A game theoretical approach for modeling competition and cooperation strategy in a maritime supply chain

Abstract

To accommodate the structural changes in logistics, competitions and cooperation along a maritime supply chain are more intensive than before. Firstly, a number of shipping line companies have been vertically integrated with port operations through shareholdings in or full ownership of dedicated terminal facilities. Secondly, a number of shipping lines have set up strategic alliance to gain competitiveness over the competitors. Thirdly, inter-port competition is of important concerns in the maritime studies and port charge plays a crucial role in capturing the shipping lines' call and improving the port competitiveness. This paper aims to examine the cooperation and competitions among the major players along a maritime supply chain, including port authorities and shipping line companies, by applying a two-stage non-cooperative game theoretical approach. In the first stage, Bertrand game is applied to model the interactions between two shipping line companies. When the shipping line decides which port to call, a multinomial Logit model applied. In the second stage, Bertrand game is applied to model the competitions between two ports. Nash equilibrium is derived by solving the Bertrand games. A numerical example is provided as the case study**NotteboomandYap**

Keywords: Game theory, Bertrand game, equilibrium, core, competition and cooperation.

word counts = 6000 words + 3 Figures + 11 Tables

1. Introduction

Maritime freight industry has developed significantly with the expansion of global economy. According to UNCAD based on Container Trade Statistics (2013), 52% of the goods were transported through containers, and the total throughput of the ports in the world were 572.8 million TEUs (20 feet equivalent union) in 2011. The Bureau of Transportation Statistics (2014) indicated that the world container traffic has increased from 137.2 million TEUs in 1995 to 677.5 million TEUs in 2013, showing a growth of 393%. The dramatic increases in global container trades have resulted in fundamental changes in the structures of the maritime supply chain.

To accommodate the structural changes in the maritime supply chain, competitions and cooperation among players involved are more intensive than before. The players considered here are shipping line companies and port authorities. The shipping line companies determine the service charges and delivery routes while the port authorities suggest competitive service charges and conditions to attract shipping line companies. Firstly, a number of shipping line companies have been vertically integrated with port operations through shareholdings in or full ownership of dedicated terminal facilities. For example, Maersk Sealand fully owns APM Terminals Rotterdam, in order to secure sufficient and long-term capacity of the port to keep growing. At the same time, Maersk's container volumes can be handled in Rotterdam and do not need to be moved to other ports. The vertical integration strategies of the shipping line companies towards the inclusion of land-side operations take various forms, including the direct ownership, joint ventures between shipping lines and independent port authorities, majority and minority shareholdings of shipping line companies and strategic alliances in terminal.

Secondly, a number of shipping line companies have set up strategic cooperation to gain competitiveness over the competitors. A better understanding of the process of competition, cooperation and decision-making in cooperative strategic alliances is of practical importance. Strategic alliances among the shipping line companies can take various forms, such as vessel sharing agreements or slot sharing agreements or even the full integration in the form of merger or acquisition. In recent years, the strategic alliance has been accelerated by technological, operational and economic forces to address the challenges of uncertainty, allocation of resources and the market penetration. There are certain advantages in the formation of strategic alliance, including profits maximization, cost reduction, entry in new markets, wider geographical scope, higher service level, and increase in service frequency. In the context of shipping line competition, the following questions arise: How are the equilibrium service charges determined when each shipping line company sets its charge independently? What are the market share for each shipping line company with equilibrium price? To solve those two research questions, this paper proposes a non-cooperative game model with two shipping line companies (namely, SL_1 and SL_2) competing with each other in the market.

Thirdly, inter-port competition is of important concerns in the maritime studies and port charge plays a crucial role in capturing the shipping lines' call and improving the port competitiveness. Ports such as those in East Asia compete to obtain the position of a hub port by attracting transshipment cargo and increasing throughput. To improve the competitiveness of a port, the service charge and performance play an important role in capturing container cargoes. Shipping line companies are exploring options which are more cost-saving due to the pressures from the global customers on lowering the shipping fees. For example, Maersk Sealand relocated its major transshipment operations from Singapore to Malaysia. This shift has reduced the cost of Evergreen by between US$5.7 million and US$30 million per annum **Kleywegt** Consequently, it is important for each port to determine levels of port charges and service levels from the point view of port competitions. In the context of port competition, the following questions arise: How are the equilibrium port charges determined when each port sets its charge independently and strategically, to compete for the calls of shipping lines? What are the characteristics of equilibrium port charges? To solve those two research questions, this paper proposes a non-cooperative game model with two ports (namely, $Port_1$ and $Port_2$) competing with each other in the market.

Game theory is concerned with the study of decision problems of two or more players and the prediction of outcomes of their games. Each shipping line company in the particular trade market has to consider what the port authorities and other shipping line companies will do before setting up a strategy and during the implementation process of the strategy. This is due to the reason that a decision made by one player has a direct impact on the strategies adopted by the other players within the game, and will therefore change the outcomes of the game. Consequently, the insights of the game theory approach is suitable for understanding of the non-cooperative and cooperative strategies in the maritime supply chain.

The maritime supply chain is rather complex and several different types of interactions among key players need to be considered. Shipping line companies and port authorities are considered as major players in this research. Shipping line company delivers freight via waterways and make calls on ports considering the sea transportation network and port service levels. In the first stage, each shipper has to decide which shipping line company to use. In the second stage, each shipping line company has to decide which port to call on. Port service conditions affect the routing of the shipping line company, while port throughput are influenced by the decisions of the shipping line company. Obviously, port authority suggests competitive service charges and service levels to attract more shipping line companies. Shipping line companies deliver goods through an ocean transportation network and choose the visiting ports among the alternative ones by considering the port determinants such as the port location and service level. This hierarchical decision process is captured by a two-stage game approach.

The main objective of this study is to investigate the behavior of port authorities and shipping line companies along a maritime supply chain in which both vertical and horizontal interactions exist, based on a two-stage non-cooperative game theoretical approach. Very few papers have tackled the games among 4 players in the maritime studies before.

2. Literature Review

One of the main contribution of this paper is that it applies non-cooperative game theoretical approach to the context of vertical and horizontal interactions among the ports and shipping line companies along the maritime supply chain.

With respect to port competitions, which is now one of the most important issues in transportation economics, several types of papers are found. The first type of the study is to define the conceptual framework of port competition and competitiveness. For example, **NotteboomandYap** Notteboom and Yap (2012) analyzed container port competition in various regions and found that ports compete not only with neighbors but also with other ports located in the wider region. In addition, they suggested that immediate hinterland served as good base for inter-port rivalry (Notteboom and Yap, 2012). The second type of the study is to appraise port efficiency and performance based on the empirical approach. Slack (1985) investigated the container transport between North American Midwest and Western Europe and identified 11 optional port factors affecting the decision-making of exporters and forwarders. Hayuth (1993) utilized the DEA method to appraise the European ports. Tongzon (1995) adopted a polynomial model to evaluate the port competitiveness and implemented the sensitivity analysis on the port performance index. And Tongzon (2001) further used the DEA method to compare the performances of ports from Asian, European and Oceania based on the real data. Dong (2002) examined the possible competition and cooperation of the adjacent container ports in Hong Kong and South China from a strategic perspective, mainly in the Pearl River Delta region. The third is to apply game theoretical models to study port competition. Our study is classified into this category. Imai *et al* (2006) examined the economic viability of deploying container mega vessels to obtain the optimal strategy, considering the interactions between carriers and shipping line companies in the context of a non-zero sum game with two players. Anderson *et al* (2008) adopted a two-player game to understand how each competing port would respond to the developments of the rival port. They analyzed the competition currently occurring between ports of Busan and Shanghai, to identify whether the port would be able to defend market share by building additional capacity . Saeed and Larsen (2010) discussed the competitions among three container terminals in Pakistan by using a two-stage game, where the first stage is a cooperative game and the second stage is a Bertrand game.

In order to investigate the relationship between port and shipper in decision making process, Zhang *et al* (2009) proposed two types of container transport supernetwork equilibrium models and developed a diagonlization algorithm to find the approximate solution for the numerical example. Murphy and Hall (1997) found that different participants had different concerns on all the factors of a port. Maclchow *et al* (2001 and 2004) applied a discrete model to describe the choice behavior of different types of cargo by a port through a Logit model.

Bi-level optimization was first realized in the field of game theory as Stackelberg game. An equilibrium problem with equilibrium constraints is one of the mathematical programs which often arise in engineering and economics applications (Ehrenmann, 2005). One important application of the equilibrium problem with equilibrium constraints is the multi-leader-follower game proposed by Pang and Fukushima (2009). Stackelberg game, i.e. singleleader-follower game, is a well-known formulation of the equilibrium problem with equilibrium constraints in non-cooperative game theory, and some efficient solution algorithms have been proposed (Luo *et al*, 1996). It is the first realization of bi-level programming model in the field of game theory.

Game theory is a well-known mathematical framework that describes interactions between multiple rational agents to achieve optimal payoffs. The related research could be traced back to Nash (1950). However, research related to the application of game theory to ports is very limited. Anderson *et al* (2008) developed a game-theoretic best response framework for understanding how competing ports would respond to development at a focus port and whether the focus port would be able to capture or defend market share by building additional capacity. The model was applied to two competing ports, i.e. Bushan port and Shanghai port. Based on a game-theoretic response framework, the opportunities for one port to respond to investments of another port, with an objective of defending an appropriate market share. Instead of adopting a two-stage Bertrand competition model, the authors focused on the pricing game. Kaselimi and Reeven (2008) applied the Hotelling location model to the port competitions. The model was used to develop a framework for linking strategic interdependence among ports. Zhang (2008) investigated the relationship between hinterland access conditions and port competition, considering the price competition among the ports. De Borger *et al* (2008) analyzed the interaction between the pricing behavior of the ports and optimal investment policies in port and hinterland capacity, based on a two-stage game in capacities and prices.

3. Model Description - Non-cooperative games

There are three non-cooperative games in the proposed model. In a competitive situation with few players and an inhomogeneous product, the outcome in terms of market shares and prices can often be treated as the result of a game where each player maximizes profits, but with due consideration of the expected reaction of its competitors. When the competitor's actions are confined to setting the prices of their own products or services, the outcome can be modeled as Bertrand equilibrium (Pindyck and Rubinfeld, 2001). In the first stage, Bertrand games are modeled between two shipping line companies and the equilibrium prices for each shipping line company are derived. The vertical interactions between the shipping line and the port, is modeled through a multinomial Logit model, to describe the choice behavior of shipping line companies. In the second stage, two Bertrand games are used to model the competitions between two ports and each shipping line company, and the Bertrand equilibrium prices for each port are derived. The existing literature has suggested that the port charge is one of the most important factor when shipping line company selects a port. Figure 1 describes the conceptual framework of the proposed model. Table 1 presents some notations used in this article.

3.1. Customer Choice

Before introducing the explicit model, customer choice between two shipping line companies need to be explained. When the container goods are transported from origin to destination (only one OD pair is considered in this model), customers need to decide which shipping line company to choose. Obviously, the two shipping line companies here will compete for the market share, in terms of container traffic (TEU).

The classic multinomial logit model is adopted in this study in order to derive the relationship between service charge of shipping line company and the quantity of the containers to be handled by it. Consequently, the market share for each shipping line company could be derived as:

$$\begin{aligned} Q_1 &= \frac{1}{1 + e^{(\beta_1 - \beta_2) - \alpha(p_1 - p_2)}}\, Q \\ &= \frac{e^{-(\beta_1 + \alpha p_1)}}{e^{-(\beta_1 + \alpha p_1)} + e^{-(\beta_2 + \alpha p_2)}} Q \end{aligned} \tag{1}$$

$$Q_2 = \frac{e^{-(\beta_2 + \alpha p_2)}}{e^{-(\beta_1 + \alpha p_1)} + e^{-(\beta_2 + \alpha p_2)}} Q \tag{2}$$

Multinomial logit model has been found to fit mode choice making behavior quite well and is computationally tractable. The probability that a certain choice will be taken is proportional to e raised to the utility over the sum of e raised to the utility (Greene, 2003).

As indicated by the Equation (1) and (2), the price set by the shipping line company will influence its market share in terms of container volume to be transported. Obviously, the higher the price, the lower the market share. The value of α is very important because it will directly impact on the equilibrium price and quantity. Consequently, a detailed sensitivity analysis will be undertaken in this study.

3.2. Decision Variables

There are ten decision variables in the proposed model (Table 2). The decision variables include the service charges determined by each shipping line company and each port, and also the binary variables for the choice behavior of shipping line's call on port.

3.3. Mathematical Programming Model

The horizontal and vertical non-cooperative games between shipping line companies and ports could be described by four individual mathematical models: model of shipping line company 1 and 2, model of port 1 and 2, respectively.

Shipping Line Company 1 Model

$$\pi_1 = (p_1 - c_1 - u_{11})Q_1^1\, x_1^1 + (p_1 - c_1 - u_{12})Q_1^2\, x_1^2 \tag{3}$$

s.t.

$$\underline{p}_1 \le p_1 \le \bar{p_1} \tag{4}$$

$$x_1^1\, Q_1^1 + x_1^2\, Q_1^2 = Q_1 \tag{5}$$

$$x_2^1\, Q_2^1 + x_2^2\, Q_2^2 = Q_2 \tag{6}$$

$$Q_1 + Q_2 = Q \tag{7}$$

$$x_1^1 \in \{0, 1\} \tag{8}$$

$$x_1^2 \in \{0, 1\} \tag{9}$$

$$x_2^1 \in \{0, 1\} \tag{10}$$

$$x_2^2 \in \{0, 1\} \tag{11}$$

$$0 \le u_{11} \le u \tag{12}$$

$$0 \le u_{12} \le u \tag{13}$$

Equation (3) indicates that the SL_1 aims to maximize its profits, which are associated with the price charged per TEU p_1 and the volume of containers to be transported, either through Port_1 (by Q_1^1) and/or Port_2 (by Q_1^2). Equation (4) specifies the lower and upper bound for the service charge of SL_1. The value of p is important in this study in terms of results interpretation and will be derived from a wide range of literature review and interviews with shipping line companies. Equation (5) indicates that all the container goods transported by SL_1 either travel through Port_1 and/or Port_2. Namely, each shipping line company has to include one or two ports in its route design in this model. Equation (7) ensures that all the container goods have to be transported from the origin to the destination. Equation (8)-(11) indicate that the binary decision variable for the shipping line company to visit the port. Equation (12) and (13) specify the lower and upper bound for service charge of port, u. Similarly to p, the value of u is derived from a wide range of annual reports and interviews with shipping line companies and port authorities in this research.

The solution of mathematical programming for SL_1 will provide the equilibrium service charge p_1 and shipping line's call on the port, as a result of the game.

Shipping Line Company 2 Model

$$\pi_2 = (p_2 - c_2 - u_{21})Q_2^1\, x_2^1 + (p_2 - c_2 - u_{22})Q_2^2\, x_2^2 \tag{14}$$

s.t.

$$\underline{p}_2 \leq p_2 \ \leq \bar{p_2} \tag{15}$$

$$x_1^1\, Q_1^1 + x_1^2\, Q_1^2 = Q_1 \tag{16}$$

$$x_2^1\, Q_2^1 + x_2^2\, Q_2^2 = Q_2 \tag{17}$$

$$Q_1 + Q_2 = Q \tag{18}$$

$$x_{\frac{1}{1}} \in \{0, 1\} \tag{19}$$

$$x_{\frac{2}{1}} \in \{0, 1\} \tag{20}$$

$$x_{\frac{1}{2}} \in \{0, 1\} \tag{21}$$

$$x_{\frac{2}{2}} \in \{0, 1\} \tag{22}$$

$$0 \leq u_{21} \ \leq u \tag{23}$$

$$0 \leq u_{22} \ \leq u \tag{24}$$

Similar to the payoff function of SL_1, Equation (14) indicates that the profits of SL_2 are associated with the price charged per TEU p_2 and the volume of containers to be transported, either through $Port_1$ (Q_2^1) and/or $Port_2$ (Q_2^2). Equation (15) specifies the lower and upper bound for the service charge of shipping line company. Equation (17) indicates that all the container goods transported by SL_2 either travel through $Port_1$ and/or $Port_2$. Equation (18) ensures that all the container goods have to be transported from the origin to the destination. Equation (19)-(22) indicate that the binary decision variables for shipping line's call on the port. Equation (23) and (24) specify the lower and upper bound for the service charge of the port.

The solution of mathematical programming for SL_2 will provide the equilibrium price p_2 and shipping line's call on the port, as a result of the game.

It is noted here that the bounds of service charge for each shipping line company might be different, i.e. $\bar{p_1} \neq \bar{p_2}$ and $\underline{p}_1 \neq \underline{p}_2$. That means the players of the game have different characteristics. If $\bar{p_1} = \bar{p_2}$ and $\underline{p}_1 = \underline{p}_2$, and other parameters are the same, the players have identical roles. Each scenario will be discussed in this paper to identify the impacts of $\triangle\bar{p}$, $\triangle\underline{p}$, $\triangle\bar{u}$, $\triangle\underline{u}$ on the equilibrium results.

<u>Port 1 Model</u>

$$\varepsilon_1 = (u_{11} - s_1)Q_1^1\, x_1^1 + (u_{21} - s_1)Q_2^1\, x_2^1 \tag{25}$$

s.t.

$$\underline{u}_1 \leq u_{11} \ \leq \bar{u_1} \tag{26}$$

$$\underline{u}_1 \leq u_{21} \ \leq \bar{u_1} \tag{27}$$

$$x_1^1\, Q_1^1 + x_1^2\, Q_1^2 = Q_1 \tag{28}$$

$$x_2^1\, Q_2^1 + x_2^2\, Q_2^2 = Q_2 \tag{29}$$

$$Q_1 + Q_2 = Q \tag{30}$$

Equation (25) indicates that the profits of $Port_1$ is determined by the service charge of the port u_{11} and u_{21} (per TEU) and the container volume handled by the port (Q_1^1 and Q_2^1). Equation (26) and (27) ensure the upper and lower bound of the port charge. Equation (28) and (29) ensure that the containers transported to the destination are either through $Port_1$

and/or Port$_2$. Equation (30) indicates that all the container goods have to be transported from origin to destination.

The solution of mathematical programming for Port$_1$ will provide the equilibrium prices u_{11} and u_{21}, and the container volume handled by Port$_1$, as a result of the game.

Port 2 Model

$$\varepsilon_1 = (u_{12} - s_2)Q_1^2\, x_1^2 + (u_{22} - s_2)Q_2^2\, x_2^2 \tag{31}$$

s.t.

$$\underline{u}_2 \leq u_{12} \;\leq \bar{u_2} \tag{32}$$

$$\underline{u}_2 \leq u_{22} \;\leq \bar{u_2} \tag{33}$$

$$x_1^1\, Q_1^1 + x_1^2\, Q_1^2 = Q_1 \tag{34}$$

$$x_2^1\, Q_2^1 + x_2^2\, Q_2^2 = Q_2 \tag{35}$$

$$Q_1 + Q_2 = Q \tag{36}$$

Similar to the payoff function of Port$_1$, Equation (31) indicates that the profits of Port$_2$ is determined by the service charge of the port u_{12} and u_{22} (per TEU), and the container volume handled by the Port$_2$. Equation (32) and (33) ensure the upper and lower bound of the service charge of the port. It is noted here that the bounds of service charge for each port might be different. Equation (34) and (35) ensure that the containers transported to the destination either pass through Port$_1$ and/or Port$_2$. Equation (36) indicates that all the container goods have to be transported from origin to destination.

The solution of mathematical programming for Port$_2$ will provide the equilibrium prices u_{12} and u_{22}, and the container volume handled by Port$_2$, as a result of the game.

Choice of shipping line company on the port

As stated in Section 3.1, two shipping line companies compete for the container volume from the shipper. It is basically a two-player game. Once the container volume received by each shipping line company is determined based on the result of the game, the company has to decide which port to use. Obviously, it is again a two-player game, with two ports competing with each other. To model the relationship between quantity of containers received by each port and the service charge of each port, a multinomial Logit model is introduced by Equation (37)-(40).

$$
\begin{aligned}
Q_1^1 &= \frac{1}{1 + e^{(\theta_1-\theta_2)-\gamma(u_{11}-u_{12})}}\ Q_1 \\
&= \frac{e^{-(\theta_1+\gamma u_{11})}}{e^{-(\theta_1+\gamma u_{11})} + e^{-(\theta_2+\gamma u_{12})}} Q_1
\end{aligned} \tag{37}
$$

$$
Q_1^2 = \frac{e^{-(\theta_2+\gamma u_{12})}}{e^{-(\theta_1+\gamma u_{11})} + e^{-(\theta_2+\gamma u_{12})}} Q_1 \tag{38}
$$

$$
Q_2^1 = \frac{e^{-(\theta_1+\gamma u_{21})}}{e^{-(\theta_1+\gamma u_{21})} + e^{-(\theta_2+\gamma u_{22})}} Q_2 \tag{39}
$$

$$
Q_2^2 = \frac{e^{-(\theta_2+\gamma u_{22})}}{e^{-(\theta_1+\gamma u_{21})} + e^{-(\theta_2+\gamma u_{21})}} Q_2 \tag{40}
$$

As shown in the mathematical models above, port charges, i.e. u_{11}, u_{12}, u_{21} and u_{22} play an important part in shipping line company's payoff function, i.e. Equation (3) and (14). That reflects the reality that when shipping line selects the port to call, the port charge is an important factor to be considered. Offering lower service price is undoubtedly one of key strategies used to attract shipping lines that are themselves under big pressure to reduce total shipping costs. Many ports attempt to secure cargo volume by lowering port charges for container lines using their terminals as load centers. Similarly, port charges become a major area where shipping lines are attempting to cut total operating costs, therefore shipping line companies usually prefer ports that can offer relatively low service charges. Ports often become least-cost providers to achieve competitive advantages and this is in line with the findings of some research papers **Marlow2003189 doi:10.1080/03088830119197 TheoENotteboom portevolution** Of course, price is not the only factor to be considered in reality. Other factors, for example, customer royalty, access to multimodal transportation, and port location, could be modeled in the future research.

The shipping line company aims to maximize its profits by minimizing its production cost c_1, selecting a port with lower charge u_{ij}, setting its own charge to the customer with lower value p and increasing the quantity of the container cargo to be transported. At the same time, the port itself tries to increase its charge to the shipping line company u_{ij}, reduce its the production cost s, and attract more container cargo to be operated to optimize its profits. Obviously, there are games between ports, shipping line companies, at both vertical and horizontal levels.

4. Solution Method

It has been widely acknowledged that it will be both mathematically and computationally difficult to obtain an exact formulation and solution of a four-player game. Consequently, this study proposes an enumeration-based algorithm to solve the problem. It will provide a starting point for the future research. It is noted here that the main contribution of current paper is to propose a game theoretical model to describe the vertical and horizontal interactions among the players along the maritime supply chain. The proposed algorithm is implemented in Matlab R2012a.

The algorithm was developed based on the definition of equilibrium. Let $(S,\ f)$ be a game with n players, where S_i is the strategy set for player i. Let $f = (f_1(x), f_2(x)...f_n(x))$ be the payoff function for $x \in S$. A strategy profile $x^* \in S$ is a Nash equilibrium if no unilateral deviation in strategy by any single player is profitable for that player, that is:

$$\forall i, x_i \in S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*) \tag{41}$$

Consequently, for our model, Nash equilibrium could be found when the following equations hold. Here the model for Port_1 is used as an example.

$$\begin{aligned} &\varepsilon_1(u_{11}^*, u_{21}^*, u_{12}^*, u_{22}^*, x_{11}^*, x_{12}^*, x_{21}^*, x_{22}^*, p_1^*, p_2^*) \geq \\ &\quad \varepsilon_1(u_{11}, u_{21}, u_{12}^*, u_{22}^*, x_{11}^*, x_{12}^*, x_{21}^*, x_{22}^*, p_1^*, p_2^*) \end{aligned} \tag{42}$$

For the Port_1, the equilibrium is obtained when the payoff function of Port_1 is maximized when the decision variables of Port_1 model are identified as u_{11}^* and u_{21}^*.

By solving the equilibrium of the Bertrand game, the pricing rule set by the shipping line companies and the ports could be obtained. However, certain assumptions and the values of parameters p, u, α, s and c, need to be made before solving the Bertrand equlibrium (see Table 3).

4.1. Assumed values for $\underline{p}$, $\bar{p}$, $\underline{u}$ and $\bar{u}$

The values of price bounds are important in this research in terms of results interpretation and are derived from a wide range of annual reports and interviews with shipping line companies and port authorities. Based on the interview results, we adopt ($800, $1100) per TEU for container shipping line from Asia to Europe as the bounds for the service charge. ($400, $650) per TEU is used as the bounds for the service charge of ports along that shipping line.

4.2. Assumed values for α

No research has been undertaken in which this value has been estimated, by multinominal logit model. However, there are some researches, similar in many ways to this paper, in which its value was estimated.

Polydoropoulou and Litinas (2007) adopted a multinomial logit model to model the choice among different types of shipping lines and airlines **Polydoropoulou2007297** Ohashi *et al* used an aggregate form of multinomial logit model to identify the critical factors influencing the choice of air cargo transshipment route decision **Ohashi2005149** Saeed (2009) estimated a logit model for container terminal selection by shipping companies on a dataset for the 4 terminals **Saeed2009phd** To test the implications of α value on the equilibrium results, a sensitivity analysis is implemented in this research and an appropriate value of α is proposed.

4.3. Assumed values for c and s

Port Authority of Singapore suggested that the average operating cost was around US $60 per TEU, which is in line with the findings of Saeed (2010) **Saeed2010393** Interviews with Maersk Line indicated that the average operating cost per TEU for Asia-Europe shipping line is around US $220, which is in line with some research findings **Song200299** **Kleywegt** Consequently, 60 and 220 are adopted for p and u respectively in this research.

5. Computational Results

With the available information, the equilibrium of four models consisting Equations (3)-(40) are solved using the enumeration algorithm implemented by Matlab R2012a.

5.1. Games with Identical Player Roles

In game theory, the players with identical player roles compete for the same objective (that of winning the game). Under this scenario, it is assumed that the characteristics of SL_1 (or $Port_1$) are identical to SL_2 (or $Port_2$), in terms of price bounds and operating cost. Bertrand equilibrium is presented in Table 4. Each of the four players chooses the lower bound of the service charge as the equilibrium price and share half of the market.

In order to test the impacts of price bounds of each player on the equilibrium results, a range of values are tested here with results presented in Figure 2. Each scenario is defined with different values of price bounds for the shipping line companies and the ports. For example, in scenario 1, $p_1 \in (900, 1000)$, $p_2 \in (900, 1000)$, $u_1 \in (400, 550)$, and $u_2 \in (400, 550)$. The Bertrand equilibrium results suggest that each player of the game will always choose the lowest price to maintain its market share and profits.

The market share and the equilibrium profits for each scenario are presented in Table 5. The results suggested that when each player of the game chooses its lowest-possible price, it obtains the best-possible profits and occupies 50% of the market share, i.e., yielding the Nash equilibrium. Obviously, if one player changes its price, it will not be able to gain more profits. Furthermore, when the lowest-possible price for the player increases, the profits under Nash equilibrium will increase as well.

To further study the impacts of bounds of each player's service on the equilibrium results, four scenarios are implemented with the results presented in Table 6 and 7.

Table 6 suggest that for the same level of upper/lower bounds of the shipping line service charge, two ports always choose the lower bounds of its service charge as the Bertrand equilibrium prices. When the lower bound of port price increases, the equilibrium price of each shipping line company increases too. Bertrand equilibrium is realized when each of two players sets the same price.

Table 7 indicate that for the same level of the upper/lower bounds of the port service charge, two shipping line companies always choose the lower bounds of its service charge as the Bertrand equilibrium prices. The price rules are also applicable to the two ports. Bertrand equilibrium is realized when each of two players sets the same price.

α, as the coefficient of multinomial logit model, will directly impact on the relationship between the price and the quantity of containers to be transported when two shipping line companies compete for the market share. When the Bertrand game in which the players have identical roles yield a Nash equilibrium, each player always chooses the lowest-possible price and obtains 50% of market share. That means when $\triangle(p_1 - p_2) = 0$ or $\triangle(u_1 - u_2) = 0$, Equation (1) and Equation (37) can be rewritten as $Q_1 = Q_2 = \frac{1}{2}Q$ and $Q_{\frac{1}{1}} = Q_{\frac{2}{1}} = \frac{1}{2}Q_1$. Consequently, when identical shipping line companies and identical ports are involved in a game, α doesn't impact on the equilibrium results.

5.2. Games with Different Player Roles

Under this scenario, the players with different player roles compete for the same objective (that of winning the game). Here, it is assumed that the characteristics of SL_1 (or Port_1) are different from SL_2 (or Port_2), in terms of price bounds and operating cost. Bertrand equilibrium result is presented in Table 8. α here is used as 0.1.

The results suggest that the shipping line company SL_1 with a choice of higher service charge, will choose a equilibrium price, which is slightly higher than the lower bound of its service charge. This will ensure it for the best-possible profits and market share. When the upper/lower bounds of price for SL_1 increase from (800, 1000) to (850, 1050), the equilibrium price of SL_1 also increases from 880 to 940 ($/TEU), in order to achieve a higher amount of profits. For the shipping line company SL_2 with constrained price option, it has to set the equilibrium price at the lower level than SL_1, in order to maintain a small fraction of market share. The findings can also be applied to the pricing rules set by the ports. Noted that the equilibrium price for the port with a constrained option of bounds, is its lowest-possible price.

To further study the impacts of bounds of each player's service on the equilibrium results, four scenarios are implemented with the results presented in Table 9 and 10.

Table 9 indicate when the upper/lower bounds of service charge of the shipping line companies are fixed, two ports always choose the lower bounds of its service charge as the Bertrand equilibrium prices. And the equilibrium prices of two shipping line companies are slightly higher than their individual lower bounds.

Table 10 indicate that when the upper/lower bounds of the service charges of the two ports are fixed, Bertrand equilibrium prices of each shipping line company are slightly higher than their lower bounds. Two ports always choose the lower bounds of its service charge as the equilibrium prices.

As stated above, α, as the coefficient of multinomial logit model, will directly impact on the relationship between the price and the quantity of containers to be transported when two shipping line companies compete for the market share. To test the impacts of α on the equilibrium results, a variety of α values are tested here. The input parameters for those tests are represented in Table 11. The Bertrand equilibrium results are presented in Figure 4.

The results suggest that the value of α will generate significant influences on the Bertrand equilibrium results. When α equals to 0.1, each player of the game chooses the lowest-possible price as Bertrand equilibrium price and each player shares half of the market in terms of container volume transported. When α is greater than 0.1, the player with higher equilibrium price occupies a small proportion of market share in terms of container volume. However, when α is smaller than 0.1, the player with lower equilibrium price occupies nearly the whole market, which is not reflecting the reality. Consequently, it is recommended that the value of α should set between 0.1 and 0.2. Very few researches have implemented the sensitivity analysis on the game results and proposed an appropriate value of α.

6. Conclusions

This paper has adopted a two-stage non-cooperative game theoretical approach to model the horizontal and vertical interactions among two shipping line companies and two ports. In the first stage, Bertrand game is used to model the interactions between two shipping line companies. When the shipping line decides which port to call, a multinomial Logit model applied to quantify the relationship between quantity and price. In the second stage, Bertrand game is applied to model the competitions between two ports. Nash equilibrium is derived by solving the Bertrand games. Very few papers have tackled the games among 4 players in the maritime studies before.

The computational results suggested that:

(1) For the games with identical players, Bertrand equilibrium prices for each of the four players are its lowest-possible service charge. Each player shares 50% of the market. α doesn't impact on the equilibrium results in this case.

(2) For the games with different players, the shipping line company which has constrained price option, sets the equilibrium price at the lowest-possible level. Another shipping line company with a choice of higher service charge, will choose a equilibrium price, which is slightly higher than the lower bound of its price. Bertrand equilibrium price for the port with a constrained option of bounds, is its lowest-possible price. It is recommended that the value of α should set between 0.1 and 0.2, to reflect the reality.

The main contributions of this research is that it applies non-cooperative game theoretical approach to the context of vertical and horizontal interactions among the ports and shipping line companies along the maritime supply chain. The pricing rules obtained from this study will be helpful for a better understanding of the process of competition, cooperation and decision-making in maritime supply chain.

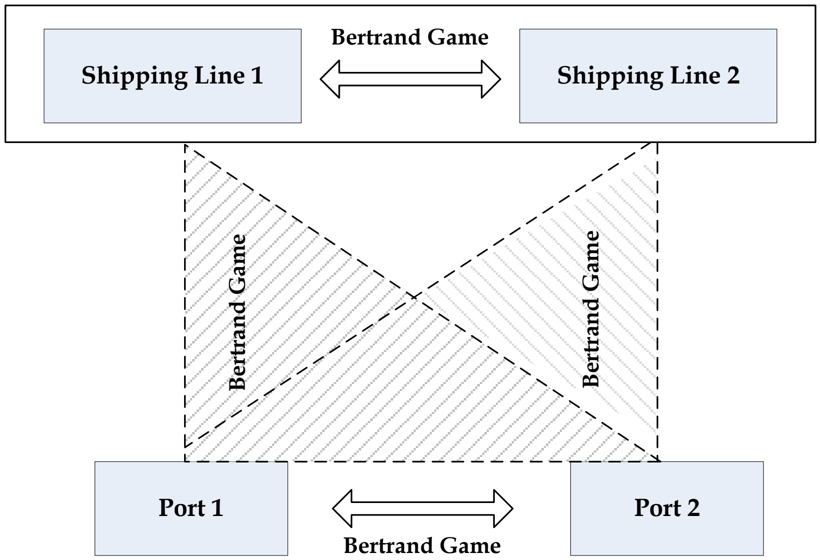

Figure 1: Conceptual framework for the two-stage model

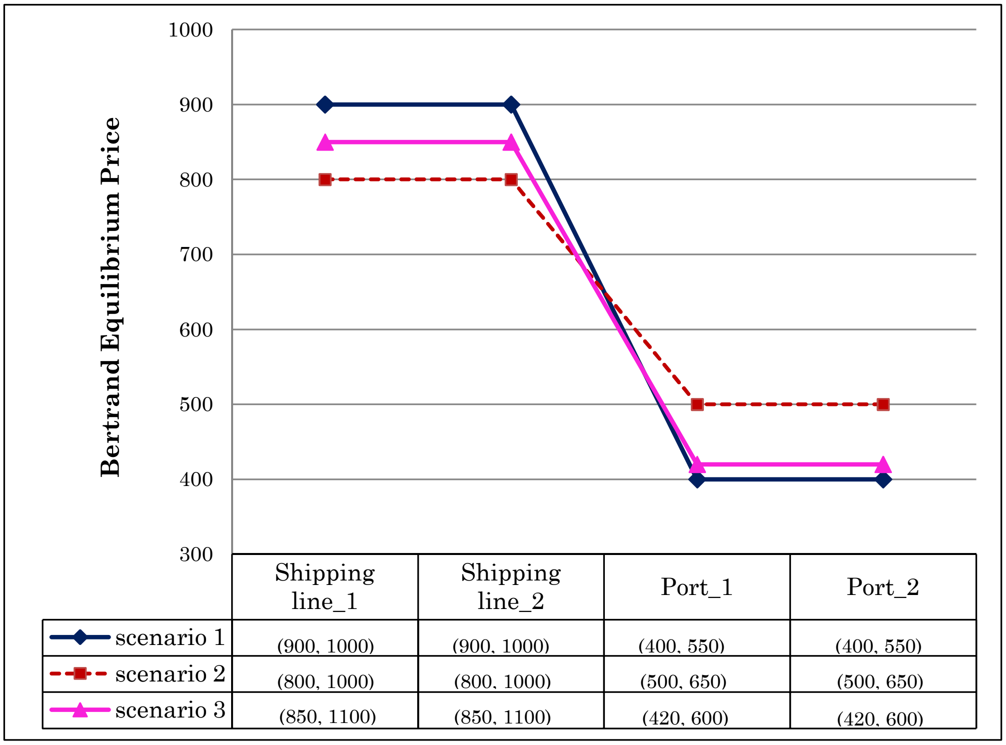

Figure 2: Impacts of lower/upper bounds on the equilibrium results of Bertrand game, when players have identical roles

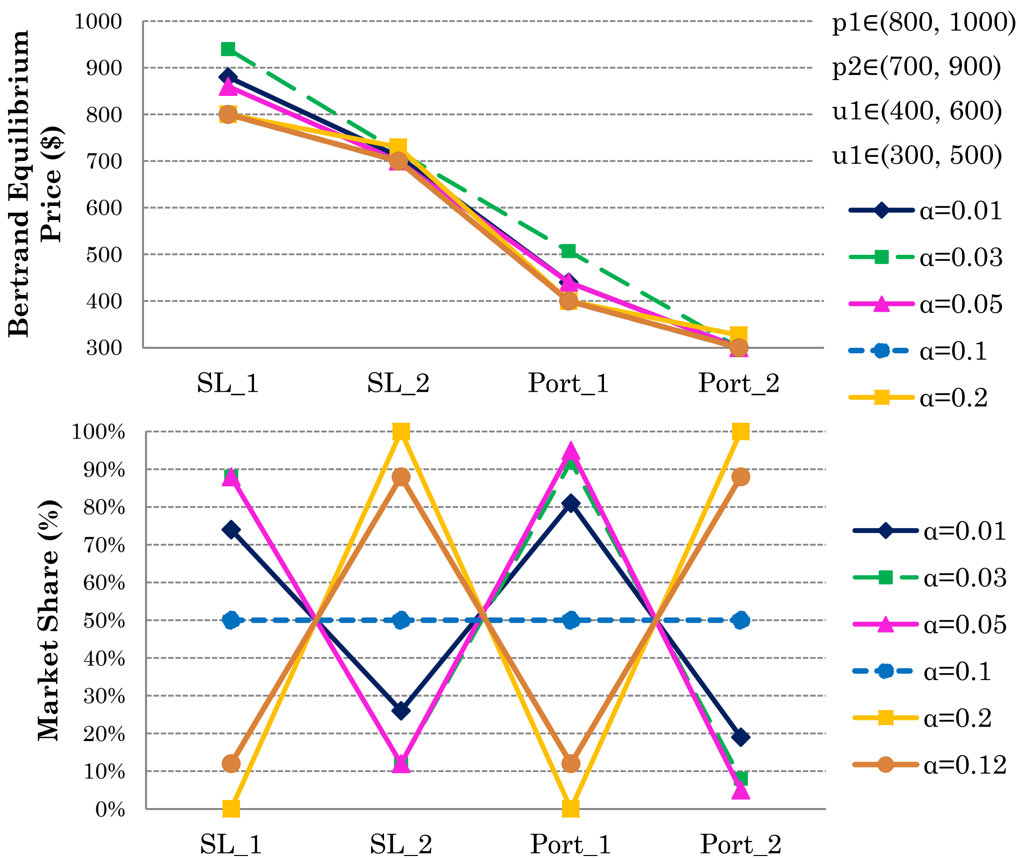

Figure 3: Impacts of α on the equilibrium results of Bertrand game, when players have different roles

α	Elasticity factor in Logit model
β_1	Coefficient of SL_1 in Logit model
β_1	Coefficient of SL_2 in Logit model
γ	Elasticity factor in Logit model
θ_1	Coefficient of $Port_1$ in Logit model
θ_2	Coefficient of $Port_2$ in Logit model
Q_1	Market share by SL_1 (container volume transported)
Q_2	Market share by SL_2 (container volume transported)
Q	Total container volume transported by two shipping line companies
Q_1^1	Container volume handled by $Port_1$, as part of container volume shipping by SL_1
Q_1^2	Container volume handled by $Port_2$, as part of container volume shipping by SL_1
Q_2^1	Container volume handled by $Port_1$, as part of container volume shipping by SL_2
Q_2^2	Container volume handled by $Port_2$, as part of container volume shipping by SL_2
p_1	Price set by SL_1
p_2	Price set by SL_2
$\underline{p}_1$,$\bar{p_1}$	Lower and upper bounds of price of SL_1
$\underline{p}_2$,$\bar{p_2}$	Lower and upper bounds of price of SL_2
$\underline{u}_1$,$\bar{u_1}$	Lower and upper bounds of price of $Port_1$
$\underline{u}_2$,$\bar{u_2}$	Lower and upper bounds of price of $Port_2$
c_1	Marginal cost of SL_1
c_2	Marginal cost of SL_2
s_1	Marginal cost of $Port_1$
s_2	Marginal cost of $Port_2$
π_1	Payoff function of SL_1
π_2	Payoff function of SL_2
ε_1	Payoff function of $Port_1$
ε_2	Payoff function of $Port_2$
u_{11}	Price set by $Port_1$ if SL_1 chooses $Port_1$
u_{12}	Price set by $Port_1$ if SL_1 chooses $Port_2$
u_{21}	Price set by $Port_2$ if SL_2 chooses $Port_1$
u_{22}	Price set by $Port_2$ if SL_2 chooses $Port_2$
x_1^1	1, if SL_1 chooses $Port_1$; 0 otherwise
x_1^2	1, if SL_1 chooses $Port_2$; 0 otherwise
x_2^1	1, if SL_2 chooses $Port_1$; 0 otherwise
x_2^2	1, if SL_2 chooses $Port_2$; 0 otherwise

Table 1: Notations used in the proposed model

p_1	Price set by SL_1
p_2	Price set by SL_2
u_{11}	Price set by $port_1$ if SL_1 chooses $Port_1$
u_{12}	Price set by $port_1$ if SL_1 chooses $Port_2$
u_{21}	Price set by $port_2$ if SL_2 chooses $Port_1$
u_{22}	Price set by $port_2$ if SL_2 chooses $Port_2$
x_1^1	1, if SL_1 chooses $Port_1$; 0 otherwise
x_1^2	1, if SL_1 chooses $Port_2$; 0 otherwise
x_2^1	1, if SL_2 chooses $Port_1$; 0 otherwise
x_2^2	1, if SL_2 chooses $Port_2$; 0 otherwise

Table 2: Decision variables used in the proposed model

Number of ports	2
Number of shipping line companies	2
Bounds of service charge of shipping line p (\$/TEU)	(800,1100)
Bounds of service charge of port u (\$/TEU)	(400,650)
Multinominal logit model α	0.01
Production cost s_1 (\$/TEU)	60
Production cost s_2 (\$/TEU)	60
Production cost c_1 (\$/TEU)	220
Production cost c_2 (\$/TEU)	220

Table 3: Model parameters

	SL_1	SL_2	$Port_1$	$Port_2$
Bounds of price $/TEU	(900, 1000)	(900, 1000)	(400, 550)	(400, 550)
Operating cost $/TEU	220	220	60	60
Equilibrium price $/TEU	900	900	400	400
Equilibrium profits $ million	1.4	1.4	1.7	1.7
Market share %	50	50	50	50

Table 4: Bertrand equilibrium when players have identical roles

Scenario 1	SL_1	SL_2	$Port_1$	$Port_2$
Equilibrium price $/TEU	900	900	400	400
Profits million $	1.4	1.4	1.7	1.7
Market share $	50%	50%	50%	50%
Scenario 2				
Equilibrium price $/TEU	800	800	500	500
Profits million $	0.4	0.4	2.2	2.2
Market share $	50%	50%	50%	50%
Scenario 3				
Equilibrium price $/TEU	850	850	420	420
Profits million $	1.05	1.05	1.8	1.8
Market share $	50%	50%	50%	50%

Table 5: Impacts of lower/upper bounds on the equilibrium results of Bertrand game, when players have identical roles

SL price bound	Port price bound	p_1^*	p_2^*	u_1^*	u_2^*
(800, 1000)	(400,600)	800	800	400	400
(800, 1000)	(500,650)	880	880	500	500
(800, 1000)	(450,650)	830	830	450	450
(800, 1000)	(420,620)	800	800	420	420

Table 6: Impacts of port charge bounds on Bertrand equilibrium results

SL price bound	Port price bound	p_1^*	p_2^*	u_1^*	u_2^*
(800,1000)	(420,620)	800	800	420	420
(900,1100)	(420,620)	900	900	420	420
(850,1150)	(420,620)	850	850	420	420
(950,1150)	(420,620)	950	950	420	420

Table 7: Impacts of shipping line charge bounds on Bertrand equilibrium results

scenario 1	SL_1	SL_2	$Port_1$	$Port_2$
Bounds of price $/TEU	(800, 1000)	(700, 900)	(400, 600)	(300, 500)
Operating cost $/TEU	220	200	60	50
Equilibrium price $/TEU	880	710	440	300
Equilibrium profits $ million	1.82	0.25	3.10	0.47
Market share %	74	26	81	19
scenario 2				
Bounds of price $/TEU	(850, 1050)	(700, 900)	(420, 620)	(300, 500)
Operating cost $/TEU	220	200	60	50
Equilibrium price $/TEU	940	720	460	300
Equilibrium profits $ million	2.15	0.23	3.25	0.47
Market share %	74	26	81	19
scenario 3				
Bounds of price $/TEU	(800, 1000)	(700, 900)	(420, 620)	(300, 500)
Operating cost $/TEU	220	200	60	50
Equilibrium price $/TEU	900	720	460	300
Equilibrium profits $ million	1.78	0.26	3.25	0.47
Market share %	71	29	81	19

Table 8: Bertrand equilibrium when players have different roles

SL_1	SL_2	$Port_1$	$Port_1$	p_1^*	p_2^*	u_1^*	u_2^*
(800, 1000)	(700, 900)	(400,600)	(420,620)	880	710	400	420
(800, 1000)	(700, 900)	(500,650)	(420,620)	950	780	500	420
(800, 1000)	(700, 900)	(450,650)	(420,620)	920	750	450	420
(800, 1000)	(700, 900)	(420,620)	(400,600)	880	710	420	400

Table 9: Impacts of port charge bounds on Bertrand equilibrium results

Port_1	Port_2	SL_1	SL_2	p_1^*	p_2^*	u_1^*	u_2^*
(420,620)	(450,650)	(800,1000)	(700,900)	920	750	420	450
(420,620)	(450,650)	(900,1100)	(700,900)	1000	750	420	450
(420,620)	(450,650)	(850,1150)	(700,900)	1045	750	420	450
(420,620)	(450,650)	(950,1150)	(700,900)	1050	750	420	450

Table 10: Impacts of shipping line charge bounds on Bertrand equilibrium results

	SL_1	SL_2	$Port_1$	$Port_2$
Bounds of price $/TEU	(800, 1000)	(700, 900)	(400, 600)	(300, 500)
Operating cost $/TEU	220	200	60	50

Table 11: Input parameters for the tests of a variety of α values