Cruise Itinerary Schedule Design

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Abstract: The Cruise Itinerary Schedule Design (CISD) problem determines the optimal sequence of a given set of ports of call (a port of call is an intermediate stop in a cruise itinerary) and the arrival and departure times at each port of call for maximizing the monetary value of the utility at ports of call minus the fuel cost. To solve the problem, in view of the practical observations that most cruise itineraries do not have many ports of call, we first enumerate all sequences of ports of call and then optimize the arrival and departure times at each port of call by developing a dynamic programming approach. To improve the computational efficiency, we propose effective bounds on the monetary value of each sequence of ports of call, eliminating non-optimal sequences without invoking the dynamic programming algorithm. Extensive computational experiments are conducted and the results show that, first, using the bounds on the profit of each sequence of ports of call considerably improves the computational efficiency; second, the total profit of the cruise itinerary is sensitive to the fuel price and hence an accurate estimation of the fuel price is highly desirable; third, the optimal sequence of ports of call is not necessarily the sequence with the shortest voyage distance, especially when the ports do not have a naturally geographical sequence.

Keywords: Cruise shipping; Schedule design; Itinerary planning; Dynamic programming;

1. Introduction

A cruise itinerary is a cruise route operated by a cruise company: A cruise ship deployed on a cruise route picks up passengers at an embarkation port, calls at several ports of call where passengers alight from the ship to visit the port cities, and finally returns to a disembarkation port where passengers get off the cruise ship and finish the trip. The ship that is deployed on the itinerary, the embarkation port, the sequence of ports of call, the disembarkation port, and the time schedule are all pre-determined in a cruise itinerary. Cruise ships are different from other ships such as

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tankers, bulk carriers and containerships in that transportation is not the purpose of cruise ships.

The cruising industry has maintained a steady increase in supply for the past 20 years. In 2014, the number cruise passengers reached a total of 22.04 million and the global cruise industry generated revenues of 37.1 billion U.S. dollars (Statista, 2015). Meantime, the world cruise fleet had 296 ships (Cruise Industry News, 2015) with a total of 482,000 lower berths[1] (Statista, 2015). The Caribbean and the Mediterranean areas are the most important cruising destinations and hence they are also where most ship capacity is deployed. Cruise passengers are mainly from developed countries: for example, among the 22.04 million cruise passengers in 2014, 12.16 million (55%) were from North America and 6.39 million (29%) were from Europe; only 3.49 million (16%) were from the other parts of the world. However, Cruising is an oligopolistic industry: Carnival, Royal Caribbean, and Norwegian Cruise Lines are the three largest companies with market shares of 41.8%, 21.8%, and 8.2%, respectively (Statista, 2015)

A few strategic decisions have a long-lasting effect on the profitability of a cruise company (Veronneau and Roy, 2009). The first one is cruise fleet planning. A large cruise ship may have more than 5000 lower berths and the construction cost can be as high as one billion US dollars. Companies book new ships in order to replace the old ones, to cater to the rising demand in the cruising market, and to provide extra capacity to block potential entrants to the market (Wang et al., 2016). The second one is ship deployment. Some cruise ships are repositioned from the Caribbean to the Mediterranean in summer and from the Mediterranean to the Caribbean in winter. Recently, a number of mass market cruise ships were relocated to Asia as the Asian market grows fast. The third one is itinerary planning. A cruise itinerary is similar to a container liner service (Fransoo and Lee, 2013; Pang and Liu, 2014): both have fixed sequences of ports of call and fixed schedules (arrival and departure time at each port of call); the itineraries are announced in advance to attract bookings and cruise ships have to adhere to the announced itineraries irrespective of whether they are full or not. Moreover, both industries are markedly capital-intensive and characterized by high fixed costs for operators, who seek high volume of bookings to fill their capacity (Wang et al., 2015).

Most itineraries are loops with a home port: the itinerary starts and ends at the home port and

[1] It is often considered that one cabin has two beds (two lower berths) when calculating the capacity of cruise ships. Any extra beds in a cabin are referred to as “upper berths”. The actual average number of beds per cabin in a cruise ship is usually higher than two.

most cruise passengers embark and disembark at the home port. The choice of home ports by cruise companies depends on many factors, such as the size of the passenger market, the air-lift capacity of the port city as many passengers fly to home ports from other places, and the infrastructure and services of the port. Typical examples of home ports include Miami and Barcelona. Some itineraries are one-way in that they start and end at different home ports: for instance, trans-Atlantic itineraries. A cruise company also needs to determine which ports of call to include into an itinerary for a cruise ship. Ports of call are chosen based on the attractions of the port cities, the infrastructure and services of the ports, and the proximity to other ports in the itinerary. Under the background, the sequence of visiting the ports of call and the arrival and departure times at the ports of call need be determined.

This study assumes that the home port (or home ports in case of one-way itinerary) and the ports of call have been chosen in advance and addresses the Cruise Itinerary Schedule Design (CISD) problem that determines the optimal sequence of the ports of call to visit and the arrival and departure times at the ports of call. The optimal sequence to visit the ports of call is mainly affected by their locations. In general, a shorter overall itinerary distance means less fuel consumption per itinerary and therefore more fuel cost savings. As reported by Statista (2015), the fuel cost was 220 US dollars per cruise passenger on average, which is 15% of the cruise expenses. Therefore, one percent reduction in the fuel cost is translated to savings of 48 million US dollars (220 US dollars per cruise passenger times 1% and then times 22.04 million cruise passengers in 2014) for the industry.

Factors other than just the overall itinerary distance should be accounted for when determining the sequence of ports of call. For example, different sequences may not have much effect on the overall distance when the ports of call are close to each other, as is the case for the Caribbean and Mediterranean areas (Wang et al., 2016). Moreover, some itineraries are along the coast of a continent (e.g., from Sydney to the north along the east coast of Australia) and whether a port of call is included in the direction away from the home port or back to the home port does not affect the overall itinerary distance. These observations motivate the development of more sophisticated models that formulate factors beyond the port distances to determine the sequence of visiting the ports of call. In particular, we take into account the arrival and departure times at each port of call. A cruise ship generally visits a port of call in the early morning and departs in the late afternoon so that cruise passengers can go onshore to have a tour to the port city. In extreme cases, a cruise ship may stay at a port of call for as short as two hours or as long as two days. In reality, it may not be possible for a cruise ship to visit all of the ports of call at the same time of a day because it will mean the ship often has to sail very fast or very slowly from the previous port of call. In other words, although it is preferable for cruise passengers to spend more hours in the daytime at each port of call, it comes at the cost of reducing the sailing time at sea, resulting in higher sailing speed and possibly higher fuel consumption.

Based on the above analysis, this paper presents an explorative study on the CISD problem, in which the optimal sequence of visiting a given set of ports of call and the arrival time and departure time at each port of call are to be determined. In view of the practical observations that most cruise itineraries do not have many ports of call, we first enumerate all sequences of ports of call and then optimize the arrival and departure times at each port of call by developing a dynamic programming approach. To improve the computational efficiency, we propose effective bounds on the profit of each sequence of ports of call, eliminating non-optimal sequences without invoking the dynamic programming algorithm. Extensive computational experiments are conducted and the results show that, first, using the bounds on the profit of each sequence of ports of call considerably improves the computational efficiency; second, the total profit of the cruise itinerary is sensitive to the fuel price and hence an accurate estimation of the fuel price is highly desirable; third, determining the sequence of ports of call solely by minimizing the overall voyage distance leads to significant reduction in the total profit when the ports do not have a naturally geographical sequence.

The existing researches on cruise shipping, such as the above-mentioned works, are mainly descriptive. There are few quantitative studies on cruise shipping such like Maddah et al. (2010). Given that cruise itineraries have fixed sequences of ports of call and fixed schedules, optimization-based service planning tools should be able to increase the profit or save the cost for cruise shipping companies and improve the service quality for cruise passengers. Such tools are urgent in view of the fast growing cruise market and the gigantism of cruise ships. This paper develops quantitative models on the CISD problem and thus contributes to the state-of-the-art research and practice by developing such a tool.

2. Literature Review

We first review academic literature on cruise shipping. As the work is also related to maritime freight transportation and land transportation, we also relate our work to these studies after introducing some research works on cruise operations.

2.1. Cruise operations

There is not much research about cruise shipping in the academic literature. This might be attributed to the reason that tourism researchers have not paid much attention as worldwide cruise ship tourism accounts for just about 2% of world tourism (Gui and Russo, 2011), and maritime researchers mainly focus on freight transportation.

Soriani et al. (2009) investigated the structural aspects and evolutionary trends for cruising in the Mediterranean region. They identified three crucial issues for the future development of the region. Gui and Russo (2011) introduced a global value chain framework for the cruise industry. Veronneau and Roy (2009) were amongst the first to lay a descriptive theoretical foundation of a cruise ship supply chain. They pointed out that a unique feature to the cruise industry is that the itinerary planning will affect the travel demand. Rodrigue and Notteboom (2013) conducted a deep market investigation in the cruise industry. They found that cruise ship deployment and itinerary design decisions are influenced by both market considerations and operational constraints.

Revenue management (RM) in the cruise industry was a hot topic among researchers. According to Kimies (1989), cruise lines, just like hotels and airlines, can be deemed as traditional RM industries. Talluri and van Ryzin (2004) stated that the cruise ships are nothing more than floating hotels. However, Biehn (2006) strongly disagreed with the common idea that running a cruise ship is identical to managing a hotel, and claimed that hotel management guidelines should not be directly used for cruise lines in terms of RM strategies. Meanwhile, He proposed a deterministic linear programming to maximize revenue for a cruise ship considering the capacity limitations on the number of cabins and the number of lifeboat seats. Based on his study, Maddah et al. (2010) built a discrete-time dynamic capacity control model to improve the profit of cruise ships, in which the orders from arrival customers follow a stochastic process and request one type of cabin combined with one or more lifeboats. Further review on the cruise operations can be referred to Wang et al. (2016).

2.2. Maritime freight transportation

Cruise shipping is akin to container liner services as both of them have fixed port rotation and schedule. Moreover, the fuel consumption of cruise ships and container ships are both related to the speeds of the ships. We refer to Meng et al. (2014) for a review of research on container liner services. Generally, there are two major differences between the modeling approaches for the two types of operations. First, one liner service alone is usually not sufficient to transport containers from their origins to their destinations as containers are often transshipped during their trips (Ng, 2014, 2015). As a result, a liner service cannot be designed independently without considering other services, and cargo routing among multiple shipping service routes is critical (Song and Dong, 2012). However, in cruising shipping, only one cruise ship is deployed on a cruise itinerary and passengers do not transfer between different itineraries. Therefore, the schedule design for an itinerary can be implemented separately. Second, the purpose of docking at ports by container ships is to load and unload containers. Consequently, it is always desirable for a container ship to spend less time at ports (Song and Dong, 2011; Du et al., 2015). Different from container shipping, the purpose of docking at ports by cruise ships is for cruise passengers to visit the port city and hence a longer port time could be advantageous.

Compared with limited research papers on the cruise shipping, tremendous research works have been devoted into the container liner shipping. Take the research topic of route design and schedule design in the container liner shipping as an example: Shintani et al. (2007) proposed a problem for liner shipping networks design, which consists of dozens of shipping routes. Qi and Song (2012) worked on a problem of designing an optimal schedule in order to minimize the total fuel consumption. For the uncertainties in port operations, Wang and Meng (2012a) studied a robust schedule design problem. Meantime, Wang and Meng (2012b) further considered sea contingency time for the schedule design problem. Song and Dong (2013) combined both ship deployment and empty container repositioning into a long-haul shipping route design. Regarding container terminal operations, Zhen et al. (2016) addressed a container terminal allocation problem and Zhen (2016) analyzed a yard operations planning problem.

2.3. Land transportation

Land transportation, such as the travelling salesman problem (Applegate et al., 2011) and the vehicle routing problem (Toth and Vigo, 2001), is also relevant to cruise shipping in that a vehicle/vessel visits several locations. However, land transportation is different from maritime transportation because the travel speed in land transportation is largely determined by the traffic conditions and vehicles usually travel at the highest possible safe speed in the exogenous traffic conditions. On the contrary, ships can sail freely at sea without congestion and they do not often sail at their highest speed mainly for economic reasons: a ship burns more fuel when it sails faster. As the relation between speed and fuel consumption is nonlinear, optimization models for cruise itinerary are also nonlinear. A more relevant category of research is vehicle routing problems with time windows (VRPTW) (Cordeau et al., 2001). The time window at a customer in VRPTW is a time interval, e.g., 9:00 am to 2:00 pm, and the planning horizon in vehicle routing problems is usually one day. However, the cruise ship schedule design problem covers a planning horizon of many days and the cruise ship can visit a port in the daytime on any day; hence the "time window" at a port is a set of disconnected time intervals. Cruise ship schedule design is also relevant to the travelling salesman problem with profits (Feillet et al., 2005) as passengers gain extra utility by spending time at ports. The difference is: the amount of extra utility gained by cruise passengers at a port depends on the time of visit and duration of visit, rather than a fixed value.

The above literature review shows that existing research on cruise shipping is mainly descriptive with just a few exceptions (e.g. Maddah et al., 2010). Moreover, cruise shipping modeling is inherently different from other maritime transportation analysis and land transportation formulations. Given that cruise services have fixed sequences of ports of call and fixed schedules, optimization-based service planning tools should be able to increase the profit or save the cost for cruise shipping companies and improve the service quality for cruise passengers. Such tools are urgent in view of the fast growing cruise market and the gigantism of cruise ships. This paper develops a quantitative solution approach on the CISD problem and thus contributes to the state-of-the-art research and practice by developing such a tool.

3. Problem Description

The CISD problem designs a schedule for a cruise itinerary, which has a given home port (or two home ports in case of one-way itineraries) and a given set of ports of call. The decision variables include (i) the sequence of the ports of call to visit and (ii) the arrival and departure time at each port of call. In the CISD, the deployed cruise ship departs from the home port, denoted by Port 1, visits a set of given ports of call, denoted by Ports $2, \cdots, N-1$, the sequence of which is to be determined, and finally returns to the home port, denoted by Port N, which is the same port as

Port 1 in looped itineraries or is a different port in one-way itineraries. We use $P_c := \{2, \cdots, N-1\}$ to represent the set of ports of call, and $P := \{1, \cdots, N\}$ to represent the set of all of the ports (both the home ports and ports of call). For the example of a cruise itinerary in Figure 1, Miami is the home port (i.e., Port 1 and Port $N = 6$), at which the cruise itinerary starts and terminates; there are four selected ports of call for the cruise itinerary and we can define Cozumel as Port 2, Belize as Port 3, Mahogany Bay as Port 4, and Grand Cayman as Port 5. Figure 1 shows that the cruise ship on the cruise itinerary starts from the home port at 4:00 pm (Day 1), visits Cozumel at 8:00 am (Day 3), spends nine hours at Cozumel, departs at 5:00 pm (Day 3), visits Belize (Day 4), Mahogany Bay (Day 5), Grand Cayman (Day 6), and returns to the home port (Day 8). As shown in Figure 1, an instance of the solution for the CISD problem is presented, in which the sequence of the ports of call and the times when the cruise ship arrives at and departs from each port of call are displayed.

DAY	PORT	ARRIVE	DEPART
Day 1	Miami, FL		4:00 PM
Day 2	... Fun Day At Sea	---	---
Day 3	Cozumel, Mexico	8:00 AM	5:00 PM
Day 4	Belize	8:00 AM	5:00 PM
Day 5	Mahogany Bay, Isla Roatan	7:00 AM	3:00 PM
Day 6	Grand Cayman, Cayman Islands	10:00 AM	6:00 PM
Day 7	... Fun Day At Sea	---	---
Day 8	Miami, FL	8:00 AM	

Figure 1: The itinerary of 7 Day Western Caribbean of Carnival

(Source: https://www.carnival.co.uk/itinerary-classic/7-day-western-caribbean-cruise/miami/glory/7-days/wek/?numGuests=2&destination=all-destinations&dest=any&datFrom=102016&datTo=092018&dur=D2&shipCode=GL)

In the CISD problem, the following information is required as inputs: (i) The departure time when a cruise ship leaves the home port and its arrival (return) time at the home port for termination; (ii) The utility distribution at each port of call for the cruise passengers to experience (for instance, the utility for the cruise passengers to spend time at a port city at 3:00 am is marginal); (iii) The relationship between bunker consumption and speed on each leg (a leg is the voyage from one port to the next port). Then, based on these inputs, we make two critical decisions: (i) The sequence of ports of call for the cruise ship to visit; and (ii) The arrival and departure time at each port of call. The sequence displays the order list of ports, in which ports of call must be visited by the cruise ship one by one. The arrival and departure times confirm the staying time that the ship spends at each port of call and the voyage time on each leg. The objective of the CISD problem is to maximize the total monetary value from the utilities brought to cruise passengers at port cities minus the bunker fuel cost of the cruise ship.

3.1. Departure time from the home port and the return time

We assume that the cruise ship departs from the home port and returns to the home port at pre-determined time. This assumption does not restrict the model but simply aims to simplify the notation. Without loss of generality, we define that the cruise ship departs at Time 0 and returns at Time T. Hence, there are a total of $T+1$ time points to complete the cruise, denoted by set $\mathbb{T} = \{0, \cdots, T\}$. Here, one time period could be set as one hour, as using one hour in the schedule for cruise itineraries is precise enough (our model can also handle other time periods, e.g., half an hour). Note that when we mention "at time $t \in \mathbb{T}$" we refer to the time at the end of the tth time period (or equivalently, at the beginning of the $(t+1)$th time period).

3.2. Utility distribution at ports

Regarding the time spent at port cities, we notice that cruise ships generally visit a port in the morning and leave in the evening so that cruise passengers can have a tour in the port city. Evidently, if a cruise ship visits a port at e.g. 3:00 am, then there is no transport available for the cruise passengers and there is no place for the cruise passengers to visit.

To capture the impact of arrival and departure times on the cruise itinerary, we need to know the utility of a port in different hours of a day. One example of the utility distribution at a port is showed in Figure 2. In the daytime hours the utility is positive. The utility is zero when the port is closed, for instance, at night. The utility for each hour can be estimated by expert judgment or by analyzing existing cruise itineraries, details of which will be discussed in Section 6.1 and Section 7.

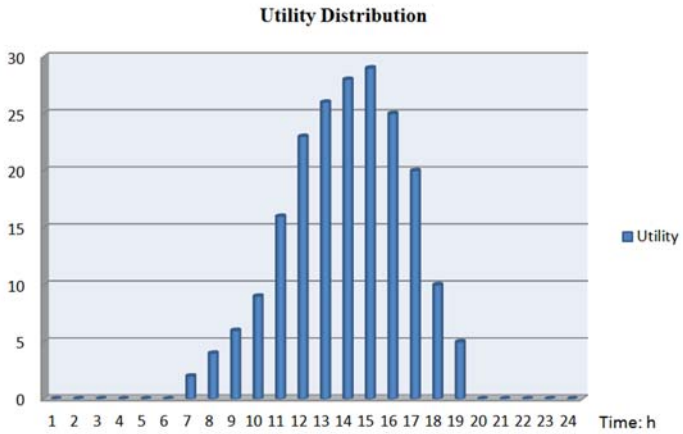

Figure 2: The utility distribution for one day (Wang et al., 2016)

We make three comments on the utility shown in Figure 2. (i) The utility mentioned here actually refers to the extra utility by spending time at port cities compared with spending time at sea on cruise ships (as cruise passengers also have a lot of fun at sea). (ii) Different ports have different utility distributions. For instance, a world-renowned city like Rome should have high utilities; cities in which people tend to go to bed and wake up early, have different profiles from those in which people stay late at nights in bars. (iii) It is possible that the ship stays at a port when the utility is zero. For instance, in Figure 3, when two ports are very close, e.g, two hours' sailing, it is possible that the ship stays in Port j overnight, when there is no utility, and leaves the port at 8:00 am on the next day.

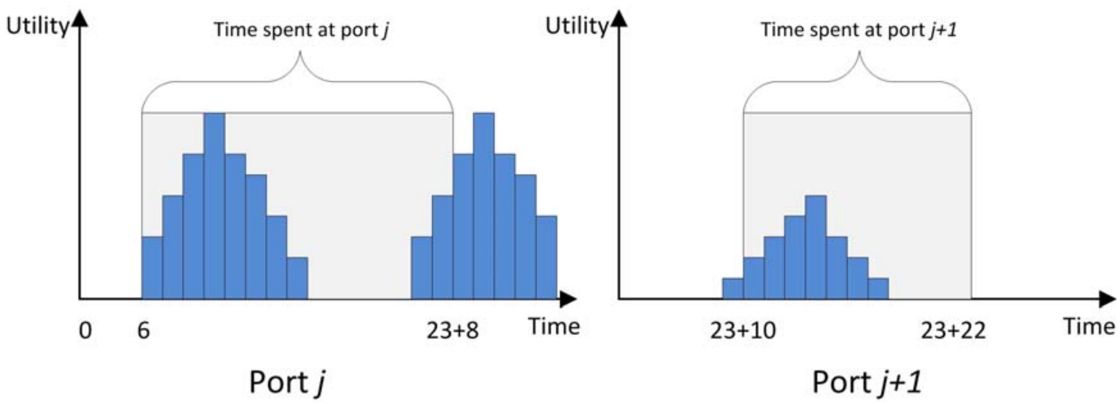

Figure 3: Utility distributions and time spent at two ports

To facilitate the solution approach development, we define $g_i(t)$ as the utility at Port i in Time period $t \in \{1,2,\cdots,T\}$. $g_i(t)$ can be derived based on the daily distribution of the utility. Considering the example in Figure 2, if the cruise ship leaves the home port at 4:00 pm (which, by our definition, is Time 0), then 6:00-7:00 am of the next day corresponds to Time period 15 and we thus have $g_i(15) = 2$ as the utility for the hour from 6:00 am to 7:00 am is two. Similarly, we have $g_i(15+24)=2$ and $g_i(15+48)=2$ in which 24 means one day and 48 means two days.

To synthesize passengers' utility with the bunker cost in one objective function, we denote p^u as the monetary value for the cruise company from one unit of utility. The product of the unit monetary value (i.e., p^u) and the total utility that the cruise passengers would experience at ports of call is the total monetary value for visiting the ports of call during the cruise.

3.3. Fuel consumption

Different arrival and departure times at ports affect the sailing speed, which impacts the fuel consumption by the main engine of the cruise ship. According to the research conducted by Du et al. (2011), the fuel consumption rate of a ship is determined by the speed and can be estimated as follows.

$$k + k' \cdot (v)^s \tag{1}$$

Here k and k' are regression coefficients, v is the sailing speed, and $s \in \{3.5, 4, 4.5\}$. For feeders, $s = 3.5$; for medium-sized vessels, $s = 4$; and for jumbo vessels, $s = 4.5$. To calculate the fuel consumption of a leg, let d be the distance of the leg and τ be the sailing time on the voyage, implying that the speed is $v = d/\tau$. Therefore, the bunker consumption on the leg, denoted by function $\tilde{F}(d,\tau)$, is

$$\tilde{F}(d,\tau) = [k + k' \cdot (v)^s] \cdot \tau = k\tau + k' d^s \tau^{1-s} \tag{2}$$

There exists an optimal sailing speed, denoted by v^*, to save the fuel, which can be derived by minimizing the consumption in Eq. (2). The calculation for the optimal sailing speed is:

$$v^* = \left(\frac{k}{k' \cdot (s-1)}\right)^{1/s} \tag{3}$$

The cruise ship can decelerate (or accelerate) if its sailing speed exceeds (or is lower than) the optimal speed in order to save fuel.

Given d and τ, the average speed is d/τ. If $d/\tau \geq v^*$, the ship should sail at constant speed that is equal to d/τ for saving fuel consumption. Otherwise, the ship should sail at its optimal speed to the destination and then wait at the destination (Wang et al., 2013). Thus, given d and τ, the minimum fuel consumption, denoted by function $F(d,\tau)$, is denoted as:

$$F(d,\tau) = \begin{cases} \tilde{F}(d,\tau) & , if \quad d/\tau \geq v^* \\ [k + k' \cdot (v^*)^s] \cdot (d/v^*) & , otherwise \end{cases} \tag{4}$$

In the CISD, we define d_{ij} and τ_{ij} as the voyage distance and voyage time between Port i and Port j. d_{ij} is an input data, which can be easily obtained from geographical database. τ_{ij} is meaningful only if the cruise ship visits Port j directly after Port i; if this is the case, τ_{ij} is the time interval between the departure from Port i and the arrival at Port j, and hence is a decision variable. Given d_{ij} and τ_{ij}, the minimum bunker consumption between Port i and Port j can be calculated by $F(d_{ij}, \tau_{ij})$ in Eq. (4). We further define c^F as the unit price of fuel to compute the fuel costs.

3.4. Sequence of ports of call

Determining the sequence of ports of call is a crucial decision that we should make for the CISD problem. To represent the sequence in the manner of mathematical models, here, we use the same way to define it as many vehicle routing problems (VRPs) do: Set x_{ij} to one if the cruise ship visits Port j immediately after visiting Port i, and zero otherwise.

It is worthwhile to mention that visa restrictions of cruise passengers should be considered when designing the sequence of ports of call. Specifically, some ports of call belong to the same country and must be visited without interruption. For instance, if the cruise itinerary is Shanghai (China) →Nagoya (Japan) →Busan (Korea) →Kobe (Japan) →Shanghai (China), then the cruise passengers from China must obtain a tourist visa for Japan that allows multiple entries to Japan. If this is difficult for the cruise passengers, the two Japanese ports should be visited without interruption, for example, Shanghai (China) →Busan (Korea) →Nagoya (Japan) →Kobe (Japan) →Shanghai (China). To capture this practical consideration, we define $\mathbb{H}$ as the set of countries that can only be entered once, H_r as the set of ports belonging to Country $r \in \mathbb{H}$, and N_r as the number of ports in Country r, $N_r := |H_r|$.

3.5. Arrival and departure times at ports of call

For arrival time and departure times at ports of call, it does not make sense for a cruise ship to arrive at a time when the port is closed, for instance, at 3:00 am, or leave too late. Therefore, we define sets of possible arrival and departure times based on realistic situations as follows. First, we note that different ports may be located in different time zones and ignoring the difference in time zones will lead to incorrect decisions. Second, given the opening hours in a day for a port of call (evidently, the opening hours refer to the local time zone), the time zones of the home port and the port of call, we can define the time windows of the port of call in the planning horizon (i.e., from Time 0 to Time T). For instance, if a cruise ship departs from the home port (time zone: UTC+8) at 8:00 pm (i.e., Time 0 in our model), Port of call i is in time zone UTC+9 and opens every day from 7:00 am to 3:00 pm, then, the time windows of Port of call i is $\mathcal{T}_i = [10, 18] \cup [(10 + 24), (18 + 24)] \cup [(10 + 48), (18 + 48)] \cdots$. The arrival and departure times of the cruise ship at Port of call i, denoted by a_i and b_i, respectively, must be in the set, $a_i \in \mathcal{T}_i, b_i \in \mathcal{T}_i, i \in P_c$.

We define m_i as the minimum time (e.g., five consecutive hours) that the cruise ship should stay at Port i before the port closes when it arrives at a port during its opening hours $\mathcal{T}_i$. During the m_i hours, the cruise ship could replenish consumables or fuel and the cruise passengers could have a tour around the city. Given minimal staying hours in ports, we can refine the set of arrival time windows at Port of call i, for instance, if $m_i = 6$ and $\mathcal{T}_i = [10, 18] \cup [(10 + 24), (18 + 24)] \cup [(10 + 48), (18 + 48)] \cdots$, the set of possible arrival times (i.e., a_i) is $\mathcal{T}_i' = [10, 12] \cup [(10 + 24), (12 + 24)] \cup [(10 + 48), (12 + 48)] \cdots$.

3.6. Notations for the problem

Based on above problem description, to clarify the solution approach that will be elaborated in the next section, some notations for the problem are listed below.

Input parameters:

i	index of a port
t	index of a time period
P	set of all ports of call and home ports, $P = \{1,2, \cdots, N-1, N\}$, where 1 and N represent home ports
P_c	set of all ports of call, $P_c = \{2,3, \cdots, N-2, N-1\}$, excluding home ports
$\mathbb{T}$	set of all time periods in one cruise, $\mathbb{T} = \{0,1,2, \cdots, T-1, T\}$
$F(d,\ \tau)$	fuel consumption (tones) of the cruise ship if the voyage distance is d and the sailing time is τ
c^F	unit fuel price of the cruise ship
d_{ij}	voyage distance between Port i and Port j
$g_i(t)$	number of the utility at Port i for Time period t
m_i	minimum time that the cruise ship should stay in Port of call i
p^u	unit monetary value of the utility
v^{max}	maximum speed of the cruise ship
$\mathcal{T}_i$	set of all possible visiting time windows of Port i

$\mathcal{T}_i'$	set of all possible arrival time windows of Port i
H_r	set of ports that belong to Country $r \in \mathbb{H}$
$\mathbb{H}$	set of countries that can only be entered once
N_r	number of ports in H_r
M	a sufficiently large number

Decision variables:

a_i	time when the cruise ship arrives at Port i
b_i	time when the cruise ship departs from Port i
x_{ij}	binary, set to one if the cruise ship visits Port j immediately after visiting Port i, and zero otherwise.
θ_{ij}	sailing time on the leg from Port i to Port j

Some other auxiliary decision variables will also be introduced in the next section.

4. Solution Approach for Cruise Itinerary Schedule Design (CISD)

In this section, we develop an efficient solution algorithm to obtain optimal solutions by analyzing some special features of the problem.

5.1. Complexity of the CISD problem

Proposition 1: The CISD problem is NP-hard.

Proof: Suppose that all of the utilities $g_i(t)$ are zero, all of the minimum staying times m_i are zero, all of the time windows $\mathcal{T}_i' = \mathcal{T}_i = \mathbb{T}$, and the fuel consumption function $F(d,\ t)$ is proportional to the distance d and there is no visa restriction. Then the CISD problem becomes the one that finds the shortest distance to visit all of the ports of call from the home port and then returns to the home port, which is exactly the travelling salesman problem (TSP). Since the TSP is NP-hard (Cormen et al., 2009), the general version of the CISD problem is also NP-hard. ■

5.2. Dynamic programming for the model

Despite the NP-hardness of the problem in nature, we find that in realistic cases the number of ports of call is not large. Figure 4 shows the statistics on the trips in 2016 of the biggest cruise ship—The Carnival Vista—owned by Carnival Cruise Line. From the figure, we note that the number of ports of call and cycle time in a cruise itinerary is not large, ranging from two to nine and 5 days to 13 days, respectively. Thus, we could enumerate all of the sequences of visiting the ports of call for one specific cruise itinerary. Meanwhile, as can be seen in the third part of the figure, the ports of call among the trips are normally located in several countries. When considering the visa restrictions, some infeasible sequences for the ports of call can be deleted directly without exploring.

The total number of possible sequences for $(N-2)$ ports of call is $(N-2)!$. Let s denote the index of a sequence, $s \in S = \{1,2,\cdots,(N-2)!-1,(N-2)!\}$. Then, for each sequence s, we develop a dynamic programming (DP) approach to find the optimal arrival and departure times for each port of call. As mentioned in Section 3, the sequence (denoted by $x_{ij}, \forall i \in P\backslash\{N\}, j \in P\backslash\{1\}, i \neq j$) and the arrival and departure times of each port of call (i.e.,a_i and b_i,$\forall i \in P_c$) are two critical decisions in the CISD. The combined enumeration and DP approach could hence obtain the optimal solution for the CISD problem.

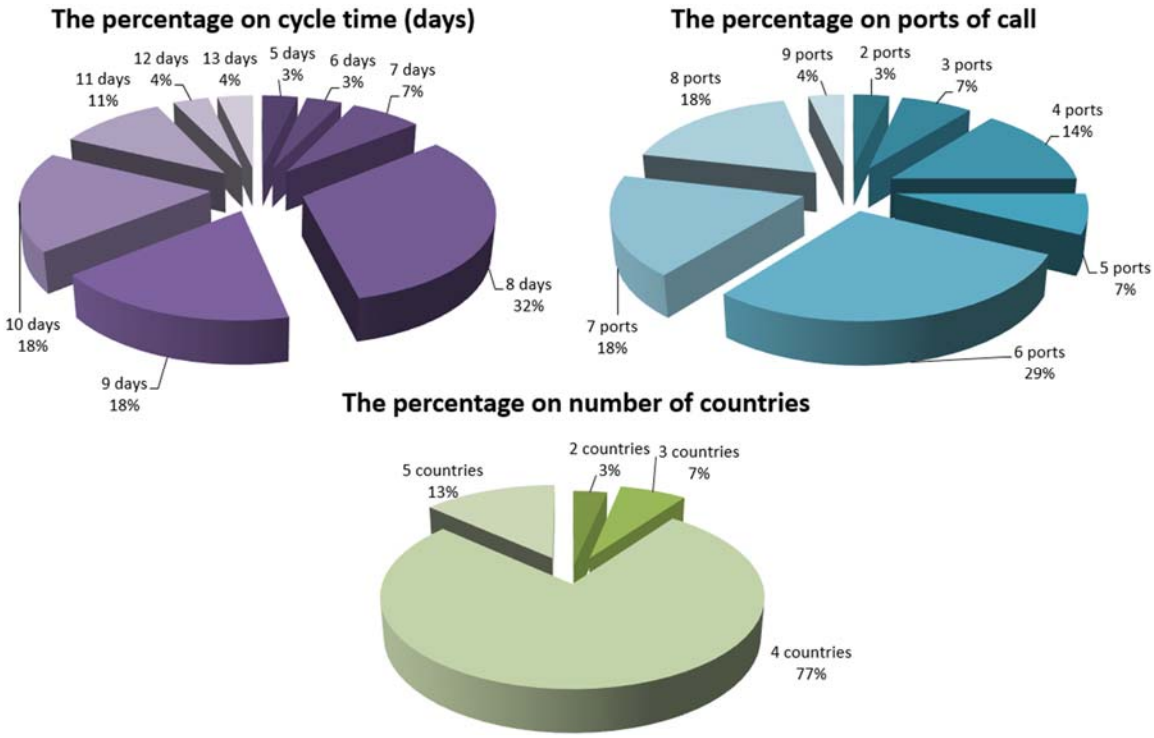

Figure 4: The statistic on the count of trips for the Carnival Vista (Cruise Ship Schedule, 2016)

We now consider a special sequence s that visits the ports of call according to their IDs, i.e., $1,\cdots j, j+1 \cdots N$, in which 1 and N refer to the home ports and j is the j^{th} port visited on the cruise itinerary. The cruise ship leaves Port 1 at Time 0 (i.e., $b_1 = 0$), and returns to Port N at Time T (i.e., $a_N = T$). We need to determine the optimal arrival and departure times at each Port $2,\cdots j, j+1 \cdots N-1$. The purpose of examining this special case is for notational convenience;

any sequence can be addressed using the same DP approach.

To apply DP, we firstly define $U_j(t)$ as the maximum possible profit (to be determined) from Time t to T, if the cruise ship arrives at Port j at Time t, and $V_j(t)$ as the maximum possible profit (to be determined) from Time t to T, if the cruise ship leaves Port j at Time t. Then, we define τ_j^v as the voyage time from the j^{th} port to the $(j+1)^{th}$ port, and τ_j^p as the staying time at the j^{th} port. The τ_j^v and τ_j^p are two decision variables in the DP algorithm. Third, since the cruise ship has a maximum sailing speed v^{max} and the minimum time spent at Port j is m_j, the earliest possible arrival time at Port j can be computed by assuming the cruise ship sails at its highest speed and spends the least time at previous ports of call:

$$a_j^{min} = \begin{cases} \sum_{k=1}^{j-1} \left\lceil \frac{d_{k,k+1}}{v^{max}} \right\rceil & ,j = 2 \\ \sum_{k=1}^{j-1} \left\lceil \frac{d_{k,k+1}}{v^{max}} \right\rceil + \sum_{k=2}^{j-1} m_k & ,j = \{3,4,\cdots N-1\} \end{cases} \tag{5}$$

The latest departure time is:

$$b_j^{max} = \begin{cases} T - \left(\sum_{k=j}^{N-1} \left\lceil \frac{d_{k,k+1}}{v^{max}} \right\rceil + \sum_{k=j+1}^{N-1} m_k \right) & ,j = \{2,3,\cdots N-2\} \\ T - \sum_{k=j}^{N-1} \left\lceil \frac{d_{k,k+1}}{v^{max}} \right\rceil & ,j = N-1 \end{cases} \tag{6}$$

Define

$$a_j^{max} := b_j^{max} - m_j,\ b_j^{min} := a_j^{min} + m_j \qquad \forall j \in P_c \tag{7}$$

To be feasible, the arrival time at Port j must be in the interval $\left[a_j^{min}, a_j^{max}\right]$ and the departure time must be in $\left[b_j^{min}, b_j^{max}\right]$.

In the DP approach, we have the boundary conditions:

$$U_N(t) = \begin{cases} -\infty, t \neq T \\ 0 \quad ,t = T \end{cases} \qquad \forall t \in \mathbb{T} \tag{8}$$

and the recursive relations between $V_j(t)$ and $U_j(t)$ for all $t \in \mathbb{T}$, $j \in \{1, 2, \cdots N-1\}$ are:

$$V_j(t) = \begin{cases} max_{\lceil d_{j,j+1}/v^{max} \rceil \leq \tau_j^v \leq T-t} \left\{ -c^F \cdot F\left(d_{j,j+1}, \tau_j^v\right) + U_{j+1}\left(t + \tau_j^v\right) \right\}, t \in \mathcal{T}_j \cap \left[b_j^{min}, b_j^{max}\right] \\ -\infty \qquad ,otherwise \end{cases} \tag{9}$$

$$U_j(t) = \begin{cases} \max_{m_j \leq \tau_j^p \leq T-t} \left\{ p^u \cdot \sum_{h=t}^{t+\tau_j^p-1} g_j(h) + V_j\left(t + \tau_j^p\right) \right\}, t \in \mathcal{T}_j' \cap \left[a_j^{min}, a_j^{max}\right] \\ -\infty \qquad ,otherwise \end{cases} \tag{10}$$

Eq. (9) is used to maximize the profit from Time t to T given that the cruise ship leaves Port

j at Time t. Here, the possible departure times at Port j should be restricted in the intersection of $\mathcal{T}_j$ and $[b_j^{min}, b_j^{max}]$. The $\mathcal{T}_j$ is the set of opening hours of Port j. To enforce that the ship must leave Port j within its opening hours (i.e., time windows), we also need to include $\mathcal{T}_j$ in Eq. (9). The $[b_j^{min}, b_j^{max}]$ is the possible departure time range defined by Eq. (5), (6) and (7) based on the speed of the ship and minimal staying hours at the ports of call. It is impossible for the cruise ship to depart from Port j at Time t if the t is out of the range (i.e., $[b_j^{min}, b_j^{max}]$). The intersection of these two parts offers a strict limit on the possible departure time at Port j. This is crucial for Eq. (9), because the $V_j(t)$ with impossible Time t should be set to $-\infty$ in order to avoid unexpected problems in processing DP. For all possible Time t, the $V_j(t)$ is calculated by $U_{j+1}(t+\tau_j^v)$ and $-c^F \cdot F(d_{j,j+1}, \tau_j^v)$. The former is the maximal profit from Time $t+\tau_j^v$ to T, if the cruise ship arrives at Port $j+1$ at time $t+\tau_j^v$. As this maximal profit of Port $j+1$ has been obtained beforehand in the DP process, the $V_j(t)$ can be easily achieved by adding the bunker fuel profit (i.e., $-c^F \cdot F(d_{j,j+1}, \tau_j^v)$) between Port j and Port $j+1$. For the maximum $V_j(t)$, we need to determine the optimal voyage time (i.e., τ_j^v) between two ports, which is selected within the possible range (i.e., $\lceil d_{j,j+1}/v^{max} \rceil \le \tau_j^v \le T-t$). Here, we notice that the range may be infeasible when time t is large. For instance, when $t=T$, the upper bound of the range is zero, which is less than the lower bound of the range. To avoid these infeasible cases, we set $V_j(t)$ to $-\infty$ directly without calculation when the upper bound is less than the lower bound (i.e., $T-t < \lceil d_{j,j+1}/v^{max} \rceil$).

Eq. (10) is used to maximize the profit from Time t to T given that the cruise ship arrives at Port j at Time t. Similar to Eq. (9), the possible arrival time set at port j is also to be defined before calculation, which is the intersection of $\mathcal{T}_j'$ and $[a_j^{min}, a_j^{max}]$. The possible arrival time range (i.e., $[a_j^{min}, a_j^{max}]$) based on the speed of the ship and minimal staying hours at the ports of call is also achieved by Eq. (5), (6) and (7). For all possible arrival Time t, the $U_j(t)$ is determined by $V_j(t+\tau_j^p)$ and $p^u \cdot \sum_{h=t}^{t+\tau_j^p-1} g_j(h)$. The former is the maximal profit from time $t+\tau_j^p$ to T, if the cruise ship leaves Port j at time $t+\tau_j^p$. As this maximal profit of Port j in terms of departure times has been obtained previously, the $U_j(t)$ can be easily obtained by adding the utility profit (i.e., $p^u \cdot \sum_{h=t}^{t+\tau_j^p-1} g_j(h)$) at Port j. For the maximum $U_j(t)$, we need to determine the optimal staying time (i.e., τ_j^p) at Port j, which is chose within the possible range (i.e., $m_j \leq \tau_j^p \leq T - t$). Similar to Eq. (9), the range could also be feasible. Thus, we set $U_j(t)$ to minus infinity when the upper bound of the range is less than the lower bound of the range (i.e., $T - t < m_j$).

$V_1(0)$ represents the maximum possible profit from Time 0 to T if the cruise ship departs from Port 1 at Time 0 and the value of $V_1(0)$ is obtained by executing the above algorithm. In essence, the value of the $V_1(0)$ is the optimal objective value for the sequence. The optimal arrival and departure times at each port of call under this sequence can be calculated by using the optimal decision variables $(\tau_j^v)^*$ and $(\tau_j^p)^*$ in DP, which is shown as follow.

$$a_j^* = \begin{cases} \sum_{k=1}^{j-1} (\tau_j^v)^* & ,j = 2 \\ \sum_{k=1}^{j-1} (\tau_j^v)^* + \sum_{k=2}^{j-1} (\tau_j^p)^* & ,j = 3, 4, \cdots N-1 \end{cases} \tag{11}$$

$$b_j^* = a_j^* + (\tau_j^p)^* \qquad ,j = 2,3, \cdots N-1. \tag{12}$$

The detailed algorithm for the above-mentioned DP is given in **Appendix A**, denoted as *Algorithm 1*.

5.3 Improving the enumeration

When we enumerate all of the sequences, we may stop once we know that this sequence cannot be better than the incumbent best one. To this end, we develop an efficient method to find a high-quality upper bound on the profit of a sequence.

To begin with, the total cruise rotation time (i.e., T) is divided into the sailing time $\hat{T}$ (i.e., the total voyage time at sea) and port time $T - \hat{T}$ (i.e., the total staying hours in the ports of call). The optimal division is to be determined. Then, given a sequence s, we let $j(s)$ be the ID of the physical port of the j^{th} port visited on s. Here, we need to note that there is an underlying bound for the total sailing time (i.e., $\hat{T}$) when considering the speed restriction on sail (i.e., the speed cannot exceed v^{max}) and staying time restriction in ports (i.e., the stating in each port of call must exceed m_i). The bound for the total sailing time in each sequence s is.

$$\sum_{j=1}^{N-1} \lceil d_{j(s),(j+1)(s)} / v^{max} \rceil \leq \hat{T} \leq T - \sum_{i=1}^{N-1} m_i \tag{13}$$

where we notice that for some sequences s, the upper bound in (13) (i.e., $T - \sum_{i=1}^{N-1} m_i$) could be less than the lower bound in (13) (i.e., $\sum_{j=1}^{N-1} \lceil d_{j(s),(j+1)(s)} / v^{max} \rceil$), which means under these sequences, it is impossible for the cruise ship to return to the home port on time even if it sails at the maximum speed in the whole cruise. To improve the enumeration, we just drop these sequences and calculate the next one.

An upper bound on the profit from the negative fuel cost when the total sailing time is $\hat{T}$, denoted by $UB_s(\hat{T}, s)$, can be calculated as follow.

$$UB_s(\hat{T}, s) = \begin{cases} -c^F \cdot F\left(\sum_{j=1}^{N-1} d_{j(s),(j+1)(s)}, \hat{T}\right) & , \hat{T} \text{ satisfies (28)} \\ -\infty & , \text{otherwise} \end{cases} \quad (14)$$

where the fuel consumption is the lowest when the cruise ship sails at a constant speed.

An upper bound on the monetary value of the total utilities when the total port time is $T - \hat{T}$, denoted by $UB_p(T - \hat{T})$, can be calculated by solving an integer-linear program. First, we let

$$G_j(\delta_j) := max_{0 \le \tau \le 23} \sum_{h=\tau}^{\tau+\delta_j-1} p^u \cdot g_j(h) \quad (15)$$

That is, $G_j(\delta_j)$ is the maximum monetary value from the utility for spending δ_j consecutive hours at Port j. To obtain $UB_p(T - \hat{T})$, we let binary variable $z_{j\delta_j}$ be one if and only if the time spent at Port j is δ_j. Since the minimum time spent at Port j is m_j, to have a feasible port time solution, the time spent at Port j must be between m_j and $T - \hat{T} - \sum_{i \in P_c \setminus \{j\}} m_i$. The model for obtaining $UB_p(T - \hat{T})$ is:

$$[\boldsymbol{M1}] \quad UB_p(T - \hat{T}) = max \sum_{j \in P_c} \sum_{\delta_j = m_j}^{T - \hat{T} - \sum_{i \in P_c \setminus \{j\}} m_i} G_j(\delta_j) z_{j\delta_j} \quad (16)$$

subject to:

$$\sum_{\delta_j = m_j}^{T - \hat{T} - \sum_{i \in P_c \setminus \{j\}} m_i} z_{j\delta_j} = 1 \qquad \forall j \in P_c \quad (17)$$

$$\sum_{j \in P_c} \sum_{\delta_j = m_j}^{T - \hat{T} - \sum_{i \in P_c \setminus \{j\}} m_i} \delta_j z_{j\delta_j} = T - \hat{T} \quad (18)$$

$$z_{j\delta_j} \in \{0,1\} \qquad \forall j \in P_c, \delta_j \in \left\{m_j, m_j + 1, \cdots, T - \hat{T} - \sum_{i \in P_c \setminus \{j\}} m_i\right\} \quad (19)$$

Note that although $M1$ is an integer linear program, we only need to solve it once for each possible value of $T - \hat{T}$ in one CISD problem.

An upper bound on the profit of a sequence s can now be easily obtained:

$$UB(s) = max_{\sum_{j=1}^{N-1}\lceil d_{j(s),(j+1)(s)}/v^{max}\rceil \leq \hat{T} \leq \; T-\sum_{i=1}^{N-1} m_i} \left[UB_s(\hat{T}, s) + UB_p(T - \hat{T})\right] \quad (20)$$

Based on above-mentioned definitions and formulations, the algorithm for improving the enumeration can be designed (denoted as *Algorithm 2*), and is shown in **Appendix B**.

When the procedure for the *Algorithm 2* is finished, the optimal profit for the CISD problem is achieved, which is UB_{best}, and the best sequence s^* is recorded. The details of arrival times and departure times can also be checked in the results of the DP for the sequence s^*.

6. Computational Experiment

In order to validate the effectiveness of the proposed model $M1$ and the efficiency of the developed solution method, we conduct numerical experiments by using a PC (Intel Core i5, 2.1G Hz; Memory, 4G). The solution method is implemented by Matlab R2013b. The integer linear program $M1$ is solved by CPLEX12.1 with concert technology of C# (VS2008).

Before conducting experiments, we need to notice that the objective of our model does not represent the final profit for a cruise. To calculate the final profit, the incomes (e.g., tickets for cruise passengers) and the costs (e.g., operating cost of the cruise ship) should be included. To combine these incomes and costs together, a margin for per cruise passenger per day is assumed, denoted by p^m. In practice, this margin can be easily achieved by cruise companies to analyze the previous profit reports of cruises. Here, we take US$100 as the value of p^m. The total number of cruise passengers and the cycle time for one cruise are denoted by φ and π respectively. Thus, the final profit can be calculated by $\text{profit} = p^m \times \varphi \times \pi + Z$, in which Z is the optimal result obtained by the proposed method. In the following experiments, the final profit will be used as the optimal profit to be displayed in tables.

For the bunker cost function of the cruise ship deployed on the itinerary, according to Du et al (2011), the coefficients in bunker consumption function (i.e., k, k' and s in Eq. (2)) are related to the size of cruise ships. Here, we take the Explorer of the Seas (a cruise ship that belongs to the Royal Caribbean) as the example, which is a jumbo ship with 138,000 deadweight tons. We set the coefficients in Eq. (2) as $k = 698$, $k' = 0.000865$ and $s = 4.5$. The fuel price for the ship is assumed to be US$251.50 per metric ton, which is the price of IFO 380 at the port of Singapore on 8 September 2015 according to Ship & Bunker (2015).

6.1. Estimation of utility distribution

When using the above-mentioned method to design itineraries, cruise companies may find it difficult to evaluate the monetary value of the utilities in ports of call. To facilitate their designing, we propose a rational idea to help them derive the utility profit. This utility profit is calculated based on analyzing the announced itineraries of their own or other cruise companies. For example, if we are helping Carnival Cruise Line to design a new itinerary, we could analyze the itineraries from the Royal Caribbean, which is the biggest competitor for Carnival Cruise Line, in order to obtain the adopted utility profit.

Our idea to derive the utility profit of ports of call comes from the observation of the different arrival times at these ports in announced itineraries. Take the "*11 Night Middle East & Asia Cruise*" operated by the Royal Caribbean International as an example. In Day 5 of the itinerary, the cruise ship arrives at the port of Mormugao, India at 6:00 am. In Day 10 of the itinerary, the ship arrives at the port of Penang, Malaysia at 12:00 pm. Here, we have the question: why does not the ship arrive at these ports one hour earlier or one hour later? For the port of Mormugao, the ship cannot arrive at 5:00 am because the port closes for service at that time, but it is possible to arrive at 7:00 am. However, the cruise ship does not postpone the arrival time for one hour even if this means a saving of US$780.65 in the bunker cost, which is calculated by the bunker cost function. One reasonable explanation for this is that the cruise ship could earn more than US$780.65 from the utility in the time period from 6:00 am to 7:00 am at the port of Mormugao. For the port of Penang, the cruise ship could arrive at 11:00 am (or 1:00 pm) if possible, which increases (or decreases) the bunker cost by US$855.02 (or US$784.45). This means that the utility profit of the port of Penang from 11:00 am to12:00 pm is less than US$855.02 (or from 12:00 pm to 1:00 pm is more than US$784.45).

According to the above analysis, the main idea for the method is: we derive the utility profit based on the announced itineraries operated by the other cruise companies. This method for estimating the utility profit might not be the best, but a rational alternative for the estimation of utility distribution. In Section 7, we will also propose a potential marketing method to estimate the utility.

6.2. Impact of different units of time period

In Section 3.1, we mentioned that we use one hour as a time period when implementing the CISD and the method can also handle time periods, e.g., half an hour. Here, we conduct some experiments on two types of settings of the time unit, including one hour and half an hour setting. The input data for testing the settings is randomly generated. Based on the above-mentioned method of the estimation, we randomly generate the utility distribution under the principles that bigger ports have higher utilities than small ports and noon hours have higher utilities than other opening hours. The results of the experiments are shown in Table 1.

From Table 1, we observe that the half an hour setting brings more profit than that of the one-hour setting. On average, the former increases the profit by 0.36%. However, the smaller time unit causes trouble in computational time. In the half an hour setting, the time to find the optimal results is longer than that of the one-hour setting. The average ratio between two settings is 5.31. Moreover, the ratio keeps increasing when the scale of instances becomes larger. For the cruise companies, half an hour setting can still be used to improve the total profit, as the problem is a strategic decision problem. In following experiments, in order to save the CPU time, we will use the one-hour setting rather than half an hour setting to solve the CISD problem.

Table 1: Comparison between different settings of time unit

Instance			**One hour**		**Half an hour**		**Comparison**	
# of ports of call	Cycle time	Instance id	Z_o	T_o	Z_h	T_h	$(Z_h - Z_o) / Z_o$	$\frac{T_h}{T_o}$
3	6	3_6	1.237	2	1.240	5	0.25%	2.50
	7	3_7	2.115	4	2.122	11	0.33%	2.75
	8	3_8	3.196	5	3.208	16	0.38%	3.20
	9	3_9	3.690	8	3.710	26	0.54%	3.25
5	8	5_8	1.545	6	1.553	29	0.51%	4.83
	9	5_9	2.440	10	2.447	41	0.30%	4.10
	10	5_10	3.374	13	3.392	70	0.53%	5.38
	11	5_11	4.070	15	4.079	90	0.22%	6.00
7	10	7_10	1.934	16	1.939	113	0.23%	7.06
	11	7_11	2.646	22	2.653	173	0.24%	7.86
	12	7_12	3.573	53	3.583	434	0.27%	8.19
	13	7_13	4.404	85	4.426	732	0.49%	8.61
						Average:	**0.36%**	**5.31**

Note: (1) "# of ports of call" column denotes the total number of ports of call involved in one cruise. This does not include two home ports. (2) "Cycle time" column indicates the total days for one cruise. (3) " Z_o" and " Z_h" columns list the optimal profits in the two settings of time unit with the unit of one million US dollars. (4) " T_o" and " T_h" columns show the CPU time (seconds) to solve the problem.

6.3.Performance of the enumeration improving method

We have proposed two exact solution methods. One enumerates all sequences of ports of call and applies DP to calculate all these sequences in order to find the optimal one; while the other one considers the enumeration improving (i.e., *Algorithm 2*). In order to test the efficiency of the method with the enumeration improving, we conduct experiments under different instance-scales by using the two methods. The comparisons are listed in Table 2.

As can be seen in Table 2, both two methods obtain the optimal results. This is verified by the same optimal profits shown in the column " Z_0" and the column " Z_1". However, the method without using the enumeration improving is extremely time-consuming. The average ratio of CPU time between this enumeration method and the method with the enumeration improving is on average 4.92, which implies the proposed method saves approximately 80% of computational time on average. More importantly, the time ratio between two methods increases dramatically with the growth of the instance-scale, which is shown in the last column of the table. This demonstrates that enumeration improved method is quite efficient to find the optimal solutions for the problem.

Table 2: Computational efficiency with and without using enumeration improving

Instance			**Enumeration improving method**		**No enumeration improving**		**Time ratio**
# of ports of call	Cycle time	Instance id	Z_0	T_0	Z_1	T_1	$\frac{T_1}{T_0}$
3	7	3_7	2.115	4	2.115	5	1.25
	8	3_8	3.196	5	3.196	8	1.60
	9	3_9	3.690	8	3.690	20	2.50
5	9	5_9	2.440	10	2.440	14	1.40
	10	5_10	3.374	13	3.374	25	1.92
	11	5_11	4.096	15	4.096	43	2.87
7	11	7_11	2.646	22	2.646	62	2.82
	12	7_12	3.573	53	3.573	198	3.74
	13	7_13	4.404	85	4.404	552	6.49
9	13	9_13	3.803	236	3.803	2035	8.62
	14	9_14	4.697	374	4.697	3961	10.59
	15	9_15	5.413	642	5.413	9753	15.19
						Average:	**4.92**

Note: (1) " Z_0" and " Z_1" columns list the optimal profits obtained by two methods with the unit of one million US dollars. (2) " T_0" and " T_1" columns show the CPU time (seconds) to solve the problem.

6.4. Sensitivity analysis for a real case with a natural geographical sequence

As we have already tested the efficiency of the proposed method, we will use this method to conduct sensitivity analysis in terms of three inputs, which are fuel price for the ship, minimal staying hours of ports of call and opening hours of ports of call. Here, we take a popular cruise line from the Royal Caribbean, “*14 Night Singapore to Fremantle Cruise*”, as an example, and suppose that we are helping the Carnival Cruise Line to design a similar one-way cruise. The nine ports involved in this cruise are shown in Figure 5, and it can easily be seen that this itinerary has a naturally geographical sequence, under which we may design a good visiting sequence by direct observation.

For this cruise, assume that we deploy the Carnival Vista, which is the biggest cruise ship of the Carnival Cruise Line. It has 133,500 deadweight tons. Same as the “*14 Night Singapore to Fremantle Cruise*” operated by the Royal Caribbean International, in this cruise, the cruise ship departs from the port of Singapore at 5:00 pm on Day 1 and arrives at the port of Fermantle, Australia at 7:00 am on Day 15. Singapore and Fermantle are the home ports. The information about the seven selected ports of call (i.e., Phuket, Langkawi, Kuala Lumpur, Geraldton, Bali, Lombok and Broome) and the two home ports are shown in Table 3. We assume that each port opens for the cruise ship at 7:00 am and closes at 7:00 pm on a daily basis. The fuel price for the cruise ship has been mentioned in Section 6.1, which is US$251.50 per metric ton.

Figure 5: The map for the ports in “*14 Night Singapore to Fremantle Cruise*”

Table 3: The information of selected ports in a real case

Port index	Port	Country	Latitude (N+,S–)	Longitude (W–,E+)	Minimal staying hours
1	Singapore	Singapore	1.35	103.82	***Home Port***
2	Phuket	Thailand	7.88	98.39	9

3	Langkawi	Malaysia	6.35	99.80	6
4	Kuala Lumpur	Malaysia	3.14	101.69	6
5	Geraldton	Australia	–28.77	114.61	6
6	Bali	Indonesia	–8.41	115.19	9
7	Lombok	Indonesia	–8.65	116.32	7
8	Broome	Australia	–17.95	122.24	6
9	Fremantle	Australia	–31.95	115.86	***Home Port***

The above-mentioned information is deemed as the baseline setting for the following sensitivity analysis. The sensitivity analysis on the fuel price is implemented at first, which is shown in Table 4. As it is shown in the table, there are two port sequences for different fuel prices. For the optimal profit, we notice that with the rising of the fuel price, the profit decreases evidently. If the fuel price increases by 20%, the profit drops by 0.81%. The number of voyage hours also changes in different settings of the fuel price. It keeps rising and stays unchanged for 268 hours when the fuel price increases by more than 40% compared with the baseline setting. The increasing tendency for the voyage hours in response to the rising fuel price is reasonable. This is because that lower fuel price may induce the cruise ship to sail faster in order to gain more utility profit from the ports of call, and the higher fuel price may impede the cruise ship to sail faster, because the bunker consumption increases significantly when speeding up, meaning that the extra bunker cost is higher at a higher fuel price.

Table 4: The sensitivity analysis on fuel price

Differentiation	**Solved optimal port sequence**	**Optimal profit**	**Deviation**	**Voyage hours**
0.40	[1 4 3 2 6 7 8 5 9]	5.969	2.52%	240
0.60	[1 4 3 2 6 7 8 5 9]	5.918	1.64%	263
0.80	[1 4 3 2 6 7 8 5 9]	5.870	0.82%	264
Baseline setting	[1 4 3 2 6 7 8 5 9]	5.822	0.00%	265
1.20	[1 4 3 2 6 7 8 5 9]	5.775	–0.81%	267
1.40	[1 2 3 4 6 7 8 5 9]	5.728	–1.62%	268
1.60	[1 2 3 4 6 7 8 5 9]	5.681	–2.43%	268

Note: (1) the coefficients in "Differentiation" column (i.e., {0.40, 0.60, 0.80, 1.20, 1.40, 1.60}) are used to multiply the baseline setting of the fuel price (i.e., US$251.50 per metric ton) to represent the price in each instance, e.g., $0.40 \times 251.50 = 100.60$. 2) "Deviation" column lists the gap between the current setting and the baseline setting with respect to the optimal profit. (2) "Optimal profit" column list the optimal profits obtained under different settings with the unit of one million US dollars.

Table 5: The sensitivity analysis on minimum staying hours

Differentiation	**Solved optimal port sequence**	**Optimal profit**	**Deviation**	**Staying hours**

–3	[1 2 3 4 6 7 8 5 9]	5.823	0.02%	55
–2	[1 2 3 4 6 7 8 5 9]	5.823	0.02%	55
–1	[1 2 3 4 6 7 8 5 9]	5.823	0.01%	57
Baseline setting	[1 4 3 2 6 7 8 5 9]	5.822	0.00%	60
+1	[1 4 3 2 6 7 8 5 9]	5.821	–0.02%	61
+2	[1 4 3 2 6 7 8 5 9]	5.819	–0.06%	64
+3	[1 4 3 2 6 7 8 5 9]	5.814	–0.14%	70

Note: (1) the coefficients in "Differentiation" column (i.e., $\{-3,-2,-1,+1,+2,+3\}$) are used to add the baseline setting of the minimal staying hours of each port call (i.e., the last column in Table (4) to obtain the minimal staying hours in each instance, e.g., for Port *2:* $-3+9=6$.

Table 5 shows the results of sensitivity analysis on minimal staying hours for the cruise ship dwelling in all ports of call. With the decreasing of minimal staying hours, the optimal profit increases steadily until the minimal staying hours of all ports of call decrease by two hours compared with the baseline setting. This is the point when the minimal staying hours lose its effect as the restriction. It is also observed from the table that the cruise ship tends to reduce its total staying hours in all ports of call with the minimal staying hours decreasing. However, it stands at 55 hours even when the minimal staying hours decrease further, which means the cruise ship still needs to stay in the ports of call for enough hours in order to obtain decent profits from the utility of these ports.

The results of sensitivity analysis on opening hours of the ports of call are given in Table 6. In the baseline setting, all ports of call open to the cruise ship for twelve hours daily, which starts at 7:00 am and ends at 7:00 pm. Here, in the table, we change the daily opening hours for all ports of call by one hour each time. As can be seen from the table, the cruise ship tends to earn more profit as the ports open for longer time. However, the effect on the profit increasing from the increase of opening hours is not obvious, which is generally less than 0.05% per hour in the instances shown in the table. Therefore, it is highly recommended that cruise companies need to be considerate when they want to increase the profit by requiring more opening hours from the ports of call. In the table, the total staying hours also increase steadily with more opening hours offered by the ports of call, which is the direct reason for the increasing of the optimal profit as more utility profit is earned from the ports of call.

Table 6: The sensitivity analysis on opening hours

Differentiation	Solved optimal port sequence	Optimal profit	Deviation	Staying hours
–3	[1 4 3 2 6 7 8 5 9]	5.817	–0.09%	55

–2	[1 4 3 2 6 7 8 5 9]	5.820	–0.04%	56
–1	[1 4 3 2 6 7 8 5 9]	5.820	–0.04%	58
Baseline setting	[1 4 3 2 6 7 8 5 9]	5.822	0.00%	60
+1	[1 4 3 2 6 7 8 5 9]	5.822	0.00%	60
+2	[1 2 3 4 6 7 8 5 9]	5.823	0.02%	61
+3	[1 2 3 4 6 7 8 5 9]	5.823	0.02%	66

Note: (1) The coefficients in "Differentiation" column (i.e., $\{-3, -2, -1, +1, +2, +3\}$) are used to add the baseline setting of the opening hours of each port call (i.e., twelve hours) to achieve the opening hours in each instance, e.g., $-3 + 12 = 9$.

In sum, if the ports on an itinerary have a naturally geographical sequence, then port distance will dominate the design of the itinerary and other parameters have a marginal effect on the results. In the next section, we will examine cases in which the ports do not have a naturally geographical sequence.

6.5. Further analysis for cases without a natural geographical sequence

In some cruise areas with scattered ports and islands, such as the Caribbean Sea area (see Figure 6), the ports of call in some cruise itineraries may not have a natural geographical sequence, and there are many potentially good sequences given a set of ports of call. In this section, we examine the value of sophisticated models for the itinerary design in such cruise areas, in which a geographical sequence may not be easy to derive by direct observation.

Figure 6: The map for the city ports around the Caribbean Sea

As the bunker cost is one of the major concerns for cruise companies, we would like to further conduct analysis on the fuel price to see how important an optimal port sequence is needed when the fuel price fluctuates. In order to conduct such analysis, we select 16 real cruise services operated by Royal Caribbean International in the Caribbean Sea area (Royal Caribbean International, 2016).

The itinerary maps of the 16 selected cruise services are given in **Appendix C**. Before conducting the analysis, we first redesign the itinerary schedules for the 16 real cruise service. The comparisons in terms of port sequence and port staying hours between the 16 designed itineraries and the 16 actual itineraries are listed in Figure 7. Here, the y-axis shows the total port staying hours for each itinerary, and the x-axis indicates the index of each cruise service. Symbol "Y" (Symbol "N") above pairs of bars indicates that the designed itinerary and the actual itinerary have the same port sequence (different port sequences) for the corresponding cruise service.

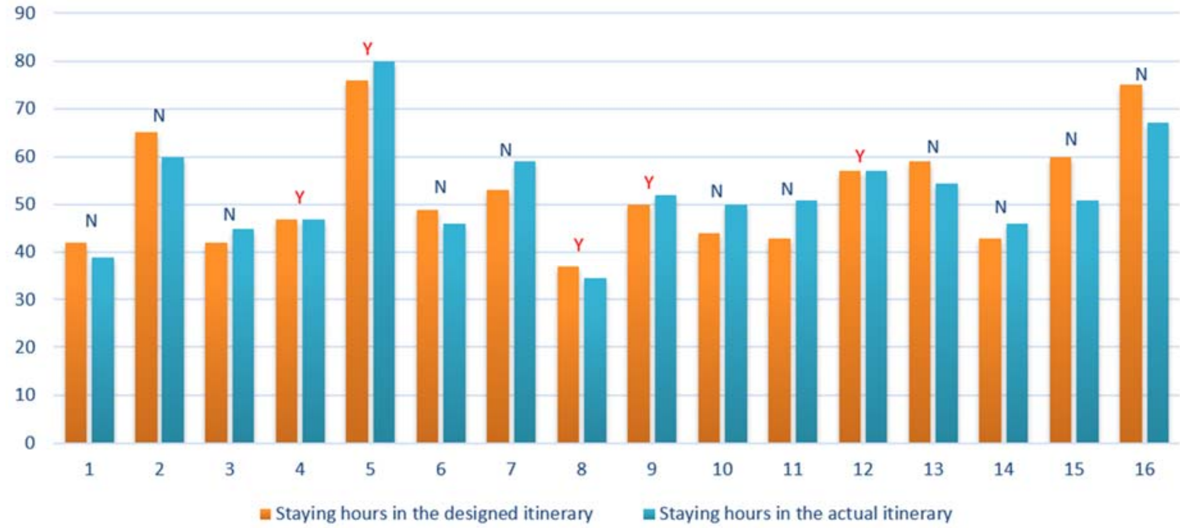

Figure 7: Comparisons between the designed itineraries and the actual itineraries

For the further analysis on the fuel price, based on the above-mentioned 16 cruise services, the optimal sequence under the baseline setting of the fuel price is obtained by using the proposed method at first. Then, the profits under other fuel price settings of this optimal sequence (i.e., Baseline itinerary) are calculated, which are defined as Baseline itinerary profit. The optimal sequences and the optimal profits under other fuel price settings are also re-optimized by using the proposed method, which are compared with Baseline itinerary and Baseline itinerary profit. The results of the comparisons are reported in Table 7.

From Table 7, we can observe that when the fuel price fluctuates, the optimal sequences for the itineraries are highly likely to change. Especially for large degrees of fluctuation, such as the 0.4-differentiation and the 1.6-differentiation on the fuel price, the majority of optimal sequences in the 16 cruise services are different from the optimal sequences obtained under the baseline setting of the fuel price. However, when the fuel price fluctuates, if the optimal sequence obtained in the baseline setting is fixed for the itinerary, there is significantly profit loss based on the average deviation in the table. In extreme cases, for instance, with 0.4-differentiation on the fuel price, the average profit loss is 4.52%. Therefore, it is critical for the cruise companies to re-optimize the visiting sequence when the fuel price fluctuates significantly in order to achieve a higher profit.

Table 7: Further analysis on fuel price

Differentiation on the fuel price	Number of instances having the same optimal port sequence as the baseline setting	Average optimal profit	Average Baseline itinerary profit	Average deviation
0.4	3	5.022	4.805	4.52%
0.6	6	4.894	4.772	2.56%
0.8	8	4.779	4.721	1.23%
Baseline setting	16	4.663	4.663	0.00%
1.2	10	4.538	4.490	1.07%
1.4	6	4.424	4.317	2.48%
1.6	5	4.322	4.159	3.92%

Note: (1) "Average optimal profit" column list the average profit among the 16 cruise services under each fuel price setting. (2) "Average optimality gap" column shows the gap between the average optimal profit and the average Baseline itinerary profit.

Table 8: The sensitivity analysis on the utility distributions

Differentiation on the standard deviation	Number of instances having the same optimal port sequence as the baseline setting	Average optimal profit	Average baseline itinerary profit	Average deviation
0.8	12	4.551	4.526	0.55%
0.9	14	4.615	4.601	0.30%
Baseline setting	16	4.663	4.663	0.00%
1.1	14	4.675	4.667	0.17%
1.2	13	4.717	4.694	0.49%

Note: (1) the coefficients in "Differentiation" column (i.e., {0.80, 0.90, 1.10, 1.20}) are used to multiply the baseline setting of the standard deviations (SDs) of the utility distributions of all ports of call to represent the SDs in each instance.

Based on the 16 selected cruise services from Royal Caribbean International, we also conduct some sensitivity analysis on the utility distributions of the ports of call. For each port of call in one instance, its utility distribution among hours in one day may have different values of mean and standard deviation (SD). The SD indicates the degree of variation of the utility among 24 hours. Here, we change the SD of the utility distribution for each port of call by multiplying a coefficient, which generates a new case of the instance for the sensitivity analysis. Given the 16 cruise services, we test each instance in different SDs. The effects on the optimal sequence and the optimal profit by different SDs (i.e., different variations of the utility distribution) are shown in Table 8. As can be seen from Table 8, the differentiation on the SD of the utility distribution has limited impacts on the optimal sequence and the optimal profit. For the majority of the 16 cruise services, the optimal sequence under different SDs is still the same as the optimal sequence under the baseline setting. Meanwhile, if we insist on using the baseline optimal sequence under different SDs in large differentiations such as 0.8-differentiation and 1.2 differentiation, the profit loss is around 0.5%.

6.6. Comparison between the proposed method and two heuristics

To test how important the proposed method is needed for the CISD, we compare our method with two heuristics, which are the minimum fuel cost heuristic and the shortest path heuristic. For the both heuristics, we firstly find the sequence with the shortest voyage distance. Then, in the minimum fuel cost heuristic, we design the itinerary schedule (i.e., the arrival and departure times at each port of call) by minimizing the total fuel consumption among all the voyage legs in the sequence without considering the port staying profit. In the shortest path heuristic, assuming that we also consider the port staying profit, the itinerary schedule is derived by using *Algorithm 1* based on the sequence. The total profits of the itinerary schedules obtained by the proposed method and the two heuristics under different problem scales are compared in Table 9.

Table 9: Comparisons between the proposed method and two heuristics

Instance group		**The proposed method**	**Minimum fuel cost heuristic**		**The shortest path heuristic**	
# of ports of call	Cycle time	Average profit	Average profit	Average deviation	Average profit	Average deviation
5	9	2.352	2.225	5.40%	2.325	1.15%
	10	3.249	3.044	6.31%	3.184	2.00%
	11	3.894	3.551	8.81%	3.764	3.34%
7	11	2.501	2.352	5.96%	2.457	1.76%
	12	3.297	3.084	6.46%	3.194	3.12%
	13	4.235	3.814	9.94%	4.074	3.80%
9	13	3.712	3.478	6.30%	3.604	2.91%
	14	4.494	4.121	8.30%	4.284	4.67%
	15	5.108	4.578	10.38%	4.818	5.68%
Average:				**7.54%**		**3.16%**

Note: (1) In each instance group, there are 10 instances, which are randomly generated based on major ports or islands in the Caribbean Sea area. (2) The average profit of 10 instances in one group is calculated for the proposed method and the two heuristics. (3) "Average deviation" shows the deviations between the average profit by the proposed method and the average profits by the heuristics.

As can be seen in Table 9, the deviation in the sense of the average profit between the proposed method and the minimum fuel cost heuristic is 7.54% on average. Such deviation suggests that when designing the itinerary schedule, only focusing on the fuel cost minimization would lead to significant profit loss, as the port staying profit is ignored in the optimization. Although the port staying profit is considered in the shortest path heuristic, there is still an average of 3.16% deviation to the optimal profit, which implies that the sequence with the shortest voyage is not necessarily the optimal sequence, especially when the ports do not have a naturally geographical sequence.

6.7. Managerial implications

The above numerical experiments shed lights on the nature of the CISD problem and enable us to discover a number of useful managerial implications for cruise companies summarized as follow.

First, when designing itineraries, using smaller time unit causes an increase in computational time, but it could bring more profits. As the planning of cruise itineraries is a strategic decision, it is recommended that cruise companies use smaller time unit in order to obtain more profits. The resulting schedule could be an "internal" schedule and the published schedule could be the one that rounds the internal schedule to one hour or half an hour. For instance, if it is calculated that the ship should arrive at a port of call at 6:50 am, then the cruise company can inform the captain to try to arrive at 6:50 am, and can inform the cruise passengers that the ship arrives at 7:00 am.

Second, when the ports of call on an itinerary have a naturally geographical sequence, then the distances between ports dominate the design of the schedule. In other words, the sequence of the ports of call with the shortest total distance is generally the optimal choice, and the impacts of minimum staying hours at ports and the opening hours at ports are marginal. The bunker fuel price has a larger effect on the total profit in the following way: when the fuel price is higher, the voyage time is longer, leading to lower fuel consumption and thereby lower fuel costs, and vice versa.

Third, when the ports of call on an itinerary do not have a naturally geographical sequence, as is the case for the largest cruise destination—the Caribbean Sea area, there are many potentially good schedules. We find that the fuel price has a much larger impact on the design of schedule. Specifically, when the fuel price deviates from the estimated price by 60%, sticking to the optimal schedule based on the estimated price will lead to a profit reduction of around 4% (cf. Table 7).

Hence, it is highly desirable for a cruise company to have an accurate estimation of the fuel price.

7. Utility estimation by marketing techniques

In this paper, a big challenge to implement the model in practice is the utility distribution estimation. We have proposed a method for the estimation in Section 6.1. Such method might not be the best, but a rational alternative. In this section, we further construct a potential utility estimation approach by using some marketing techniques. Here we design a *conjoint analysis* (Green et al., 1996; Ding et al., 2009) for evaluating customers' preference on cruise itinerary schedules. A choice-based conjoint experiment (Toubia et al., 2007; Gilbride et al., 2008) is conducted to obtain conjoint data. Then, the conjoint data is analyzed by a basic *multinomial logit model* (Hongmin and Woonghee, 2011; Li, 2011; Paat and Huseyin, 2012), and henceforth the utility distribution can be obtained. Note that this approach will not be implemented in this paper, as it involves tremendous research efforts in collecting conjoint data by interviewing many potential cruise passengers. However, this approach will be explored and developed in our future study.

7.1. Choice data collection

Before illustrating our analysis procedure, we would introduce the background of the analysis. Our conjoint experiment is conducted for the cruise itinerary schedules in a specific region (e.g., Asia region) rather than the global. It is due to that: firstly, loop cruise itineraries only traverse a set of ports in the same region, and secondly, the customers from different regions have different preferences on cruise itinerary schedules. For instance, the China-Japan-Korea cruise services are popular in China. In those services, cruise passengers in China would leave from the home port in China (such as Shanghai and Tianjin), visit some ports of call in Japan and Korea (such as Nagasaki (Japan), Fukuoka (Japan) and Busan (Korea)), and return to the home port finally. In the background of China-Japan-Korea area, many cruise itinerary schedules can be designed based on the cruise passengers' preference in China.

The first step in the *conjoint analysis* is to define the attributes (or factors) of a cruise service (or a cruise itinerary schedule) that have effects on customers' preference on cruise itinerary schedules. A straightforward attribute is the ticket price for the cruise service, which is an explanatory variable on how the attribute motivates the customers to buy the service. Each attribute has different levels, i.e., the possible values for the attributes. For example, for the ticket price attribute, there are possible levels in \$800, \$850, \$900 and so on. Xie et al. (2012) have proposed several attributes of a cruise ship that affect the customers' preference. In our analysis, we focus on some attributes related to the itinerary schedule design.

Table 10: Attributes and levels used in the conjoint analysis

Attributes	Levels
Price	A continuous variable (such as \$800, \$850 and \$900)
Rotation time	A discrete variable (such as 5 days, 6 ports and 7 ports)
Home port	A discrete variable (1 for Shanghai, and 2 for Tianjin)
Staying hours at Nagasaki (Japan): 6:00 – 7:00 am 7:00 – 8:00 am 8:00 – 9:00 am and so on	A dummy binary variable for each hourly time period (for example, 6:00 – 7:00 am in Nagasaki (Japan): 1 for the cruise ship staying at the port in 6:00 – 7:00 am; otherwise 0)
Staying hours at Fukuoka(Japan): 6:00 – 7:00 am 7:00 – 8:00 am 8:00 – 9:00 am and so on	A dummy binary variable for each hourly time period (for example, 6:00 – 7:00 am in Fukuoka(Japan): 1 for the cruise ship staying at the port in 6:00 – 7:00 am; otherwise 0)
Staying hours at Busan(Korea): 6:00 – 7:00 am 7:00 – 8:00 am 8:00 – 9:00 am and so on	A dummy binary variable for each hourly time period (for example, 6:00 – 7:00 am in Busan(Korea): 1 for the cruise ship staying at the port in 6:00 – 7:00 am; otherwise 0)

Note: (i) In "Staying hours at Nagasaki(Japan)" attributes, each hourly time period (e.g., 6:00 – 7:00 am in Nagasaki(Japan)) has a corresponding attribute. (ii) For "Staying hours in Nagasaki(Japan)" attribute, if the cruise ship for a cruise schedule arrives at Nagasaki(Japan) at 7:00 am and departs from the port at 11:00 am. Then, this schedule has the dummy binary variables (corresponding to 7:00 – 8:00 am, 8:00 – 9:00 am, 9:00 – 10:00 am and 10:00 – 11:00 am) equaling to one, and all other dummy binary variables equaling to zero.

Table 10 lists the attributes and levels used in our analysis under the background of China-Japan-Korea area. Apart from some regular variables (such as price, home port, and rotation time), there are some important hourly dummy binary variables (i.e., time-of-day variables), which are defined to show whether the cruise ship stays at a certain port of call during a certain hourly time period (e.g., 6:00 – 7:00 am) or not. The purpose of defining the hourly dummy binary variables is to estimate the utility distribution in each port of call (see Koppelman et al. (2008) for the application of time-of-day variables). Here. we denote E as the set of all the regular variables, and R as the set of all the hourly dummy binary variables. Assume that there is a mock-up cruise schedule: the ticket price is $850, the rotation time is 6 days, the home port is Shanghai, and it only has one port of call (Fukuoka (Japan), arrival time: 7:00 am, departure time: 5:00 pm). Then based on Table 10, the cruise service can be depicted in: "Price" with value 850; "Rotation time" with value 6; "Home port" with value 1; the binary variables in "Staying hours in Fukuoka(Japan)" corresponding to the staying hours from 7:00 am to 5:00 pm with value 1, and all other variables with value 0.

As we have decomposed the cruise schedule into such attributes and levels in Table 10. The next step is to generate some mock-up cruise schedules, which will be used in the interview with potential cruise passengers. $\mathbb{S}$ denotes the set for all generated cruise schedules. Those generated mock-up cruise schedules will be clustered into some choice sets. The schedule generation and cluster process can be done by Efficient Factorial Design in the statistical package of SAS (Kuhfeld, 2010). Each choice set contains a certain number of mock-up cruise schedules (denoted as J). Thereafter, the conjoint choice data is collected by interviewing potential cruise passengers (i.e., respondents), and asking them to choose one preferred cruise schedule (i.e., alternative) form each choice set. Here, we denote w and q as the index for the choice set and respondent, respectively, where $w \in \mathbb{W}$ and $q \in \mathbb{Q}$. For instance, we have generated 20 (i.e., $|\mathbb{S}| = 20$) unique mock-up cruise schedules, which are clustered into 12 choice sets (i.e., $|\mathbb{W}| = 12$) with 3 alternatives in each set ($J = 3$). Note that a mock-up cruise schedule can exist in several choice sets. Assuming that the choice set 1 contains the alternative 1, 2 and 4, each respondent q will be asked to choose one of the three alternatives for the choice set.

7.2. Choice data analysis

After collecting the conjoint choice data, the following step is to analyze the conjoint data. The most popular model to analyze the choice data is the *multinomial logit model* (Vermeulen et al., 2008). By using aggregate logit share techniques (Koppelman et al., 2008), we can derive the utility experienced by the respondent q when facing the jth alternative ($j \in \{1, .., J\}$) in the choice set w as follows:

$$\mathbb{U}_{qwj} = \sum_{e \in E} \alpha_e X_{wje} + \sum_{r \in R} \beta_r Y_{wjr} + \varepsilon_{qwj} \tag{21}$$

where α_e represents the partial utility (i.e., the coefficient or "partworths") for the eth regular variable (e.g., the variable for the price), and X_{wse} is the value of the eth no time-of-day variable for the jth cruise schedule in the choice set w (e.g., \$850 for the price). Similarly, β_r and Y_{wjr} are corresponding to the rth hourly dummy binary variable. ε_{qwj} is the error term. Note that $\beta_r, \forall r \in R$ indicate the utility distribution for all the ports of call.

Then, all the error terms are assumed to be i.i.d., under which we can calculate the probability that the cruise passenger q will choose the jth alternative in the choice set w:

$$P_{qwj} = \frac{\exp(\sum_{e\in E} \alpha_e X_{wje} + \sum_{r\in R} \beta_r Y_{wjr})}{\sum_{j'\in\{1,2,\dots,J\}} \exp(\sum_{e\in E} \alpha_e X_{wj'e} + \sum_{r\in R} \beta_r Y_{wj'r})} \quad ((22)$$

Based on the probability function, we can derive its log-likelihood function:

$$\ln(F(\alpha,\beta)) = \sum_{q\in\mathbb{Q}} \sum_{w\in\mathbb{W}} \sum_{j\in\{1,2,\dots,J\}} Z_{qwj} \ln(P_{qwj}) \quad (23)$$

where Z_{qwj} shows the conjoint choice data collected in Section 7.1, which equals one if and only if the cruise passenger q choose the jth alternative in the choice set w. In order to estimate $\hat{\alpha}_e, \forall e \in E$ and $\hat{\beta}_r, \forall r \in R$, we can maximize the above log-likelihood function by using the conjoint choice data as the input parameters. This procedure can also be done by SAS (Kuhfeld, 2000), and some technical issues are discussed in Vermeulen et al. (2008). Till now, the estimated coefficient $\hat{\beta}_r, \forall r \in R$ are obtained and indicate the utility distribution for each hour in each port of call.

Based on the estimated coefficient $\hat{\alpha}_e, \forall e \in E$ and $\hat{\beta}_r, \forall r \in R$, we further estimate the possible demand for a newly designed cruise schedule (denoted as γ): firstly, we investigate all the existing cruise schedule $s \in \mathbb{S}$ in the specific region (e.g., the China-Japan-Korea area) as well as the total regional market share (denoted as $\mathbb{M}$). The total regional market share can be easily obtained from some industry reports, such as Statista (2015). Then, the utility is calculated for each existing cruise schedule and the newly designed cruise schedule by: $\mathbb{U}_s = \sum_{e\in E} \hat{\alpha}_e X_{se} + \sum_{r\in R} \hat{\beta}_r Y_{sr}$ and $\mathbb{U}_\gamma = \sum_{e\in E} \hat{\alpha}_e X_{\gamma e} + \sum_{r\in R} \hat{\beta}_r Y_{\gamma r}$ (X and Y parameters here have the same meanings with that in Eq.(21)). Thereafter, the probability that potential cruise passengers in the regional market will choose the newly designed cruise schedule is $P(\gamma) = \frac{\exp(\mathbb{U}_\gamma)}{\sum_{s\in\mathbb{S}} \exp(\mathbb{U}_s) + \exp(\mathbb{U}_\gamma)}$. Next, the demand in the regional market for the newly designed cruise schedule γ is estimated as: $\mathbb{M} \cdot P(\gamma)$.

8. Conclusions

This paper addresses the Cruise Itinerary Schedule Design (CISD) problem that determines the optimal sequence of a given set of ports of call and the arrival and departure times at each port to maximize the monetary value of the utility minus the fuel cost. In view of the practical observations that most cruise itineraries do not have many ports of call, we first enumerate all sequences of ports of call and then optimize the arrival and departure times at each port of call by developing a dynamic programming approach. To improve the computational efficiency, we propose effective bounds on the profit of each sequence of ports of call, eliminating non-optimal sequences without invoking the dynamic programming algorithm. The computational experiments show that, first, the proposed bounds on the profit of each sequence of ports of call can considerably improve the computational efficiency; second, the total profit of the cruise itinerary is sensitive to the fuel price and hence, it is acceptable to use the shortest voyage distance method to design the schedule when the ports of call have a naturally geographical distance; in contrast, determining the sequence of ports of call solely by minimizing the overall voyage distance frequently leads to a significant reduction in the total profit when the ports do not have a naturally geographical sequence.

Given that cruise itineraries have fixed sequences of ports of call and fixed schedules, optimization-based itineraries planning tools should be able to increase the profit or save the cost for cruise shipping companies and improve the service quality for cruise passengers. Compared with other areas of transportation such as a truck, rail, and air (we note that transportation is not the purpose of cruising, but the cruise shipping is a part of transportation), there are few quantitative studies on cruise shipping. Nevertheless, cruise shipping has its own characteristics that need to be explored by industrial engineers/operations researchers. Moreover, the cruise market has maintained steady growth over the past 20 years despite the economic crisis in 2008 and cruising companies have ordered a number of large cruise ships to serve the mass market of cruising. We believe that there is a broad range of research topics in cruise shipping.

In this study, we have a limitation on assuming that the utility increase for the cruise passengers is additive over the port staying hours. However, in reality, cruise passengers' utility may depend on the time period that they want to stay in the ports, and the incremental utility of an extra port staying hour may decrease over time. Under this circumstance, the proposed solution can still be applied by analyzing the utility for each possible port staying period (e.g., 7:00 am to 6:00 pm) rather than each port staying hour. Our future study will further explore the relation between the utility increase for cruise passengers and the port staying hour increase

For the future research topics on the cruise industry, there are some recommendations. (1) The cruise itinerary design topic: based on the limitation of this study, we do not consider some practical constraints for the cruise ship to dwell in the ports of call, such as berth availability and tide effects in ports. Therefore, a more general cruise itinerary deign problem can be studied. (2) The cruise ship fleet deployment topic: cruise ships are frequently repositioned from one region to another region. From the perspective of the cruise lines, there are several decision problems on how cruise ships are repositioned. (3) The cruise ticket pricing topic: in airlines, many pricing policies, and strategies have been developed to increase revenue. However, the pricing in cruise shipping is not so flexible than that of the airlines. Thus, future studies can be conducted in the cruise pricing by borrowing some ideas from the pricing of the airlines.

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Appendices

Appendix A: Detailed Procedure for Algorithm 1

Algorithm 1: Dynamic programming

Initialize $U_N(t), \forall t \in \mathbb{T}$ according to Eq. (8);
For $j = N-1 \ to \ j = 1$
  If $j \neq 1$ **Then**
    For $t = 0 \ to \ t = T$
      If $T - t \geq \lceil d_{j,j+1}/v^{max} \rceil$ **Then**
        Calculate $V_j(t)$ by using Eq. (9);
      Else
        $V_j(t) = -\infty$;
      End If
    End For
    For $t = 0 \ to \ t = T$
      If $T - t \geq \ m_j$ **Then**
        Calculate $U_j(t)$ by using Eq. (10);
      Else
        $U_j(t) = -\infty$;
      End If
    End For
  Else // $j = 1$
    Calculate $V_j(0)$ by using Eq. (9), output the optimal values of the decision variables τ_j^v and τ_j^p, and return.
  End If
End For

Appendix B: Detailed Procedure for Algorithm 2

Algorithm 2: Improving the enumeration:

Define incumbent best profit: $UB_{best} = -\infty$;
Calculate $UB_p(T - \hat{T})$ for all possible values of $T - \hat{T}$;
Enumerate each sequence: sequence $s \in S$;
For $s = 1 \ to \ s = (N-2)!$
  If the sequence s violates the visa restriction **Then**
    Continue;
  End If
  If $T - \sum_{i=1}^{N-1} m_i < \sum_{j=1}^{N-1} \lceil d_{j(s),(j+1)(s)} / v^{max} \rceil$ **Then**
    Continue;
  End If
  Calculate the upper bound on the profit of the sequence s by Eq. (20)
  If $UB(s) \ \leq UB_{best}$ **Then**
    Continue;
  Else
    Use DP (i.e., *Algorithm 1*) to find the maximum profit of the sequence s, denoted

by UB_s^{DP}.

If $UB_{best} < UB_s^{DP}$ **Then**

Set $UB_{best} = UB_s^{DP}$ and record the sequence s;

End If

End If

End For

Appendix C: 16 Selected Cruise Itinerary from Royal Caribbean International

Source: https://secure.royalcaribbean.com/cruises?destinationRegionCode_CARIB=true&currentPage=0&action=update

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