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Formulating cargo inventory costs for liner shipping network design

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Abstract

This study examines how to incorporate the inventory costs of containerized cargoes into existing liner service planning models such that the designed networks could be improved while not causing extra modeling/computational burden. Two approaches are compared: (i) not considering the inventory costs at all, and (ii) incorporating the inventory costs associated with on board time and those related to transshipment by assuming a fixed connection time. The two models are compared with the ideal model capturing the exact inventory costs on a route choice problem and a capacity planning problem based on extensive randomly generated and practical numerical experiments. The results show that: first, ignoring the inventory costs in service planning models may lead to network design with much higher costs (poor network design decisions); second, in service planning models assuming weekly frequency, the inventory costs associated with on board time could be formulated exactly, and those related to the connection time of weekly services could be approximated by assuming fixed connection time of 3.5 days for ports with 1 day's minimum connection time and 4.5 days for ports with 2 days' minimum connection time.

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1. Introduction

Container shipping lines transport containers on regularly serviced liner routes similar to bus services (Liu et al., 2016). To this end, a container shipping line makes three levels of decisions: strategic, such as fleet size and mix; tactical, such as fleet deployment and schedule design; and operational, such as container routing and schedule recovery. Higher-level decisions have a bearing on lower-level decisions. For instance, container routing at the operational level depends on the shipping network determined at the strategic/tactical level. Therefore, a shipping line must account for lower-level decisions when making higher-level plans. In particular, researchers usually take into account container routing in network design or fleet deployment models by minimizing the sum of operating costs, voyage costs, and container handling costs. In reality, however, there is another cost component associated with container routing, that is, the inventory costs of the containerized cargoes. The inventory costs are proportional to the transit time, the volume of containers, and the time-sensitivity of cargoes. Although the inventory costs are not directly borne by shipping lines, shipping lines could design better services when taking them into consideration because customers are willing to pay more for shorter transit time.

1.1. Challenge of formulating inventory costs

The challenge of formulating inventory costs lies in the formulation of connection time, which is part of the overall transit time. The transit time of a container is the time interval between the departure from the origin port to the arrival at the destination port. The transit time consists of the time on board ships and the connection time at transshipment ports. This study uses the four routes (services) with weekly schedules operated by American President Lines (APL, 2014)[1] in Fig. 1 to illustrate how to formulate the transit time. In Fig. 1, "SIN(1)[Tue,Wed]" means that Singapore is defined as the first port of call and that a ship arrives at Singapore on Tuesday and leaves on Wednesday. Hence, the ship stays at Singapore for one day. The numbers on the arrows represent the transit time of each leg. For example, the transit time from Singapore to Karachi is 7 days (the time interval between the departure from Singapore on Thursday to the arrival at Karachi on the next Thursday).

For a given shipping network, the transit time of each leg is known, and the connection time of a transshipment container spent at a transshipment port can also be calculated based on the schedules of the services. Hence, it is relatively easy to account for the inventory costs in container routing models. Suppose that the minimum connection time at Singapore, defined as the minimum time interval required between the arrival of ship A to the departure of ship B for containers to be transshipped from A to B, is 1 day. Then, the connection time at Singapore for a container transported from SEM to KHI on routes 3 and 1 is 2 days (Tuesday to Thursday), from SEM to KHI on routes 4 and 1 is 5 days (Saturday to the next Thursday), from KHI to SEM on routes 1 and 3 is 1 day (Tuesday to Wednesday), and from KHI to SEM on routes 1 and 4 is 5 days (Tuesday to the next Sunday). If the minimum connection time at Singapore is 2 days, then the connection time for a container from KHI to SEM on routes 1 and 3 is 8 days (Tuesday to the next Wednesday).

[1]APL is wholly owned subsidiary of Neptune Orient Lines, a global transportation and logistics company engaged in shipping and related businesses based in Singapore. APL was the world's ninth largest container shipping company on 1 January 2014, having 121 ships with a total capacity of 629,479 twenty-foot equivalent units (TEUs) (UNCTAD, 2014).

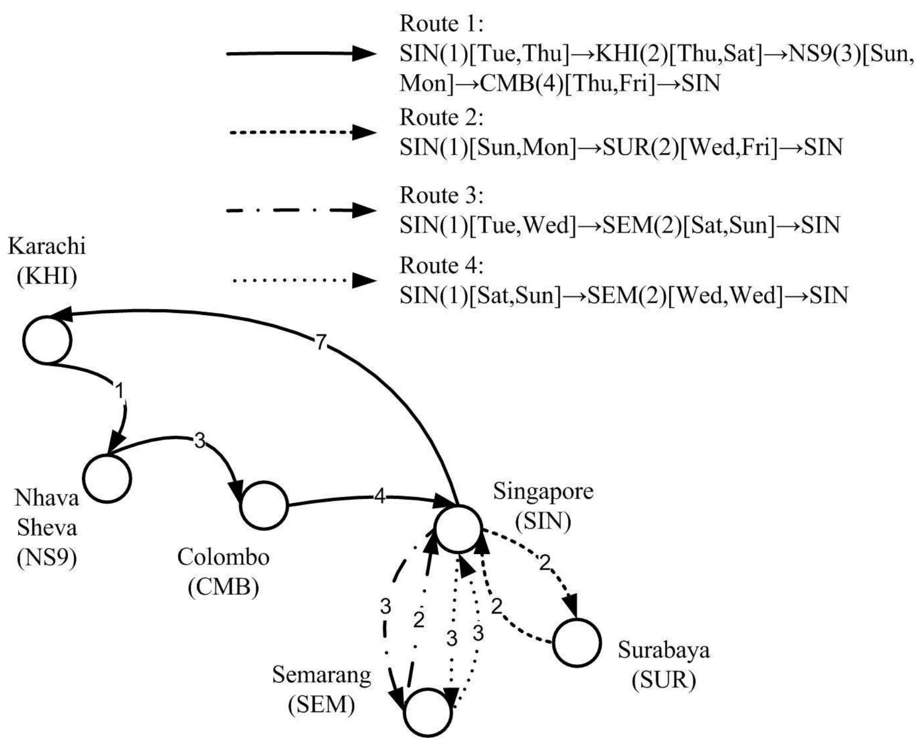

Figure 1: A liner shipping network with four routes

By contrast, it is extremely challenging to take into account the inventory costs when making tactical-level decisions with container routing. This is because the network is not designed yet, or at least some information regarding the network, e.g., the arrival and departure dates at each port of call, is not available. If the sailing speeds of the ships are given, the transit time between two ports for a ship is determined. Consequently, the inventory costs associated with the sailing time are proportional to the volume of containers transported between the two ports and easy to formulate and optimize. If the sailing speeds of the ships are to be determined, the inventory costs between two ports will be proportional to the volume of containers transported divided by the sailing speed. Since both the volume of containers and the speed are decision variables, the optimization model is non-convex, posing significant challenges for algorithm design.

Incorporating the inventory costs of transshipped containers is much more challenging than those associated with containers directly shipped between ports. For the ease of exposition, consistent with Bell et al. (2011, 2013); Wang et al. (2015a), this study assumes that all of the services are weekly[2]. Suppose that the minimum connection time at Singapore is $t_{\text{SIN}}^{\min} = 1$ day, and suppose that the arrival date at each port of call on the four routes shown in Fig. 1 is to be optimized. Let $\hat{t}_{1,1}$ be an integer variable representing the arrival date at Singapore on route 1, and $\hat{t}_{3,1}$ the arrival date at Singapore on route 3. Since a ship spends two days at Singapore on route 1, the departure date from Singapore on route 1 is $\hat{t}_{1,1} + 2$. Defining $\mathbb{Z}$ as the set of integers and $\mod (a, b)$ the remainder of the division of a by b, the connection time (days) at Singapore for a container transported from SEM to KHI on routes 3 and 1 is

$$\begin{cases} \mod (\hat{t}_{1,1} + 2 - \hat{t}_{3,1}, 7), & \text{if} \quad \mod (\hat{t}_{1,1} + 2 - \hat{t}_{3,1}, 7) \geq t_{\text{SIN}}^{\min} \\ \mod (\hat{t}_{1,1} + 2 - \hat{t}_{3,1}, 7) + 7, & \text{otherwise.} \end{cases} \tag{1}$$

Note that Eq. (1) is valid for calculating any connection time for weekly services as long as the minimum connection time is not greater than 7 days (1 week). Defining k as an auxiliary integer variable, the connection time in Eq. (1) can be reformulated

[2]In reality, some liner services are not weekly. However, the models proposed in the paper are also applicable to other frequencies by minimal modification as long as all of the services have the same frequency. If the services do not have the same frequency, e.g., a weekly service and a service which visits each port of call every 5 days, then they can be considered them as having the same frequency—every 35 day—by repeating the port rotation of the first service 5 times and repeating the second service 7 times.

as:

$$\min\{\hat{t}_{1,1} + 2 - \hat{t}_{3,1} + 7k \mid \hat{t}_{1,1} + 2 - \hat{t}_{3,1} + 7k \geq t^{\min}_{\mathrm{SIN}}, k \in \mathbb{Z}\}. \quad (2)$$

It can be seen that to formulate the connection time of transshipment containers using conventional operations research techniques, not only need decision variables regarding the arrival dates at each port of call are needed, but also auxiliary integer variables to capture the cyclic nature of the liner services are required. Moreover, the connection time (2) has to be multiplied by the decision variable representing the volume of transshipment containers to formulate the inventory costs in network design models with container routing. As a result, most network design models do not consider the inventory costs of containers.

1.2. Objectives and contributions

This study examines how to incorporate the inventory costs into existing liner service planning models such that: First, minimal modifications of the existing service planning models/algorithms are required. Second, the new service planning models should outperform the existing models that do not consider inventory costs, and hopefully, the new models produce near-optimal solutions compared with ideal models capturing the exact inventory costs (such ideal models are too hard to solve; otherwise they should be used).

Two approaches will be compared. The first approach, called [P1], does not consider the inventory costs at all. [P1] is widely used in the literature on network design. The second method [P2] incorporates the inventory costs associated with on board time and assumes the connection time at each port to be a fixed positive number, in other words, [P2] uses an approximate value in place of the exact connection time expressed by Eq. (2). [P2] could easily be incorporated into the existing network design models.

This study tries to compare the two modeling approaches using exact network design algorithms rather than heuristics. However, to date, only very small-scale network design problems have been solved to optimality as they are strongly NP-hard. To circumvent this difficulty, this study examines two less general and easier-to-solve service planning problems. In the first one, a set of candidate routes is already available and the only network design decisions are which routes to use. This problem is called the route choice (RC) problem. The second one is borrowed from Dong et al. (2015): the routes are jointly operated by several shipping lines and a shipping line could book whatever proportion of ship capacity it needs on each route. This problem is named the capacity planning (CP) problem.

In the RC/CP problem, it is assumed that the schedules for the candidate routes/routes whose slots are to be booked are available. Hence, [P1] and [P2] could not only be compared with each other, but also be compared with the ideal model, called [P0], that captures the exact inventory costs for the RC/CP problem. [P0] produces the optimal network design decisions and is a benchmark model. For a problem instance, this study first uses each of the three models [P0]–[P2] to obtain the network design decisions, i.e., which routes to operate in the RC problem and how much capacity on each route to book in the CP problem. This study then fixes the network design decisions and solves a container routing problem with the exact inventory costs. Note that whereas solving a general network design problem with exact inventory costs is extremely challenging, solving a container routing problem with exact inventory costs for a given network with known schedules is relatively easy as it is a multi-commodity network flow problem. The three models are compared on the basis of the sum of network-design related costs and container-routing related costs including inventory costs.

This study conducts extensive numerical experiments based on randomly generated networks, networks from the literature, and practical networks operated by APL to evaluate the performance of the two approximate models [P1] and [P2]. The results show that [P1], the model that ignores the inventory costs, is significantly inferior to [P2], the model that captures the inventory costs associated with on board time and suitable fixed connection time at transshipment ports. Moreover, in [P2] the fixed connection time could be set at 3.5 days for ports with 1 day's minimum connection time and 4.5 days for those with 2 days' minimum connection time. Although the modeling approaches in this paper are not too challenging from a mathematical point of view, the results are surprisingly elegant. The results are useful for planning decisions such as network design, fleet deployment, and speed optimization. Considering that shipping lines have been struggling to improve their shipping services since the economic crisis of 2008, our research is of timely and significant contributions.

The remainder of the paper is organized as follows. Section 2 reviews relevant literature. Section 3 describes the container routing problem. Section 4 presents two modeling approaches on the inventory costs for service planning problems. Section 5 reports the results of extensive numerical experiments for comparison of the models based on randomly generated networks and realistic networks. Section 6 concludes.

2. Literature review

Literature on maritime studies can be divided into the stream of port operations (Du et al., 2011, 2015; Zhen, 2015, 2016) and shipping operations (Luo et al., 2009; Meng and Wang, 2010; Wang et al., 2013a,b; Woo and Moon, 2014; Yin et al., 2014;

Luo et al., 2014; Lindstad et al., 2015; Meng et al., 2015; Zhang and Lam, 2015; Zhen et al., 2016). Our study falls into the category of shipping operations, for which Christiansen et al. (2013) and Meng et al. (2014) have provided excellent reviews. There are some quantitative models for estimating the inventory costs. Notteboom (2006) estimated that one day delay of a fully loaded 4,000-TEU ship from Asia to Europe implies a total time associated cost of €57,000, assuming the cargo value of €40,000/TEU, 3% opportunity cost, and 10% depreciation per year. Supposing €1 = US$1.36, the inventory cost rate is US$19.38/TEU/day. Wang et al. (2015c) examined how shipping lines balance the trade-off between fuel costs and inventory costs. They defined the term "perceived value of transit time (perceived VOTT)", which is the inventory cost rate adopted by shipping lines. They found that the perceived VOTT for APL and Maersk Line is generally between US$5/TEU/day and US$30/TEU/day. This means that a shipping line would be willing to pay an extra cost between $50,000 and $300,000 if it could reduce the transit time of 10,000 TEUs by one day.

Some studies have investigated the operational-level container routing problem with given liner services. Brouer et al. (2011) presented a space-time network to formulate container flow, which had ground arcs representing container storage at ports and travel arcs representing transporting containers between ports. A link-flow model and a path-flow model were developed. They did not explicitly mention the inventory costs of cargoes, however, a unit flow cost on each arc was incorporated. Song and Dong (2012) formulated a path-flow model requiring that a laden container was transshipped at most twice. They considered the handling costs, waiting costs at transshipment ports (storage costs), and lost-sale costs of laden containers, and handling, transportation and storage costs of empty containers. Although the inventory costs of the cargoes were not mentioned, they could easily be incorporated into the model. In sum, given a liner shipping network, the exact inventory costs could be captured in container routing formulations.

Incorporating the inventory costs of containers into network design problems when there is no transshipped container is possible. However, once transshipment is allowed, it immediately becomes extremely challenging to formulate the inventory costs in network design. For example, Alvarez (2009), Mulder and Dekker (2014), and Wang et al. (2015b) have all designed networks without considering the inventory costs.

In sum, the liner shipping professionals have realized the importance of inventory costs. Shipping lines have taken inventory costs into account in some aspects of their service planning. However, the importance of inventory costs has not yet aroused sufficient attention from the research community as not many models have explicitly considered the inventory costs. It is interesting to note that, on one side, it is extremely challenging to capture the exact inventory costs related to connection time at transshipment ports and hence it is easy to understand that most research does not consider them; on the other hand, however, the component of the inventory costs associated with on board time is relatively easy to formulate, and most research does not consider this component, at least not explicitly, either.

3. Container routing problem with given liner routes

In this section, it is assumed that a liner shipping network has been designed and the schedules are known. This study focuses on container routing taking into account the inventory costs, which is a subproblem for network design. For better readability, the symbols used are listed below:

Sets

$\mathcal{A}$	Set of arcs in the operational network
$\mathcal{A}_n^+$	Set of arcs incident from node $n \in \mathcal{N}$
$\mathcal{A}_n^-$	Set of arcs incident to node $n \in \mathcal{N}$
$\mathcal{A}^p$	Set of port visit arcs
$\mathcal{A}^r$	Set of rejection arcs
$\mathcal{A}^s$	Set of source/sink arcs
$\mathcal{A}^t$	Set of transshipment arcs
$\mathcal{A}^v$	Set of voyage arcs
$\mathcal{A}_r^p$	Set of port visit arcs associated with route $r \in \mathcal{R}$
$\mathcal{A}_r^v$	Set of voyage arcs associated with route $r \in \mathcal{R}$
I_r	Set of ports of call on route $r \in \mathcal{R}$
I_{rp}	Set of ports of call on route $r \in \mathcal{R}$ that correspond to port p
$\mathcal{N}$	Set of nodes in the operational network
$\mathcal{P}$	Sets of ports
$\mathcal{R}$	Sets of routes
$\mathcal{R}_p$	Sets of routes that visit port $p \in \mathcal{P}$
$\mathcal{W}$	Sets of origin-destination (OD) port pairs with demand
$\mathbb{Z}^+$	Set of nonnegative integers

Parameters

β^w	Perceived VOTT (US$/TEU/day) for OD pair $w \in \mathcal{W}$
C_r	Operating cost (US$/week) of route r in the RC problem
$\check{c}_r$	Cost US$/TEU/week for booking one TEU slot on route $r \in \mathcal{R}$ in the CP problem
c_a	Handling or penalty cost (US$/TEU) of arc $a \in \mathcal{A}$
$\hat{c}_p$	Loading cost (US$/TEU) at port $p \in \mathcal{P}$
$\tilde{c}_p$	Discharge cost (US$/TEU) at port $p \in \mathcal{P}$
$\bar{c}_p$	Transshipment cost (US$/TEU) at port $p \in \mathcal{P}$
c^w	Penalty (US$/TEU) for rejecting containers between OD pair $w \in \mathcal{W}$

d^w	Destination port of OD pair $w \in \mathcal{W}$
E_r	Capacity of a ship deployed on route $r \in \mathcal{R}$
N_r	Number of ports on route $r \in \mathcal{R}$
o^w	Origin port of OD pair $w \in \mathcal{W}$
p_a	Physical port that corresponds to transshipment arc $a \in \mathcal{A}^t$
p_{ri}	Physical port that corresponds to the ith port of call on route r
q^w	Number of containers to transport (TEUs/week) between OD pair $w \in \mathcal{W}$
$< r, i >$	Port of call i on route r
s_a	Capacity (TEU/week) of arc $a \in \mathcal{A}$
t_a	Duration (day) of arc $a \in \mathcal{A}$
$\bar{t}_p$	Fixed connection time (days) assumed for port $p \in \mathcal{P}$
$\bar{t}_{ri}$	Time (day) a ship spends at $< r, i >$
$\hat{t}_{ri}$	Arrival date at $< r, i >$
$\hat{t}_{r,N_r+1}$	Date on which the first port of call is visited again
$\tilde{t}_{ri}$	Departure date from $< r, i >$
$\tilde{t}_{rij}$	Transit time (days) of a container from $< r, i >$ to $< r, j >$
$\tilde{t}_{rsij}$	Connection time (days) of a container transshipped from $< r, i >$ to $< s, j >$, $< r, i > \neq < s, j >$
$t_p^{\min}$	Minimum connection time (days) required at port $p \in \mathcal{P}$

Decision variables

f_a^w	Number of containers (TEUs/week) from OD pair $w \in \mathcal{W}$ transported on arc $a \in \mathcal{A}$
x_r	Binary variable that equals 1 if and only if route $r \in \mathcal{R}$ is operated in the RC problem
y_r	Continuous variable representing the capacity (TEUs/week) booked on route $r \in \mathcal{R}$ in the CP problem

Consider a liner container shipping company that operates a number of routes (services), denoted by the set $\mathcal{R}$, regularly serving a group of ports denoted by the set $\mathcal{P}$. A route can have any structure (e.g., a simple cycle, a butterfly route, or even more complex). The port rotation of a route $r \in \mathcal{R}$ can be expressed as:

$$p_{r1} \rightarrow p_{r2} \rightarrow \ldots \rightarrow p_{rN_r} \rightarrow p_{r1}, \tag{3}$$

where N_r is the number of ports of call on the route and $p_{ri} \in \mathcal{P}$ is the physical port corresponding to ith port of call. Let I_r be the set of ports of call of route $r \in \mathcal{R}$, i.e., $I_r = \{1, 2, \cdots, N_r\}$. For brevity, define $< r, i >$ as port of call i on route r. Defining $p_{r,N_r+1} = p_{r1}$, the voyage from p_{ri} to $p_{r,i+1}$ is called *leg* i, $i \in I_r$. The port rotations of the four routes in Fig. 1 are:

$$\begin{aligned}
r = 1, N_r = 4: &\quad p_{1,1}(\text{SIN}) \rightarrow p_{1,2}(\text{KHI}) \rightarrow p_{1,3}(\text{NS9}) \rightarrow p_{1,4}(\text{CMB}) \rightarrow p_{1,1}(\text{SIN}),\\
r = 2, N_r = 2: &\quad p_{2,1}(\text{SIN}) \rightarrow p_{2,2}(\text{SUR}) \rightarrow p_{2,1}(\text{SIN}),\\
r = 3, N_r = 2: &\quad p_{3,1}(\text{SIN}) \rightarrow p_{3,2}(\text{SEM}) \rightarrow p_{3,1}(\text{SIN}),\\
r = 4, N_r = 2: &\quad p_{4,1}(\text{SIN}) \rightarrow p_{4,2}(\text{SEM}) \rightarrow p_{4,1}(\text{SIN}).
\end{aligned}$$

This study further defines $\mathcal{R}_p$ as the set of routes that visit port $p \in \mathcal{P}$, and define I_{rp} as the set of ports of call on route $r \in \mathcal{R}_p$ that correspond to port p. In the above example, $\mathcal{R}_{\text{SIN}} = \{1, 2, 3, 4\}$, $I_{1,\text{CMB}} = \{4\}$, $I_{2,\text{SIN}} = \{1\}$, and $I_{3,\text{CMB}} = \varnothing$. The loading, discharge, and transshipment costs (US\$/TEU) at port $p \in \mathcal{P}$ are $\hat{c}_p$, $\tilde{c}_p$, and $\bar{c}_p$, respectively. The capacity of a ship deployed on route $r \in \mathcal{R}$ is E_r (TEUs).

Each route has a fixed weekly schedule, as shown in Fig. 1. Define a particular Sunday on which a ship visits Singapore on route 2 as day 0, and let $\hat{t}_{ri}$ be the arrival date at the port of call $< r, i >$. Hence, $\hat{t}_{1,1} = 2, \hat{t}_{1,2} = 11, \hat{t}_{1,3} = 14, \hat{t}_{1,4} = 18, \hat{t}_{2,1} = 0, \hat{t}_{2,2} = 3$. This study further defines $\hat{t}_{r,N_r+1}$ as the date on which the first port of call is visited again. Hence, $\hat{t}_{1,5} = 23, \hat{t}_{2,3} = 7$. The number of days spent at $< r, i >$ is denoted by $\bar{t}_{ri}$. Therefore, the departure time from $< r, i >$ is $\tilde{t}_{ri} := \hat{t}_{ri} + \bar{t}_{ri}$.

There is a set of origin-destination (OD) port pairs with demand, denoted by $\mathcal{W} \subset \mathcal{P} \times \mathcal{P}$. Each OD pair $w \in \mathcal{W}$ has information about the origin port o^w, the destination port d^w, the number of containers to transport q^w (TEUs/week), and the perceived VOTT β^w (US\$/TEU/day). The shipping line is allowed to reject some containers at the cost of c^w (US\$/TEU) for OD pair $w \in \mathcal{W}$. The container routing problem aims to determine how to transport the containers to minimize the sum of handling costs, inventory costs and rejection penalty.

3.1. Analysis of the transit time and connection time

The transit time of a container from $< r, i >$ to $< r, j >$, $j \neq i$, is

$$t_{rij} = \begin{cases} \hat{t}_{rj} - \tilde{t}_{ri}, & i < j \\ \hat{t}_{r,N_r+1} - \tilde{t}_{ri} + \hat{t}_{rj} - \hat{t}_{r1}, & i > j \end{cases}$$

For instance, on route 1 $t_{1,2,4} = 18 - (11 + 2) = 5$ and $t_{1,4,2} = \hat{t}_{1,5} - \tilde{t}_{1,4} + \hat{t}_{1,2} - \hat{t}_{1,1} = 23 - (18 + 1) + 11 - 2 = 13$.

Denote by $t_p^{\min}$ the minimum connection time (days) required at port $p \in \mathcal{P}$. The connection time of a container transshipped from $< r, i >$ to $< s, j >$, $< r, i > \neq < s, j >$, provided that $p_{ri} = p_{sj}$, is formulated below after introducing an auxiliary integer variable k_{rsij}

$$t_{rsij} = \min_{k_{rsij} \in \mathbb{Z}} \{\tilde{t}_{sj} - \hat{t}_{ri} + 7k_{rsij} \mid \tilde{t}_{sj} - \hat{t}_{ri} + 7k_{rsij} \geq t_{p_{ri}}^{\min}\}. \tag{4}$$

The above equation shows that the connection time is related to the time difference

$\tilde{t}_{sj} - \hat{t}_{ri}$ and the the minimum connection time $t_p^{\min}$. This study illustrates the connection time of a container transshipped from $< r, i >$ to $< s, j >$. Without loss of generality, it is assumed that $\hat{t}_{ri} = 0$. The connection time, which depends on $t_{p_{ri}}^{\min}$, $\bar{t}_{sj}$, and $\hat{t}_{sj}$, is shown in Table 1. Note that since it is independent of $\bar{t}_{ri}$, the entry "1(2)" in the column "$\bar{t}_{ri}$" means that $\bar{t}_{ri}$ is 1 or 2. It can be seen that, in theory, the worst schedule for a service leads to six days more inventory costs related to connection time as those of the optimal schedule.

Table 1: The connection time under different port time and minimum connection time

			$\hat{t}_{sj}$						
$t_{p_{ri}}^{\min}$	$\bar{t}_{ri}$	$\bar{t}_{sj}$	0	1	2	3	4	5	6
1	1(2)	1	1	2	3	4	5	6	7
1	1(2)	2	2	3	4	5	6	7	1
2	1(2)	1	8	2	3	4	5	6	7
2	1(2)	2	2	3	4	5	6	7	8

3.2. An operational network representation

To formulate container flow in a shipping network and capture the handling and inventory costs, this study transforms the shipping network to an operational network. The operational network corresponding to the shipping network in Fig. 1 is shown in Fig. 2. Each port of call is represented by an arrival node and a departure node, connected by a port visit arc representing the stay at the port. A leg is represented by a voyage arc, connecting the departure node of a port of call to the arrival node of the next port of call. Containers can be transshipped at a port that is visited more than once a week. This study hence connects the arrival node of a port of call to each of the departure nodes corresponding to the same physical port but different ports of call using a transshipment arc. Each physical port further has a dummy source node and a dummy sink node. For simplicity, this paper only draws one dummy source and one dummy sink node to reflect the only OD pair (SEM, KHI). A dummy source node is connected to each departure node at the port, and the dummy sink node is connected from each arrival node at the port. A dummy source node is connected to the dummy sink node by a rejection arc to represent the containers that are rejected. Containers from the OD pair (SEM, KHI) originate from the dummy source node for SEM and are sent to the dummy sink node for KHI. In sum, there are three types of nodes: arrival nodes, departure nodes, and dummy nodes; and there are five sets of arcs: set of port visit arcs $\mathcal{A}^p$, set of voyage arcs $\mathcal{A}^v$, set of transshipment arcs $\mathcal{A}^t$, set of source/sink arcs $\mathcal{A}^s$ and set of rejection arcs $\mathcal{A}^r$.

The container shipment demand must be fulfilled by the operational network. Each arc $a \in \mathcal{A}^p \cup \mathcal{A}^v \cup \mathcal{A}^t \cup \mathcal{A}^s \cup \mathcal{A}^r$ has the properties of duration t_a (day), unit handling or rejection cost c_a (US\$/TEU), and capacity s_a (TEUs/week), summarized in Table 2.

Table 2: Properties of the arcs

Arc type	Duration t_a	Unit cost c_a	Capacity s_a
Port visit arc $< r, i >$	t_{ri}	0	E_r
Voyage arc $< r, i >$ to $< r, i+1 >$	$\hat{t}_{r,i+1} - \tilde{t}_{ri}$	0	E_r
Transshipment arc $< r, i >$ to $< s, j >$	Eq. (4)	$\bar{c}_{p_{ri}}$	∞
Source/sink arc at $p \in \mathcal{P}$	0	$\hat{c}_p / \tilde{c}_p$	∞
Rejection arc for $w \in \mathcal{W}$	0	c^w	∞

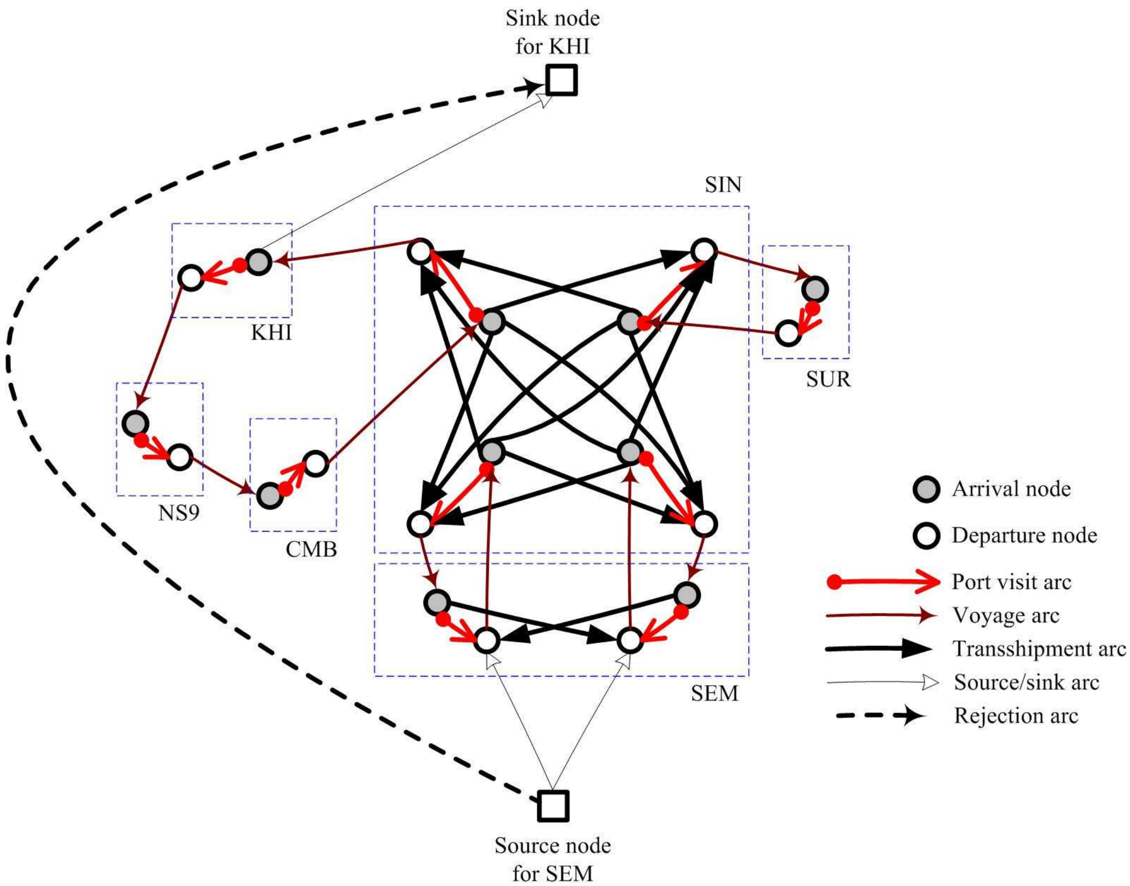

Figure 2: An operational network

Comment 1: One cannot use one node to represent a port of call for transshipment purposes. For instance, suppose that ships on route 1 are at Singapore on both Sunday and Monday, ships on route 2 are on both Monday and Tuesday, and ships on route 3 are on both Tuesday and Wednesday. Suppose further that $t_{\text{SIN}}^{\min} = 1$ day. Then $t_{3,1,1,1} = 7, t_{3,2,1,1} = 1, t_{2,1,1,1} = 1$, as shown in Fig. 3a. If container flow from SEM to KHI is analyzed using one node to represent each port of call, as shown in Fig. 3b, then the container might be transshipped from $< 3, 1 >$ to $< 2, 1 >$ and then to $< 1, 1 >$, i.e., containers are transshipped from service 3 to service 2 and then to service 1 at the same port, leading to a total connection time of 2 days. Suppose that the transshipment cost $\bar{c}_{\text{SIN}} =$ US\$80/TEU and the perceived VOTT is US\$20/TEU/day, then this incorrect flow incurs US\$160 of transshipment cost and US\$40 of inventory costs. By contrast, the correct transshipment from $< 3, 1 >$ directly to $< 1, 1 >$ incurs US\$80 of transshipment cost and US\$140 of inventory costs. As a consequence, in the incorrect optimal solution the container will be transshipped from $< 3, 1 >$ to $< 2, 1 >$ and then to $< 1, 1 >$.

Comment 2: One can use one node to represent a port of call for a physical port with no transshipment containers.

3.3. Container routing model

Let $\mathcal{N}$ and $\mathcal{A} := \mathcal{A}^p \cup \mathcal{A}^v \cup \mathcal{A}^t \cup \mathcal{A}^s \cup \mathcal{A}^r$ be the sets of nodes and arcs in the operational network, respectively. Denote by $\mathcal{A}_n^+$ the set of arcs incident from node $n \in \mathcal{N}$, i.e., $\mathcal{A}_n^+ := \{(n, m) \in \mathcal{A} | m \in \mathcal{N}\}$, and $\mathcal{A}_n^-$ the set of arcs incident to node n. With a little abuse of notation, this paper also uses $o^w \in \mathcal{N}$ to represent the dummy source node for the origin port of OD pair $w \in \mathcal{W}$, and d^w the dummy sink node of the destination node.

To optimize the container flow in the given operational network $(\mathcal{N}, \mathcal{A})$, let f_a^w be the decision variable representing the number of containers between OD pair $w \in \mathcal{W}$ transported on arc $a \in \mathcal{A}$. The model is:

[ContainerRouting]:

$$\min_{f_a^w} \sum_{a \in \mathcal{A}} \sum_{w \in \mathcal{W}} c_a f_a^w + \sum_{a \in \mathcal{A}} \sum_{w \in \mathcal{W}} t_a \beta^w f_a^w \tag{5}$$

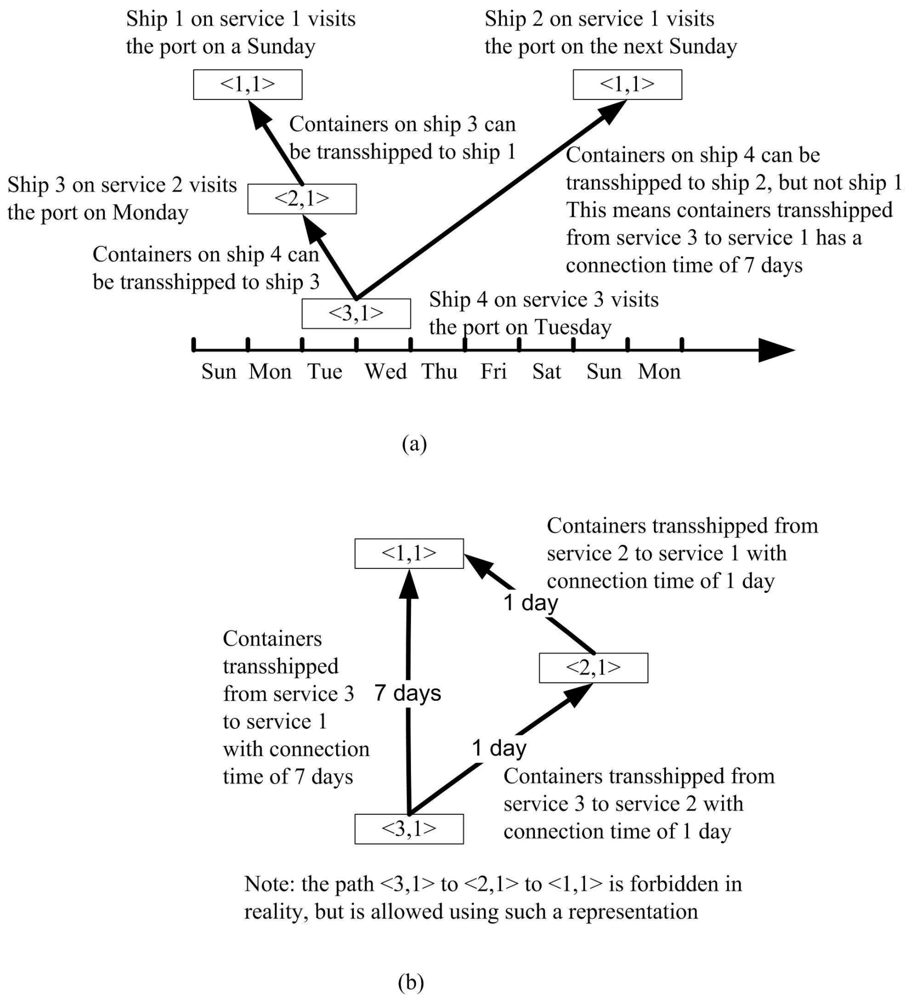

Figure 3: The problem of formulating connection time using one node to represent a port of call

subject to:

$$\sum_{a \in \mathcal{A}_n^+} f_a^w - \sum_{a \in \mathcal{A}_n^-} f_a^w = \begin{cases} q^w, & n = o^w \\ -q^w, & n = d^w \\ 0, & \text{otherwise} \end{cases} \quad , \quad \forall n \in \mathcal{N}, \forall w \in \mathcal{W} \tag{6}$$

$$\sum_{w \in \mathcal{W}} f_a^w \leqslant s_a \quad , \quad \forall a \in \mathcal{A} \tag{7}$$

$$f_a^w \geqslant 0 \quad , \quad \forall a \in \mathcal{A}, \forall w \in \mathcal{W}. \tag{8}$$

The objective function (5) minimizes the sum of handling and rejection costs and inventory costs. Constraints (6) impose the flow conservation conditions. Constraints (7) enforce the capacity on each arc. Constraints (8) define f_a^w as nonnegative decision variables.

Comment 3: If the perceived VOTT β^w is the same for all OD pairs $w \in \mathcal{W}$, a more compact origin-based formulation can be used in place of the OD-based formulation [ContainerRouting].

4. Modeling inventory costs for network design problems

It is convenient to analyze the impact of inventory costs by comparing the network design solution without considering inventory costs with the ideal model that takes the exact inventory costs into account. However, liner shipping network design problems are strongly NP-hard (Plum et al., 2014). To the best of our knowledge, only very small-scale problems have been solved to optimality without considering the inventory costs.

To circumvent this difficulty, this study examines two less general and easier-to-solve service planning problems. In the first one, each route r in the set $\mathcal{R}$ with given schedule is optional. If r is operated, an operating cost denoted by C_r (US\$/week), including the operating costs of the ships deployed on the route and the voyage costs, will be incurred. This problem is easier as determining the port rotation or the arrival dates at each port of call is not required. This problem is called the route choice (RC) problem. The second problem is borrowed from Dong et al. (2015): the routes are jointly operated by several shipping lines and a shipping line could book whatever proportion of ship capacity it needs on each route. This problem is named the capacity planning (CP) problem.

4.1. Formulating the inventory costs for the route choice problem

This section presents two models on how to formulate the inventory costs for the RC problem. Both models have the following decision variables: binary variable x_r equal to 1 if and only if route $r \in \mathcal{R}$ is operated and continuous variable f_a^w representing the number of containers between OD pair $w \in \mathcal{W}$ transported on arc $a \in \mathcal{A}$.

4.1.1. Minimizing handling costs and inventory costs with given schedules

Ideally, the schedule of each route is known and a model that captures the inventory costs is developed. Define $\mathcal{A}_r^p$ and $\mathcal{A}_r^v$ the sets of port arcs and voyage arcs associated with route $r \in \mathcal{R}$, respectively. The model is:

[P0]:

$$\min_{x_r, f_a^w} \sum_{r \in \mathcal{R}} C_r x_r + \sum_{a \in \mathcal{A}} \sum_{w \in \mathcal{W}} c_a f_a^w + \sum_{a \in \mathcal{A}} \sum_{w \in \mathcal{W}} t_a \beta^w f_a^w \tag{9}$$

subject to

$$\sum_{w \in \mathcal{W}} f_a^w \leqslant s_a x_r \quad , \quad \forall r \in \mathcal{R}, \forall a \in \mathcal{A}_r^p \cup \mathcal{A}_r^v \tag{10}$$

$$x_r \in \{0, 1\} \quad , \quad \forall r \in \mathcal{R} \tag{11}$$

and Eqs. (6) and (8).

The first term of the objective function (9) is the operating costs of the routes, the second term is the handling and rejection costs, and the third term is the inventory costs. Constraints (10) impose the ship capacity constraints. Constraints (11) define x_r as binary variables.

Let $(x_r^0, r \in \mathcal{R}; f_a^{w0}, a \in \mathcal{A}, w \in \mathcal{W})$ be the optimal solution to model [P0]. Then $(x_r^0, r \in \mathcal{R})$ is the optimal route choice decision for the problem. Note that the schedules for general network design problems are unknown and hence model [P0] is simply used as a benchmark. To compare with other models, this paper uses $C(\text{P0}) = C(x_r^0, f_a^{w0})$ to represent the optimal objective function value of model [P0], i.e., the "quality" of solution $(x_r^0, r \in \mathcal{R})$.

4.1.2. Minimizing handling costs only

In general network design problems it is impossible to know the schedule of a route because the routes themselves are to be determined yet. Therefore, most network design models consider the handling and rejection costs only and do not consider the inventory costs, leading to the following model:

[P1]:

$$\min_{x_r, f_a^w} \sum_{r \in \mathcal{R}} C_r x_r + \sum_{a \in \mathcal{A}} \sum_{w \in \mathcal{W}} c_a f_a^w \tag{12}$$

subject to Eqs. (6), (8) and (10)–(11).

Let $(x_r^1, r \in \mathcal{R}; f_a^{w1}, a \in \mathcal{A}, w \in \mathcal{W})$ be the optimal solution to model [P1]. The shipping line then determines how to deliver containers for a given network consisting of the chosen ship routes $\{r \in \mathcal{R} | x_r^1 = 1\}$ using model [ContainerRouting]. Therefore, to compare the solution $(x_r^1, r \in \mathcal{R})$ with the optimal one $(x_r^0, r \in \mathcal{R})$, model [ContainerRouting] under the new operational network of the routes $\{r \in \mathcal{R} | x_r^1 = 1\}$ must be solved:

[ContainerRoutingAfterRC]:

$$\min_{f_a^w} \sum_{a\in\mathcal{A}} \sum_{w\in\mathcal{W}} c_a f_a^w + \sum_{a\in\mathcal{A}} \sum_{w\in\mathcal{W}} t_a \beta^w f_a^w$$

subject to:

$$\sum_{a\in\mathcal{A}_n^+} f_a^w - \sum_{a\in\mathcal{A}_n^-} f_a^w = \begin{cases} q^w, & n = o^w \\ -q^w, & n = d^w \\ 0, & \text{otherwise} \end{cases} \quad , \quad \forall n \in \mathcal{N}, \forall w \in \mathcal{W}$$

$$\sum_{w\in\mathcal{W}} f_a^w \leqslant s_a x_r^1 \quad , \quad \forall r \in \mathcal{R}, \forall a \in \mathcal{A}_r^p \cup \mathcal{A}_r^v$$

$$f_a^w \geqslant 0 \quad , \quad \forall a \in \mathcal{A}, \forall w \in \mathcal{W}.$$

The quality of solution $(x_r^1, r \in \mathcal{R})$, denoted by $C(\text{P1}) = C(x_r^1, f_a^{w1})$, is equal to the sum of the route operating costs $\sum_{r\in\mathcal{R}} C_r x_r^1$ and the optimal objective function value of model [ContainerRoutingAfterRC].

4.1.3. Minimizing handling costs and inventory costs associated with on board time and fixed connection time

Although it is extremely difficult to formulate the connection time in network design models, it is relatively easy to formulate the on board time when the ship speed is fixed. Despite the difficulty in formulating the connection time, the number of transshipped containers at each port is captured by most network design models. Therefore, a fixed connection time at port $p \in \mathcal{P}$, denoted by $\bar{t}_p$, can be assumed. This study will discuss how to estimate $\bar{t}_p$ in the numerical experiments of the next section. Define p_a as the physical port of the transshipment arc $a \in \mathcal{A}^t$. The model with on board time and fixed connection time is:

[P2]:

$$\min_{x_r, f_a^w} \sum_{a\in\mathcal{A}} \sum_{w\in\mathcal{W}} c_a f_a^w + \sum_{a\in\mathcal{A}^p\cup\mathcal{A}^v} \sum_{w\in\mathcal{W}} t_a \beta^w f_a^w + \sum_{a\in\mathcal{A}^t} \sum_{w\in\mathcal{W}} \bar{t}_{p_a} \beta^w f_a^w \tag{13}$$

subject to Eqs. (6), (8) and (10)–(11).

Let $(x_r^2, r \in \mathcal{R}; f_a^{w2}, a \in \mathcal{A}, w \in \mathcal{W})$ be the optimal solution to model [P2]. The quality of solution $(x_r^2, r \in \mathcal{R})$, denoted by $C(\text{P2}) = C(x_r^2, f_a^{w2})$, can be evaluated after solving the corresponding model [ContainerRouting].

4.2. Formulating the inventory costs for the capacity planning problem

Similar to the RC problem, the two different models for the CP problem can also be formulated. For conciseness, this paper only presents the ideal model minimizing the sum of handling, rejection and inventory costs with given schedules. Let $\check{c}_r$ be the cost (US$/TEU/week) for booking one TEU slot on route $r \in \mathcal{R}$. The model has the following decision variables: continuous variable y_r represents the capacity (TEUs/week) booked on route $r \in \mathcal{R}$, and continuous variable f_a^w represents the number of containers (TEUs/week) between OD pair $w \in \mathcal{W}$ transported on arc $a \in \mathcal{A}$. The model is:

[P0-CP]:

$$\min_{y_r, f_a^w} \sum_{r\in\mathcal{R}} \check{c}_r y_r + \sum_{a\in\mathcal{A}} \sum_{w\in\mathcal{W}} c_a f_a^w + \sum_{a\in\mathcal{A}} \sum_{w\in\mathcal{W}} t_a \beta^w f_a^w \tag{14}$$

subject to

$$\sum_{w\in\mathcal{W}} f_a^w \leqslant y_r \quad , \quad \forall r \in \mathcal{R}, \forall a \in \mathcal{A}_r^p \cup \mathcal{A}_r^v \tag{15}$$

$$0 \leqslant y_r \leqslant E_r \quad , \quad \forall r \in \mathcal{R} \tag{16}$$

and Eqs. (6) and (8).

The first term of the objective function (14) is the total slot booking costs. Constraints (15) impose the capacity constraints. Constraints (16) require that the number of booked slots is nonnegative and cannot exceed the ship capacity.

5. Numerical experiments

This section reports the results of numerical experiments that are aimed at assessing the performance of models [P1] and [P2] relative to model [P0]. All of the models are solved using CPLEX12.1. This study uses off-the-shelf solvers rather than designs heuristic algorithms because optimal solutions are desirable for the sake of comparison (Li et al., 2012; Wang and Du, 2013; Du and Wang, 2014; He et al., 2015; Wu et al., 2015; Zhen and Wang, 2015; Zhen et al., 2015).

In [P2], the fixed connection time $\bar{t}_p$ at all ports is 4 days. The demand between an OD pair is randomly generated. As a result, it is possible that there are a large number of rejected containers. In reality, not many containers are rejected because the network is designed or altered according to the demand. Hence, for instance, to evaluate the performance of [P1], this study uses the relative error (RE) defined as

$$RE := \frac{C(\text{P1}) - C(\text{P0})}{C(\text{P0}) - \sum_{a\in\mathcal{A}^r} c_a \sum_{w\in\mathcal{W}} f_a^{w0}},$$

where the second term in the denominator of the right-hand side is the total penalty for rejecting containers in the optimal solution to [P0]. Hence, RE means the percentage of total cost increase of model [P1] or [P2] relative to model [P0]. (Because the penalty costs for rejecting containers may be high in [P0] due to randomly generated parameters, this paper deducts the penalty costs in the denominator.)

5.1. Performance of the models for random networks

Test instances are randomly generated as follows. A total of 20 ports are randomly located in a 5,000×5,000 (nautical miles^2) grid. Ten routes with schedules are randomly designed, with the number of ports of call between 2 and 15 on each route. Five types of ships—1,500-TEU, 3,500-TEU, 5,000-TEU, 8,000-TEU, and 10,000-TEU—are deployed in the network to provide weekly services on each route. Deploying large ships leads to economies of scale. The loading cost at a port is randomly chosen between 100 and 200 US$/TEU. The discharge cost is the same as the loading cost. The transshipment cost is randomly chosen between 150 and 250 US$/TEU. A quarter of the port pairs have demand. The perceived VOTT for each OD pair is between 5 and 30 US$/day/TEU. The demand for each OD pair is randomly chosen between 1 and a number related to the capacity of the network. The penalty cost for rejecting a container depends on the distance between the origin port and the destination port and the perceived VOTT for the OD pair.

This study first compares the performance of models [P1] and [P2] for the RC problem. Three groups of tests are carried out with the minimum connection time $t_p^{\min} = 1$ for all ports, $t_p^{\min}$ being randomly chosen from 1 and 2 for each port, and $t_p^{\min} = 2$ for all ports. This study generates 100 instances with different networks and demand for each group. Table 3 reports the number of instances for which a model does not produce the optimal route choice decision (#RE> 0), the average RE and the maximum RE of the 100 instances.

Table 3: RC results for random networks

$t_p^{\min}$	Criteria	[P1]	[P2]
1 day	#RE> 0	51	13
	average RE	0.403%	0.017%
	max RE	2.439%	0.362%
1 or 2 days	#RE> 0	47	9
	average RE	0.405%	0.016%
	max RE	4.912%	0.352%
2 days	#RE> 0	57	15
	average RE	0.660%	0.022%
	max RE	4.531%	0.450%

Table 3 shows that [P1] performs much worse than [P2]: it fails to find the optimal route choice decisions in half of the instances, the average RE is between 0.4% and 0.7%, and the maximum RE can be as high as 5%. [P2] has an average RE of ca. 0.02%.

This study generates another 300 instances with different networks and demand and compare the models for the CP problem. The results are shown in Table 4, which also include the median RE. As the capacity planning decisions are continuous, neither [P1] nor [P2] could produce the optimal solution for an instance. Again, [P2] considerably outperforms [P1].

5.2. Tests using the LINER-LIB instances

This study then tests the results using instances from LINER-LIB (Plum et al., 2014). LINER-LIB is a free benchmark suite for liner shipping network design problems that is developed by Technical University of Denmark. The suite is designed

Table 4: CP results for random networks

$t_p^{\min}$	Criteria	[P1]	[P2]
1 day	#RE> 0	100	100
	average RE	0.913%	0.062%
	median RE	0.769%	0.050%
	max RE	4.486%	0.237%
1 or 2 days	#RE> 0	100	99
	average RE	0.865%	0.062%
	median RE	0.752%	0.048%
	max RE	3.312%	0.251%
2 days	#RE> 0	100	100
	average RE	0.931%	0.059%
	median RE	0.794%	0.048%
	max RE	3.351%	0.218%

based on a large number of realistic inputs and hence is an invaluable database for liner shipping researchers. This study uses the Mediterranean network in the suite, consisting of 39 ports and 369 OD pairs. This study generates 30 instances with different routes and compare the models for the CP problem. The results, shown in Table 5, also demonstrate that [P2] is preferable.

5.3. Performance of the models for APL's networks

To compare the performance of the models on realistic networks, this study considers APL's intra-Asia and Middle East services, excluding services for Oceania (APL, 2014). There are a total of 13 linehaul services and 25 feeder services, calling at 68 ports. The details of the services could be found in the website of APL (2014). Other information such as ship deployment, demand, and costs is randomly generated similar to Section 5.1.

Table 5: CP results for the Mediterranean network in LINER-LIB

$t_p^{\min}$	Criteria	[P1]	[P2]
1 day	#RE> 0	10	9
	average RE	98.041%	0.230%
	median RE	82.869%	0.161%
	max RE	198.084%	0.546%
1 or 2 days	#RE> 0	10	10
	average RE	90.281%	0.182%
	median RE	68.988%	0.209%
	max RE	247.211%	0.295%
2 days	#RE> 0	10	10
	average RE	91.259%	0.205%
	median RE	87.005%	0.162%
	max RE	186.630%	0.636%

5.3.1. Performance of the models on APL's services

To examine the RC problem, this study randomly generates 300 instances, in each of which 10 routes are randomly chosen from APL's 34 routes and the demand and costs are also randomly generated. The results for the RC problem are shown in Table 6, and the results for the CP problem based on another 300 instances are shown in Table 7. Again, [P2] considerably outperforms [P1].

Table 6: RC results for APL networks

$t_p^{\min}$	Criteria	[P1]	[P2]
1 day	#RE> 0	34	12
	average RE	1.089%	0.024%
	max RE	61.916%	0.620%
1 or 2 days	#RE> 0	31	15
	average RE	0.333%	0.020%
	max RE	11.072%	0.515%
2 days	#RE> 0	41	16
	average RE	0.275%	0.022%
	max RE	4.588%	0.436%

Table 7: CP results for APL networks

$t_p^{\min}$	Criteria	[P1]	[P2]
1 day	#RE> 0	99	91
	average RE	1.783%	0.069%
	median RE	1.369%	0.032%
	max RE	7.637%	0.973%
1 or 2 days	#RE> 0	98	93
	average RE	1.728%	0.054%
	median RE	1.352%	0.032%
	max RE	5.217%	0.600%
2 days	#RE> 0	100	93
	average RE	2.384%	0.060%
	median RE	1.863%	0.044%
	max RE	19.217%	0.370%

5.3.2. Sensitivity of the fixed connection time

Since [P2] outperforms [P1], this study examines whether 4 days is the best connection time to fix. Similar to the previous tests, this study generates 50 instances based on APL's network for each of the three groups. Each instance is solved by model [P2] with the fixed connection time set at 3, 3.5, 4, 4.5, and 5 days. The results for the RC problem are shown in Table 8, and the results for the CP problem based on another 150 instances are shown in Table 9.

The two tables show that, in general, a smaller fixed connection time should be used when the minimum connection time is 1 day, and a larger fixed connection time is suitable when the minimum connection time is 2 days. This observation is intuitively true. Therefore, this study conducts another test: for instances in which the minimum connection time is randomly chosen from 1 and 2 days, this study fixes the connection time for those ports with $t_p^{\min} = 1$ at 3.5 days and those with $t_p^{\min} = 2$ at 4.5 days. The results are shown in the last columns of Table 8 and Table 9, which

Table 8: RC results for APL networks with different fixed connection time

$t_p^{\min}$	Criteria	3.0	3.5	4.0	4.5	5.0	3.5/4.5
	#RE> 0	4	3	3	4	5	
1 day	average RE	0.016%	0.008%	0.008%	0.020%	0.027%	
	max RE	0.386%	0.238%	0.238%	0.568%	0.568%	
	#RE> 0	2	3	4	4	7	2
1 or 2 days	average RE	0.006%	0.009%	0.017%	0.010%	0.021%	0.006%
	max RE	0.216%	0.216%	0.515%	0.216%	0.417%	0.216%
	#RE> 0	8	7	7	5	6	
2 days	average RE	0.018%	0.013%	0.020%	0.013%	0.011%	
	max RE	0.247%	0.233%	0.361%	0.361%	0.214%	

Table 9: CP results for APL networks with different fixed connection time

$t_p^{\min}$	Criteria	3.0	3.5	4.0	4.5	5.0	3.5/4.5
	#RE> 0	49	49	48	47	47	
1 day	average RE	0.061%	0.060%	0.061%	0.067%	0.074%	
	median RE	0.029%	0.028%	0.033%	0.042%	0.047%	
	max RE	0.439%	0.540%	0.352%	0.311%	0.326%	
	#RE> 0	47	46	48	48	49	48
1 or 2 days	average RE	0.060%	0.052%	0.064%	0.075%	0.097%	0.057%
	median RE	0.035%	0.034%	0.042%	0.057%	0.077%	0.049%
	max RE	0.320%	0.241%	0.600%	0.537%	0.535%	0.195%
	#RE> 0	49	49	48	46	46	
2 days	average RE	0.091%	0.074%	0.064%	0.047%	0.051%	
	median RE	0.069%	0.054%	0.051%	0.028%	0.031%	
	max RE	0.325%	0.325%	0.295%	0.242%	0.220%	

outperform the results by fixing the connection time of all ports at 4 days. If there is no available information, this study thus recommends fixing the connection time for those ports with $t_p^{\min} = 1$ at 3.5 days and those with $t_p^{\min} = 2$ at 4.5 days. Of course, there are many factors that affect the results and fixing the connection time for those ports with $t_p^{\min} = 1$ at 3.5 days and those with $t_p^{\min} = 2$ at 4.5 days is not necessarily the optimal choice.

6. Conclusions

In view of the difficulty of formulating the exact inventory costs of containers for liner service planning, this study has examined two approaches on incorporating the inventory costs into existing service planning models: the first model does not consider the inventory costs at all, and the second model incorporates the inventory costs associated with on board time and assumes the connection time to be a fixed positive number for each port. The two models are compared with the exact solution on two less general and easier-to-solve service planning problems: the route choice problem and the capacity planning problem. Extensive numerical experiments based on randomly generated networks and practical networks show that the model that ignores the inventory costs produces significantly worse network design decisions than the model that captures the inventory costs associated with on board time and suitable fixed connection time at transshipment ports. Based on the limited results of our research, this study finds that: first, the inventory costs in service planning models should not be ignored simply because it is too difficult to capture the exact connection time; second, in service planning models, the inventory costs associated with on board time could be formulated exactly, and those related to connection time of weekly services could be approximated by assuming fixed connection time of 3.5 days for ports with 1 day's minimum connection time and 4.5 days for ports with 2 days' minimum connection time.

The research has the following shortcomings that should be overcome in the future. First, to make the results more meaningful, the comparisons should be based on shipping lines' real container shipment demand patterns, whereas shipping lines are generally reluctant to share data due to the concern of leaking their confidential information to competitors. Second, the general network design problem that determines the port rotations is much more complex than the route choice problem and the capacity planning problem. How to compare different modeling approaches for the inventory costs in general network design problems is a worthwhile future research direction.

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