Distributed LCMV beamformer design by randomly permuted ADMMDistributed LCMV beamformer design by randomly permuted ADMM

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Abstract

In recent years, distributed beamforming has attracted a lot of attention. Since each node has its own processing power, one significant advantage is the capability of distributed computing. In general, almost all distributed beamforming approaches are solving certain multi-block optimization problems. However, additional conditions are usually required to ensure convergence. In this paper, a new distributed beamforming algorithm is proposed. We first introduce the augmented Lagrangian method to implement the centralized LCMV beamformer design. Then, we propose an effective blockwise optimization method for the design of distributed LCMV beamformer based on the randomly permuted alternating direction method of multiplier (RP-ADMM). The expected convergence is obtained for distributed LCMV beamformer design without additional conditions. Numerical experiments are conducted to illustrate the performance of the proposed method.

Keywords: Distributed LCMV beamformer, speech enhancement, blockwise optimization, ADMM, random permutation

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1. Introduction

In traditional beamforming techniques, a dedicated device (the "fusion center") is assumed to gather all the sensor observations for further processing. This approach is often referred to as the centralized beamforming and it requires large communication bandwidths and significant computing power at the fusion center. However, for many applications, the availability of a fusion center could be a major problem. Therefore, the distributed computing where each node has its own processing unit to update data independently and exchange compressed signal with other nodes, has been widely studied in the last few years [1, 2, 3, 4, 5, 6, 7, 8, 9].

In more recent works [10, 11, 8, 12, 13], a family of distributed beamformer design methods have been investigated based on convex optimization with different structures. For example, in [10], a distributed minimum variance (DMV) algorithm was proposed for robust linearly constrained minimum variance (LCMV) beamforming by casting it as a distributed convex optimization problem. In [11], a sparse distributed beamformer that trades off SNR performance against reduced inter-node power consumption was designed by using the bi-alternating direction method of multipliers (Bi-ADMM). In [8], a distributed MVDR beamforming technique was developed based on the primal-dual method of multipliers (PDMM) [12]. A stochastic ADMM approach for coordinated multicell beamforming (CMBF) was developed for a smart grid powered coordinated multicell downlink system in [13]. It is noted that most of the distributed approaches for speech enhancement are based on blockwise optimization techniques. However, the theoretical convergence results on ADMM-like algorithms (Bi-ADMM, stochastic ADMM) for such multi-block optimization problems have remained unclear for a very long time, except for the two-block optimization model.

The important proof of convergence for extension of ADMM beyond two-block convex minimization problems were unclear until 2016. In [14] it was shown that ADMM may actually fail to converge for blocks greater than or equal to three via a simple three block counterexample. To resolve the issue, a popular method called the randomized block-coordinate descent (RBCD) method was first developed for a class of unconstrained composite minimization problems [15, 16]. Subsequently, a variety of randomized strategies have been investigated to tackle multi-block optimization problems. More recent attempts can be found in [17, 18, 19, 20]. At the same time, many blockwise techniques and splitting methods for solving the constrained composite optimization problem have also been developed for separable optimization problems with applications in signal and imaging processing, machine learning, statistics, and engineering [21, 14, 22, 23, 24, 25, 26, 27, 28]. Indeed, numerous experiments have demonstrated that they are powerful for solving large-scale optimization problems arising in machine learning [15, 16, 29]. Among different randomized techniques, the randomly permuted alternating direction method (RP-ADMM) of multipliers developed in [30, 31] is noticeably important, which is convergent in expectation for cases witj blocks greater than or equal to three for nonseparable quadratic programming problems, under the condition that the block diagonal matrix related to the coefficient matrices of objective and constraint is positive definite.

In this paper, we propose a new distributed LCMV beamformer design method based on augmented Lagrangian method together with the blockwise technique. The LCMV beamformer design is formulated into a blockwise optimization problem with each block related to a specific node. Then a simple but efficient alternative iteration algorithm is proposed to solve the problem by employing the RP-ADMM. This method transforms the original optimization problem into a class of sequential subproblems, and then distributes the computational workload to each node of the sensor array. It obtains a great reduction in the computational complexity and makes parallel computing achievable. Another contribution of this paper is to show that the proposed method is applicable to cases with blocks bigger than 3. In establishing the expected convergence of the proposed algorithm, we prove that the necessary condition for convergence is satisfied without further assumption. Finally, the complexity of the proposed distributed LCMV is discussed and compared with the centralized LCMV via both theoretical and numerical analysis.

The outline of this paper is as follows. In Section II, we describe the LCMV beamformer design problem and introduce an iteration algorithm based on augmented Lagrangian method. In Section III, we present the blockwise framework for the D-LCMV beamformer design, as well as the algorithm and the convergence analysis. In Section IV, we describe the application of the proposed algorithm for wireless acoustic sensor network and provide experimental results for evaluation. Conclusions are drawn in Section V.

2. Problem statement

Consider a wireless acoustic sensor network of N nodes equipped with a microphone array consisting of M microphones in forming a wide-band acoustic beamformer. Our goal is to enhance the desired speech signal from a point source that is contaminated by noise and other interfering sources. The desired and the interference-plus-noise components are assumed to be uncorrelated. In the sequel, we will denote by $\boldsymbol{x}_i$, $i \in \{1, \ldots, M\}$, the received signal at the i-th microphone, and by $\boldsymbol{s}_j$, $j \in \{1, \ldots, J\}$, the speech signal from the j-th source location. The discrete signal model in time domain is

$$\boldsymbol{x}_i(k) = \sum_{j=1}^{J} (\boldsymbol{h}_{ij} * \boldsymbol{s}_j)(k) + \boldsymbol{v}_i(k), \tag{1}$$

where $*$ is the linear convolution operator, $\boldsymbol{h}_{ij}$ is the acoustic transfer function between the position of i-th microphone and location of the j-th acoustic source, $\boldsymbol{v}_i$ is the noise (NOI) component captured by the i-th microphone, $i = 1, \ldots, M$.

Assume each microphone is equipped with an L-tap FIR filter $\boldsymbol{w}_i$, $i = 1, \ldots, M$. The beamformer output can be written as

$$\boldsymbol{y}(k) = \sum_{i=1}^{M} (\boldsymbol{w}_i * \boldsymbol{x}_i)(k) = \sum_{i=1}^{M} \sum_{j=1}^{J} (\boldsymbol{w}_i * \boldsymbol{h}_{ij} * \boldsymbol{s}_j)(k) + \sum_{i=1}^{M} (\boldsymbol{w}_i * \boldsymbol{v}_i)(k).$$

The goal of design is to find the filter coefficients $\boldsymbol{w}$ such that a constant response is obtained after beamforming for signal of interest (SOI), and zero responses are generated for all the other sources (interferences, INT). Without loss of generality, assume the SOI comes from the first source position, the beamforming conditions on the filter coefficients can be written as

$$\sum_{i=1}^{M} \boldsymbol{h}_{i1} * \boldsymbol{w}_i = \boldsymbol{g}_1, \tag{2}$$

$$\sum_{i=1}^{M} \sum_{j=2}^{J} \boldsymbol{h}_{ij} * \boldsymbol{w}_i = \mathbf{0}. \tag{3}$$

In LCMV beamforming, the variance of the noise component can be defined as

$$J = \boldsymbol{E}\{\boldsymbol{v}^2\} = \boldsymbol{w}^T \boldsymbol{R}_{\boldsymbol{vv}} \boldsymbol{w}, \tag{4}$$

where $\boldsymbol{v}$ denotes the noise component, $\boldsymbol{E}\{\cdot\}$ is the expectation operator, $\boldsymbol{w}$ is the filter coefficients, and $\boldsymbol{R}_{\boldsymbol{vv}} = \boldsymbol{E}\{\boldsymbol{v}\boldsymbol{v}^T\}$ is the correlation matrix. The beamformer response is steered towards the SOI and is given by

$$\boldsymbol{H}_S \boldsymbol{w}(k) = \boldsymbol{g}_1, \tag{5}$$

where $\boldsymbol{H}_S$ is the convolution matrix generated by the impulse responses from the speaker to each array element corresponding to the condition (2). It is given by

$$\boldsymbol{H}_S = [\boldsymbol{H}_{11} \quad \boldsymbol{H}_{21} \quad \cdots \quad \boldsymbol{H}_{M1}], \tag{6}$$

where

$$\boldsymbol{H}_{i1} = \begin{pmatrix} h_{i1}(0) & 0 & 0 & \cdots & 0 \\ h_{i1}(1) & h_{i1}(0) & 0 & \cdots & 0 \\ h_{i1}(2) & h_{i1}(1) & h_{i1}(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \cdots & h_{i1}(L_i) \end{pmatrix},$$

with L_i being the length of the impulse response from the SOI to the i-th microphone element. Similarly constraints correspond to condition (3) can be expressed as

$$\boldsymbol{H}_I \boldsymbol{w}(k) = \mathbf{0}, \tag{7}$$

with $\boldsymbol{H}_I$ being the convolution matrix generated by the interference impulse responses. Putting the constraints together and denoting $\boldsymbol{H} = [\boldsymbol{H}_S^T \ \boldsymbol{H}_I^T]^T$ and $\boldsymbol{g} = [\boldsymbol{g}_1^T \ \boldsymbol{0}^T]^T$, the LCMV beamformer design problem can be formulated into the following optimization model

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \frac{1}{2}\boldsymbol{w}^T \boldsymbol{R}_{\boldsymbol{vv}} \boldsymbol{w} \\ s.t. \quad & \boldsymbol{H}\boldsymbol{w} = \boldsymbol{g}. \end{aligned} \tag{8}$$

The Lagrange function for (8) is defined as

$$\mathcal{L}(\boldsymbol{w}, \boldsymbol{\lambda}) = \frac{1}{2}\boldsymbol{w}^T \boldsymbol{R}_{\boldsymbol{vv}} \boldsymbol{w} - \boldsymbol{\lambda}^T (\boldsymbol{H}\boldsymbol{w} - \boldsymbol{g}), \tag{9}$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier. The extrema of $\mathcal{L}(\boldsymbol{w}, \boldsymbol{\lambda})$ satisfy

$$\nabla \mathcal{L}(\boldsymbol{w}, \boldsymbol{\lambda}) = \boldsymbol{0},$$

i.e., the Karush-Kuhn-Tucker (KKT) conditions:

$$\begin{cases} \nabla_{\boldsymbol{w}} \mathcal{L}(\boldsymbol{w}, \boldsymbol{\lambda}) = \boldsymbol{R}_{\boldsymbol{vv}} \boldsymbol{w} - \boldsymbol{H}^T \boldsymbol{\lambda} = \boldsymbol{0}, \\ \nabla_{\boldsymbol{\lambda}} \mathcal{L}(\boldsymbol{w}, \boldsymbol{\lambda}) = \boldsymbol{H}\boldsymbol{w} - \boldsymbol{g} = \boldsymbol{0}. \end{cases} \tag{10}$$

Thus, a closed-form solution (also called a KKT point) of (10) is obtained as

$$\boldsymbol{w}^* = \boldsymbol{R}_{\boldsymbol{vv}}^{-1} \boldsymbol{H}^T (\boldsymbol{H} \boldsymbol{R}_{\boldsymbol{vv}}^{-1} \boldsymbol{H}^T)^{-1} \boldsymbol{g}. \tag{11}$$

In consideration of the computational complexity of the inverse matrix, and merging penalty function with the primal-dual and Lagrangian philosophy, a class of augmented Lagrangian methods of multipliers can be developed to solve problem (8) via iterative approaches. Define the augmented Lagrangian function as

$$\mathcal{L}_{\mathcal{A}}(\boldsymbol{w}, \boldsymbol{\lambda}) = \frac{1}{2}\boldsymbol{w}^T \boldsymbol{R}_{\boldsymbol{vv}} \boldsymbol{w} - \boldsymbol{\lambda}^T (\boldsymbol{H}\boldsymbol{w} - \boldsymbol{g}) + \frac{\beta_1}{2} \|\boldsymbol{H}\boldsymbol{w} - \boldsymbol{g}\|_2^2, \tag{12}$$

where $\boldsymbol{\lambda}$ is the Lagrangian multiplier and $\beta_1 > 0$. Denote by

$$\eta_1 = \left| \frac{1}{2}\boldsymbol{w}^T \boldsymbol{R}_{\boldsymbol{vv}} \boldsymbol{w} \right|, \quad \eta_2 = \|\boldsymbol{H}\boldsymbol{w} - \boldsymbol{g}\|_2, \tag{13}$$

the residuals of objective function η_1 and constraint violation η_2, respectively; they will be used to define the termination criterion. An iterative algorithm based on successive minimization of the augmented Lagrangian $\mathcal{L}_{\mathcal{A}}(\boldsymbol{w}, \boldsymbol{\lambda})$ with respect to $\boldsymbol{w}$, with updates of $\boldsymbol{\lambda}$ can be developed accordingly.

Algorithm 1 Centralized LCMV beamformer design

1: Given $(\boldsymbol{w}^0, \boldsymbol{\lambda}^0)$, $\beta_1 > 0$, $\epsilon > 0$ and $K > 0$, set $k = 0$.

2: Substituting $\boldsymbol{w}^0$ to (13) to calculate η_1 and η_2.

3: **while** $k < K$ and $\max\{\eta_1, \eta_2\} > \epsilon$ **do**

4: Update $\boldsymbol{w}$ by

$$\boldsymbol{w}^{k+1} = \arg\min_{\boldsymbol{w}} \mathcal{L}_{\mathcal{A}}(\boldsymbol{w}, \boldsymbol{\lambda}^k). \tag{14}$$

5: Update Lagrangian multiplier

$$\boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k - \beta_1(\boldsymbol{H}\boldsymbol{w}^{k+1} - \boldsymbol{g}). \tag{15}$$

6: Substituting $\boldsymbol{w}^{k+1}$ to (13) to calculate η_1 and η_2, and set $k = k + 1$.

7: **end while**

In Algorithm 1, once the primal variable $\boldsymbol{w}^k$ has been updated, the constraint violation $\boldsymbol{H}\boldsymbol{w}^{k+1} - \boldsymbol{g}$ is approximately equal to $-\beta_1(\boldsymbol{\lambda}^* - \boldsymbol{\lambda}^k)$, where $\boldsymbol{\lambda}^*$ denotes the unknown optimal Lagrangian multiplier. Thus, an accurate approximation for $\boldsymbol{\lambda}^{k+1}$ is calculated via (15) immediately. More detailed discussion on the derivation of augmented Lagrangian method can be found in Chapter 17 of the seminal monograph by Nocedal and Wright [32].

The solution (11) for problem (8) developed in [33, 34] is also called the centralized LCMV (C-LCMV) beamformer due to all nodes transmitting their microphone array signals to an FC (fusion center) for further processing. However, this results in a large communication cost and hence a fast battery depletion at the nodes. Furthermore, the FC must have sufficient processing power to collect and process M microphone signals. If the resulting beamformer output signal should also be locally available at the nodes, there is an additional communication cost to transmit this signal from the FC back to various nodes.

3. Distributed LCMV beamformer design

In this section, we develop an implementation of the design for distributed beamformer. We establish the optimization problem for distributed LCMV beamformer design, and a numerical algorithm is proposed for solving the problem, as well as convergence analysis in a concise form.

Suppose each node has its own processing unit and ability to exchange signals with others, and the n-th node is equipped with M_n microphones, and total number is $M = \sum_{n=1}^{N} M_n$ (see Fig. 1).

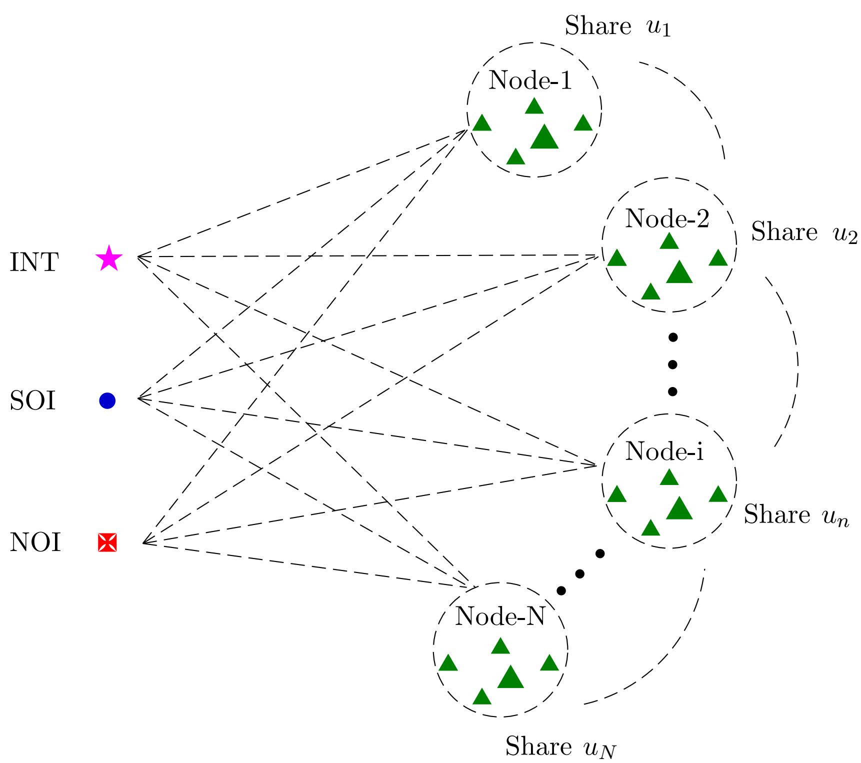

Figure 1: Diagram of distributed beamforming.

Recall the LCMV beamformer design problem (8) which can be rewritten as

$$\begin{aligned} \min_{\boldsymbol{u}_1,\ldots,\boldsymbol{u}_N} \quad & \frac{1}{2}\left[\boldsymbol{u}_1^T \cdots \boldsymbol{u}_N^T\right] \boldsymbol{R} \left[\boldsymbol{u}_1^T \cdots \boldsymbol{u}_N^T\right]^T \\ s.t. \quad & \sum_{n=1}^{N} \boldsymbol{H}_n \boldsymbol{u}_n = \boldsymbol{g}, \end{aligned} \tag{16}$$

where $\boldsymbol{R} = \boldsymbol{R}_{\boldsymbol{vv}} + \varepsilon \boldsymbol{I}$ is the regularized matrix of $\boldsymbol{R}_{\boldsymbol{vv}}$ with small $\varepsilon > 0$ and identity matrix $\boldsymbol{I}$, $\boldsymbol{u}_n$ denotes the stacked beamformer coefficients related to the n-th node, and $\boldsymbol{H}_n$ is the submatrix of $\boldsymbol{H}$ corresponding to $\boldsymbol{u}_n$, $i = 1, \ldots, N$, respectively. It is noted that problem (16) is a linearly constrained minimization problem with a coupled quadratic objective function. In the sequel, we propose a simple but efficient algorithm for solving (16) based on RPADMM in detail.

Rewrite the augmented Lagrangian function of (16) as

$$\begin{aligned} \mathcal{L}_{\mathcal{A}}\left(\boldsymbol{u}_1, \ldots, \boldsymbol{u}_N, \boldsymbol{\lambda}\right) \;=\; & \frac{1}{2}\left[\boldsymbol{u}_1^T \cdots \boldsymbol{u}_N^T\right] \boldsymbol{R} \left[\boldsymbol{u}_1^T \cdots \boldsymbol{u}_N^T\right]^T - \boldsymbol{\lambda}^T \left(\sum_{n=1}^{N} \boldsymbol{H}_n \boldsymbol{u}_n - \boldsymbol{g}\right) \\ & + \frac{\beta_2}{2} \left\| \sum_{n=1}^{N} \boldsymbol{H}_n \boldsymbol{u}_n - \boldsymbol{g} \right\|_2^2, \end{aligned} \tag{17}$$

where $\boldsymbol{\lambda}$ is the Lagrangian multiplier and $\beta_2 > 0$. Denote by

$$\Gamma \triangleq \{\sigma \mid \sigma \text{ is a permutation of } \{1, \ldots, N\}\}, \tag{18}$$

the permutation set of index set, and

$$\eta_3 = \left| \frac{1}{2}\left[\boldsymbol{u}_1^T \cdots \boldsymbol{u}_N^T\right] \boldsymbol{R} \left[\boldsymbol{u}_1^T \cdots \boldsymbol{u}_N^T\right]^T \right|, \quad \eta_4 = \left\| \sum_{n=1}^{N} \boldsymbol{H}_n \boldsymbol{u}_n - \boldsymbol{g} \right\|_2, \tag{19}$$

the residuals of objective function and constraint violation. The schedule of RPADMM is to update the node filters $\boldsymbol{u}_n$, $n = 1, \ldots, N$, in the order of one permutation $\sigma \in \Gamma$ at random in each iteration (see Algorithm (2) for details).

As shown in Algorithm 2, the variable $\boldsymbol{u}$ in the distributed LCMV beamformer design problem (16) is separated into N blocks, and to be solved sequentially. For each block n, a subproblem is solved to update a partial variable $\boldsymbol{u}_n$ with a small size ($1/N$ of $\boldsymbol{u}$), which has lower computational complexity. During the whole iterative process, only partial variables $\boldsymbol{u}_n$, $n = 1, 2, \ldots, N$, are required to be shared among the nodes in the network of microphone array.

Algorithm 2 Distributed LCMV beamformer design

1: Given $(\boldsymbol{u}_1^0, \ldots, \boldsymbol{u}_N^0, \boldsymbol{\lambda}^0)$, $\beta_2 > 0$, $\epsilon > 0$ and $K > 0$, set $k = 0$.

2: Substituting $(\boldsymbol{u}_1^0, \ldots, \boldsymbol{u}_N^0, \boldsymbol{\lambda}^0)$ to (19) to calculate η_3 and η_4.

3: **while** $k < K$ and $\max\{\eta_3, \eta_4\} > \epsilon$ **do**

4: Pick a permutation $\sigma \in \Gamma$ uniformly at random.

5: **for** $n = 1, \ldots, N$ **do**

$$\boldsymbol{u}_{\sigma(n)}^{k+1} = \arg \min_{\boldsymbol{u}_{\sigma(n)}} \ \mathcal{L}_{\mathcal{A}}(\boldsymbol{\xi}). \tag{20}$$

with $\boldsymbol{\xi} = (\boldsymbol{u}_{\sigma(1)}^{k+1}, \ldots, \boldsymbol{u}_{\sigma(n-1)}^{k+1}, \boldsymbol{u}_{\sigma(n)}, \boldsymbol{u}_{\sigma(n+1)}^{k}, \ldots, \boldsymbol{u}_{\sigma(N)}^{k}, \boldsymbol{\lambda}^k)$.

6: **end for**

7: Update the Lagrangian multiplier by

$$\boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k - \beta_2 \left(\sum_{n=1}^{N} \boldsymbol{H}_n \boldsymbol{u}_n^{k+1} - \boldsymbol{g} \right). \tag{21}$$

8: Substituting $(\boldsymbol{u}_1^k, \ldots, \boldsymbol{u}_N^k)$ to (19) to calculate η_3 and η_4, and set $k = k + 1$.

9: **end while**

Generally speaking, some essential conditions are necessary for ensuring the convergence of RPADMM. For instance, nonsingularity of the coefficient matrix for the square system of linear equations [30], the positive definiteness of a generated diagonal matrix for quadratic programming [14]. Fortunately, the required conditions for the distributed LCMV beamformer design problem (16) is satisfied automatically. To prove it and make the convergence analysis tractable, we first prove the following lemma for the proposed Algorithm 2.

Lemma 1. *The matrix*

$$\begin{bmatrix} \boldsymbol{H}_1^T \boldsymbol{H}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{H}_2^T \boldsymbol{H}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{H}_N^T \boldsymbol{H}_N \end{bmatrix} + \begin{bmatrix} \boldsymbol{R}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{R}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{R}_N \end{bmatrix} \succ 0, \tag{22}$$

where $\boldsymbol{R}_i$, $i = 1, \ldots, N$, is the submatrix of $\boldsymbol{R}$ corresponding to the terms $\boldsymbol{u}_i^T \boldsymbol{u}_i$ in quadratic objective function of (16)*, respectively.*

Proof. Since the sensor signal correlation matrix

$$\boldsymbol{R}_{\boldsymbol{vv}} = E\{\boldsymbol{v}\boldsymbol{v}^T\},$$

is symmetric positive semidefinite. Then, we have

$$\boldsymbol{R} = \boldsymbol{R}_{\boldsymbol{vv}} + \varepsilon \boldsymbol{I} \succ 0,$$

with $\varepsilon > 0$, and

$$\boldsymbol{R}_i + \boldsymbol{H}_i^T \boldsymbol{H}_i \succ 0,$$

for all $i = 1, \ldots, N$. As a result, the matrix defined in (22) is symmetric positive definite, which completes the proof. □

The above Lemma 1 shows that the diagonal matrix generated by block coefficients $\boldsymbol{H}_i$ and correlation matrix $\boldsymbol{R}_i$ is positive definite. Thus, the expected convergence of Algorithm (2) can be obtained together with Lemma 1 as follows.

Theorem 1. *Suppose the Algorithm* (2) *is employed to solve the distributed LCMV beamformer design problem* (16)*, the expected iterative sequence converges to a KKT point.*

The proof of Theorem 1 is directly based on the results in [31] and is omitted here. The reader is referred there for the proofs of the lemmas and theorems, and for more information concerning the mathematical derivation. Moreover, the $O(1/n)$ convergence rate results of general ADMM scheme can be found in the seminal papers [35, 36], and reference therein.

In the sequel, we discuss the computational complexity of Algorithm 2 comparing with Algorithm 1 for each iteration. Suppose the matrices $\boldsymbol{H}$ and $\boldsymbol{H}_n$, $n = 1, 2, \ldots, N$ are positive semidefinite, then both the C-LCMV beamformer design problem (8) and the D-LCMV beamformer design problem (16) are convex. It has been proven that the convex quadratic programming can be solved in polynomial time with either the ellipsoid method or interior point method [37, 38]. Assume the length of input signal $\mathbb{L} > ML$ (where ML is the size of matrix $\boldsymbol{H}$), it was shown that the ellipsoid method can solve the C-LCMV beamformer design problem (8) in $O(M^2L^2\mathbb{L})$ iterations, where each iteration (14) in Algorithm 1 requires $O\left((ML)^2\right)$ arithmetic operations in fusion center [39, 40]. However, each iteration (20) in Algorithm 2 only requires $O\left((ML/N)^2\right)$ arithmetic operations in each node for the solving of D-LCMV beamformer design problem (16). It shows great reduction in computational complexity for D-LCMV method as the number of nodes N increases.

4. Numerical experiments

In this section, we develop a multi-source scenario to evaluate the performance of the proposed D-LCMV beamformer design method, and compare with the C-LCMV beamformer design method.

4.1. System illustration and algorithm implementation

In the example, we employ a simple 2-D microphone array system for testing the beamformer designs. The sources of SOI, INT and NOI are placed at $(0,0)$, $(-1m, -0.5m)$ and $(1m, -0.5m)$, respectively. The microphone array are fully connected and consists of $N = 3$ nodes (each node have the same M_0 microphone elements). They are placed in the front, left and right of the acoustic sources and are 2 meters away from the nearest source. The schematic illustration of the system layout is shown in Fig. 2.

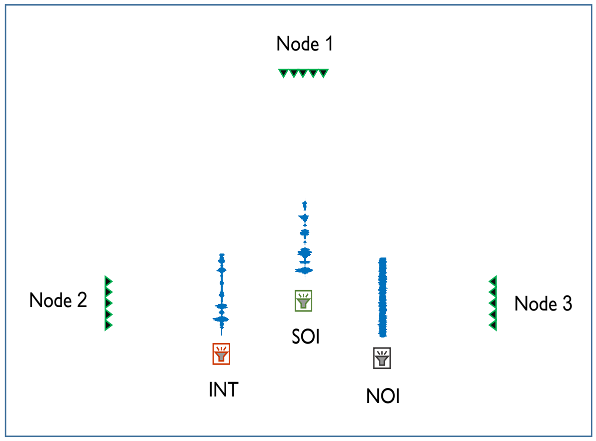

Figure 2: Layout of 2-D microphone array system.

We evaluate our proposed beamformer design methods based on [41], where a male speech "*Dots of light betrayed the black cat*" is chosen as the SOI signal, a female speech "*She had your dark suit in greasy wash water all year*" is chosen as the INT signal, and a white Gaussian noise is generated as the NOI signal.

Over the past few decades, various objective measurement indicators have been derived to verify the performance of speech enhancement methods quantitatively [42]. These indicators include the signal-to-noise ratio (SNR) [43], the perceptual evaluation of speech quality (PESQ) [44], and the normalized noise suppression and interference suppression [45]. Apparently, SNR is the mostly used measure of comparing the signal and noise power, whereas the PESQ measure is the most complex to compute and is the one recommended by ITU-T for speech quality assessment of narrow-band handset telephony and speech codecs [44, 46]. As described in [46], the PESQ score is computed as a linear combination of the average disturbance value D_{ind} and the average asymmetrical disturbance value A_{ind} as follows:

$$\text{PESQ} = a_0 + a_1 D_{\text{ind}} + a_2 A_{\text{ind}},$$

where $a_0 = 4.5$, $a_1 = -0.1$, and $a_2 = -0.0309$, which were optimized for speech processed through networks. The range of the PESQ score is 0.5 to 4.5, although for most cases the output range will be a Mean opinion score (MOS)-

like score, i.e., a score between 1.0 and 4.5. On the whole, these measurement indicators were shown to be consistent in [47]. Therefore, we only introduce the segmental signal-to-noise-plus-interference ratio ($SNIR_{seg}$) and the PESQ score to evaluate the performance of the proposed algorithm.

In general, the penalty parameters β_1 and β_2 in Algorithm 1 and Algorithm 2 are dependent on problem, and must be gradually reduced to small values. In the current design problem, we found it is sufficient to define $\beta_1 = \beta_2 = 10^{-3}$ for the implementation of both algorithms.

4.2. Performance evaluation for $M_0 = 5$

In this subsection, we define the number of microphone elements $M_0 = 5$ per node to verify the overall performance of the proposed D-LCMV beamformer design method, as well as the comparison with the C-LCMV method. By setting the filter length $L = 15$, we can have a glimpse of the speech quality enhancement from both the C-LCMV method and the D-LCMV method. As shown in Fig. 3, a great enhancement of speech quality can be obtained after beamforming. Moreover, the beamformers designed by both methods achieve similar performance on the enhancement of speech quality. This means that both the D-LCMV method and the C-LCMV method are effective in the design of beamformer.

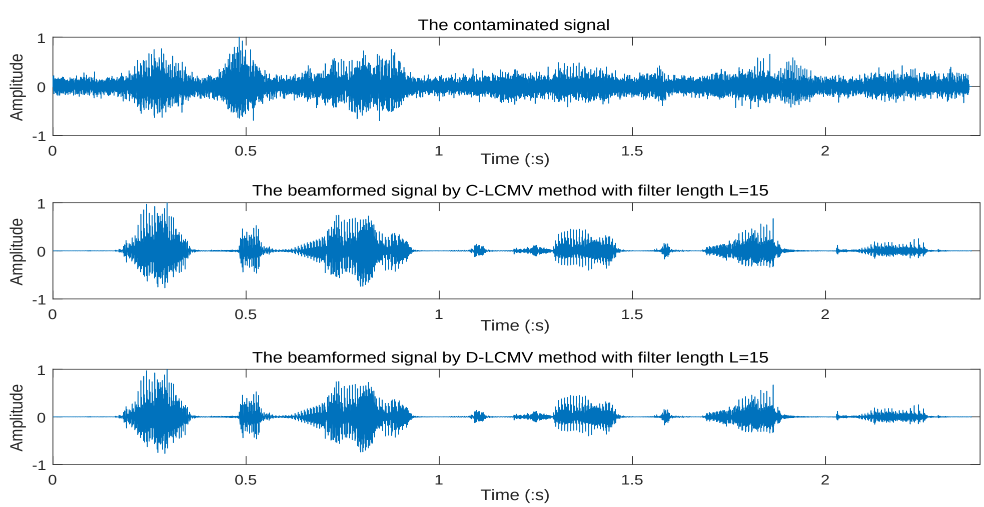

Figure 3: The contaminated signal and beamformed signals with filter length $L = 15$.

For further experiments, we choose two filter lengths of $L = 15$ and $L = 25$ for the evaluation of the efficiency of D-LCMV method and C-LCMV method. As shown in Fig. 4 and Fig. 5, we see that the converged value of indicators $SNIR_{seg}$ (dB) and PESQ score for D-LCMV method and C-LCMV are the same. Furthermore, the convergence of beamforming performance using the D-LCMV method is faster than that by using the C-LCMV method. This means that the D-LCMV method is more efficient than the C-LCMV method for designing beamformers.

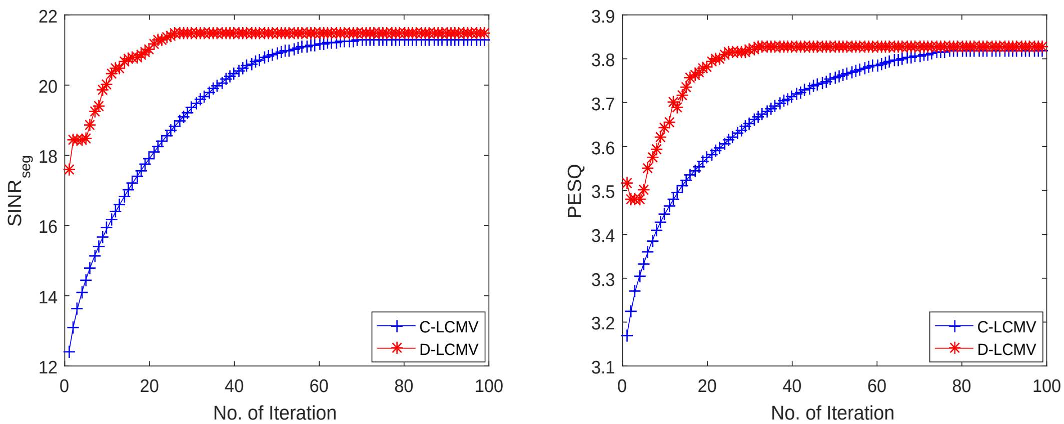

Figure 4: Convergence of the beamformers by using C-LCMV and D-LCMV methods with filter length $L = 15$.

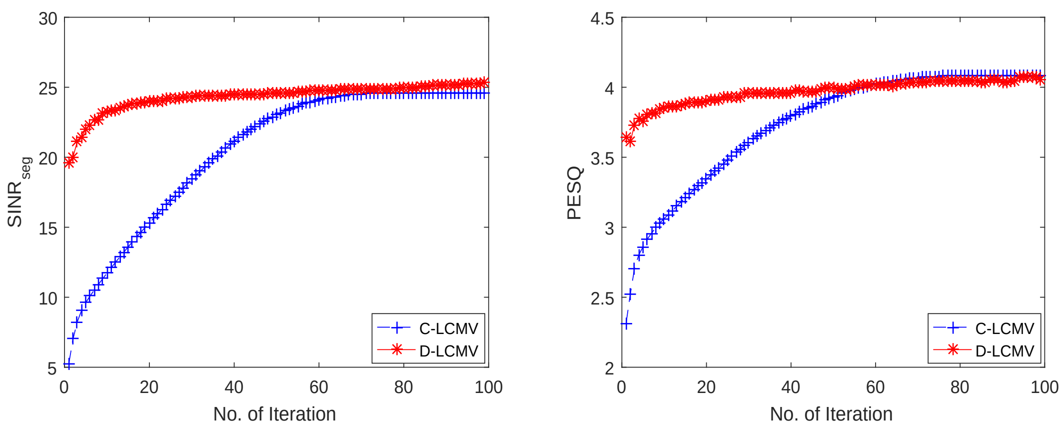

Figure 5: Convergence of the beamformers by using C-LCMV and D-LCMV methods

with filter length $L = 25$.

We conduct further experiments on the beamformer design by using both the C-LCMV method and the D-LCMV method with a collection of filter length L from 5 to 25, to evaluate the overall performance of speech enhancement. As shown in Fig. 6, the longer the filter length, the better the speech quality enhancement for both methods, and the performance are very close. Thus, we can conclude that the D-LCMV beamformer design method can achieve similar performance on speech enhancement as the C-LCMV method.

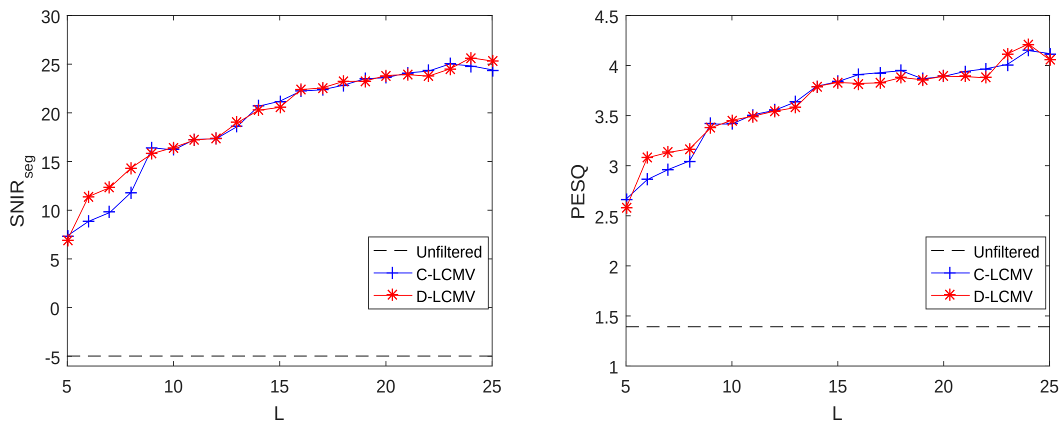

Figure 6: Overall performance of beamforming with respect to different filter length selection.

4.3. Comparison results on different selection of M_0

Finally, we give a simple evaluation of the proposed D-LCMV beamformer design with respect to different selection of microphone elements per node Num_{Mic} and filter length L in the following.

As shown in Table 1, we can see that both the beamforming performance measures, $SNIR_{seg}$ and PESQ score, increase as the number of microphone elements and filter length increases. This is in line with our expectation.

4.4. A simple comparison of RPADMM with direct ADMM

As a final experiment, we give a simple example to show that the direct application of ADMM (Dir-ADMM) to D-LCMV beamformer design will fail

Table 1: Performance of D-LCMV beamformer with respect to Num_{Mic} and L.

	Num_{Mic}	L		
		15	20	25
$SNIR_{seg}$	3	15.9958	20.5613	23.7284
	5	16.6906	23.8347	26.0809
	7	17.7116	25.3219	29.1011
PESQ	3	3.4014	3.8310	3.9296
	5	3.4689	3.8930	4.0653
	7	3.5409	4.0587	4.3157

while the RPADMM remains effective. As shown in Fig. 7, a six-elements microphone array with 3-nodes are placed in front of acoustic sources (include SOI, INT and NOI).

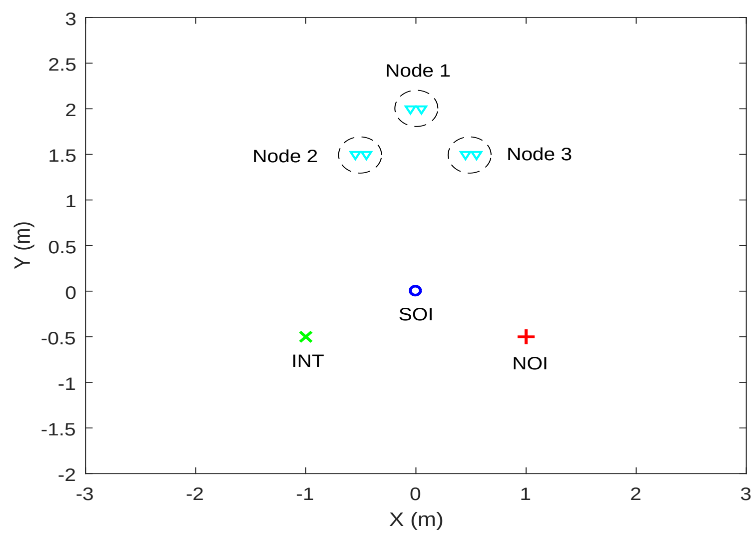

Figure 7: A simple beamforming layout.

In the experiment, the distributed beamformers with filter lengths $L = 10$ are designed by using Dir-ADMM and RPADMM. From the beamforming performance based on the evaluation of $SNIR_{seg}$ (dB) and PESQ score (shown in

Fig. 8), we see that the Dir-ADMM fails to converge to the required achievable beamforming performance as the number of iteration increases compared with RPADMM.

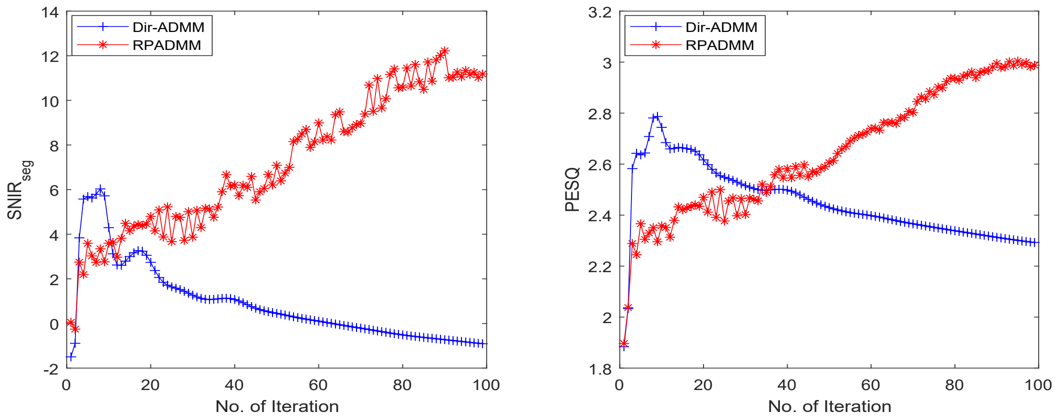

Figure 8: Performance comparison of RPADMM with Dir-ADMM.

5. Conclusion

In this paper, we have developed a novel multi-block framework for the distributed LCMV beamformer design together with a randomly permuted alternating direction algorithm, which implements a blockwise iterative strategy. It is based on the augmented Lagrangian technique which is popular in solving large scale optimization problems. The expected convergence of this algorithm has also been established, as well as discussion of the computational complexity. We have provided simulation results to demonstrate the effectiveness and efficiency of the proposed method for speech enhancement in a wireless acoustic sensor network.

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