Mixed-integer linear programming on work-rest schedule design for construction sites in hot weather

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Abstract: *An appropriate design of work-rest schedule is recognized as an efficient way in providing better ergonomic environment, improving labour productivity as well as safety. Construction workers usually undertake physically demanding tasks in an outdoor environment, with awkward postures and repetitive motions. This study proposes a mixed-integer linear programming approach to optimize the work-rest schedule for construction workers in hot weather for the objective of maximizing the total productive time. The model takes into consideration the physical and physiological conditions of the workers, the working environment, the nature of the jobs and the minimum rest duration of the government regulation. The results of numerical experiments show that the proposed model outperforms a default work-rest schedule by up to 10% in terms of total productive time. This implies considerable cost savings for the construction industry.*

1 INTRODUCTION

A large number of workers are exposed to heat on the jobsite. In the US, at least 2,630 workers suffered from heat-related illnesses and 18 people died from heat on the job (US Department of Labor, 2016a). Construction workers are found to be more susceptible to the adverse effect of heat stress than other industries because they have to repetitively perform physically demanding tasks at awkward postures with excessive energy expenditure (Japan International Center for Occupational Safety and Health, 2001). The risk of heat stress for construction workers is even higher for construction sites in the Northern Hemisphere in summer or in hot and humid countries or regions such as Singapore, Hong Kong, and North Africa (SG Forums, 2010).

Numerous syndromes and illnesses are associated with heat, such as heat cramps (e.g., tonic contraction of muscles in the legs), heat exhaustion (e.g., dizziness or fainting) and heat stroke (e.g., unconsciousness) (Workplace Safety and Health Council of Singapore, 2012). Heat stress on construction workers may further lead to increased injuries and accidents due to fatigue and non-concentration (Christensen et al., 2000). Chronical exposure to heat stress can cause physiological and physical symptoms to construction workers (Bates and Schneider, 2008).

Many government agencies and organizations have expressed concerns on preventing heat stress in the construction industry and have promulgated guidelines to combat heat stresses. The National Institute for Occupational Safety and Health (2016) of the US recommends limiting time spent in hot environments and/or increasing the recovery time spent in a cool environment, reducing the physical demands of the jobs, and using special tools to minimize manual strain. The Workplace Safety and Health Council of Singapore (2012) requires that workers who are from colder countries must undergo a 14-day heat acclimatisation program since those workers might not adapt to the hot and humid climate. To protect workers from heat stroke, some measures are suggested including drinking sufficient water, adopting alternate work and rest periods, and providing shaded areas for workers who have to work under direct sunlight for a long time. The Construction Industry Council (2013) of Hong Kong promulgated a guideline on site safety in hot weather, which designates that "apart from the regular 30-minute rest period for construction workers during the afternoon work session, an additional 15-minute rest period should be allowed for workers during hot summer months (from May to

September every year)". The Laborers' Health and Safety Fund of North America (2014) suggests that employers allow workers to acclimatize by gradually increasing heat exposure, provide cool water to drink, allow rest breaks, and schedule heavy workloads during the coolest time of the day.

Having proper rest has been recognized as an effective approach for construction workers to recover from heat stress (US Department of Labor, 2016b). Proper work-rest cycles allow the body an opportunity to get rid of the excess heat and to slow down the production of internal body heat, thus improving the human comfort, occupational health, and work productivity. Kamon et al. (1983) analyzed the heat acclimatization effect of work-rest schedules for hot ambient conditions. Christensen et al. (2000) used electrophysiological signs of muscle fatigue to compare the physiological responses between fast meat cutters who finish their jobs early and then take a rest and slow meat cutters who finish their jobs just on time, and found no differences in measured physiological responses. Tiwari and Gite (2006) conducted an experiment to study the influence of four work-rest schedules on physical workload during a power tiller operation. Their results indicated that the work-rest schedules have a bearing on the heart rates and postural discomfort of the workers. Dababneh et al. (2001) studied the impact of frequent but short rest periods on the productivity and well-being of workers in the meat-processing industry and showed that workers do not readily accept fragmentation of rest time into short frequent breaks. Kakarot et al. (2012) investigated the impact of ergometer cycling under two situations with the same total rest time but with different activity–rest schedules. They found that heart rate was lower in the frequent-short-break situation. Yi and Chan (2013) used a Monte Carlo simulation technique to study the work-rest schedule of construction workers during summer time in Hong Kong. Based on the simulation model, Yi and Chan (2014) compared 21 work patterns with different start and finish times and proposed having a 20-min rest in the morning and a 30-min rest in the afternoon.

A number of optimization methods have been used in civil and infrastructure related research. Genetic algorithm (GA), a metaheuristic inspired by the process of natural selection, is arguably the most commonly used heuristic algorithm to generate high-quality solutions. GA has been used in the contexts of high-rise housing/skyscraper construction (Tam and Tong, 2003; Koo et al., 2016), steel space-frame roof structures (Kociecki and Adeli, 2014), structural damage detection (Cha and Buyukozturk, 2015), retrofit design (Park et al., 2015), vehicle ID - matching (Li and Souleyrette, 2016), and freeway safety (Li et al., 2016). Simulated annealing (SA), inspired from annealing in metallurgy, is a probabilistic technique for approximating the global optimum and is often used when the search space is discrete. SA has been applied in taxi dispatching (Jung et al., 2016) and management of disruptions in water distribution (Nayak and Turnquist, 2016).

A major breakthrough in solving large-scale nonlinear civil and infrastructure optimization problems is the neural dynamics model of Park and Adeli (1995, 1997). The method was used by Adeli and Karim (1997, 2003) and Karim and Adeli (1999) to solve project scheduling problems, by Senouci and Adeli (2001) on resource scheduling, by Adeli and Kim (2001), Tashakori and Adeli (2002), and Ahmadkhanlou and Adeli (2005) for the design of composite beams, steel space structures, and reinforced concrete slabs, respectively.

This study examines the optimal design of work-rest schedules (i.e., when to have a rest and how long the rest lasts) for construction sites in hot weather with the objective of maximizing the productivity. The contribution of the paper is twofold. First, to the best of our knowledge, this is the first study that develops a mathematical programming model to optimize the work-rest schedules for construction workers. The advantage of mathematical programming approaches over simulation-based heuristics lies in that the former approaches can obtain the optimal work-rest schedules, meaning that the productivity is maximized. Second, we have conducted numerical experiments and the results show that the optimal work-rest schedule outperforms a default schedule and increases the total productive time of the workers by up to 10%, implying considerable cost savings.

The remainder of this paper is organized as follows: Section 2 gives a detailed description for the problem. Section 3 formulates a mathematical model, discusses a few extensions, and proposes a solution algorithm. Computational experiments are reported in Section 4. Final conclusions are drawn in the last section.

2 PROBLEM DESCRIPTION

Consider a working day from 8:00am to 5:30pm shown in Figure 1 for construction workers. At the beginning of the day (i.e., at 8:00am), the workers are fully recovered from heat stress after a night's sleep. The morning shift is from 8:00am to 12:00pm. Then the workers have a one-hour lunch. After lunch (at 1:00pm), the workers are fully recovered from heat stress again. The afternoon shift is from 1:00pm to 5:30pm. Governments have regulations regarding the minimum duration of rest in a day. For example, as mentioned above, the Construction Industry Council (2013) of Hong Kong requires a minimum rest of 45 min in a whole day during hot summer months. We will examine, for a construction site, how to design the optimal work-rest schedule that maximizes the total productive time of the workers while satisfying the governments' requirement of rest.

A key term related to heat stress is heat tolerance time (HTT), which is the maximum duration that a worker can perform tasks without hurting his health (Chan et al., 2012). The HTT of a worker who is fully recovered from heat stress, denoted by HTT_0, depends on the worker's physical and physiological conditions, the working environment, and the nature of the job. HTT_0 (min) can be estimated by the equation below (Chan et al., 2012):

$$\mathrm{HTT}_0 = 60 \times (12.43 - 0.11\mathrm{WBGT} - 0.10\mathrm{API} - 0.06\mathrm{Age} - 2.28\mathrm{ADH} - 0.50\mathrm{SH} - 0.14\mathrm{EC} - 0.16\mathrm{RER} + 0.07\mathrm{PBF} + 0.01\mathrm{RHR})/1.40 \quad (1)$$

where

WBGT: wet bulb globe temperature (°C)
API: air pollution index
Age: age of the worker
ADH: alcohol drinking habit ("none"=0, "occasionally"=1, "usually"=2)
SH: smoking habit ("none"=0, "occasionally"=1, "usually"=2)
EC: energy consumption
RER: respiratory exchange rate
PBF: percentage of body fat (%)
RHR: resting heart rate (bpm).

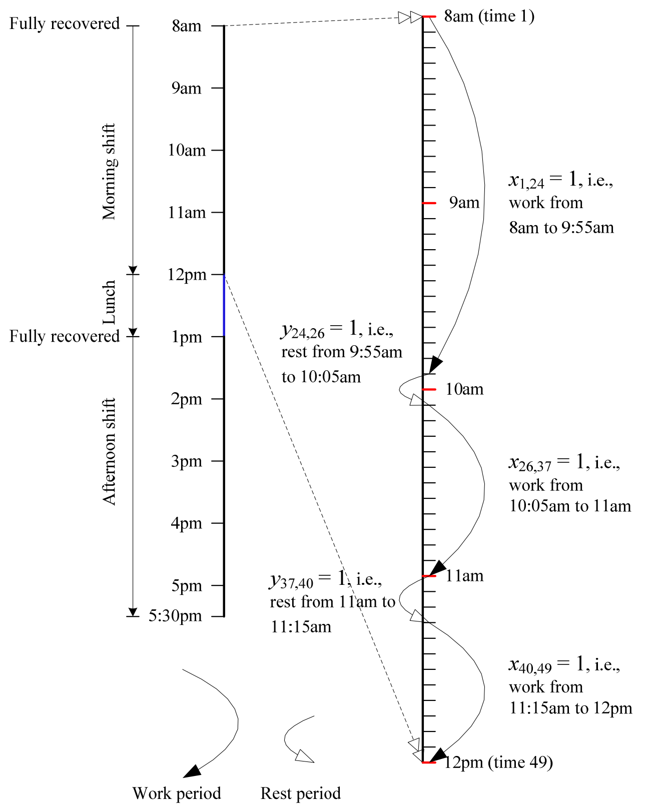

Figure 1 An example of a work-rest schedule

A construction team has several workers. Suppose that a worker's HTT_0 is 120 min and he works from 8:00am to 9:55am, then his productive time during the period is 115 min; suppose that another worker's HTT_0 is 108 min, then his productive time during the period is 108 min and the remaining 115–108=7 min is unproductive. In sum, the productive time is the smaller one of the heat tolerance time and the work duration.

A worker will recover from rest, either partially or fully, depending on the duration of the rest. Chan et al. (2012) conducted experiments and showed that a construction worker achieves 58% energetic recovery after a 5-min rest, 68% after 10 min, 78% after 15 min, 84% after 20 min, 88% after 25 min, 92% after 30 min, 93% after 35 min, and 94% after 40 min, as shown in Figure 2. For example, a worker whose HTT_0 is 120 min works from 8:00am to 9:55am and then takes a 10-min rest, his HTT at 10:05am will be 68%×120=81.6 min; if the worker works from 8:00am to 8:30am and then takes a 10-min rest, since he can still work for 120–30 = 90 min without rest, his HTT at 8:40am will be 90 min instead of 81.6 min; if he takes a 15-min rest from 8:30am to 8:45am, his HTT at 8:45am will be 78%×120=93.6 min. Note that because of lack of experimental data, it is hard to quantify, e.g., what is the HTT of a worker if he works for 1 min and then rests for 5 min, what is his HTT if he works for 2 min and then rests for 5 min, what is his HTT if he works for 3 min and then rests for 5 min? Therefore, we assume that there is no energetic recovery for the workers when they take a rest at a point where they have a minor degree of exhaustion.

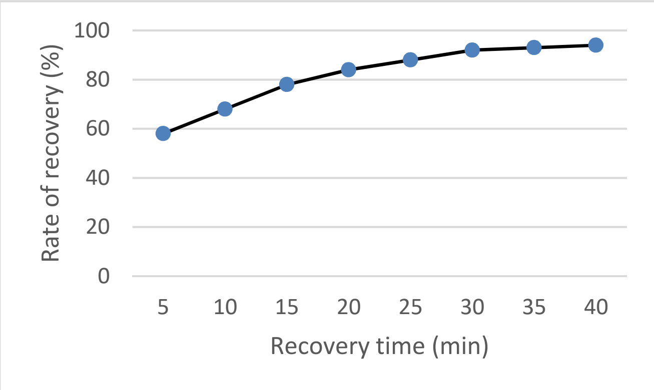

Figure 2 Rate of recovery

At the beginning of a day, the leader of the construction team will estimate the HTT_0 of the workers in the team. Then the leader needs to determine the optimal work-rest schedule that maximizes the total productive time of the workers while satisfying the governments' requirement of rest. The productive time of a worker is the duration from the start (e.g. 8:00am) to the end (e.g. 5:30pm) of a working day, minus the lunch time (e.g. 1 hour), the unproductive time after the heat tolerance time is reached, and the rest time. The total productive time is the sum of productive time of all of the workers at the construction site.

In this research we will develop models for the construction team leaders to obtain the optimal work-rest schedule. We choose to formulate a mixed-integer linear optimization model because in the operations research community, the solution techniques for mixed-integer linear optimization have gained significant advances. Combined with the advancement of information technology, solvers, such as CPLEX and GUROBI, can solve mixed-integer linear optimization models in an efficient manner to optimality.

3 MODEL FORMULATION AND ALGORITHM

We will first examine how to determine the optimal work-rest schedule for the morning shift, based on which the optimal work-rest schedule for a whole day can be derived. Section 3.1 lists the notation; Section 3.2 presents a model for designing the optimal work-rest schedule for the morning shift; Section 3.3 describes linearization techniques that are used to transform the model into a mixed-integer linear programming one; Section 3.4 determines the big-M's in the mixed-integer linear programming model; Section 3.5 discusses some extensions to the model; Section 3.6 presents an algorithm for determining the optimal work-rest schedule for a whole day.

3.1 Notation

***Indices*:**

w: categories of workers;

t_1, t_2, t: time points;

j: number of time intervals in a rest period;

***Input parameters*:**

Δ: number of minutes of a time interval; $\Delta = 5$ min

W: number of categories of workers;

n_w: number of workers in category w;

T: number of time points in the morning or afternoon; $T = 49$ for the morning shift and $T = 55$ for the afternoon shift

$\tau_{0\mathrm{am}}^w$: HTT_0 of a worker in category w at the beginning of the morning shift of a working day;

$\tau_{0\mathrm{pm}}^w$: HTT_0 of a worker in category w at the beginning of the afternoon shift of a working day;

J: maximum duration of a rest period (in terms of the number of time intervals); we assume that a rest period lasts at most 40 min, i.e., $J = 8$

ρ_j: percentage of HTT recovery after resting for j time intervals; $\rho_1 = 58\%, \rho_2 = 68\%, \rho_3 = 78\%, \rho_4 = 84\%, \rho_5 = 88\%, \rho_6 = 92\%, \rho_7 = 93\%, \rho_8 = 94\%$

Θ_{am}: minimum total duration of the rest periods (min) in the morning required by the government; $\Theta_{\mathrm{am}} = 0$ if there is no such requirement

Θ_{pm}: minimum total duration of the rest periods (min) in the afternoon required by the government; $\Theta_{\mathrm{pm}} = 0$ if there is no such requirement

Θ: minimum total duration of the rest periods (min) in a day required by the government; $\Theta = 0$ if there is no such requirement

M: a sufficiently large positive number;

***Decision variables*:**

$x_{t_1 t_2} \in \{0, 1\}$: set to 1 if a work period starts at time t_1 and finishes at time t_2, and zero otherwise;

$y_{t_1t_2} \in \{0,1\}$: set to 1 if a rest period starts at time t_1 and finishes at time t_2, and zero otherwise;

$p^w_{t_1t_2}$: productive time (min) of a worker in category w during the work period from time t_1 to time t_2; $p^w_{t_1t_2} = 0$ if no work period starts at time t_1 and finishes at time t_2

s^w_t: HTT (min) of a worker in category w at time t, provided that a rest period finishes at time t; the value of s^w_t does not matter if no rest period finishes at time t

r^w_t: remaining time (min) that a worker in category w can still be productive after time t, provided that a work period finishes at time t; the value of r^w_t does not matter if no work period finishes at time t

$\pi^w_t \in \{0,1\}$: an auxiliary variable for calculating the HTT after a rest period.

3.2 Mathematical model for optimal work-rest schedule design for the morning shift

Before describing the model, we first summarize the assumptions. First, we assume that time is discretized into 5-min intervals, and a rest period must start and finish at e.g. 8:35 (time 8) or 8:40 (time 9) but not at e.g. 8:36 or 8:37. Second, we assume that the construction company has already measured relevant parameters of the construction workers, the working environment, and the job, so that we can calculate the HTT. Third, all of the workers have the same work-rest schedule.

As shown in Figure 1, we discretize the 4-hour morning shift into 48 time intervals; each interval has $\Delta = 5$ min. We therefore have 49 time points; for example, time 1 is 8:00am, time 2 is 8:05am, and time 49 is 12:00pm. We assume that a rest must start and finish at e.g. 8:35 (time 8) or 8:40 (time 9) but not at e.g. 8:36 or 8:37. To formulate the work-rest schedule, we define binary variable $x_{t_1t_2}$ that is equal to one if a work period starts at time $t_1 = 1,2,\ldots,49$ and finishes at time $t_2 = 1,2,\ldots,49$, and zero otherwise, and binary variable $y_{t_1t_2}$ that is equal to one if a rest period starts at time t_1 and finishes at time t_2, and zero otherwise. Take Figure 1 as an example. If workers work from 8:00am to 9:55am (that is a work period), take a rest to 10:05 (this rest period is from 9:55am to 10:05am), work to 11:00am, rest to 11:15, and work to 12:00pm, then we have $x_{1,24} = x_{26,37} = x_{40,49} = y_{24,26} = y_{37,40} = 1$ and all the other $x_{t_1t_2}$ and $y_{t_1t_2}$ variables are 0.

Without loss of generality, we classify the workers in the team into W categories. There are n_w workers in category $w = 1,\ldots,W$ (note that if all workers are different, we can set W to be equal to the number of workers and set $n_w = 1$). The HTT_0 of a worker in category w at the beginning of the morning shift is denoted by $\tau^w_{0\text{am}}$. The workers are heterogeneous but they must have the same work-rest schedule. That means it is possible that during work some workers are no longer productive (the work duration exceeds their heat tolerance time) while the other workers are still productive (the work duration does not exceed their heat tolerance time). We assume that a rest period lasts at most 40 min, i.e., 8 time intervals. Without loss of generality, we assume that there is a minimum total duration of the rest periods (min) in the morning denoted by Θ_{am} (note that if there is no such requirement, we can simply set $\Theta_{\text{am}} = 0$). The work-rest schedule design problem for the morning shift can be formulated as:

$$\text{[M1]} \quad \max \sum_{w=1}^{W} n_w \sum_{t_1=1}^{T-1} \sum_{t_2=t_1+1}^{T} p^w_{t_1t_2} \tag{2}$$

subject to:

$$y_{1,t_2} = 0, t_2 = 2, \ldots T \tag{3}$$

$$y_{t_1,T} = 0, t_1 = 1, \ldots T-1 \tag{4}$$

$$\sum_{t_2=2}^{T} x_{1,t_2} = 1 \tag{5}$$

$$\sum_{t_1=1}^{T-1} x_{t_1,T} = 1 \tag{6}$$

$$\sum_{t_1=1}^{t-1} x_{t_1,t} = \sum_{t_2=t+1}^{T} y_{t,t_2}, t = 2, \ldots, T-1 \tag{7}$$

$$\sum_{t_1=1}^{t-1} y_{t_1,t} = \sum_{t_2=t+1}^{T} x_{t,t_2}, t = 2, \ldots, T-1 \tag{8}$$

$$s^w_1 = \tau^w_{0\text{am}}, w = 1, \ldots, W \tag{9}$$

$$\text{If } y_{t_1t_2} = 1 \text{ then } s^w_{t_2} = \max\{r^w_{t_1}, \rho_{t_2-t_1} \cdot \tau^w_{0\text{am}}\}, w = 1, \ldots, W, 1 \le t_1 < t_2 \le T \tag{10}$$

$$\text{If } x_{t_1t_2} = 1 \text{ then } p^w_{t_1t_2} = \min\{s^w_{t_1}, \Delta(t_2 - t_1)\}, w = 1, \ldots, W \text{ and } r^w_{t_2} = s^w_{t_1} - p^w_{t_1t_2}, w = 1, \ldots, W, \ 1 \le t_1 < t_2 \le T \tag{11}$$

$$\text{If } x_{t_1t_2} = 0 \text{ then } p^w_{t_1t_2} = 0, w = 1, \ldots, W, 1 \le t_1 < t_2 \le T \tag{12}$$

$$\sum_{t_1=1}^{T-1} \sum_{t_2=t_1+1}^{T} \Delta(t_2 - t_1) y_{t_1t_2} \ge \Theta_{\text{am}} \tag{13}$$

$$x_{t_1t_2} \in \{0,1\}, 1 \le t_1 < t_2 \le T \tag{14}$$

$$y_{t_1t_2} \in \{0,1\}, 1 \le t_1 < t_2 \le T \tag{15}$$

$$y_{t_1t_2} = 0, 1 \le t_1 \le T - J - 1, \ t_1 + J + 1 \le t_2 \le T \tag{16}$$

$$p^w_{t_1t_2} \ge 0, 1 \le t_1 < t_2 \le T, w = 1, \ldots, W \tag{17}$$

$$s^w_t \ge 0, t = 1, \ldots, T, w = 1, \ldots, W \tag{18}$$

$$r^w_t \ge 0, t = 1, \ldots, T, w = 1, \ldots, W. \tag{19}$$

In the above model, Eq. (2) maximizes the total productive time of all of the workers in the team; here $p^w_{t_1t_2}$ is nonzero if a work period starts at time t_1 and finishes at time t_2; $\sum_{t_1=1}^{T-1}\sum_{t_2=t_1+1}^{T} p_{t_1t_2}^{w}$ is the productive time of one worker in category w from time 1 to time T. Constraints (3) require that a period that starts at 8:00am cannot be a rest period, i.e., all workers must start to work at 8:00am rather than start to rest at 8:00am. Constraints (4) require that a period that finishes at 12:00pm cannot be a rest period, i.e., all workers must be working from 11:55am to 12:00pm rather than resting. Constraints (5) impose that there must be a work period that starts at 8:00am. Constraints (6) impose that there must be a work period that finishes at 12:00pm. Constraints (7) state that if a work period finishes at time t, then a rest period must start at time t. Constraints (8) state that if a rest period finishes at time t, then a work period must start at time t. Constraints (9) define the HTT at time 1 for each category of workers. Constraints (10) compute the HTT of a worker in category w after a rest period. Constraints (11) compute the productive time during a work period and the remaining time that a worker can still be productive after a work period. Constraints (12) set $p_{t_1t_2}^{w}$ at zero if no work period starts at time t_1 and finishes at time t_2. Constraints (13) enforce the condition that the total duration of the rest periods is at least Θ_{am}. Constraints (14) and (15) define $x_{t_1t_2}$ and $y_{t_1t_2}$ as binary variables, respectively. Constraints (16) require that a rest period does not exceed J time intervals. Constraints (17), (18) and (19) define $p_{t_1t_2}^{w}$, s_t^{w} and r_t^{w} as nonnegative decision variables, respectively.

3.3 Linearization of the model

This problem cannot be solved using dynamic programming because there is the "curse-of-dimensionality" associated with the number of categories of workers: at each time (stage), we have to record the states of the laborers, which include how many minutes workers in each category have worked for. The state space increases exponentially with the number of categories of workers. Hence, dynamic programming is not applicable. We can use some metaheuristic methods such as genetic algorithm. However, genetic algorithm does not guarantee optimality. We choose to formulate a mixed-integer linear optimization model because in the operations research community, the solution techniques for mixed-integer linear optimization have gained significant advances. Combined with the advancement of information technology, solvers, such as CPLEX and GUROBI, can solve mixed-integer linear optimization models in an efficient manner to optimality. However, model [M1] has logical operators such as "max" and "if", rendering it unsuitable to be solved by mathematical programming techniques. Therefore, we propose approaches below to transform the model into a mixed-integer linear program which can then be solved to optimality.

3.3.1 Linearization of the constraints related to the rest periods

Constraints (10) have the "max" operator. Since $s_{t_2}^{w}$ is to be maximized at the optimal solution, constraints (10) are inherently nonconvex and we have to introduce additional binary variables π_t^{w} to linearize the constraints. First, we note that the constraints can be replaced by

If $y_{t_1t_2}=1$ then $s_{t_2}^{w}\le r_{t_1}^{w}+M\pi_{t_2}^{w}, w=1,\dots,W$, $1\le t_1<t_2\le T$ (20)

If $y_{t_1t_2}=1$ then $s_{t_2}^{w}\le \rho_{t_2-t_1}\cdot\tau_{0\text{am}}^{w}+M(1-\pi_{t_2}^{w}), w=1,\dots,W, 1\le t_1<t_2\le T$ (21)

$\pi_t^{w}\in\{0,1\}, t=1,\dots,T, w=1,\dots,W.$ (22)

Constraints (20) to (22) imply that if $y_{t_1t_2}=1$, then $s_{t_2}^{w}$ has to be smaller than at least one of $r_{t_1}^{w}$ and $\rho_{t_2-t_1}\cdot\tau_{\text{am}}^{w}$. As $s_{t_2}^{w}$ is to be maximized at the optimal solution, it will be equal to the larger one of $r_{t_1}^{w}$ and $\rho_{t_2-t_1}\cdot\tau_{\text{am}}^{w}$. Constraints (20) and (21) can be linearized as follows:

$$s_{t_2}^{w}\le r_{t_1}^{w}+M\pi_{t_2}^{w}+M(1-y_{t_1t_2}), w=1,\dots,W, \quad 1\le t_1<t_2\le T \tag{23}$$

$$s_{t_2}^{w}\le \rho_{t_2-t_1}\cdot\tau_{0\text{am}}^{w}+M(1-\pi_{t_2}^{w})+M(1-y_{t_1t_2}), w=1,\dots,W, 1\le t_1<t_2\le T. \tag{24}$$

Constraints (23) imply that if $y_{t_1t_2}=0$, then there is no constraint on $s_{t_2}^{w}$; if $y_{t_1t_2}=1$, then constraints (23) are reduced to constraints (20). Constraints (24) have a similar transformation to constraints (23).

3.3.2 Linearization of the constraints related to the work periods

Constraints (11) have the "min" operator and constraints (12) have the "if" operator. Since $p_{t_1t_2}^{w}$ is to be maximized at the optimal solution, the relations $p_{t_1t_2}^{w}=\min\{s_{t_1}^{w}, t_2-t_1\}$ are inherently convex. Hence, constraints (12) and the first half of constraints (11) can be replaced by

$$p^w_{t_1t_2} \le s^w_{t_1}, w = 1, \dots, W, 1 \le t_1 < t_2 \le T \quad (25)$$

$$p^w_{t_1t_2} \le \Delta(t_2 - t_1), w = 1, \dots, W, 1 \le t_1 < t_2 \le T \quad (26)$$

$$p^w_{t_1t_2} \le M x_{t_1t_2}, w = 1, \dots, W, 1 \le t_1 < t_2 \le T. \quad (27)$$

Since $r^w_{t_2}$ in the second half of constraints (11) is to be maximized at the optimal solution, the relations "if $x_{t_1t_2} = 1$ then $r^w_{t_2} = s^w_{t_1} - p^w_{t_1t_2}, w = 1, \dots, W, 1 \le t_1 < t_2 \le T$" can be replaced by

$$\text{If } x_{t_1t_2} = 1 \text{ then } r^w_{t_2} \le s^w_{t_1} - p^w_{t_1t_2}, w = 1, \dots, W, \quad 1 \le t_1 < t_2 \le T. \quad (28)$$

Constraints (28) can be linearized as

$$r^w_{t_2} \le s^w_{t_1} - p^w_{t_1t_2} + M(1 - x_{t_1t_2}), w = 1, \dots, W, 1 \le t_1 < t_2 \le T. \quad (29)$$

Based on the above newly added decision variables and constraints, model [M1] has been transformed to a mixed-integer linear programming formulation:

[M2] Objective (2)

subject to: Constraints (3) to (9), (13) to (19), (22) to (27), and (29).

3.4 Determining the values of the big-M's

Proper values of the big-M's in Constraints (23), (24), (27) and (29) must be computed before model [M2] can be solved. The big-M's in constraints (23) and (24) can be set to $\tau^w_{0\text{am}}$ for each worker category w as $s^w_{t_2}$ is never greater than $\tau^w_{0\text{am}}$; the big-M's in constraints (27) can be set to $\min\{\Delta(t_2 - t_1), \tau^w_{0\text{am}}\}$ as $p^w_{t_1t_2}$ is never greater than $\min\{\Delta(t_2 - t_1), \tau^w_{0\text{am}}\}$; the big-M's in constraints (29) can be set to $\tau^w_{0\text{am}}$ for each worker category w as $r^w_{t_2}$ is never greater than $\tau^w_{0\text{am}}$.

Model [M2] can be solved by a mixed-integer linear programming solver such as CPLEX (González et al., 2016; Maghrebi et al., 2016; Rashidi et al., 2016; Xie and Jiang, 2016; Zhao et al., 2016; Zockaie et al., 2016; Yu et al., 2017).

3.5 Practical extensions to model [M2]

Model [M2] can be revised to handle other practical considerations. For instance, if the team leader requires that there is only one rest period in the morning, then we can add the following constraint to the model:

[M3]: Model [M2] with the following constraint

$$\sum_{t_1=1}^{T-1} \sum_{t_2=t_1+1}^{T} y_{t_1t_2} = 1. \quad (30)$$

If workers are working far away from the place to have a rest (usually a room with air-conditioner) and have to e.g. walk for 2.5 min to the room, meaning that 5 min is lost during each rest period, we can replace constraints (24) by

$$s^w_{t_2} \le \rho_{t_2-t_1-1} \cdot \tau^w_{0\text{am}} + M(1 - \pi^w_{t_2}) + M(1 - y_{t_1t_2}), w = 1, \dots, W, 1 \le t_1 < t_2 \le T, \quad (31)$$

where ρ_0 is defined to be 0.

3.6 Algorithm for work-rest schedule design for a whole day

Model [M2] solves the work-rest schedule design problem for the morning shift with a minimum total duration of the rest periods Θ_{am}. The work-rest schedule design problem for the afternoon shift can be solved similarly after replacing Θ_{am} by Θ_{pm}, changing the number of time points T from 49 (4 hours) to 55 (4 hours 30 minutes), and using the new $\tau^w_{0\text{pm}}$ instead of $\tau^w_{0\text{am}}$ because the working environment of the afternoon is different from that in the morning (the afternoon is usually hotter).

If governments have regulations on the minimum total duration of the rest periods (min) in a day (denoted by Θ) instead of separate requirements on the morning shift and the afternoon shift, then the work-rest schedule design problem for the morning shift and the problem for the afternoon shift cannot be solved independently. We note that there are not many combinations of $(\Theta_{\text{am}}, \Theta_{\text{pm}})$ satisfying $\Theta_{\text{am}} + \Theta_{\text{pm}} \ge \Theta$ that need to be evaluated; based on this observation, we propose the following algorithm for optimizing the work-rest schedule for a whole day with minimum total duration of the rest periods Θ:

Algorithm 1. *Optimal work-rest schedule for a whole day*

Step 0: Solve model [M2] for the morning problem without constraints (13) and obtain the optimal total duration of the rest periods in the morning, denoted by Θ_{am}^*. Solve model [M2] for the afternoon problem without constraints (13) and obtain the optimal total duration of the rest periods in the afternoon, denoted by Θ_{pm}^*. If $\Theta_{\text{am}}^* + \Theta_{\text{pm}}^* \geq \Theta$, then the solutions to the above two problems form the optimal work-rest schedule for the whole day, and stop.

Step 1: Define $K \leftarrow (\Theta - \Theta_{\text{am}}^* - \Theta_{\text{pm}}^*)/\Delta$ (round up if the right-hand side is not an integer). Solve the following $K + 1$ groups of problems: (i) model [M2] for the morning problem with constraints (13) where $\Theta_{\text{am}} = \Theta_{\text{am}}^* + k \cdot \Delta$, and (ii) model [M2] for the afternoon problem with constraints (13) where $\Theta_{\text{pm}} = \Theta_{\text{pm}}^* + (K - k) \cdot \Delta$, $k = 0,1, \dots, K$. Note that the solutions to the morning and afternoon problems in each group comprise a feasible work-rest schedule for a whole day because the constraints guarantee that the total rest duration in the day is at least Θ.

Step 2: The solutions to the group solved in Step 1 with the highest productive time in the day among the $K + 1$ groups form the optimal work-rest schedule for the day.

4 COMPUTATIONAL EXPERIMENTS

In this section, we report the results of numerical experiments that are used to validate the effectiveness of the proposed model. The experiments are run on a PC equipped with 3.60GHz of Intel Core i7 CPU and 16GB of RAM. The mixed-integer programming models [M2] and [M3] are solved by CPLEX 12.6.3.

We compare the total productive time using model [M2], the total productive time using model [M3] (i.e., [M2] with only one rest period in the morning and one rest period in the afternoon), and the default work-rest schedule by the Construction Industry Council (2013) with a 15-min rest from 10:00am to 10:15am in the morning and a 30-min rest from 3:00pm to 3:30pm in the afternoon.

4.1 One category of workers

We first consider a morning shift with $\Theta_{\text{am}} = 0$ and a team of 10 homogeneous workers. The parameters are as follows: WBGT = 25, API = 35, ADH = 1, SH = 1, EC = 2.0, RER = 1.0, PBF = 14.3, RHR = 77.8. We solve four instances with Age = 25, 30, 35, 40. Table 1 reports the results including the worker's HTT_0 (min, the row "$\tau_{0\text{am}}^w$"), the total productive time under the default work-rest schedule of a rest period from 10:00am to 10:15am (min, the row "Default"), the total productive time of the work-rest schedule obtained by [M2] (min, the row "[M2]"), the percentage of improvement in total productive time (row "[M2]" – row "Default") / row "Default", the total productive time of the work-rest schedule obtained by [M3] (min, the row "[M3]"), the duration of the rest period (min, the row "Rest duration"), and the percentage of improvement in total productive time (row "[M3]" – row "Default") / row "Default".

The results in Table 1 show that the optimized work-rest schedule by [M2] considerably improves over the default work-rest schedule with large percentages of productive time increase. In the optimal work-rest schedule by [M2], each rest period lasts only 5 min; this is because a worker will recover 58% of his energy after a 5-min rest, but just recover 68% after a 10-min rest. The more realistic results given by [M3] improve over the ones of the default schedule by 0.9% for the instance with workers of 25 years old and by 9.0% for the instance with workers of 40 years old. We can further see from the row "Rest duration" that if a construction team comprises young workers, then a short rest period will be more productive; by contrast, a team of older workers should have a longer rest duration to increase the productive time. Of course, the rest period should never be shorter than that required by the governments; for example, if a minimum rest period of 15 min is required for the morning shift, then in the column for Age = 25 the rest duration should be 15 min and the productive time will be 2250 min.

Table 2 reports the results of four instances with the same setting as those in Table 1 except that the WBGT is 30 instead of 25 (i.e., the working environment for the settings in Table 2 is much hotter). Comparing the two tables, we can see that construction workers should have more rest when working in hotter environment. In fact, allocating more rest time will increase rather than reduce the productive time. This further emphasize that there is need to tailor the work-rest schedule based on the physical and physiological conditions of the workers and the working environment rather than just adopt the default work-rest schedule.

Table 1 Comparison under WBGT = 25

Age	25	30	35	40
τ_{0am}^{w}	135	123	110	97
Default	2250	2156	1952	1723
[M2]	2300	2300	2297	2250
Improvement	2.2%	6.7%	17.7%	30.6%
[M3]	2271	2200	2097	1878
Rest duration	10	20	30	40
Improvement	0.9%	2.0%	7.4%	9.0%

Table 2 Comparison under WBGT = 30

Age	25	30	35	40
τ_{0am}^{w}	112	99	86	73
Default	1990	1761	1533	1304
[M2]	2300	2250	2249	2200
Improvement	15.6%	27.8%	46.7%	68.7%
[M3]	2102	1920	1670	1421
Rest duration	25	40	40	40
Improvement	5.6%	9.0%	8.9%	9.0%

4.2 More than one category of workers

In this section we examine instances with heterogeneous workers. The parameters are the same as the ones used in Table 1, except that the 10 workers are in five different age categories; for instance, in the instance shown by the column of Average age = 30 in Table 3, two workers are 30 years old, two are 25, two are 20, two are 35, and two are 40. The HTT_0 of workers of 20, 25, 30, 35, and 40 years old are 148, 135, 123, 110, and 97 min, respectively.

Table 3 reports the results of four instances with Average age = 30, 35, 40, 45. We can see that the results given by [M3] improves over the default one by 0.68% for the instance with workers of 30 years old on average and by 7.76% for the instance with workers of 45 years old on average.

Table 3 Comparison with heterogeneous workers

Average age	30	35	40	45
Average τ_{0am}^{w}	123	110	97	84
Default	2066	1915	1718	1494
[M3]	2080	1954	1807	1610
Rest duration	20	30	30	40
Improvement	0.68%	2.04%	5.18%	7.76%

We conduct some large-scale experiments. The parameters are similar to the ones for Table 3, but each experiment has 220 workers in 11 categories. Table 4 reports the results of four instances with Average age = 30, 35, 40, 45. For example, if the average age is 30, then 20 workers are 20 years old, 20 are 22 years old, etc., and 20 workers are 40 years old. We can see that the results given by [M3] improves over the default one by 0.80% for the instance with workers of 30 years old on average and by 8.02% for the instance with workers of 45 years old on average.

Table 4 Comparison with heterogeneous workers

Average age	30	35	40	45
Default	45738	42321	37862	32879
[M3]	46106	43338	39972	35515
Improvement	0.80%	2.40%	5.57%	8.02%

4.3 Schedule design for a whole day

In this section we examine the work-rest schedule design for a whole day. The parameters are the same as the ones used in Table 3, except that (i) we consider both the morning shift (4 hours) and the afternoon shift (4.5 hours), (ii) the minimum total duration of the rest periods in the morning and the afternoon required by the government is $\Theta = 45$ min, and (iii) four instances are examined with the WBGTs in the morning and the afternoon being 25 and 25, 25 and 30, 30 and 25, and 30 and 30.

Table 5 reports the results. We can see that the total productive time of the work-rest schedule given by [M3] is 0.4% to 2.5% larger than that by the default one. The improvement is more evident when the morning is hot (WBGT = 30). Although both the default schedule and model [M3] suggest having a 30-min rest in the afternoon, the default one is from 3:00pm to 3:30pm, while [M3] suggests having a rest from 3:05pm to 3:35pm (for both WBGT = 25 and WBGT = 30); this shift of rest period slightly increases the productive time in the afternoon.

Table 5 Comparison for a whole day

WBGT am	25	25	30	30
WBGT pm	25	30	25	30
Default am	2066	2066	1752	1752
Default pm	2218	1890	2218	1890
Default total	4284	3956	3970	3642
[M3] am	2080	2080	1832	1832
Rest duration am	20	20	30	30
[M3] pm	2223	1900	2223	1900
Rest duration pm	30	30	30	30
[M3] total	4303	3980	4055	3732
Improvement	0.4%	0.6%	2.1%	2.5%

5 CONCLUSIONS

This study has proposed a mixed-integer linear programming approach to optimize the work-rest schedule for workers at a hot construction site with the objective of maximizing the total productive time. We have proposed a few techniques to linearize the constraints, enabling the model to be solved by off-the-shelf mixed-integer linear programming solvers. To use the model, a team leader needs to have the physical and physiological data of the workers available; on a working day, the leader input the environment parameters based on weather reports and onsite measurements and the information of the tasks to be performed; then the model will calculate the optimal work-rest schedule that satisfies the government regulation on the minimum rest duration and maximizes the total productive time. The schedule provided by the model can be put to use without much modification. The results of numerical experiments show that the proposed model outperforms a default work-rest schedule by up to 10% in terms of total productive time. This implies considerable cost savings for the construction industry. Future research will be directed at field-testing of the model to see how much efficiency and cost savings would actually be saved by the model. Another future research direction is jointly optimizing the work-rest schedule and the assignment of workers to the construction tasks.

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