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Willingness to Board: A Novel Concept for Modeling Queuing Up Passengers

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Abstract

This paper addresses an innovative concept, termed as queuing passengers' willingness to board (WTB) the transit vehicles. In the peak hours, some queuing passengers cannot board a crowded bus/train, but when the same vehicle arrives at the next stop, some other passengers could still get on. This phenomenon reflects that passengers at different queuing locations have heterogeneous level of ambitions to board. A methodological framework is proposed for the quantitative investigation of WTB. First, a general model is proposed, together with a new least square method (LSM) for the calibration. Then, a parametric model is developed, which is also calibrated by the LSM. To refine the calibration method and deal with the biasness of survey data, a weighted least square method is further developed. Based on real survey data, the calibration results clearly support the existence of WTB, which can be used to estimate the capacity of transit vehicles. This paper also sheds some lights on the practical applications of the quantitative WTB.

Keywords: transit passengers; willingness to board; parametric model; weighted least square method; mixed-integer programming.

1 Introduction

Urban transportation network is a huge and complex system, involving millions of travelers. The network operation condition is an outcome of the travelers' decision making, interaction and decision adjustments. To improve the network efficiency and to make wise/balanced use of network resources, accurate predictions are usually needed for the network flows (Liu and Wang, 2015). For decades, the transport scholars have endeavored to develop more accurate tools for flow predictions; for instance, the well-known user equilibrium theory is a widely used means for flow prediction.

Improvements of such sort of models usually drive from better understanding of the travelers' behaviors; for instance, stochastic user equilibrium is an extension of UE, which was developed by addressing the travelers' perception errors on travel time. To this end, this paper focuses on an interesting psychological factor of the travelers, which is termed as willingness to board (WTB). This idea comes from the observations in practice on an interesting phenomenon: a crowded bus or train comes to a stop, and only a proportion of the queuing passengers can get on board; however, when this bus or train arrives at the next stop and no one alights, some passengers still can get on! Such a phenomenon is very common at the metro stations in the dense mega cities in the peak hours. This phenomenon gives us the impression that different queuing passengers have different levels of ambitions to get on board. Hence, this paper aims to holistically define this concept, and then quantitatively calibrate this value based on real survey data.

Section 2 gives an in-depth discussion of the WTB, which is followed by two mathematical models in Sections 3 and 4, respectively. The basic concept for the calibration is developed from the least square method (LSM). However, in view of the unique features of WTB, the existing LSMs are not suitable to be directly adopted. Hence, as another major contribution of this paper, two new LSMs are developed to calibrate the WTB. These two new LSMs also contribute to the state-of-the-art statistical regression methods.

1.1 Literature review

Based on real survey data, LSM aims to calibrate the parameters of assumed function types that best fit the data (May, 1990). The least square method (LSM) and weighted least square method (WLSM) that was proposed by Aitken (1935) have been widely applied in economics, aerospace design, urban studies, political studies, etc. They of course also play a significant role in transportation analysis problems.

Lewandowski and Protzel (2001) established a local linear model with adaptive kernel functions to obtain a well fitted function with consideration of computing costs. Local linear regression model applied to short-term traffic prediction was proposed by Sun et al. (2003). Then the parameters of bandwidth and covariate vector dimension were chosen by optimizing an overall average square error between computed values by the cross validation method and observed values. Time-varying coefficient linear regression model was also applied to traffic prediction (Rice and Van, 2004), where the difference of departure time is used to get the weighting function.

The relationship between dwell times and number of alighting and boarding passengers at bus bays was established by using linear regression approaches through a survey study, however, the relevant low coefficient of determination and high root mean square error were acquired by least square method. Then a probabilistic approach was developed that could consider the interactions among buses, arriving passengers and traffic in estimation of bus dwell time, which were not estimated by the linear regression approaches (Meng and Qu, 2013).

The raw traffic data is usually not well distributed among each section; for instance, the traffic counting data of a suburban road with light traffic are mainly small values. In such case, if LSM is directly employed, then the small values will be dominated by these small values, thus giving rise to inaccurate results. To cope with this issue, Qu et al. (2015) proposed a weighted least square method (WLSM) to fit the empirical data for six well-known single-regime models. Based on large sample tests, the WLSM performs excellently both in light-traffic conditions and congested conditions.

The concept of WTB proposed in this paper is similar to the economics term, willingness to pay (WTP). Willingness to pay (WTP) is the maximum amount of money that an individual would pay to obtain a desired good or service. Consumer preference/welfare theory was further developed quantitatively by Rosen (1974) with the calculation of willingness to pay. Shin (2015) carried out a detailed analysis of consumer preference for alternative fuel types and technology options. Jou and Chen (2015) determined the compensation caused by traffic accidents of different types based on willingness to pay of parties to traffic accidents for loss of productivity and consolation. Schniederjans (2014) suggested consumer attitude and peer pressure were positively associated with intention which was positively relevant with willingness to pay for green transportation. Moreover, Lera-López (2014) indicated that younger, better educated and more environmentally-aware citizens are more willing to pay to reduce noise and air pollution. Lanzini (2015) investigated key-determinants of drivers' willingness to pay for biofuels in Northern Italy. Gupta (2016) showed that the analysis of Indian willingness to pay for effective implementation of carbon tax in road passenger transport from three metropolitan cities. However, these studies have not provided any clear methodology to address the heterogeneity of the travelers' willingness to pay on a particular transport system, which is addressed in this paper.

To sum up, the contribution of this paper is to (a) provide an in-depth discussion of WTB with a clear conceptual framework, and (b) to propose a methodology for the quantitative measurement of WTB using real survey data. It should be pointed out that although this paper focuses on the queuing phenomenon of transit passengers, the proposed concept and methodology have much wider applications to other queue related topics in transport and traffic engineering studies. Most of the existing studies on the queuing problems/systems treat each individual as homogeneous, while WTB considers the heterogeneity of the queuing ones in terms of their location. For example, at a non-signalized intersection, the gap acceptance of queuing/waiting vehicles on the minor roads are heterogeneous; the first vehicle uses longer critical gaps but the queuing vehicles frequently take the risk to accept shorter critical gaps. The proposed methodology is suitable for and can easily be adjusted to address the heterogeneity of these queue-related problems.

2 Problem Descriptions

The following notations are adopted in this paper:

B	Total number of new boarding passengers. It is used to represent a general B^i.
$\hat{B}$	The maximum number of boarding passengers, i.e., the largest B^i.
B^i	Total number of new boarding passengers of the data record i.
$C(k)$	Willingness to board of the k th passenger in the queue.
i	ID of a collected data record.
k	The passenger in the k th position in the queue.
K	Total number of waiting passengers. It is used to represent a general K^i.
$\hat{K}$	The max value of K^i in the collected data.
K^i	Total number of waiting passengers of the data record i.
m_j	Number of data records with j passengers boarding.
n	Total number of collected data records.
N	Data set of collected data that neither nobody nor all the queuing passengers will board.
N_0	Data set of collected data that nobody will board at the stop.

N_K Data set of collected data that all the queuing passengers will board.

x Number of passengers in the vehicle that do not alight at the stop when the vehicle arrives. It is used to represent a general x^i.

x^i Number of passengers in the vehicle that do not alight at the stop of the data record i.

ϖ_i Weight of the data record i for calibration.

2.1 Willingness to board

This idea is developed based on the existence of the passengers' willingness (desire/anxiety) to board, abbreviated as WTB.

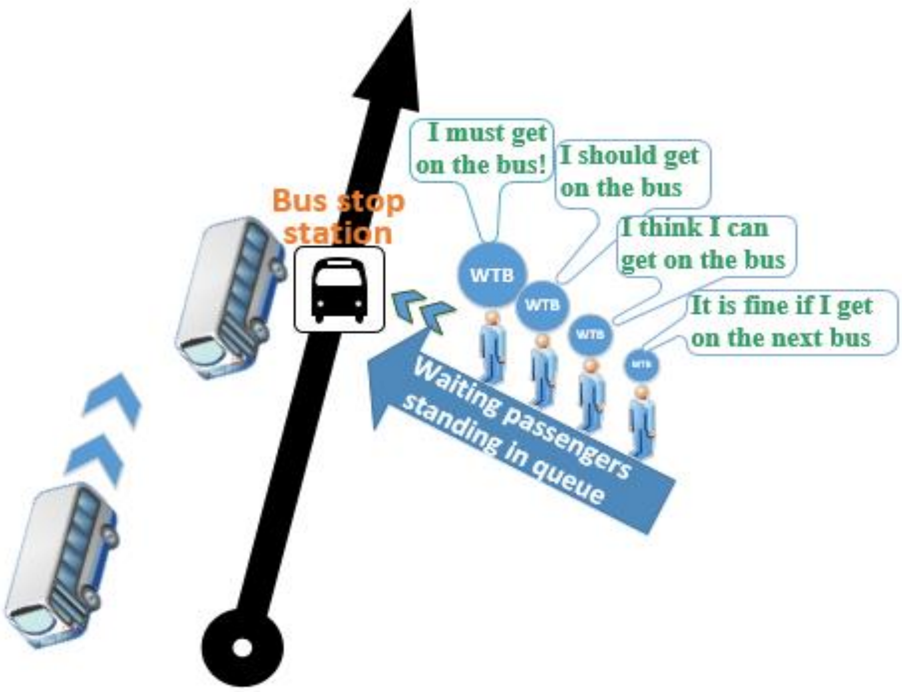

Figure 1. Illustration of the level of passengers' WTB

From the example in Figure 1, we take a very representative case: a busy transit line running from suburbs to the city in the morning peak, with high demand of passengers heading to the city. At a suburban stop S, the passengers are queuing to board the vehicle. One interesting phenomenon is usually observed: at such a bus stop there are totally K persons waiting and only $B \le K$ of them successfully get on the vehicle. Then, when $B < K$, it means someone(s) at Stop S fails to board; but, in such case, when the vehicle arrives at Stop S+1, assuming no one gets off the vehicle, some passengers could still get on board.

This phenomenon is because of one psychological effect: if the *first* passenger waiting at Stop S+1 cannot board the vehicle, it would give him/her the impression that he/she would never get on any following vehicles. So, this passenger will have much larger motivation to board the vehicle. What we observe is, even if the vehicle is already very crowded, the passenger still gets on it.

The above example shows the WTB of passengers in different positions of a queue is different. Hence, it is of considerable interests and importance to quantitatively estimate the values of different passengers' WTB.

At any stop S, the waiting passengers are numbered by their sequence in the queue, and the total number of waiting passengers is K. We use $C(k)$ to denote the WTB of passenger k ($1 \le k \le K$) waiting at a stop. To simply the initial analysis of such a new topic, it is assumed that $C(k)$ is the same for all bus stops. Note that in the same queue, it is reasonable to assume that the front waiting passengers have larger WTB than the ones at the back, namely,

Assumption 1. $C(k) \ge C(k+1), 1 \le k \le K-1$.

2.2 Boarding the vehicle

When the queuing passenger k at a stop is about to board the vehicle (all the $k-1$

predecessors already got on board), whether he/she can successfully board the vehicle is determined by not only $C(k)$, but also the current available space in the vehicle. With given capacity, the existing space in the vehicle can be reflected by the number of passengers in the vehicle. Note that the "capacity" here refers to the maximum number of passengers that the bus could carry, instead of the number of seats. Let x denote the number of passengers in the vehicle that do not alight at the stop when the vehicle arrives. Then, when passenger k is trying to board the vehicle, the total number of passengers already in the vehicle is $x+k-1$. We thus have the following assumption:

Assumption 2. If $C(k) \geq x+k-1$, the queuing passenger k will board the vehicle; if $C(k) < x+k-1$, he/she will not board the vehicle.

The existence of Assumption 2 in practice is obvious, which is reflected by the common phenomenon that not all the queuing passengers can board an arriving bus/train. The number of aboard passengers in the vehicles is a good measure of its capability to further accommodate any passengers. Note that as the number of passengers is an integer value, we can model $C(k)$ as an integer. In this case, Assumption 2 implies that the queuing passenger k will not board the vehicle if and only if $C(k) \leq x+k-2$.

Based on Assumption 2, we know that at a stop, given x passengers in the vehicle that do not alight at the stop when the vehicle arrives and given K passengers waiting, the total number of new boarding passengers, denoted by B, equals

$$B = \begin{cases} 0, \text{ if } C(1) < x \\ \max\left\{k = 1, 2 \cdots K \mid C(k) \geq x+k-1\right\}, \text{ otherwise} \end{cases} \quad (1)$$

Hence, at any stop, we can observe three values, which are x, B and K. We ignore the stops with no waiting passengers, which are $K=0$. Suppose that we have collected n data records. Then, each observed record should follow any of the three possible cases:

Case 1. $B=0$, it implies that $C(1)<x$, which is $C(1)\le x-1$ assuming $C(1)$ takes an integer value, and nobody will board at the stop. All these data records are grouped in set N_0.

Case 2. $B=K$, we should know that $C(K)\ge x+K-1$ and all the queuing passengers will board. All these data records are collected in set N_K.

Case 3. The data in set N with $0<B<K$, it means that

$$C(B)\ge x+B-1 \tag{2}$$

and

$$C(B+1)<x+B \tag{3}$$

If we model $C(B+1)$ as an integer, Eq. (3) is equivalent to:

$$C(B+1)\le x+B-1 \tag{4}$$

We have $|N_0|+|N_K|+|N|=n$. A data record $i=1,2\cdots n$ has the following information: the number of passengers in the vehicle that do not alight at the stop when the vehicle arrives, denoted by x^i; the number of passengers waiting denoted by K^i; the number of passengers who successfully board the bus, denoted by B^i.

3 Quantitative analysis of WTB $C(k)$

3.1 A general model of $C(k)$

If the specific value of $C(k)$ can be obtained, then the WTB of passengers at each stop could be predetermined, and hence the total number of boarding passengers can be predicted. This can subsequently be used to estimate the dwell time of a transit vehicle before it approaches the stop, which is of considerable significance for the transit operation controls as well as transit traveler information systems. Hence, in this section, we discuss about the calibration of $C(k)$ using survey data.

From the discussions in Section 2, in this section we assume that $C(k)$ is a monotonically decreasing integer-valued function of k. We first provide a general form of $C(k)$. Define $\hat{K} := \max\{K^i, i = 1, 2 \cdots n\}$, then, the general regression model is to calibrate using real survey data the following parameters:

$$C(k), 1 \le k \le \hat{K} \tag{5}$$

A new calibration method is developed in the following sub-section.

3.2 A New Least Square Method

The least square method (LSM) is the most well-known calibration method. However, LSM could not be directly employed, as the addressed problem is not a simple linear or nonlinear regression problem. Specifically, the WTB problem is different from conventional calibration problems. In conventional calibration problems, given an input or given inputs for multiple input values, there is an output that can be observed. However, in the WTB problem, given the inputs of x and K, we cannot observe the WTB of any passenger; all we can observe is that the WTB of a passenger is larger than or smaller than a particular value. In this sense, the information available in the WTB problem is less than that in conventional calibration problems. Following the concept of minimizing the square of errors, we developed an innovative regression method in this sub-section.

Based on the collected data x^i, K^i, B^i, $i = 1, 2 \cdots n$, the sum of square of errors is defined as:

$$\begin{aligned} \mathrm{Z}_1 = & \sum_{i \in N_0} \left[\max\left(C(1) - (x^i - 1), 0 \right) \right]^2 + \sum_{i \in N_K} \left[\min\left(C(K^i) - (x^i + K^i - 1), 0 \right) \right]^2 \\ & + \sum_{i \in N} \left\{ \left[\min\left(C(B^i) - (x^i + B^i - 1), 0 \right) \right]^2 + \left[\max\left(C(B^i + 1) - (x^i + B^i - 1), 0 \right) \right]^2 \right\} \end{aligned} \tag{6}$$

Take the first term as an example. If nobody boards the transit vehicle, then we have $C(1) \le x^i - 1$. Therefore, if $C(1) - (x^i - 1)$ is non-positive, then there is no error (i.e., no inconsistency) between the observed information and the estimation. Otherwise, we minimize the sum of square errors between the observed information and the estimated WTB. Then, the new least square method aims to minimize Z_1 and solve the following model [M1].

[M1]

$$\min Z_1 \tag{7}$$

S.T.

$$C(k) \text{ are nonnegative integers}, 1 \le k \le \hat{K} \tag{8}$$

$$C(k) \ge C(k+1), 1 \le k \le \hat{K} - 1 \tag{9}$$

It can be seen that the objective function is non-differentiable but convex. We have the following Theorem 1.

Theorem 1: Define the following three sets of decision variables: $C(k)$, $1 \le k \le \hat{K}$; $u^i, i \in N_0 \cup N$; $v^i, i \in N_K \cup N$. Then, [M1] is equivalent to the following convex quadratic mixed-integer program [M1']:

[M1']

$$\min Z_2 = \sum_{i \in N_0} (u^i)^2 + \sum_{i \in N_K} (v^i)^2 + \sum_{i \in N} [(v^i)^2 + (u^i)^2] \tag{10}$$

S.T.

$$C(k) \text{ are nonnegative integers, } 1 \le k \le \hat{K} \quad (11)$$

$$C(k) \ge C(k+1), 1 \le k \le \hat{K} - 1 \quad (12)$$

$$u^i \ge C(1) - (x^i - 1), u^i \ge 0, i \in N_0 \quad (13)$$

$$v^i \le C(K^i) - (x^i + K^i - 1), v^i \le 0, i \in N_K \quad (14)$$

$$v^i \le C(B^i) - (x^i + B^i - 1), v^i \le 0, i \in N \quad (15)$$

$$u^i \ge C(B^i + 1) - (x^i + B^i - 1), u^i \ge 0, i \in N \quad (16)$$

The model [M1’] is evidently a convex quadratic mixed-integer program, which can be conveniently solved by some off-the-shelf solvers, such as CPLEX.

4 Parametric models and calibration

4.1 Function form of $C(k)$

We proceed to provide some interesting discussions of the form of $C(k)$ and its calibration. In Section 3, $C(k)$ appears as a general form with no parameter but independent values. In this section, we assume that $C(k)$ has a specific function form and we also assume that $C(k)$ can be a fractional quantity. The purpose of this analysis is twofold. First, if we know a function form of $C(k)$, then we can estimate the WTB of any passenger even if our data records do not contain information of some passengers, e.g., the 200th passenger in the queue. Second, a (continuously differentiable) function form is easier to be incorporated into an optimization model for public transit planning than the discrete values in Section 3.

From Assumption 1, we also know that $C(k)$ is a monotone function of k. Hence, the following simple form can be provided for $C(k)$, which is a power function plus a constant:

$$C(k) = a + \frac{b}{k^c} \tag{17}$$

where a, b and c are parameters, $b > 0$ and $c > 0$. With the given function form in Eq. (17), the problem now becomes to calibrate the parameters a, b and c in the function.

4.2 A Least Square Method for the parametric models

The sum of square of errors in terms of Eq. (17) is defined as:

$$\begin{aligned} Z_3 = &\sum_{i \in N_0} \left[\max\left(C(1) - (x^i - 1), 0 \right) \right]^2 + \sum_{i \in N_K} \left[\min\left(C(K^i) - (x^i + K^i - 1), 0 \right) \right]^2 \\ &+ \sum_{i \in N} \left\{ \left[\min\left(C(B^i) - (x^i + B^i - 1), 0 \right) \right]^2 + \left[\max\left(C(B^i + 1) - (x^i + B^i - 1), 0 \right) \right]^2 \right\} \end{aligned} \tag{18}$$

Hence, the least square method shall solve the following model [M2]

[M2]

$$\min Z_3 \tag{19}$$

S.T.

$$b > 0; c > 0 \tag{20}$$

$$C(k) = a + \frac{b}{k^c} \tag{21}$$

We further define an equivalent objective function Z_4 as follows:

$$\begin{aligned} Z_4 = &\sum_{i \in N_0} \left[\max\left(a + \frac{b}{1^c} - (x^i - 1), 0 \right) \right]^2 + \sum_{i \in N_K} \left[\min\left(a + \frac{b}{(K^i)^c} - (x^i + K^i - 1), 0 \right) \right]^2 \\ &+ \sum_{i \in N} \left\{ \left[\min\left(a + \frac{b}{(B^i)^c} - (x^i + B^i - 1), 0 \right) \right]^2 + \left[\max\left(a + \frac{b}{(B^i + 1)^c} - (x^i + B^i - 1), 0 \right) \right]^2 \right\} \end{aligned}$$

The model [M2] is then equivalently transformed to the following model [M2']:

[M2']

$$\min Z_4 \tag{22}$$

S.T.

$$b > 0; c > 0 \tag{23}$$

The function Z_4 is not convex. To see this point, we can look at part of its second term $a + b(K^i)^{-c} - (x^i + K^i - 1)$. The Hessian of this term is

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -(\ln K^i)(K^i)^{-c} \\ 0 & -(\ln K^i)(K^i)^{-c} & b(\ln K^i)^2(K^i)^{-c} \end{bmatrix} \tag{24}$$

The determinant of the principal minor $\begin{bmatrix} 0 & -(\ln K^i)(K^i)^{-c} \\ -(\ln K^i)(K^i)^{-c} & b(\ln K^i)^2(K^i)^{-c} \end{bmatrix}$ is negative, and therefore the Hessian is not positive semi-definite.

Although [M2'] is a non-convex optimization problem, it has only three variables. We therefore can use the nonlinear optimization tools of MATLAB with different starting points to obtain a near-optimal solution.

4.3 Weighted Least Square Method for calibration

The LSM simply provides the best fitted function for *all the* collected raw data. However, the data collection process may be biased unintentionally. For instance, the expressway traffic flow data collected by detectors are dominated by the free-flow conditions/data. Hence, if LSM is directly used, it may give rise to a biased model that does not accurately depict the relationship between dependent and independent variables. The collected data for calibrating the WTB may also biased. For instance, if we have 100 data records, then it cannot be the case that exactly 10 records have no boarding passenger, exactly 10 records have one boarding passenger, etc. By contrast, it is possible, for example, that 5 records have no boarding passenger, 23 records have one boarding passenger, etc.

In this regard, a weighted least square method (WLSM) was proposed by Qu et al. (2015) to cope with the sample selection bias problem. The basic logic of WLSM is to add a reasonable weight ϖ_i for each data record i. Following the same logic as Qu et al. (2015), we further developed a WLSM for the calibration of WTB. For the general case (Section 3.2) and parametric case (Section 4.2), the WLSM model can be obtained by simply replacing the objective functions Z_2 and Z_4, by the following $\bar{Z}_2$ and $\bar{Z}_4$, respectively:

$$\bar{Z}_2 = \sum_{i\in N_0} \varpi_i (u^i)^2 + \sum_{i\in N_K} \varpi_i (v^i)^2 + \sum_{i\in N} \varpi_i [(v^i)^2 + (u^i)^2]$$

$$\begin{aligned}\bar{Z}_4 = &\sum_{i\in N_0} \varpi_i \left[\max\left(a + \frac{b}{1^c} - (x^i - 1), 0\right)\right]^2 + \sum_{i\in N_K} \varpi_i \left[\min\left(a + \frac{b}{(K^i)^c} - (x^i + K^i - 1), 0\right)\right]^2 \\ &+ \sum_{i\in N} \varpi_i \left\{\left[\min\left(a + \frac{b}{(B^i)^c} - (x^i + B^i - 1), 0\right)\right]^2 + \left[\max\left(a + \frac{b}{(B^i+1)^c} - (x^i + B^i - 1), 0\right)\right]^2\right\}\end{aligned}$$

Regarding the weight ϖ_i, we propose the following weight determination method enlightened by the idea of Qu et al. (2015). Define $\hat{B}$ as the maximum number of boarding passengers, $\hat{B} := \max\{B^i, i \in N_0 \cup N_K \cup N\}$. Define m_j as the number of data records satisfying $B^i = j$, $j = 0,1,2\cdots\hat{B}$. If all m_j are positive, then the weight can be obtained by:

$$\varpi_i = \frac{1}{m_{x^i}}, i \in N_0 \cup N_K \cup N \quad (25)$$

The above equation means, if for example there are many data records with no boarding passengers, then the weight of these data records should be small.

If some m_j are 0, i.e., we do not have any data record of particular numbers of boarding passengers, and then to determine the weight, we define j^+ and j^- as follows:

$$j^{+} := \underset{j'=j+1,j+2\cdots\hat{B}}{\arg\min}\{m_{j'} \geq 1\},\ j = 0,1,2\cdots\hat{B}-1 \tag{26}$$

$$j^{-} := \underset{j'=0,1\cdots j-1}{\arg\max}\{m_{j'} \geq 1\},\ j = 1,2\cdots\hat{B} \tag{27}$$

For instance, if in our data there are only records of no boarding passenger, 2 boarding passengers, 3 boarding passengers, and 10 boarding passengers, then $0^{+} = 2, 2^{+} = 3, 3^{+} = 10$ and $2^{-} = 0, 3^{-} = 2, 10^{-} = 3$. Since very few records are between 3 and 10, the weight of data records with 3 or 10 boarding passengers should be larger. Mathematically, the weight can be obtained by:

$$\varpi_i = \begin{cases} \dfrac{1}{m_0}(0^{+} - 0), B^i = 0 \\ \dfrac{1}{m_{x^i}} \dfrac{(B^i)^{+} - (B^i)^{-}}{2}, B^i = 1,2,\cdots,\hat{B}-1, \\ \dfrac{1}{m_{\hat{B}}}(\hat{B} - \hat{B}^{-}), B^i = \hat{B} \end{cases} \qquad i = 1,2\cdots n \tag{28}$$

Note that in the case of Eq. (25), $j^{+} = j+1$ and $j^{-} = j-1$. Therefore, Eq. (25) is a special case of Eq. (28).

5 Calibration using field data

Real survey data is collected to verify the proposed models and methods and we report the results in this section.

5.1 Data collection

The bus line Express Route 2 of Suzhou City in China is taken as the target bus line. It connects Southeast University Suzhou Campus (where the authors are based) and the Suzhou Railway Station in the city, as shown in Figure 2. This bus line has 20 stops, 18 of which are in the suburban area. It is a typical suburban line, with most of the passengers travelling to the city. The bus line has a fleet size of 17 vehicles and is operated at a frequency of 10 vehicles/hour at peak period (a headway of 6 mins) and 6 vehicles/hour at other time (a headway of 10 mins). All the buses have the same vehicle size (12m*2.5m) and capacity. In each vehicle, there are 31 passenger seats, and in a crowded case it can totally accommodate about 85 passengers. Note that evidently all waiting passengers' WTB is larger than 31; therefore, we only consider standing passengers in the calibration when we report the WTB. In the morning peak, the vehicles become very crowded after the first five to eight stops. Therefore, this bus line is quite suitable for the analysis of WTB.

Figure 2. Itinerary of the Express Route 2 in Suzhou

The data of trips to the city direction are collected. We collected data for six bus trips from Southeast University Suzhou Campus to Suzhou Railway Station during morning peak hours (7:30 to 9:30am). After removing the data in which nobody waits, we obtain a total of 74 valid observations.

Model [M1'] is solved by CPLEX 12.2 coded in MATLAB on a 3.5 GHz Four Core laptop with 4 GB of RAM. Model [M2'] is solved by the constrained minimizing function of MATLAB. Both models can be efficiently solved.

5.2 Calibration results

5.2.1 Results for the general model

We first use the general model proposed in Section 3.1. The results are shown in Table 1 and Figure 3. The results also provide a comparison of LSM and WLSM.

Table 1. Calibrated General Model $C(k)$ of the WTB

Queue Sequence k	1	2 to 4	5 to 13	14 to 26	27 to 32
LSM	58	53	52	43	33
WLSM	53	53	53	48	33

The results in Table 1 and Figure 3 show that the $C(k)$ is monotone (non-increasing) with regards to k. With the change of queuing sequence k, there is an obvious gap on the value of $C(k)$; for instance, from the results obtained by LSM, the $C(k)$ of the second queuing passenger is 53, which is 8.7% less than that of the first queuing passenger. The calibration results clearly show the existence of WTB in practice.

Regarding the comparison of LSM and WLSM, we can see a clear difference between these two methods from Figure 3, especially when k is 1 or between 14 and 26. This difference exists because the numbers of boarding passengers in the survey data records are not uniformly distributed. Note that the WTB estimates for many passengers are the same. This is because of the limited quantity of data. If large survey data can be used in the future, more accurate calibration results can be obtained.

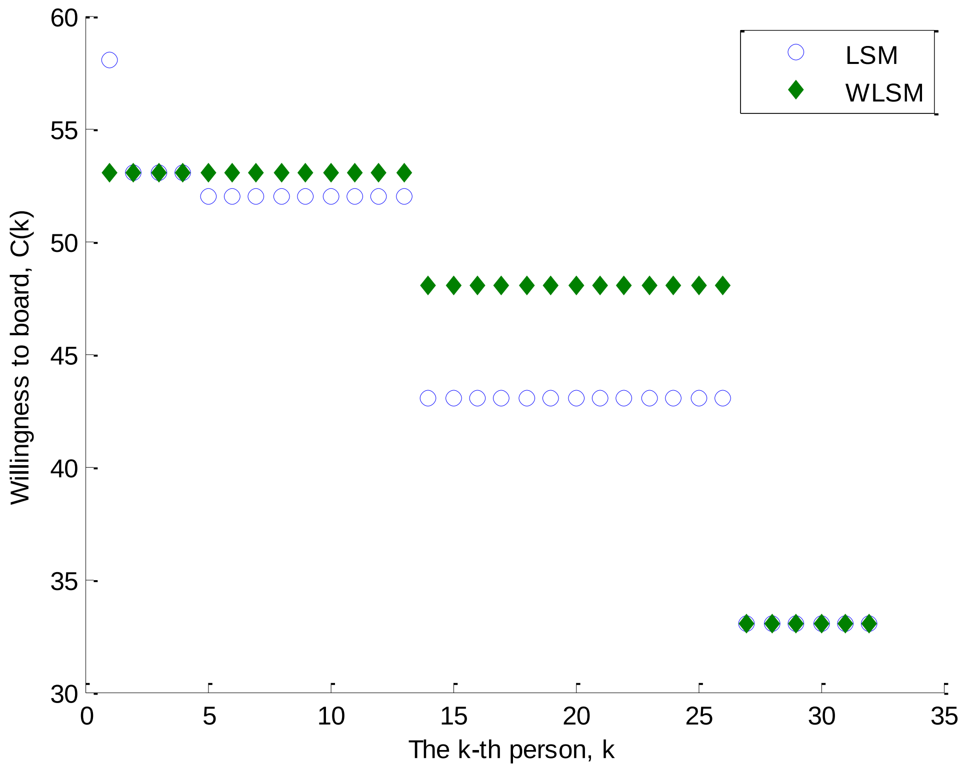

Figure 3. Calibration Results of the WTB General Model

5.2.2 Results for the parametric models

Using the same set of survey data, we then calibrate the parametric models proposed in Section 4.1. Assuming that $C(k) = a + \frac{b}{k^c}$, the LSM and WLSM are also used here. Table 2 shows the calibrated parameters and Figure 4 indicates the shape of the optimal model. The optimal models obtained by the two methods are both sharply decreasing functions, which means that the queuing sequence will largely affect their boarding behavior. Hence, the parametric model also supports the existence of WTB.

Table 2. Calibrated Parametric Model the WTB

Queue Sequence k	a	b	c
LSM	−299.994	356.412	0.010
WLSM	−300.000	353.251	0.007

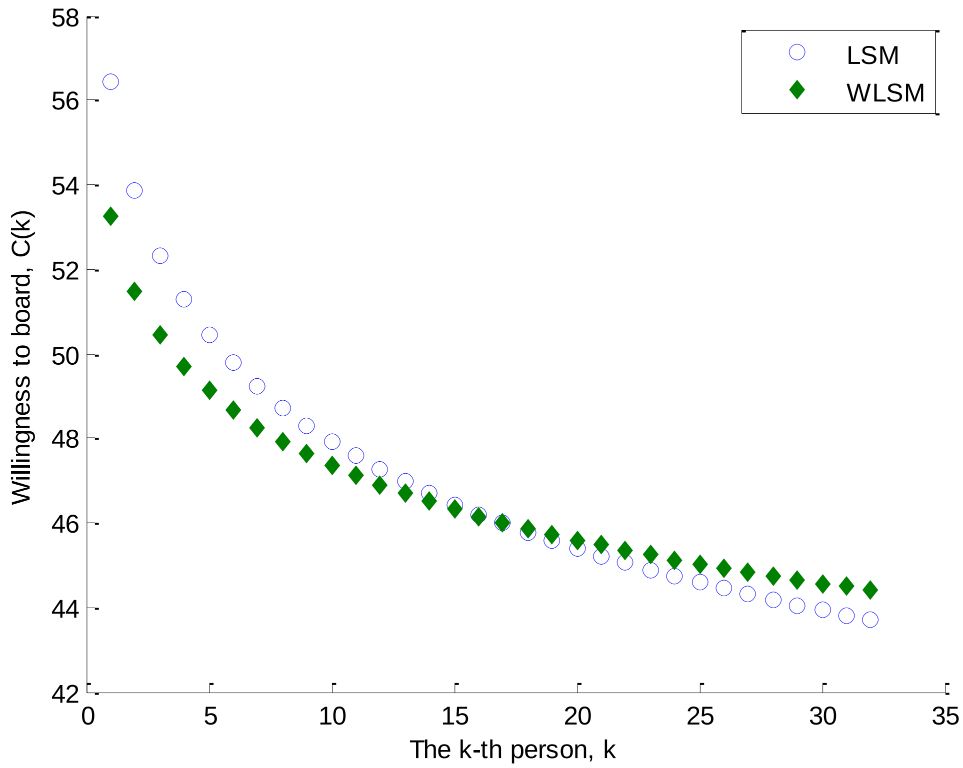

Figure 4. Calibration Results of the WTB Parametric Model

Similar to the general case, the LSM and WLSM methods also give quite different results. The gap is more evident when k takes very small values or very large values. From Figure 4, it can be seen that when k is smaller, WTB has a more sharply change in terms of k, i.e., larger slope. This property is reasonable; for instance, the different between the 9th and 10th passengers should be smaller than that between the 1st and 2nd ones.

5.3 Discussions

It is easy to see that the calibrated WTB values are random as the observed data records are random. To see the dispersion of the results, it is not difficult to estimate the variance of the calibrated parameters in a simple linear regression model. However, our models are highly nonlinear and there is no available approach to estimate the variance of the calibrated WTB values. To address this problem, we take a numerical approach. More data are collected for another seven bus trips from Southeast University Suzhou Campus to Suzhou Railway Station during morning peak hours (7:30 to 9:30am). Combining with the previous six bus trip, we obtain a total of 150 valid observations. We take 20 samples from the 150 valid observations, each with a size of 120 observations. Then, we use the LSM to calibrate the parameter model using each sample. As a result, we obtain 20 estimates of the WTB value for each passenger k. A box and whisker diagram is drawn in Figure 5, which gives a rough idea of the dispersion of the WTB values. Moreover, we can observe that the WTB values decrease quickly for the first few passengers, but the rate of decrease is dramatically reduced when k is large.

During the field survey, we observed that passengers' boarding behaviors are also affected by some side factors, for instance, body size of each passenger, their carrying luggage. The driver at occasions may encourage or discourage the passengers to further get on board, which is an important psychological factor for the passengers to make decisions. Also, in the previous studies of public transport, the influences of lateness penalty, crowdedness effects on passengers' boarding behaviors are also discussed. In this paper, the passengers' queuing sequence is taken as the only influencing factor of WTB. The current study is a complement rather than substitute of previous studies on other factors. However, compared with the queuing sequence, it is difficult to accurately collect the data of these other factors. If other factors are taken into account, their influence on the WTB should be taken as a random variable, namely, the value B^i in our model should be a random variable.

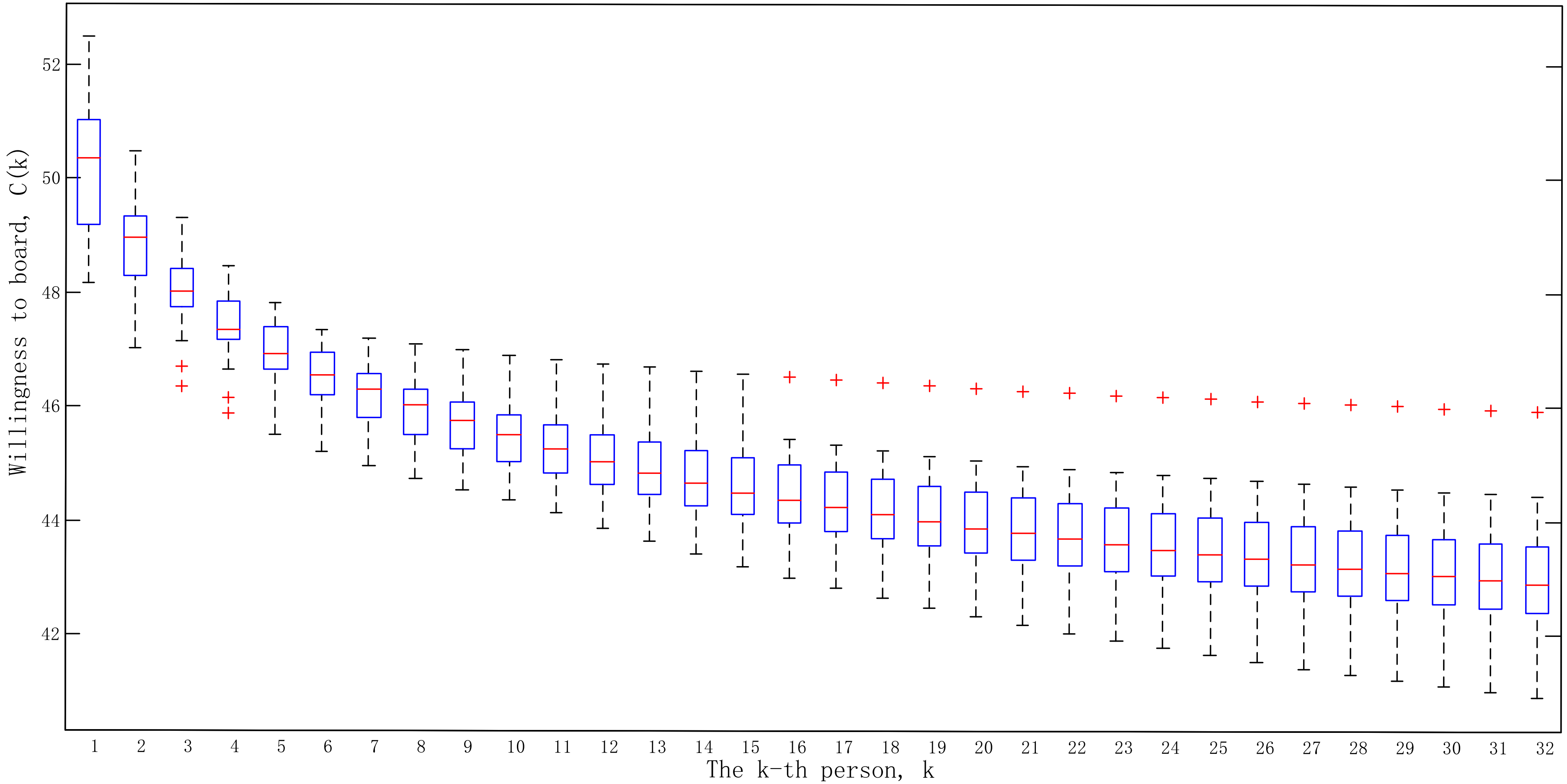

Figure 5. Box and Whisker Diagram of the WTB Using LSM for the Parameter Model

Besides the phenomena observed during the survey, we also think about another endogenous factor that may affect the boarding actions, which is the queuing passengers' value of time (VOT). The VOT does not coincide with the queuing location. Thus, those who have higher VOT would be more ambitious to board. In the morning peak, the effects of VOT are more obvious as the passengers are more urgent to go to their work place. However, VOT is a very diverse value among different commuters, and each commuter does not accurately know his VOT value. Hence, it is very challenging to consider VOT in our survey or data analysis, which will make the addressed topic too complicated to handle. VOT is thus omitted in our analysis.

6 Applications

As to such a new concept for the travel behaviors and travel psychology, it is of considerable significance to discuss about the practical value of WTB. In general, WTB provides a quantitative measure of a bus vehicle's maximum capacity, which helps to give more accurate predictions/estimations of number of boarding passengers at each stop. Hence, it brings convenience to both passengers and bus operators. In this section, we present some initial discussions of the WTB's practical applications.

6.1 A three-bus-stop case study

We consider a toy example. A bus route visits three bus stops S1, S2, and S3. Assume that all passengers waiting at S1 alight at S3. Suppose that the number of seats in the bus is 10 (we assume such a small number because otherwise the figure will be hard to read). Then, using the WTB calibrated by the LSM in Section 5.2.2, we can see that at S1 at most 52 passengers can board, because the 52nd passenger's WTB is greater than 41 (there are 10 passengers who sit rather than stand) and the 53rd passenger's WTB is smaller than 42. When the bus arrives at S2, a maximum of another 7 passengers can board, because the 7th passenger's WTB is greater than 48 (42+7−1) and the 8th passenger's WTB is smaller than 49. As a result, if we use the conventional model with fixed bus capacity, then we will conclude that the bus capacity is 52 if we count at S1, and conclude the capacity is 59 if we count at S2. In both models (whether we count at S1 or S2), the total number of passengers boarding at S1 and S2 cannot exceed the bus capacity. The area enclosed between the green dotted line (the red dashed line, respectively) and the two axes in Figure 6 shows the feasible numbers of boarding passengers at the two bus stops when the bus capacity is 52 (59, respectively).

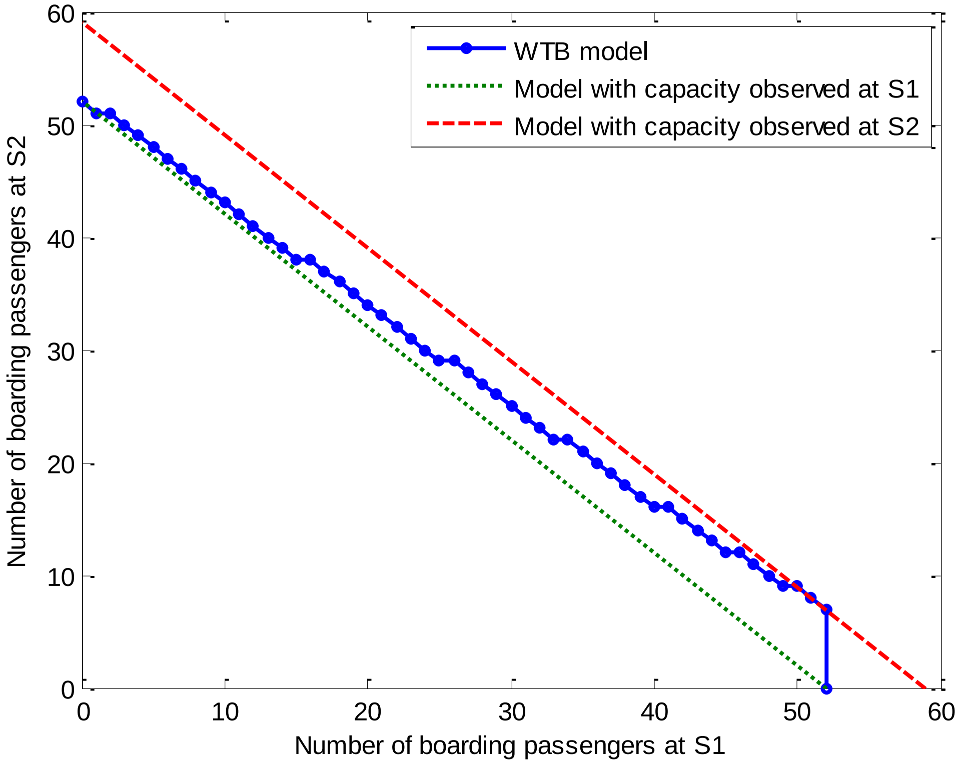

Figure 6. Comparison of the WTB Model and Conventional Bus Capacity Models

However, when we use the WTB model, we can see the area of feasible numbers of boarding passengers lies between the above-mentioned two areas. In particular, when 52 passengers board at S1, it is possible that 0 to 7 passengers board at S2, which is shown by the vertical blue continuous line segment in Figure 6. One end point (52, 0) is on the line with capacity 52, and the other end point (52, 7) is on the line with capacity 59. When nobody boards at S1, then at most 52 passengers can board at S2, and the point (0, 52) is on the line with capacity 52. The general trend of the curve of the WTB model is the same as the fixed capacity models: when one more passenger boards at S1, the number of passengers who can board at S2 will generally be reduced by 1. However, there are several “flat steps” on the curve of the WTB model. Take the left-most step as an example: if one passenger boards at S1, at most 51 can board at S2; however, if two passengers board at S1, still at most 51 can board at S2. This characteristic cannot be captured by fixed capacity models. In sum, in the WTB model, when more passengers board at S1, the maximum number of passengers who can board at S2 decreases monotonically, but may not decrease strictly monotonically.

It should be noted the data in this case study are collected at the morning peaks of 10 different working days. Hence, this case study only captures the WTB of morning peaks. However, the passengers’ WTB may vary at different time in the day due to the different circumstances (e.g., different crowd level, different temperature, etc.); also the WTB of home-work trip in the morning would be larger than the work-home trip in the evening. Hence, it is more suitable to separately calibrate the WTB of different time within the day.

6.2 Other applications

Some other possible impacts of WTB are briefly discussed here. First, with known passenger demand at each stop (can be observed by the detection cameras), WTB helps to accurately make decisions on some bus operation strategies; for instance, for bus stop-skipping problems (Liu et al. 2013), if the number of in-vehicle passengers is larger than $C(1)$ and no one intends to alight at the forthcoming stop, the driver could directly skip this stop. With the advanced information system equipped at the bus stops and vehicles, it would not be difficult for the transit operators to handle such sort of operation incidents.

Second, it enables a more precise traveler information system. At each stop with variable message board, besides indicating the arrival time of forthcoming buses, the system can also indicate the maximum number of boarding passengers at this stop. It consequently helps the waiting passenger to get a more accurate idea of whether they could board the coming vehicle and their waiting time. With such sort of detailed information, the passengers could wisely make their route plans. If they know that it is impossible for them to board the next vehicle, some of them may directly change to other travel modes.

Third, the detailed traveler information could also be put online, such that the potential passengers could check the remaining capacity of the coming bus. If the estimated number of possible boarding passenger at each stop is very small during a peak hour, then these passengers could stay at home instead of going to the transit stop and waste longer time on waiting.

Fourth, as mentioned, WTB provides a more accurate measure of the real capacity of each bus, which helps the bus operators to make more accurate decisions. For instance, at some dense cities with very large demand, additional vehicles are usually deployed to serve certain bus lines. With estimated demand pattern, WTB then helps the operators to accurately decide how many vehicles are needed in different periods.

7 Conclusions

This paper addressed a new concept of queuing up passengers, focusing on their willingness to board the transit vehicles. It derives from some interesting observations from real life that passengers at different queuing locations have different level of ambitions to get on board. Such a phenomenon has impacts on both queuing passengers and bus operators, which needs an in-depth quantitative analysis. Therefore, the first major contribution of the research is to identify and analyze such a new topic. It should be stressed that there are very few studies examining the psychology in transportation, except drivers' driving behaviors.

The second major contribution lies in the technical aspect. This paper developed two calibration models: a general model and a parametric model. As a result of the nature of the WTB problem, we cannot directly observe the WTB of a passenger, which prevents us from directly using the LSM. To overcome this difficulty, an innovative calibration method, inspired by the LSM, was proposed to calibrate these two models, using real survey data. Moreover, the calibration of the general model is proved to be a convex optimization problem. The proposed calibration method can also be used for other transportation problems. For example, one may use regression models to calibrate the relations between road/traffic factors and road accidents. If some road accidents are not reported, then the real number of accidents will be larger than or equal to the number that has been reported. Our proposed calibration method could thus be used in such a context. To further refine the new calibration method, a WLSM was also proposed, which copes with the biasness of the survey data. From the numerical test, it clearly showed the existence of WTB among the queuing passengers.

For the practicality of the proposed research, this paper further discussed about the applications of the qualitative analysis of WTB, which could be used to accurately estimate the real capacity of transit vehicles and also improve the service of transit information systems. Future efforts are needed to further get large scale survey data for this topic, which can help to filter the outliers in the data and thus enhance the accuracy of calibration results.

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