0_A$	routes ② $\dashrightarrow$ ①
HKG $\to$ SIN	15	$0_H \to 1'_H \to 1^a_H \to 1^d_H \to 1^a_S \to 0_S$	route ①
	16	$0_H \to 2'_H \to 2^a_H \to 2^d_H \to 3^a_S \to 0_S$	route ②
HKG $\to$ RTD	17	$0_H \to 1'_H \to 1^a_H \to 1^d_H \to 1^a_S \dashrightarrow 3^d_S \to 1^a_R \to 0_R$	routes ① $\dashrightarrow$ ②
	18	$0_H \to 2'_H \to 2^a_H \to 2^d_H \to 3^a_S \to 3^d_S \to 1^a_R \to 0_R$	route ②
SIN $\to$ SHA	19	$0_S \to 1'_S \to 1^a_S \to 1^d_S \to 1^a_A \to 0_A$	route ①
	20	$0_S \to 2'_S \to 2^a_S \to 2^d_S \to 2^a_A \to 0_A$	route ②
SIN $\to$ HKG	21	$0_S \to 1'_S \to 1^a_S \to 1^d_S \to 1^a_A \to 1^d_A \to 1^a_H \to 0_H$	routes ①
	22	$0_S \to 1'_S \to 1^a_S \to 1^d_S \to 1^a_A \dashrightarrow 2^d_A \to 2^a_H \to 0_H$	routes ① $\dashrightarrow$ ②
	23	$0_S \to 2'_S \to 2^a_S \to 2^d_S \to 2^a_A \to 2^d_A \to 2^a_H \to 0_H$	route ②
	24	$0_S \to 2'_S \to 2^a_S \to 2^d_S \to 2^a_A \dashrightarrow 1^d_A \to 1^a_H \to 0_H$	routes ② $\dashrightarrow$ ①
SIN $\to$ RTD	25	$0_S \to 3'_S \to 3^a_S \to 3^d_S \to 1^a_R \to 0_R$	route ②
RTD $\to$ SHA	26	$0_R \to 1'_R \to 1^a_R \to 1^d_R \to 2^a_S \dashrightarrow 1^d_S \to 1^a_A \to 0_A$	routes ② $\dashrightarrow$ ①
	27	$0_R \to 1'_R \to 1^a_R \to 1^d_R \to 2^a_S \to 2^d_S \to 2^a_A \to 0_A$	route ②
RTD $\to$ HKG	28	$0_R \to 1'_R \to 1^a_R \to 1^d_R \to 2^a_S \dashrightarrow 1^d_S \to 1^a_A \to 1^d_A \to 1^a_H \to 0_H$	routes ② $\dashrightarrow$ ①
	29	$0_R \to 1'_R \to 1^a_R \to 1^d_R \to 2^a_S \dashrightarrow 1^d_S \to 1^a_A \to 2^d_A \to 2^a_H \to 0_H$	routes ② $\dashrightarrow$ ① $\dashrightarrow$ ②
	30	$0_R \to 1'_R \to 1^a_R \to 1^d_R \to 2^a_S \to 2^d_S \to 2^a_A \to 2^d_A \to 2^a_H \to 0_H$	route ②
	31	$0_R \to 1'_R \to 1^a_R \to 1^d_R \to 2^a_S \to 2^d_S \to 2^a_A \to 1^d_A \to 1^a_H \to 0_H$	routes ② $\dashrightarrow$ ①
RTD $\to$ SIN	32	$0_R \to 1'_R \to 1^a_R \to 1^d_R \to 2^a_S \to 0_S$	route ②

†: for the OD pair HKG $\to$ SHA, the candidate shipping paths that contain a long voyage loop SIN $\to$ RTD $\to$ SIN are dropped, although perhaps without transshipments.

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