Flexible Capacity Strategy in an Asymmetric Oligopoly Market with Competition and Demand Uncertainty

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ABSTRACT

This paper studies flexible capacity strategy (FCS) under oligopoly competition with uncertain demand. Each firm utilizes either the FCS or inflexible capacity strategy (IFCS). Flexible firms can postpone their productions until observing the actual demand, whereas inflexible firms cannot. We formulate a new asymmetrical oligopoly model for the problem, and obtain capacity and production decisions of the firms at Nash equilibrium. It is interesting to verify that cross-group competition determines the capacity allocation between the two groups of firms, while intergroup competition determines the market share within each group. Moreover, we show that the two strategies coexist among firms only when cost differentiation is medium. Counterintuitively, flexible firms benefit from increasing production cost when the inflexible competition intensity is sufficiently high. This is because of retreat of inflexible firms, flexibility effect, and the corresponding high price. We identify conditions under which FCS is superior than IFCS. We also demonstrate that flexible firms benefit from increasing demand uncertainty. However, when demand variance is not very large, flexible firms may be disadvantaged. We further investigate the effects of cross-group and intergroup competition on individual performance of the firms. We show that as flexible competition intensity increases, inflexible firms are mainly affected by the cross-group competition first and then by the intergroup competition, whereas flexible firms are mainly affected by the intergroup competition. Finally, we examine endogenous flexibility and identify its three drivers: cost parameters, cross-group competition, and intergroup competition.

Key words: asymmetric oligopoly, flexible capacity strategy, demand uncertainty, competition intensity

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1. INTRODUCTION

Volume flexibility refers to a firm's ability to adjust its production quantity up or down to meet the demand (Gerwin, 1993; Goyal and Netessine, 2011). Given the advances in outsourcing, flexible manufacturing systems, and postponement strategy, volume flexibility has become a prevailing strategy, especially when demand fluctuates widely. Several successful businesses have demonstrated the advantages of volume flexibility, such as Anheuser–Busch (Heizer and Render, 2008), Snapper mowers (Heizer and Render, 2008), and Dell (Magretta, 1998).

To explore the advantages of volume flexibility, firms can adopt various operational approaches. Training employees to perform multiple tasks, applying advanced manufacturing technology, and design for manufacturing are the top three practices for acquiring volume flexibility (Hallgren and Olhager, 2009). Some data show that volume flexibility is applied widely in different industries. A survey indicates that volume flexibility is preferred in 31 of 42 industries (Buxey, 2005).[1] Researchers have observed that firms with and without volume flexibility coexist in many industries (Suarez et al., 1996; Vickery et al., 1997; Chang et al., 2003; Hallgren and Olhager, 2009). Furthermore, the effects of volume flexibility can differ considerably among industries. For example, volume flexibility is applied to a lesser extent in the food industry than in the electronics, automotive, and clothing industries (Van Hoek, 1999).

These observations indicate that competition intensity may be a significant factor determining the effects of volume flexibility. Such a competition effect is more pronounced when two types of firms, with and without volume flexibility, coexist in a market. For example, when other computer manufacturers adopted traditional strategies without volume flexibility, Dell achieved extreme success through its volume flexibility strategy (Magretta, 1998). However, when most computer manufacturers implemented volume flexibility to some extent, the benefit of volume flexibility decreased. This indicates that the competition intensity of different groups (with and without volume flexibility) may play different roles in affecting volume flexibility.

In this paper, we explore the effect of competition intensity on volume flexibility under demand uncertainty. We consider two types of capacity strategy, the flexible capacity strategy (FCS) and inflexible capacity strategy (IFCS). Each firm utilizes either FCS or IFCS in an oligopoly. All firms decide their capacity volumes at the beginning of the planning horizon, with anticipation of demand uncertainty and competition pressure. Flexible firms can

[1] In the study, volume flexibility is embedded in a chase strategy or modified chase strategy in operations.

postpone their production until demand information is available so that they can adjust their production quantities to respond to demand changes. By contrast, inflexible firms begin production before obtaining demand information.

We construct an asymmetric oligopoly model consisting of r flexible and s inflexible firms. Each firm engages in two types of competition simultaneously: cross-group competition (flexible group vs. inflexible group) and intergroup competition (competition between firms with the same strategy). This general model enables us to capture both competition intensity effects and the strategy differentiation effect. We verify that the cross-group competition determines the allocation of market potential between the two groups, whereas the intergroup competition determines the individual market share within each group. We characterize the Nash equilibrium and demonstrate that two strategies coexist only when the cost differentiation effect is medium. Under certain conditions, flexible firms can be driven out of the market when the inflexible competition intensity is sufficiently high, and vice versa. This is because the cross-group competition effect dominates the situation.

Contrary to the intuition that increasing costs are always harmful to a firm's profit, we determine that flexible firms benefit from increasing production cost when the inflexible competition intensity is sufficiently high. This finding is attributable to three factors: first, fierce intergroup competition enhances inflexible firms' retreat, potentially leading to a greater market share in the flexible group; second, compared with inflexible firms, flexible firms receive a less negative impact of increasing production costs because they can avoid overproduction through postponement; third, a reduction of total product quantity leads to a higher market price. Consequently, flexible firms are better off.

We identify conditions under which FCS is advantaged. The advantage is attributed to the combination of costing parameters and competition intensities. Comparing the effects of capacity cost differentiation and production cost differentiation, we ascertain that capacity cost differentiation guarantees a production cost advantage for flexible firms, whereas production cost differentiation does not. Furthermore, production cost differentiation induces flexible firms to invest in excess capacity more aggressively.

We determine that flexible firms benefit from demand uncertainty. However, when the market is relatively certain, flexible firms can be disadvantaged. A flexible firm attains a higher profit than that of an inflexible firm only when demand variance is large. This is because a relatively certain market encourages inflexible firms to engage in more production and diminishes the advantages of the postponement ability.

Differing from the traditional monopoly and duopoly models, a two-coexisting-strategy oligopoly model enables us to investigate both cross-group and intergroup competition effects simultaneously. We determine that as more firms shift to the flexible strategy, the flexible group acquires an advantaged position in cross-group competition. However, because of increasing flexible competition intensity, each flexible firm receives a negative impact from intergroup competition. Moreover, flexible firms have more ability to hedge against external changes. Accordingly, flexible firms are mainly affected by intergroup competition: as more firms switch to the flexible strategy, each flexible firm's capacity and profit decrease. For inflexible firms, we observe that when fewer flexible firms exist in the market, the cross-group competition impact dominates; when many inflexible firms shift to the flexible strategy, the intergroup competition effect dominates. Therefore, as more firms switch to the flexible strategy, each inflexible firm's capacity and profit decrease first and then increase.

We also investigate endogenous flexibility, where each firm freely switches its strategy until no firm can increase its profit. This stable status is referred to as final equilibrium (FE). We identify the conditions of FE and propose an approach to determining FE. We characterize FE with numerical examples and illustrate the existence and uniqueness of FE. We identify three drivers of FE: (i) the cost advantage or disadvantage, (ii) the cross-group competition effect, and (iii) the intergroup competition effect.

The rest of the paper is organized as follows: We review the relevant literature in Section 2, and provide some system features in Section 3 and model construction in Section 4. The equilibrium of the two-coexisting-strategy oligopoly is analytically characterized in Section 5. Sensitivity analyses of different effects are given in Section 6. We further investigate the FE where firms are allowed to switch their strategies in Section 7. Finally, we present a conclusion and provide further research directions in Section 8.

2. LITERATURE REVIEW

The literature contains extensive research about flexible capacity investment in various forms (Gupta and Somers, 1996; Vokurka and O'Leary-Kelly, 2000). A comprehensive literature review is provided by Van Mieghem (2003). Gerwin (1993) establishes a research agenda for flexibility. In the following, we review the relevant literature in two streams: (i) flexible capacity in a monopoly, and (ii) flexible capacity in competition.

2.1 Flexible capacity in monopoly

Several studies investigate the FCS in a monopoly. From the timing perspective, delayed differentiation (i.e., postponement) is one method for practicing flexibility under demand uncertainty (e.g., Mathews and Syed, 2004). Van Mieghem and Dada (1999) investigate six flexible capacity strategies with postponement in a decision-making process composed of capacity, production, and pricing. Aviv and Federgruen (2001a, 2001b) investigate the benefits of delayed differentiation in a two-phase multi-item inventory system setting, with consideration of seasonally fluctuating demand. Recently, Yang et al. (2011, 2014) study the FCS with consideration of technology investment. Yang and Ng (2014) consider flexible capacity investment in a multiple production horizon with financial budget constraints.

From the viewpoint of product scope, the focus of the FCS is resource flexibility that enables a firm to manufacture multiple products (Bish and Wang, 2004). Fine and Freund (1990) study the trade-offs between the costs and benefits of employing flexible production technology. Considering the role of price and cost mix differentials, Van Mieghem (1998) investigates the optimal strategy of flexible resource investment for a two-product firm. Introducing responsive pricing, Bish et al. (2005) and Chod and Rudi (2005) study how the FCS is applied to address demand uncertainty in a two-product model setting. Goyal and Netessine (2011) address volume flexibility and product flexibility according to the role of demand correlation and product substitution.

2.2 Flexible capacity in competition

Studies modeling the FCS in competition under demand uncertainty are limited in the literature on operations management (Van Mieghem and Dada, 1999; Goyal and Netessine, 2007; Anand and Girotra, 2007; Anupindi and Jiang, 2008). Some papers investigate the FCS under a certain demand. Röller and Tombak (1993) study the FCS among firms in a market with a certain demand. Furthermore, Boyer and Moreaux (1997) examine the impacts of market scope on firms' FCS choice in competition under demand certainty. Considering demand uncertainty, Van Mieghem and Dada (1999) evaluate the FCS in symmetrical oligopoly competition in which all firms adopt the FCS. Goyal and Netessine (2007) compare the monopoly and duopoly models to emphasize the influence of a firm's FCS decision on its competitors' decisions. In a duopoly setting, Anand and Girotra (2007) demonstrate that only under certain conditions do firms benefit from the FCS in a competitive market. Anupindi and Jiang (2008) examine the effect of demand type—additive or multiplicative—on firms' decisions in a competitive duopoly market. However, there is a substantial difference between duopoly and competition involving two coexisting strategies. In a duopoly model, a firm competes with only one rival. Therefore, the competition intensity effect is neglected. Furthermore, regardless of the competitor's strategy, a firm utilizes only one strategy in competition. In a competitive setting where two strategies coexist, a firm competes with both rivals employing the same strategy and those employing a different strategy. Competition pressure from different strategies can affect a firm's performance in different ways. Such interplay among firms is unknown, especially under demand uncertainty. To the best of our knowledge, research on volume flexibility in an oligopoly involving two strategies is very limited in the literature. Wu and Zhang (2014) investigate the trade-off between the responsiveness and cost in an oligopoly: each firm could choose either efficient or responsive sourcing strategy. Despite the similarity in the basic idea, our paper differs from Wu and Zhang (2014) in some significant perspectives. First, the two papers have different foci. Our paper focuses on the effects of two types of competition intensity, i.e., cross-group and intergroup competition; whereas Wu and Zhang (2014) focus on the information correlation effects. Second, decision stage settings are very different. In this paper, each firm needs to make two sequential decisions, i.e., capacity volume and production quantity; while each firm only makes production quantity decision in Wu and Zhang (2014). Third, firms' decision behaviors are different. Wu and Zhang (2014) assume that each firm makes its decision based on its own demand signals. In this paper, demand distribution is a common knowledge. An inflexible firm decides its production quantity based on demand distribution whereas a flexible firm makes its decision on the basis of actual demand. Fourth, production decision timings are different. All firms make ex ante production decision in Wu and Zhang (2014). In our model, a flexible firm makes an ex post production decision. Accordingly, this paper provides distinctive insights.

In the economics literature, numerous studies investigate flexibility in competition. Gal-Or (1985) addresses the first-mover and second-mover advantages in a duopoly model. The study illustrates that the advantages of the leader and follower depend on the reaction functions of the players. Ellingsen (1995) demonstrates that even when the two firms move simultaneously at the first stage, the mere opportunity to wait out the rival's move can be as harmful as the second-mover disadvantage. By using a duopoly model to simplify an oligopoly, Spencer and Brander (1992) focus on the trade-off between precommitment and flexibility in competition. Because duopoly and oligopoly involving two coexisting strategies differ markedly as discussed previously, these studies cannot demonstrate cross-group and intergroup competition effects on firms' individual decisions. Other studies consider flexibility a cost parameter that represents the additional marginal cost of a firm producing more than its commitment. Klemperer and Meyer (1989) study a firm's ability in determining its supply function, rather than a fixed price or a fixed quantity, in an oligopoly under uncertainty. However, the study does not address the impact of postponement operation on the firm's decision. Vives (1986) studies the trade-off between commitment and flexibility in an n-firm oligopoly. The results show that at equilibrium, as all firms shift from the inflexible to the flexible strategy, the resulting price ranges from the Cournot price to the Bertrand price. Vives (1989) also investigates two specific models, the cost reduction model and plant design model, in which flexibility is addressed as a cost parameter. In these two studies, all firms have the same cost structure and are ex ante identical. Moreover, all firms are identical in making decisions during the decision process. Differing from these studies, our paper addresses the competition effects for firms having different costing environments and production timing. Thus, our model is asymmetrical in production timing and cost structure.

3. SYSTEM FEATURES

Consider a competitive market consisting of r flexible firms and s inflexible firms. The total number of firms is n. Such competition is expressed as $n \sim (r, s)$ throughout this paper. All firms are assumed to be rational.[2] The superscripts F and N are used to specify the variables of flexible and inflexible firms, respectively. Subscripts are used to represent characteristics including the firm index i, quantity at equilibrium e, and best response b.

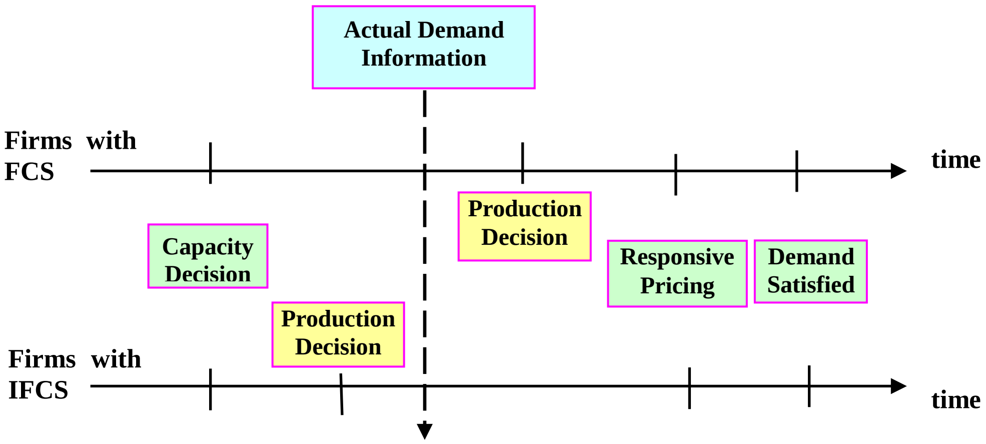

Figure 1: Decision-making processes of firms with FCS and IFCS.

The sequence of events is illustrated in Figure 1. At the beginning of the planning horizon, each firm determines its capacity to maximize the expected profit throughout the process (i.e., capacity decision stage). At this stage, all firms make their capacity volume decisions simultaneously, but they are aware of the competition intensity and the other firms' strategies (i.e., firms are aware that r flexible firms and s inflexible firms exist in the market). In other words, the capacity strategy is exogenous in our main model. We extend our model

[2] Being rational, each firm considers its best response to any given parameter of the market (i.e., given the decisions of all firms and, if applicable, given demand). In mathematics, when we write "given" values, we mean that we consider all possible values.

to endogenize the capacity strategy in Section 7. Because the capacity decision is often a middle- to long-term decision, it is not changed easily or frequently.

As the selling season approaches, firms begin their production (i.e., production decision stage) with knowledge of the other firms' capacity volumes. Inflexible firms begin production before demand information is revealed. Because there is no need to invest in excess capacity, each inflexible firm implements production at its full capacity. Differing from inflexible firms, flexible firms postpone their production until observing the actual demand. Consequently, flexible firms can adjust their production quantities in response to demand changes. At the pricing stage, all firms compete in Cournot competition. Note that pricing is a consequence of the total production quantity in the market. Here, we consider pricing a stage to emphasize the interplay among multiple firms and linkage among supply, demand, and pricing. In a competitive market, each firm can affect but not determine the market price.

We adopt the additive inverse demand function $p(\alpha, Q) = (\alpha - Q)^+$, where p is the responsive price, Q is the total production quantity in the market, and α is a specific realization of an uncertain demand, which follows a distribution with the probability density function $f(\cdot)$ and cumulative distribution function $F(\cdot)$. The mean and variance of α are μ and σ^2, respectively. We assume that all firms sell all products in the market (i.e., "market clearance" rule). We define k and q as the capacity volume and production quantity, respectively. We define $\Omega = \{1,2,\ldots,n\}$, $\Omega^F = \{1,2,..., r\}$, and $\Omega^N = \{r+1, r+2,..., n\}$ as the index sets of all firms, flexible firms, and inflexible firms, respectively. Let $Q^F = \sum_{i\in\Omega^F} q_i^F$ and $Q^N = \sum_{j\in\Omega^N} q_j^N$ be the total production quantities of flexible and inflexible firms, respectively. Moreover, $Q = Q^F + Q^N$ is the total production quantity of all firms. Similarly, we denote by k^F and k^N the total flexible and inflexible capacity volumes, respectively. To facilitate the discussion, we define $L(x) = \int_x^\infty (\alpha - x) f(\alpha) d\alpha$, $x \in [0, \infty)$. The inverse function of L is denoted by $X(c)$, for all $c \in (0, \mu]$. Consider x as the total production level in the market. Then, we derive $L(x) = \int_0^x 0 f(\alpha) d\alpha + \int_x^\infty (\alpha - x) f(\alpha) d\alpha = \int_0^\infty (\alpha - x)^+ f(\alpha) d\alpha = E[p(x)]$. This equation indicates that the physical meaning of $L(x)$ is the expected market price with a total production level x. We denote by C_F and C_N the unit flexible and inflexible capacity costs, respectively, and denote by $\beta \geq 0$ the unit production cost for all firms.

4. MODEL DESCRIPTION

To determine the equilibrium of competition, we first apply backward induction to identify the best response functions of each firm at the production and capacity stages. We then combine all firms' best response functions in Section 5.

4.1 Pricing stage

At the pricing stage, all firms compete in Cournot competition. The market price is determined by $p(\alpha,Q)=(\alpha-Q)^{+}=(\alpha-Q^{F}-Q^{N})^{+}$. Note that we consider market price a function of α and Q; we do not know α and Q.

4.2 Production decision stage

At the production decision stage, the capacity strategies and capacity volumes of all firms are public information. Because of the difference in production timing, flexible and inflexible firms have different operations. When each flexible firm makes its production decision after demand realization, it considers any given decisions of all firms and the demand. Subject to a capacity constraint, a flexible firm sets an optimal production quantity to maximize its ex post profit, in response to a specific demand realization and other firms' production decisions. This can be formulated as

$$\text{Max } \pi_i^F(q_i^F \mid \alpha, q_j, \forall j\in\Omega\setminus\{i\}) = q_i^F p(\alpha,Q)-\beta q_i^F, \text{ s.t. } 0\le q_i^F \le k_i^F,\ i\in\Omega^F. \quad (1)$$

We can derive the optimal production quantity q_i^{F*} of a flexible firm i as described in Lemma 1.

Lemma 1 Given the demand realization α and all other flexible firms' production levels q_j, $j\in\Omega\setminus\{i\}$, firm i's optimal production level is $q_i^{F*}=\begin{cases}0, & p(\alpha,Q)\le\beta\\ q_{ib}^F, & \beta<p(\alpha,Q)\le k_i^F+\beta\\ k_i^F, & k_i^F+\beta<p(\alpha,Q)\end{cases}$,

where $p=p(\alpha,Q)=\alpha-\sum_{j\neq i}q_j(\alpha)$ and $q_{ib}^F=\frac{1}{2}(\alpha-\sum_{h\neq i}q_h^F(\alpha)-Q^N-\beta)$, $h\in\Omega^F$. □

Lemma 1 shows that the flexible firm i's optimal production quantity is a function of all other firms' production levels and the specific demand realization α. The driver inherent is the trade-off between the market price and production cost. Each flexible firm starts production only when the price is higher than the production cost. Lemma 1 demonstrates that flexible firms can respond to demand changes while considering other firms' decisions. Generally, if the demand is very low or the other firms set high production levels, firm i engages in no production because the revenue cannot offset the production cost; if the demand is very high or the other firms set low production levels, it manufactures products at full capacity to capture market potential as much as possible; if the demand is medium, it can determine the optimal production level to avoid production waste or capture excess market potential.

Contrary to flexible firms, inflexible firms begin production before observing demand information. When they make production decisions, they do not know the production quantities as they do not know the demand. Yet they are aware that flexible firms will make responses. Because there is no need to invest in excess capacity at the first stage, each inflexible firm fully utilizes its capacity and sets its production level equal to its capacity volume (i.e., $q_i^N = k_i^N$, $i \in \Omega^N$).

4.3 Capacity decision stage

At the capacity decision stage, all firms decide capacity volumes simultaneously. Each firm determines its optimal capacity decision to maximize its expected profit, for any given capacity decisions of the other firms.

For flexible firms:

$$\text{Max} \quad \Pi_i^F(k_i^F \,|\, k_j, \forall j \in \Omega \setminus \{i\}) = \int_0^\infty q_i^F((\alpha - Q^F(\alpha) - k^N)^+ - \beta) f(\alpha) d\alpha - C_F k_i^F,$$

$$\text{s.t.} \quad k_i^F \geq 0, \; i \in \Omega^F. \tag{2}$$

For inflexible firms:

$$\text{Max} \quad \Pi_i^N(k_i^N \,|\, k_j, \forall j \in \Omega \setminus \{i\}) = \int_0^\infty q_i^N((\alpha - Q^F(\alpha) - k^N)^+ - \beta) f(\alpha) d\alpha - C_N k_i^N$$

$$\text{s.t.} \quad k_i^N \geq 0, \; i \in \Omega^N. \tag{3}$$

The production quantity of each firm in (2) and (3) is the optimal solution at the production stage. Because of the symmetry among the flexible (inflexible) firms, we can prove that all flexible (inflexible) firms make the same decision at equilibrium. The optimal capacity of each flexible and inflexible firm in response to other firms' capacity decisions (i.e., with given capacity volumes of other firms) are presented in Propositions 1 and 2, respectively.

Proposition 1 Given the total capacity of inflexible firms k^N, the optimal capacity and production decisions of the flexible firm $i \in \Omega^F$ at equilibrium are

(i) If $k^N \geq X(C_F) - \beta$, then $k_i^{F*} = q_i^{F*} = 0$ for all $i \in \Omega^F$.

(ii) If $k^N < X(C_F) - \beta$, then $k_i^{F*} = k_e^F = k^F / r > 0$ for all $i \in \Omega^F$; furthermore, we have $k^N + (r+1)k_e^F = X(C_F) - \beta$. The corresponding production decision is

$$q_e^F = \begin{cases} 0, & 0 \le \alpha \le \beta + k^N \\ (\alpha - \beta - k^N)/(r+1), & \beta + k^N < \alpha \le X(C_F) \\ k_e^F, & X(C_F) < \alpha \end{cases}. \quad \square$$

Proposition 1 illustrates that there are two types of competition coexisting in this oligopoly market (i.e., cross-group competition and intergroup competition). Cross-group competition is the competition between flexible and inflexible groups. If the inflexible group sets a very high capacity, the flexible group sets a total capacity of zero, because the market is not profitable at this moment. If the inflexible group sets a medium capacity, the flexible group sets a certain capacity level and competes with the inflexible group. In particular, the total capacity of the flexible group is $k^F = \frac{r}{r+1}[X(C_F) - \beta - k^N]$. As more flexible firms join the market, the total capacity of the flexible group increases and tends toward the boundary $[X(C_F) - \beta - k^N]$. Each flexible firm pursues its profit; therefore, intergroup competition occurs within the flexible group. Because of symmetry, all flexible firms implement production at the same capacity $k_e^F = \frac{1}{r+1}[X(C_F) - \beta - k^N]$ at equilibrium. These principles imply that each flexible firm's capacity decision is jointly affected by cross-group competition and intergroup competition. Specifically, cross-group competition intensity determines market potential allocation between the two groups, whereas the intergroup competition intensity determines each flexible firm's market share within the flexible group.

Through the production postponement ability, each flexible firm implements production only when it can receive a positive marginal net profit. Because all inflexible firms reach production level k^N in total before observing the demand information, the minimum demand should be higher than k^N to obtain a price higher than zero. Furthermore, this price should be higher than the unit production cost so that the firm can attain a positive marginal net profit. Therefore, the minimum demand condition is $\alpha - k^N > \beta$. Note that flexibility helps flexible firms avoid production waste when demand is low. It also helps a firm set a proper production level in response to various market conditions. When demand is medium, each flexible firm uses part of its capacity to achieve the optimal production level while leaving some capacity idle. The optimal production is a function of the specific demand realization α. When demand is very high, each flexible firm achieves its full capacity utilization.

Proposition 1 also shows that there is a trade-off between flexibility and commitment.

The drawback of flexibility is that it hinders fulfilling a commitment for a certain production level. According to Proposition 1, the production level of a flexible firm is $\min\{\frac{(\alpha-\beta-k^N)^+}{r+1},\frac{(X(C_F)-\beta-k^N)^+}{r+1}\}$. Therefore, if a firm commits to a certain production level u, then it can fulfill its commitment only when $\min\{\frac{(\alpha-\beta-k^N)^+}{r+1},\frac{(X(C_F)-\beta-k^N)^+}{r+1}\}\geq u$. Suppose that $\frac{(X(C_F)-\beta-k^N)^+}{r+1}\geq u$; otherwise, the firm must not be able to fulfill its commitment. Then, the probability of fulfilling the commitment is $P(\alpha\geq(r+1)u+\beta+k^N)$. In other words, the probability of not fulfilling the commitment is $F((r+1)u+\beta+k^N)$, which is greater than zero and increases in r and u. This illustrates the inability of a flexible firm to commit. When demand is very low, the flexible firm may lose its market share. As a consequence of implementing production before receiving information, inflexible firms have a premium on the market share. Because of the privilege of production postponement, flexible firms implement production only when demand is higher than the minimum threshold ($\beta+k^N$). When demand is low, flexible firms have low production, or even zero production, and thus they lose their market share. This can harm flexible firms in the long run.

The capacity decision of each flexible firm follows a threshold policy. Each flexible firm makes its capacity investment only when $k^N<X(C_F)-\beta$, which can be represented as $\int_{k^N+\beta}^{\infty}(\alpha-k^N)f(\alpha)d\alpha>C_F+\beta\overline{F}(\beta+k^N)$. Recall that each flexible firm implements production only when demand is higher than $(\beta+k^N)$. This principle implies that the likelihood that a flexible firm implements production is $\overline{F}(\beta+k^N)$. Therefore, the expected production cost is $\beta\overline{F}(k^N+\beta)$. Together with capacity cost, the total expected unit cost for flexible firms is $C_F+\beta\overline{F}(k^N+\beta)$. Note that the term $\int_{k^N+\beta}^{\infty}(\alpha-k^N)f(\alpha)d\alpha$ represents the expected product price. Therefore, flexible firms invest in capacity only when the expected product price is higher than the expected product cost (including capacity and production costs).

Differing from flexible firms, inflexible firms cannot make any changes after demand realization. In production, they have the first-mover advantage but lack adaptability.

Proposition 2 Given the total production of flexible firms Q^F, the optimal capacity decisions of inflexible firms are:

(1) If $\int_{Q^F<\alpha}(\alpha - Q^F)f(\alpha)d\alpha \le C_N + \beta$, then $k_i^{N*} = 0$, for all $i \in \Omega^N$.

(2) If $\int_{Q^F<\alpha}(\alpha - Q^F)f(\alpha)d\alpha > C_N + \beta$, then $k_i^{N*} = k_e^N = k^N/s > 0$, for all $i \in \Omega^N$;

furthermore, $\int_{Q^F+k^N<\alpha}(\alpha - Q^F - \frac{s+1}{s}k^N)f(\alpha)d\alpha = C_N + \beta$. □

Similar to flexible firms, each inflexible firm competes with flexible firms in cross-group competition and with inflexible firms in intergroup competition. Each inflexible firm follows a threshold policy. The threshold is $\int_{Q^F<\alpha}(\alpha - Q^F)f(\alpha)d\alpha = C_N + \beta$. Because inflexible firms lack the ability to adjust production, each unit capacity is transferred into production with a 100% chance. Therefore, the constant expected unit cost is $C_N + \beta$. Note that the term $\int_{Q^F<\alpha}(\alpha - Q^F)f(\alpha)d\alpha$ is the expected product price when inflexible firms begin making capacity investments. Therefore, only when the expected product price is higher than the constant marginal product cost can inflexible firms invest in capacity.

Although flexible and inflexible firms have different costs and production timing, they follow the same threshold policy. Table 1 provides a comparison of the costs of the FCS and IFCS. The difference is the expected marginal product cost (including the capacity and production stages).

Table 1: Cost comparison between FCS and IFCS

	FCS	**IFCS**
Minimum demand to make production	$\beta + k^N$	0
Probability of making production	$\overline{F}(\beta + k^N)$	100%
Unit production cost	β	β
Unit capacity cost	C_F	C_N
Marginal product cost	$C_F + \beta\overline{F}(k^N + \beta)$	$C_N + \beta$
Price condition to invest in capacity	$p(\alpha, Q^N) > C_F + \beta\overline{F}(k^N + \beta)$	$p(\alpha, Q^F) > C_N + \beta$

In the preceding sections, we characterize each firm's response to the other firms' decisions. To obtain the equilibrium of $n \sim (r,s)$ competition, we must combine the best response functions of all firms, as discussed in Section 5.

5. EQUILIBRIUM OF $n \sim (r,s)$ COMPETITION

According to Propositions 1 and 2, in response to the other firms' action, each firm determines whether to make a capacity investment at the capacity decision stage. Four possible equilibriums are listed in Table 2.

Table 2: Four equilibriums of oligopoly competition

		Inflexible firm	
		Making capacity investment	Quitting the market
Flexible firm	Quitting the market	**Case B**	**Case A**
	Making capacity investment	**Case D**	**Case C**

Because of symmetry within intergroup competition, $k^F = rk_e^F$ and $k^N = sk_e^N$ always hold for all cases. Proposition 3 characterizes the equilibrium of $n \sim (r, s)$ competition with demand uncertainty.

Proposition 3 Suppose that $r > 0$ flexible firms and $s > 0$ inflexible firms are given. At equilibrium, $k^F = rk_e^F$ and $k^N = sk_e^N$, where k_e^F and k_e^N together with Π_e^F and Π_e^N in different regions of $R = \{(C_N, C_F): 0 < C_N < \infty,\ 0 < C_F < \infty\}$ are as follows:

***Case A*:** If $\mu - \beta \le C_N$ and $L(\beta) \le C_F$, then $k_e^F = k_e^N = 0$;

***Case B*:** If $C_N < \mu - \beta$ and $L(\beta + k_w) \le C_F$, then $k_e^F = 0$, and k_e^N satisfies

$$C_N + \beta = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha;$$

***Case C*:** If $\mu - (C_N + \beta) \le \frac{r}{r+1}(L(\beta) - C_F)$ and $L(\beta) > C_F$, then

$k_e^N = 0$, and $k_e^F = \frac{1}{r+1}(X(C_F) - \beta)$;

***Case D*:** If $L(\beta + k_w) > C_F$ and $\mu - (C_N + \beta) > \frac{r}{r+1}(L(\beta) - C_F)$, then

$k_e^F = \frac{1}{r+1}(X(C_F) - \beta - sk_e^N)$, and

k_e^N satisfies $C_N + \beta = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha - \frac{r}{r+1}(L(\beta + sk_e^N) - C_F)$,

where k_w is defined as the unique value satisfying $C_N + \beta = \int_{k_w}^{\infty} \left(\alpha - \frac{s+1}{s} k_w\right) f(\alpha) d\alpha$. □

Proposition 3 analytically characterizes the equilibrium of two-strategy coexisting oligopoly competition. Given a costing environment, the equilibrium solution is unique under the assumption $xf(x) < \overline{F}(x)$.[3] Contrary to the belief that flexibility is preferred by

[3] This assumption has been adopted and discussed by a few previous studies (e.g., Van Mieghem and Dada, 1999; Anupindi and Jiang, 2008). For a uniform distribution x~ U[0, b], the condition $xf(x) < \overline{F}(x)$ holds for $x \in [0, b/2)$. For an exponential distribution with parameter λ, the condition $xf(x) < \overline{F}(x)$ holds for $x \in [0, 1/\lambda)$. As discussed in Anupindi and Jiang (2008), with *x* considered as total production in the market, the condition $xf(x) < \overline{F}(x)$ implies that the probability of a positive price is relatively inelastic with respect to the total production in the market. Furthermore, the

managers, we demonstrate that depending on the conditions, both flexible and inflexible strategies can be beneficial or harmful. Because each firm aims to maximize its expected profit through the capacity decision, the demand uncertainty effect is internalized into the equilibrium. We plot the equilibrium results in Figure 2 to provide an intuitive understanding. Define Curve 1 and Curve 2 as $C_{F1} = L(\beta + k_w)$ and $C_{F2} = \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$, respectively. The two curves are boundaries of Region D.[4] The corresponding individual profits of flexible and inflexible firms are shown in the Appendix.

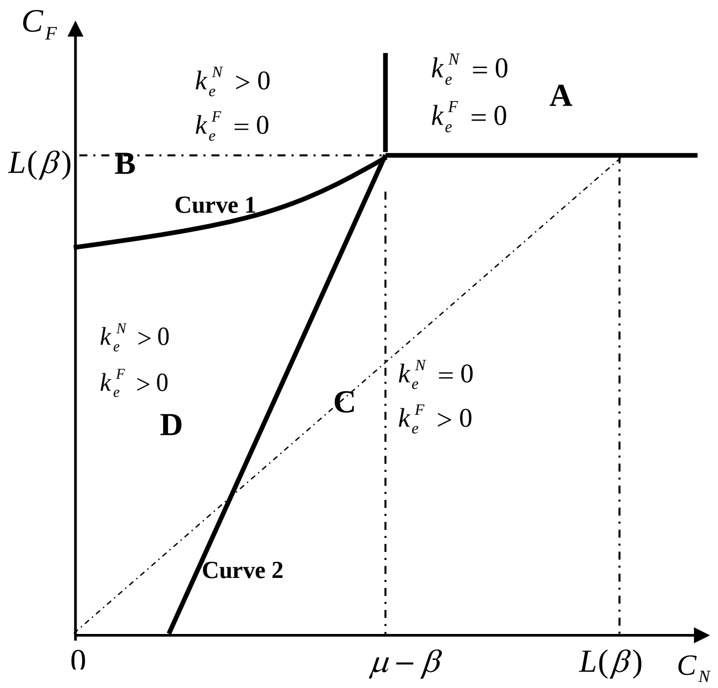

Figure 2: Nash equilibrium of oligopoly.

Apart from demand uncertainty, the competition intensity and costing environment are two major drivers of the equilibrium. As observed, two strategies coexist only in Case D, in which both strategies are profitable but neither has a dominant cost advantage. In other words, the two strategies coexist in the market only when cost differentiation is medium. In Case B (Case C), inflexible (flexible) firms have a leading cost advantage and, therefore, all flexible (inflexible) firms exit the market and the competition becomes a symmetrical oligopoly. In Case A, all firms exit the market because of extremely high capacity costs.

Figure 2 illustrates that Curve 1 is the boundary between Case B and Case D. When the

condition in the model can imply a sensible bound of cost parameters (see Van Mieghem and Dada, 1999; Anupindi and Jiang, 2008).

4 It can be proved that under the assumption $\overline{F}(x) - xf(x) > 0$, in both Curve 1 and Curve 2, C_F is strictly increasing in C_N, and within $C_N \in [0, \mu - \beta]$, Curve 1 is always above Curve 2, except where they intersect at $(\mu - \beta,\ L(\beta))$.

marginal flexible capacity cost is higher than C_{F1}, the FCS is not profitable, and thus, flexible firms exit the market ($k_e^F = 0$). Unexpectedly, although Curve 1 determines whether flexible firms achieve any profits, it depends only on the inflexible competition intensity (i.e., number of inflexible firms *s*). In other words, Curve 1 is independent of the flexible competition intensity (i.e., number of flexible firms *r*). Specifically, with a fixed C_N, C_{F1} is decreasing in *s*. A similar characteristic is found in Curve 2. Although Curve 2 is the boundary determining whether the IFCS is profitable, it is independent of the inflexible competition intensity *s*.

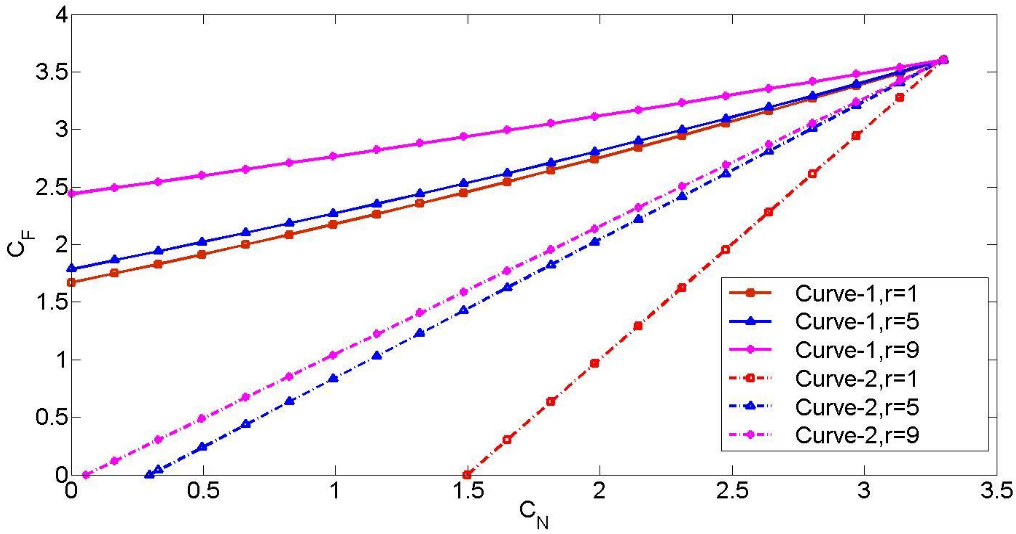

Figure 3: Curve 1 and Curve 2 with different numbers of flexible firms *r*.

With a fixed number of firms $n=10$, Figure 3 depicts the equilibriums with different numbers of flexible firms *r*. We consider three cases, namely $10 \sim (1,9)$, $10 \sim (5,5)$, and $10 \sim (9,1)$. We use the superscript (r,s) to refer the curves of the case $n \sim (r,s)$. As more firms shift from the inflexible to the flexible strategy, both Curve 1 and Curve 2 move upward. The area between Curve $1^{(1,9)}$ and Curve $1^{(9,1)}$ changes from equilibrium with only the IFCS (Region B) to equilibrium with two coexisting strategies (Region D). This change indicates that in this area, the total profit of flexible firms changes from zero to a positive value, while inflexible firms always attain a positive profit. It also indicates that in the area between Curve $1^{(1,9)}$ and Curve $1^{(9,1)}$, as more firms change to flexible firms, the flexible group enhances its competitiveness and the cross-group competition effect dominates. Thus, inflexible firms cannot enjoy a single-strategy market. Conversely, in the area between Curve $2^{(1,9)}$ and Curve $2^{(9,1)}$, the equilibrium changes from a market with two coexisting strategies (Region D) to a market with only the flexible strategy (Region C) as more firms become flexible. This change implies that, in cross-group competition, as the competitiveness of the flexible group increases (i.e., more flexible firms exist in the market), inflexible firms are driven out of the market. As more firms join the market, the region where two strategies coexist (i.e., Region D) becomes smaller and competition becomes fiercer.

The two types of competition have different influences among the cases. In Case B (Case C), cross-group competition drives the flexible (inflexible) firms out of the market, and the intergroup competition intensity determines the profit share of firms surviving in the market. In Case D, each firm is affected by cross-group competition and intergroup competition simultaneously. As observed in practice, multiple flexible and inflexible firms coexist in many industries (Suarez et al., 1996; Vickery et al., 1997; Chang et al., 2003; Hallgren and Olhager, 2009). However, to the best of our knowledge, investigation of simultaneous competition effects (cross-group and intergroup) is very limited in the literature. We compare our general model with models applied in similar contexts in the operations management literature (as shown in Table 3). Our generalized model enables us to investigate both cross-group and intergroup competition effects coexisting in the same market, whereas the previous studies capture either the cross-group or intergroup competition effect.

Table 3: Particular cases of asymmetric oligopoly model in the literature

Model setting	r	s	Competition effects	References
Flexible monopoly model	1	0	No competition	Van Mieghem & Dada, 1999
Inflexible monopoly model	0	1	No competition	Van Mieghem & Dada, 1999
Flexible duopoly model	2	0	Inter-group competition	Anupindi & Jiang, 2008
Inflexible duopoly model	0	2	Inter-group competition	Anupindi & Jiang, 2008
Flexible vs. Inflexible duopoly model	1	1	Cross-group competition	Anupindi & Jiang, 2008
Symmetrical flexible oligopoly model	n	0	Inter-group competition	Van Mieghem & Dada, 1999
Asymmetrical flexible oligopoly model	**r**	**s**	**Cross-group and inter-group competitions coexist**	**The current paper**

6. DISCUSSION

6.1 Production cost effects

It is widely believed that an increasing production cost damages firms' profit. This is true in one-strategy-only competition (i.e., Cases B and C). However, Lemma 2 demonstrates that flexible firms may benefit from an increasing production cost under certain conditions, because of their production postponement ability.

Lemma 2 Given capacity costs $(C_N,\ C_F)$, within Region D of the $n \sim (r,s)$ competition equilibrium, (i) inflexible firms' individual profit is decreasing in production cost (i.e., $\Pi_e^{N(1)}(\beta) < 0$), and (ii) flexible firms' individual profit is increasing in production cost when competition intensity within the inflexible group is sufficiently high (i.e., there exists an s_0, and we have $\Pi_e^{F(1)}(\beta) > 0$ when $s > s_0$). □

Intuitively, an increasing cost diminishes firms' profit, regardless of their strategies. However, Lemma 2 surprisingly demonstrates that this intuition is not always true for flexible firms. When the inflexible competition intensity is high (i.e., s is sufficiently high), each flexible firm can benefit from an increasing production cost. By contrast, inflexible firms' individual profit always decreases as the production cost increases. This means it is possible that increasing cost benefits the flexible firms but harms the inflexible firms, which enhances the flexible firms' advantage. Several factors contribute to this phenomenon. In response to an increasing production cost, each inflexible firm reduces its capacity volume as well as production quantity. Fierce intergroup competition also enhances inflexible firms' retreat. This retreat may leave a greater market share to the flexible group. Differing from inflexible firms, which have a higher risk of overproduction, flexible firms can avoid overproduction through postponement. Therefore, compared with inflexible firms, flexible firms receive a less negative impact of an increasing production cost. Furthermore, increasing production cost forces firms to reduce their production quantities, leading to a higher market price. This also helps flexible firms gain an advantaged position in the competition.

6.2 Capacity cost effects

Note that neither strategy dominates in the region where the two strategies coexist (Region D). Proposition 4 compares the individual profits of flexible and inflexible firms in Region D.

Proposition 4 Within Region D at the equilibrium of $n \sim (r,s)$ competition,

(i) there exists a unique Curve 3 satisfying $\Pi_e^F(C_N, C_{F3}) = \Pi_e^N(C_N, C_{F3})$, in which C_{F3} is increasing in C_N; and

(ii) with a fixed $C_N \in [0, \mu - \beta)$, flexible individual profit is higher than inflexible individual profit $\Pi_e^F > \Pi_e^N > 0$ when $C_F < C_{F3}$; otherwise, each inflexible firm earns a higher individual profit (i.e., $\Pi_e^N > \Pi_e^F > 0$). □

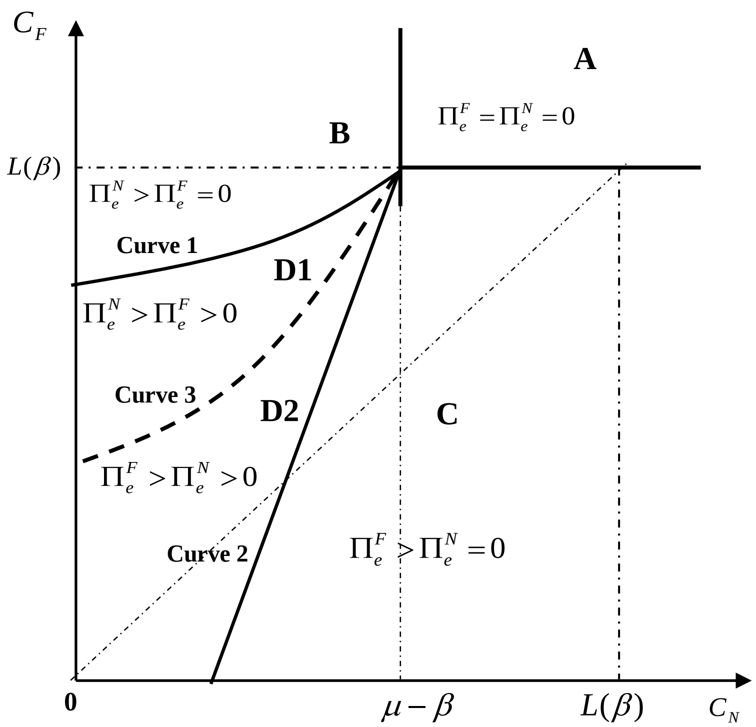

Figure 4: Individual profit comparison.

Proposition 4 proves the existence and uniqueness of a threshold that leads to flexible and inflexible firms having the same individual profits, as depicted in Figure 4. Denote by Region D1 (Region D2) the area above (below) Curve 3. The division curve is jointly determined by both the costing parameters and competition intensity of each group. A high capacity cost and intense intergroup competition reduce a firm's profit. As discussed previously, flexible firms have a lower expected production cost than that of inflexible firms. Therefore, flexible firms are advantaged when the capacity cost difference ($C_F - C_N$) is small. When the capacity cost difference becomes large, either the FCS or IFCS can provide an advantage. Our results are in line with the observation that the effects of volume flexibility can vary substantially among industries (Van Hoek, 1999). Our model explains that multiple factors affect volume flexibility, including costing effects and competition intensity effects.

6.3 Production cost differentiation effect

Thus far, our analyses are based on the assumption that the two types of firms have different capacity costs but the same production cost. In this subsection, we examine what occurs if the cost differentiation effect is embodied in the production cost, rather than the capacity cost. We assume that flexible and inflexible firms incur different production costs (β_F and β_N) and the same capacity cost C_C. We derive the equilibrium as shown in the Appendix.

We plot the equilibrium in Figure 5. Curve 1a and Curve 2a are the boundaries of Case D.

With a fixed inflexible production cost $\beta_N = \beta_{N1}$ (which is less than $\mu - C_C$), the equilibrium changes from Case C, to Case D, and further to Case B. This implies that when the production cost difference increases, the flexible strategy is less profitable. This effect is similar to that in the case of capacity cost differentiation. However, when the capacity cost approaches zero, flexible firms tend to create infinite capacity (i.e., k_e^F tends to infinity), whereas inflexible firms set only a certain level of capacity (i.e., k_e^N is finite). This is because flexible firms do not need to consider production cost when making their capacity decision, because they can postpone production. It implies that a low capacity cost induces flexible firms to invest in considerable capacity that may not be necessary when the demand is not high. This result does not hold at the equilibrium of capacity cost differentiation.[5]

Recall that in the case of capacity cost differentiation, flexible firms always have an advantage in the expected production cost (as $\beta\overline{F}(k^N + \beta) < \beta$). In the case of production cost differentiation, a production cost advantage is not guaranteed.

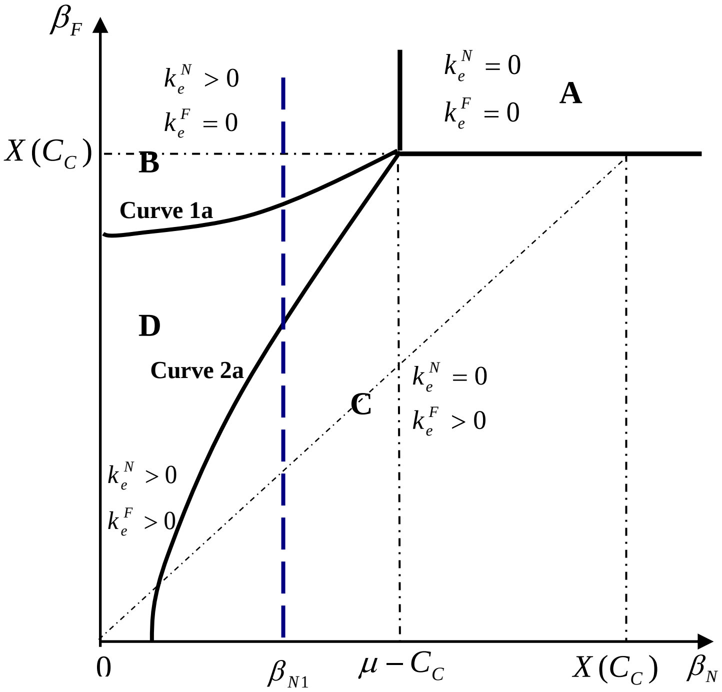

Figure 5: Nash equilibrium of oligopoly with different production costs.

6.4 Demand uncertainty effect

Although flexibility equips a firm with the ability to hedge against demand changes, it also induces the firm to invest in excess capacity. A flexible firm can adjust its production in response to demand changes through postponement (which is a positive effect); conversely,

[5] This indicates that in the case of capacity cost differentiation, both flexible and inflexible firms' capacities are bounded when the production cost is zero.

excess capacity may incur a high investment cost (which is a negative effect). The advantage of flexibility is therefore closely related to demand uncertainty. Figures 4 and 5 illustrate the effect of demand uncertainty. (We use a uniform distribution with parameters set as follows: $\mu = 10$, $\beta = 5$, $C_N = 1$, and $C_F = 1.33$. The market consists of 10 firms, namely six flexible and four inflexible firms.)

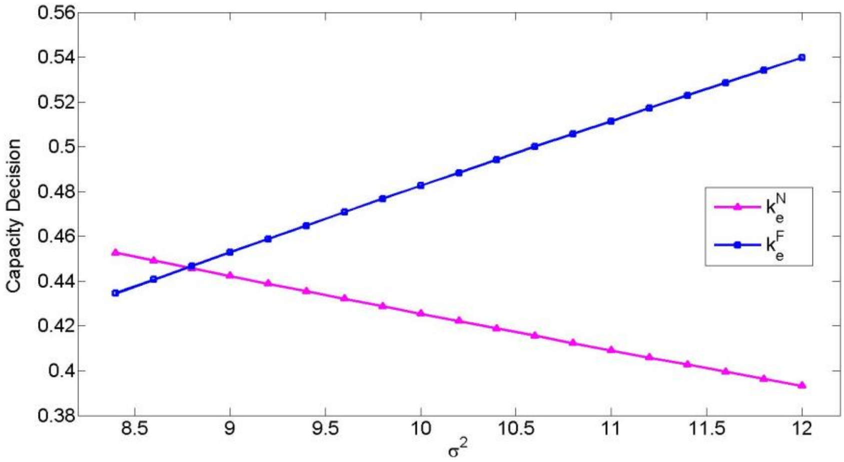

Figure 6: Effect of demand uncertainty on firms' capacity decisions.

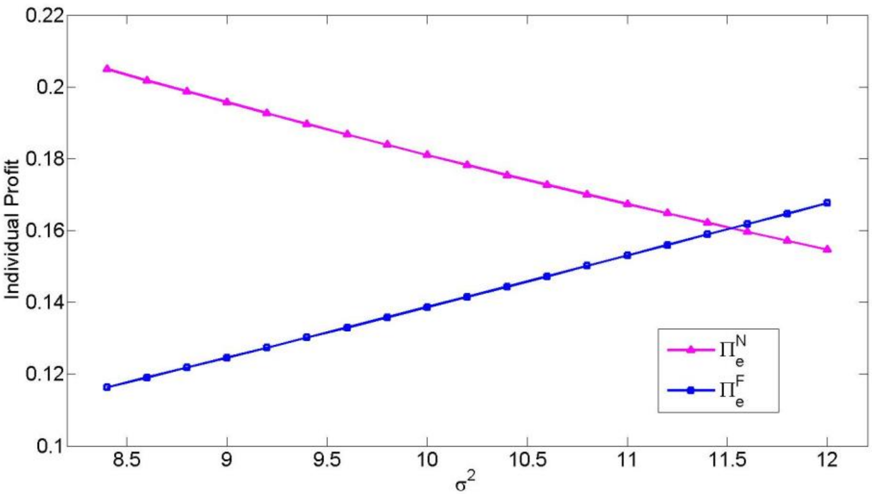

Figure 7: Effect of demand uncertainty on individual profit.

Figure 6 shows that as the market becomes more uncertain, each flexible firm increases its capacity while each inflexible firm reduces its capacity. This is because when demand is low, flexible firms can avoid overproduction; when demand is high, they explore market potential by increasing capacity. In contrast to flexible firms, inflexible firms become more conservative, because they lack the ability to make any adjustment after demand realization. Figure 7 illustrates that flexible firms benefit from demand uncertainty. However, when the market is relatively certain, flexible firms can be disadvantaged. In other words, each flexible firm attains a higher profit than that of an inflexible firm only when demand variance is large.

Conversely, the expected profit of each flexible firm decreases as the market becomes more certain. This is due to two factors: first, in a more certain market, the postponement ability loses its advantages; second, inflexible firms have more confidence regarding market potential and, therefore, they implement more inflexible production. This leads to flexible firms having a small market share. In an extreme case where the market is certain, inflexible firms complete all production that the market demands.

6.5 Competition intensity effect

Two forces determine a firm's capacity decision and expected profit. The first force is the competition from firms with a different strategy. The second force is the competition from firms with the same strategy. This force diminishes each firm's profit within the same group. We use numerical examples to illustrate the effects of the number of flexible firms.

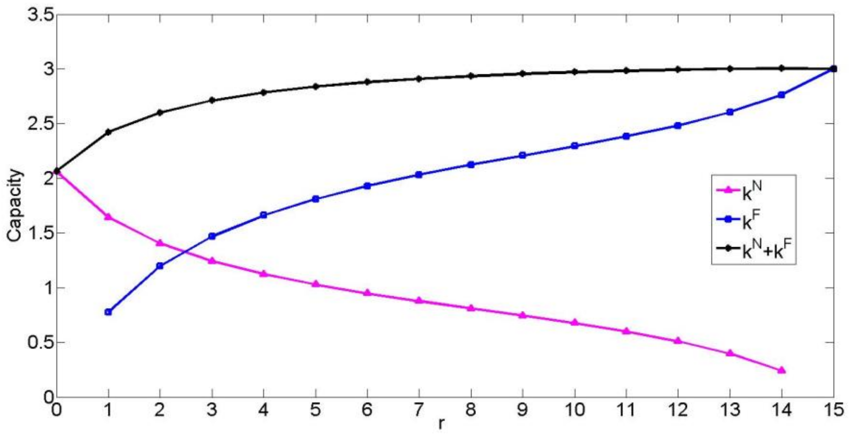

Figure 8: Effects of r on total capacity and total flexible and inflexible capacity.

With a fixed number of firms *n*, an increase in *r* indicates more firms switching from the inflexible strategy to the flexible strategy. Consider a market with 15 firms. Figure 8 illustrates that as the number of flexible firms changes from 1 to 14 (i.e., when the two strategies coexist in the market), the total flexible capacity increases while the total inflexible capacity decreases. In addition, the total capacity of all firms increases slightly. This result clearly demonstrates that the cross-group competition effect determines the market potential allocation between the two groups.

Figures 9 and 10 show the effect of the number of flexible firms on each individual firm. The figures evidently have a similar pattern: as more firms switch from the inflexible to the flexible strategy, each flexible firm's capacity and profit are reduced, and each inflexible firm's capacity and profit decrease first and then increase. This is because of the coexistence of the two types of competition. As *r* increases, the flexible group becomes larger and the inflexible group becomes smaller. These changes imply that the flexible group's position becomes more advantaged in cross-group competition. Therefore, cross-group competition benefits the flexible group as more firms become flexible. Moreover, as *r* increases, the intergroup competition intensity increases in the flexible group, and the inflexible group experiences less intense intergroup competition. In other words, each flexible (inflexible) firm receives a negative (positive) impact. Moreover, flexible firms have greater ability to hedge against changes outside the group. Therefore, flexible firms are mainly affected by intergroup competition. As *r* increases, inflexible firms encounter two situations: when *r* is relatively low, cross-group competition has the dominant impact; when *r* is relatively high, intergroup competition has the dominant impact.

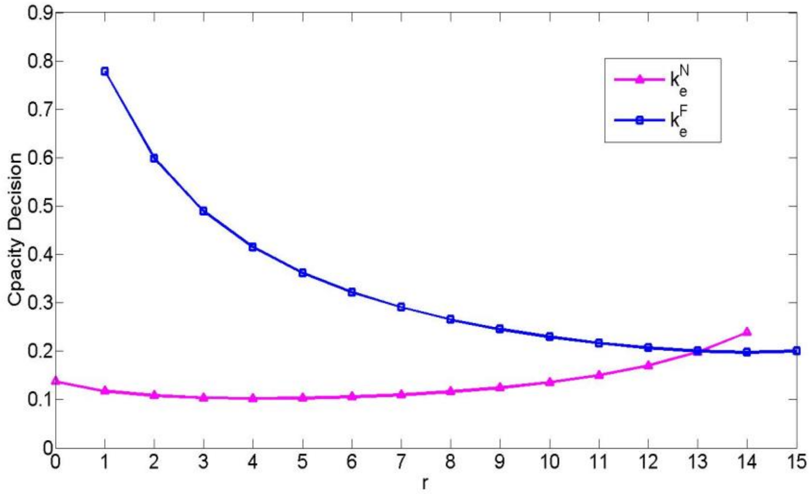

Figure 9: Effects of r on individual capacity.

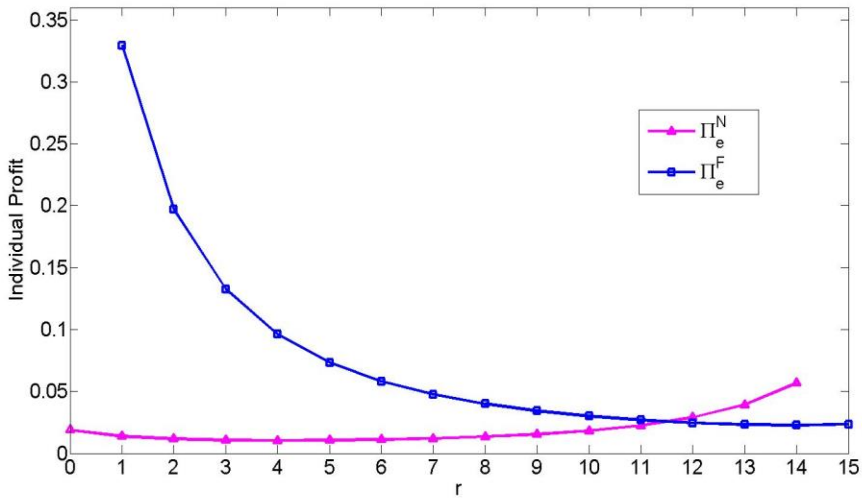

Figure 10: Effects of r on individual profit.

7. EXTENSION: ENDOGENOUS FLEXIBILITY

Thus far we assume that the numbers of flexible and inflexible firms are given. In this section, we endogenize *r* and *s* and identify the equilibrium. In other words, all firms are allowed to freely switch their strategies to maximize individual profits. This process can be regarded as a strategy competition involving multiple players. Eventually, the status becomes stable where no firm has an incentive to change its strategy. We refer this status as FE. To streamline the analysis, we focus on the case in which both strategies exist (Case D).

7.1 Conditions of final equilibrium

We first identify the conditions of FE. Consider a competitive market involving *n* firms, $n \sim (r, s)$. The expected flexible and inflexible individual profits are $\Pi_e^F(r,s)$ and $\Pi_e^N(r,s)$, respectively. If a firm switches from the flexible to the inflexible strategy, the expected individual profits become $\Pi_e^F(r-1, s+1)$ and $\Pi_e^N(r-1, s+1)$. Similarly, if a firm switches from the inflexible strategy to the flexible strategy, the expected individual profits become $\Pi_e^F(r+1, s-1)$ and $\Pi_e^N(r+1, s-1)$. When no firm can enlarge its profit, the status becomes stable. Therefore, the necessary and sufficient conditions of FE at $n \sim (r, s)$ are $\Pi_e^F(r,s) \ge \Pi_e^N(r-1, s+1)$ and $\Pi_e^N(r,s) \ge \Pi_e^F(r+1, s-1)$. We define $D(r) = \Pi_e^F(r, n-r) - \Pi_e^N(r-1, n-r+1)$, $1 \le r \le n$. The conditions for FE can be represented as $D(r) \ge 0$ and $D(r+1) \le 0$. Given $r = r_0$, $r_0 \in [1,\ n]$, we define $G(C_N,\ C_F | r_0) = D(r_0) = 0$ in terms of C_N and C_F.

Lemma 3 Given $r = r_0$, $r_0 \in [1,\ n]$, there exists a unique curve satisfying $G(C_N,\ C_F |\ r_0) = D(r_0) = 0$, on which C_F increases as C_N increases; with a fixed $C_N \in [0, \mu - \beta)$, $D(r_0)$ is decreasing in C_F. □

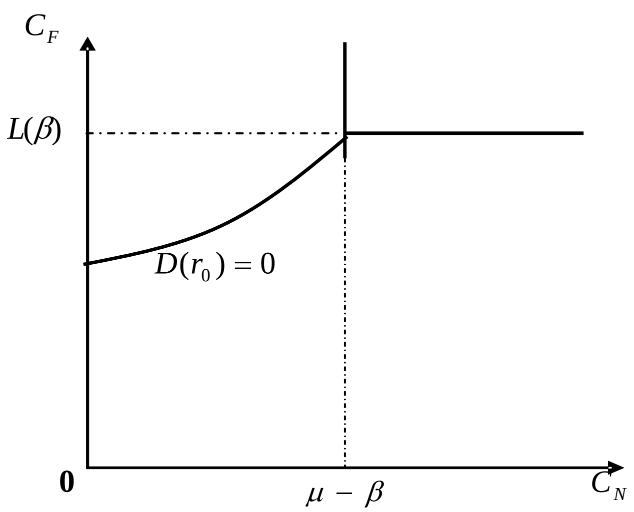

Figure 11: Curve $D(r_0) = 0$**.**

Note that all curves $D(r) = 0$, $r \in [1,\ n]$, intersect at point $(C_N,\ C_F) = (\mu - \beta,\ L(\beta))$. Note that Lemma 3 implies that in areas above the curve $D(r_0) = 0$, $r_0 \in [1,\ n-1]$, we have

$G(C_N,\ C_F|\ r_0) = D(r_0) < 0$; in areas below the curve $D(r_0) = 0$, we have $G(C_N,\ C_F|\ r_0) = D(r_0) > 0$. Therefore, according to the conditions of FE, we have Proposition 5.

Proposition 5 Given n firms, for every $r_0 \in [1,\ n-1]$, consider the curves $G(C_N,\ C_F|\ r_0) = D(r_0) = 0$ and $G(C_N,\ C_F|\ r_0+1) = D(r_0+1) = 0$ within the area $\{(C_N, C_F): C_N \le \mu - \beta\ \&\ C_F \le L(\beta)\ \&\ 0 \le C_N \le C_F\}$; then, FE $n = (r_e,\ s_e)$ can be categorized into one of the following five scenarios.

(i) In areas below the curve $G(C_N,\ C_F|\ r_0) = D(r_0) = 0$ and above the curve $G(C_N,\ C_F|\ r_0+1) = D(r_0+1) = 0$, FE is $n = (r_e,\ s_e) = (r_0,\ n-r_0)$.

(ii) In areas above the curve $G(C_N,\ C_F|\ r_0) = D(r_0) = 0$ and below the curve $G(C_N,\ C_F|\ r_0+1) = D(r_0+1) = 0$, $r = r_0$ is not FE.

(iii) If these two curves overlap, with respect to points on the curves, FE is either $n = (r_e,\ s_e) = (r_0,\ n-r_0)$ or $n = (r_e,\ s_e) = (r_0+1,\ n-r_0-1)$.

(iv) In areas above all curves $G(C_N,\ C_F|\ r_0) = D(r_0) = 0$, $r_0 \in [1,\ n]$, FE is $n = (r_e = 0,\ s_e = n) = (0,\ n)$;

(v) In areas below all curves $G(C_N,\ C_F|\ r_0) = D(r_0) = 0$, $r_0 \in [1,\ n]$, FE is $n = (r_e = 0,\ s_e = n) = (n,\ 0)$. □

Proposition 5 shows that the equivalent condition of FE (i.e., $D(r) \ge 0$ and $D(r+1) \le 0$) can be reflected by the relative position of the two curves $D(r) = 0$ and $D(r+1) = 0$. This relationship ensures that theoretically FE can be determined. With application of Proposition 6 under a certain demand distribution, FE can be practically determined by plotting all curves $G(C_N,\ C_F|\ r_0) = D(r_0) = 0$, where $r_0 \in [1,\ n]$.

7.2 Numerical examples

As a demonstration, we apply the proposed approach to a market with 10 firms under a uniform distribution (parameters: $\mu = 9$, $\sigma^2 = 12$, $\beta = 5.7$). We plot all curves $G(C_N,\ C_F|\ r_0) = D(r_0) = 0$, $r_0 = (1,2,...,10)$ in Figure 12.

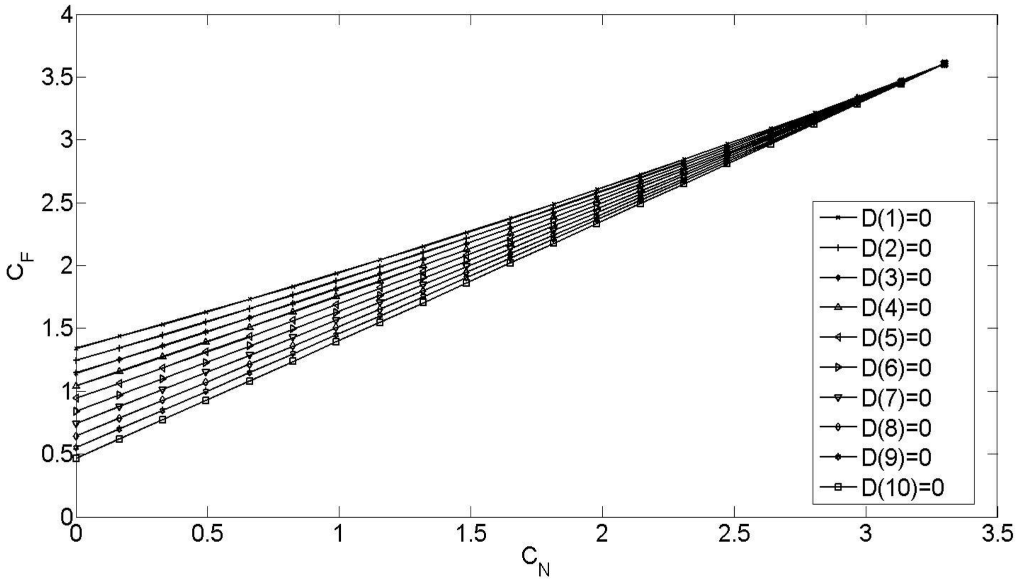

Figure 12: Final equilibrium of a 10-firm market.

We find that all 10 curves intersect only at point $(C_N,\ C_F)=(\mu-\beta,\ L(\beta))$. Furthermore, we find that the curve $D(r)=0$ moves downward when *r* increases. These two observations are critical, because they guarantee the existence and uniqueness of FE in an oligopoly market. The existence and uniqueness of FE means that the market is in a stable status and no firm changes its strategies. FE is fully determined and characterized as follows:

(i) With respect to points on the curves $D(r_0)=0$, $r_0=(1,2,...,10)$, FE remains at $10\sim(r_0,10-r_0)$.

(ii) In the area between $D(r_0)=0$ and $D(r_0+1)=0$, $r_0=(1,2,...,9)$, FE remains at $10\sim(r_0,10-r_0)$.

(iii) In the area above the curve $D(1)=0$, FE remains at $n\sim(0,10)$.

(iv) In the area below the curve $D(10)=0$, FE remains at $n\sim(10,0)$.

As Figure 12 shows, the two strategies coexist only when the inflexible strategy has a cost advantage; otherwise, all firms adopt the flexible strategy. As the cost advantage of the inflexible strategy increases, more firms adopt the inflexible strategy. Therefore, the cost advantage or disadvantage is a major driver of the strategy choice. In addition, in the area between the curves $D(1)=0$ and $D(10)=0$, although the inflexible strategy has a cost disadvantage, some flexible firms remain in the market. This is because each firm is also affected by the competition between and within groups. As more firms shift from the flexible to the inflexible strategy, flexible firms' intergroup competition pressure is relieved (i.e., *r* decreases). This ensures that each flexible firm obtains a higher market share within the flexible group, which may positively affect a flexible firm's profit. Meanwhile, inflexible firms have a cost advantage but more intense intergroup competition. Furthermore, more firms shifting from the flexible to the inflexible strategy may benefit the inflexible group in cross-group competition. Consequently, jointly affected by the costing environment, intergroup competition, and cross-group competition, the market achieves a stable status, that is, FE.

Our results are in line with our observation that different industries may have different flexibility levels within the industry. For example, volume flexibility is applied to a lesser extent in the food industry than in the electronics, automotive, and clothing industries (Van Hoek, 1999). Fewer firms may adopt volume flexibility in the food industry because the life cycle is very short and, therefore, flexibility can incur very high costs (processing cost and transportation cost).

8. CONCLUSIONS

This paper investigates the FCS in an oligopoly consisting of r flexible firms and s inflexible firms under demand uncertainty. We conclude our paper by summarizing its contributions.

We first characterize two types of competition coexisting in the market simultaneously: cross-group competition and intergroup competition. Cross-group competition determines the market share allocation between flexible and inflexible groups, whereas intergroup competition determines the individual profit within each group. We observe that the driver inherent in the decision-making of each firm is the trade-off between the market price and marginal product cost (including capacity and production stages): only when the market price is higher than the respective marginal production cost does each flexible and inflexible firm invest in capacity. Our general model enables us to capture the effects of the two types of competition simultaneously; such a model gets little attention in the literature. Furthermore, we analytically characterize the equilibrium and determine that the two strategies coexist only when the cost differentiation effect is medium.

Contrary to the intuition that increasing the cost always damages the profits of firms, we show that flexible firms benefit from an increasing production cost when the inflexible competition intensity is sufficiently high. This may be attributable to three factors: First, fierce intergroup competition enhances inflexible firms' retreat, which may leave a greater market share to flexible firms. Second, flexible firms receive a less negative impact of the increasing production cost because they can avoid overproduction. Third, the reduction of total product quantity leads to a higher market price. This also helps flexible firms gain an advantaged position in the competition.

Focusing on the two-coexisting-strategy environment, we compare the individual profits of flexible and inflexible firms. We identify the conditions under which a flexible firm is advantaged. A higher capacity cost or higher intergroup competition intensity harms a firm's profit. Because flexible firms have a lower expected production cost than that of inflexible firms, they are advantaged relative to inflexible firms when the capacity cost difference is small. We also investigate the effects of production cost differentiation. The results show that the production cost advantage of flexible firms cannot be guaranteed. Moreover, production cost differentiation induces flexible firms to invest in excess capacity more aggressively.

We study the demand uncertainty effect and competition intensity effects. We observe that flexible firms benefit from increasing demand uncertainty. However, when demand variance is not very large, inflexible firms may be advantaged. This is because a less uncertain market encourages more inflexible production and diminishes the advantage of flexible firms' postponement ability. With a fixed number of firms, we observe that as more firms become flexible, each flexible firm is affected mainly by intergroup competition, and both its capacity and expected profit are reduced. However, each inflexible firm's capacity and expected profit decrease first and then increase. This is due to the combination of the effects of intergroup and cross-group competition. When many inflexible firms exist, cross-group competition has the dominant effect; otherwise, each inflexible firm is mainly affected by intergroup competition.

Finally, we extend our model by endogenizing r and s, thus allowing firms to switch their strategies to increase profits. We determine that the market tends to be in a stable status, FE, and no firm changes its strategy. We identify the conditions of FE and propose an approach to determining FE. We numerically demonstrate the existence and uniqueness of FE. We observe that the three main drivers of FE are the costing environment, cross-group competition, and intergroup competition.

In this paper, we consider a homogenous product. An interesting research direction is to investigate production differentiation, especially the production substitutability effect and demand correlation effect. We assume that each firm is either flexible or inflexible. However, a firm may gain an advantage by adopting partial flexibility, which is another direction worthy of investigation.

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APPENDIX

Proof of Lemma 1

Let $x = \alpha - \sum_{j \neq i} q_j^F - k^N$. Define $\pi_i^F(q_i^F | \alpha, q_j, \forall j \in \Omega \setminus \{i\}) = q_i^F(\alpha - Q)^+ - \beta q_i^F$. We have $m_i^F(q_i^F) = q_i^F(x - q_i^F)^+ - \beta q_i^F$. There are three cases. (i) If $x < 0$; (ii) If $0 \le x < k_i^F$ (iii) If $k_i^F \le x$. We solve each case and combine the three cases, we have $q_i^{F*} = \begin{cases} 0, & x \le \beta \\ q_{ib}^F, & \beta < x \le 2k_i^F + \beta \\ k_i^F, & 2k_i^F + \beta < x \end{cases}$.

Note that $p(\alpha, Q) = x - q_i^F$, then we can derive Lemma 1. This completes the proof of Lemma 1.□

Consider any feasible firm $i \in \Omega^F$. Suppose that k_j , $j \in \Omega \setminus \{i\}$, and $q_j^F(\alpha)$, $j \in \Omega^F \setminus \{i\}$, are given. Let $A^F(k_i^F) = \Pi_i^F(k_i^F | k_j^F,\ q_j^F(\alpha)\ \forall j \neq i, \text{ and } k_l^N,\ l \in \Omega^N)$ be the expected profit of firm *i*. The objective of firm *i* is to maximize $A^F(k_i^F)$.

Lemma A1 In an oligopoly market competition with $r > 0$ flexible firms and $s \ge 0$ inflexible firms, the optimal capacity of flexible firm $i \in \Omega^F$, i.e., k_i^{F*} , is either (i) $k_i^{F*} = 0$ and $A^{F(1)}(0) \le 0$; or (ii) $k_i^{F*} > 0$ and $A^{F(1)}(k_i^{F*}) = 0$.

Proof By (2), $A^F(k_i^F) = \int_0^\infty q_i^{F*}((\alpha - Q^F(\alpha) - k^N)^+ - \beta) f(\alpha) d\alpha - C_F k_i^F$. (a1)

Together with Lemma1, we have $A^F(k_i^F) = \int_0^\infty q_i^{F*}(\alpha - Q^F(\alpha) - k^N - \beta) f(\alpha) d\alpha - C_F k_i^F$. Let $y = q_{ib}^F = (x - \beta)/2$, then $q_i^{F*} = \begin{cases} 0 & y \le 0 \\ y & 0 < y \le k_i^F \\ k_i^F & k_i^F < y \end{cases}$. $A^F(k_i^F) = \int_0^\infty q_i^{F*}(2y - q_i^{F*}) f(\alpha) d\alpha - C_F k_i^F$

$= \int_{0<y} y^2 f(\alpha) d\alpha - \int_{k_i^F<y} (y - k_i^F)^2 f(\alpha) d\alpha - C_F k_i^F$. (a2)

Let $B(k_i^F) = \int_{0<y} y^2 f(\alpha) d\alpha$, and $C(k_i^F) = \int_{k_i^F<y} (y - k_i^F)^2 f(\alpha) d\alpha$. Note that $B(k_i^F)$ is independent of k_i^F , i.e., $B^{(1)}(k_i^F) = 0$. Therefore, with respect to k_i^F , by (a2) we have

$A^{F(1)}(k_i^F) = 2 \int_{k_i^F<y} (y - k_i^F) f(\alpha) d\alpha - C_F$ and $A^{F(2)}(k_i^F) = -2 \int_{k_i^F<y} f(\alpha) d\alpha \le 0$. Therefore, $A^F(k_i^F)$ is concave. Note that $C(k_i^F) \ge 0$ and $C_F > 0$. It follows that, as $k_i^F \to \infty$, $A^F(k_i^F) \to -\infty$. Therefore, there exists an $S > 0$, such that $A^F(k_i^F) < 0$ for all $k_i^F \ge S$. Since $A^F(0) = 0$, we can restrict our search for the optimal k_i^F in $[0, S]$. Therefore, either (i) $k_i^{F*} = 0$ and $A^{F(1)}(0) \le 0$; or (ii) $k_i^{F*} > 0$ and $A^{F(1)}(k_i^{F*}) = 0$. □

Lemma A2 At the equilibrium of an oligopoly market competition with $r > 0$ flexible firms and $s \ge 0$ inflexible firms, the optimal capacities of flexible firms are either $k_i^{F*} = 0$, for all $i \in \Omega^F$; or $k_i^{F*} > 0$, for all $i \in \Omega^F$; further, (i) $k_i^{F*} = 0$, for all $i \in \Omega^F$, is equivalent to $k^N \ge X(C_F) - \beta$; (ii) $k_i^{F*} > 0$, for all $i \in \Omega^F$, is equivalent to $k^N < X(C_F) - \beta$.

Proof We follow the notations in the proof of Lemma A1. Let $\theta = 2y - q_i^{F*} = \alpha - Q^F(\alpha) - k^N - \beta$, which is independent of i. By Lemma A1, there are two cases of k_i^{F*}.

In case (i) $k_i^{F*} = 0$ and $A^{F(1)}(0) \le 0$, we have $q_i^{F*} = 0$ and

$$A^{F(1)}(0) = 2\int_{0<y} yf(\alpha)d\alpha - C_F \le 0, \text{ i.e., } \int_{0<\theta} \theta f(\alpha)d\alpha \le C_F. \quad \text{(a3)}$$

In case (ii) $k_i^{F*} > 0$, $A^{F(1)}(k_i^{F*}) = 0$, we have

$$A^{F(1)}(k_i^F) = 2\int_{k_i^F<y} (y - k_i^F) f(\alpha)d\alpha - C_F = 0, \text{ i.e., } C_F/2 = \int_{k_i^F<y} (y - k_i^F) f(\alpha)d\alpha. \quad \text{(a4)}$$

When $y > k_i^{F*}$, $q_i^{F*} = k_i^{F*}$. Therefore, $y = (\theta + k_i^{F*})/2 > k_i^{F*}$ is equivalent to $\theta > k_i^{F*}$. By (a4), we have $\int_{k_i^{F*}<\theta} (\theta - k_i^{F*}) f(\alpha)d\alpha = C_F$. Note that $\int_{k_i^F<\theta} (\theta - k_i^F) f(\alpha)d\alpha \le \int_{0<\theta} \theta f(\alpha)d\alpha - k_i^F \int_{k_i^F<\theta} f(\alpha)d\alpha$.

If $\int_{k_i^F<\theta} f(\alpha)d\alpha = 0$, then $0 \le \int_{0<\theta-k_i^F\le t} (\theta - k_i^F) f(\alpha)d\alpha \le t \int_{0<\theta-k_i^F\le t} f(\alpha)d\alpha \le t \int_{0<\theta-k_i^F} f(\alpha)d\alpha = 0$ for any $t > 0$. So, $C_F = \int_{k_i^F<\theta} (\theta - k_i^F) f(\alpha)d\alpha = \lim_{t\to\infty} \int_{0<\theta-k_i^F\le t} (\theta - k_i^F) f(\alpha)d\alpha = 0$, which is a contradiction.

Therefore, $\int_{k_i^F<\theta} f(\alpha)d\alpha > 0$. Since $k_i^{F*} > 0$, we have $C_F = \int_{k_i^F<\theta} (\theta - k_i^F) f(\alpha)d\alpha < \int_{0<\theta} \theta f(\alpha)d\alpha$,

$$\text{i.e., } \int_{0<\theta} \theta f(\alpha)d\alpha > C_F. \quad \text{(a5)}$$

Since (a3) and (a5) are contradictory to each other and independent of i, we have at equilibrium either $k_i^{F*} = 0$, for all $i \in \Omega^F$; or $k_i^{F*} > 0$, for all $i \in \Omega^F$. We discuss these two cases respectively.

Case-I $k_i^{F*} = 0$, for all $i \in \Omega^F$, then $q_i^{F*} = 0$. By (a3),

we have $\int_{k^N+\beta}^{\infty} (\alpha - k^N - \beta) f(\alpha)d\alpha \le C_F$, i.e., $k^N \ge X(C_F) - \beta$.

Case-II $k_i^{F*} > 0$, for all $i \in \Omega^F$, then by (a5),

we have $\int_{\alpha > Q^F(\alpha)+k^N+\beta} (\alpha - Q^F(\alpha) - k^N - \beta) f(\alpha)d\alpha > C_F$. Therefore,

$$\int_{\alpha>k^N+\beta} (\alpha - k^N - \beta) f(\alpha)d\alpha \ge \int_{\alpha>Q^F(\alpha)+k^N+\beta} (\alpha - k^N - \beta) f(\alpha)d\alpha$$

$$\ge \int_{\alpha>Q^F(\alpha)+k^N+\beta} (\alpha - Q^F(\alpha) - k^N - \beta) f(\alpha)d\alpha > C_F, \text{ i.e., } L(k^N + \beta) > C_F.$$

Equivalently, $k^N < X(C_F) - \beta$. Therefore, (1) $k_i^{F*} = 0$, for all $i \in \Omega^F$, is equivalent to $k^N \ge X(C_F) - \beta$; (2) $k_i^{F*} > 0$, for all $i \in \Omega^F$, is equivalent to $k^N < X(C_F) - \beta$. □

Proof of Proposition 1

We use the same notations as in the proof of Lemma A1. For any two flexible firms $j \ne h$, $j, h \in \Omega^F$. Without loss of generality, we assume $k_j^{F*} \le k_h^{F*}$. By (a5), we have $\int_{k_j^{F*}<\theta} (\theta - k_j^{F*}) f(\alpha)d\alpha = \int_{k_h^{F*}<\theta} (\theta - k_h^{F*}) f(\alpha)d\alpha$. Assume that $k_j^{F*} < k_h^{F*}$. Since $\int_{k_h^{F*}<\theta} f(\alpha)d\alpha > 0$, we have

$$\int_{k_j^{F*}<\theta}(\theta-k_j^{F*})f(\alpha)d\alpha \geq \int_{k_h^{F*}<\theta}(\theta-k_j^{F*})f(\alpha)d\alpha = \int_{k_h^{F*}<\theta}\theta f(\alpha)d\alpha - k_j^{F*}\int_{k_h^{F*}<\theta}f(\alpha)d\alpha$$

$> \int_{k_h^{F*}<\theta}\theta f(\alpha)d\alpha - k_h^{F*}\int_{k_h^{F*}<\theta}f(\alpha)d\alpha = \int_{k_h^{F*}<\theta}(\theta-k_h^{F*})f(\alpha)d\alpha$. This is a contradiction. Hence, $k_j^{F*}=k_h^{F*}$. Therefore, we have $k_j^{F*}=k_e^F>0$ for all $j\in\Omega^F$. Therefore, for any feasible firm $i\in\Omega^F$, together with Proposition 1, we have $q_i^{F*}=\begin{cases}0, & \theta\leq 0\\ \theta, & 0<\theta\leq k_e^F\\ k_e^F, & k_e^F<\theta\end{cases}$, where $\theta=\alpha-Q^F(\alpha)-k^N-\beta$ is independent of *i*. So all $q_i^{F*}, i\in\Omega^F$, are equal. That means $q_i^{F*}=q_e^F$ for all $i\in\Omega^F$, and we have $Q^F=rq_e^F$. Since there are *r* flexible firms, $\theta=\alpha-rq_e^F-k^N-\beta$.

Therefore, $q_i^{F*}=q_e^F$ can be expressed as $q_e^F=\begin{cases}0, & 0\leq\alpha\leq\beta+k^N\\ \dfrac{\alpha-\beta-k^N}{r+1}, & \beta+k^N<\alpha\leq\beta+(r+1)k_e^F+k^N\\ k_e^F, & \beta+(r+1)k_e^F+k^N<\alpha\end{cases}$.

By (a5), we have $\int_{k^N+\beta+(r+1)k_e^F<\alpha}(\alpha-k^N-\beta-(r+1)k_e^F)f(\alpha)d\alpha=C_F$, i.e.,

$L(k^N+\beta+(r+1)k_e^F)=C_F$. So, we have $k^N+(r+1)k_e^F=X(C_F)-\beta$,

i.e., $k_e^F=\frac{1}{r+1}[X(C_F)-\beta-k^N]$. (a6)

We consider the individual expected profit of each flexible firm. By (a2) and (a4),

$$A^F(k_i^F)=\int_{0<y\leq k_i^F}y^2 f(\alpha)d\alpha+\int_{k_i^F<y}k_i^F(2y-k_i^F)f(\alpha)d\alpha-2k_i^F\int_{k_i^F<y}(y-k_i^F)f(\alpha)d\alpha$$

$$=\int_{0<y\leq k_i^F}y^2 f(\alpha)d\alpha+\int_{k_i^F<y}(k_i^F)^2 f(\alpha)d\alpha. \quad \text{(a7)}$$

Note that $y=(x-\beta)/2=\frac{1}{2}[\alpha-(r-1)q_e^F-k^N-\beta]$. Then

(1) If $0\leq\alpha\leq\beta+k^N$, then $q_e^F=0$, $y=(\alpha-k^N-\beta)/2\leq 0$.

(2) If $\beta+k^N<\alpha\leq\beta+(r+1)k_e^F+k^N$, then $q_e^F=\frac{1}{r+1}(\alpha-k^N-\beta)$; $0<y=\frac{1}{r+1}(\alpha-k^N-\beta)\leq k_e^F$.

(3) If $\beta+(r+1)k_e^F+k^N<\alpha$, then $q_e^F=k_e^F$, $y=\frac{1}{2}[\alpha-(r-1)k_e^F-k^N-\beta]>k_e^F$.

Therefore, by (a7) we have

$A^F(k_i^F)=\int_{k^N+\beta}^{k^N+\beta+(r+1)k_e^F}\frac{1}{(r+1)^2}(\alpha-k^N-\beta)^2 f(\alpha)d\alpha+\int_{k^N+\beta+(r+1)k_e^F}^{\infty}(k_e^F)^2 f(\alpha)d\alpha$. Together with (a6), we have $k^N+\beta+(r+1)k_e^F=X(C_F)$ and so

$$A^F(k_i^F)=\int_{k^N+\beta}^{X(C_F)}\frac{1}{(r+1)^2}(\alpha-k^N-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(k_e^F)^2 f(\alpha)d\alpha$$

$$=\frac{1}{(r+1)^2}\left(\int_{k^N+\beta}^{X(C_F)}(\alpha-k^N-\beta)^2 f(\alpha)d\alpha+\int_{X(C_F)}^{\infty}(X(C_F)-k^N-\beta)^2 f(\alpha)d\alpha\right).$$

This completes the proof of Proposition 1. □

Consider any infeasible firm $i\in\Omega^N$. Suppose that k_j, $j\in\Omega\setminus\{i\}$, and $q_j^F(\alpha)$, $j\in\Omega^F$, are given. Let $A^N(k_i^N)=\Pi_i^N\left(k_i^N \middle| k_j^N,\ \forall j\neq i, \text{and } k_l^F, q_l^F(\alpha)\ l\in\Omega^F\right)$ be the expected profit of firm *i*. The objective of firm *i* is to maximize $A^N(k_i^N)$.

Lemma A3 In an oligopoly market competition with $r \ge 0$ flexible firms and $s > 0$ inflexible firms, the optimal capacity of inflexible firm $i \in \Omega^N$, i.e., k_i^{N*}, is either (1) $k_i^{N*} = 0$ and $A^{N(1)}(0) \le 0$; or (2) $k_i^{N*} > 0$, $A^{N(1)}(k_i^{N*}) = 0$ and $A^{N(2)}(k_i^{N*}) \le 0$.

Proof By (3), $A^N(k_i^N) = \int_0^\infty k_i^N(\alpha - Q^F(\alpha) - \sum_{j \ne i} k_j^N - k_i^N)^+ f(\alpha)d\alpha - (C_N + \beta)k_i^N$

$$\le \int_0^\infty k_i^N(\alpha - k_i^N)^+ f(\alpha)d\alpha - (C_N + \beta)k_i^N$$

$$= \int_{k_i^N}^\infty k_i^N(\alpha - k_i^N) f(\alpha)d\alpha - (C_N + \beta)k_i^N = k_i^N(L(k_i^N) - \beta - C_N).$$

Let $u = \alpha - Q^F(\alpha) - \sum_{j \ne i} k_j^N$. Then, $A^N(k_i^N) = \int_{k_i^N < u} k_i^N(u - k_i^N) f(\alpha)d\alpha - (C_N + \beta)k_i^N$. (a8)

$A^{N(1)}(k_i^N) = \int_{k_i^N < u} (u - 2k_i^N) f(\alpha)d\alpha - (C_N + \beta)$. It is noted that as $k_i^N \to \infty$, $A^N(k_i^N) \le k_i^N(L(k_i^N) - \beta - C_N) \to -\infty$. Therefore, there exists $S_1 > 0$, such that $A^N(k_i^N) < 0$ for all $k_i^N \ge S_1$. Since $A^N(0) = 0$, to find the optimal k_i^{N*}, if any, we can restrict our search to the range $[0, S_1]$. Because $A^N(k_i^N)$ is continuous on $[0, S_1]$. Hence, there exists $k_i^{N*} \in [0, S_1]$ to maximize $A^N(k_i^N)$. Therefore, either (1) $k_i^{N*} = 0$ and $A^{N(1)}(0) \le 0$; or (2) $k_i^{N*} > 0$, $A^{N(1)}(k_i^{N*}) = 0$ and $A^{N(2)}(k_i^{N*}) \le 0$. □

Lemma A4 At the equilibrium of an oligopoly market competition with $r \ge 0$ flexible firms and $s > 0$ inflexible firms, the optimal capacities of inflexible firms are either (1) $k_i^{N*} = 0$, for all $i \in \Omega^N$; or (2) $k_i^{N*} > 0$, for all $i \in \Omega^N$; further, we have

(i) $k_i^{N*} = 0$, for all $i \in \Omega^N$, is equivalent to $\int_{0<v} vf(\alpha)d\alpha \le C_N + \beta$;

(ii) $k_i^{N*} > 0$, for all $i \in \Omega^N$, is equivalent to $\int_{0<v} vf(\alpha)d\alpha > C_N + \beta$, where $v = \alpha - Q^F(\alpha) - k^N$.

Proof We follow the notations in the proof of Lemma A3. By Lemma A3, there are two cases.
In case (1) $k_i^{N*} = 0$ and $A^{N(1)}(0) \le 0$, then

$$A^{N(1)}(k_i^N) = \int_{0<v} vf(\alpha)d\alpha - (C_N + \beta) \le 0, \text{ i.e., } \int_{0<v} vf(\alpha)d\alpha \le C_N + \beta. \quad \text{(a9)}$$

In case (2) $k_i^{N*} > 0$ and $A^{N(1)}(k_i^{N*}) = 0$, then $\int_{0<v} (v - k_i^N) f(\alpha)d\alpha = C_N + \beta$. (a10)

By the proof of Lemma A3, we get $\int_{0<v} f(\alpha)d\alpha > 0$. Since $k_i^{N*} > 0$, we have

$$C_N + \beta = \int_{0<v} (v - k_i^{N*}) f(\alpha)d\alpha = \int_{0<v} vf(\alpha)d\alpha - k_i^{N*} \int_{0<v} f(\alpha)d\alpha < \int_{0<v} vf(\alpha)d\alpha,$$

i.e., $\int_{0<v} vf(\alpha)d\alpha > C_N + \beta$. (a11)

Since (a9) and (a11) are contradictory to each other and independent of *i*, we have, at equilibrium, either $k_i^{N*} = 0$, for all $i \in \Omega^N$; or $k_i^{N*} > 0$, for all $i \in \Omega^N$. Therefore, at equilibrium, there are two cases.

Case-I $k_i^{N*} = 0$, for all $i \in \Omega^N$, then by (a9) we have $\int_{0<v} vf(\alpha)d\alpha \le C_N + \beta$.

Case-II $k_i^{N*} > 0$, for all $i \in \Omega^N$, then by (a11)we have $\int_{0<v} vf(\alpha)d\alpha > C_N + \beta$.

Therefore, $k_i^{N*} = 0$, for all $i \in \Omega^N$, is equivalent to $\int_{0<v} vf(\alpha)d\alpha \le C_N + \beta$; $k_i^{N*} > 0$, for all $i \in \Omega^N$, is equivalent to $\int_{0<v} vf(\alpha)d\alpha > C_N + \beta$. □

Proof of Proposition 2

Following the notations in Lemma A4, we consider two inflexible firms $j \ne h$, $j,h \in \Omega^N$. By (a10) we have $\int_{0<v}(v-k_j^{N*})f(\alpha)d\alpha = \int_{0<v}(v-k_h^{N*})f(\alpha)d\alpha$. This implies that $(k_h^{N*}-k_j^{N*})\int_{0<v} f(\alpha)d\alpha = 0$.

To show $\int_{0<v} f(\alpha)d\alpha > 0$: By (a10), we have $\int_{0<v}(v-k_j^{N*})f(\alpha)d\alpha = C_N + \beta$. If $\int_{0<v} f(\alpha)d\alpha = 0$, then for any $t > k_j^{N*}$, $\int_{0<v\le t}(v-k_j^{N*})f(\alpha)d\alpha \ge -k_j^{N*}\int_{0<v\le t} f(\alpha)d\alpha \ge -k_j^{N*}\int_{0<v} f(\alpha)d\alpha = 0$ and $\int_{0<v\le t}(v-k_j^{N*})f(\alpha)d\alpha \le (t-k_j^{N*})\int_{0<v\le t} f(\alpha)d\alpha \le (\alpha)\int_{0<v} f(\alpha)d\alpha = 0$, implying $\int_{0<v\le t}(v-k_j^{N*})f(\alpha)d\alpha = 0$. So, $C_N + \beta = \int_{0<v}(v-k_j^{N*})f(\alpha)d\alpha = \lim_{t\to\infty}\int_{0<v\le t}(v-k_j^{N*})f(\alpha)d\alpha = 0$, which is a contradiction. Therefore, $\int_{0<v} f(\alpha)d\alpha > 0$. Hence, we get $k_h^{N*} = k_j^{N*}$. Therefore, we have $k_j^{N*} = k_e^N > 0$ for all $j \in \Omega^N$. Since there are s inflexible firms, we have $k^N = sk_e^N > 0$, $v = \alpha - rq_e^F - sk_e^N$ and $\int_{0<v} vf(\alpha)d\alpha = \int_{rq_e^F + sk_e^N < \alpha}(\alpha - rq_e^F - sk_e^N)f(\alpha)d\alpha$. Therefore, by Lemma A4,

$k_i^{N*} = 0$, for all $i \in \Omega^N$, is equivalent to $\int_{rq_e^F + sk_e^N < \alpha}(\alpha - rq_e^F - sk_e^N)f(\alpha)d\alpha \le C_N + \beta$.

$k_i^{N*} > 0$, for all $i \in \Omega^N$, is equivalent to $\int_{rq_e^F + sk_e^N < \alpha}(\alpha - rq_e^F - sk_e^N)f(\alpha)d\alpha > C_N + \beta$.

If $v = \alpha - rq_e^F - sk_e^N > 0$, then $\alpha > rq_e^F + sk_e^N$. By (a10) we have $\int_{rq_e^F + sk_e^N < \alpha}(\alpha - rq_e^F - (s+1)k_e^N)f(\alpha)d\alpha = C_N + \beta$. Also, $v = u - k_i^N = \alpha - Q^F(\alpha) - k^N$, by (a10) we have $\int_{k_i^N < u}(u - 2k_i^N)f(\alpha)d\alpha = C_N + \beta$. Together with (a8), we have

$$A^N(k_i^N) = \int_{k_i^N<u} k_i^N(u-k_i^N)f(\alpha)d\alpha - (C_N+\beta)k_i^N$$

$$= \int_{k_i^N<u} k_i^N(u-k_i^N)f(\alpha)d\alpha - k_i^N\int_{k_i^N<u}(u-2k_i^N)f(\alpha)d\alpha = \int_{sk_e^N<\alpha-rq_e^F}(k_i^N)^2 f(\alpha)d\alpha$$

$$= (k_e^N)^2\int_{sk_e^N<\alpha-rq_e^F} f(\alpha)d\alpha \tag{a12}$$

By Proposition 1,
$$rq_e^F = \begin{cases} 0, & 0 \le \alpha \le \beta + sk_e^N \\ \dfrac{r(\alpha-\beta-sk_e^N)}{r+1}, & \beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N \\ rk_e^F, & \beta + (r+1)k_e^F + sk_e^N < \alpha \end{cases}. \tag{a13}$$

There are three cases as follows.

(1) If $0 \le \alpha \le \beta + sk_e^N$ and $\alpha - rq_e^F > sk_e^F$, then $rq_e^F = 0$, $\alpha - rq_e^F = \alpha > sk_e^N$, so we have $sk_e^N < \alpha \le \beta + sk_e^N$.

(2) If $\beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N$ and $\alpha - rq_e^F > sk_e^N$, then $rq_e^F = \frac{r}{r+1}(\alpha - \beta - sk_e^N)$, $\alpha - rq_e^F = \frac{1}{r+1}[\alpha + r(\beta + sk_e^N)] > sk_e^N$ and so $\alpha > sk_e^N - r\beta$. Therefore, we have $\beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N$.

(1) If $\beta + (r+1)k_e^F + sk_e^N < \alpha$ and $\alpha - rq_e^F > sk_e^N$, then $rq_e^F = rk_e^F$, $\alpha - rq_e^F = \alpha - rk_e^F > sk_e^N$, and so $\alpha > rk_e^F + sk_e^N$. Therefore, we have $\beta + (r+1)k_e^F + sk_e^N < \alpha$.

Combine these three cases, the range of α for $\alpha - rq_e^F > sk_e^F$ is $sk_e^N < \alpha$. Therefore, from (a12) we have $A^N(k_i^N) = (k_e^N)^2 \int_{sk_e^N}^{\infty} f(\alpha) d\alpha = (k_e^N)^2 \overline{F}(sk_e^N)$.

This completes the proof of Proposition 2. □

Let $R = \{(C_N, C_F): 0 < C_N < \infty,\ 0 < C_F < \infty\}$ be the feasible region of all C_N and C_F. According to Propositions 1 and 2, region $R = \{(C_N, C_F): 0 < C_N < \infty,\ 0 < C_F < \infty\}$ can be divided into four regions, corresponding to four cases as listed in the following table.

Table A1: Four equilibriums of an oligopoly competition.

	$k_e^N > 0$	$k_e^N = 0$
$k_e^F = 0$	Case B	Case A
$k_e^F > 0$	Case D	Case C

Lemma A5 Given $r > 0$ flexible firms and $s > 0$ inflexible firms, within the area $R = \{(C_N, C_F): 0 < C_N \le C_F < \infty\}$. For Case A that $k_e^F = k_e^N = 0$, we have $\Pi_e^F = \Pi_e^N = 0$ and a necessary condition for Case A is: $L(\beta) \le C_F$ and $\mu - \beta \le C_N$.

<u>Proof</u> We define Case A as the case with solution $k_e^F = k_e^N = 0$. By Propositions 1 and 2, we have

$$k^N \ge X(C_F) - \beta; \tag{a14}$$

$$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha) d\alpha \le C_N + \beta, \tag{a15}$$

By (a14), we have $L(\beta) \le C_F$; by (a15), we have $\mu - \beta \le C_N$. The individual expected profits of both flexible and inflexible firms are $\Pi_e^F = \Pi_e^N = 0$. □

Lemma A6 Given $r > 0$ flexible firms and $s > 0$ inflexible firms, within the area $R = \{(C_N, C_F): 0 < C_N \le C_F < \infty\}$. For Case B that $k_e^F = 0$, $k_e^N > 0$, we have $\Pi_e^F = 0$, $\Pi_e^N = \frac{1}{s^2}(k^N)^2 \overline{F}(k^N)$, and (i) a necessary condition for Case B is: $L(\beta + k_w) \le C_F$ and $C_N < \mu - \beta$; (ii) $k_e^N > 0$ satisfies $\int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha = C_N + \beta$, $k^N = sk_e^N$.

<u>Proof</u> We define Case B as the case with solution that $k_e^F = 0$, $k_e^N > 0$.

By Propositions 1 and 2, the conditions of Case B are: $sk_e^N \ge X(C_F) - \beta$; (a16)

$$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha) d\alpha > C_N + \beta. \tag{a17}$$

The solution satisfies $\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - (s+1)k_e^N) f(\alpha) d\alpha = C_N + \beta$. (a18)

Since $k_e^F = 0$, we get $q_e^F = 0$. By (a17), we have $\mu \geq L(sk_e^N) > C_N + \beta$. By (a18),

$$Z(sk_e^N) = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha = C_N + \beta. \quad \text{(a19)}$$

Therefore, by (a16) and (a19), $X(C_F) - \beta \leq sk_e^N = k_w$. Thus, $C_F \geq L(k_w + \beta)$. By Proposition 2, the individual expected profit of inflexible firms is $\Pi_e^N = \frac{1}{s^2}(k^N)^2 \overline{F}(k^N)$ and $k^N = sk_e^N$. A necessary condition for Case B is: $L(\beta + k_w) \leq C_F$ and $C_N < \mu - \beta$. □

Lemma A7 Given $r > 0$ flexible firms and $s > 0$ inflexible firms, within the area $R = \{(C_N, C_F) : 0 < C_N \leq C_F < \infty\}$. For Case C that $k_e^F > 0$, $k_e^N = 0$, we have $\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta}^{X(C_F)} (\alpha - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - \beta)^2 f(\alpha) d\alpha\right)$, $\Pi_e^N = 0$, and (i) a necessary condition for Case C is: $L(\beta) > C_F$ and $\mu - (C_N + \beta) \leq \frac{r}{r+1}(L(\beta) - C_F)$; (ii) $k_e^F = \frac{1}{r+1}(X(C_F) - \beta)$.

<u>Proof</u> We define Case C as the case with solution of $k_e^F > 0$, $k_e^N = 0$.

By Propositions 1 and 2, the conditions of Case C are $k^N < X(C_F) - \beta$; (a20)

$$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha) d\alpha \leq C_N + \beta. \quad \text{(a21)}$$

The solution satisfies $(r+1)k_e^F = X(C_F) - \beta$, i.e., $k_e^F = \frac{1}{r+1}(X(C_F) - \beta)$. (a22)

By (a20), we have $C_F < L(\beta)$.

By (a21), we have $\int_{rq_e^F < \alpha} (\alpha - rq_e^F) f(\alpha) d\alpha \leq C_N + \beta$. (a23)

By Proposition 1, we have $rq_e^F = \begin{cases} 0, & 0 \leq \alpha \leq \beta \\ \frac{r}{r+1}(\alpha - \beta), & \beta < \alpha \leq \beta + (r+1)k_e^F \\ rk_e^F, & \beta + (r+1)k_e^F < \alpha \end{cases}$. Therefore,

$\alpha - rq_e^F = \begin{cases} \alpha, & 0 \leq \alpha \leq \beta \\ (\alpha + r\beta)/(r+1), & \beta < \alpha \leq \beta + (r+1)k_e^F \\ \alpha - rk_e^F, & \beta + (r+1)k_e^F < \alpha \end{cases}$. Thus, $\alpha - rq_e^F > 0$ for any $\alpha > 0$.

Therefore, by (a23) we have

$$\int_0^{\beta} \alpha f(\alpha) d\alpha + \int_{\beta}^{\beta + (r+1)k_e^F} (\alpha - \tfrac{r}{r+1}(\alpha - \beta)) f(\alpha) d\alpha + \int_{\beta + (r+1)k_e^F}^{\infty} (\alpha - rk_e^F) f(\alpha) d\alpha \leq C_N + \beta$$

i.e., $\mu - \frac{r}{r+1}(L(\beta) - C_F) \leq C_N + \beta$. Hence, $\mu - (C_N + \beta) \leq \frac{r}{r+1}(L(\beta) - C_F)$.

By Proposition 1, the individual expected profit of flexible firms is

$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta}^{X(C_F)} (\alpha - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - \beta)^2 f(\alpha) d\alpha\right)$. Since $k_e^N = 0$, $\Pi_e^N = 0$.

A necessary condition for Case C is: $L(\beta) > C_F$ and $\mu - (C_N + \beta) \leq \frac{r}{r+1}(L(\beta) - C_F)$. □

Lemma A8 Given $r > 0$ flexible firms and $s > 0$ inflexible firms, within the area $R = \{(C_N, C_F) : 0 < C_N \leq C_F < \infty\}$. For Case D that $k_e^F > 0$, $k_e^N > 0$, we have $\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta + sk_e^N}^{X(C_F)} (\alpha - sk_e^N - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - sk_e^N - \beta)^2 f(\alpha) d\alpha\right)$; and

$\Pi_e^N = (k_e^N)^2 \overline{F}(sk_e^N)$. The solution of Case D is: $k_e^F = \frac{1}{r+1}(X(C_F) - \beta - sk_e^N)$ and k_e^N satisfies $C_N + \beta = \int_{sk_e^N}^{\infty}(\alpha - (s+1)k_e^N) f(\alpha)d\alpha - \frac{r}{r+1}(L(\beta + sk_e^N) - C_F)$. A necessary condition for Case D is: $\mu - (C_N + \beta) > \frac{r}{r+1}(L(\beta) - C_F)$ and $C_F < L(\beta + k_w)$.

Proof We define Case D as the case with solution that $k_e^F > 0$, $k_e^N > 0$.

By Propositions 1 and 2, the conditions of Case D are $k^N < X(C_F) - \beta$ (a24)

$$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - sk_e^N) f(\alpha)d\alpha > C_N + \beta \quad \text{(a25)}$$

The solutions of Case D satisfy $sk_e^N + (r+1)k_e^F + \beta = X(C_F)$; (a26)

and $$\int_{rq_e^F + sk_e^N < \alpha} (\alpha - rq_e^F - (s+1)k_e^N) f(\alpha)d\alpha = C_N + \beta. \quad \text{(a27)}$$

By (a13), we have $$rq_e^F = \begin{cases} 0, & 0 \le \alpha \le \beta + sk_e^N \\ \dfrac{r(\alpha - \beta - sk_e^N)}{r+1}, & \beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N \\ rk_e^F, & \beta + (r+1)k_e^F + sk_e^N < \alpha \end{cases}.$$

By (a27), we have the following three cases.

Case 1: $0 \le \alpha \le \beta + sk_e^N$ and $\alpha - rq_e^F - sk_e^N > 0$.

In this case, $rq_e^F = 0$ and $\alpha - rq_e^F - sk_e^N = \alpha - sk_e^N > 0$. Therefore, we have $sk_e^N < \alpha \le \beta + sk_e^N$ and $\alpha - rq_e^F - (s+1)k_e^N = \alpha - (s+1)k_e^N$.

Case 2: $\beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N$ and $\alpha - rq_e^F - sk_e^N > 0$.

In this case, $rq_e^F = \frac{r}{r+1}(\alpha - \beta - sk_e^N)$ and $\alpha - rq_e^F - sk_e^N = \frac{1}{r+1}[\alpha - (sk_e^N - r\beta)]$. So, $\alpha \ge sk_e^N - r\beta$. Therefore, we have $\beta + sk_e^N < \alpha \le \beta + (r+1)k_e^F + sk_e^N$ and $\alpha - rq_e^F - (s+1)k_e^N = \frac{1}{r+1}[\alpha - (s+r+1)k_e^N + r\beta]$.

Case 3: $\beta + (r+1)k_e^F + sk_e^N < \alpha$ and $\alpha - rq_e^F - sk_e^N > 0$.

In this case, $rq_e^F = rk_e^F$ and $\alpha - rq_e^F - sk_e^N = \alpha - rk_e^F - sk_e^N > 0$. So, $\alpha > rk_e^F + sk_e^N$. Therefore, $\beta + (r+1)k_e^F + sk_e^N < \alpha$ and $\alpha - rq_e^F - (s+1)k_e^N = \alpha - rk_e^F - (s+1)k_e^N$.

Hence, by (a26) and (a27), we have

$$C_N + \beta = \int_{sk_e^N}^{\beta + sk_e^N} (\alpha - (s+1)k_e^N) f(\alpha)d\alpha + \frac{1}{r+1}\int_{\beta + sk_e^N}^{X(C_F)} (\alpha + r\beta - (r+s+1)k_e^N) f(\alpha)d\alpha$$
$$+ \int_{X(C_F)}^{\infty} (\alpha - rk_e^F - (s+1)k_e^N) f(\alpha)d\alpha.$$

Therefore, $C_N + \beta = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha)d\alpha - \frac{r}{r+1}(L(\beta + sk_e^N) - C_F)$.

Let $R(k_e^N) = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha)d\alpha - \frac{r}{r+1}(L(\beta + sk_e^N) - C_F)$. With respect to k_e^N, $R^{(1)}(k_e^N) = sk_e^N f(sk_e^N) - (s+1)\overline{F}(sk_e^N) + \frac{rs}{r+1}\overline{F}(\beta + sk_e^N) < 0$. So, $R(k_e^N)$ is decreasing as k_e^N increases. Since $k_e^F > 0$ and $k_e^N > 0$, by (a26) we have $0 < k_e^N = \frac{X(C_F) - \beta - (r+1)k_e^F}{s} < \frac{X(C_F) - \beta}{s}$, and so $R(\frac{X(C_F) - \beta}{s}) < R(k_e^N) < R(0)$. $R(0) = \int_0^{\infty} \alpha f(\alpha)d\alpha - \frac{r}{r+1}(L(\beta) - C_F) = \mu - \frac{r}{r+1}(L(\beta) - C_F)$.

$R(\frac{X(C_F) - \beta}{s}) = \int_{X(C_F) - \beta}^{\infty} (\alpha - \frac{s+1}{s}(X(C_F) - \beta)) f(\alpha)d\alpha - \frac{r}{r+1}(L(\beta + X(C_F) - \beta) - C_F)$

$= \int_{X(C_F)-\beta}^{\infty} (\alpha - \frac{s+1}{s}(X(C_F)-\beta)) f(\alpha) d\alpha$. Therefore,

$\int_{X(C_F)-\beta}^{\infty} (\alpha - \frac{s+1}{s}(X(C_F)-\beta)) f(\alpha) d\alpha < C_N + \beta < \mu - \frac{r}{r+1}(L(\beta) - C_F)$.

By (a24), $0 < k^N < X(C_F) - \beta$ and so $L(\beta) > C_F$. Therefore, we have

$$\mu - (C_N + \beta) > \frac{r}{r+1}(L(\beta) - C_F) > 0, \quad \text{(a28)}$$

and $Z(X(C_F) - \beta) < C_N + \beta = Z(k_w)$. Therefore, $X(C_F) - \beta > k_w$,

$$\text{i.e., } C_F < L(\beta + k_w). \quad \text{(a29)}$$

By Propositions 1 and 2, the individual expected profit of flexible and inflexible firms are

$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta + sk_e^N}^{X(C_F)} (\alpha - sk_e^N - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - sk_e^N - \beta)^2 f(\alpha) d\alpha \right)$;

$\Pi_e^N = (k_e^N)^2 \overline{F}(sk_e^N)$, respectively. Therefore, the solution of Case D is

$k^N = sk_e^N$; k_e^N satisfies $C_N + \beta = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha - \frac{r}{r+1}(L(\beta + sk_e^N) - C_F)$;

$k^F = rk_e^F$; $k_e^F = \frac{1}{r+1}(X(C_F) - \beta - sk_e^N)$; A necessary condition for Case D is: $C_F < L(\beta + k_w)$ and $\mu - (C_N + \beta) > \frac{r}{r+1}(L(\beta) - C_F)$. □

Proof of Proposition 3

From Lemmas A5-A8, it is noted that in Case D, if $k^N = 0$, then $k^F = \frac{r}{r+1}(X(C_F) - \beta)$, which is the same as Case C; If $k^F = 0$, then k^N satisfies $C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha$, which is the same as Case B. It is easy to check that any two of the necessary conditions for Case A to Case D do not overlap, except Case B and Case C pair. In the following, we will show that even for the Case B and Case C pair, their necessary conditions do not overlap.

The necessary condition for Case B is: $L(\beta + k_w) \le C_F$ and $C_N < \mu - \beta$.

The necessary condition for Case C is: $L(\beta) > C_F$ and $\mu - (C_N + \beta) \le \frac{r}{r+1}(L(\beta) - C_F)$.

Suppose that they overlap, i.e., there exists (C_N, C_F) such that the above conditions hold. Then, $L(\beta + k_w) \le C_F \le \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$. Let $C_{F1} = L(\beta + k_w)$ and $C_{F2} = \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$ be two functions of $C_N \in [0, \mu - \beta]$. The curves of them are called Curve 1 and Curve 2, respectively.

Recall that $\int_{k_w}^{\infty} (\alpha - \frac{s+1}{s} k_w) f(\alpha) d\alpha = C_N + \beta$. With respect to k_w, we have $\frac{dC_{F1}}{dC_N} = \frac{dC_{F1}}{dk_w} \cdot \frac{dk_w}{dC_N}$;

$\frac{dC_{F1}}{dk_w} = -\overline{F}(\beta + k_w) < 0$; $\frac{dC_N}{dk_w} = -\frac{1}{s}[(s+1)\overline{F}(k_w) - k_w f(k_w)]$. So $\frac{dC_{F1}}{dC_N} = \frac{s\overline{F}(\beta + k_w)}{(s+1)\overline{F}(k_w) - k_w f(k_w)}$. Therefore, under the assumption that $\overline{F}(x) - xf(x) > 0$, we have $0 < \frac{dC_{F1}}{dC_N} = \frac{s\overline{F}(\beta + k_w)}{(s+1)\overline{F}(k_w) - k_w f(k_w)} < \frac{\overline{F}(\beta + k_w)}{\overline{F}(k_w)} \le 1$. Thus, C_{F1} is increasing in C_N, and its slope is strictly bounded above by 1. On the other hand, $\frac{dC_{F2}}{dC_N} = \frac{r+1}{r} > 1$. Therefore, C_{F2} is also increasing in C_N, but strictly bounded below by 1.

Let $\Delta = C_{F1} - C_{F2}$ for $C_N \in [0, \mu - \beta]$. Then, $\frac{d\Delta}{dC_N} = \frac{dC_{F1}}{dC_N} - \frac{dC_{F2}}{dC_N} < 1 - 1 = 0$. Thus, Δ is decreasing in C_N. When $C_N = \mu - \beta$, $\Delta = L(\beta + k_w) - L(\beta)$. Since $\int_{k_w}^{\infty} (\alpha - \frac{s+1}{s} k_w) f(\alpha) d\alpha = \mu - \beta + \beta = \mu$, we have $k_w = 0$. So, $\Delta = 0$. Therefore, for all $C_N \in [0, \mu - \beta)$, $\Delta = C_{F1} - C_{F2} > 0$. This is a contradiction. Hence, even for the Case B and Case C pair, their necessary conditions do not overlap.

Therefore, any two of the necessary conditions for Case A to Case D do not overlap. This implies that the four conditions are necessary and sufficient conditions for Case A to Case D, respectively. They partition the region $R = \{(C_N, C_F) : 0 < C_N \le C_F < \infty\}$ into four parts. Hence, given r flexible firms and s inflexible firms, we have the following conclusions on equilibrium within R:

(I) Case A occurs if and only if $L(\beta) \le C_F$ and $\mu - \beta \le C_N$;

(II) Case B occurs if and only if $L(\beta + k_w) \le C_F$ and $C_N < \mu - \beta$,;

(III) Case C occurs if and only if $L(\beta) > C_F$ and $\mu - (C_N + \beta) \le \frac{r}{r+1}(L(\beta) - C_F)$;

(IV) Case D occurs if and only if $C_F < L(\beta + k_w)$ and $\mu - (C_N + \beta) > \frac{r}{r+1}(L(\beta) - C_F)$.

The corresponding individual profit of flexible and inflexible firms is shown in Table A2.

Table A2: Individual profit of flexible and inflexible firms

Individual profit for flexible firms Π_e^F	**Individual profit for inflexible firms** Π_e^N
$\frac{1}{(r+1)^2} \left(\int_{\beta + sk_e^N}^{X(C_F)} (\alpha - sk_e^N - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - sk_e^N - \beta)^2 f(\alpha) d\alpha \right)$	$(k_e^N)^2 \overline{F}(sk_e^N)$
Note: The expression fits all cases. The firm attains a positive profit only when its capacity is larger than zero; otherwise, the profit is zero.	

This completes the proof of Proposition 3. □

<u>Proof of Lemma 2</u>

Part (i) can be obtained directly by Proposition 3.

Region B By Proposition 3, w.r.t. β, we have $k^{N(1)}(\beta) = -s / [(s+1)\overline{F}(k^N) - k^N f(k^N)] < 0$.

Therefore, $\Pi_e^{N(1)}(\beta) = \frac{1}{s^2} k^N [2\overline{F}(k^N) - k^N f(k^N)] \cdot k^{N(1)}(\beta) < 0$.

Region C By Proposition 3, w.r.t. β, we have $\Pi_e^{F(1)}(\beta) = -\frac{2}{(r+1)^2}[L(\beta) - C_F] < 0$.

Region D By Proposition 3, w.r.t. β, we have

$k^{N(1)}(\beta) = -\frac{1 - \frac{r}{r+1}\overline{F}(k^N + \beta)}{\overline{F}(k^N) - \frac{r}{r+1}\overline{F}(k^N + \beta) + [\overline{F}(k^N) - k^N f(k^N)]/s} < 0$. Then we have

$\Pi_e^{N(1)}(\beta) = \frac{1}{s^2} k^N [2\overline{F}(k^N) - k^N f(k^N)] \cdot k^{N(1)}(\beta) < 0$;

$\Pi_e^{F(1)}(\beta) = -\frac{2}{(r+1)^2}[L(k^N + \beta) - C_F] \cdot (1 + k^{N(1)}(\beta))$.

$$\Pi_e^{F(1)}(\beta) = -\frac{2}{(r+1)^2}[L(k^N + \beta) - C_F] \cdot \frac{[(s+1)\overline{F}(k^N) - k^N f(k^N)]/s - 1}{\overline{F}(k^N) - \frac{r}{r+1}\overline{F}(k^N + \beta) + [\overline{F}(k^N) - k^N f(k^N)]/s}.$$

Therefore, we have (1) if $[(s+1)\overline{F}(k^N) - k^N f(k^N)]/s < 1$, then $\Pi_e^{F(1)}(\beta) > 0$; (2) if $[(s+1)\overline{F}(k^N) - k^N f(k^N)]/s > 1$, then $\Pi_e^{F(1)}(\beta) < 0$.

Further, it is noted that $[(s+1)\overline{F}(k^N) - k^N f(k^N)]/s < 1$ is equivalent to $\overline{F}(k^N) < 1 - \frac{1}{s+1}(1 - k^N f(k^N))$. As $s \to \infty$, $1 - \frac{1}{s+1}(1 - k^N f(k^N)) \to 1$. Therefore, there must exist a s_0, for any $s > s_0$, we have $[(s+1)\overline{F}(k^N) - k^N f(k^N)]/s < 1$, so that $\Pi_e^{F(1)}(\beta) > 0$.

This completes the proof of Lemma 2. □

Proof of Proposition 4

By Proposition 3, in Case D $C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha - \frac{r}{r+1}(L(\beta + k^N) - C_F)$; (a30)

$\Pi_e^F = \frac{1}{(r+1)^2}\left(\int_{\beta+k^N}^{X(C_F)} (\alpha - k^N - \beta)^2 f(\alpha) d\alpha + \int_{X(C_F)}^{\infty} (X(C_F) - k^N - \beta)^2 f(\alpha) d\alpha \right)$;

$\Pi_e^N = \frac{1}{s^2}(k^N)^2 \overline{F}(k^N)$. Note that $0 \le C_N < \mu - \beta$ in Case D.

Let $M_e(C_N, C_F) = \Pi_e^F - \Pi_e^N$. With respect to C_F, it can be proved that in Case D $\Pi_e^{F(1)}(C_F) < 0$, $\Pi_e^{N(1)}(C_F) > 0$ with given $C_N \in [0, \mu - \beta)$, and so $M_e^{(1)}(C_F) = \Pi_e^{F(1)}(C_F) - \Pi_e^{N(1)}(C_F) < 0$. Therefore, for each given C_N, M_e is decreasing in C_F.

Recall that Curve 1: $C_{F1} = L(\beta + k_w)$, Curve 2: $C_{F2} = \frac{r+1}{r} C_N - \frac{r+1}{r}(\mu - \beta) + L(\beta)$, (a31)

where $C_N + \beta = \int_{k_w}^{\infty} (\alpha - \frac{s+1}{s} k_w) f(\alpha) d\alpha$. When $C_F = C_{F1}$, we have $\Pi_e^F = 0$ and $\Pi_e^N > 0$. Therefore, $M_e(C_N, C_{F1}) < 0$. When $C_F = C_{F2} < C_{F1}$, clearly $\Pi_e^F > 0$. (Otherwise, $\Pi_e^F = 0$ implies $X(C_{F2}) = k^N + \beta$, i.e., $C_{F2} = L(k^N + \beta)$. Thus, $C_N + \beta = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha$ and $k^N = k_w$, so $C_{F2} = C_{F1}$, contradiction.) On the other hand, by (a30) and (a31) $\mu = \int_{k^N}^{\infty} (\alpha - \frac{s+1}{s} k^N) f(\alpha) d\alpha - \frac{r}{r+1}(L(\beta + k^N) - L(\beta))$. Let the right hand side be $V(k^N)$, $k^N \ge 0$. Then, $V^{(1)}(k^N) = -\frac{1}{s}(\overline{F}(k^N) - k^N f(k^N)) - (\overline{F}(k^N) - \frac{r}{r+1}\overline{F}(\beta + k^N)) < 0$ implies that $V(k^N)$ is strictly decreasing. Since $V(0) = \mu$, we have $k^N = 0$. Therefore, $\Pi_e^N = 0$. Thus, $M_e(C_N, C_F) = \Pi_e^F - \Pi_e^N > 0$. Hence, when $C_N \in [0, \mu - \beta)$ is given, there exists a unique $C_F \in (C_{F2}, C_{F1})$ such that $M_e(C_N, C_F) = 0$, i.e., $\Pi_e^F = \Pi_e^N$. Clearly, when $C_N = \mu - \beta$, we can take $C_F = C_{F2} = C_{F1} = L(\beta)$ and $\Pi_e^F = \Pi_e^N$. Thus, we obtain C_F as a function of C_N so that $M_e(C_N, C_F) = 0$, $C_N \in [0, \mu - \beta]$. Differentiating both sides w.r.t. C_N, we have $\Pi_e^{F(1)}(C_N) = \Pi_e^{N(1)}(C_N)$.

$$\Pi_e^{F(1)}(C_N) = -\frac{2}{(r+1)^2}(X(C_F) - k^N - \beta) \cdot C_F^{(1)}(C_N) - \frac{2}{(r+1)^2}(L(k^N + \beta) - C_F) \cdot k^{N(1)}(C_N)$$

$$= -\frac{2A_1}{(r+1)^2} \cdot C_F^{(1)}(C_N) - \frac{2A}{(r+1)^2} \cdot k^{N(1)}(C_N)$$

$$\Pi_e^{N(1)}(C_N) = \frac{1}{s^2} k^N (2\overline{F}(k^N) - k^N f(k^N)) \cdot k^{N(1)}(C_N) = \frac{B}{s^2} \cdot k^{N(1)}(C_N),$$

where $A_1 = X(C_F) - k^N - \beta > 0$, $A = L(k^N + \beta) - C_F > 0$, $B = k^N(2\overline{F}(k^N) - k^N f(k^N)) > 0$.

So, $-\frac{2A_1}{(r+1)^2} C_F^{(1)}(C_N) = (\frac{B}{s^2} + \frac{2A}{(r+1)^2}) \cdot k^{N(1)}(C_N)$ (a32)

By (a31), differentiating with respect to C_N, we have $1 = -B_1 \cdot k^{N(1)}(C_N) + \frac{r}{r+1} \cdot C_F^{(1)}(C_N)$, (a33)

where $B_1 = \frac{1}{s}(\overline{F}(k^N) - k^N f(k^N)) + \overline{F}(k^N) - \frac{r}{r+1}\overline{F}(k^N + \beta) > 0$. Therefore, using (a31) and (a31), $C_F^{(1)}(C_N) = \dfrac{\frac{B}{s^2} + \frac{2A}{(r+1)^2}}{\frac{2A_1 B_1}{(r+1)^2} + \frac{r}{r+1}(\frac{B}{s^2} + \frac{2A}{(r+1)^2})} > 0$. This shows that Curve 3 is an increasing function of C_N. Furthermore, since for each given C_N, M_e is decreasing in C_F, we obtain that in the area above Curve 3, $\Pi_e^N > \Pi_e^F > 0$, and in the area below Curve 3, $\Pi_e^F > \Pi_e^N > 0$.
This completes the proof of Proposition 4. □

Equilibrium of competition with production cost differentiation

Lemma A9: Assume that flexible and inflexible firms incur different production costs (β_F and β_N) and the same capacity cost C_C, at the equilibrium of $n \sim (r, s)$ competition, we have the following cases:

***Case A*:** If $\mu - C_C \le \beta_N$ and $X(C_C) \le \beta_F$, then $k_e^F = k_e^N = 0$;

***Case B*:** If $\beta_N < \mu - C_C$ and $X(C_C) - k_{w1} \le \beta_F$, then $k_e^F = 0$, k_e^N satisfies

$$C_C + \beta_N = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha;$$

***Case C*:** If $\beta_N \ge \mu - \frac{1}{r+1}C_C - \frac{r}{r+1}L(\beta_F)$ and $\beta_F < X(C_C)$, then $k_e^N = 0$, and $k_e^F = \frac{1}{r+1}(X(C_C) - \beta_F)$;

***Case D*:** If $\beta_F < X(C_C) - k_{w1}$ and $\beta_N < \mu - \frac{1}{r+1}C_C - \frac{r}{r+1}L(\beta_F)$, then $k_e^F = \frac{1}{r+1}(X(C_C) - \beta_F - sk_e^N)$, and k_e^N satisfies

$C_C + \beta_N = \int_{sk_e^N}^{\infty} (\alpha - (s+1)k_e^N) f(\alpha) d\alpha - \frac{r}{r+1}(L(\beta_F + sk_e^N) - C_C)$, where k_{w1} is defined as the unique value satisfying $C_C + \beta_N = \int_{k_{w1}}^{\infty} \left(\alpha - \frac{s+1}{s} k_{w1}\right) f(\alpha) d\alpha$. □

<u>**Proof**</u>

Similar to the proof of Proposition 3, we can use backward induction method to obtain the bet response function of each firm, and then derive the Nash equilibrium. □

<u>**Proof of Lemma 3**</u>

Let $s_0 = n - r_0$ and $r_0 \in [1,\ n-1]$. Considering two cases $n = (r_0,\ s_0)$ and $n = (r_0 - 1,\ s_0 + 1)$, we have $G(C_N,\ C_F \mid r_0) = D(r_0) = \Pi_e^F(r_0, n - r_0) - \Pi_e^N(r_0 - 1, n - r_0 + 1)$. (a34)

It is noted that given totally number of n firms, Curve 1 and Curve 2 depend on value of $(r,\ s)$. Given C_N , then Curve 1 is $C_{F1} = L(\beta + k_w)$, where k_w satisfies $\int_{k_w}^{\infty} (\alpha - \frac{s+1}{s} k_w) f(\alpha) d\alpha = C_N + \beta$; Curve 2 is $C_{F2} = \frac{r+1}{r}(C_N + \beta - \mu) + L(\beta)$.

We define Curve 1a and Curve 2a to be the curves corresponding to the case $n = (r_0,\ s_0)$, and Curve 1b and Curve 2b to be the curves corresponding to the case $n = (r_0 - 1,\ s_0 + 1)$. These four curves can be presented as: Curve 1a $C_{F1a} = L(\beta + k_{w1a})$, where $\int_{k_{w1a}}^{\infty} (\alpha - \frac{s_0+1}{s_0} k_{w1a}) f(\alpha) d\alpha = C_N + \beta$; Curve 2a $C_{F2a} = \frac{r_0+1}{r_0}(C_N + \beta - \mu) + L(\beta)$; Curve 1b $C_{F1b} = L(\beta + k_{w1b})$, where $\int_{k_{w1b}}^{\infty} (\alpha - \frac{s_0+2}{s_0+1} k_{w1b}) f(\alpha) d\alpha = C_N + \beta$; Curve 2b $C_{F2b} = \frac{r_0}{r_0-1}(C_N + \beta - \mu) + L(\beta)$. Given *n* firms, referring to Curve 1, it can be proved that C_F is decreasing in *s,* i.e., increasing in r; referring to Curve 2, C_F is decreasing in *r*. Therefore, given *n* firms and $C_N \in [0,\ \mu - \beta]$, $C_{F1a} > C_{F1b} > C_{F2a} > C_{F2b}$. Three sub-areas are created between Curve 1a and Curve 2b. Define these three sub-areas as: (i) Area D1 is the area between Curve 2b and Curve 2a; (ii) Area D2 is the area between Curve 2a and Curve 1b; (iii) Area D3 is the area between Curve 1b and Curve 1a. Figure A1 shows these three areas. In the following, each sub-area is discussed.

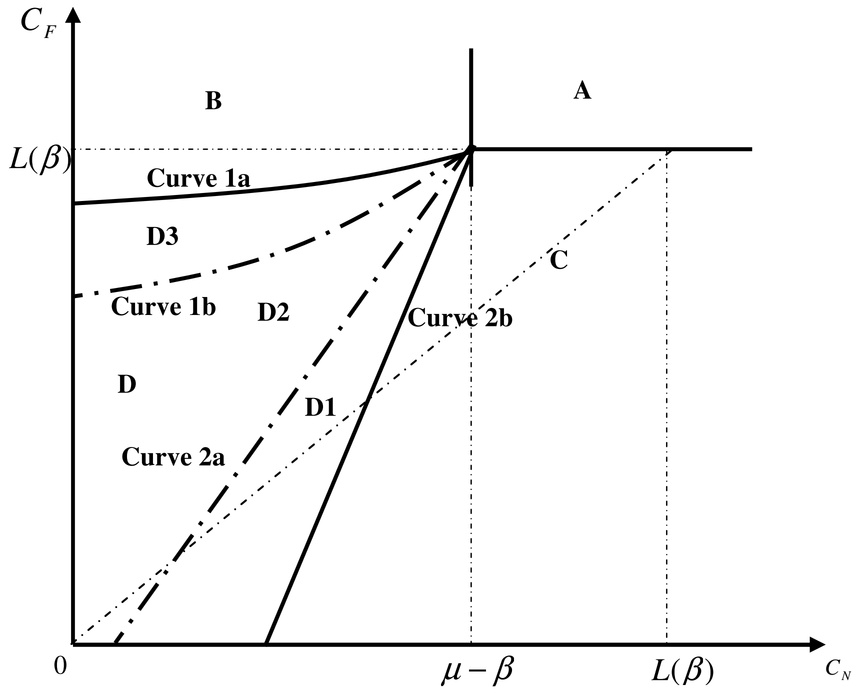

Figure A1: Three areas created by Curve 1a~2b.

(i) Area D1

In this area, case $n = (r_0,\ s_0)$ occurs as Case C, and case $n = (r_0 - 1,\ s_0 + 1)$ occurs as Case D. By Proposition 3, we have:

In case $n = (r_0,\ s_0)$, we get $\Pi_e^F(r_0, n - r_0) = \frac{1}{(r_0+1)^2} \Pi_0^F(0)$; (a35)

In case $n = (r_0 - 1,\ s_0 + 1)$, we get $\Pi_e^N(r_0 - 1, n - r_0 + 1) = \frac{1}{(s_0+1)^2} \Pi_0^N(k^1)$, (a36)

where k^1 satisfies $C_N + \beta = \int_{k^1}^{\infty} (\alpha - \frac{s_0+2}{s_0+1} k^1) f(\alpha) d\alpha - \frac{r_0 - 1}{r_0}(L(\beta + k^1) - C_F)$. (a37)

(i-1) situation: Given C_N, with respect to C_F, by (a37) we have $\frac{dk^1}{dC_F} > 0$. Together with (a34)~(a36), we have

$$\frac{dD(r_0)}{dC_F} = -\frac{2}{(r_0+1)^2}(X(C_F)-\beta) - \frac{1}{(s_0+1)^2}k^1[2\overline{F}(k^1)-k^1 f(k^1)]\cdot\frac{dk^1}{dC_F} < 0.$$ **(i-2) situation:**

Given C_F, with respect to C_N, by (a35) we have $\frac{dk^1}{dC_N} < 0$. Together with (a34)~(a36), we have

$$\frac{dD(r_0)}{dC_N} = -\frac{1}{(s_0+1)^2}k^1[2\overline{F}(k^1)-k^1 f(k^1)]\cdot\frac{dk^1}{dC_N} > 0.$$

(ii) Area D2

If both cases $n=(r_0,\ s_0)$ and $n=(r_0-1,\ s_0+1)$ are in Region D2, then the two cases occur as Case D. By Proposition 3, we have

$$\Pi_e^F(r_0, n-r_0) = \frac{1}{(r_0+1)^2}\Pi_0^F(k^0), \tag{a38}$$

and $\Pi_e^N(r_0-1, n-r_0+1) = \frac{1}{(s_0+1)^2}\Pi_0^N(k^1)$, (a39)

where k^0 satisfies $C_N + \beta = \int_{k^0}^{\infty}(\alpha - \frac{s_0+1}{s_0}k^0)f(\alpha)d\alpha - \frac{r_0}{r_0+1}(L(\beta+k^0)-C_F)$, (a40)

as the total inflexible capacity of case $n=(r_0,\ s_0)$;

k^1 satisfies $C_N + \beta = \int_{k^1}^{\infty}(\alpha - \frac{s_0+2}{s_0+1}k^1)f(\alpha)d\alpha - \frac{r_0-1}{r_0}(L(\beta+k^1)-C_F)$, (a41)

as the total inflexible capacity of case $n=(r_0-1,\ s_0+1)$.

(ii-1) situation: Given C_N, with respect to C_F, by (a40) and (a41), we have $\frac{dk^0}{dC_F} > 0$ and $\frac{dk^1}{dC_F} > 0$. Together with (a34), (a38) and (a39), we have

$$\frac{dD(r_0)}{dC_F} = \frac{1}{(r_0+1)^2}[-2(L(\beta+k^0)-C_F)\cdot\frac{dk^0}{dC_F} - 2(X(C_F)-\beta-k^0)]$$
$$-\frac{1}{(s_0+1)^2}k^1[2\overline{F}(k^1)-k^1 f(k^1)]\cdot\frac{dk^1}{dC_F} < 0.$$

(ii-2) situation: Given C_F, with respect to C_N, by (a40) and (a41), we have $\frac{dk^0}{dC_N} < 0$ and $\frac{dk^1}{dC_N} < 0$. Together with (a34), (a38) and (a39), we have

$$\frac{dD(r_0)}{dC_N} = -\frac{2}{(r_0+1)^2}(L(\beta+k^0)-C_F)\cdot\frac{dk^0}{dC_N} - \frac{1}{(s_0+1)^2}k^1[2\overline{F}(k^1)-k^1 f(k^1)]\cdot\frac{dk^1}{dC_N} > 0$$

(iii) Area D3

In this area, case $n=(r_0,\ s_0)$ occurs as Case D, and case $n=(r_0-1,\ s_0+1)$ occurs as Case B.

By Proposition 3, we have $\Pi_e^F(r_0, n-r_0) = \frac{1}{(r_0+1)^2}\Pi_0^F(k^0)$, (a42)

and $\Pi_e^N(r_0-1, n-r_0+1) = \frac{1}{(s_0+1)^2}\Pi_0^N(k^1)$, (a43)

k^0 satisfies $C_N + \beta = \int_{k^0}^{\infty} (\alpha - \frac{s_0+1}{s_0} k^0) f(\alpha) d\alpha - \frac{r_0}{r_0+1} (L(\beta + k^0) - C_F)$; (a44)

k^1 satisfies $C_N + \beta = \int_{k^1}^{\infty} (\alpha - \frac{s_0+2}{s_0+1} k^1) f(\alpha) d\alpha$. (a45)

(iii-1) situation: Given C_N, with respect to C_F, by (a44), $\frac{dk^0}{dC_F} > 0$. Note that in Case B, $\frac{dk^1}{dC_F} = 0$. Together with (a34), (a42) and (a43), we have $\frac{dD(r_0)}{dC_F} < 0$.

(iii-2) situation: Given C_F, with respect to C_N, by (a44) and (a45), we have $\frac{dk^0}{dC_N} < 0$ and $\frac{dk^1}{dC_N} < 0$. Together with (a34), (a42) and (a43), we have

$$\frac{dD(r_0)}{dC_N} = -\frac{2}{(r_0+1)^2}(L(\beta + k^0) - C_F) \cdot \frac{dk^0}{dC_N} - \frac{1}{(s_0+1)^2} k^1 [2\overline{F}(k^1) - k^1 f(k^1)] \cdot \frac{dk^1}{dC_N} > 0 .$$

Considering these three sub-areas together, given *n* firms, in the areas between Curve 1a and Curve 2b, we have

(1) $\frac{dD(r_0)}{dC_F} < 0$ with given $C_N \in [0, \ \mu - \beta]$; (a46)

(2) $\frac{dD(r_0)}{dC_N} > 0$ with given $C_F \in [0, \ L(\beta)]$. (a47)

Consider the curve $D(r_0) = 0$, $r_0 \in [1, \ n-1]$, given *n* firms. With respect to C_N, we have

$$\frac{\partial D(r_0)}{\partial C_F} \cdot \frac{dC_F}{dC_N} + \frac{\partial D(r_0)}{\partial C_N} = 0, \text{ so that } \frac{dC_F}{dC_N} = -\frac{\partial D(r_0)}{\partial C_N} \Big/ \frac{\partial D(r_0)}{\partial C_F}. \quad \text{(a48)}$$

By (a46) and (a47), we have $\frac{dD(r_0)}{dC_F} < 0$ and $\frac{dD(r_0)}{dC_N} > 0$ in the areas between Curve 1a and Curve 2b. Therefore, by (a48) we have $\frac{dC_F}{dC_N} > 0$.

Consider areas between Curve 1a and Curve 2b. By (a46), $\frac{dD(r_0)}{dC_F} < 0$. By (a34), on Curve 2b, $D(r_0) = \Pi_e^F(r_0, n-r_0) - \Pi_e^N(r_0 - 1, n - r_0 + 1) = \frac{1}{(r_0+1)^2} \Pi_0^F(0) > 0$; on Curve 1a, $D(r_0) = \Pi_e^F(r_0, n-r_0) - \Pi_e^N(r_0 - 1, n - r_0 + 1) = 0 - \frac{1}{(s_0+1)^2} \Pi_0^N(k^1) < 0$, where k^1 satisfies $C_N + \beta = \int_{k^1}^{\infty} (\alpha - \frac{s_0+2}{s_0+1} k^1) f(\alpha) d\alpha$.

Curve 1a and Curve 2b intersect at point $(C_N, \ C_F) = (\mu - \beta, \ L(\beta))$. Note that at this point, $\Pi_e^F(r_0, n-r_0) = \Pi_e^N(r_0 - 1, n - r_0 + 1) = 0$. Therefore, we have $D(r_0) = 0$. The point $(C_N, \ C_F) = (\mu - \beta, \ L(\beta))$ is a common end of curves $D(r_0) = 0$, $r_0 \in [1, \ n-1]$. Therefore, in area between Curve 1a and Curve 2b, there exists a unique curve which satisfies $G(C_N, \ C_F \big| \ r_0) = D(r_0) = 0$.

By (a46), given a C_N, $D(r_0)$ is decreasing in C_F. Therefore, in areas above the curve $D(r_0) = 0$, we have $D(r_0) < 0$; in areas below the curve $D(r_0) = 0$, we have $D(r_0) > 0$.
This completes the proof of Lemma3. □

Proof of Proposition 6

By conditions of FE and Lemma 3, Proposition 6 can be achieved directly. □