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Pricing Strategy for A Two-Echelon Supply Chain with Optimized Return Effort Level

Yue Xie * Allen H. Tai † Wai-Ki Ching ‡ Tak-Kuen Siu §

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Abstract

This paper studies a supply chain consisting of one supplier and n retailers. The market demand for each retailer is assumed to be dependent on the difference between the retail price and the average retail price. The supplier considers two wholesale price strategies. In the first strategy, Strategy I, the wholesale prices to all n retailers are the same. In the second strategy, Strategy II, different wholesale prices are given to the retailers on the basis of the effort levels they required on products. The retailers who face retail price-dependent demand have different unit sales costs and determine their effort levels according to their different unit return service costs. We first model the retail price competition behavior of n retailers under the two wholesale price strategies. Then the retailers' optimal retail prices and the supplier's optimal wholesale price in each model are derived using a game theoretic approach. The effects of some key parameters on supply chain decisions and profit are investigated. The properties of retailers' (supplier's) optimal profits under the two wholesale price strategies are studied.

Keywords: Supply Chain; Multiple Retailers; Price-dependent Demand; Pricing; Game theory.

*College of Economics and Management, Zhejiang University of Technology, P.R. China. Email: xieyue1988moon@163.com

†The Hong Kong Polytechnic University, Hung Hom, Hong Kong. Email: allen.tai@polyu.edu.hk

‡Advanced Modeling and Applied Computing, Department of Mathematics, The University of Hong Kong. Email: wching@hku.hk

§Department of Applied Finance and Actuarial Studies, Faculty of Business and Economics, Macquarie University, Sydney, Australia. E-mail: ktksiu2005@gmail.com

1 Introduction

In modern supply chain systems, suppliers and retailers would try to optimize their own profit functions by improving their services and/or product quality with a view to establishing brand loyalty and compete with each other in the market. These competitions in supply chains have gradually received much attention. For example, Chen et al. [6] modeled the competition process between two retailers whose demands depend on warranty periods. Retailers' optimal warranty periods under different wholesale pricing strategies were derived using game theory. Wu et al. [20] investigated the equilibrium behavior of two competing supply chains with uncertain demand. They found that applying vertical integration in both chains is the unique Nash Equilibrium over one period decision. Narayanan et al. [14] modeled a manufacturer that contracts with two competing retailers. They showed that if the manufacturer can subsidize the retailers' leftover inventory, first-best retail prices and fill rates can be achieved. For details, interested readers may refer to [2, 15, 18, 22] and the relevant references therein. Also, a number of strategies, such as buy-back contracts, credit payment options and wholesale price discounts, have been proposed to enhance the coordination between suppliers (or manufacturers) and retailers, see for example [1, 7, 11].

A production process is not deemed to be perfect and a product may be defective or non-conforming. A number of inventory models have been proposed for this important problem involving an imperfect production process, see for example [12, 17, 19, 21, 23] amongst others. After retailers receive the items from the suppliers, they may only conduct some sample check, and some defective items may be identified. The rest of the defective items will then be sold together with the normal items to the customers. The returns of defective products will affect the retailers' credibility and reputation, and this may affect the profitability of the retailers. To maintain both credibility and reputation, the retailers have to adopt some feasible strategies to avoid customers from getting the defective product and/or provide after-sales service to protect consumers' interest.

Improving the product qualification ratio is one of the efficient ways to reduce the likelihood of the returns of defective products. In fact, the issue about production quality has been studied quite widely in the literature. For example, Banker et al. [3] investigated how quality is influenced by competitive intensity in a given industry and Reyniers et al. [16] recognized that quality supply and quality inspection would result in opposing interests between the supplier and the retailer. In [8], Ferguson et al. showed that false failure returns can be reduced if the retailer put more effort on sales service. Then they designed a target rebate contract to encourage the retailer to put more effort on sales services. It is found that their contract contributes to the profit improvement of both parties. Furthermore, Huang et al. [10] proposed a quantity discount contract to motivate the retailer to put more effort on sales services since the payment from the supplier to the retailer in the contract is exponentially decreasing with the number of false failure returns. Thus, based on [8] and [10], we would consider the case that retailers can purchase better quality products by making an extra effort on products, which aims to reduce the number of returns of defective products.

In general, it would be beneficial for the retailers to improve quality control by making an extra effort on products. On one hand, retailers who value the quality of their products typically provide a full credit to consumers. A high level of product quality may give consumers confidence in purchasing the product and may stimulate sales. This may then lead to an increase in profit. On the other hand, if the cost spent on improving quality of products (hence reducing the number of returns) is less than the cost that can be saved, then investment in the production quality may be able to reduce the total cost.

In practice, retailers usually provide return service as one of the after-sales services. This allows consumers to return the defective products and to request for a new one. The return service cost is usually undertaken by both the retailer and the supplier. In this paper, since possible costs, such as the goodwill cost and the transportation cost, depending on the situations of retailers, could be different, the return service costs incurred by retailers are assumed to be different from each other.

This paper studies a supply chain with one supplier and n retailers. The main focus is the price competition behavior of the n retailers. We assume that the retailers have different unit sales costs and different unit return service costs since they may have different sales and management efficiencies. The product quality can be improved by making an extra effort. The demand rate of each retailer is assumed to be dependent on her retail price and also affected by the difference between her retail price and the average retail price among all retailers. For the supplier, there are two wholesale price strategies.

Strategy I is that the supplier gives a single wholesale price to all the n retailers. Strategy II is that the supplier sets different wholesale prices according the retailers' effort levels on products. It seems that most of supply chain models consider only one retailer or two competing retailers. Bernstein et al. [2] analysed the situation that a supplier distributes a product to multiple competing retailers, while they did not give a closed form solution of the optimal retail price for each retailer. Our study contributes to the literature by giving a simple closed-form solution for each retailer's optimal retail price under each of the two wholesale pricing strategies. The optimal wholesale price decided by the supplier is also given for each strategy. The Law of Large Numbers adopted in [11, 13] is then applied to simplify the calculations and which then gives rise to an approximate solution.

The rest of this paper is organized as follows. Section 2 presents the model description and assumptions. Section 3 presents the model of one supplier and n retailers under the wholesale pricing Strategy I. Furthermore, Section 4 presents a model of one supplier and n retailers under the wholesale pricing Strategy II. The effect of different pricing strategies on decision making of retailers and the supplier are presented in Section 5. Finally, the last section concludes the paper.

2 Model Assumptions

A single-period two-echelon supply chain with one supplier and n competing retailers is discussed here. The supplier's unit ordering cost of a product is m and two wholesale price strategies are adopted by the supplier. Strategy I is that the supplier gives the same wholesale price w to the n retailers. Strategy II is that the supplier sets different wholesale price w_i for each retailer according to the effort level they required on products. The retailers purchase products from the supplier and then sell them to the consumers at the retail prices p_i $(i = 1, \ldots, n)$, where p_i is the retail price of the i'th retailer. Similar to the model in [2], we assume that the supplier replenishes her inventory from some sources/manufacturers with an ample supply, and all demands and all retailers' orders are satisfied.

Retailer i's unit sales cost of a product is u_i. Because of the differences in sales efficiencies and strategies, it may not be unreasonable to assume that the retailers have different unit sales costs. The consumers are allowed to return the products to the retailer and receive a new one if the item is defective. The returns incur a cost, such as the goodwill cost, the transportation cost, and the re-processing cost, etc. The unit cost due to the return incurred by the supplier is denoted as s and the unit cost due to the return incurred by the ith retailer is denoted as k_i ($k_i \neq k_j$ if $i \neq j$). The situation that a consumer receives a defective item and chooses to walk away or makes a repurchase from other retailers is not considered here.

We assume that to reduce the return service cost caused by defective items, the retailers can purchase products with better quality from the supplier. The additional cost on one product paid by a retailer is $l(q)$, which depends on the effort level q. It is assumed that $q \geq 1$, i.e., a minimal level of effort to be exerted is one. The effort level q may be thought of the inspection time on each product which is required by the retailer. Similar assumptions can refer to Ferguson et al. [8] and Huang et al. [10].

Given an effort level q, the retailer's unit effort cost of the product is defined by

$$l(q) = \frac{b}{2}q^2, \ b > 0.$$

In this definition, b can be explained as the retailer's marginal cost of effort under the minimum effort level $q = 1$. In the "toy" modeling set up considered here, the supplier is given the flexibility to order products from different manufacturers. The qualities of the products from different manufacturers may vary. To meet the quality requirements of the retailers, the supplier may order products with better quality from different manufacturers. Alternatively, the supplier may require a particular manufacturer to supply products with improved quality. In either one of the two cases, the supplier may have to pay an additional cost for products with better quality. To capture this additional cost, we assume that the additional cost for each product, denoted by $h(q)$, has the following form:

$$h(q) = \frac{c}{2}q^2, \ c > 0,$$

where c can be explained as the supplier's marginal cost of effort under the minimum effort level $q = 1$. Here the quadratic functions of $l(q)$ and $h(q)$ are convex in q. These expressions indicate that as q increases, to further enhance the effort level by a specific amount would become more expensive.

The probability that a consumer gets a defective item is $Z(q)$, which decreases as the effort level q increases. Let β be the probability of getting a defective item when the minimal level of effort is exerted, which means $\beta = Z(1) \in (0,1)$ and we suppose that

$$Z(q) = \frac{\beta}{q} \in (0,1).$$

This parametrization for the probability is for algebraic convenience and it makes intuitive sense.

Furthermore, we assume that the demand of the i'th retailer is given by

$$D_i = \begin{cases} a - \gamma p_i & \text{if } n = 1 \\ a - \gamma p_i - g\left(p_i - \dfrac{1}{n}\displaystyle\sum_{j=1}^{n} p_j\right) & \text{if } n > 1, \end{cases} \tag{2.1}$$

where $a > 0$, $\gamma > 0$ and $g > 0$. Similar demand functions can be found in Bernstein et al. [2] and Wu et al. [20]. However, in this supply chain, the demand rate of each retailer depends on her retail price and the average retail price of all retailers. This is different from other supply chain models. For example, in [7], the demand rate of the retailer just depends on her own retail price. In [2], Bernstein et al. assumed that the demand function of each retailer is a linear combination of all retailers' retail prices. In [4, 5], each retailer was assumed to experience stochastic and independent demand. Here a is the primary demand of each retailer, γ is the consumers' sensitivity to the retail price of the product, and g is the competitive factor generated by the difference between Retailer i's retail price and the average retail price. The linear and symmetric demand model represents a situation in which retailers have equal competing power in the market.

To facilitate our discussion with the one-supplier and n-retailer model, a summary of notations is given in Table 1.

Table 1: List of Notations

a	the primary demand of each retailer;
b	the retailer's marginal cost of effort under the minimum effort level;
c	the supplier's marginal cost of effort under the minimum effort level;
g	the competitive factor;
m	the unit ordering cost of the product;
s	the unit return service cost of the supplier.
β	the probability of getting a defective when minimum effort is exerted;
γ	the rate of change of demand with respect to price;
D_i	the demand of Retailer i;
k_i	the unit return service cost of Retailer i;
p_i	the retail price of Retailer i;
q_i	the effort level required by Retailer i;
u_i	the unit sales cost of Retailer i;
w_i	the wholesale price to Retailer i.

3 One-supplier and n-retailer Model with Wholesale Pricing Strategy I

In this section, the general situation of one supplier and n retailers is considered. The sequence of events is as follows. Retailer i first decides the optimal effort level q_{ni} according to her unit return service cost k_{ni}. Next the supplier sets the optimal wholesale price w_n for all of the retailers. Finally Retailer i determines the optimal retail price p_{ni} as a function of w_n and q_{nj}, $j = 1, 2, \ldots, n$, following the Nash's equilibrium.

In order to simplify the notations in this section, we let

$$G_{ni} = m + \frac{c-b}{2} q_{*ni}^2 + s\frac{\beta}{q_{*ni}} \quad \text{and} \quad F_{ni} = \frac{b}{2} q_{*ni}^2 + k_{ni}\frac{\beta}{q_{*ni}} + u_{ni}. \tag{3.2}$$

When n is sufficiently large, by the Law of Large Numbers, denote

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} G_{ni} = E(G_{ni}) = \bar{G} \quad \text{and} \quad \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} F_{ni} = E(F_{ni}) = \bar{F} \tag{3.3}$$

3.1 The Retailer Problem

Retailer i ($i = 1, 2, \ldots, n$) has to define the optimal effort level and retail price so as to maximize her profit. Again, Retailer i's profit is a function of two variables, denoted as $\Pi^r_{ni}(p_{ni}, q_{ni})$,

$$\Pi^r_{ni}(p_{ni}, q_{ni}) = p_{ni}D_{ni} - \left(w_n + \frac{b}{2}q^2_{ni}\right) D_{ni} - k_{ni}\frac{\beta}{q_{ni}}D_{ni} - u_{ni}D_{ni}, \tag{3.4}$$

where the demand function D_{ni} is given by

$$D_{ni} = a - \gamma p_{ni} - g\left(p_{ni} - \frac{1}{n}\sum\nolimits_{j=1}^{n} p_{nj}\right).$$

Therefore, Retailer i's ($i = 1, \ldots, n$) decision problem is

$$\max_{p_{ni}, q_{ni}} \Pi^r_{ni}(p_{ni}, q_{ni}).$$

The following proposition gives the optimal solution and the proof can be found in Appendix.

Proposition 1. *In the case of one-supplier and n-retailer, Retailer i's ($i = 1, \ldots, n$) optimal effort level is given by*

$$q_{*ni} \quad = \quad max\left(\left(\frac{k_{ni}\beta}{b}\right)^{1/3} 1\right). \tag{3.5}$$

Then under the supplier's optimal wholesale price w_{*n} which will be explained in Proposition 2, Retailer i's ($i = 1, \ldots, n$) optimal retail price is given by

$$\begin{aligned} p_{*ni} \quad = \quad & \frac{a}{2\gamma + g - \frac{g}{n}} + \frac{(\gamma + g - \frac{g}{n})g}{(2(\gamma + g) - \frac{g}{n})(2\gamma + g - \frac{g}{n})} \frac{\sum_{j=1}^{n}(w_{*n} + F_{nj})}{n} \\ & + \frac{(\gamma + g - \frac{g}{n})}{2(\gamma + g) - \frac{g}{n}}\left(w_{*n} + F_{ni}\right). \end{aligned} \tag{3.6}$$

When n is sufficiently large, based on Eq. (3.3), by the Law of Large Numbers, it has

$$p_{*ni} = \frac{a}{2\gamma+g} + \frac{g}{2(2\gamma+g)}(w_{*n}+\bar{F}) + \frac{1}{2}(w_{*n}+F_{ni}). \tag{3.7}$$

This proposition shows that when n is sufficiently large, $a/(2\gamma+g)$ is a basic retail price for each retailer, which is decided by the market. Then Retailer i's optimal retail price is affected by the average total unit cost of the retailers $(w_{*n}+\bar{F})$ and her total unit cost $(w_{*n}+F_{ni})$, with weighting factors $g/(2(2\gamma+g))$ and $1/2$, respectively. We note that $1/2 > g/(2(2\gamma+g))$. This means that Retailer i's optimal retail price is more sensitive to her total unit cost. Furthermore, the mean retail price is given by:

$$\bar{P}_n = E(p_{*ni}) = \lim_{n\to\infty} \frac{\sum_{i=1}^n p_{*ni}}{n} = \frac{a}{2\gamma+g} + \frac{\gamma+g}{2\gamma+g}(w_{*n}+\bar{F}).$$

From Eq. (3.4), Retailer i's optimal profit is

$$\Pi^r_{ni}(p_{*ni}, q_{*ni}) = (p_{*ni} - w_{*n} - F_{ni})\left(a - \gamma p_{*ni} - g\left(p_{*ni} - \frac{1}{n}\sum_{i=1}^n p_{*ni}\right)\right). \tag{3.8}$$

When n is sufficiently large, it can be rewritten as

$$\Pi^r_{ni}(p_{*ni}, q_{*ni}) = (p_{*ni} - w_{*n} - F_{ni})\left(a - \gamma p_{*ni} - g\left(p_{*ni} - \bar{P}_n\right)\right). \tag{3.9}$$

3.2 The Supplier Problem

Given that Retailer i's optimal effort level q_{*ni} has been decided, the supplier has to determine a wholesale price w_n so as to maximize her expected profit. The supplier's profit, $\Pi^m_n(w_n)$, is given by

$$\Pi^m_n(w_n) = \sum_{i=1}^n \left\{ w_n D_{*ni} - \left(m + \frac{c-b}{2}q^2_{*ni}\right) D_{*ni} - \frac{s\beta}{q_{*ni}} D_{*ni} \right\}, \tag{3.10}$$

where

$$D_{*ni} = a - \gamma p_{*ni} - g\left(p_{*ni} - \frac{1}{n}\sum_{j=1}^n p_{*nj}\right).$$

Here the supplier provides a wholesale price w_n to all of the retailers. Hence based on

Proposition 1, Retailer i's optimal retail price in Eq. (3.6) is denoted as

$$\begin{aligned} p_{*ni} &= \frac{a}{2\gamma + g - \frac{g}{n}} + \frac{(\gamma + g - \frac{g}{n})g}{(2(\gamma+g) - \frac{g}{n})(2\gamma + g - \frac{g}{n})} \frac{\sum_{j=1}^{n}(w_n + F_{nj})}{n} \\ &\quad + \frac{(\gamma + g - \frac{g}{n})}{2(\gamma + g) - \frac{g}{n}} (w_n + F_{ni}) . \end{aligned} \tag{3.11}$$

The first term in Eq. (3.10) is the sales revenue and the second term is the total ordering cost, where the unit cost consists of the unit ordering cost m and the unit additional ordering cost with effort level q_{*ni}. The last term is the expected cost due to returns of products. Therefore, the supplier's decision problem is

$$\max_{w_n} \Pi_n^m(w_n).$$

The following proposition gives the supplier's optimal wholesale price. The proof is given in Appendix.

Proposition 2. *In the case of one-supplier and n-retailer, the supplier's optimal wholesale price satisfies*

$$w_{*n} = \frac{a}{2\gamma} - \frac{1}{2}\frac{\sum_{i=1}^{n} F_{ni}}{n} + \frac{1}{2}\frac{\sum_{i=1}^{n} G_{ni}}{n}. \tag{3.12}$$

When n is sufficiently large, based on Eq. (3.3), by the Law of Large Numbers, we have

$$w_{*n} = \frac{a}{2\gamma} - \frac{\bar{F} - \bar{G}}{2}. \tag{3.13}$$

As n is getting large, then by Eq. (3.13), w_{*n} can be rewritten (approximate) as:

$$w_{*n} = \frac{a}{\gamma} - (\bar{F} + w_{*n} - \bar{G}).$$

Note that the second item is the difference between retailers' average total unit cost and the supplier's average total unit cost for the retailers, when the optimal effort level and the optimal wholesale price are adopted. Then, from Eq. (3.10), the supplier's optimal profit is given by

$$\Pi_n^m(w_{*n}) = \sum_{i=1}^{n}(w_{*n} - G_{ni})D_{*ni}. \tag{3.14}$$

The following proposition presents some properties of the supplier's profit function. The proof can be found in Appendix.

Proposition 3. *Suppose that* $k_{ni} > \dfrac{b}{\beta}$ *and* n *is sufficiently large.*

1. *With supplier's optimal wholesale price in proposition 2,* $\Pi_{ni}^r(p_{*ni}, q_{*ni})$ *is a decreasing function in* s*.*

2. *By the Law of Large Numbers,*

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}\frac{1}{q_{*ni}} = E\left[\frac{1}{q_{*ni}}\right] = \frac{1}{\bar{q}_n}.$$

   If $q_{*ni} < 2\bar{q}_n$ *for each* $i = 1, 2, \ldots, n$ *then* $\Pi_n^m(w_{*n})$ *is also a decreasing function in* s*.*

Proposition 3 implies that if q_{*ni} is less than twice the harmonic mean of q_{*ni} for any i ($i = 1, 2, \ldots, n$) then by reducing the unit cost s of the supplier's return service, the net profits of supplier and retailers will increase. Hence, it is crucial to reduce the unit cost s.

4 One-supplier and "sufficiently large" n-retailer Model with Pricing Strategy II

In this section, a hypothetical situation where the supplier provides different wholesale prices to retailers based on the their effort levels is considered. The sequence of events is assumed to be as follows. Retailer i first decides the optimal effort level $q_{\tilde{n}i}$ according to her unit return service cost $k_{\tilde{n}i}$. Next the supplier sets the optimal wholesale price $w_{\tilde{n}i}$ for Retailer i. Finally, according to the Nash equilibrium, Retailer i determines her optimal retail price $p_{\tilde{n}i}$ as a function of $w_{\tilde{n}j}$ and $q_{\tilde{n}i}$, $j = 1, 2, \ldots, n$. (Here $k_{\tilde{n}i} = k_{ni}$ and both of them represent the unit return service cost of Retailer i. $u_{\tilde{n}i} = u_{ni}$ and both of them represent the unit sales cost of the product of Retailer i).

In order to simplify the notations in this section, we let

$$G_{\tilde{n}i} = m + \frac{c-b}{2} q_{*\tilde{n}i}^2 + s\frac{\beta}{q_{*\tilde{n}i}} \quad \text{and} \quad F_{\tilde{n}i} = \frac{b}{2} q_{*\tilde{n}i}^2 + k_{\tilde{n}i}\frac{\beta}{q_{*\tilde{n}i}} + u_{\tilde{n}i}. \tag{4.15}$$

Since for Retailer i, we have $k_{\tilde{n}i} = k_{ni}$, and $u_{\tilde{n}i} = u_{ni}$, then $q_{*\tilde{n}i} = q_{*ni}$, $F_{\tilde{n}i} = F_{ni}$ and $G_{\tilde{n}i} = G_{ni}$. Thus, we define in Sections 3 and 4 that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} F_{\tilde{n}i} = E(F_{\tilde{n}i}) = \bar{F} = E(F_{ni}) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} F_{ni} \tag{4.16}$$

and

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} G_{\tilde{n}i} = E(G_{\tilde{n}i}) = \bar{G} = E(G_{ni}) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} G_{ni}. \tag{4.17}$$

When n is sufficiently large, by the Law of Large Numbers, denote the mean wholesale price as:

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} w_{*\tilde{n}i} \quad = \quad E(w_{*\tilde{n}i}) = \bar{W}_{*\tilde{n}}, \tag{4.18}$$

where $w_{*\tilde{n}i}$ is the optimal wholesale price given by the supplier to Retailer i.

4.1 The n-Retailer Problem

Retailer i has to determine the optimal effort level and retail price so as to maximize her profit. Retailer i's profit is a function of two variables, which is denoted by $\Pi_{\tilde{n}i}(p_{\tilde{n}i}, q_{\tilde{n}i})$ where

$$\Pi_{\tilde{n}i}(p_{\tilde{n}i}, q_{\tilde{n}i}) = p_{\tilde{n}i} D_{\tilde{n}i} - \left(w_{\tilde{n}i} + \frac{b}{2} q_{\tilde{n}i}^2 \right) D_{\tilde{n}i} - k_{\tilde{n}i} \frac{\beta}{q_{\tilde{n}i}} D_{\tilde{n}i} - u_{\tilde{n}i} D_{\tilde{n}i}. \tag{4.19}$$

Here the demand function $D_{\tilde{n}i}$ is

$$D_{\tilde{n}i} = a - \gamma p_{\tilde{n}i} - g \left(p_{\tilde{n}i} - \frac{\sum_{j=1}^{n} p_{\tilde{n}j}}{n} \right).$$

Therefore, Retailer i's $(i = 1, \ldots, n)$ optimization problem is

$$\max_{p_{\tilde{n}i}, q_{\tilde{n}i}} \Pi_{\tilde{n}i}(p_{\tilde{n}i}, q_{\tilde{n}i}).$$

The optimal solution is given in the following proposition and the proof can be found in Appendix.

Proposition 4. *In the case of one supplier and n retailers with different wholesale prices, Retailer i's $(i = 1, \ldots, n)$ optimal effort level is given by*

$$q_{*\tilde{n}i} = max\left(\left(\frac{k_{\tilde{n}i}\beta}{b}\right)^{1/3}, 1\right). \tag{4.20}$$

Then under the supplier's optimal wholesale prices $w_{*\tilde{n}i}$ $(i = 1, \ldots, n)$ which will be explained in Proposition 5, Retailer i's $(i = 1, \ldots, n)$ optimal retail price is given by

$$\begin{aligned} p_{*\tilde{n}i} &= \frac{a}{2\gamma + g - \frac{g}{n}} + \frac{g(\gamma + g - \frac{g}{n})}{(2(\gamma + g) - \frac{g}{n})(2\gamma + g - \frac{g}{n})} \frac{\sum_{j=1}^{n}(w_{*\tilde{n}j} + F_{\tilde{n}j})}{n} \\ &+ \frac{\gamma + g - \frac{g}{n}}{2(\gamma + g) - \frac{g}{n}} (w_{*\tilde{n}i} + F_{\tilde{n}i}) . \end{aligned} \tag{4.21}$$

When n is sufficiently large, based on Eq. (4.16) and Eq. (4.18), by the Law of Large Numbers, it has

$$p_{*\tilde{n}i} = \frac{a}{2\gamma + g} + \frac{g}{2(2\gamma + g)}(\bar{W}_{*\tilde{n}} + \bar{F}) + \frac{1}{2}(w_{*\tilde{n}i} + F_{\tilde{n}i}). \tag{4.22}$$

The optimal retail price in Eq. (4.22) is similar to that in Eq. (3.7). The difference is that Retailer i's wholesale price $w_{*\tilde{n}i}$ given by the supplier in Eq. (4.22) is dependent on the retailer's effort level and is different from those of other retailers. While in Eq. (3.7), all of the retailers' wholesale prices are the same. Furthermore, from Eq. (4.22), the mean retail price is

$$\bar{P}_{\tilde{n}} = E(p_{*\tilde{n}i}) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} p_{*\tilde{n}i} = \frac{a}{2\gamma + g} + \frac{\gamma + g}{2\gamma + g}(\bar{W}_{*\tilde{n}} + \bar{F}).$$

From Eq. (4.19), Retailer i's optimal profit is

$$\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i}) = (p_{*\tilde{n}i} - w_{*\tilde{n}i} - F_{\tilde{n}i})\left(a - \gamma p_{*\tilde{n}i} - g\left(p_{*\tilde{n}i} - \frac{\sum_{i=1}^{n} p_{*\tilde{n}i}}{n})\right)\right). \tag{4.23}$$

When n is sufficiently large, it can be rewritten (approximate) as follows:

$$\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i}) = (p_{*\tilde{n}i} - w_{*\tilde{n}i} - F_{\tilde{n}i}) \left(a - \gamma p_{*\tilde{n}i} - g \left(p_{*\tilde{n}i} - \bar{P}_{\tilde{n}}\right)\right). \quad (4.24)$$

4.2 The Supplier Problem

Given that Retailer i's optimal effort level $q_{*\tilde{n}i}$ has been decided, the supplier has to determine the optimal wholesale prices $w_{\tilde{n}i}$ $(i = 1, 2, \ldots, n)$ so as to maximize her expected profit. The supplier's profit, denoted by $\Pi_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n})$, is given by

$$\Pi^m_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n}) = \sum_{i=1}^{n} \left\{ w_{\tilde{n}i} D_{*\tilde{n}i} - (m + \frac{c-b}{2} q^2_{*\tilde{n}i}) D_{*\tilde{n}i} - s \frac{\beta}{q_{*\tilde{n}i}} D_{*\tilde{n}i} \right\} \quad (4.25)$$

where

$$D_{*\tilde{n}i} = a - \gamma p_{*\tilde{n}i} - g \left(p_{*\tilde{n}i} - \frac{1}{n} \sum_{j=1}^{n} p_{*\tilde{n}j} \right).$$

Here the supplier provides a wholesale price w_{ni} to Retailer i. Hence based on Proposition 4, Retailer i's optimal retail price in Eq. (4.21) is denoted as:

$$\begin{aligned} p_{*\tilde{n}i} &= \frac{a}{2\gamma + g - \frac{g}{n}} + \frac{g(\gamma + g - \frac{g}{n})}{(2(\gamma+g) - \frac{g}{n})(2\gamma + g - \frac{g}{n})} \frac{\sum_{j=1}^{n}(w_{\tilde{n}j} + F_{\tilde{n}j})}{n} \\ &\quad + \frac{\gamma + g - \frac{g}{n}}{2(\gamma+g) - \frac{g}{n}} (w_{\tilde{n}i} + F_{\tilde{n}i}). \quad (4.26) \end{aligned}$$

In Eq. (4.25), the unit wholesale price satisfies $w_{\tilde{n}i} \neq w_{\tilde{n}j}$ if $i \neq j$, which is different from that in Eq. (3.10). Therefore, the supplier's optimization problem is

$$\max_{w_{\tilde{n}i}} \Pi_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n}).$$

With regards to the supplier's optimal wholesale price, we have the following propositions and their proofs can be found in Appendix.

Proposition 5. *Assume that n is sufficiently large. In the case of one supplier and n*

retailers with different prices, the supplier's optimal wholesale price $w_{*\tilde{n}i}$ satisfies

$$\begin{aligned} w_{*\tilde{n}i} &= \frac{1}{2\gamma+g}a + \frac{g}{2(2\gamma+g)}(\bar{W}_{*\tilde{n}} + \bar{F}) - \frac{1}{2}(F_{\tilde{n}i} - G_{\tilde{n}i}) \\ &= \frac{2a}{4\gamma+g} + \frac{g}{2(4\gamma+g)}(\bar{F} + \bar{G}) - \frac{1}{2}(F_{\tilde{n}i} - G_{\tilde{n}i}), \end{aligned} \tag{4.27}$$

and the mean wholesale price is

$$\bar{W}_{*\tilde{n}} = \frac{2a}{4\gamma+g} - \frac{2\gamma\bar{F}}{4\gamma+g} + \frac{(2\gamma+g)\bar{G}}{4\gamma+g}. \tag{4.28}$$

Different to the case in Section 3, the optimal wholesale price in this model is affected by the competitive factor g among the retailers. In Eq. (4.27), the sum of the first and second terms is a basis wholesale price for each retailer. There the first term depends on the market and the second term depends on retailers' average total unit cost with weighting factor $g/(2(2\gamma+g))$. The last term is the difference between Retailer i's unit cost except wholesale cost and the supplier's total unit cost for Retailer i, with weighting factor 1/2, when the optimal effort level is adopted. It is found that the retailer with a larger value of $(F_{\tilde{n}i} - G_{\tilde{n}i})$ will need to pay a lower wholesale price. More precisely, the retailer with $(F_{\tilde{n}i} - G_{\tilde{n}i}) > 0$ will pay a lower wholesale price than the one with $(F_{\tilde{n}j} - G_{\tilde{n}j}) < 0$. This may imply that if Retailer i's unit cost except wholesale cost $F_{\tilde{n}i}$ is greater than the supplier's total unit cost for Retailer i $G_{\tilde{n}i}$, then to incentivise the retailers, the supplier would set a lower wholesale price so that the price of the retailer falls into her acceptable range. If $F_{\tilde{n}i}$ is less than $G_{\tilde{n}i}$, the supplier would set a higher wholesale price which may not affect the retailer's purchasing decision.

Then, from Eq. (4.25), the supplier's optimal profit is given by

$$\Pi^m_{\tilde{n}}(w_{*\tilde{n}i,i=1,2,\ldots,n}) = \sum_{i=1}^{n}(w_{*\tilde{n}i} - G_{\tilde{n}i})D_{*\tilde{n}i}. \tag{4.29}$$

The following propositions discuss the effects of the unit return service cost and sales on retailers and the supplier's profits.

Proposition 6. *Assume that $k_{\tilde{n}i} > \dfrac{b}{\beta}$ $(i = 1, 2, \ldots, n)$ and n is sufficiently large. With supplier's optimal wholesale price in Proposition 5, the following statements hold:*

1. *If* $0 < \beta < \dfrac{2b+c}{s}$ *then* $\dfrac{sb}{2b+c} < k_{\tilde{n}i}$. *Then* $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ *decreases in* $k_{\tilde{n}i}$.

2. *If* $\dfrac{2b+c}{s} < \beta < 1$, *the following statements hold:*

   (a) *If* $\dfrac{b}{\beta} < k_{\tilde{n}i} < \dfrac{sb}{2b+c}$ *then* $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ *increases in* $k_{\tilde{n}i}$.

   (b) *If* $\dfrac{sb}{2b+c} < k_{\tilde{n}i}$ *then* $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ *decreases in* $k_{\tilde{n}i}$.

   (c) *If* $k_{\tilde{n}i} = \dfrac{sb}{2b+c}$ *then* $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ *attains its maximum value with respect to* $k_{\tilde{n}i}$.

Here $\beta < \frac{2b+c}{s}$ can be written as $\beta s < 2b + c$, where βs is the expected unit return service cost for the supplier, and it is less than the sum of twice the retailer i's marginal cost of effort under the minimum effort level and the supplier's marginal cost of effort under the minimum effort level. Proposition 6 implies that if $\beta < \frac{2b+c}{s}$, retailers can enhance their profits by reducing their unit return service cost k. However, if $\frac{2b+c}{s} < \beta < 1$, which means the unit return service cost for the supplier s is large, then retailers can enhance their profits by making their unit return service cost k tends to $\frac{sb}{2b+c}$.

Proposition 7. *Assume that* $k_{\tilde{n}i} > \dfrac{b}{\beta}$ $(i = 1, 2, \ldots, n)$ *and* n *is sufficiently large. With supplier's optimal wholesale price in Proposition 5,* $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ *decreases in* $u_{\tilde{n}i}$.

Proposition 8. *Assume that* $k_{\tilde{n}i} > \dfrac{b}{\beta}$ $(i = 1, 2, \ldots, n)$ *and* n *is sufficiently large. By the Law of Large Numbers,*

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \frac{1}{q_{*\tilde{n}i}} = E[\frac{1}{q_{*\tilde{n}i}}] = \frac{1}{\bar{q}_{\tilde{n}}}.$$

Then with supplier's optimal wholesale price in Proposition 5, the following results hold.

1. (a) *If* $q_{*\tilde{n}i} < \frac{4\gamma+g}{g}\bar{q}_{\tilde{n}}$ *then* $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ *decreases in* s;

   (b) *If* $q_{*\tilde{n}i} > \frac{4\gamma+g}{g}\bar{q}_{\tilde{n}}$ *then* $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ *increases in* s;

   (c) *If* $q_{*\tilde{n}i} = \frac{4\gamma+g}{g}\bar{q}_{\tilde{n}}$ *then* $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ *does not change with respect to* s;

2. *If* $q_{*\tilde{n}i} < \frac{4\gamma+g}{g}\bar{q}_{\tilde{n}}$ *for each* $i = 1, 2, \ldots, n$, *then* $\Pi^m_{\tilde{n}i}(w_{*\tilde{n}i,i=1,2,\ldots,n})$ *decreases in* s.

Similar to Proposition 3, Proposition 8 implies that if $q_{*\tilde{n}i}$ is less than $(g + 4\gamma)/g$ times the harmonic mean of $q_{*\tilde{n}i}$ for any i $(i = 1, 2, \ldots, n)$, then by reducing the unit cost of the supplier's return service s, the net profit of supplier will increase. However, it shows that if $q_{*\tilde{n}i}$ is greater than $(g+4\gamma)/g$ times the harmonic mean of $q_{*\tilde{n}i}$, Retailer i's profit will increase in s. That is because when $q_{*\tilde{n}i} > \frac{4\gamma+g}{g}\bar{q}_{\tilde{n}}$, the rate of change of Retailer i's optimal retail price with respect to s is greater than that of the corresponding optimal wholesale price, i.e., $\frac{\partial p_{*\tilde{n}i}}{\partial s} - \frac{\partial w_{*\tilde{n}i}}{\partial s} > 0$, and the demand of Retailer i increases in s, i.e., $\frac{\partial D_{*\tilde{n}i}}{\partial s} > 0$, which are shown in the proof of Proposition 8.

5 Discussions

For the supplier, there are two possible pricing strategies. Strategy I is that supplier gives a single wholesale price to retailers. Strategy II is that supplier provides different wholesale prices to retailers based on their required effort levels. This section compares the results obtained in Sections 3 and 4, focusing on the effects of different supplier's wholesale price strategies on the retailers' optimal sale price and their optimal profit, supplier's wholesale price and her optimal profit. The following propositions can be obtained.

Proposition 9. *Assume that* $k_{ni} > \dfrac{b}{\beta}$ $(i = 1, 2, \ldots, n)$ *and* n *is sufficiently large. From Propositions 1, 2, 4 and 5,* $w_{*n} > \bar{W}_{*\tilde{n}}$ *and* $E(p_{*ni}) > E(p_{*\tilde{n}i})$.

Suppose the supplier is allowed to adopt wholesale pricing Strategy II. The proposition means that when n is sufficiently large and $k_{ni} > \dfrac{b}{\beta}$, both the mean of the optimal wholesale price and the mean of the optimal retailer price of Strategy I can be reduced by adopting Strategy II.

Proposition 10. *Assume that* $k_{ni} > \dfrac{b}{\beta}$ $(i = 1, 2, \ldots, n)$ *and* n *is sufficiently large. We note*

$$YY = \frac{1}{(2\gamma+g)(4\gamma+g)}(-2ga + (8\gamma^2 + 8\gamma g + g^2)\bar{F} - (8\gamma^2 + 4\gamma g + g^2)\bar{G})$$

and

$$ZZ = \frac{1}{(2\gamma+g)(4\gamma+g)}((16\gamma + 6g)a + (8\gamma^2 + 12\gamma g + 3g^2)\bar{F} - (8\gamma^2 - g^2)\bar{G}).$$

From Propositions 1, 2, 4 and 5, the following relationships hold.

Case 1: If

$$\begin{cases} F_{ni} - G_{ni} < YY \\ 3F_{ni} + G_{ni} < ZZ \end{cases} \quad \text{or} \quad \begin{cases} F_{ni} - G_{ni} > YY \\ 3F_{ni} + G_{ni} > ZZ \end{cases}$$

then

$$\Pi^r_{ni}(p_{*ni}, q_{*ni}) > \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i}).$$

Case 2: If

$$\begin{cases} F_{ni} - G_{ni} < YY \\ 3F_{ni} + G_{ni} > ZZ \end{cases} \quad \text{or} \quad \begin{cases} F_{ni} - G_{ni} > YY \\ 3F_{ni} + G_{ni} < ZZ \end{cases}$$

then

$$\Pi^r_{ni}(p_{*ni}, q_{*ni}) < \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i}).$$

Case 3: If $F_{ni} - G_{ni} = YY$ *or* $3F_{ni} + G_{ni} = ZZ$ *then* $\Pi^r_{ni}(p_{*ni}, q_{*ni}) = \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$.

Recall that F_{ni} is Retailer i's unit cost except wholesale cost and G_{ni} is the supplier's total unit cost for Retailer i. This proposition indicates that if the conditions in Case 1 are satisfied then the retailers would prefer the supplier to adopt the wholesale pricing Strategy I, since the retailers can gain a higher profit under the wholesale pricing Strategy I than Strategy II. On the other hand, if the conditions in Case 2 are satisfied then the retailers do not expect the supplier to adopt the pricing Strategy I. If the conditions in Case 3 are satisfied then Retailer i will get the same profit under the two pricing strategies. So Retailer i is indifference to the two pricing strategies.

Proposition 11. *Assume that* $k_{ni} > \dfrac{b}{\beta}$ $(i = 1, 2, \ldots, n)$ *and* n *is sufficiently large. The following proposition follows from Propositions 1, 2, 4 and 5. If*

$$\begin{aligned} \frac{1}{8}E\{(F_{ni} - G_{ni})^2\} \quad \leq \quad & \frac{1}{2(4\gamma + g)^2}\left(2a + \frac{g}{2}\bar{F} + \frac{g}{2}\bar{G}\right)^2 + T_{\tilde{n}}\bar{W}_{*\tilde{n}} + (T_n - T_{\tilde{n}})\bar{G} \\ & +(w_{*n} - \bar{W}_{*\tilde{n}})\left(\frac{1}{\gamma + g}a - \frac{g}{2\gamma + g}\bar{G}\right) - \frac{1}{\gamma + g}(\gamma w_{*n}\bar{P}_n + g\bar{W}_{*\tilde{n}}\bar{P}_{\tilde{n}}), \end{aligned} \tag{5.30}$$

where

$$T_n = \frac{a}{2\gamma + g} + \frac{\gamma + g}{2\gamma + g}w_{*n} + \frac{g}{2(2\gamma + g)}\bar{F}$$

and

$$T_{\tilde{n}} = \frac{a}{2\gamma + g} + \frac{g}{2(2\gamma + g)}\bar{W}_{*\tilde{n}} + \frac{g}{2(2\gamma + g)}\bar{F},$$

then

$$E(\Pi_n^m(w_{*n})) \geq E(\Pi_{\tilde{n}}^m(w_{*\tilde{n}i,i=1,2,\ldots,n}));$$

otherwise

$$E(\Pi_n^m(w_{*n})) < E(\Pi_{\tilde{n}}^m(w_{*\tilde{n}i,i=1,2,\ldots,n})).$$

Note that all of the expectation operators above are interpreted as the limits of the corresponding (sample) averages based on the Law of Large Numbers. Here w_{*n} ($\bar{W}_{*\tilde{n}}$) is the (average) wholesale price when the supplier chooses wholesale pricing strategy I (II) when n is sufficiently large. Moreover, $\bar{P}_n$ ($\bar{P}_{\tilde{n}}$) is the average retail price when the supplier chooses wholesale pricing strategy I (II), $\bar{G}$ is the average total unit cost that the supplier spends on the product for the retailers and $\bar{F}$ is the retailers' average unit cost except wholesale cost. The supplier can decide her pricing strategy according to this proposition. If Eq. (5.30) is satisfied then the supplier gains higher profit by setting a single wholesale prices to retailers. Otherwise, the supplier gains higher profit by adopting pricing Strategy II.

6 Conclusions

A supply chain consisting of one supplier and n retailers is considered while typical models usually discuss one or two retailers. In this supply chain, the retailers have different unit sales costs, and in order to reduce the returns of defective products, the retailers can request the supplier to supply higher quality products by making an extra effort on products. The supplier may decide her wholesale price from two strategies. The first one is setting the same wholesale price for n retailers. The second one is setting different wholesale prices to the retailers on the basis of their required effort levels on products. Then we model the retail price competition behavior of the n retailers under different wholesale price strategies, with retail price-dependent demands. We assume that the market demand of Retailer i not only decreases in her retail price p_i but also depends on the difference between p_i and the average retail price of all retailers. If p_i is higher (lower) than the average retail price, the difference between these two prices will decrease (increase) the market demand for Retailer i. The retailers' optimal retail prices and the supplier's optimal wholesale price in each model have been derived using game theory.

In addition, the effects of some key costs such as unit return service cost and sales cost, on supply chain decisions and profit have been investigated. We compare the retailers' (supplier's) optimal profit under the two wholesale price strategies. The main results under the assumptions that the number of retailers n is sufficiently large and Retailer i's unit return service cost satisfies $k_i > \dfrac{b}{\beta}$ $(i = 1, 2, \ldots, n)$ are summarized as follows.

(1) Retailer i's profit decreases in k_i under wholesale pricing Strategy I. Whereas Proposition 6 case 2(a) shows that retailer i's profit may increase in k_i with wholesale pricing Strategy II. This is because the optimal retail price in Eq. (4.22) under Strategy II decreases as the unit return service cost k_i increases in that region, which is shown in the proof of Proposition 6. Then the market demand increases as the optimal retail price decreases. This process may enhance the profit when an increase in income is greater than an increase in cost.

(2) Propositions 3 and 8 show that an increase in s may decrease the profits of the supplier and the retailers under each wholesale pricing strategy. Because an increase in s will lead to an increase in the wholesale price, which would increase the cost of the retailers. However, if $q_{*\tilde{n}i}$ is greater than $(g + 4\gamma)/g$ times the harmonic mean of $q_{*\tilde{n}i}$, Retailer i's profit may increase in s.

(3) Proposition 9 indicates that both the average wholesale price and the average retail price with wholesale pricing Strategy I would be higher than the corresponding prices under wholesale pricing Strategy II.

(4) In case 1 (2) of Proposition 10, Retailer i's profit is higher (lower) under wholesale pricing Strategy I than that under wholesale pricing Strategy II. This implies that retailers would like the supplier to adopt wholesale pricing strategy I (II).

(5) If Eq. (5.30) in Proposition 11 is satisfied then the supplier should choose pricing Strategy I since he could gain higher profit by adopting pricing Strategy I. Otherwise, the supplier could gain higher profit by adopting pricing Strategy II.

The number of returns caused by defective items would affect a retailer's reputation. For further research, this model may be extended to incorporate the effects of the returns on the market demand of each retailer.

7 Appendix

Proof of Proposition 1

Proof. Based on the sequence of events in the one supplier and n retailers model, the retailer's decision problem can be written as

$$\max_{p_{ni},q_{ni}} \Pi^r_{ni}(p_{ni}, q_{ni}) = \max_{p_{ni}}\{\max_{q_{ni}} \Pi^r_{ni}(p_{ni}, q_{ni})\}.$$

First, we wish to find an optimal effort level so as to maximize the retailer's profit. Differentiating Eq. (3.4) gives:

$$\frac{\partial^2 \Pi^r_{ni}(p_{ni}, q_{ni})}{\partial q^2_{ni}} = -bD_{ni} - 2k_{ni}\frac{\beta}{q^3_{ni}}D_{ni} < 0,$$

so $\Pi^r_{ni}(p_{ni}, q_{ni})$ is concave in q_{ni}. Set $\frac{\partial \Pi^r_{ni}(p_{ni}, q_{ni})}{\partial q_{ni}} = 0$, solving it gets the retailer's optimal effort level in Eq. (3.5), here $q_{ni} \geq 1$ is always assumed. Second, under the supplier's optimal wholesale price w_{*n} which will be explained in Proposition 2, we wish to find Retailer i's optimal sales price so as to maximize the retailer's profit. After taking differentiation, we get

$$\frac{\partial^2 \Pi^r_{ni}(p_{ni}, q_{*ni})}{\partial p^2_{ni}} = -2(\gamma + g - \frac{g}{n}) < 0.$$

Thus $\Pi^r_{ni}(p_{ni}, q_{*ni})$ is concave in p_{ni}. Setting $\frac{\partial \Pi^r_{ni}(p_{ni}, q_{*ni})}{\partial p_{ni}} = 0$ for each $i = 1, 2, \cdots, n$, the linear matrix equation is obtained:

$$MP = B,$$

where

$$M = \begin{bmatrix} 2(\gamma+g-\frac{g}{n}) & -\frac{g}{n} & \cdots & -\frac{g}{n} & -\frac{g}{n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{g}{n} & -\frac{g}{n} & \cdots & -\frac{g}{n} & 2(\gamma+g-\frac{g}{n}) \end{bmatrix},$$

$$P = \begin{bmatrix} p_{n1} \\ \vdots \\ p_{n,n} \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} a+(w_{*n}+F_{n1})(\gamma+g-\frac{g}{n}) \\ \vdots \\ a+(w_{*n}+F_{n,n})(\gamma+g-\frac{g}{n}) \end{bmatrix}.$$

Furthermore, M can be rewritten as: $M = (2(\gamma+g) - \frac{g}{n})I - \frac{g}{n}\mathbf{e}\mathbf{e}^T$, where I is the $n \times n$ identity matrix and $\mathbf{e}$ is the $n \times 1$ vector with all ones. According to Sherman-Morrison formula [9]

$$(A + b\mathbf{u}\mathbf{v}^T)^{-1} = A^{-1} - \frac{b}{1 + b\mathbf{v}^T A^{-1}\mathbf{u}} A^{-1}\mathbf{u}\mathbf{v}^T A^{-1}.$$

Then

$$M^{-1} = \frac{1}{2(\gamma+g) - \frac{g}{n}} \left(I + \frac{g}{n(2\gamma+g-\frac{g}{n})}\mathbf{e}\mathbf{e}^T \right).$$

Thus $P = M^{-1}B$, and the result of p_{*ni} in Eq. (3.6) is obtained. This completes the proof. □

Proof of Proposition 2

Proof. First, recalling Proposition 1, we observe that the value of q_{*ni}. Suppose that w_n is the wholesale price given by the supplier to all the retailers. Then set

$$d_n = \frac{\gamma+g-\frac{g}{n}}{2\gamma+g-\frac{g}{n}},$$

$$L_{ni} = \frac{1}{2\gamma+g-\frac{g}{n}}a + \frac{\gamma+g-\frac{g}{n}}{2(\gamma+g)-\frac{g}{n}} \left(F_{ni} + \frac{g\sum_{j=1}^{n} F_{nj}}{n(2\gamma+g-\frac{g}{n})} \right),$$

and obtain that $p_{*ni} = d_n w_n + L_{ni}$ based on Eq. (3.6) in Proposition 1 and Eq. (3.11).

We wish to find an optimal price w_n so as to maximize the supplier's profit written in

Eq. (3.10). The second-order condition shows that

$$\frac{\partial^2 \Pi_n^m(w_n)}{\partial w_n^2} = -2n\gamma d_n < 0,$$

so $\Pi_n^m(w_n)$ is concave in w_n. Let $\dfrac{\partial \Pi_n^m(w_n)}{\partial w_n} = 0$. Solving it and we get the optimal wholesale price w_{*n} in Eq. (3.12). This completes the proof. □

Proof of Proposition 3

Proof. With retailers' optimal retail prices in Proposition 1 and supplier's optimal wholesale price in Proposition 2, we have

$$w_{*n} = \frac{a}{2\gamma} - \frac{\bar{F} - \bar{G}}{2}, \tag{7.31}$$

$$p_{*ni} = \frac{a}{2\gamma + g} + \frac{g}{2(2\gamma + g)}(w_{*n} + \bar{F}) + \frac{1}{2}(w_{*n} + F_{ni}), \tag{7.32}$$

$$\bar{P}_n = E(p_{*ni}) = \lim_{n\to\infty} \frac{\sum_{i=1}^n p_{*ni}}{n} = \frac{a}{2\gamma + g} + \frac{\gamma + g}{2\gamma + g}(w_{*n} + \bar{F}), \tag{7.33}$$

$$D_{*ni} = a - \gamma p_{*ni} - g(p_{*ni} - \bar{P}_n). \tag{7.34}$$

Then based on the above equations, it shows that

$$\frac{\partial G_{ni}}{\partial s} = \beta \frac{1}{q_{*ni}} > 0, \quad \frac{\partial \bar{G}}{\partial s} = \beta E(\frac{1}{q_{*ni}}) = \beta \frac{1}{\bar{q}_n} > 0, \quad \frac{\partial w_{*n}}{\partial s} = \frac{1}{2}\frac{\partial \bar{G}}{\partial s} > 0,$$

$$\frac{\partial p_{*ni}}{\partial s} = \frac{\gamma + g}{2\gamma + g}\frac{\partial w_{*n}}{\partial s} > 0, \quad \frac{\partial \bar{P}_n}{\partial s} = \frac{\gamma + g}{2\gamma + g}\frac{\partial w_{*n}}{\partial s} > 0$$

and

$$\frac{\partial D_{*ni}}{\partial s} = -(\gamma + g)\frac{\partial p_{*ni}}{\partial s} + g\frac{\partial \bar{P}_n}{\partial s} = -\frac{\gamma(\gamma + g)}{2\gamma + g}\frac{\partial w_{*n}}{\partial s} < 0.$$

Then

$$\frac{\partial \Pi_{ni}^r(p_{*ni}, q_{*ni})}{\partial s} = -\frac{\gamma}{2\gamma + g}\frac{\partial w_{*n}}{\partial s}D_{*ni} + (p_{*ni} - w_{*n} - F_{ni})\frac{\partial D_{*ni}}{\partial s} < 0.$$

Therefore, $\Pi_{ni}^r(p_{*ni}, q_{*ni})$ decreases in s.

If $q_{*ni} < 2\bar{q}_n$ (i.e., $\frac{1}{q_{*ni}} > \frac{1}{2\bar{q}_n}$,) $i = 1, 2, \ldots, n$, then

$$\frac{\partial \Pi_n^m(w_{*n})}{\partial s} = \sum_{i=1}^{n} \left\{ (\beta \frac{1}{2\bar{q}_n} - \beta \frac{1}{q_{*ni}}) D_{*ni} + (w_{*n} - G_{ni}) \frac{\partial D_{*ni}}{\partial s} \right\} < 0.$$

Therefore, $\Pi_n^m(w_{*n})$ decreases in s. This completes the proof. □

Proof of Proposition 4

Proof. First, similar to the optimal effort level in Proposition 1, the retailer i's optimal effort level is $q_{*\tilde{n}i} = \max\left(\left(\frac{k_{\tilde{n}i}\beta}{b}\right)^{1/3}, 1 \right)$. Second, under the supplier's optimal wholesale prices $w_{*\tilde{n}i}$ $(i = 1, \ldots, n)$ which will be explained in Proposition 5, we wish to find an optimal sales price so as to maximize the retailer i's profit. Again similar to the proof of Proposition 1, the problem can be transformed to a matrix form. The key issue is that the wholesale price provided by the supplier to each retailer is different. Then $P = M^{-1}B$ and Retailer i's optimal retail price $p_{*\tilde{n}i}$ is given in Eq. (4.21). This completes the proof. □

Proof of Proposition 5

Proof. First, recall that in Proposition 4, the values of $q_{*\tilde{n}i}$, $i = 1, 2, \ldots, n$, are observed. Second, we wish to find optimal prices $w_{\tilde{n}i}$ $(i = 1, 2, \ldots, n)$ so as to maximize the supplier's profit which is presented in Eq. (4.25). Here $w_{\tilde{n}i}$ is the wholesale price given by the supplier to Retailer i. Then based on Eq. (4.21) in Proposition 4, Retailer i's $(i = 1, \ldots, n)$ corresponding optimal retail price $p_{*\tilde{n}i}$ is given in Eq. (4.26). By the Law of Large Numbers, denote

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} w_{\tilde{n}i}}{n} = E(w_{\tilde{n}i}) = \bar{W}_{\tilde{n}}.$$

Then when n is sufficiently large

$$\bar{P}_{\tilde{n}} = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} p_{*\tilde{n}i}}{n} = E(p_{*\tilde{n}i}) = \frac{a}{2\gamma + g} + \frac{\gamma + g}{2\gamma + g}(\bar{W}_{\tilde{n}} + \bar{F}).$$

Then the first-order and second-order conditions are:

$$\frac{\partial \Pi_{\tilde{n}}^m(w_{\tilde{n}i, i=1,2,\ldots,n})}{\partial w_{\tilde{n}i}} = \frac{\gamma + g}{2\gamma + g}(a + \frac{g}{2}\bar{W}_{\tilde{n}} + \frac{g}{2}\bar{F}) + \frac{\gamma + g}{2} G_{\tilde{n}i} - (\gamma + g) w_{\tilde{n}i} - \frac{\gamma + g}{2} F_{\tilde{n}i}$$

and

$$\begin{aligned}
\frac{\partial^2 \Pi^m_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n})}{\partial w^2_{\tilde{n}i}} &= -(\gamma+g) < 0, \\
\frac{\partial^2 \Pi^m_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n})}{\partial w_{\tilde{n}i}\partial w_{\tilde{n}j}} &= 0, \; i \neq j.
\end{aligned}$$

Then

$$\begin{vmatrix}
\frac{\partial^2 \Pi^m_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n})}{\partial w^2_{\tilde{n}1}} & \cdots & \frac{\partial^2 \Pi^m_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n})}{\partial w_{\tilde{n}1}\partial w_{\tilde{n}n}} \\
\vdots & \ddots & \vdots \\
\frac{\partial^2 \Pi^m_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n})}{\partial w_{\tilde{n}n}\partial w_{\tilde{n}1}} & \cdots & \frac{\partial^2 \Pi^m_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n})}{\partial w^2_{\tilde{n}n}}
\end{vmatrix} = (-1)^n(\gamma+g)^n.$$

Since $(-1)^m(\gamma+g)^m < 0$ for m is odd and $(-1)^m(\gamma+g)^m > 0$ for m is even, thus $\Pi^m_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n})$ is concave in $w_{\tilde{n}i}, i = 1, 2, \ldots, n$. Set $\frac{\partial \Pi^m_{\tilde{n}}(w_{\tilde{n}i,i=1,2,\ldots,n})}{\partial w_{\tilde{n}i}} = 0$ and get the optimal wholesale price $w_{*\tilde{n}i}$ in Eq. (4.27). Given that n is sufficiently large, by the Law of Large Numbers,

$$\begin{aligned}
\bar{W}_{*\tilde{n}} &= E(w_{*\tilde{n}i}) = \lim_{n\to\infty} \frac{\sum_{i=1}^n w_{*\tilde{n}i}}{n} \\
&= \frac{1}{2\gamma+g}a + \frac{g}{2(2\gamma+g)}\bar{W}_{*\tilde{n}} - \frac{\gamma}{2\gamma+g}\bar{F} + \frac{1}{2}\bar{G}.
\end{aligned}$$

The results of $\bar{W}_{*\tilde{n}}$ and $w_{*\tilde{n}i}$ can be obtained. This completes the proof. □

Proof of Proposition 6

Proof. Assume that $k_{\tilde{n}i} > \frac{b}{\beta}$, hence $q_{*\tilde{n}i} = (\frac{k_{\tilde{n}i}\beta}{b})^{1/3}$, then

$$\frac{\partial F_{\tilde{n}i}}{\partial k_{\tilde{n}i}} = b^{1/3}\beta^{2/3}k_{\tilde{n}i}^{-1/3} > 0$$

and

$$\frac{\partial G_{\tilde{n}i}}{\partial k_{\tilde{n}i}} = -\frac{1}{3}(1 - cb^{-1} + sk_{\tilde{n}i}^{-1})b^{1/3}\beta^{2/3}k_{\tilde{n}i}^{-1/3} = -\frac{1}{3}(1 - cb^{-1} + sk_{\tilde{n}i}^{-1})\frac{\partial F_{\tilde{n}i}}{\partial k_{\tilde{n}i}}.$$

From Propositions 4 and 5, when n is sufficiently large,

$$\begin{aligned}
w_{*\tilde{n}i} &= \frac{2a}{4\gamma+g} + \frac{g}{2(4\gamma+g)}(\bar{F}+\bar{G}) - \frac{1}{2}(F_{\tilde{n}i} - G_{\tilde{n}i}), && (7.35)\\
\bar{W}_{*\tilde{n}} &= E(w_{*\tilde{n}i}) = \frac{2}{4\gamma+g}a - \frac{2\gamma}{4\gamma+g}\bar{F} + \frac{2\gamma+g}{4\gamma+g}\bar{G}, && (7.36)\\
p_{*\tilde{n}i} &= \frac{a}{2\gamma+g} + \frac{g}{2(2\gamma+g)}(\bar{W}_{*\tilde{n}} + \bar{F}) + \frac{1}{2}(w_{*\tilde{n}i} + F_{\tilde{n}i}), && (7.37)\\
\bar{P}_{\tilde{n}} &= E(p_{*\tilde{n}i}) = \frac{a}{2\gamma+g} + \frac{\gamma+g}{2\gamma+g}(\bar{W}_{*\tilde{n}} + \bar{F}), && (7.38)\\
D_{*\tilde{n}i} &= a - (\gamma+g)p_{*\tilde{n}i} + g\bar{P}_{\tilde{n}}. && (7.39)
\end{aligned}$$

Then

$$\begin{aligned}
\frac{\partial w_{*\tilde{n}i}}{\partial k_{\tilde{n}i}} &= \frac{1}{2}\frac{\partial G_{\tilde{n}i}}{\partial k_{\tilde{n}i}} - \frac{1}{2}\frac{\partial F_{\tilde{n}i}}{\partial k_{\tilde{n}i}},\\
\frac{\partial p_{*\tilde{n}i}}{\partial k_{\tilde{n}i}} &= \frac{1}{2}\frac{\partial w_{*\tilde{n}i}}{\partial k_{\tilde{n}i}} + \frac{1}{2}\frac{\partial F_{\tilde{n}i}}{\partial k_{\tilde{n}i}} = \frac{1}{4}\frac{\partial G_{\tilde{n}i}}{\partial k_{\tilde{n}i}} + \frac{1}{4}\frac{\partial F_{\tilde{n}i}}{\partial k_{\tilde{n}i}},\\
\frac{\partial D_{*\tilde{n}i}}{\partial k_{\tilde{n}i}} &= -(\gamma+g)\frac{\partial p_{*\tilde{n}i}}{\partial k_{\tilde{n}i}}.
\end{aligned}$$

It shows that

$$\frac{\partial \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})}{\partial k_{\tilde{n}i}} = -\frac{\partial p_{*\tilde{n}i}}{\partial k_{\tilde{n}i}}\left(\frac{1}{2}D_{*\tilde{n}i} + (\gamma+g)(p_{*\tilde{n}i} - w_{*\tilde{n}i} - F_{\tilde{n}i})\right).$$

1. If $0 < \beta < \dfrac{2b+c}{s}$, then $\dfrac{sb}{2b+c} < k_{\tilde{n}i}$ and $(1 - cb^{-1} + sk_{\tilde{n}i}^{-1}) < 3$. Then

$$-\frac{\partial G_{\tilde{n}i}}{\partial k_{\tilde{n}i}} < \frac{\partial F_{\tilde{n}i}}{\partial k_{\tilde{n}i}}, \quad \frac{\partial p_{*\tilde{n}i}}{\partial k_{\tilde{n}i}} > 0, \quad \text{and} \quad \frac{\partial \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})}{\partial k_{\tilde{n}i}} < 0.$$

   Therefore, $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ decreases in $k_{\tilde{n}i}$.

2. If $\dfrac{2b+c}{s} < \beta < 1$, then $\dfrac{b}{\beta} < \dfrac{sb}{2b+c}$.

   (a) If $\dfrac{b}{\beta} < k_{\tilde{n}i} < \dfrac{sb}{2b+c}$, then $(1 - cb^{-1} + sk_{\tilde{n}i}^{-1}) > 3$. Then

$$\frac{\partial F_{\tilde{n}i}}{\partial k_{\tilde{n}i}} < -\frac{\partial G_{\tilde{n}i}}{\partial k_{\tilde{n}i}}, \quad \frac{\partial p_{*\tilde{n}i}}{\partial k_{\tilde{n}i}} < 0, \text{and} \quad \frac{\partial \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})}{\partial k_{\tilde{n}i}} > 0.$$

   Therefore, $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ increases in $k_{\tilde{n}i}$.

(b) If $\dfrac{sb}{2b+c} < k_{\tilde{n}i}$, then $(1 - cb^{-1} + sk_{\tilde{n}i}^{-1}) < 3$. Then

$$-\frac{\partial G_{\tilde{n}i}}{\partial k_{\tilde{n}i}} < \frac{\partial F_{\tilde{n}i}}{\partial k_{\tilde{n}i}}, \quad \frac{\partial p_{*\tilde{n}i}}{\partial k_{\tilde{n}i}} > 0, \quad \text{and} \quad \frac{\partial \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})}{\partial k_{\tilde{n}i}} < 0.$$

Therefore, $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ decreases in $k_{\tilde{n}i}$.

(c) If $k_{\tilde{n}i} = \dfrac{sb}{2b+c}$, then $(1 - cb^{-1} + sk_{\tilde{n}i}^{-1}) = 3$.

$$-\frac{\partial G_{\tilde{n}i}}{\partial k_{\tilde{n}i}} = \frac{\partial F_{\tilde{n}i}}{\partial k_{\tilde{n}i}}, \quad \frac{\partial p_{*\tilde{n}i}}{\partial k_{\tilde{n}i}} = 0, \quad \text{and} \quad \frac{\partial \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})}{\partial k_{\tilde{n}i}} = 0.$$

From cases (a) and (b), it shows that $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ attains its maximum value with respect to $k_{\tilde{n}i}$ at the point $\dfrac{sb}{2b+c}$.

This completes the proof. □

Proof of Proposition 7

Proof. Based on the equations from Eq. (7.35) to Eq. (7.39),

$$\begin{aligned}
\frac{\partial w_{*\tilde{n}i}}{\partial u_{\tilde{n}i}} &= -\frac{1}{2}\frac{\partial F_{\tilde{n}i}}{\partial u_{\tilde{n}i}} = -\frac{1}{2}, \\
\frac{\partial p_{*\tilde{n}i}}{\partial u_{\tilde{n}i}} &= \frac{1}{2}\frac{\partial w_{*\tilde{n}i}}{\partial u_{\tilde{n}i}} + \frac{1}{2}\frac{\partial F_{\tilde{n}i}}{\partial u_{\tilde{n}i}} = \frac{1}{4}, \\
\frac{\partial D_{*\tilde{n}i}}{\partial u_{\tilde{n}i}} &= -(\gamma + g)\frac{\partial p_{*\tilde{n}i}}{\partial u_{\tilde{n}i}} = -\frac{(\gamma + g)}{4}.
\end{aligned}$$

Hence

$$\frac{\partial \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})}{\partial u_{\tilde{n}i}} = -\frac{1}{4}D_{*\tilde{n}i} - (p_{*\tilde{n}i} - w_{*\tilde{n}i} - F_{\tilde{n}i})\frac{\gamma+g}{4} < 0.$$

Therefore, $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ decreases in $u_{\tilde{n}i}$. This completes the proof. □

Proof of Proposition 8

Proof. Given that $k_{\tilde{n}i} > \frac{b}{\beta}$, then $q_{*\tilde{n}i} = \left(\frac{k_{\tilde{n}i}\beta}{b}\right)^{1/3}$. Based on the equations from Eq. (7.35) to Eq. (7.39),

$$\frac{\partial w_{*\tilde{n}i}}{\partial s} = \frac{g}{2(4\gamma+g)}\frac{\partial \bar{G}}{\partial s} + \frac{1}{2}\frac{\partial G_{\tilde{n}i}}{\partial s} \quad , \frac{\partial \bar{W}_{*\tilde{n}}}{\partial s} = \frac{2\gamma+g}{4\gamma+g}\frac{\partial \bar{G}}{\partial s},$$

$$\frac{\partial p_{*\tilde{n}i}}{\partial s} = \frac{g}{2(2\gamma+g)}\frac{\partial \bar{W}_{*\tilde{n}}}{\partial s} + \frac{1}{2}\frac{\partial w_{*\tilde{n}i}}{\partial s}, \quad \frac{\partial \bar{P}_{\tilde{n}}}{\partial s} = \frac{\gamma+g}{2\gamma+g}\frac{\partial \bar{W}_{*\tilde{n}}}{\partial s}$$

and

$$\frac{\partial D_{*\tilde{n}i}}{\partial s} = -(\gamma+g)\frac{\partial p_{*\tilde{n}i}}{\partial s} + g\frac{\partial \bar{P}_{\tilde{n}}}{\partial s} = \frac{\gamma+g}{2}\left(\frac{g}{2(4\gamma+g)}\frac{\partial \bar{G}}{\partial s} - \frac{1}{2}\frac{\partial G_{*\tilde{n}i}}{\partial s}\right).$$

Hence

$$\frac{\partial \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})}{\partial s} = (\frac{g}{2(4\gamma+g)}\frac{\partial \bar{G}}{\partial s} - \frac{1}{2}\frac{\partial G_{\tilde{n}i}}{\partial s})\left(\frac{1}{2}D_{*\tilde{n}i} + \frac{\gamma+g}{2}(p_{*\tilde{n}i} - w_{*\tilde{n}i} - F_{\tilde{n}i})\right).$$

It has

$$\frac{\partial G_{\tilde{n}i}}{\partial s} = \beta\frac{1}{q_{*\tilde{n}i}} \quad \text{and} \quad \frac{\partial \bar{G}}{\partial s} = \beta E(\frac{1}{q_{*\tilde{n}i}}).$$

If

$$\frac{1}{q_{*\tilde{n}i}} > \frac{g}{4\gamma+g}E(\frac{1}{q_{*\tilde{n}i}}) \text{ or } q_{*\tilde{n}i} < \frac{4\gamma+g}{g}\bar{q}_{\tilde{n}}$$

then

$$\frac{g}{2(4\gamma+g)}\frac{\partial \bar{G}}{\partial s} - \frac{1}{2}\frac{\partial G_{\tilde{n}i}}{\partial s} < 0.$$

Thus the properties of $\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i})$ with respect to s can be obtained.

$$\frac{\partial \Pi^m_{\tilde{n}}(w_{*\tilde{n}i,i=1,2,\ldots,n})}{\partial s} = \sum_{i=1}^{n}\left(\frac{g}{2(4\gamma+g)}\frac{\partial \bar{G}}{\partial s} - \frac{1}{2}\frac{\partial G_{\tilde{n}i}}{\partial s}\right)\left(D_{*\tilde{n}i} + \frac{\gamma+g}{2}(w_{*\tilde{n}i} - G_{\tilde{n}i})\right),$$

then if

$$q_{*\tilde{n}i} < \frac{4\gamma+g}{g}\bar{q}_{\tilde{n}}, \quad i = 1, 2, \ldots, n,$$

then

$$\frac{\partial \Pi^m_{\tilde{n}}(w_{*\tilde{n}i})}{\partial s} < 0.$$

This means that $\Pi^m_{\tilde{n}i}(w_{*\tilde{n}i,i=1,2,\ldots,n})$ is decreasing in s. This completes the proof. □

Proof of Proposition 9

Proof. Based on the equations from Eq. (7.31) to Eq. (7.34) and from Eq. (7.35) to Eq. (7.38),

$$
\begin{aligned}
w_{*n} - \bar{W}_{*\tilde{n}} &= \frac{g}{2\gamma(4\gamma+g)}(a - \gamma\bar{F} - \gamma\bar{G}), \\
E(p_{*ni}) - E(p_{*\tilde{n}i}) &= \frac{\gamma+g}{2\gamma+g}(w_{*n} - \bar{W}_{*\tilde{n}}).
\end{aligned}
$$

For the supplier with Strategy I, both of the optimal wholesale and the net profit per unit of products that the supplier sells to Retailer i should be positive, i.e., $w_{*n} > 0$ and $w_{*n} - G_{ni} > 0$ $(i = 1, 2, \ldots, n)$. Thus $w_{*n} - \bar{G} > 0$, which requires $\frac{a}{\gamma} > \bar{F} + \bar{G}$. Similarly for the supplier with Strategy II, $w_{*\tilde{n}i} > 0$ and $w_{*\tilde{n}i} - G_{\tilde{n}i} > 0$ $(i = 1, 2, \ldots, n)$. Thus $\bar{W}_{*\tilde{n}} - \bar{G} > 0$, which requires $\frac{a}{\gamma} > \bar{F} + \bar{G}$. Thus $a - \gamma\bar{F} - \gamma\bar{G} > 0$, $w_{*n} > \bar{W}_{*\tilde{n}}$ and $E(p_{*ni}) > E(p_{*\tilde{n}i})$. This completes the proof. □

Proof of Proposition 10

Proof. The means of the optimal retail prices under the two wholesale price strategies are

$$
\begin{aligned}
\bar{P}_n &= E(p_{*ni}) = \lim_{n\to\infty} \frac{\sum_{i=1}^n p_{*ni}}{n}, \\
\bar{P}_{\tilde{n}} &= E(p_{*\tilde{n}i}) = \lim_{n\to\infty} \frac{\sum_{i=1}^n p_{*\tilde{n}i}}{n}.
\end{aligned}
$$

Then from Propositions 1, 2, 4 and 5, Retailer i's profit functions under the two wholesale pricing strategies are

$$
\begin{aligned}
\Pi^r_{ni}(p_{*ni}, q_{*ni}) &= (p_{*ni} - w_{*n} - F_{ni})(a - (\gamma+g)p_{*ni} + g\bar{P}_n), \\
\Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i}) &= (p_{*\tilde{n}i} - w_{*\tilde{n}i} - F_{\tilde{n}i})(a - (\gamma+g)p_{*\tilde{n}i} + g\bar{P}_{\tilde{n}}).
\end{aligned}
$$

Denote

$$
A = \frac{-\gamma}{2\gamma+g}w_{*n} + \frac{-g}{2(2\gamma+g)}\bar{W}_{*\tilde{n}}
$$

then based on the equations from Eq. (7.31) to Eq. (7.34) and from Eq. (7.35) to Eq. (7.39),

$$
\begin{aligned}
(p_{*ni} - w_{*n} - F_{ni}) - (p_{*\tilde{n}i} - w_{*\tilde{n}i} - F_{\tilde{n}i}) &= A + \frac{w_{*\tilde{n}i}}{2}, \\
(a - (\gamma + g)p_{*ni} + g\bar{P}_n) - (a - (\gamma + g)p_{*\tilde{n}i} + g\bar{P}_{\tilde{n}}) &= (\gamma + g)(A + \frac{w_{*\tilde{n}i}}{2}).
\end{aligned}
$$

Then

$$
\Pi^r_{ni}(p_{*ni}, q_{*ni}) - \Pi^r_{\tilde{n}i}(p_{*\tilde{n}i}, q_{*\tilde{n}i}) = \frac{(\gamma + g)}{16}(YY - F_{\tilde{n}i} + G_{\tilde{n}i})(ZZ - 3F_{\tilde{n}i} - G_{\tilde{n}i}),
$$

where

$$
\begin{aligned}
YY &= \frac{1}{(2\gamma + g)(4\gamma + g)}(-2ga + (8\gamma^2 + 8\gamma g + g^2)\bar{F} - (8\gamma^2 + 4\gamma g + g^2)\bar{G}), \\
ZZ &= \frac{1}{(2\gamma + g)(4\gamma + g)}((16\gamma + 6g)a + (8\gamma^2 + 12\gamma g + 3g^2)\bar{F} - (8\gamma^2 - g^2)\bar{G}).
\end{aligned}
$$

Then the results can be obtained. This completes the proof. □

Proof of Proposition 11

Proof. Based on the equations from Eq. (7.31) to Eq. (7.34) and from Eq. (7.35) to Eq. (7.39),

$$
\begin{aligned}
p_{*ni} &= T_n + \frac{1}{2}F_{ni}, \\
p_{*\tilde{n}i} &= T_{\tilde{n}} + \frac{1}{2}(w_{*\tilde{n}i} + F_{\tilde{n}i}), \\
w_{*\tilde{n}i} &= I - \frac{1}{2}(F_{*\tilde{n}i} - G_{\tilde{n}i}).
\end{aligned}
$$

where

$$
\begin{aligned}
T_n &= \frac{a}{2\gamma + g} + \frac{\gamma + g}{2\gamma + g}w_{*n} + \frac{g}{2(2\gamma + g)}\bar{F}, \\
T_{\tilde{n}} &= \frac{a}{2\gamma + g} + \frac{g}{2(2\gamma + g)}(\bar{W}_{*\tilde{n}} + \bar{F}), \\
I &= \frac{2a}{4\gamma + g} + \frac{g}{2(4\gamma + g)}(\bar{F} + \bar{G}).
\end{aligned}
$$

Again based on these equations,

$$\begin{aligned}
E(G_{ni}p_{*ni}) &= T_n\bar{G} + \frac{1}{2}E(G_{ni}F_{ni}),\\
E(w_{*\tilde{n}i}p_{*\tilde{n}i}) &= T_{\tilde{n}}\bar{W}_{*\tilde{n}} + \frac{1}{2}(I^2 + \frac{1}{4}E(G_{ni}^2) - \frac{1}{4}E(F_{ni}^2) + I\bar{G}),\\
E(G_{*\tilde{n}i}p_{*\tilde{n}i}) &= T_{\tilde{n}}\bar{G} + \frac{1}{2}(I\bar{G} + \frac{1}{2}E(G_{ni}^2) + \frac{1}{2}E(G_{ni}F_{ni})),\\
E((w_{*n} - G_{ni})D_{*ni}) &= w_{*n}a - \gamma w_{*n}\bar{P}_n - a\bar{G} - g\bar{P}_n\bar{G} + (\gamma+g)E(G_{ni}p_{*ni}),\\
E((w_{*\tilde{n}i} - G_{\tilde{n}i})D_{*\tilde{n}i}) &= \bar{W}_{*\tilde{n}}a + g\bar{P}_{\tilde{n}}\bar{W}_{*\tilde{n}} - a\bar{G} - g\bar{P}_{\tilde{n}}\bar{G} - (\gamma+g)E(w_{*\tilde{n}i}p_{*\tilde{n}i}) + (\gamma+g)E(G_{\tilde{n}i}p_{*\tilde{n}i}).
\end{aligned}$$

Consequently,

$$\begin{aligned}
&E((w_{*n} - G_{ni})D_{*ni}) - E((w_{*\tilde{n}i} - G_{\tilde{n}i})D_{*\tilde{n}i})\\
&= \Big((\gamma+g)(\frac{1}{2}I^2 + T_{\tilde{n}}\bar{W}_{*\tilde{n}} - (T_{\tilde{n}} - T_n)\bar{G}) + (w_{*n} - \bar{W}_{*\tilde{n}})(a - \frac{g(\gamma+g)}{2\gamma+g}\bar{G}) - (\gamma w_{*n}\bar{P}_n + g\bar{W}_{*\tilde{n}}\bar{P}_{\tilde{n}})\Big)\\
&-\frac{1}{8}(\gamma+g)(E\{(G_{ni} - F_{ni})^2\}.
\end{aligned}$$

When n is sufficiently large,

$$\begin{aligned}
E(\Pi_n^m(w_{*n})) &= nE((w_{*n} - G_{ni})D_{*ni}),\\
E(\Pi_{\tilde{n}}^m(w_{*\tilde{n}i,i=1,2,\cdots,n})) &= nE((w_{*\tilde{n}i} - G_{\tilde{n}i})D_{*\tilde{n}i}).
\end{aligned}$$

Then the results can be obtained. This completes the proof. □

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