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Models on ship scheduling in transshipment hubs with considering bunker cost

Abstract: This paper studies a decision problem for scheduling ships' starting operation time in a transshipment hub with considering the bunker cost and the transshipment cost of containers. Based on the information about the handing capacity of the container port in each period, the port operator makes a proper decision and informs the shipping companies about the start operation days of each liner route. The optimized decision may help shipping companies significantly reduce the cost. This study proposes a mixed integer programming model. The NP-hardness of the problem is proved. A local branching based solution method is also developed for solving the model. Extensive numerical experiments demonstrate the optimality of the solution by the proposed method is less than 2% for large-scale problem instances.

Keywords: Port operations; Shipping line; Bunker cost; Maritime transportation

1. Introduction

Container shipping is vital to global trade. The international ocean freights are mainly containerized especially in the transshipment hubs. There are two major players in container shipping: container shipping lines and container port operators (Talley and Ng, 2013). The benefits and cost issues on their sides (port operators & shipping lines) are intertwined. Efficient operations on one of the two sides will benefit the other side. For example, berth allocation is one of the most important decisions on the side of port operators. The berth allocation plan has influence on the voyage arrangement of shipping lines. On the other hand, when planning the berth allocation decision, the factors for shipping lines should also be considered. For example, the bunker cost of shipping lines needs to be taken into account when making the berth allocation plans (Du et al., 2011; Hu et al., 2014). Berths, quay cranes (QCs), yard cranes (YCs), yard trucks are the resources in a port (He et al., 2010, 2013). Among these resources, the allocation of the berths to arrival ships may be one of the most important decisions. The berth allocation decision affects the berthing time of ships, and hence influences the scheduled arrival time of the ships when the shipping line makes the voyage schedule for the ships. From a tactical-level perspective, the decision on which berth is allocated to a ship is not as important as the decision on when the ship could moor at a berth. Therefore how to make a tactical-level decision on when each ship (route or service) should be scheduled to moor at a port is an important decision for both the shipping line and the port, especially some mega transshipment hubs. This paper focuses on port operation problems in transshipment hubs, as the transshipment has become more and more important in terminals around the world. This trend is expected to continue especially when ocean liners are using larger ships and visiting fewer ports. The maximum size of containerships was 4300 twenty-foot equivalent units (TEUs) in 1988, up to 7100 TEUs in 1996, then 15,500 TEUs in 2006, and is 19,000 TEUs now (Fan et al., 2012; Tran et al., 2015). Hence, the study on ship scheduling problem in mega transshipment hubs with considering the bunker cost as well as transshipment related cost will become even more important in the future.

The remainder of the paper is organized as follows. Section 2 reviews the related works in the literature. Section 3 gives a description of the studied problem. Section 4 formulates a mathematical model for making decisions about the starting operation time of containerships. Section 5 presents the algorithm. Section 6 states the results of numerical experiments. Some possible extension is discussed in Section 7. Closing remarks are then given in the last section.

2. Related works

There are numerous studies on the shipping problems and port operation problems in the past decades. For knowing the advances of the related studies in these fields, we recommend readers the review works given by Christiansen et al. (2013), Meng et al. (2014) and Tran and Haasis (2015). They gave comprehensive literature surveys on the container routing, fleet management and network design problems in container liner shipping. The theme of this paper is about the decisions on starting operation time (date) with considering some realistic factors. It is a tactical level decision problem. This study highlights that the handling time of containerships in a port is associated with the capacity of terminal and the number of ships in the port at the same time. The collaborative mechanism was proposed between shipping companies and container terminal operators to obtain accurate information. Thus, this section mainly reviews the related studies through the following two streams: container liner shipping, and port operation (berth/yard) management problems.

The theme of this study is optimization for shipping lines with considering bunker cost. Fuel costs occupy more than half of a liner shipping company's total operation costs (Yao et al., 2012). Many liner shipping companies have sought to save fuel by making some operational adjustments, e.g., speed adjustment, consolidation of services, reduction of navigation resistance. Most works in this research area are about solving bunkering port selection problems (Mazraati, 2011). Some studies on shipping line are related to design liner shipping network, e.g., Wang and Meng (2012b) proposed a model on ship sailing speed optimization a liner shipping network, which has the similarity with this study in that both works optimizing the sailing speed. However, from the application point of view, Wang and Meng (2012b) focuses on a tactical-level problem faced by a shipping line. They only determine the sailing speed on each leg, without considering the arrival and departure time at each port of call. By contrast, this study focuses on the collaboration between a shipping line and a terminal operator, which is a practical problem when the shipping line operates the terminal. We also consider the limited capacity of the terminal: limited numbers of berths and QCs, which were ignored in Wang and Meng (2012b). Wang and Meng (2014) further developed a mixed-integer programming (MIP) model which is associated with liner shipping network design with deadlines, and they believe that a large proportion of the operating cost is determined at the stage of shipping network design. Mulder and Dekker (2014) studied a strategic liner shipping network planning problem, which combines fleet-design, ship-scheduling and cargo-routing problem with limited availability of ships in liner shipping. In addition, Brouer et al. (2014) investigated a model for the liner shipping network design problem, which is to maximize the revenue of cargo transport, while minimizing the cost of operating the network. Zheng et al. (2015) studied a liner hub-and-spoke shipping network design problem. In addition, transit time constraints are considered in single liner shipping service design (Plum et al., 2014) and liner ship route schedule design (Wang and Meng, 2012a). Pang et al. (2011) studied a ship routing problem to coordinate the routing and the berthing time of its vessels. An algorithm based on the set partitioning formulation and column generation techniques was developed for solving their model. Another research branch of liner shipping is the fleet deployment problem. There are many other works related to liner shipping network management. Ng (2014) proposed a probability distribution free liner fleet deployment model, which only requires the specification of the mean, standard deviation and an upper bound on the shipping demand. Branchini et al. (2015) designed a MIP model of Routing and fleet deployment based on the directed graph, in which contractual and spot voyages are denoted as nodes. Wang and Meng (2015) proposed a robust bunker management model for liner shipping networks. They developed a mixed-integer nonlinear optimization model to minimize the total cost consisting of ship operating cost and bunker cost. Huang et al. (2015) developed a MIP model on fleet deployment with empty container repositioning. Their model incorporated several constraints, such as weekly frequency, the transshipment of cargo between two or more service routes, and transport time. Xia et al. (2015) proposed a strategic level decision model on fleet deployment, speed optimization, and cargo allocation. Their model considers a general fuel consumption function that depends on speed and load.

Besides the research problems of shipping lines, this study is more related to the well-known berth allocation problem (BAP) in container ports, especially in transshipment hubs. Two excellent literature review works have been conducted on this problem (Bierwirth and Meisel, 2010 and 2015). Some scholars have studied this problem. For example, Giallombardo et al. (2010) investigated the tactical berth allocation problem for transshipment hubs. They integrated at the tactical level two decision problems arising in ports: the berth allocation problem, which consists of assigning and scheduling incoming containerships to berthing positions, and the QC assignment problem. Zhen et al., (2011a) proposed a decision model for berth allocation under uncertain arrival time and uncertain operation time of ships. Imai et al. (2014) proposed a strategic berth template problem. They considered a decision environment that if a terminal operator has to deal with excessive calling requests simultaneously, the operator can choose which ship requests to serve and arrange their berth-windows within a limited planning horizon. Zhen et al. (2011b) combined the berth allocation and the yard template problems for transshipment hubs. Jin et al. (2015)

also studied a tactical berth and yard template, and proposed a column generation based solution method. Terminal allocation, a more tactical level problem than the berth allocation, was also investigated by Lee et al. (2011). Their team also studied a scheduling problem especially for feeders at transshipment hubs (Lee and Jin, 2013).

This study proposes a decision model for scheduling the starting operation time for arrival ships so as to minimize the bunker cost for shipping lines and the storage cost for the transshipped containers in the transshipment hub. A mixed-integer nonlinear optimization model is developed. The model is transformed to a linear optimization model. In view of the NP-hardness of the problem, we develop a local branching based heuristic. Extensive numerical experiments demonstrate the optimality gap of the solution by the proposed method is less than 3% for large-scale problem instances.

3. Problem background

This study is about scheduling ships' starting operation time at transshipment hubs. A more exact description on "ships' starting operation time" should be "services' starting operation time" (Cordeau et al., 2007). Here the service is the 'ship route', which is a loop containing a sequence of ports. Each service is fulfilled by a fleet of ships, the number of which equals the total round trip time of the loop divided by the cycle time such as one week. However, as the majority of BAP related studies usually use the term 'ship' as the object for allocating and the BAP is similar as the scheduling ships' starting operation time to some extent, this study uses the term 'ships' instead of 'services or routes' in the remainder of this paper.

This study assumes the shipping line is concerned about the bunker cost. A lower sailing speed implies a lower bunker cost because daily bunker consumption is approximately proportional to the speed cubed (i.e., bunker consumption per unit distance is approximately proportional to the speed squared). However, a lower speed means that containership will need more time in transportation. Suppose that the bunker consumption (ton/nautical-mile) is 0.001 times the sailing speed squared, and the bunker price is $500/ton. The highest sailing speed of the ships is 25 knots. Each period is 8 hours. The total costs of Ship i, including the bunker cost on the first leg (voyage from the previous port to the port of study) and the bunker cost on the second leg (voyage from the port of study to the next port), can be calculated as follow:

$$u_{i,t} = 500 \times h_i \times 0.001 \times \left(\frac{h_i}{t\times 8}\right)^2 \quad (1)$$

$$w_{i,t,d} = 500 \times k_i \times 0.001 \times \left(\frac{k_i}{(P_i-d-t)\times 8}\right)^2 \quad (2)$$

The above formulae are formulated according to the regression results of a previous study (Wang and Meng, 2012b). In the formulae, the parameters are described as follows:

$u_{i,t}$ bunker cost for the first leg of Ship i if it arrives at the port in t.

$w_{i,t,d}$ bunker cost for the second leg of Ship i if it arrives at the port in t and dwells for d periods.

d index of the number of periods when a ship dwells in this port.

h_i length of the first leg (nautical-mile).

k_i length of the second leg (nautical-mile).

P_i total time for Ship i sailing from its previous port to its next port through the port.

We give a simple instance to show the impact of different handing and starting operation times on shipping lines. Consider the start operation day at Singapore with a weekly liner service route that includes Shenzhen, Singapore, and Colombo. A fleet of 8000-TEU ships are deployed on this route and visit each port of call on the same day every week. Suppose that ships on the route leave Shenzhen on Sunday (day 0), and arrive at Colombo on Friday about two weeks later (day 12). We define one period t be 8 hours. Then total time from Shenzhen to Colombo is 24 periods. A ship spends one period, two periods, or even up to three periods at Singapore for handling containers. 8000 TEUs are carried from Shenzhen to Singapore (the distance is 2700 nautical miles) and 6000 TEUs are carried from Singapore to Colombo (the distance is 3060 nautical miles).

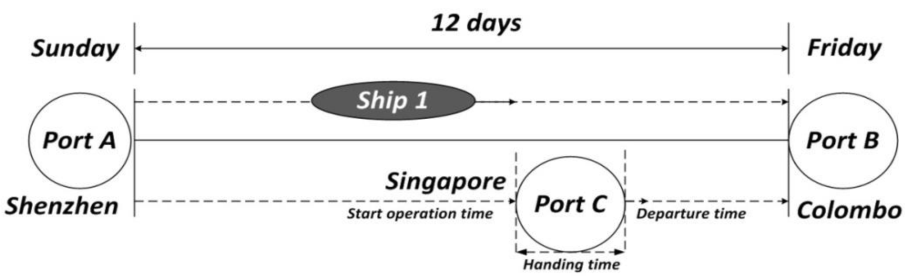

Figure 1: An example of a ship route

If the ship arrives at Singapore port at Period 13, the sailing speed (denoted by v knots) of the ship from Shenzhen to Singapore is $v = \frac{sailing\ distance}{sailing\ time} = \frac{2700}{13\times8} \approx 25.96$. The speed is greater than the maximum speed 25 knots. If the ship arrives at Singapore port at Period 14, the sailing speed of the ship from Shenzhen to Singapore is $v = \frac{sailing\ distance}{sailing\ time} = \frac{2700}{14\times8} \approx 24.11$, which is smaller than the maximum speed 25 knots. If the ship arrives at Singapore port at Period 19 and dwells in the Singapore port for two periods, the sailing speed of the ship from Shenzhen and Singapore is $v = \frac{sailing\ distance}{sailing\ time} = \frac{2700}{19\times8} \approx 17.76$, which is smaller than the maximum speed 25 knots; but the sailing speed of the ship from Singapore to Colombo, denoted by v' (knots), is $v' = \frac{sailing\ distance}{sailing\ time} = \frac{3060}{(36-19-2)\times8} \approx 25.5$, which is greater than the maximum speed 25 knots. It is possible for the ship to arrive at Singapore at period 14, and dwell in the Singapore port for one period. The total bunker costs on the two legs can be calculated as $500 \times 2700 \times 0.001 \times v^2 + 500 \times 3060 \times 0.001 \times (v')^2 \approx 1.29 \times 10^6$. Similar to the above calculation, Figure 2 shows the bunker cost of the ship under different starting operation time and handing time at the Singapore port. The curves in Figure 2 indicate the importance of reducing handing time in the port. In addition, the starting operation time also has significant influence on the cost.

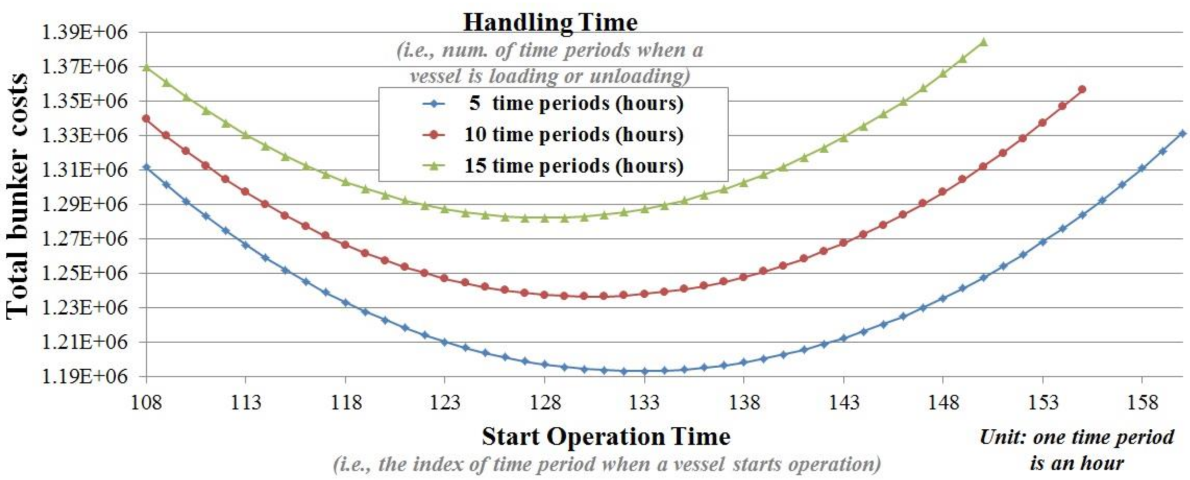

Figure 2: The total bunker costs under different handing time and starting operation time

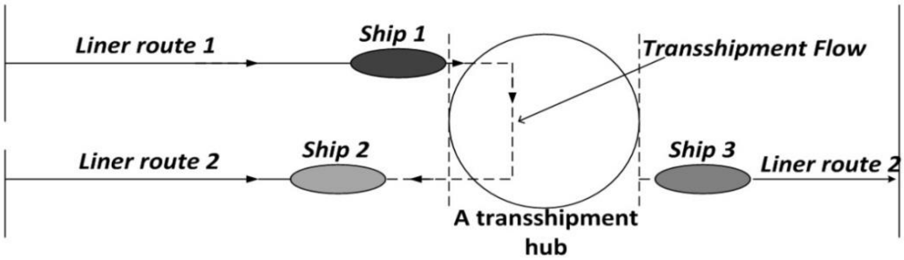

Figure 3: The flow of transshipment in a terminal

The starting operation time not only affects the bunker costs, but also affects the storage time at transshipment ports. The storage time is important for both the carriers and the customers as it is related to the storage costs and delivery time of containers. Figure 3 shows an example of container transshipment between two routes in a transshipment hub. When the ship 1 (deployed in the liner route 1) arrives at the terminal, the ship 3 (deployed in the liner route 2) has left. The containers will be stored in the transshipment hub, and wait for being loaded onto ship 2 (deployed in the liner route 2). The waiting time of containers depends on the starting operation time of ship 1 and ship 2. Therefore, the holding cost of the transshipment container due to waiting at the terminal is also considered in this study. In this study, the unit holding cost is $ 0.5 per TEU per hour (Wang et al., 2015).

This study thus focuses on a tactical-level ship scheduling problem at a transshipment port that determines the starting operation time of each ship. The objective is to minimize the total bunker costs on the previous and next legs and holding costs of transshipment containers while considering the capacity constraints in terms of the number of berths and number of QCs at the terminal.

4. Model formulation

4.1 Notations

Indices and parameters:

i, j index of ship.

I set of ships.

t, τ index of period when ships arrive at the port.

d index of number of periods when ships dwell in the port.

T set of periods when ships arrive at the port. We consider one hour as one period and hence $|T| = \{1,2 \cdots 168\}$ represents weekly services (168 hours = 1 week).

D set of periods when ships dwell in the port for handling containers. For example, $D = \{1, 2, \cdots, 48\}$ means a ship may dwell for one period (hour), two periods up to 48 periods.

A_i set of periods when Ship i could arrive at the port, $A_i \subseteq T$.

$Z_{\tau,d}$ subset of the set T, which is determined by the parameter τ and the parameter d.
$$Z_{\tau,d} = \begin{cases} \{\tau + 1 - d \cdots \tau\}, & if \tau + 1 - d \geq 0 \\ \{\tau + 1 - d + T \cdots T\} \cup \{1 \cdots \tau\}, & if \tau + 1 - d < 0 \end{cases}.$$
That is, $Z_{\tau,d}$ is the set of starting operation times such that if the starting operation time of a ship $t \in Z_{\tau,d}$ and the ship is berthed for d periods, then the ship occupies a berth in period τ. For example, $Z_{4,2} = \{3, 4\}$, $Z_{2,4} = \{20, 21, 1, 2\}$.

q unit holding cost of storing one container for one period in the port.

c handling capacity of the port in terms of the number of QC moves in each period.

b total number of berths in the port.

$u_{i,t}$ bunker cost for Ship i from its previous port to the port, if Ship i arrives at the port in Period t.

$w_{i,t,d}$ bunker cost for Ship i from the port to its next port, if Ship i arrives at this port in Period t, and dwells in the port for d periods.

$n_{i,j}$ number of containers that need to be transshipped from Ship i to Ship j.

$F_{t,\tau}$ number of periods for transshipping containers from Ship i to Ship j, and Ship i and Ship j arrive at this port in Period t and Period τ, respectively. $F_{t,\tau} = \begin{cases} |T| + (\tau - t), & if\ \tau < t \\ \tau - t, & if\ \tau \geq t \end{cases}.$

Decision Variables

$\alpha_{i,t} \in \{0,1\}$ set to one if Ship i arrives at the port in Period t; and zero otherwise.

$\beta_{i,t,d} \in \{0,1\}$ set to one if Ship i arrives at the port in Period t, and dwells in this port for d periods; and zero otherwise.

4.2 Mathematical model

[**M0**] *Minimize* $\sum_{i \in I, t \in T} u_{i,t}\alpha_{i,t} + \sum_{i \in I, t \in T, d \in D} w_{i,t,d}\beta_{i,t,d} + \sum_{i \in I, j \in I, t \in T, \tau \in T} q n_{i,j} F_{t,\tau} \alpha_{i,t} \alpha_{j,\tau}$ (3)

Subject to:

$$\sum_{t\in A_i, d\in D} \beta_{i,t,d} = 1 \qquad \forall i \in I$$

(4)

$$\sum_{t\in T\setminus A_i, d\in D} \beta_{i,t,d} = 0 \qquad \forall i \in I$$

(5)

$$\alpha_{i,t} = \sum_{d\in D} \beta_{i,t,d} \qquad \forall i \in I; \forall t \in T$$

(6)

$$\sum_{i\in I, d\in D, t\in Z_{\tau,d}} \beta_{i,t,d} (1/d)\left(\sum_{j\in I} n_{i,j} + \sum_{j\in I} n_{j,i}\right) \le c \qquad \forall \tau \in T$$

(7)

$$\sum_{i\in I, d\in D, t\in Z_{\tau,d}} \beta_{i,t,d} \le b \qquad \forall \tau \in T$$

(8)

$$\alpha_{i,t}, \beta_{i,t,d} \in \{0,1\} \qquad \forall i \in I; \forall t \in T; d \in D$$

(9)

Objective (3) is to minimize the total bunker costs of ships and the holding cost of the transshipped containers in the port. Constraints (4-6) are the definition of the starting operation time and dwell time of ships. Constraints (7) ensure that the number of the handled containers does not exceed the capacity of the port in each period, in which $(1/d)\left(\sum_{j\in I} n_{i,j} + \sum_{j\in I} n_{j,i}\right)$ is the number of containers handled for Ship i in one period if Ship i is berthed for d days. Constraints (8) guarantee the number of ships that dwell in the port simultaneously does not exceed the number of the available berths in the port.

In the above model, the binary variable $\alpha_{i,t}$ can be relaxed without affecting the optimal solution to the problem. According to Constraints (6) and the definition that $\beta_{i,t,d}$ is a binary variable, it is easy to see that $\alpha_{i,t} = \sum_{d\in D} \beta_{i,t,d}$ is a nonnegative integer. Moreover, Constraints (4-5) indicate $\alpha_{i,t} = \sum_{d\in D} \beta_{i,t,d} \le 1$. Hence, there is no need to limit $\alpha_{i,t}$ as a binary variable. Then constraints $\alpha_{i,t} \in \{0,1\}$, $\forall i \in I$, $\forall t \in T$, can be dropped without affecting the optimal solution to the problem.

In reality, there may exist the situation of 'free time of transshipment containers'. In this case, the above model just needs to be revised slightly. More specifically, the port has a policy that the transshipment containers can be stored in the yard freely for k periods. We define a variable $\gamma_{t,\tau} \geq 0$, and add a constraint $\gamma_{t,\tau} \geq F_{t,\tau-k}$ ($F_{t,\tau-k}$ is defined to be 0 if $\tau - k \leq 0$). Then the third part of Objective (3) is revised to '$\sum_{i\in I, j\in I, t\in T, \tau\in T} qn_{i,j}\gamma_{t,\tau}\alpha_{i,t}\alpha_{j,\tau}$', in which '$\gamma_{t,\tau}$' replaces '$F_{t,\tau}$' in the original objective.

There are some remarks on the proposed model in the case that a port is not very busy and has sufficient berths and QCs.

(1) All of the ships will be served within their minimum possible times. That is, the optimal d will take the smallest value in the set D.

The reason is: if there are sufficient berths and QCs, then Constraints (7-8) can be dropped. Hence, the choice of $d \in D$ no longer affects the feasibility and only impacts the choice of $w_{i,t,d}$ in the Objective (3). A smaller d dictates a smaller $w_{i,t,d}$ due to a longer sailing time from the port of study to the next port. That is, the optimal d will take the smallest value in the set D.

(2) Assume that there are sufficient QCs. Once the arrival time of each ship is determined, the number of berths, as long as it is feasible, does not affect the optimal decision.

The reason is: if the arrival time of each ship is determined (i.e., the variables $\alpha_{i,t}$ are determined), the total bunker cost on the first leg ($\sum_{i\in I, t\in T} u_{i,t}\alpha_{i,t}$) and the holding costs of transshipment containers ($\sum_{i\in I, j\in I, t\in T, \tau\in T} qn_{i,j}F_{t,\tau}\alpha_{i,t}\alpha_{j,\tau}$) are determined. To minimize the total bunker cost on the second leg, all of the d will take the smallest possible value. This is possible because on one side, there are sufficient QCs, and on the other side, the smallest possible values of d imply the smallest number of berths to use in each period (recall that the arrival time of each ship is determined). In other words, as long as the number of berths is feasible, we could adopt the smallest possible values of d for the variables $\beta_{i,t,d}$. So once the arrival time of each ship is determined, the number of berths does not affect the optimal decision.

4.3NP-hardness and linearization

Despite the properties of the ship scheduling problem in the above two propositions, the problem is NP-hard in its nature.

Proposition 1: The ship scheduling problem described by model [M0] is NP-hard.

Proof: It is well-known that minimizing a submodular function with a simple cover constraint is NP-hard. We will demonstrate that if the ship scheduling problem could be solved in polynomial time, then minimizing a submodular function with a simple cover constraint could also be solved in polynomial time.

Suppose that (i) there are sufficient QCs; (ii) $D = \{1\}$, i.e., each ship spends exactly one period for container handling; (iii) $A_i = \{8, 9\}$, i.e., a ship arrives either in Period 8 or Period 9; (iv) the bunker consumption functions of the first leg and the second leg are the same for all of the ships; the total bunker consumption of Ship i arriving in Period 8 is much larger than that if it arrives in Period 9; the bunker consumption difference dominates the holding costs of transshipment containers. Therefore, exactly b ships will arrive in Period 8 and which b ships from the total of $|I|$ ships arrive in Period 8 does not affect the total bunker consumption as we assume the bunker consumption functions of the first leg and the second leg are the same for all of the ships.

The problem now becomes, choosing $|I| - b$ ships, denoted by set $S \subseteq I$, that arrive in Period 9, to minimize the total holding costs of transshipment containers. The holding costs of transshipment containers between ship $i \in S$ that arrives in Period 9 and ship $j \in I \backslash S$ that arrives in Period 8 is $Q_{i,j} \coloneqq q\left[(|T| - 1)n_{i,j} + 1 \cdot n_{j,i}\right]$.

The problem is hence:

Minimize $f(S) \coloneqq \sum_{i \in S} \sum_{j \in I \backslash S} Q_{i,j}$

(10)

Subject to: $S \subseteq I, |S| = |I| - b$

(11)

The above problem is evidently minimizing a set function with a cover constraint. We now need to prove that the function $f(S)$ is submodular. To this end, we only need to prove that for any $S \subseteq I, k \in I \backslash S, l \in I \backslash S$, we have $f(S \cup \{l\}) - f(S) \geq f(S \cup \{k, l\}) - f(S \cup \{k\})$. By definition of function $f(S)$, we have

$f(S \cup \{l\}) - f(S) = \sum_{j \in I \backslash (S \cup \{l\})} Q_{l,j} - \sum_{i \in S} Q_{i,l}$

(12)

$f(S \cup \{k,l\}) - f(S \cup \{k\}) = \sum_{j \in I \setminus (S \cup \{k,l\})} Q_{l,j} - \sum_{i \in S \cup \{k\}} Q_{i,l}$

(13)

It is now easy to see that $f(S \cup \{l\}) - f(S) \geq f(S \cup \{k,l\}) - f(S \cup \{k\})$, and this completes the proof of the proposition.■

Since the problem is NP-hard, we cannot expect to solve all of the large-scale problem instances efficiently. To take advantage of state-of-the-art mixed-integer linear programming solvers and develop integer programming based heuristics, we need to transform the problem into a mixed-integer linear program. In particular, Objective (3) contains nonlinear part '$\alpha_{i,t}\alpha_{j,\tau}$', which is linearized by defining new variables $\varphi_{i,j,t,\tau} \geq 0$ and some auxiliary constraints as follows:

$\varphi_{i,j,t,\tau} \geq \alpha_{i,t} + \alpha_{j,\tau} - 1 \qquad \forall i \in I; j \in I; \forall t \in T; \forall \tau \in T$

(14)

$\varphi_{i,j,t,\tau} \geq 0 \qquad \forall i \in I; j \in I; \forall t \in T; \forall \tau \in T$

(15)

Then the model turns to:

Minimize $\sum_{i \in I, t \in T} u_{i,t}\alpha_{i,t} + \sum_{i \in I, t \in T, d \in D} w_{i,t,d}\beta_{i,t,d} + \sum_{i \in I, j \in I, t \in T, \tau \in T} F_{t,\tau} \cdot q \cdot n_{i,j}\varphi_{i,j,t,\tau}$

(16)

Subject to: (4-9; 14-15).

4.4 More practical considerations

The model can incorporate a number of other practical considerations.

(1) The shipping line may not only be interested in the optimal solution, but also interested in obtaining the best few solutions and then choosing one from them. The shipping line may not choose the optimal solution as it may have other considerations that are not modeled.

Algorithm for obtaining the best K solutions

Step 0: Set $k \leftarrow 1$. Solve [M0] and obtain the optimal solution denoted by $(\alpha_{i,t}^{(1)}, \beta_{i,t,d}^{(1)})$

Step 1: Solve [M0] by adding the following constraints:

$\sum_{i \in I, t \in A_i, d \in D}(1 - \beta_{i,t,d}^{(\kappa)})\beta_{i,t,d} \geq 1 \qquad \forall \kappa \in \{1,2 \cdots k\} \qquad (17)$

The optimal solution to the problem is the $(k+1)th$ best solution denoted by

$(\alpha_{i,t}^{(k+1)}, \beta_{i,t,d}^{(k+1)})$.

Step 2: Set $k \leftarrow k+1$. If $k = K$, stop. Otherwise, go to Step 1. ■

(2) If a ship i'cannot arrive at the port in a particular period t', for instance, because a major shipper objects the arrival time t', then we could easily incorporate this consideration by adding the following constraint:

$$\alpha_{i',t'} = 0 \tag{18}$$

(3) If Ship $i'$and Ship i'' must arrive at the port at the same period t' for commercial reasons, then we could add the following constraint:

$$\alpha_{i',t'} = \alpha_{i'',t''} \tag{19}$$

(4) The shipping line may want to know, given the arrival time of each ship $(\alpha_{i,t}^{(*)})$, if in a particular week the demand is higher than the forecasted values $n_{i,j}$, how many extra containers can be handled. Suppose that an extra θ (percentage) of containers can be handled, then we can solve:

$$\textit{Maximize} \quad \theta \tag{20}$$

Subject to:

$$\sum_{t \in T - A_i, d \in D} \beta_{i,t,d} = 0 \qquad \forall i \in I \tag{21}$$

$$\sum_{d \in D} \beta_{i,t,d} = \alpha_{i,t}^{(*)} \qquad \forall i \in I; \forall t \in T \tag{22}$$

$$\sum_{i \in I, d \in D, t \in Z_{\tau,d}} \beta_{i,t,d} (1/d) \cdot \left(\sum_j n_{i,j} + \sum_j n_{j,i}\right) \cdot (1+\theta) \le c \qquad \forall \tau \in T \tag{23}$$

$$\sum_{i \in I, d \in D, t \in Z_{\tau,d}} \beta_{i,t,d} \le b \qquad \forall \tau \in T \tag{24}$$

$$\beta_{i,t,d} \in \{0,1\} \qquad \forall i \in I; \forall t \in T; d \in D \tag{25}$$

The above model is a mixed-integer nonlinear formulation. It is equivalent to the following integer linear program:

Minimize θ'

(26)

Subject to:

Constraints (21-22; 24-25) and

$\sum_{i\in I,d\in D,t\in Z_{\tau,d}} \beta_{i,t,d} (1/d)\cdot(\sum_j n_{i,j} + \sum_j n_{j,i}) \leq \theta' \qquad \forall \tau \in T$

(27)

Then the optimal θ, denoted by θ^*, can be derived from the optimal θ' denoted by θ'^*:

$\frac{c}{1+\theta^*} = \theta'^*$

(28)

5. Local branching based solution method

The model [M0] is a mixed-integer programming model containing both continuous variables $\alpha_{i,t}$ and $\varphi_{i,j,t,\tau}$ and binary variables $\beta_{i,t,d}$. As we know linear programming is polynomially solvable while general integer optimization problems are NP-hard, the bottleneck for solving model [M0] lies in how to efficiently handle the large feasible set of decision variables $\beta_{i,t,d}$. We do not expect to solve all of the large-scale instances of [M0] to optimality in reasonable time, but apply an effective approach that could obtain high-quality solutions efficiently. In view of the development of the state-of-the-art mixed-integer linear programming solvers, we apply a local branching strategy to solve the model (Fischetti and Lodi, 2003).

The core idea of our local branching based method is to use the CPLEX solver as a black-box 'tactical' tool for exploring suitable solution subspaces, which are defined and controlled at a 'strategic' level by a simple external branching framework. Different from the conventional branch and bound search process, local branching uses a highly imbalanced search tree: one branch has a very small feasible set and is solved to optimality by CPLEX; the other branch has a feasible set that is too large to be explored completely. After the smaller branch is solved, the larger branch is branched again into two imbalanced branches.

The branching strategy favors early updating of the incumbent solution, thereby producing improved solutions at early stages of the computation. In other words, high-quality solutions are obtained early and hence the algorithm could stop in reasonable time.

5.1 Idea of local branching

To start the local branching process, we need a feasible solution of the integer variables β. This feasible solution is denoted by $\beta^{(0)}$. In our problem, we can find such a feasible initial solution by using the CPLEX solver to solve the original model [M0] for a certain period of time (e.g., one min). The CPLEX solver usually cannot obtain the optimal solution within such a short time, but can return a feasible and good solution, which is used as the initial solution for the local branching based solution method in this study. In local branching we first explore a small neighborhood of $\beta^{(0)}$, hoping that we can find a better solution. To this end, we confine the feasible set in the smaller branch (called node 1) to be:

$$|\beta - \beta^{(0)}| \leq e \tag{29}$$

Here e is a predetermined number (e.g., 5), and the left hand side is the 1-norm defined as

$$\left|\beta - \beta^{(0)}\right| \coloneqq \sum_{i \in I, t \in T, d \in D} |\beta_{i,t,d} - \beta_{i,t,d}^{(0)}| \tag{30}$$

The larger branch (called node 2) has the constraint

$$\left|\beta - \beta^{(0)}\right| \geq e + 1 \tag{31}$$

Therefore, Constraint (29) actually confines the domain of β similar to a hypercube centered at $\beta^{(0)}$ with a radius e. Constraint (29) can be linearized as follows:

$$\sum_{i \in I, t \in T, d \in D} [(1 - \beta_{i,t,d}^{(0)})\beta_{i,t,d} + \beta_{i,t,d}^{(0)}(1 - \beta_{i,t,d})] \leq e \tag{32}$$

Therefore, node 1, i.e., model [M0] with Constraint (32), can be solved by CPLEX. The optimal solution is denoted by $\beta^{(1)}$. Note that since $\beta^{(0)}$ satisfies Constraint (32), $\beta^{(1)}$ is as least as good as $\beta^{(0)}$.

If $\beta^{(1)}$ is different from $\beta^{(0)}$($\beta^{(1)}$ is better than $\beta^{(0)}$, or they are both optimal but different), we can then center at $\beta^{(1)}$ and explore another part of the tree. In detail, node 2 is branched into a smaller branch (node 2.1) with the following constraint:

$$|\beta - \beta^{(1)}| \leq e$$
(33)

and a larger branch (node 2.2) with the following constraint:

$$\left|\beta - \beta^{(1)}\right| \geq e + 1$$
(34)

Note that node 2.1 and node 2.2 still have Constraint (31), as this constraint defines node 2, the parent node of nodes 2.1 and 2.2. We then use CPLEX to solve node 2.1.

If $\beta^{(1)} = \beta^{(0)}$, we have no new feasible point to center at. In this case, we increase the radius by 1 and branch node 2 into a smaller branch (node 2.1) with the following constraint:

$$\left|\beta - \beta^{(0)}\right| \leq e + 1$$
(35)

and a larger branch (node 2.2) with the following constraint:

$$\left|\beta - \beta^{(0)}\right| \geq e + 2$$
(36)

Note that both node 2.1 and node 2.2 have Constraint (31). We then use CPLEX to solve node 2.1. The above procedure is repeated.

5.2 Procedure of the local branching based method

The procedure of the local branching based is as follows:

Local branching based solution method

Input: Model [M0], a feasible solution $\beta^{(0)}$, radius e, and K that represents the number of iterations without improvement for the algorithm to stop

Step 0: Let $k := 0$ be the number of iterations without improvement, $\Delta := 0$ be the increment of the radius, and $\beta^* := \beta^{(0)}$ be the incumbent solution. Branch model [M0] into two problems: a smaller branch with the constraint:

$$|\beta - \beta^*| \leq e \quad (37)$$

and a larger branch with the constraint:

$$|\beta - \beta^*| \geq e + 1 \quad (38)$$

Step 1: Solve the smaller branch to optimality using CPLEX and let $\hat{\beta}$ be the optimal solution.

Step 2: If $\hat{\beta}$ has the same objective function value as β^*, set $k \leftarrow k + 1$. If $k \geq K$, output the incumbent solution β^* and stop.

Step 3: If $\hat{\beta} \neq \beta^*$, set $\beta^* \leftarrow \hat{\beta}$, $\Delta \leftarrow 0$, and branch the larger branch into two problems: a smaller branch with the constraint:

$$|\beta - \beta^*| \leq e \tag{39}$$

and a larger branch with the constraint:

$$|\beta - \beta^*| \geq e + 1 \tag{40}$$

Go to Step 1.

Step 4: If $\hat{\beta} = \beta^*$, set $\Delta \leftarrow \Delta + 1$, and branch the larger branch into two problems: a smaller branch with the constraint:

$$|\beta - \beta^*| \leq e + \Delta \tag{41}$$

and a larger branch with the constraint:

$$|\beta - \beta^*| \geq e + 1 + \Delta \tag{42}$$

Go to Step 1. ■

The local branching based method terminates if the incumbent best objective value has not been improved for K consecutive iterations. Although the method is not an exact solution approach, it can obtain a satisfying solution in a more efficient way than the widely used heuristics such as the genetic algorithms and simulated annealing.

6. Numerical experiments

We conduct some experiments to validate the effectiveness of the proposed model. The mathematic model is implemented by CPLEX12.1 with technology of C# (VS2008) on a PC (Intel Core i3, 2.67 GHz; Memory, 4G). The computational experiments in this study include three parts: the first part investigates the influence of resources on the results; the second part is the performance analysis on the proposed local branching based solution approach; the last part is about the experiments on some large-scale instances.

6.1 Influence of the resources on the results

In this study, the planning horizon is one week (seven days). Each time period is one hour. It means the planning horizon contains 168 periods. Each route needs to load and unload 1000 TEUs to 19000 TEUs. For the ships (routes), the maximum and minimum journey time between two ports is about thirty days and one day, respectively. The maximum speed of ships' voyage is 25 nautical miles per hour.

The number of resources such as berths and QCs is very important for the operation efficiency of the ports. We conduct some experiments to investigate the influence of the resources on the optimal objective value. The results are shown in Figure 4. It should be noted that the number of berths is three when the QCs' handling capacity is changing; while the QCs' handling capacity in set as 1000 TEUs per hour when the number of QCs is changing in the experiments. In the left part of Figure 4, the horizontal axis is about the total handling capacity of all the QCs. Here one QC's handling capacity is about 50 TEUs per hour (i.e., 25 Forty-foot equivalent containers per hour).

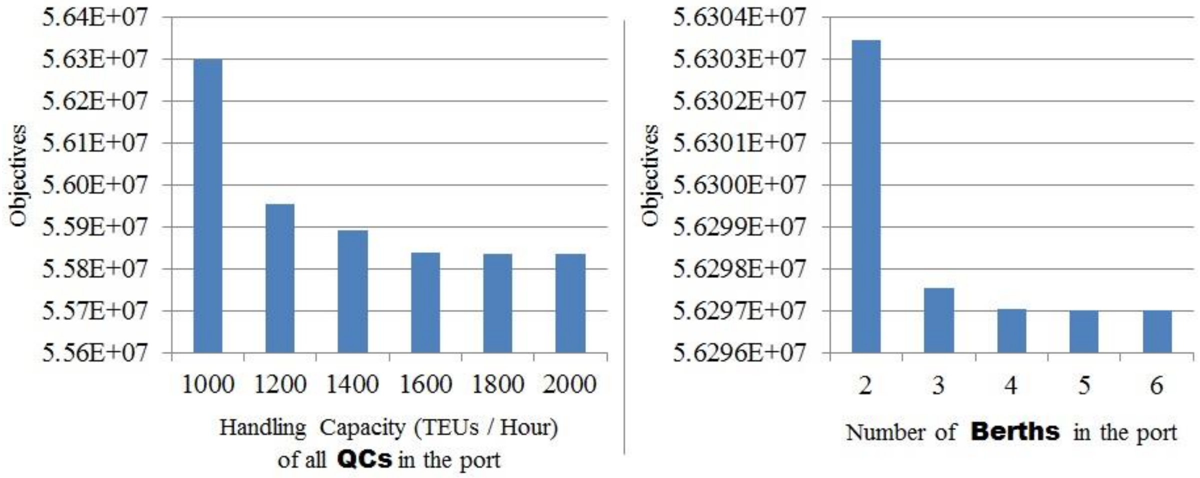

Figure 4: The influence of the resources in the port

From the above results, we can see that sufficient port resources can reduce the cost of the port operations concerned in this problem. When the number of resources (e.g., the berths in the right part of Figure 4) exceeds some threshold value, the cost will not be reduced any more.

6.2 Performance of the local branching based solution method

For evaluating the performance of the local branching based solution method, we compare it with the optimal solution directly obtained by the widely used CPLEX solver. In addition, we also compare with another meta-heuristic based on a given sequence of ships. This heuristic is called sequence based method (heuristic) and was proposed by Lee et al. (2006) and widely used in solving some optimization models on port operations. Table 1 shows the comparative experimental results between the local branching based solution method and the optimal solutions by CPLEX as well as the sequential heuristic.

Table 1: Comparative experiments between different methods

Case id	CPLEX		Local branching			Sequential heuristic		
	Optimal Objective value OBJ_{CPLEX}	CPU time T_{CPLEX}	Objective value OBJ_{LocBra}	CPU time (s) T_{LocBra}	Gap1(%)	Objective value OBJ_{SeqHeu}	CPU time T_{SeqHeu}	Gap2(%)
20-1	5.630×10^7	4235	5.638×10^7	697	0.142	5.668×10^7	132	0.675
20-2	5.029×10^7	5216	5.034×10^7	602	0.099	5.041×10^7	127	0.239
20-3	5.212×10^7	4862	5.216×10^7	586	0.077	5.222×10^7	145	0.192
20-4	5.850×10^7	4475	5.855×10^7	625	0.085	5.862×10^7	152	0.205
20-5	5.703×10^7	3892	5.710×10^7	523	0.123	5.736×10^7	138	0.579
20-6	5.605×10^7	4677	5.609×10^7	638	0.071	5.630×10^7	124	0.446
20-7	5.426×10^7	4079	5.432×10^7	587	0.111	5.458×10^7	146	0.590
20-8	5.023×10^7	5986	5.031×10^7	721	0.159	5.063×10^7	153	0.796
20-9	5.125×10^7	2946	5.128×10^7	584	0.059	5.150×10^7	146	0.488
20-10	5.275×10^7	3617	5.279×10^7	501	0.076	5.305×10^7	129	0.569
20-11	5.524×10^7	4265	5.530×10^7	815	0.109	5.560×10^7	137	0.652
20-12	5.426×10^7	4372	5.433×10^7	846	0.129	5.454×10^7	148	0.516
20-13	5.027×10^7	3945	5.033×10^7	701	0.119	5.045×10^7	134	0.358
20-14	5.644×10^7	4705	5.649×10^7	637	0.089	5.669×10^7	139	0.443
20-15	5.764×10^7	4573	5.769×10^7	604	0.087	5.790×10^7	144	0.451
20-16	5.635×10^7	4681	5.640×10^7	625	0.089	5.662×10^7	132	0.479
20-17	5.763×10^7	5426	5.770×10^7	746	0.121	5.796×10^7	161	0.573
20-18	5.244×10^7	4039	5.248×10^7	624	0.076	5.268×10^7	142	0.458
20-19	5.373×10^7	3046	5.376×10^7	599	0.056	5.383×10^7	128	0.186
20-20	5.462×10^7	6247	5.470×10^7	975	0.146	5.502×10^7	167	0.732
Avg.		4464		662	0.101		141	0.481

Note: (1) Case id '50-#' means that there are 50 ships in each problem instance; '#' denotes the index of the instances. (2) Gap1 $=\frac{OBJ_{LocBra}-OBJ_{CPLEX}}{OBJ_{CPLEX}}\times 100\%$, Gap2 $=\frac{OBJ_{SeqHeu}-OBJ_{CPLEX}}{OBJ_{CPLEX}}\times 100\%$.

The results in Table 1 indicate that the local branching based solution method can obtain satisfying solutions with about 0.101% optimality gap in a much shorter time period than solving the model by the CPLEX directly. The local branching based solution method can save about eight times of the computation time in comparison to solving the model directly by the CPLEX. Moreover, the local branching based solution method also demonstrates its outperformance to the previously proposed meta-heuristics (i.e., sequential heuristic). The optimality gap of the local branching based solution method is just one fourth of the gap of the sequential heuristic. However, the computation time is the price that the local branching based solution method needs to pay for its superiority to the sequential heuristic in terms of solution quality.

6.3 Experiments of large-scale problem instances

Some experiments are conducted to validate the efficiency of the local branching based solution method when solving some large-scale problem instances. As the CPLEX solver cannot solve some large-scale instances within a reasonable time, the results obtained by the local branching method should be compared with some lower bounds so as to evaluate the quality of the obtained solution. Here we use a lower bound by relaxing the part related to the container transshipment cost and QC capacity constraints. The model for solving the lower bound is as follows:

[**LB**] $Minimize Z_1 = \sum_{i\in I,t\in T} u_{i,t}\alpha_{i,t} + \sum_{i\in I,t\in T,d\in D} w_{i,t,d}\beta_{i,t,d}$

(29)

Subject to:

Constraints (4-6; 8-9)

Before comparing the results of the local branching with the lower bound, we need to evaluate the optimality gap of the lower bound. The results are shown in Table 2 as follows. We can see that the average optimality gap for these instances with 20 ships is about 1.704%.

Table 2: The optimality gap of the lower bound for the instances with 20 ships

Case id	Optimal results by CPLEX OBJ_{CPLEX}	Lower bound LB	Gap3 (%)
20-1	5.630×10^7	5.532×10^7	1.741
20-2	5.029×10^7	4.947×10^7	1.631
20-3	5.212×10^7	5.114×10^7	1.880
20-4	5.850×10^7	5.764×10^7	1.470
20-5	5.703×10^7	5.606×10^7	1.701
20-6	5.605×10^7	5.503×10^7	1.820
20-7	5.426×10^7	5.323×10^7	1.898
20-8	5.023×10^7	4.942×10^7	1.613
20-9	5.125×10^7	5.038×10^7	1.698
20-10	5.275×10^7	5.191×10^7	1.592
Avg.			1.704

Note: Gap3 $=\frac{OBJ_{CPLEX}-LB}{LB}\times 100\%$

We conduct some experiments on large-scale problem instances with 40 ships and compare the obtained results with the lower bound. Results in Table 3 indicate that the average gap is about 1.915%, which is not small but is still acceptable if we recall that the optimality gap of the lower bound has been about 1.704%. Therefore, the actual optimality gap for the local branching based solution method is much smaller than the '1.915%'.

Table 3: Comparison between the local branching based method with the lower bounds

Case id	Local Branching Objective Value OBJ_{LocBra}	Local Branching CPU Time (s) T_{LocBra}	Lower bound LB	Gap4 (%)
40-1	9.214×10^7	2637	9.036×10^7	1.970
40-2	9.635×10^7	2708	9.457×10^7	1.882
40-3	9.573×10^7	2430	9.383×10^7	2.025
40-4	9.024×10^7	2119	8.875×10^7	1.679
40-5	9.728×10^7	2435	9.560×10^7	1.757
40-6	9.903×10^7	3081	9.690×10^7	2.198
40-7	9.507×10^7	2496	9.348×10^7	1.701
40-8	9.267×10^7	2276	9.084×10^7	2.015
40-9	9.364×10^7	2538	9.191×10^7	1.882
40-10	9.406×10^7	2872	9.218×10^7	2.039
Avg.		2559		1.915

Note: Gap4 $=\frac{OBJ_{LocBra}-LB}{LB}\times 100\%$

We increase the problem scale and conduct some more experiments with 50 ships. From Table 4, we can see that the gap between the results obtained by the local branching based method is not large, about 2.094%. Moreover, the computation time for such large-scale problem instances is still acceptable since this problem is a tactical level decision. These results validate the efficiency of the proposed local branching based method.

Table 4: Comparative experiments on large-scale problem instances (50 ships)

Case id	Local Branching Objective Value OBJ_{LocBra}	 CPU Time (s) T_{LocBra}	Lower bound LB	Gap4 (%)
50-1	1.085×10^8	6249	1.064×10^8	1.974
50-2	1.067×10^8	6791	1.045×10^8	2.105
50-3	1.057×10^8	6874	1.035×10^8	2.126
50-4	1.025×10^8	6209	1.004×10^8	2.092
50-5	1.042×10^8	6634	1.020×10^8	2.157
50-6	1.033×10^8	7328	1.010×10^8	2.277
50-7	1.068×10^8	6537	1.048×10^8	1.908
50-8	1.090×10^8	6401	1.067×10^8	2.156
50-9	1.077×10^8	5976	1.055×10^8	2.085
50-0	1.092×10^8	6104	1.070×10^8	2.056
Avg.		6510		2.094

Note: Gap4 $=\frac{OBJ_{LocBra}-LB}{LB}\times 100\%$

7. Discussion and extension

7.1 Extension for considering more ports

The port operator and the shipping line must collaborate closely to achieve the overall system optimization (i.e., the minimization of fuel cost and transshipment container holding cost). Otherwise, the port operator may be more concerned about its own interest. The port operator and the shipping line can make decisions in a holistic manner if the terminal is a dedicated one operated/used by the shipping line. This is common in reality, and this is the motivation of our study. For example, APL has eight dedicated terminals; OOCL operates two dedicated terminals; APM Maersk has ten dedicated terminals in Europe, thirteen dedicated terminals in Asia, and five dedicated terminals in North America.

It should be pointed out that a large shipping line such as APL and OOCL serves over 200 ports and Maersk Line serves over 400 ports. Therefore, it is very rare (if any) that all of the ports visits by a service have dedicated terminals for the shipping line. That is why we focus on one dedicated terminal, rather than a whole service.

In this section, we propose a model for the case where two neighboring ports both have dedicated terminals, for instance, Bremerhaven and Rotterdam for Maersk Line. We consider three types of ship routes: routes that only visit the first dedicated terminal (e.g., Bremerhaven), ship routes that only visit the second dedicated terminal (e.g., Rotterdam), and ship routes that visit the first terminal and then the second one. Note that we can easily incorporate ship routes that visit the second terminal and then the first one; however, to simplify the presentation, we do not consider this type of ship routes.

New indices and parameters:

I_1 set of ship routes that only visit the first terminal.

I_2 set of ship routes that only visit the second terminal.

I_{12} set of ship routes that visit the first terminal and then the second terminal.

W set of periods in a week; e.g., $W = \{0, 1, 2 \cdots 20\}$ if a period is 8 hours.

T set of periods of possible arrival times at the two terminals .

$Z_{\tau,d}$ subset of the set T, which is the set of starting operation times such that if the starting operation time of a ship $t \in Z_{\tau,d}$ and the ship is berthed for d periods, then the ship occupies a berth in period $\tau \in W$.

c_1 handling capacity of the first terminal in terms of the number of QC moves in each period.

c_2 handling capacity of the second terminal in terms of the number of QC moves in each period.

b_1 total number of berths in the first terminal.

b_2 total number of berths in the second terminal.

$u_{i,t}$ bunker cost for Ship $i \in I_1 \cup I_{12}$ from its previous port to the first terminal, if Ship i arrives at the first terminal in Period t.

$u'_{i,t}$ bunker cost for Ship $i \in I_2$ from its previous port to the second terminal, if Ship i arrives at the second terminal in Period t.

$w_{i,t,d}$ bunker cost for Ship $i \in I_1$ from the first terminal to its next port, if Ship i arrives at the first terminal in Period t, and dwells at the first terminal for d periods.

$w'_{i,t,d}$ bunker cost for Ship $i \in I_2 \cup I_{12}$ from the second terminal to its next port, if Ship i arrives at the second terminal in Period t, and dwells at the second terminal for d periods.

$w''_{i,\tau,t,d}$ bunker cost for Ship $i \in I_{12}$ from the first terminal to the second terminal, if Ship i arrives at the first terminal in Period τ, arrives at the second terminal in Period t, and dwells at the first terminal for d periods.

$F_{t,\tau}$ number of periods for transshipping containers from Ship i to Ship j, if Ship i and Ship j arrive at this port in Period t and Period τ, respectively. $F_{t,\tau} = \min_{k \in Z}\{\tau - t + k|W|:\ 0 \le \tau - t + k|W| \le |W| - 1\}$.

New decision variables

$\alpha_{i,t} \in \{0,1\}$ set to one if Ship $i \in I_1 \cup I_{12}$ arrives at the first terminal in Period t; and zero otherwise.

$\beta_{i,t,d} \in \{0,1\}$ set to one if Ship $i \in I_1 \cup I_{12}$ arrives at the first terminal in Period t, and dwells in this terminal for d periods; and zero otherwise.

$\alpha'_{i,t} \in \{0,1\}$ set to one if Ship $i \in I_2 \cup I_{12}$ arrives at the second terminal in Period t; and zero otherwise.

$\beta'_{i,t,d} \in \{0,1\}$ set to one if Ship $i \in I_2 \cup I_{12}$ arrives at the second terminal in Period t, and dwells in this terminal for d periods; and zero otherwise.

[M1] *Minimize* $\sum_{i \in I_1 \cup I_{12}, t \in T} u_{i,t}\alpha_{i,t} + \sum_{i \in I_2, t \in T} u'_{i,t}\alpha'_{i,t} + \sum_{i \in I_1, t \in T, d \in D} w_{i,t,d}\beta_{i,t,d} + \sum_{i \in I_2 \cup I_{12}, t \in T, d \in D} w'_{i,t,d}\beta'_{i,t,d} + \sum_{i \in I_{12}, t \in T, d \in D} w''_{i,\tau,t,d}\alpha'_{i,t}\beta_{i,\tau,d} + \sum_{i \in I_1 \cup I_{12}, j \in I_1 \cup I_{12}, t \in T, \tau \in T} qn_{i,j}F_{t,\tau}\alpha_{i,t}\alpha_{j,\tau} + \sum_{i \in I_2 \cup I_{12}, j \in I_2 \cup I_{12}, t \in T, \tau \in T} qn_{i,j}F_{t,\tau}\alpha'_{i,t}\alpha'_{j,\tau}$

(30)

Subject to:

$\sum_{t \in T, d \in D} \beta_{i,t,d} = 1 \qquad \forall i \in I_1 \cup I_{12}$

(31)

$\sum_{t\in T,d\in D}\beta'_{i,t,d}=1 \qquad \forall i\in I_2\cup I_{12}$

(32)

$\alpha_{i,t}=\sum_{d\in D}\beta_{i,t,d} \qquad \forall i\in I_1\cup I_{12};\forall t\in T$

(33)

$\alpha'_{i,t}=\sum_{d\in D}\beta'_{i,t,d} \qquad \forall i\in I_2\cup I_{12};\forall t\in T$

(34)

$\sum_{i\in I_1\cup I_{12},d\in D,t\in Z_{\tau,d}}\beta_{i,t,d}(1/d)\left(\sum_{j\in I}n_{i,j}+\sum_{j\in I}n_{j,i}\right)\le c_1 \quad \forall\tau\in T$

(35)

$\sum_{i\in I_2\cup I_{12},d\in D,t\in Z_{\tau,d}}\beta'_{i,t,d}(1/d)\left(\sum_{j\in I}n_{i,j}+\sum_{j\in I}n_{j,i}\right)\le c_2 \quad \forall\tau\in T$

(36)

$\sum_{i\in I_1\cup I_{12},d\in D,t\in Z_{\tau,d}}\beta_{i,t,d}\le b_1 \qquad \forall\tau\in T$

(37)

$\sum_{i\in I_2\cup I_{12},d\in D,t\in Z_{\tau,d}}\beta'_{i,t,d}\le b_2 \qquad \forall\tau\in T$

(38)

$\alpha_{i,t},\beta_{i,t,d}\in\{0,1\} \qquad \forall i\in I_1\cup I_{12};\forall t\in T;d\in D$

(39)

$\alpha'_{i,t},\beta'_{i,t,d}\in\{0,1\} \qquad \forall i\in I_2\cup I_{12};\forall t\in T;d\in D$

(40)

The first term in the objective function is the fuel cost for ships in set $I_1\cup I_{12}$ from their previous port to the first terminal; the second term is the fuel cost for ships in set I_2 from their previous port to the second terminal; the third term is the fuel cost for ships in set I_1 from the first terminal to the next port; the fourth term is the fuel cost for ships in set $I_2\cup I_{12}$ from the second terminal to the next port; the fifth term is the fuel cost for ships in set I_{12} from the first terminal to the second terminal; the sixth term is the holding cost of transshipment containers at the first terminal; the seventh term is the holding cost of transshipment containers at the second terminal.

7.2 Extension for considering ship charting cost

Moreover, the model proposed in Section 4 is mainly based on bunker cost. That is, we assume that the departure time from the previous port and arrival time at the next port are determined. We only adjust the arrival and departure time at the focal port. This means, on one side, if the leg from the previous port to the focal port has a longer time, then the leg from the focal port to the next port will have a shorter time; on the other hand, the total trip time does not change and hence the number of ships required does not change.

In reality, when the fuel price is very high, the sailing speed may become very low, which considerably decreases the bunker cost. Thus, slow steaming means a longer trip time, and as a result one or two additional ships may have to be inserted into the existing string to maintain a weekly frequency. In this case, chartering cost may increase as the voyage becomes longer. In this section, we slightly revise the previously proposed model so as to allow the number of ships to be a decision variable.

New indices and parameters:

m index of number of ships deployed on a ship route.

W set of periods in a week; e.g., $W = \{0, 1, 2 \cdots 20\}$ if a period is 8 hours.

T set of periods of possible arrival times at the focal port.

m_i^{min} Minimum number of ships deployed on Ship Route i.

m_i^{max} Maximum number of ships deployed on Ship Route i.

$\hat{c}_i$ Chartering cost of a ship on Ship Route i.

$Z_{\tau,d}$ subset of the set T, which is the set of starting operation times such that if the starting operation time of a ship $t \in Z_{\tau,d}$ and the ship is berthed for d periods, then the ship occupies a berth in period $\tau \in W$.

$w_{i,t,d,m}$ bunker cost for Ship i from the port to its next port, if Ship i arrives at this port in Period t, dwells in the port for d periods, and m ships are deployed on the ship route.

$F_{t,\tau}$ number of periods for transshipping containers from Ship i to Ship j, if Ship i and Ship j arrive at this port in Period t and Period τ, respectively. $F_{t,\tau} = \min_{k \in Z}\{\tau - t + k|W|:\ 0 \leq \tau - t + k|W| \leq |W| - 1\}$.

New decision variables

$\beta_{i,t,d,m} \in \{0,1\}$ set to one if Ship i arrives at the port in Period t, dwells in this port for d periods, and m ships are deployed on the ship route; and zero otherwise.

[**M2**] *Minimize* $\sum_{i\in I,t\in T} u_{i,t}\alpha_{i,t} + \sum_{i\in I,t\in T,d\in D,m_i^{min}\le m\le m_i^{max}} w_{i,t,d,m}\beta_{i,t,d,m}$

$$+ \sum_{i\in I,j\in I,t\in T,\tau\in T} q n_{i,j} F_{t,\tau}\alpha_{i,t}\alpha_{j,\tau} + \sum_{i\in I,t\in T,d\in D,m_i^{min}\le m\le m_i^{max}} \hat{c}_i\beta_{i,t,d,m} \quad (41)$$

Subject to:

$$\sum_{t\in T,d\in D,m_i^{min}\le m\le m_i^{max}} \beta_{i,t,d,m} = 1 \qquad \forall i \in I \quad (42)$$

$$\alpha_{i,t} = \sum_{d\in D,m_i^{min}\le m\le m_i^{max}} \beta_{i,t,d,m} \qquad \forall i \in I; \forall t \in T \quad (43)$$

$$\sum_{i\in I,d\in D,t\in Z_{\tau,d},m_i^{min}\le m\le m_i^{max}} \beta_{i,t,d,m}(1/d)\left(\sum_{j\in I} n_{i,j} + \sum_{j\in I} n_{j,i}\right) \le c \qquad \forall \tau \in W \quad (44)$$

$$\sum_{i\in I,d\in D,t\in Z_{\tau,d},m_i^{min}\le m\le m_i^{max}} \beta_{i,t,d,m} \le b \qquad \forall \tau \in W \quad (45)$$

$$\alpha_{i,t}, \beta_{i,t,d,m} \in \{0,1\} \qquad \forall i \in I; \forall t \in T; d \in D; m_i^{min} \le m \le m_i^{max} \quad (46)$$

The first term in the objective function of [M2] is fuel cost from the previous port to the focal port; the second term is the fuel cost from the focal port to the next port; the third term is the holding cost of transshipment containers; and the last term is the ship chartering cost.

8. Conclusions

This paper studies the method for determining the starting operation time of ships in container ports, especially in transshipment hubs. Some realistic factors such as the bunker cost and the holding cost of transshipment containers are taken into account. This study proposes a mixed integer programming model. A local branching based solution method is developed for solving the model in some large-scale problem instances. Numerical experiments are conducted to validate the effectiveness of the proposed model and the efficiency of the proposed solution method.

This study also has limitations. For example, this study only considers the time cost for the transshipped containers in a port but the transportation cost for them is not taken into account. In addition, global maritime transportation contains a lot of uncertainty, which will further complicate the current model which is mainly for a deterministic decision environment. The above issues could be some interesting research directions for our future study.

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