

Multi-objective mathematical programming approach to construction laborer assignment with equity consideration

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Abstract: *Construction laborer assignment is the assignment of laborers in a team to the tasks for a daily work in a construction project. This study proposes a mathematical programming approach to examine the optimal construction laborer assignment problem with the objectives of productivity and occupational health and safety while considering equity between laborers. Several linearization techniques are presented to transform the mathematical programming model into a mixed-integer linear program, which can be solved by off-the-shelf mixed-integer linear solvers. The proposed model is applied to extensive numerical experiments and the results show that the mathematical programming approach outperforms a conventional heuristic by over 10% in terms of job completion time. This implies considerable overhead, equipment, and manpower savings for the construction industry.*

1 INTRODUCTION

Construction is a large, dynamic, and complex sector, affecting the national productivity and economic well beings and creating employment for millions of people worldwide. The U.S. Census Bureau of the Department of Commerce (2016) announced that construction spending during 2016 was estimated at an annual rate of \$1,144.0 billion. In 2014 the construction industry in the UK contributed £103 billion in economic output, 6.5% of the total (Rhodes, 2015). Productivity is vital to the construction industry for two reasons. First, it is of critical importance to the profitability of most construction projects. Second, it is the most crucial as

well as the most flexible resource used in the assessment of the success of a construction project. Since construction is a labor-intensive industry, it can be argued that the workforce is the dominant productive resource; thus, labor productivity is a crucial productivity index because of the concentration of human labor needed to complete a specific task. Improving labor time utilization, reducing potential change/variation of project, benchmarking the productivity of different laborers, and making informed planning and scheduling decisions were recognized as effective approaches to enhance construction labor productivity (Yi and Chan 2013).

As a complex sector that seeks higher productivity, construction is also a sector with the most fatalities and high incident of non-fatal occupational injuries and illnesses involving days away from work. According to statistics from the International Labour Organization, each year in construction sites around the world at least 60,000 fatalities occur. Aside from the fatal accidents, workers in the construction industry are also subject to potential health hazards during their construction building process. Construction work is characterized as strenuous outdoor tasks that impose construction workers to overexertion. The overexertion includes high muscular loads, working and lifting in awkward postures, repetitive movements, and working with elevated arms (da Costa and Vieira, 2010). The overexertion leads to high prevalence of musculoskeletal pain in the low back, shoulder, and neck (Long et al., 2013). It was reported that as many as 30% of construction workers suffer

from back pains and/or other musculoskeletal disorders (European Agency for Safety and Health at Work, 2010).

1.1 Literature review

Many attempts and efforts have been made on the improvement of construction labor productivity at the project level. Advanced optimization techniques, such as artificial intelligence (Adeli and Karim, 1997; Elbeltagi and Hegazy 2001; Siddique and Adeli, 2013), genetic algorithm (Tam and Tong, 2003; Kociecki and Adeli, 2014; Park et al., 2015), harmony search (Siddique and Adeli, 2015a,b), mathematical programming (Gomar et al., 2002; Liu et al., 2016), neural networks (Adeli and Wu, 1998; Adeli, 2001; Senouci and Adeli, 2001; Ghosh-Dastidar and Adeli, 2009; Siddique and Adeli, 2016), particle swarm optimization (Zeng et al., 2014; Shabbir and Omenzetter, 2015), robust optimization (Shahabi et al., 2015), multiobjective optimization (Cha and Buyukozturk, 2015), and other techniques (Abuyounes and Adeli, 1986; Adeli and Chyou, 1987; Karim and Adeli, 1999a, b; Kim and Adeli, 2001; Kociecki and Adeli 2013, 2015; Chen et al., 2014; Szeto et al., 2014; Qarib and Adeli, 2015; Rafiei and Adeli, 2015; Sun and Betti, 2015) are frequently used. El-Rayes and Moselhi (2001) presented an automated and practical optimization model based on dynamic programming and a scheduling algorithm to optimize resource utilization for repetitive construction projects such as highways, high-rise buildings, and housing projects. Gomar et al. (2002) developed a linear programming model to help optimize the multiskilled workforce assignment and allocation process in a construction project. Tam and Tong (2003) focused on the structural concrete-frame construction stage of public housing projects and developed a genetic algorithm model for site facility layout and an artificial neural network model for predicting tower-crane operations. Cheng et al. (2005) proposed a hybrid mechanism whose advantage lies in that it integrates rule-based heuristic algorithms and metaheuristics to efficiently locate resource allocation for construction simulation optimization. Horman and Thomas (2005) analyzed the relationship between inventory (buffers) and construction labor performance using data collected from projects in Brazil. Their results show that buffer helps to achieve improved labor performance in the construction operations. Jang et al. (2007) compared the total travelled distances by construction material handling between the

observed data and the result of a genetic algorithm and demonstrated that optimization tools could reduce the material handling distance by 14%. Seçkiner and Kurt (2008) developed optimization models for job rotation planning to increase productivity. Yan et al. (2008) developed an integrated model that combines ready mixed concrete production scheduling and truck dispatching for a construction site. El-Rayes and Jun (2009) presented innovative resource leveling metrics to minimize the negative impact of resource requirement fluctuations on construction productivity and project cost. The new metrics are incorporated in a robust optimization model in order to generate optimal schedules taking into practical considerations that maximize the efficiency of resource utilization. Sadeghi et al. (2010) applied fuzzy Monte Carlo simulation to handle both random and fuzzy uncertainties in risk assessment of construction projects. Lim et al. (2014) proposed an approach that identifies the overlap between critical activities in order to reduce project completion time and cost.

Previous studies have mainly focused on developing optimization algorithms to maximize labor productivity and limited studies considered workforce health and safety in perspective of physical effort, strain and fatigue. Improving labor productivity and at the same time maintaining occupational health and avoiding accidents are major concerns in the construction industry. Tharmmaphornphilas et al. (2003) and Tharmmaphornphilas and Norman (2004) have developed worker rotation schedules by using mathematical programming models to reduce the likelihood of worker hearing loss. Hsie et al. (2009) presented a model to create work-rest schedules for construction workers. They included the objective of minimizing the time for completing jobs and the objective of minimizing any extra energy expended by laborers due to inappropriate work assignments. A heuristic solution approach was developed. However, these studies neglected the issue of the equity of employees in an organization where the inequality between the wages and amount of work of different workers could undermine the motivation and productivity of the workers (Leete, 2000; Kickul and Lester, 2001; Charness and Kuhn, 2007). Taking the equity between the laborers into the optimization algorithm, this study investigates the assignment of laborers

in a team to the tasks in a project accounting for both productivity and health concerns.

1.2 Objectives and contributions

This study examines the assignment of laborers in a team to the tasks for a daily work in a small-scale construction project (job). The purpose of this study is to develop a laborer assignment plan to reduce physiological strain, improve productivity, as well as account for equity. To reduce physiological strain, a laborer needs to have a break for recovery after completing a task. Given that different tasks require different levels of work intensity and work durations and laborers have different physique and physiological responses, the rest duration after completing a task required by a laborer differs for different combinations of task and laborer. Hence, the team leader needs to decide which laborers to perform which tasks and at what time each task should be started. Craft working time utilization often reflects the constraints enforced by the organization that adversely affect the improvement of construction productivity (Maloney 1990). Researchers have reported that the time spent in direct-work activities is a good labor productivity indicator as well as a useful predictor in a productivity projection model (Liou and Borcharding 1986). Therefore, the total time spent on production is considered as a yardstick to measure productivity in this study.

Two objectives are expected to achieve: one is to complete the job as short time as possible; the other is to minimize the physiological strain on the laborers (Zavadskas et al., 2006). Regarding the job completion time, the precedence relations between tasks, i.e., one task cannot start until another is completed, should be taken into account. The physiological strain on the laborers is formulated by adopting the concept of “total extra energy expenditure” (Hsie et al., 2009). Moreover, when assigning tasks to laborers, the team leader should account for the equity between the laborers: it is unfair if one laborer works for four hours while another one works for only one hour.

The contribution of the paper is threefold. First, to the best of our knowledge, this is the first study to develop a mathematical programming model to optimize the assignment of laborers in construction with the objectives of productivity and occupational health and safety while considering equity between laborers. Our work extends the research by Hsie et al.

(2009) in that we formulate the equity issue and we develop a model and a solution method. Second, we developed several linearization techniques that transform the mathematical programming model into a mixed-integer linear program, which can be solved by off-the-shelf mixed-integer linear solvers. Thus, team leaders only need to provide a few parameters and then the optimal assignment of laborers and the start time of the tasks will be produced by the model. Third, we have conducted extensive numerical experiments and the results show that the mathematical programming approach outperforms a conventional heuristic by over 10% in terms of job completion time, implying considerable overhead, equipment, and manpower savings. Given that there are 6.6 million construction laborers in the US alone (U.S. Bureau of Labor Statistics, 2016), the significance of the research to the construction industry is considerable.

The remainder of this paper is organized as follows: Section 2 gives a detailed description for the problem. A mathematical model is formulated in Section 3. In Section 4, solution methods are discussed. Extensive computational experiments are reported in Section 5. Final conclusions are drawn in the last section.

2 PROBLEM DESCRIPTION

In this section we use a simple example to describe the construction laborer assignment problem.

2.1 Job and tasks

Consider a small construction job shown in Figure 1. The job has five tasks, whose details are shown in Table 1. It is assumed that once a task begins, it must be performed to completion without interruption. Take the first task “column steel fixing” as the example. Three laborers are required to perform the task simultaneously. The duration required to complete the task is 30 min, that is, to complete the task, a period of 30 min is required during which three laborers all work on the task. A noteworthy issue is the precedence relation between tasks, as shown by the arrows in Figure 1. Some tasks cannot start before the completion of other tasks. For instance, to perform formwork building for external walls (task 5), both wall steel fixing (task 3) and hydropower pipe burying (task 4) must be completed. The column “Immediate predecessor” in Table 1 shows the precedence relation. Since

task 2 is an immediate predecessor of task 3 and task 3 is an immediate predecessor of task 5, task 5 cannot start if task 2 is not completed. In other words, task 2 is a predecessor of task 5, but not an immediate predecessor.

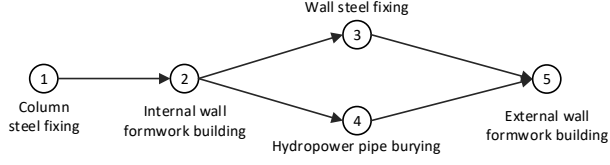


Fig. 1 An example of a construction job

Table 1 Details of the five tasks

Task ID	Description	Num of laborers	Duration (min)	Immediate predecessor	Oxygen uptake rate at work (l/min)
1	Column steel	3	30	None	2.0
2	Internal wall	3	20	1	1.5
3	Wall steel	2	40	2	2.1
4	Hydropower pipe	1	20	2	0.8
5	External wall	3	50	3, 4	1.4

2.2 Laborers

The team leader of the laborers in charge of a job needs to assign laborers to the tasks so that the job can be completed as early as possible. At the same time, the fatigue of the laborers should be minimized and the tasks should be as fairly allocated to the laborers as possible.

Different laborers are different in terms of age, gender, height, weight, exercise, and other physical factors. Different tasks also require different levels of strengths. A laborer may be tired and need to take a rest after performing a task to recover his strength. We assume that after performing a task, a laborer should take a rest whose duration is long enough for the laborer to recover and then can perform another task. We use the formulae proposed by Kuijer et al. (2004) to calculate the rest (min) required:

$$\text{Rest} = \begin{cases} \text{Work duration} \times \frac{O_w - 0.33O_m}{O_w - O_r}, & \text{if } O_w > 0.33O_m \\ 0, & \text{if } O_w \leq 0.33O_m \end{cases}$$

where O_w (l/min) is the oxygen uptake rate at work for the task (see the last column of Table 1), O_m (l/min) is the maximum

oxygen consumption rate of the laborer, and O_r (l/min) is the oxygen uptake rate of the laborer at rest. For instance, a laborer with $O_m = 3$ and $O_r = 0.34$, after performing task 1, will have to rest for a duration of at least $30 \times \frac{2 - 0.33 \times 3}{2 - 0.34} \approx 18$ min.

A laborer has a maximum acceptable work duration (MAWD, min) when performing a task, and the laborer will be tired if the duration required by the work is longer than MAWD. We use the extra energy expenditure (E) proposed by Hsie et al. (2009) to reflect the overexertion on the laborer:

$$E = 4.83 \times (\text{work duration} - \text{MAWD}) \times$$

$$O_w, \text{ if work duration} > \text{MAWD}; 0, \text{ otherwise}$$

where “4.83” means one liter of oxygen generates 4.83 kilo calorie (kcal) of energy and the MAWD (min) is calculated by (Wu and Wang, 2001):

$$\text{MAWD} = -2.09 + e^{6.59 - 5.6 \times \frac{O_w - O_r}{O_m - O_r}}$$

For the laborer with $O_m = 3$ and $O_r = 0.34$ to perform task 1, MAWD = 20 min and the total extra energy expenditure $E = 97$ kcal.

The team leader of the laborers, when assigning the tasks, also needs to take equity into consideration: the total work load of two laborers should not differ too much. With all of the above considerations regarding the tasks and the laborers, the team leader assigns the tasks to the laborers with the aim of minimizing the job completion time and the total extra energy expenditure of the laborers.

3 MODEL FORMULATION

Based on the problem background, we formulate a mathematical programming model for the construction laborer assignment problem. We list the notation in Section 3.1 and present the model in Section 3.2.

3.1 Notation

Indices:

i, i' : laborers;

j, k : tasks;

Input parameters:

I : set of laborers;

J : set of tasks;

δ_{jk} : a binary parameter that equals 1 if and only if task $j \in J$ immediately precedes task $k \in J$, and 0 otherwise;

n_j : number of laborers required to perform task $j \in J$;

d_j : working duration required by task $j \in J$;

σ_{ij} : a binary parameter that equals 1 if and only if laborer $i \in I$ is capable of performing task $j \in J$, and 0 otherwise;

e_{ij} : extra energy expenditure by laborer $i \in I$ if he performs task $j \in J$;

r_{ij} : rest time required by laborer $i \in I$ after performing task $j \in J$;

Δ : maximum allowed difference between the total work times of two laborers;

M : a sufficiently large positive number;

Decision variables:

$z_{ij} \in \{0, 1\}$: set to 1 if laborer $i \in I$ performs task $j \in J$, and zero otherwise;

t_j : time at which task $j \in J$ starts to be performed;

T : time when all of the task are completed;

E : total extra energy expenditure by all of the laborers for performing all of the tasks;

3.2 Mathematical model

$$[M1] \quad \min \begin{Bmatrix} T \\ E \end{Bmatrix} \quad (1)$$

subject to:

$$T = \max_{j \in J} \{t_j + d_j\} \quad (2)$$

$$E = \sum_{i \in I} \sum_{j \in J} e_{ij} z_{ij} \quad (3)$$

$$t_j + d_j \leq t_k + M(1 - \delta_{jk}) \quad \forall j \in J, \forall k \in J, i \neq k \quad (4)$$

$$\sum_{i \in I} z_{ij} = n_j \quad \forall j \in J \quad (5)$$

$$t_j + d_j + r_{ij} \leq t_k \text{ if } z_{ij} = z_{ik} = 1 \text{ and task } j \text{ is done before task } k, \forall i \in I, \forall j \in J, \forall k \in J, j \neq k \quad (6)$$

$$\max_{i \in I, i' \in I \setminus \{i\}} |\sum_{j \in J} d_j z_{ij} - \sum_{j \in J} d_j z_{i'j}| \leq \Delta \quad (7)$$

$$z_{ij} \in \{0, 1\} \quad \forall i \in I, \forall j \in J \quad (8)$$

$$z_{ij} \leq \sigma_{ij} \quad \forall i \in I, \forall j \in J \quad (9)$$

$$t_j \geq 0 \quad \forall j \in J \quad (10)$$

In the above model, Eq. (1) minimizes the two objectives: time when all of the task are completed, T , and total extra energy expenditure by all of the laborers for performing all of the tasks, E . Constraint (2) defines the time when all of the tasks are completed as the maximum of the completion times

of all of the tasks. Constraint (3) calculates the total extra energy expenditure by all of the laborers for performing all of the tasks. Constraints (4) impose that if task $j \in J$ immediately precedes task $k \in J$, then the completion time of task j should be not later than the start time of task k . Constraints (5) enforce the number of laborers assigned to a task meets the requirement of the task. Constraints (6) state that if a laborer performs two tasks, then the completion time of the first one plus the rest time required is not earlier than the start time of the second one. Constraint (7) requires that the difference of the total work times of two laborers does not exceed the maximum allowed difference. Constraints (8) define z_{ij} as binary decision variables. Constraints (9) ensure that a laborer may be assigned to a task only if he is capable of performing the task. Constraints (10) define the start times of all the tasks to be nonnegative.

4 SOLUTION METHOD

4.1 Linearization of the model

The above model has nonlinear and logical operators such as “max” and “if”, rendering it unsuitable to be solved by mathematical programming. We present a few techniques below to transform the model into an equivalent multi-objective mixed-integer linear program.

Constraint (2) can be replaced by the following set of linear constraints:

$$T \geq t_j + d_j \quad \forall j \in J \quad (11)$$

As we minimize T in Objective (1), the value of T will automatically be equal to the maximum of all of the $t_j + d_j$ in the optimal solution to the model with Constraints (11). Similarly, Constraint (7) is equivalent to

$$\sum_{j \in J} d_j z_{ij} - \sum_{j \in J} d_j z_{i'j} \leq \Delta \quad \forall i \in I, \forall i' \in I, i \neq i' \quad (12)$$

Constraints (6) are the most difficult to linearize. It states that if a laborer i performs two tasks j and k , then the completion time of the first one plus the rest time required is not earlier than the start time of the second one. To characterize which task is performed first, we define:

New auxiliary decision variables:

$y_{jk} \in \{0, 1\}$: set to 1 if task $j \in J$ starts earlier than task $k \in J \setminus \{j\}$, and zero otherwise;

The relation between y_{jk} and the start times of the two tasks is as follows:

$$t_k - t_j \leq M y_{jk} \quad \forall j \in J, \forall k \in J, j \neq k \quad (13)$$

$$y_{jk} + y_{kj} = 1 \quad \forall j \in J, \forall k \in J, j < k \quad (14)$$

$$y_{jk} \in \{0, 1\} \quad \forall j \in J, \forall k \in J, j \neq k \quad (15)$$

Constraints (13) impose that if $t_k > t_j$, then y_{jk} must be 1. Constraints (14) enforce that either task j starts first or task k starts first (in case of tie, either task can be considered to start first). Now Constraints (6) can be replaced by the following ones:

$$t_j + d_j + r_{ij} \leq t_k + M(3 - z_{ij} - z_{ik} - y_{jk}) \quad \forall i \in I, \forall j \in J, \forall k \in J, j \neq k \quad (16)$$

Constraints (16) state that: if laborer i performs task j ($z_{ij} = 1$) and task k ($z_{ik} = 1$) and task j starts before task k ($y_{jk} = 1$), then $t_j + d_j + r_{ij} \leq t_k$, guaranteeing that the laborer has sufficient rest time; otherwise, the corresponding constraint is redundant. Therefore, Constraints (13)–(16) fully linearize Constraints (6).

Based on above newly added decision variables and constraints, the problem can be reformulated as a multi-objective mixed-integer linear programming model:

[M2] Objective (1)

subject to Constraints (3)–(5) and (8)–(16).

4.2 Reducing the number of binary variables

The introduction of the binary variables y_{jk} in Constraints (15) will inevitably make the model cumbersome. We observe that based on the precedence of the diagram we can fix the values of some of the y_{jk} variables. For instance, if task $j \in J$ immediately precedes task $k \in J$, then y_{kj} must be 0. Next, we propose Algorithm 1 below that fixes the y_{jk} variables at 0 if task k precedes (but not necessarily immediately precedes) task j . The key idea of the algorithm is: if task k immediately precedes task j' and task j' immediately precedes task j , then task k precedes task j .

Algorithm 1. Identify all precedence relations

Initialize: Define $\theta(k)$ as a binary indicator which equals 1 if and only if task $k \in J$ has been checked and $\pi(j, k)$ as a binary indicator which equals 1 if and only if task $j \in J$ precedes task $k \in J \setminus \{j\}$. Set $\theta(k) \leftarrow 0, \pi(j, k) \leftarrow \delta_{jk}, j \in J, k \in J \setminus \{j\}$.
// Main algorithm
While (not all $\theta(k)$'s are 1)
 For each $k \in J$ **do**
 If $\theta(k) = 1$ **then**
 continue;
 End if
 // Check whether all immediate predecessor of task k have been checked
 k_has_unchecked_immediate_predecessor = **false**;
 For each $j' \in J \setminus \{k\}$ **do**
 If $\delta_{j'k} = 1$ and $\theta(j') = 0$ **then**
 k_has_unchecked_immediate_predecessor = **true**;
 break;
 End if
 End for
 If k_has_unchecked_immediate_predecessor **then**
 continue;
 End if
 // The predecessors of an immediate predecessor of task k are also predecessors of the task
 For each $j' \in J \setminus \{k\}$ **do**
 If $\delta_{j'k} = 1$ **then**
 For each $j \in J \setminus \{j', k\}$ **do**
 Set $\pi(j, k) \leftarrow 1$ if $\pi(j, j') = 1$;
 End for
 End if
 End for
 Set $\theta(k) = 1$;
 End for
End while

After identifying the precedence relation $\pi(j, k)$, we can add the following constraints to the model:

$$y_{jk} \leq 1 - \pi(k, j), \forall j \in J, \forall k \in J, j \neq k \quad (17)$$

4.3 Determining the values of big-M

Proper values of the big-M's in Constraints (4), (13), and (16) must be computed before the model can be solved. The big-M in Constraints (4) can be set to the bound of the completion time of a task; the big-M in Constraints (13) can be set to the bound of the start time of a task; the big-M in Constraints (16) can be set to the bound of the completion time

of a task plus the rest time. To simplify the presentation, we can use the same value of M for the three groups of constraints:

$$M := \max_{i \in I} \sum_{j \in J} (d_j + r_{ij}) \quad \forall j \in J, \forall k \in J, j \neq k \quad (18)$$

In Eq. (18), the value of M equals the total time required by the slowest laborer plus the final rest time to complete all of the tasks by himself. Hence, it defines a proper bound for the three groups of constraints.

4.4 Handling the two objectives

As the model has two objectives, there generally does not exist a solution that optimizes both objectives simultaneously. In this circumstance, we generally aim to find the efficient solutions. Let $(\mathbf{z}, \mathbf{t}) := (z_{ij}, t_j, i \in I, j \in J)$ be a feasible solution to the model (note that once (\mathbf{z}, \mathbf{t}) is determined, the other variables T , E , and y_{jk} are also determined, represented by $T(\mathbf{z}, \mathbf{t})$, $E(\mathbf{z}, \mathbf{t})$, and $y_{jk}(\mathbf{z}, \mathbf{t})$, respectively). An efficient solution is a solution that is not worse than any other solution; mathematically, (\mathbf{z}, \mathbf{t}) is an efficient solution if there is no feasible solution $(\mathbf{z}', \mathbf{t}')$ such that $T(\mathbf{z}', \mathbf{t}') \leq T(\mathbf{z}, \mathbf{t})$, $E(\mathbf{z}', \mathbf{t}') \leq E(\mathbf{z}, \mathbf{t})$, and at least one inequality is strict. The set of points $(T(\mathbf{z}, \mathbf{t}), E(\mathbf{z}, \mathbf{t}))$, where (\mathbf{z}, \mathbf{t}) is an efficient solution, forms the Pareto frontier. We can find an efficient solution by defining a weight $\lambda \in [0, 1]$ and solve a single-objective mixed-integer linear program with off-the-shelf solvers:

$$\begin{aligned} [\mathbf{M3}] \quad & \min \lambda T + (1 - \lambda)E \\ \text{subject to} \quad & \text{Constraints (3)–(5) and (8)–(17).} \end{aligned} \quad (19)$$

In particular, when $\lambda = 1$, we will minimize the time when all of the task are completed, and when $\lambda = 0$, we will minimize the total extra energy expenditure by all of the laborers for performing all of the tasks. In practice, it is also possible for the team leader to minimize one objective subject to a limit on the other objective. For instance, if it is required that the job must be completed in 240 min, then the team leader might minimize E subject to the constraint $T \leq 240$.

5 COMPUTATIONAL EXPERIMENT

In this section, we conduct extensive numerical experiments to validate the effectiveness and the efficiency of the proposed model. The experiments are run by a PC equipped with 3.60GHz of Intel Core i7 CPU and 16GB of RAM. All the algorithms are programmed in Matlab and the mixed-integer programming model is solved by CPLEX 12.6.3.

5.1 Illustration with a basic example

We first report the results on the basic job shown in Figure 1 to demonstrate the application of the proposed method. The parameters of the tasks are shown in Table 1. Suppose that there are a total of four laborers whose maximum oxygen consumption rates (O_m) and oxygen uptake rates at rest (O_r) are shown in Table 2 (Hsie et al., 2009). Then we can use the formula in Section 2.2 to calculate the rest time required by a laborer to perform a task (r_{ij}) and the extra energy expenditure (e_{ij}), as shown in Table 2. Furthermore, we assume that all laborers can perform all tasks, i.e., all $\sigma_{ij} = 1$, and set the maximum allowed difference between the total work times of two laborers at $\Delta = 25$ min. The weight λ in Eq. (19) is set at 0.5.

Table 2 Details of the four laborers, rest time required, and extra energy expenditure

Laborer ID	1	2	3	4
O_m	2.9	3	3.1	3.2
O_r	0.34	0.34	0.34	0.34
r_{i1}	18.8	18.3	17.7	17.1
r_{i2}	9.4	8.8	8.2	7.7
r_{i3}	26	25.2	24.5	23.7
r_{i4}	0	0	0	0
r_{i5}	20.9	19.3	17.8	16.2
e_{i1}	123.8	96.6	67.8	37.5
e_{i2}	0	0	0	0
e_{i3}	269.8	245.4	219.3	191.7
e_{i4}	0	0	0	0
e_{i5}	0	0	0	0

The optimal result of the laborer assignment is shown in Table 3, in which an entry “1” means the laborer in the row

performs the task in the column, for example, the first laborer performs tasks 2, 3 and 5. Task 1 starts at time 0. Task 2 starts at time 47.7, which is equal to the duration of task 1, 30 min, plus the rest time 17.7 min of laborer 3. Note that although laborer 2 requires more rest time (18.3 min) than that of laborer 3, laborer 2 has sufficient time to rest as he does not perform task 2 or task 3. Task 3 and task 4 start at the same time, and task 5 completes at time $143.0 + 50 = 193.0$ min. Laborer 1 works the longest time, which is 110 min, and laborer 4 works for just 90 min. The job is completed at time $T = 193.0$ min and the total extra energy expenditure by all of the laborers is $E = 663.3$ kcal.

Table 3 Optimal laborer assignment

Laborer ID	Task 1	Task 2	Task 3	Task 4	Task 5	Total work time
1		1	1		1	110
2	1			1	1	100
3	1	1			1	100
4	1	1	1			90
Task start time	0	47.7	77.0	77.0	143.0	

We then test different values of the maximum allowed difference between the total work times of two laborers $\Delta \in \{5, 10, \dots, 50\}$ min. We find that if $\Delta \leq 15$, the model has no feasible solution; if $\Delta \in \{20, 25\}$, the optimal job completion time is $T = 193.0$ min and the total extra energy expenditure is $E = 663.3$ kcal; if $\Delta \geq 30$, the optimal job completion time is $T = 190.2$ min and the total extra energy expenditure is $E = 612.8$ kcal. This experiment shows the impact of the maximum allowed difference between the total work times of two laborers.

Finally, we test different values of $\lambda \in \{0.0, 0.1, \dots, 1.0\}$ and plot the Pareto frontier (T, E) in Figure 2. Note that there are only five points in Figure 2 because different values of λ may lead to the same point. We can see that the minimum possible job completion time is $T = 190.2$ min and the minimum total extra energy expenditure is $E = 663.3$ kcal.

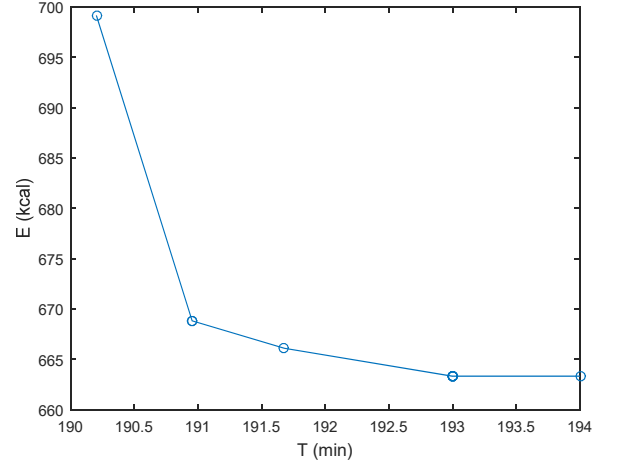


Fig. 2 Pareto frontier of the basic example

5.2 Randomly generated experiments

We generate a large number of random instances to test the effectiveness of the proposed model. In particular, we will compare the solution of the model with an intuitive heuristic assignment approach that might be adopted by many team leaders: scanning the tasks one by one and checking whether a task can be done (all of its precedence tasks are completed); if a task j can be done, then let the n_j laborers who have recovered first to perform the task.

To compare the proposed model with the above heuristic, the instances are generated as follows. We consider different combinations of the number of laborers and tasks: $(I, J) \in \{(5, 5), (5, 10), (5, 15), (10, 10), (10, 15), (10, 20), (15, 10), (15, 15), (15, 30)\}$. For each combination, we generate five instances. Regarding the tasks, the precedence relation δ_{jk} has 25% chance of being 1 if $k = j + 1, \dots, j + 4$; the number of laborers required by a task n_j is uniformly drawn from $\{1, 2, 3, 4\}$; the work duration d_j is uniformly drawn from $[10, 60]$; the oxygen uptake rate at work for a task, O_w , is uniformly drawn from $[0.5, 2.5]$ (Hsie et al., 2009). Regarding the laborers, the maximum oxygen consumption rates O_m is uniformly drawn from $[2.5, 3.5]$ and the oxygen uptake rates at rest O_r is 0.34 (Hsie et al., 2009). Furthermore, we assume that all laborers can perform all tasks, i.e., all $\sigma_{ij} = 1$, and set the maximum allowed difference between the total work times of two laborers at a very large value to ensure feasibility. We just minimize the job completion time T .

The results of the comparison are shown in Table 4, where

we report, for each combination of (I, J) , the average job completion times by the heuristic (scanning the tasks one by one and letting the laborers who have recovered first to perform the first task in the list) and the proposed model, as well as the average percentage of time reduction over the heuristic by the model (the average of the five percentages, each of which is defined as $\frac{\text{completion time by heuristic} - \text{completion time by model}}{\text{completion time by heuristic}} \times 100\%$).

We can see that the proposed model could considerably improve over the heuristic: the job completion time reduction can be as high as 20%. Moreover, we find that the quality of the result obtained by the heuristic is generally higher when the laborers are not overwhelmed by the tasks, i.e., a smaller value of J/I . When a laborer has too many tasks to perform, i.e., a larger value of J/I , using the proposed model could more dramatically reduce the job completion time.

Table 4 Comparison between the heuristic and the model

I	J	Average T by heuristic	Average T by model	Average percentage T reduction
5	5	121.5	112.5	7.3%
	10	301.0	249.8	17.1%
	15	444.8	345.4	23.4%
10	10	183.6	172.6	6.4%
	15	187.5	162.0	11.9%
	20	336.7	276.0	17.6%
15	10	140.1	138.8	0.9%
	15	223.5	221.8	0.6%
	30	296.4	259.1	12.7%

5.3 A realistic-size case study example

Finally, we apply the proposed model to a realistic building project. This project is a high-rise residential building with total construction area of 9365 m², main structure of 19 layers, and height of 57.895 m. Its structure is reinforced concrete cast-shear wall structure. The building construction has three phases, each of which consists of nine tasks. The information of the tasks is shown in Table 5, where the duration d_j has time unit “day”. A team of 30 laborers are assigned to the job. The laborers’ maximum oxygen consumption rates (O_m) are uniformly distributed between 2.55 and 3.45, and their oxygen uptake rates at rest (O_r) are

0.34 (Hsie et al., 2009). Then we can use the formula in Section 2.2 to calculate the extra energy expenditure (e_{ij}). Furthermore, we assume that all laborers can perform all tasks, i.e., all $\sigma_{ij} = 1$, and set the maximum allowed difference between the total work times of two laborers at infinity.

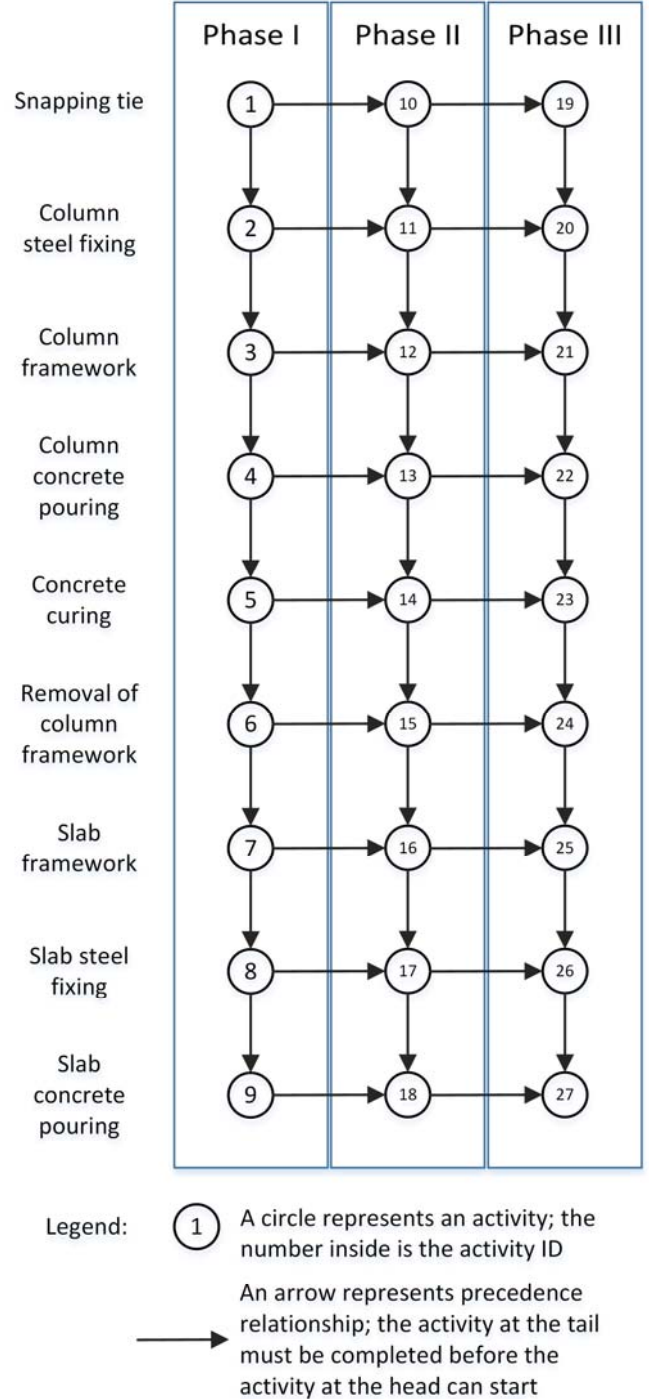


Fig. 3 Reinforced concrete cast-shear wall building

Table 5 Details of the tasks

Task	n_j	d_j	O_w
1, 10, 19	6	0.5	0.5
2, 11, 20	16	2	2.5
3, 12, 21	14	1.5	2.0
4, 13, 22	10	1	1.5
5, 14, 23	0	2	0.0
6, 15, 24	14	1.5	2.0
7, 16, 25	12	1.5	1.0
8, 17, 26	14	2	1.5
9, 18, 27	8	0.5	1.5

The computational results show that if we just minimize the job completion time, i.e., $\lambda = 1$, then the job completion time is $T = 18$ days, with minimum total extra energy expenditure $E = 1.598$ million kcal. If we just minimize the total extra energy expenditure, i.e., $\lambda = 0$, then the minimum total extra energy expenditure is $E = 1.586$ million kcal, with job completion time is $T = 37.5$ days. If we set the weight λ at 0.5, then the job completion time is $T = 19.5$ days and the minimum total extra energy expenditure is $E = 1.590$ million kcal. We can thus conclude that for this example, the total extra energy expenditure is not very sensitive to the weight λ while the job completion time is very sensitive. In the results, of course, the precedence relationships are satisfied.

The above case study is a simplified example of a real case. The developed model has been tested by a team in a construction company in Asia for three months. As the extra energy expenditure is difficult to measure, the model is used solely to reduce the job completion time. Before using the model, the team leader assigned tasks to the laborers based on his experience, and on average it took 6 days to complete one floor of a building. After using the result from the model, the team completed 11.5 floors in 65 days, meaning that the productivity is improved by 6.2% (from 0.167 floor/day to 0.177 floor/day).

6 CONCLUSIONS

This study has proposed a mathematical programming approach to optimize the assignment of laborers in construction with the objectives of productivity and occupational health and safety while considering equity

between laborers. We have presented several linearization techniques that transform the mathematical programming model into a mixed-integer linear program, which can be solved by off-the-shelf mixed-integer linear solvers. Thus, team leaders only need to provide a few parameters and then the optimal assignment of laborers and the start time of the tasks will be produced by the model. The proposed model is applied to extensive numerical experiments and the results show that the mathematical programming approach outperforms a conventional heuristic by over 10% in terms of job completion time. This implies considerable overhead, equipment, and manpower savings for the construction industry. Moreover, the quality of the result obtained by the heuristic is generally higher when the laborers are not overwhelmed by the tasks. If the number of tasks and the number of laborers are large, it may take a long time to solve the problem. If the laborer assignment is done one day in advance, then there is one night's (12 hours) time to solve the model and the problem size is generally not an obstacle. In the future, we will extend the research to the assignment of both laborers and equipment to the tasks in a construction project, which makes the problem more interesting yet challenging.

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