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Mathematically calculating the transit time of cargo through a liner shipping network with various transshipment policies

Abstract

This paper derives the mathematical expressions for the transit time of cargo through a liner shipping network. Main efforts are devoted to deriving the calculation expressions of the connection time of cargo during transshipment. For the forward and many-to-one transshipment policies, we conduct a minor correction towards the expressions by Álvarez (2012) to improve the completeness. Meanwhile, we propose an alternative but more straightforward calculation method for connection time which bypasses the complicated inductive argument in Álvarez (2012). Then we introduce two new transshipment policies: backward transshipment and one-to-many transshipment, and mathematically calculate the corresponding connection times. Numerical experiments also deliver some managerial insights into the effectiveness of backward transshipment in transit time control.

Keywords: transit time; connection time; transshipment; liner shipping; service level

1. Introduction

Liner shipping has been conveying cargoes to and from almost every place all over the world and has become the artery of the world trade. In the process of economic globalization driven by containerization (Bernhofen et al., 2013), liner shipping is the predominant service provider of containerized cargo transportation. Containerized trade volume in 2014, 1.63 billion tons, accounts for 17 per cent of global seaborne trade by weight, attaining up to 171 million twenty-foot equivalent units (TEUs) (UNCTAD, 2015). The United Nations Conference on Trade and Development (UNCTAD) compiled the so-called Liner Shipping Connectivity Index to measure the capacity of a country to transport its containerized foreign trade by shipping lines (UNCTAD, 2013). A recent study, jointly conducted by the World Bank and the United Nations Economic and Social Commission for Asia and the Pacific (UNESCAP), reveals that liner shipping connectivity shows a stronger impact on trade costs than the combined effects of logistics performance, air connectivity, costs of starting a business, and lower tariffs (Arvis et al., 2013).

As the transport service provider of liner shipping, shipping lines have been making unprecedented efforts to improve the service level to win a higher market share. There are several factors influencing the level of shipping service, among which the transit time, together with the freight rate, is often regarded as the critical one (Brouer et al., 2014; Notteboom and Vernimmen, 2009). First, the transit time, namely, the time elapsed from leaving the origin port of the cargo to arrival at its destination port, significantly determines the inventory holding cost of the cargo, which will be paid by shippers. Shippers generally desire short transit times to reduce the inventory holding cost, and ultimately curb the expenditure on nonproduction operations. Second, from a viewpoint of service diversification, the capability of providing a certain number of service combinations of freight rates and transit times concerns the market competitiveness of a shipping line. Different shippers, or one shipper at different business scenarios, would choose different combinations of high freight rate – short transit time, or low freight rate – long transit time. Moreover, the recent trend of slow steaming spurred by the slump of world trade and by bunker cost management further enhances the indispensability of transit time control in liner shipping.

Although the transit time is regarded as a tangible indicator of the level of service perceived by shippers, controlling the transit time in a shipping network, nevertheless, is not an easy job. Besides the sailing speed control, transshipment, which lies at the core of the modern liner shipping network, greatly influences the management of transit time. Transshipment for one thing provides flexibility for cargo routing and thus expands the geographical coverage of shipping services, but for another tremendously complicates the transit time control. Under the background of transshipment, transit time control is always interwoven together with container routing optimization and service schedule design (Álvarez, 2012; Wang and Meng, 2011). Schedule design involves the synchronization of different services, and determines the connection time of cargo transshipped from one service to another at transshipment ports. Container routing optimization distributes suitable container flows over the traverse routes and hence is closely related to the total transit time incurred by the containerized cargo.

Furthermore, different transshipment policies and various configurations of shipping networks bring additional complications to transit time management. For instance, in most cases, a vessel (feeding vessel) always transships its cargo at a transshipment port to a vessel on another service (connecting vessel) arriving later. This is called *forward transshipment*. However, in practice, a *backward transshipment* policy is also implemented by shipping lines, with which the connecting vessel arrives at the transshipment port earlier than its feeding vessel. If we consider the capacity difference of the feeding and connecting vessels, another two transshipment policies should be examined: *many-to-one transshipment* versus *one-to-many transshipment*. Many-to-one transshipment assumes that the connecting vessel has adequate slot capacity for the cargo transshiped from several feeding vessels, while one-to-many transshipment frequently occurs in a hub port where trunk and feeder services are connected since a mother vessel often requires multiple feeders to receive its cargo loaded (Lam and Yap, 2008).

With the importance of transit time control for service level of liner shipping and the difficulties posed by cargo transshipment in mind, this work aims to mathematically calculate the transit time of cargo through a liner shipping network, providing the network designer with a mathematical modelling tool to consider the transit time as a measure of service level. We start this work by conducting a minor correction towards the pioneering work by Álvarez (2012) for the forward and many-to-one transshipment policies and proposing a more straightforward calculation method. Meanwhile, this paper additionally examines the connection time of cargo in transshipment ports with two new transshipment policies: backward transhipment and one-to-many transshipment. Our effort on transit time (connection time) calculation in this paper could basically cover the business scenarios in liner shipping practice.

2. Literature review

Owing to the lack of profound domain knowledge and data sources caused by the conservativeness of liner shipping industry, the research topics involving mathematical modelling and solution techniques in liner shipping have not attracted the attention of researchers for a long time (Christiansen et al., 2013; Meng et al., 2014). In recent years, the continued efforts of shipping lines on decision support systems, collaboration between industry and academia, and advancements in computer technologies and optimization techniques have promoted the progress of operations research (OR) studies in liner shipping. These achievements can be seen in a wide class of relevant issues: fleet size and mix (Hoff et al., 2010), liner shipping alliance strategy (Agarwal and Ergun, 2010), network design (Brouer et al., 2014; Yang et al., 2014), frequency determination (Notteboom and Vernimmen, 2009; Ronen, 2011), fleet deployment (Álvarez, 2009; Fagerholt et al., 2009), speed optimization (Psaraftis and Kontovas, 2013; Cheaitou amd Cariou, 2012), schedule design (Karlaftis et al., 2009; Qi and Song, 2012), container assignment (Bell et al., 2011; Bell et al., 2013), cargo booking and routing (Song and Dong, 2012), vessel schedule adjustment/recovery (Brouer et al., 2013; Du et al., 2015), slot allocation (Lu and Mu, 2016), and vessel emission control (Balland et al., 2015). For a recent literature review on the closely-related topics, we refer the readers to Christiansen et al. (2013) and Meng et al. (2014).

Despite the fruitful studies on OR topics in liner shipping, recent research only scratches the surface of transit time (Brouer et al., 2014; Meng et al., 2014). Based on the background in Section 1, we could see a considerable gap between academic studies and liner shipping industry. Yin et al. (2014) analyze the impact of slow steaming on cargo's transit time and the consequent inventory cost, but do not factor cargo transshipment into their model since only one service is considered. Gelareh et al. (2010) try to design a service network for a newcomer shipping line under a competitive environment, in which the transit time is identified as a decisive factor affecting the market share possibly obtained. However, the sailing times of vessels, and the connection times consumed in transshipment ports are regarded as model parameters under the assumption that the shipping schedules are predetermined. In shipping practice, determining vessel schedules is always considered as a low-tier decision problem in the whole multi-tier decision procedure of liner network optimization, while the network design lies in the upper tier.

Wang and Meng (2011) consider the schedule design and container routing problem in liner shipping. They attempt to determine the optimal vessel schedule for each service route and optimal container flow on each container route in a liner shipping network, in order to minimize the total transit time experienced by all the containers involved. Excluding the decision of container flows among a collection of container routes, Wang and Meng (2012b) once again address the transit time issue in the liner vessel route design problem with sea contingency time and port time uncertainty. They formulate the transit time requirements as hard constraints since the transit time experienced by the cargo on each container route must be maintained at a predefined level. Wang and Meng (2011, 2012b) deal with the transit time in a weekly-frequency liner shipping network.

Álvarez (2012) conducts a pioneering study to mathematically calculate the transit time of cargo throughout a liner shipping network in a generalized setting where the service frequencies of the shipping routes involved may be different, and are not necessarily weekly. This work could well adapt to shipping networks with more complicated configurations in which the discrepancy of service frequencies among different shipping routes may exist, and to the trend of adjusting the service level by shipping lines, such as the Daily Maersk service on the Asia – North Europe trade lane (Maersk Line, 2011; Zhang and Lam, 2015). However, for the connection time calculation of cargo at a transshipment port, Álvarez (2012) only considers the forward and many-to-one transshipment policies, and does not address backward transshipment and one-to-many transshipment. This considerably restricts the application of the calculation method for transit time in various liner shipping network structures, such as the so-called hub-and-spoke network, and feeder services.

This paper relaxes the restrictions posed by the existing literature and aims to mathematically address the calculation of transit time in a more generalized sense. In details, this paper contributes to the literature in four aspects: (a) a minor correction is conducted to improve the completeness of the mathematical expressions by Álvarez (2012); (b) for the problem attacked by Álvarez (2012), we give a more straightforward calculation method for connection time bypassing the complicated inductive argument; (c) we introduce the notion of backward transshipment adopted in shipping practice to academia, mathematically calculate the connection time when backward transshipment is allowed, and experimentally evaluate the effectiveness of this transshipment policy in transit time control; and (d) we also derive the mathematical expressions for connection time (equivalently transit time) when one-to-many transshipment is operated at a hub port where trunk lines connect to feeder services.

The remainder of this paper is organized as follows. Section 3 reviews the calculation method for transit time in Álvarez (2012), conducts a minor correction, and points out the limitations of the work by Álvarez (2012). Section 4 proposes a more straightforward calculation method of transit time by avoiding the complex inductive process to mathematical expressions with a nonlinear operator of modulo. Section 5 calculates the connection time when backward transshipment is involved. Section 6 considers a one-to-many transshipment scenario since a mega-containership at a hub port always corresponds with several feeders. Section 7 assesses the effectiveness of backward transshipment in transit time control through numerical experiments, since the introduction of this transshipment policy constitutes the main contribution of this paper from the viewpoint of shipping policy and management. Concluding remarks are drawn at last in Section 8.

3. Pioneering work on transit time by Álvarez (2012) and its limitations

We firstly review the pioneering work on the transit time by Álvarez (2012) as this study aims to address a similar problem. The work by Álvarez (2012) presupposes a two-tier optimization framework for liner shipping network design problem, in which the upper tier determines fleet deployment, vessel routing, and vessel sailing speed, while the lower tier allocates container flows into shipping routes. Álvarez (2012) assumes that the decisions in the upper tier are given, focuses on the lower tier and optimizes the phases of services (intrinsically the vessel schedules) and container flows over shipping routes, including the transshipment of container flows among services. The objective is to minimize the weighted sum of the transit times for container flows over multiple services in a shipping network.

Álvarez (2012) aims to model the optimization problem described as follows. In a liner shipping network consisting of a set of services denoted by R, each service $r \in R$ is comprised of a set of ordered sailing arcs (legs) denoted by A_r and maintains an inter-arrival frequency f_r with n_r vessels deployed and sailing at the speed v_r. The sailing distance of leg, namely arc, $(i, j) \in A_r$ is represented by l_r^{ij}. In each port of call j, a vessel deployed on service r requires $\tilde{t}_r^j$ units of time for queuing, pilotage in, mooring and unloading, and $\hat{t}_r^j$ units of time for loading and pilotage out of the port. Transshipment of cargo could be operated between any two different services at any port where these services connect. To minimize the total transit time experienced by all the containers through this shipping network over a planning horizon L, three groups of decision variables are introduced: X_r^{ij} - the total volume of cargo (number of containers) carried by all the vessels over sailing leg (i, j) on service r; $T_{rs}^{ij,aj}$ - the transshipment volume of cargo from service r to service s when a vessel on service r arrives at port j after traversing leg (i, j), and another correspondence vessel on service s arrives later at the same port sailing directly from port a; ϕ_r^{ij}, the phase of service r, is the time interval between time zero and the first arrival of a vessel on service r at port j after travelling through leg (i, j), which can be equivalently regarded as the schedule of shipping service r. The objective of Álvarez (2012) is to derive the mathematical expressions for calculating the transit time in such a liner shipping network with these parameters and decision variables. In Álvarez (2012) and the study throughout this paper, it is assumed that the inter-arrival frequency f_r of any service $r \in R$ is constant.

Since the total time spent at open sea and in port calls on service r can be readily calculated as

$$\sum_{(i,j)\in A_r} X_r^{ij}\left(\frac{l_r^{ij}}{v_r}+\hat{t}_r^{i}+\tilde{t}_r^{j}\right), \quad r \in R \tag{1}$$

the efforts of Álvarez (2012) focus mainly on deriving the mathematical expressions of the connection time for cargo transshipment. Before the analytical calculation, a theorem on the connection time at a transshipment port, without considering the difference of service phases, is established as below. The theorem is also the base of our study in this paper.

Theorem 1 *Given the inter-arrival frequencies of services r and s, denoted by $f_r \in \mathbb{Z}^+$ and $f_s \in \mathbb{Z}^+$ respectively, and their greatest common divisor $\mu_{rs} = gcd(f_r, f_s)$ and least common multiplier $\gamma_{rs} = lcm(f_r, f_s)$, for every vessel on service r calling at $t = \tau_r \cdot f_r$, where $\tau_r \in \{0,1,\dots,(\gamma_{rs}/f_r - 1)\}$, there exists an earliest correspondence vessel on service s arriving $\tau_r \cdot \mu_{rs}$ units of time later to receive the transshipped cargo.*

In order to calculate the connection time when the phases of services exist, Álvarez (2012) examines four cases on phase differences: (a) $\mu_{rs} > \phi_s^{aj} \geq \phi_r^{ij} \geq 0$, (b)

$\left|\phi_s^{aj}-\phi_r^{ij}\right|=\mu_{rs}$, (c) $\left|\phi_s^{aj}-\phi_r^{ij}\right|>\mu_{rs}$, and (d) $-\mu_{rs}<\phi_s^{aj}-\phi_r^{ij}<0$. Then he inductively argues that the connection time, for the transshipment from the vessel calls at port j via leg (i,j) on service r to the vessel calls at the same port via leg (a,j) on service s in a planning horizon of length $L_{rs}=\gamma_{rs}$, can be computed as

$$T_{rs}^{ij,aj}\cdot\frac{L_{rs}}{f_r}\cdot\left(\frac{\left(f_s/\mu_{rs}-1\right)\mu_{rs}}{2}+\left(\phi_s^{aj}-\phi_r^{ij}\right)\bmod\mu_{rs}\right),\\ r\in R, s\in R\setminus\{r\},(i,j)\in A_r,(a,j)\in A_s \tag{2}$$

which contains a nonlinear operator of modulo. To eliminate the modulo operator, additional variables $\Phi_{rs}^{ij,aj}$, $x_{rs}^{ij,aj}$ are introduced, and Eq. (2) can be rewritten as

[**Álvarez_Connection time**]

$$T_{rs}^{ij,aj}\cdot\frac{L_{rs}}{f_r}\cdot\left(\frac{\left(f_s/\mu_{rs}-1\right)\mu_{rs}}{2}+\Phi_{rs}^{ij,aj}\right), r\in R, s\in R\setminus\{r\},(i,j)\in A_r,(a,j)\in A_s \tag{3}$$

subject to

$$0\le\Phi_{rs}^{ij,aj}\le\mu_{rs}-\varepsilon,\quad r\in R, s\in R\setminus\{r\},(i,j)\in A_r,(a,j)\in A_s \tag{4}$$

$$\Phi_{rs}^{ij,aj}=\phi_s^{aj}-\phi_r^{ij}-x_{rs}^{ij,aj}\cdot\mu_{rs},\ r\in R, s\in R\setminus\{r\},(i,j)\in A_r,(a,j)\in A_s \tag{5}$$

$$x_{rs}^{ij,aj}\in\mathbb{Z}^+\cup\{0\},\ r\in R, s\in R\setminus\{r\},(i,j)\in A_r,(a,j)\in A_s \tag{6}$$

In Eq. (4), ε is a small positive number.

Based on the analysis above, Álvarez (2012) obtains a mixed integer bilinear model below which well captures the inventory holding cost on service r over a planning horizon L, by using a monetary parameter θ representing the value of cargo and of money in time.

$$\min\quad\theta\cdot\left(\begin{array}{l}\sum_{(i,j)\in A_r}X_r^{ij}\left(\frac{l_r^{ij}}{v_r}+\hat{t}_r^i+\tilde{t}_r^j\right)\\+\sum_{s\in R\setminus\{r\}}\sum_{(i,j)\in A_r}\sum_{(a,j)\in A_s}T_{rs}^{ij,aj}\cdot\frac{L}{f_r}\cdot\left(\frac{\left(f_s/\mu_{rs}-1\right)\mu_{rs}}{2}+\Phi_{rs}^{ij,aj}\right)\end{array}\right),\quad r\in R \tag{7}$$

subject to constraints (4) – (6), and some constraints to define the relationship between the phases ϕ_r^{hi} and ϕ_r^{ij} for any three consecutive port calls $\langle h,i,j \rangle$ on service r. Herein, the constraints for flow conservation, transport capacity on each sailing leg, and serviced quantity of each origin-destination demand are all assumed already in the model.

A minor correction should be conducted towards constraint (5) proposed by Álvarez (2012). Constraint (5) holds when $\phi_s^{aj} - \phi_r^{ij} \geq 0$. However, when $\phi_s^{aj} - \phi_r^{ij} < 0$, constraint (5) should be revised to

$$\Phi_{rs}^{ij,aj} = \phi_s^{aj} - \phi_r^{ij} + x_{rs}^{ij,aj} \cdot \mu_{rs},\ r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s \tag{8}$$

The impact of sign of $\left(\phi_s^{aj} - \phi_r^{ij}\right)$ on the definition of $\Phi_{rs}^{ij,aj}$ is not captured by Álvarez's expressions. To reflect this, we introduce the auxiliary binary variable $\Gamma_{rs}^{ij,aj}$ to indicate the sign of $\left(\phi_s^{aj} - \phi_r^{ij}\right)$: $\Gamma_{rs}^{ij,aj} = 1$ implies $\phi_s^{aj} - \phi_r^{ij} \geq 0$, while $\Gamma_{rs}^{ij,aj} = 0$ implies $\phi_s^{aj} - \phi_r^{ij} < 0$. Then constraints (5) and (8) are formulated as:

$$\Phi_{rs}^{ij,aj} = \phi_s^{aj} - \phi_r^{ij} + \left(1 - 2\Gamma_{rs}^{ij,aj}\right) \cdot x_{rs}^{ij,aj} \cdot \mu_{rs},\ r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s \tag{9}$$

$$-M \cdot \left(1 - \Gamma_{rs}^{ij,aj}\right) \leq \phi_s^{aj} - \phi_r^{ij} < M \cdot \Gamma_{rs}^{ij,aj},\quad r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s \tag{10}$$

$$\Gamma_{rs}^{ij,aj} \in \{0,1\},\quad r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s \tag{11}$$

where M is a large constant. If we are always maintaining linear programming constraints of the whole model and not willing to see the nonlinearity in constraint (9) caused by the term $\left(1 - 2\Gamma_{rs}^{ij,aj}\right) \cdot x_{rs}^{ij,aj}$, constraint (9) can be equivalently cast as

$$\Phi_{rs}^{ij,aj} = \phi_s^{aj} - \phi_r^{ij} + \Omega_{rs}^{ij,aj} \cdot \mu_{rs},\ r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s \tag{12}$$

$$\begin{gathered} x_{rs}^{ij,aj} - M \cdot \Gamma_{rs}^{ij,aj} \leq \Omega_{rs}^{ij,aj} \leq x_{rs}^{ij,aj} + M \cdot \Gamma_{rs}^{ij,aj}, \\ r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s \end{gathered} \tag{13}$$

$$\begin{gathered} -x_{rs}^{ij,aj} - M \cdot \left(1 - \Gamma_{rs}^{ij,aj}\right) \leq \Omega_{rs}^{ij,aj} \leq -x_{rs}^{ij,aj} + M \cdot \left(1 - \Gamma_{rs}^{ij,aj}\right), \\ r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s \end{gathered} \tag{14}$$

by introducing the auxiliary decision variable $\Omega_{rs}^{ij,aj}$. This correction improves the completeness of Álvarez's expressions.

Álvarez's model provides a mathematical tool with which the planner of liner shipping network could achieve a balance between network design costs and the service level perceived by shippers, and also could well adapt to shipping networks with the discrepancy of service frequencies among different shipping routes. The main efforts of Álvarez (2012) is to derive the connection time expression (2), which needs careful examination on several cases of phase difference. In fact, we could bypass the complicated inductive argument and calculate the connection time more straightforwardly. We will elaborate on this in Section 4. Meanwhile, Álvarez (2012) only derives the mathematical expression of connection time with forward and many-to-one transshipment policies, but does not address backward transshipment and one-to-many transshipment. Sections 5 and 6 will overcome these limitations.

Since the time for sailing at open sea and for regular port operations excluding transshipment is readily calculated according to Eq. (1), the derivation hereafter only focuses on the calculation of connection time for transshipment, namely, the counterpart of the model [Álvarez_Connection time].

4. A straightforward calculation method for connection time

This straightforward calculation method for connection time is motivated by the observation that given the phases and inter-arrival frequencies of two connected services, the correspondence relationship between these two services are easily determined. Consider the transshipment at a given port j from service r to service s shown in Fig. 1(a), with the phases $\phi_r^{ij} = \phi_s^{aj} = 0$ and the inter-arrival frequencies $f_r = 6$, $f_s = 9$. The time units shown in the figures hereafter are all in days, although our derivation is independent of the unit of time. In a planning horizon of length $L_{rs} = \gamma_{rs} = \text{lcm}(f_r, f_s) = \text{lcm}(6,9) = 18$, we only need to consider the transshipment of cargo from $\gamma_{rs}/f_r = 18/6 = 3$ vessel calls on service r, according to Theorem 1. It is clear that the vessel calls at time $t = 0, 6, 12$ on service r will correspond with the vessel calls at time $t' = 0, 9, 18$ on service s, respectively. If there is a phase difference with $\phi_r^{ij} = 1$, $\phi_s^{aj} = 5$, as shown in Fig. 1(b), the vessel call at time $t = 1$ on service r will correspond with the vessel call at time $t' = 5$ on service s, while the calls at $t = 7, 13$ on service r both correspond with the call at $t' = 14$, with the implication that in many-to-one transshipment a vessel on service s has adequate capacity for the cargo transshipped from more than one vessel call on service r. Note that the correspondence relationship for the transshipment between two services are clear and with no ambiguity. So in the given planning horizon, the total connection times from service r to service s shown in Figs. 1(a) and 1(b) can be calculated as (0-0)+(9-6)+(18-12)=0+3+6=9 days, and (5-1)+(14-7)+(14-13)=4+7+1=12 days, respectively.

Let I_r and I_s denote two index sets $\{0,1,...,(\gamma_{rs}/f_r-1)\}$ and $\{0,1,...,\gamma_{rs}/f_s\}$ respectively. In a general term, in a planning horizon $L_{rs}=\gamma_{rs}=\text{lcm}(f_r,f_s)$, the cargo from a vessel call on service r at time $\phi_r^{ij}+\tau_r\cdot f_r$ with $\tau_r\in I_r$ will be transshipped to a vessel on service s arriving at the same port at time $\phi_s^{aj}+k_s\cdot f_s$ with $k_s\in I_s$ such that

$$\phi_s^{aj}+(k_s-1)\cdot f_s<\phi_r^{ij}+\tau_r\cdot f_r\le\phi_s^{aj}+k_s\cdot f_s \tag{15}$$

Eq. (15) reflects the observation that a vessel call on service r connects exactly with the earliest vessel call arriving later on service s. Solve Eq. (15) with k_s as a variable, and we find

$$k_s=\left\lceil\left(\phi_r^{ij}-\phi_s^{aj}+\tau_r\cdot f_r\right)/f_s\right\rceil \tag{16}$$

The connection time for this correspondence can thus be calculated as

$$\begin{aligned}&\phi_s^{aj}+\left\lceil\left(\phi_r^{ij}-\phi_s^{aj}+\tau_r\cdot f_r\right)/f_s\right\rceil\cdot f_s-\left(\phi_r^{ij}+\tau_r\cdot f_r\right)\\&=\left(\phi_s^{aj}-\phi_r^{ij}\right)+\left\lceil\left(\phi_r^{ij}-\phi_s^{aj}+\tau_r\cdot f_r\right)/f_s\right\rceil\cdot f_s-\tau_r\cdot f_r\end{aligned}$$

Summing the connection times up over all the connections in the planning horizon, we obtain the counterpart of expression (2) in Álvarez's model:

$$\begin{aligned}&T_{rs}^{ij,aj}\cdot\sum_{\tau_r=0}^{\gamma_{rs}/f_r-1}\left\{\left(\phi_s^{aj}-\phi_r^{ij}\right)+\left\lceil\left(\phi_r^{ij}-\phi_s^{aj}+\tau_r\cdot f_r\right)/f_s\right\rceil\cdot f_s-\tau_r\cdot f_r\right\},\\&r\in R,s\in R\setminus\{r\},(i,j)\in A_r,(a,j)\in A_s\end{aligned} \tag{17}$$

By introducing an auxiliary integer variable $\Phi_{\tau_r,s}^{ij,aj}$ to substitute for each nonlinear term $\left\lceil\left(\phi_r^{ij}-\phi_s^{aj}+\tau_r\cdot f_r\right)/f_s\right\rceil$, Eq. (17) can be rewritten as

$$[\textbf{Straightforward}] \quad T_{rs}^{ij,aj} \cdot \sum_{\tau_r=0}^{\gamma_{rs}/f_r-1} \left\{ \left(\phi_s^{aj} - \phi_r^{ij}\right) + \Phi_{\tau_r,s}^{ij,aj} \cdot f_s - \tau_r \cdot f_r \right\}, \quad r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s \tag{18}$$

subject to

$$\left(\phi_r^{ij} - \phi_s^{aj} + \tau_r \cdot f_r\right) / f_s \le \Phi_{\tau_r,s}^{ij,aj} < \left(\phi_r^{ij} - \phi_s^{aj} + \tau_r \cdot f_r\right) / f_s + 1, \quad r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s, \tau_r \in I_r \tag{19}$$

$$\Phi_{\tau_r,s}^{ij,aj} \in \mathbb{Z}^+ \cup \{0\}, \; r \in R, s \in R \setminus \{r\}, (i,j) \in A_r, (a,j) \in A_s, \tau_r \in I_r \tag{20}$$

Eqs. (18) – (20) can be regarded as the counterpart of [Álvarez_Connection time], and plugged into an objective on total transit time using service r in the same way as [Álvarez_Connection time] into expression (7), without changing the bilinearity of the objective.

Comparing the derivation process of our mathematical expressions on the connection time, i.e., [Straightforward], with those of Álvarez (2012), it can be seen that our derivation is more straightforward and simpler. For additional advantages, it will be seen in the subsequent section that this derivation also applies to more transshipment policies and scenarios like backward transshipment.

5. Calculating connection time with backward transshipment allowed

To see the notion of backward transshipment and its advantage, let us first look at an example shown in Fig. 2, with the phases $\phi_r^{ij} = 4, \phi_s^{aj} = 3$, and the inter-arrival service frequencies $f_r = 6,\ f_s = 8$ in a planning horizon of length $L_{rs} = \gamma_{rs} = \mathrm{lcm}(6,8) = 24$. If only forward transshipment is adopted, as assumed in Álvarez (2012) and in Section 4, the vessel calls at time $t = 4, 10, 16, 22$ on service r will tranship the cargo loaded to the vessel calls at time $t' = 11, 11, 19, 27$ on service s, incurring the connection times of 7, 1, 3 and 5 days respectively. Now, we allow backward transshipment, i.e., a vessel call on service r can correspond with a vessel call on service s arriving at the same port earlier. Meanwhile, let us assume in this example that the waiting time for backward transshipment consumed by each vessel call on service s does not exceed 36 hours. As indicated by the dashed arrow line in Fig. 2, the vessel call at time $t = 4$ will backward tranship its cargo loaded to the vessel call at time $t' = 3$ on service s

and incurs a connection time of 0. In this example, allowing backward transshipment reduces the total connection time by (7+1+3+5)-(0+1+3+5)=7 days at the cost of additional port dwell time of 1 day incurred by the vessel on service s arriving at time $t' = 3$.

One may argue about the additional port dwell time (1 day in this example) due to the introduction of backward transshipment. To alleviate this, in shipping practice, we could set an upper bound, denoted by $\overline{t}_{rs}$, on the additional port dwell time of each vessel call on service s waiting for the backward transshipment from service r.

As our industrial collaborator said, backward transshipment has been implemented unknowingly for a long time in shipping practice. In a shipping line, the equipment team, cargo flow team and network design team all regard backward transshipment as a measure to control the cargo connection time (or the transit time). However, this policy has not yet been captured by existing academic studies on liner shipping network analysis. To the best of our knowledge, this paper takes the first step to introduce the notion of backward transshipment (also called "*negative transshipment*" in some shipping lines) to academia, which represents one major academic contribution of this paper.

In this section, we first derive the mathematical expressions for calculating the connection time with backward transshipment involved, and then give some remarks on backward transshipment.

5.1 Calculation expressions for the connection time with backward transshipment involved

To model backward transshipment, we introduce a group of binary decision variables $\Psi_{\tau_r,\tau_s}^{ij,aj}$ such that $r \in R$, $s \in R \setminus \{r\}$, $(i,j) \in A_r$, $(a,j) \in A_s$, $\tau_r \in I_r$, $\tau_s \in I_s$: $\Psi_{\tau_r,\tau_s}^{ij,aj}=1$ indicates that the vessel call at time $t = \phi_r^{ij} + \tau_r \cdot f_r$ on service r will correspond with the vessel call at time $t' = \phi_s^{aj} + \tau_s \cdot f_s$ on service s for backward transshipment; $\Psi_{\tau_r,\tau_s}^{ij,aj}=0$, otherwise. Based on the calculation expressions (18) - (20) for regular forward transshipment and on the fact that the connection time for backward transshipment is 0, the total connection time from vessel calls at port j via leg (i,j)

on service r to vessel calls at the same port via leg (a, j) on service s in a planning horizon of length $L_{rs} = \gamma_{rs}$, with backward transshipment allowed, can be calculated as

$$T_{rs}^{ij,aj} \cdot \sum_{\tau_r=0}^{\gamma_{rs}/f_r-1} \left(1 - \sum_{\tau_s=0}^{\gamma_{rs}/f_s} \Psi_{\tau_r,\tau_s}^{ij,aj}\right) \left[\left(\phi_s^{aj} - \phi_r^{ij}\right) + \Phi_{\tau_r,s}^{ij,aj} \cdot f_s - \tau_r \cdot f_r\right], \quad (21)$$
$$r \in R, s \in R \setminus \{r\}, (i, j) \in A_r, (a, j) \in A_s$$

subject to

(19) - (20),

$$-M\left(1 - \Psi_{\tau_r,\tau_s}^{ij,aj}\right) < \left(\phi_r^{ij} + \tau_r \cdot f_r\right) - \left(\phi_s^{aj} + \tau_s \cdot f_s\right) \le \bar{t}_{rs} + M\left(1 - \Psi_{\tau_r,\tau_s}^{ij,aj}\right), \quad (22)$$
$$r \in R, s \in R \setminus \{r\}, (i, j) \in A_r, (a, j) \in A_s, \tau_r \in I_r, \tau_s \in I_s$$

$$\sum_{\tau_s=0}^{\gamma_{rs}/f_s} \Psi_{\tau_r,\tau_s}^{ij,aj} \le 1, \quad r \in R, s \in R \setminus \{r\}, (i, j) \in A_r, (a, j) \in A_s, \tau_r \in I_r \quad (23)$$

$$\Psi_{\tau_r,\tau_s}^{ij,aj} \in \{0,1\}, \quad r \in R, s \in R \setminus \{r\}, (i, j) \in A_r, (a, j) \in A_s, \tau_r \in I_r, \tau_s \in I_s \quad (24)$$

where constraints (22), in which M is an arbitrary large positive constant, impose the upper bound on the additional port dwell time of each vessel call on service s waiting for the backward transshipment from service r. Constraints (23) restrict that a vessel call on service r can correspond with at most one vessel call on service s for backward transshipment at the given port, since we are now discussing backward transshipment under the same assumption as Álvarez (2012) that one-to-many transshipment is not considered. Constraints (24) define the binary variable $\Psi_{\tau_r,\tau_s}^{ij,aj}$.

Eq. (21) includes trilinear terms formed by $T_{rs}^{ij,aj}$, $\Psi_{\tau_r,\tau_s}^{ij,aj}$, ϕ_r^{ij}, ϕ_s^{aj} and $\Phi_{\tau_r,s}^{ij,aj}$. If the bilinearity of the objective is to be maintained, we can introduce auxiliary variables $z_{\tau_r,s}^{ij,aj}$, where $r \in R$, $s \in R \setminus \{r\}$, $(i, j) \in A_r$, $(a, j) \in A_s$, $\tau_r \in I_r$, and rewrite Eq. (21) as

[Backward] $$T_{rs}^{ij,aj} \cdot \sum_{\tau_r=0}^{\gamma_{rs}/f_r-1} z_{\tau_r,s}^{ij,aj}, \quad r \in R, s \in R \setminus \{r\}, (i, j) \in A_r, (a, j) \in A_s \quad (25)$$

subject to

(19) - (20), (22) - (24)

$$-M\left(1-\sum_{\tau_s=0}^{\gamma_{rs}/f_s}\Psi_{\tau_r,\tau_s}^{ij,aj}\right)\le z_{\tau_r,s}^{ij,aj}\le M\left(1-\sum_{\tau_s=0}^{\gamma_{rs}/f_s}\Psi_{\tau_r,\tau_s}^{ij,aj}\right),\quad (26)$$
$$r\in R, s\in R\setminus\{r\}, (i,j)\in A_r, (a,j)\in A_s, \tau_r\in I_r$$

$$\phi_s^{aj}-\phi_r^{ij}+\Phi_{\tau_r,s}^{ij,aj}\cdot f_s-\tau_r\cdot f_r-M\sum_{\tau_s=0}^{\gamma_{rs}/f_s}\Psi_{\tau_r,\tau_s}^{ij,aj}\le z_{\tau_r,s}^{ij,aj}$$
$$\le\phi_s^{aj}-\phi_r^{ij}+\Phi_{\tau_r,s}^{ij,aj}\cdot f_s-\tau_r\cdot f_r+M\sum_{\tau_s=0}^{\gamma_{rs}/f_s}\Psi_{\tau_r,\tau_s}^{ij,aj},\quad (27)$$
$$r\in R, s\in R\setminus\{r\}, (i,j)\in A_r, (a,j)\in A_s, \tau_r\in I_r$$

Now the expressions [Backward] for the connection time with backward transshipment involved can be plugged into an optimization objective on the total transit time in the same way as the expressions [Álvarez_Connection time] into Eq. (7).

5.2 Some remarks on backward transshipment

Remark 1. In the planning horizon of length $L_{rs}=\gamma_{rs}$ for a setting that allows not only forward but also backward transshipment, forward transshipment and backward transshipment are operated $\sum_{\tau_r=0}^{\gamma_{rs}/f_r-1}\left(1-\sum_{\tau_s=0}^{\gamma_{rs}/f_s}\Psi_{\tau_r,\tau_s}^{ij,aj}\right)$ and $\sum_{\tau_r=0}^{\gamma_{rs}/f_r-1}\sum_{\tau_s=0}^{\gamma_{rs}/f_s}\Psi_{\tau_r,\tau_s}^{ij,aj}$ times, respectively. The total connection time, from the vessel calls at port j via leg (i,j) on service r to the vessel calls at the same port via leg (a,j) on service s in this planning horizon, reduces exactly by $T_{rs}^{ij,aj}\cdot\sum_{\tau_r=0}^{\gamma_{rs}/f_r-1}\left(\sum_{\tau_s=0}^{\gamma_{rs}/f_s}\Psi_{\tau_r,\tau_s}^{ij,aj}\right)\left(\phi_s^{aj}-\phi_r^{ij}+\Phi_{\tau_r,s}^{ij,aj}\cdot f_s-\tau_r\cdot f_r\right)$ time units due to allowing backward transshipment. Backward transshipment is often applicable when the connection time saving is remarkable but meanwhile the additional waiting times of connecting vessels do not significantly influence shipping schedules and operating costs.

Remark 2. To wait for the arrival and the unloading operations of a correspondence vessel call (say vc_r) on service r for backward transshipment, there may be several alternatives for the port behaviours of the vessel call (say vc_s) on service s: (a) vc_s goes into the anchorage ground or the anchorage area in the port basin for waiting after its unloading/loading operations at berth, and returns to berth for transshipment loading after vc_r arrives at the port and finishes the unloading of the transshipped cargo; (b) vc_s moors at the anchorage ground, and pilots into berth when it can finish its seaside operations exactly before vc_r finishes all the preparations for transshipment, and (c) vc_s pilots into berth as usual but its seaside operation time is intentionally prolonged, via the reschedule of quay cranes, to match the arrival lateness of vc_r. Our careful examination shows that expressions [Backward] will not be affected by the choices of port behaviours of vc_s.

Remark 3. Backward transshipment is mainly driven by the tight transit times stipulated by the contracts with shippers. However, several factors may restrict the implementation of backward transshipment in shipping practice. First, the additional waiting time of the connecting vessel call vc_s for correspondence might require speed-up of the vessel on the sailing leg to the next port call. This generally will increase the bunker fuel consumption over the next sailing leg, and sometimes causes the infeasibility of the shipping schedule for the remaining part of the whole voyage. Thus, backward transshipment will only be applicable when the buffer times in the transshipment port and/or over the next sailing leg are surplus and the increase of bunker fuel cost is trivial. Second, backward transshipment relies on the availability of port facilities (berthing space) and equipment (quay cranes and yard cranes). Not every port allows backward transshipment. It would be more possible to obtain the support from the port authority of a transshipment hub, such as Singapore, Tanjung Pelepas, Gioia Tauro and Algeciras. Third, whether it is worthy to implement backward transshipment over two services at a given port depends highly on the transshipment cargo volume.

6. Connection time calculation for one-to-many transshipment at a hub port

An example for many-to-one transshipment has been shown in Fig. 1(b) where the vessel calls at time $t = 7, 13$ on service r correspond with the vessel call at time $t' = 14$ on service s. However, in the regular operations of a hub port where a trunk service connects with one or more feeder services, one-to-many transshipment may often occur, based on the observation that a mega-containership always feeds the cargo loaded into several feeders running on a feeder service. In one-to-many transshipment, the capacity difference between the mega-containership and the feeder plays a critical role in the calculation of transit time. We thus use a full load of the feeder (FLF), instead of the number of containers, as the unit of transshipment volume. In this section, service r is assumed to be the trunk service on which several mega-containerships are operated, while service s is the feeder service on which the feeders running are homogenous in capacity. For simplicity, backward transshipment is not considered.

To illustrate the essentials of one-to-many transshipment, we first look at the example shown in Fig. 3, in which the phases $\phi_r^{ij} = 2, \phi_s^{aj} = 4$ and the inter-arrival service frequencies $f_r = 7,\ f_s = 3$ are maintained. In a planning horizon of length $L_{rs} = \gamma_{rs} = \mathrm{lcm}\left(f_r, f_s\right) = 21$ days, the calls of mega-containerships at the hub port j on service r, each with a load of $\frac{7}{3} = 2\frac{1}{3}$ FLF and thus needs $2\frac{1}{3}$ serving feeders, will transship the cargo loaded into the feeders on service s. Fig. 3(a), in which the number on the transshipment arrow line indicates the transshipment volume in terms of FLF, shows a feasible transshipment plan: the vessel call at time $t = 2$ on service r transships 1/3, 1 and 1 FLF of cargo to the vessels calls at time $t' = 4, 7, 10$ on service s, respectively; the vessel call at time $t = 9$ on service r transships 1, 1 and 1/3 FLF of cargo to the vessels calls at time $t' = 13, 16, 19$ on service s, respectively; the vessel call at time $t = 16$ on service r transships 2/3, 1 and 2/3 FLF of cargo to the vessels calls at time $t' = 19, 22, 25$ on service s, respectively. Fig. 3(b) shows another feasible transshipment plan. The transshipment plan in Fig. 3(a) totally incurs a connection time of 42 (FLF-days), while the plan in Fig. 3(b) reduces the connection time to 28 (FLF-days). Table 1 illustrates the calculation process of connection times for the two transshipment plans in Fig. 3.

It can be seen from the example above that compared to many-to-one transshipment, one-to-many transshipment additionally needs to determine the splitting of the load of the mega-containership and optimize the transshipment volume allocation to several feeder calls. Observations based on Fig. 3 encourage us to model one-to-many transshipment as a network flow problem with some unique features: (a) each transshipment flow from a node on service r representing a mega-containership (mother vessel) call must equal the load volume of the megaship, i.e., f_r / f_s FLFs, which will be absorbed by exactly $\lceil f_r / f_s \rceil$ nodes on service s representing the feeder calls; (b) the transshipment flow to each feeder call node on service s must be 1 FLF in order to fully use the capacity of feeders. In the following derivation, we focus on the case where f_r / f_s is not an integer, since the formulation in the other case with f_r / f_s as an integer is just a simplified version.

For ease of explanation, the following derivation will omit the superscripts identifying the transshipment port and the traverse arcs to this port. For example, ϕ_r^{ij} and ϕ_s^{aj} will be substituted by ϕ_r and ϕ_s respectively. To formulate this problem, the following additional decision variables and auxiliary variables are introduced.

$x_{\tau_r}^{\tau_s}$: the transshipment flow from the mega-containership call at time $\phi_r + \tau_r \cdot f_r$ to the feeder call at time $\phi_s + \tau_s \cdot f_s$, whose value choice could be defined by three binary variables as follows.

$\xi_{\tau_r,1}^{\tau_s}$: 1 if $x_{\tau_r}^{\tau_s} = 1$ (FLF); 0 otherwise.

$\xi_{\tau_r,2}^{\tau_s}$: 1 if $x_{\tau_r}^{\tau_s} = f_r / f_s - \lfloor f_r / f_s \rfloor$ (FLF); 0 otherwise.

$\xi_{\tau_r,3}^{\tau_s}$: 1 if $x_{\tau_r}^{\tau_s} = 1 - \left(f_r / f_s - \lfloor f_r / f_s \rfloor \right)$ (FLF); 0 otherwise.

$z_{\tau_r}^{\tau_s}$: 1 if the first feeder serving the mega-containership call at time $\phi_r + \tau_r \cdot f_r$ arrives at time $\phi_s + \tau_s \cdot f_s$; 0 otherwise.

The total connection time in the given port during a planning horizon of length $L_{rs} = \gamma_{rs} = \text{lcm}\left(f_r, f_s \right)$ can be calculated as

$$\sum_{s \in R \setminus \{r\}} \sum_{\tau_r = 0}^{\gamma_{rs}/f_r - 1} \sum_{\tau_s = 0}^{\gamma_{rs}/f_s} \left[x_{\tau_r}^{\tau_s} \cdot \left(\phi_s + \tau_s \cdot f_s - \phi_r - \tau_r \cdot f_r \right) \right], \quad r \in R \tag{28}$$

To reflect the unique features in this network flow problem, we establish the following constraints. "M"s involved in these constraints are positive constants.

(1) Feasible flow volumes

The flow (in FLF) on each transshipment arc can be fixed at one of four alternatives: 0, 1, $f_r/f_s-\lfloor f_r/f_s \rfloor$ or $1-\left(f_r/f_s-\lfloor f_r/f_s \rfloor\right)$. This can be defined by the relationship between $x_{\tau_r}^{\tau_s}$ and $\xi_{\tau_r,1}^{\tau_s}, \xi_{\tau_r,2}^{\tau_s}, \xi_{\tau_r,3}^{\tau_s}$:

$$1-M\left(1-\xi_{\tau_r,1}^{\tau_s}\right)\le x_{\tau_r}^{\tau_s}\le 1, \quad r\in R, s\in R\setminus\{r\}, \tau_r\in I_r, \tau_s\in I_s \tag{29}$$

$$\begin{gathered} f_r/f_s-\lfloor f_r/f_s \rfloor-M\left(1-\xi_{\tau_r,2}^{\tau_s}\right)\le x_{\tau_r}^{\tau_s}\le f_r/f_s-\lfloor f_r/f_s \rfloor+M\left(1-\xi_{\tau_r,2}^{\tau_s}\right), \\ r\in R, s\in R\setminus\{r\}, \tau_r\in I_r, \tau_s\in I_s \end{gathered} \tag{30}$$

$$\begin{gathered} 1-\left(f_r/f_s-\lfloor f_r/f_s \rfloor\right)-M\left(1-\xi_{\tau_r,3}^{\tau_s}\right)\le x_{\tau_r}^{\tau_s}\le 1-\left(f_r/f_s-\lfloor f_r/f_s \rfloor\right)+M\left(1-\xi_{\tau_r,3}^{\tau_s}\right), \\ r\in R, s\in R\setminus\{r\}, \tau_r\in I_r, \tau_s\in I_s \end{gathered} \tag{31}$$

$$0\le x_{\tau_r}^{\tau_s}\le M\left(\xi_{\tau_r,1}^{\tau_s}+\xi_{\tau_r,2}^{\tau_s}+\xi_{\tau_r,3}^{\tau_s}\right), \quad r\in R, s\in R\setminus\{r\}, \tau_r\in I_r, \tau_s\in I_s \tag{32}$$

$$\xi_{\tau_r,1}^{\tau_s}, \xi_{\tau_r,2}^{\tau_s}, \xi_{\tau_r,3}^{\tau_s}\in\{0,1\}, \quad r\in R, s\in R\setminus\{r\}, \tau_r\in I_r, \tau_s\in I_s \tag{33}$$

(2) Load clearance of the mega-containership

The load of the mega-containership must be completely cleared for transshipment.

$$\sum_{\tau_s=0}^{\gamma_{rs}/f_s} x_{\tau_r}^{\tau_s} = f_r/f_s, \quad r\in R, s\in R\setminus\{r\}, \tau_r\in I_r \tag{34}$$

(3) Unique first serving feeder

There is one and exactly one feeder which could be the first vessel serving the mega-containership arriving at time $\phi_r+\tau_r\cdot f_r$,

$$\sum_{\tau_s=0}^{\gamma_{rs}/f_s} z_{\tau_r}^{\tau_s}=1, \quad r\in R, s\in R\setminus\{r\}, \tau_r\in I_r \tag{35}$$

$$z_{\tau_r}^{\tau_s}\in\{0,1\}, \quad r\in R, s\in R\setminus\{r\}, \tau_r\in I_r, \tau_s\in I_s \tag{36}$$

and each feeder could be the first serving vessel of at most one mother.

$$\sum_{\tau_r=0}^{\gamma_{rs}/f_r-1} z_{\tau_r}^{\tau_s}\le 1, \quad r\in R, s\in R\setminus\{r\}, \tau_s\in I_s \tag{37}$$

(4) Consistency of arrival times in correspondence

The arrival time of the first serving feeder should not be earlier than that of its mother.

$$M\left(1-z_{\tau_r}^{\tau_s}\right)+\phi_s+\tau_s\cdot f_s\geq\phi_r+\tau_r\cdot f_r,\quad r\in R, s\in R\setminus\{r\}, \tau_r\in I_r, \tau_s\in I_s \tag{38}$$

(5) Service continuity

The feeders serving the same mother vessel call should arrive at the hub port consecutively. Namely, given the order of the three feeders by arrival time "feeder 1 $\prec$ feeder 2 $\prec$ feeder 3", it is forbidden that feeder 1 serves mother 1, feeder 2 serves mother 2 and feeder 3 once again backs to serving mother 1. This constraint can avoid long waiting times of cargo for feeder services and the possible complaint from customers.

$$M\left(1-z_{\tau_r}^{\tau_s}\right)+\sum_{\nu=0}^{\lceil f_r/f_s\rceil-1}x_{\tau_r}^{\tau_s+\nu}\geq f_r/f_s,\quad r\in R, s\in R\setminus\{r\}, \tau_r\in I_r, \tau_s\in I_s \tag{39}$$

(6) Full load of the feeder

In a planning horizon, except the first feeder serving the first mother vessel call and the last feeder serving the last mother vessel call, all the feeders should be fully loaded.

$$1-M\left(z_0^{\tau_s}+z_{\gamma_{rs}/f_r-1}^{\tau_s-(\lceil f_r/f_s\rceil-1)}\right)\leq\sum_{\tau_r=0}^{\gamma_{rs}/f_r-1}x_{\tau_r}^{\tau_s}\leq 1,\quad r\in R, s\in R\setminus\{r\}, \tau_s\in I_s \tag{40}$$

Expressions (28) - (40) address the question of connection time calculation for one-to-many transshipment. The values of "M"s in these expressions can be chosen according to Table 2.

One may question the occurrence of the case reflected by Fig. 3 in practice, in which the load of the mega-containership is completely transshiped to the feeders over service s and the capacity of each feeder is fully occupied by the cargo transshiped from service r. In fact, we can explain "the load of the mega-containership" as the cargo volume on board the mega-containership to be transshiped to service s, and explain "FLF" as the available slot space in the feeder for the cargo from service r at the focal transshipment hub. With this explanation, expressions (28)

- (40) can model a general case of one-to-many transshipment in shipping practice. One-to-many transshipment is common because the slot space in the feeder for transshiped cargo from service r at a given port often represents only a portion of the feeder's capacity. In reality, when the feeder arrives at the transshipment port, some slot space might have been occupied by the cargo over service s itself (origin and destination are both on service s) with the transshipment port as an intermediate call. Meanwhile, some slots have to be reserved for the cargo on service s with the transshipment port as the origin.

7. Numerical experiments

The main contribution of this study, from the viewpoint of shipping policy and management, is the introduction of the backward transshipment policy. This section evaluates the effectiveness of this transshipment policy in transit time control via numerical experiments. The numerical experiments will be conducted over the schedule design and container routing problem (SDCRP), due to the fact that this optimization problem for one thing contains all the essentials of shipping network analysis discussed in this study and for another provides the computational tractability for a realistic-sized liner shipping network. We also assume in the numerical experiments that all the vessels involved provide weekly services, since the weekly frequency represents the majority of liner shipping services in practice and experimental findings over weekly services are thus more meaningful.

7.1 The SDCRP and its mathematical models

For ease of mathematical formulation, this section adopts some mathematical notations which are the same as those defined through Sections 3 to 6, but replaces the superscripts/subscripts ij , aj and τ_r by p , q and r , respectively, when it is appropriate.

In the SDCRP, each service $r \in R$ is formed by a sequence of port calls

$$P_{r1} \to P_{r2} \to \cdots \to P_{rp} \to \cdot\cdot \to P_{rN_r} \to P_{r1}$$

where N_r is the number of ports on service r and the voyage between port calls P_{rp} and $P_{r(p+1)}$ represents the sailing leg $p \in \mathcal{P}_r = \{1, 2, ..., N_r\}$ of service r . The last sailing leg N_r goes back to port call P_{r1} because the service forms a loop. We denote all the port calls in the whole service network by $\mathcal{P} = \bigcup_{r \in R} \mathcal{P}_r$. To transport containers from port $o \in \mathcal{P}_r$ to port $d \in \mathcal{P}_s$, a transshipment operation must be conducted from service r to service s at a connected port $P_{rp} = P_{sq}$. That is, the containers traverse through some legs on service r, arrive at the connected port, switch onto service s, traverse further on some legs of service s, and finally arrive at its destination $d \in \mathcal{P}_s$, which actually forms a *container route*. The transshipment involved in this container route is denoted by $\langle r, s, p, q \rangle$. All the possible transshipment operations between services of the network are assumed to be given and denoted by a set

$$\mathcal{T} = \left\{ \langle r, s, p, q \rangle \middle| r \in R, s \in R \setminus \{r\}, p \in \mathcal{P}_r, q \in \mathcal{P}_s, P_{rp} = P_{sq} \right\}.$$

For the container flow for a specific O-D pair, there may exist several container routes serving it. Let us denote by $\mathcal{H}_{od}$ the set of container routes serving the O-D pair $< o, d >$, with the shipment demand d_{od}, and define $\mathcal{H} = \bigcup_{o \in \mathcal{P}, d \in \mathcal{P}} \mathcal{H}_{od}$. Theoretically, the total number of container routes in $\mathcal{H}$ could be exceedingly significant. However, if some business rules apply, a set $\mathcal{H}$ of practical container routes will become constructible. For instance, a container route involved in a very long journey or involving transshipment at too many ports is considered as practically infeasible. Thus, we assume that the set $\mathcal{H}$ is also given. Two binary parameters σ_h^{rp} and δ_h^{rspq} are additionally adopted, with $\sigma_h^{rp} = 1$ indicating that container route $h \in \mathcal{H}$ contains leg p of service r, and $\delta_h^{rspq} = 1$ indicating that container route $h \in \mathcal{H}$ involves the transhipment operation $\langle r, s, p, q \rangle \in \mathcal{T}$. The SDCRP is to determine the schedule $\left\langle \phi_r^1, \phi_r^2, \cdots, \phi_r^{N_r}, \phi_r^{N_r+1} \right\rangle$ of each service $r \in R$, and the container flow Y_h through each container route $h \in \mathcal{H}$ (or equivalently the container flow X_r^p over each leg $p \in \mathcal{P}_r$ of service $r \in R$), in order to minimize the total transit time of containers through the whole network.

If only forward transshipment is allowed, the SDCRP can be formulated as model [**SDCRP$^{\mathbf{F}}$**] (see Appendix A). With the mathematical expressions derived in Section 5, it is easy to formulate the SDCRP with backward transshipment additionally allowed as model [**SDCRP$^{\mathbf{FB}}$**] (see Appendix B). Both models are mixed-integer bilinear programming models, whose nonconvexity poses considerable computational burdens. In general, solving a bilinear programming model is NP-hard. Fortunately, the latest version of IBM ILOG CPLEX Optimizer provides the functionality of globally solving a (mixed-integer) nonconvex quadratic programming model by taking advantage of the *spatial branch-and-bound algorithm* together with *secant approximations* and *McCormick envelopes*.

7.2 A case study

We evaluate the effectiveness of backward transshipment in transit time savings over a real-case example provided by a global shipping line. In this Asia-Europe-Oceania liner shipping network, there are totally 46 ports (Fig. 4) and 11 weekly services (Table 3) involved. A total of 818 container routes, containing 71 possible transshipments, for 652 shipment O-D pairs are constructed with the aid of the global shipping line. The pilotage and berthing times, i.e., $\tilde{t}_r^p$ and $\hat{t}_r^p$, are randomly generated based on the patterns reflected by the shipping log data provided by the global shipping line. For simplicity, all the possible backward transshipment operations hold the same waiting time limit ($\overline{t}_{rs}$ is simplified as $\overline{t}$). The feasible speed interval $\left[\underline{v}_r, \overline{v}_r\right]$= [12, 23], [12, 26], and [12, 30] (knots) for 3000-TEU, 5000-TEU and 10000-TEU vessels, respectively. According to the shipping logs provided by the global shipping line, the bunker fuel consumption (metric ton, MT) of a 3000-TEU vessel in one day at the sailing speed v (knots) can be estimated as $0.0137 \cdot v^{2.892}$. For a 5000-TEU vessel and a 10000-TEU vessel, their fuel consumption rates are $0.0721 \cdot v^{2.456}$ (MT/day) and $0.3822 \cdot v^{1.943}$ (MT/day), respectively.

With the above parameter settings, we globally solve the mixed-integer bilinear programming models [SDCRP$^{\mathrm{F}}$] and [SDCRP$^{\mathrm{FB}}$] via IBM ILOG CPLEX Optimizer 12.6.3 by setting the solver parameter "*solutiontarget*" as 3. All the experiments are run on a workstation with a 12-core CPU and 32 GB of RAM. The solution time limit for each model is set as 7200 s. The experimental results of models [SDCRP$^{\mathrm{F}}$] and [SDCRP$^{\mathrm{FB}}$] are collected in Table 4. In order to analyze the influence of the waiting time limit for backward transshipment, Table 4 reports the results of model [SDCRP$^{\mathrm{FB}}$] with $\overline{t}$=12, 24, 36 and 48 h. $\overline{t} = \infty$ means that no upper limit is explicitly set for the additional waiting time of the correspondence vessel for backward transshipment but the defined schedule at the next port call (after backward transshipment) should still be maintained.

We first analyze the quality of the solutions returned by CPLEX. For the benchmark model [$SDCRP^F$], CPLEX solves it to global optimality with a gap of 1.07%. For model [$SDCRP^{FB}$] with both forward and backward transshipment, the optimality gap of the solution returned by CPLEX is less than 3% when $\bar{t} \le 48$ h. When $\bar{t} = \infty$, the difficulty in deriving good lower bounds increases, but the gap of 4.35% is still acceptable for this mixed-integer nonconvex quadratic programming model. This justifies the trustability of the following experimental findings based on these solutions.

By comparing the results of models [$SDCRP^F$] and [$SDCRP^{FB}$], we can see that 0.15% to 0.86% of the total transit time can be saved if backward transshipment is allowed in this Asia-Europe-Oceania shipping network. These savings are actually significant if we look at the numbers in the parentheses in the 5^{th} column of Table 4. For instance, when $\bar{t} = 36$ h, totally 1254 TEUs can each save one day in transit times. When the upper limit for additional waiting time for backward transshipment is ignored ($\bar{t} = \infty$), around 5000 TEUs can each save one day during shipment. These significant transit time savings are fulfilled by adopting the backward transshipment policy in a small portion (around 7-10%) of transshipment operations, according to the results in the last column of Table 4.

Savings are usually accompanied by additional cost: when the correspondence vessel additionally waits at the transshipment port for backward transshipment, it has to speed up in the following voyage to catch up the schedule at the next port call, which causes the increase of its bunker fuel consumption. In experiments, we also calculate the total bunker fuel increase of 11 vessels (over the 11 services) with each vessel completing a round trip, and report this total bunker fuel increase in Fig. 5. We can see that during the 11 round trips, only less than 110 MT of bunker fuel is additionally burned to maintain the speed increase caused by backward transshipment. This bunker fuel increase is absolutely negligible.

The last bar in Fig. 5 shows a more interesting fact: when $\bar{t} = \infty$, the introduction of backward transshipment does not bring the increase of bunker fuel consumption, but instead saves bunker fuel. This is beyond our intuition from the managerial viewpoint. In fact, this can easily be explained from the viewpoint of mathematical programming: allowing a transshipment operation to be backward transshipment with $\overline{t} = \infty$ actually greatly enlarges the feasible domain of the mathematical model of the SDCRP, which provides the possibility of finding a better solution in transit time with no or even a negative bunker fuel increase. This breaks our conjecture that backward transshipment definitely brings ships an increase in bunker fuel consumption.

8. Conclusions

From the viewpoint of improving the service level perceived by shippers, we revisit the mathematical expressions for transit times of cargo throughout the liner shipping network, based on the pioneering work by Álvarez (2012). Our main efforts are devoted to deriving the mathematical expressions for connection time under different transshipment policies with various liner network configurations: (a) we patch a minor correction towards Álvarez's expressions; (b) we bypass the complicated inductive argument and propose a more straightforward calculation method for connection time; (c) for the first time, we introduce the policy of backward transshipment to the academic community of liner shipping network analysis, and derive the expressions to calculate the connection time when backward transshipment is allowed; (d) we attack the calculation of connection time for one-to-many transshipment, which is actually a relaxation of Álvarez's assumption on the correspondence vessel's capacity. For the main contribution (c), we experimentally evaluate the effectiveness of backward transshipment over an Asia-Europe-Oceania liner shipping network, and find that allowing backward transshipment in a shipping network results in a considerable reduction in the total transit time of cargo at a negligible (or even no) cost of bunker fuel consumption increase.

It is worthy to be noted that our derivation in all scenarios of transshipment does not use the modelling convenience provided by the sense of the optimization objective, i.e., "maximize" or "minimize". Thus, the calculation expressions for connection time could also be included into model constraints instead of objectives. Additionally, in shipping practice, forward transshipment vs. backward transshipment, and many-to-one transshipment vs. one-to-many transshipment might coexist in one liner shipping network. In this case, we should mix together the mathematical expressions exploited in this study in order to calculate the connection time and transit time.

The basic liner shipping network design problem (LSNDP), without consideration of shipping schedule, is already strongly NP-hard (Brouer et al., 2014). Álvarez (2009) and Brouer et al. (2014) propose a two-tier solution approach: the upper tier solves fleet deployment, vessel routing and speed optimization via tabu search with a MIP (mixed integer programming) based neighbourhood exploring procedure, while the lower tier solves the container flow subproblem by formulating it into a multicommodity network-flow problem (MCNP). Additionally determining the optimal shipping schedule for each service in the shipping network, which is closely related to cargo transit time, will further complicate the LSNDP and greatly increase the computational burden. A possible approach decoupling the underlying computational difficulty is to extend the two-tier solution procedure by Álvarez (2009) and Brouer et al. (2014) to three tiers, by adding the third tier for the shipping schedule subproblem. Transit time requirements could be treated in any tier (or multiple tiers): the MIP-based neighbourhood search procedure in the first tier could be retrofitted to produce vessel routes with transit time constraints; the container flow and shipping schedule subproblems in the second and third tiers could also accommodate transit time requirements. Wang et al. (2016) prove that the container flow problem with transit time constraints is polynomial-time solvable by using the periodicity property (e.g. weekly) of liner shipping business, which poses a sharp contrast with the fact that the MCNP with time delay constraints is NP-hard (Holmberg and Yuan, 2003). The two-tier solution approach by Álvarez (2009) and Brouer et al. (2014) is a good example of hybrid optimization algorithms on the LSNDP. Some other kinds of hybrid optimization methods could also be tried to harness the joint strength of mathematical programming, constraint programming and metaheuristics (Blum et al., 2011). Our future research will evaluate these potential methods towards the LSNDP considering the transit time of cargo through extensive numerical experiments.

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Appendices

Appendix A. Model [SDCRP^F]

$$\begin{aligned}\min \ & \sum_{r\in R}\sum_{p\in\mathcal{P}_r} X_r^p \cdot \left(\phi_r^{p+1} - \phi_r^p - \tilde{t}_r^{\,p} + \tilde{t}_r^{\,p+1}\right) \\ & + \sum_{o\in\mathcal{P}}\sum_{d\in\mathcal{P}}\sum_{h\in\mathcal{H}_{od}}\sum_{\langle r,s,p,q\rangle\in\mathcal{T}} \delta_h^{rspq}\cdot Y_h \cdot \left(168\Phi_{rs}^{pq} + \phi_s^q - \phi_r^p\right)\end{aligned} \tag{41}$$

s.t.

$$\phi_r^p \le \phi_s^q + M\left(1-\Lambda_{rs}^{pq}\right), \quad \langle r,s,p,q\rangle\in\mathcal{T} \tag{42}$$

$$0 \le \Phi_{rs}^{pq} \le M\left(1-\Lambda_{rs}^{pq}\right), \quad \langle r,s,p,q\rangle\in\mathcal{T} \tag{43}$$

$$\phi_r^p > \phi_s^q - M\left(1-\Pi_{rs}^{pq}\right), \quad \langle r,s,p,q\rangle\in\mathcal{T} \tag{44}$$

$$\frac{\left(\phi_r^p - \phi_s^q\right)}{168} - M\left(1-\Pi_{rs}^{pq}\right) \le \Phi_{rs}^{pq} < \frac{\left(\phi_r^p - \phi_s^q\right)}{168} + 1 + M\left(1-\Pi_{rs}^{pq}\right), \quad \langle r,s,p,q\rangle\in\mathcal{T} \tag{45}$$

$$\Lambda_{rs}^{pq} + \Pi_{rs}^{pq} = 1, \quad \langle r,s,p,q\rangle\in\mathcal{T} \tag{46}$$

$$X_r^p = \sum_{h\in\mathcal{H}} \sigma_h^{rp}\cdot Y_h, \quad r\in R, p\in\mathcal{P}_r \tag{47}$$

$$X_r^p \le c_r, \quad r\in R, p\in\mathcal{P}_r \tag{48}$$

$$l_r^p / \bar{v}_r \le \phi_r^{p+1} - \left(\phi_r^p + \tilde{t}_r^p + \hat{t}_r^p\right) \le l_r^p / \underline{v}_r, \quad r \in R, p \in \mathcal{P}_r \tag{49}$$

$$\sum_{h \in \mathcal{H}_{od}} Y_h = d_{od}, \quad o \in \mathcal{P}, d \in \mathcal{P} \tag{50}$$

$$\Lambda_{rs}^{pq}, \Pi_{rs}^{pq} \in \{0,1\}, \Phi_{rs}^{pq} \in \mathbb{Z}^+ \cup \{0\}, \quad \langle r,s,p,q \rangle \in \mathcal{T} \tag{51}$$

$$0 \le \phi_r^1 < 168, \quad r \in R \tag{52}$$

$$0 \le \phi_r^{N_r+1} - \phi_r^1 \le 168 n_r, \quad r \in R \tag{53}$$

$$Y_h \ge 0, \quad h \in \mathcal{H} \tag{54}$$

$$\phi_1^1 = 0 \tag{55}$$

In the objective (41), the first term is the total time spent at sea and in port calls. The second term, together with constraints (42)-(46), calculates the connection time during transshipment. Constraint (47) relates the container flow over each sailing leg of each service to the container flows over the container routes. Constraint (48) imposes that the container flow over each sailing leg should be less than the vessel's capacity c_r. Constraint (49) ensures that the sailing speed of a vessel running on service r should always be maintained in its feasible range $[\underline{v}_r, \bar{v}_r]$. Constraint (50) requires that the shipment demand for each O-D pair should be fulfilled. Constraints (51) to (54) define the domains of decision variables. Constraint (55) is redundant but helps to break the symmetry of the feasible domain.

Appendix B. Model [SDCRP^{FB}]

$$\begin{aligned} \min \quad & \sum_{r \in R} \sum_{p \in \mathcal{P}_r} X_r^p \cdot \left(\phi_r^{p+1} - \phi_r^p - \tilde{t}_r^p + \tilde{t}_r^{p+1}\right) \\ & + \sum_{o \in \mathcal{P}} \sum_{d \in \mathcal{P}} \sum_{h \in \mathcal{H}_{od}} \sum_{\langle r,s,p,q \rangle \in \mathcal{T}} \delta_h^{rspq} \cdot Y_h \cdot z_{rs}^{pq} \end{aligned} \tag{56}$$

s.t. (42)-(55),

$$-M\left(1 - \sum_{\tau_s=0}^{n_s-1} \Psi_{r,\tau_s}^{pq}\right) \le z_{rs}^{pq} \le M\left(1 - \sum_{\tau_s=0}^{n_s-1} \Psi_{r,\tau_s}^{pq}\right), \quad \langle r,s,p,q \rangle \in \mathcal{T} \tag{57}$$

$$\begin{aligned} & 168\Phi_{rs}^{pq} + \phi_s^q - \phi_r^p - M \cdot \sum_{\tau_s=0}^{n_s-1} \Psi_{r,\tau_s}^{pq} \le z_{rs}^{pq} \\ & \le 168\Phi_{rs}^{pq} + \phi_s^q - \phi_r^p + M \cdot \sum_{\tau_s=0}^{n_s-1} \Psi_{r,\tau_s}^{pq}, \quad \langle r,s,p,q \rangle \in \mathcal{T} \end{aligned} \tag{58}$$

$$\sum_{\tau_s=0}^{n_s-1} \Psi_{r,\tau_s}^{pq} \le 1, \quad \langle r,s,p,q \rangle \in \mathcal{T} \tag{59}$$

$$\Psi_{r,\tau_s}^{pq} \in \{0,1\}, \quad \langle r,s,p,q \rangle \in \mathcal{T}, \tau_s \in \{0,1,...,(n_s-1)\} \tag{60}$$

$$\begin{aligned} -M \cdot \left(1-\Psi_{r,\tau_s}^{pq}\right) \le \phi_r^p - \left(168\tau_s + \phi_s^q\right) \le \overline{t}_{rs} + M \cdot \left(1-\Psi_{r,\tau_s}^{pq}\right), \\ \langle r,s,p,q \rangle \in \mathcal{T}, \tau_s \in \{0,1,...,(n_s-1)\} \end{aligned} \tag{61}$$

$$\begin{aligned} & l_s^q / \overline{v}_s - M \cdot \left(1-\Psi_{r,\tau_s}^{pq}\right) \le 168\tau_s + \phi_s^{q+1} - \left(\phi_r^p + \tilde{t}_s^q + \hat{t}_s^q\right) \\ & \le l_s^q / \underline{v}_s + M \cdot \left(1-\Psi_{r,\tau_s}^{pq}\right), \quad \langle r,s,p,q \rangle \in \mathcal{T}, \tau_s \in \{0,1,...,(n_s-1)\} \end{aligned} \tag{62}$$

In this model, $\sum_{\tau_s=0}^{n_s-1} \Psi_{r,\tau_s}^{pq} = 1$ indicates the transshipment $\langle r,s,p,q \rangle$ adopts the backward transshipment policy. The introduction of z_{rs}^{pq} and constraints (57) to (61) has already been discussed in Section 5. Constraint (62) ensures that a vessel experiencing additional waiting time for backward transshipment should still maintain the punctuality of its arrival time at the next port.

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Table 1. Connection time calculation for transshipment plans in Fig. 3

Vessel call time (service r)		2			9			16			Total
Transshipment plan 1	Load split [a]	1/3	1	1	1	1	1/3	2/3	1	2/3	
	Receiving time [b]	4	7	10	13	16	19	19	22	25	
	Connection time [c]	2	5	8	4	7	10	3	6	9	
	Weighted connection time [d]	2/3	5	8	4	7	10/3	2	6	6	42
Transshipment plan 2	Load split [a]	1	1	1/3	2/3	1	2/3	1/3	1	1	
	Receiving time [b]	4	7	10	10	13	16	16	19	22	
	Connection time [c]	2	5	8	1	4	7	0	3	6	
	Weighted connection time [d]	2	5	8/3	2/3	4	14/3	0	3	6	28

Note: [a] Unit: FLF. [b] Vessel call time on service s receiving the corresponding cargo split. [c] "Receiving time" minus "Vessel call time (service r)". [d] "Load split" times "Connection time", in terms of FLF-days.

Table 2. Values of "*M*"s in expressions (28) - (40)

Eq. number	Value of M	
	Left-hand side	Right-hand side
(29)	1	—
(30)	$f_r / f_s - \lfloor f_r / f_s \rfloor$	$1-\left(f_r / f_s - \lfloor f_r / f_s \rfloor \right)$
(31)	$1-\left(f_r / f_s - \lfloor f_r / f_s \rfloor \right)$	$f_r / f_s - \lfloor f_r / f_s \rfloor$
(32)	—	1
(38)	$(\tau_r + 1) \cdot f_r - \tau_s \cdot f_s$	—
(39)	f_r / f_s	—
(40)	$1 - \min\left(f_r / f_s - \lfloor f_r / f_s \rfloor, 1-\left(f_r / f_s - \lfloor f_r / f_s \rfloor \right) \right)$	—

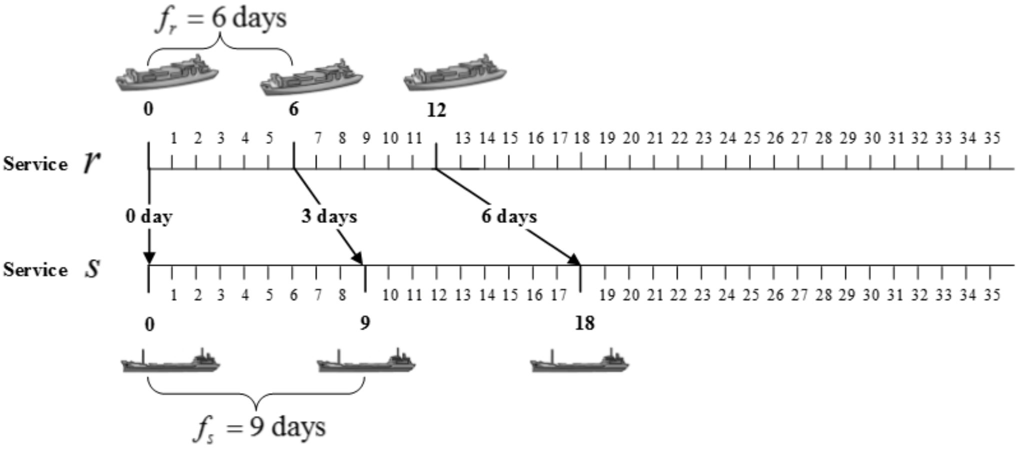
Table 3. Ship services. Source: Wang and Meng (2012a).

No.	Vessel type	Ports of call
1	5000-TEU	Singapore → Brisbane → Sydney → Melbourne → Adelaide → Fremantle
2	5000-TEU	Xiamen → Chiwan → Hong Kong → Singapore → Port Klang → Salalah → Jeddah → Aqabah → Salalah → Singapore
3	3000-TEU	Yokohama → Tokyo → Nagoya → Kobe → Shanghai
4	3000-TEU	Ho Chi Minh → Laem Chabang → Singapore → Port Klang
5	3000-TEU	Brisbane → Sydney → Melbourne → Adelaide → Fremantle → Jakarta → Singapore
6	3000-TEU	Manila → Kaohsiung → Xiamen → Hong Kong → Yantian → Chiwan → Hong Kong
7	3000-TEU	Dalian → Xingang → Qingdao → Shanghai → Ningbo → Shanghai → Kwangyang → Busan
8	3000-TEU	Chittagong → Chennai → Colombo → Cochin → Nhava Sheva → Cochin → Colombo → Chennai
9	5000-TEU	Sokhna → Aqabah → Jeddah → Salalah → Karachi → Jebel Ali → Salalah
10	10000-TEU	Southampton → Thamesport → Hamburg → Bremerhaven → Rotterdam → Antwerp → Zeebrugge → Le Havre
11	10000-TEU	Southampton → Sokhna → Salalah → Colombo → Singapore → Hong Kong → Xiamen → Shanghai → Busan → Dalian → Xingang → Qingdao → Shanghai → Hong Kong → Singapore → Colombo → Salalah

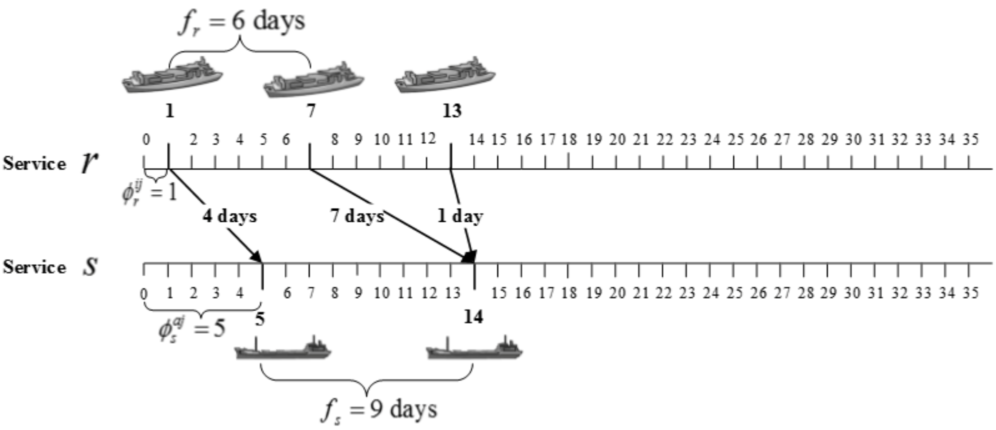
Table 4. Transit time savings by introduction of backward transshipment

	$\bar{t}$ [a]	Transit time (TEU-h)	Optimality Gap (%)	Improvement	# Transshipments
SDCRP^{F}	-	13754155(9782175+3971980) [b]	1.07	-	65+0 [c]
SDCRP^{FB}	12	13733925(9764889+3969036)	2.31	0.15% (20230/843) [d]	60+5
	24	13733925(9764889+3969036)	2.75	0.15% (20230/843)	60+5
	36	13724061(9762030+3962031)	2.97	0.22% (30094/1254)	59+6
	48	13724061(9762030+3962031)	2.51	0.22% (30094/1254)	59+6
	∞	13635867(9931996+3703871)	4.35	0.86% (118288/4929)	61+5

Note: [a] Waiting time limit for backward transshipment (h). [b] Transit time (Port and sailing time + Connection time). [c] Number of forward transshipments + number of backward transshipments. [d] Improvement of model [SDCRP^{FB}] against model [SDCRP^{F}] over the objective of transit time; A(B/C): A-transit time saving percentage, B-transit time savings in TEU-h, C-transit time savings in TEU-day.



(a) Without a phase difference



(b) With a phase difference

Fig. 1. An example for connection time calculation

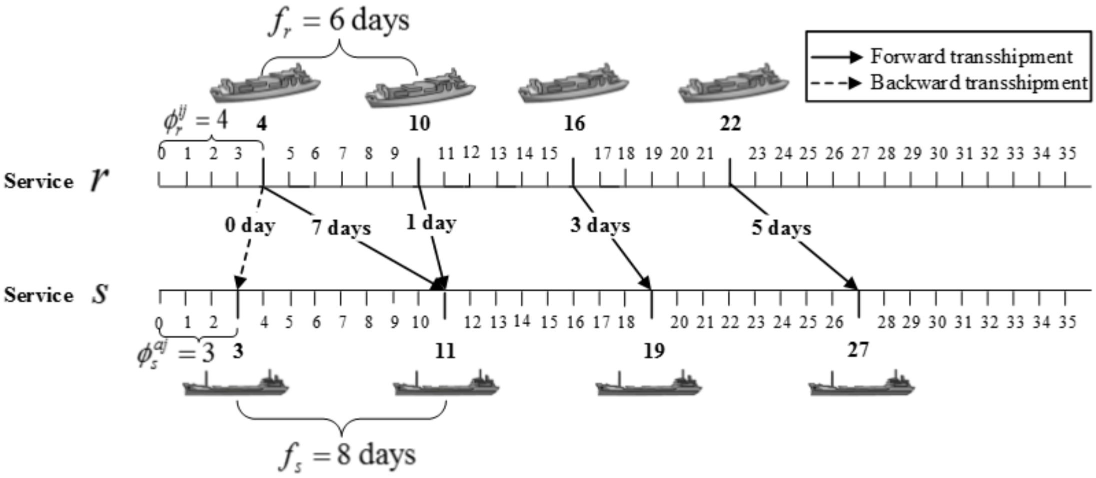
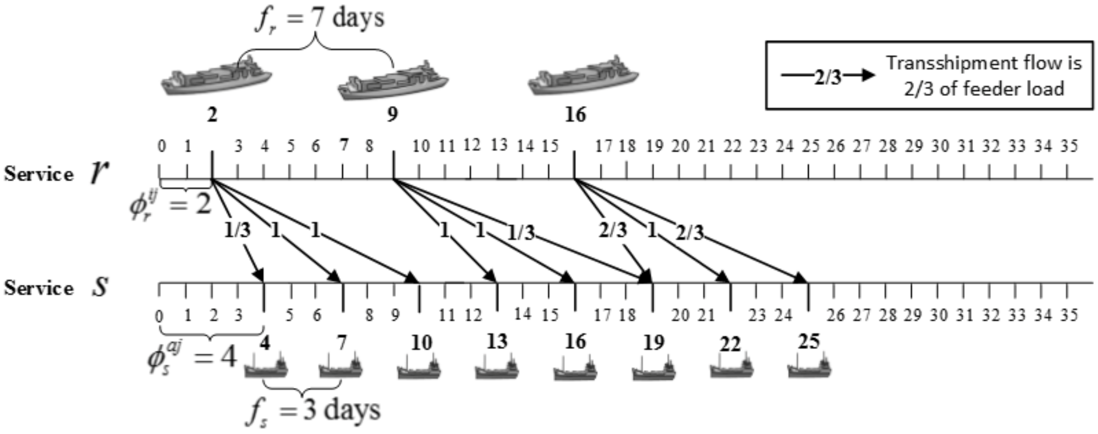
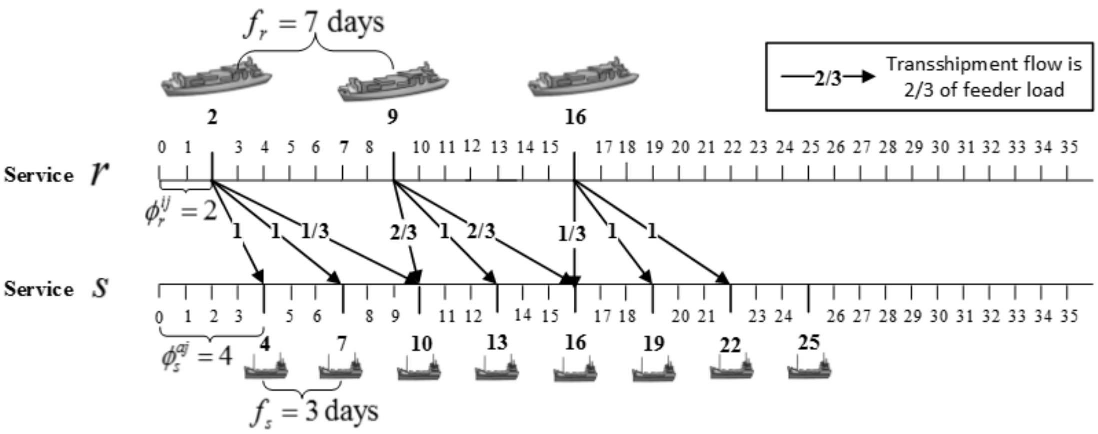


Fig. 2. An example for connection time calculation when backward transshipment is allowed



(a) Transshipment plan 1



(b) Transshipment plan 2

Fig. 3. An example for one-to-many transshipment in a hub port

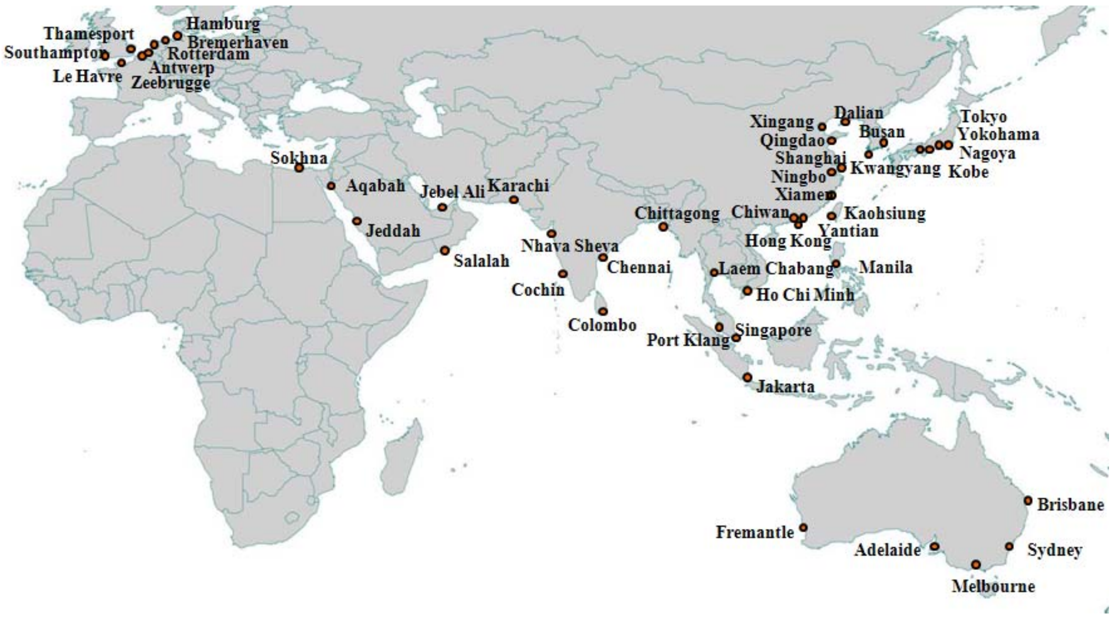


Fig. 4. Ports in an Asia-Europe-Oceania liner shipping network. Source: Wang and Meng (2011).

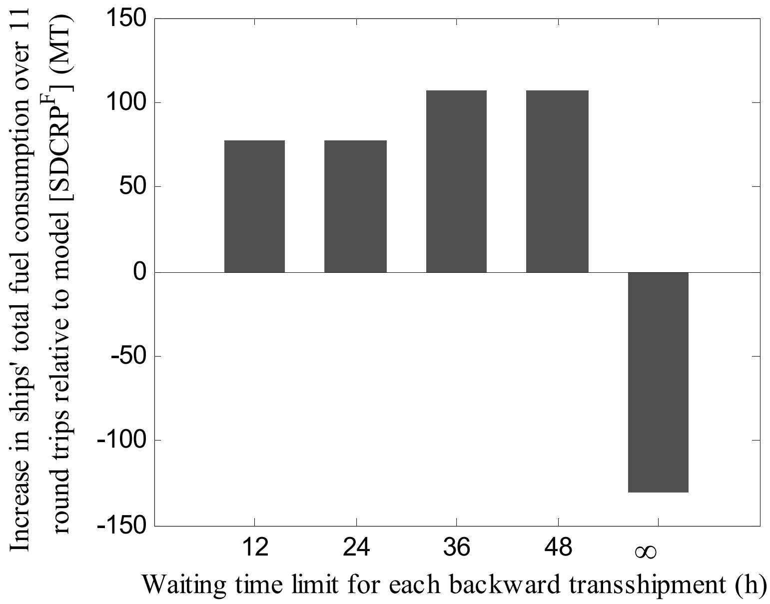


Fig. 5. Vessels' fuel consumption increase over 11 round trips when backward transshipment is allowed