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Weekly Container Delivery Patterns in Liner Shipping Planning Models

Abstract

This paper addresses a fundamental question related to nearly all container liner shipping planning models: whether the implicit assumption of identical container delivery pattern every week is valid in a situation of identical shipping services and identical cargo demand every week. We prove that when the number of containers transported from one port to the next is formulated as a continuous variable, the resulting mathematical model with an identical container delivery pattern is equivalent to the model with general container delivery patterns which can be different in different weeks. When the number of containers transported is formulated as an integer variable, the model with an identical container delivery pattern is not equivalent to the model with general container delivery patterns. However, the difference between the optimal objective values of the two models is negligible for practical applications. In sum, little, if not nothing, is lost by assuming an identical container delivery pattern in liner shipping planning models.

Key Words: liner shipping planning; space-time network; container delivery pattern

1 Introduction

Global container transportation is vital to international trade (Qu et al., 2011; Qu and Meng, 2012). As reported by UNCTAD (2015), containerized trade volumes reached 171 million twenty-foot equivalent units (TEUs) in 2014. Containerized cargos include fruits, meat, dairy products, electronics, clothes, appliances, etc., which generally have a high unit value. Assuming the cargo value of €40,000/TEU (Notteboom, 2006), the total value of containerized cargos transported in 2014 was €6.8 trillion.

Containers between ports are transported by ships deployed on liner services operated by shipping lines (Du et al., 2011; Yang et al., 2014; Zhang and Lam, 2015; Zhen, 2015; Lindstad et al., 2016). A liner service has a fixed port rotation and schedule, and transports containers on a regular (usually weekly) basis (Zheng et al., 2015; Liu et al., 2016a), similar to public transport services (Liu et al., 2016b). Fig. 1 shows a trans-Pacific service named Central China 1 (CC1) operated by Orient Overseas Container Line (OOCL, 2014). The service has a fixed sequence of ports of call and a fixed arrival day in each week at each port of call. Ships deployed on the service have to adhere to the announced port rotation and schedule. OOCL may redesign or alter a service every three to six months.

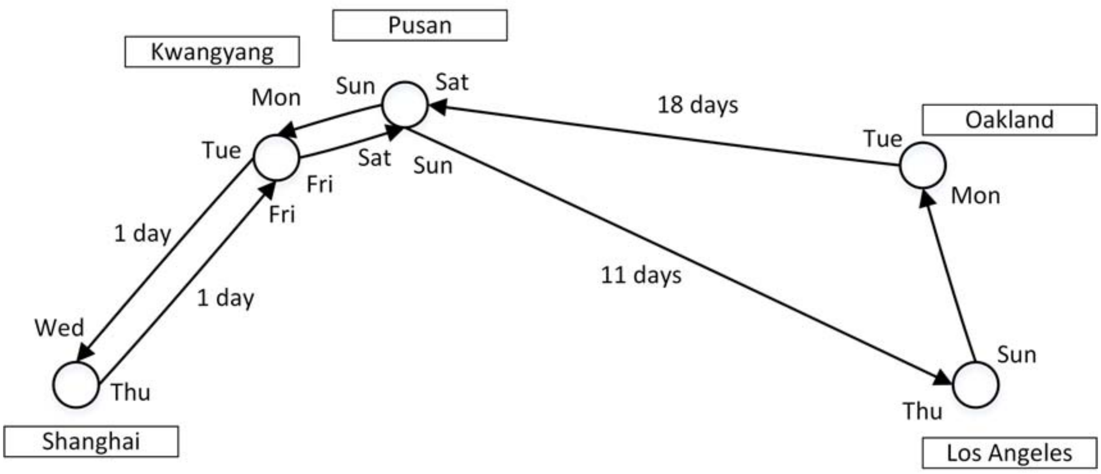

Fig. 1 CC1 service by OOCL (2014)

Unlike tramp shipping where ships are operated in reaction to known cargo demand and a tramp company handles a very small number of cargos, a liner company operates many regular services that form a shipping network to transport containers for a large number of customers. Since the liner services are fixed, they must be designed based on estimated cargo demand. The quality of the liner services determines to a large extent the profitability of a shipping line (Feng and Chang, 2008; Ng and Kee, 2008; Du et al., 2015; Zhen et al., 2016).

Liner service design is especially significant in the present industrial situation. On one hand, the sizes of container ships are becoming larger and larger. For example, in 2012, the largest containerships with a capacity of 18,300 TEUs were delivered and put to service by Maersk Line; later, China Shipping Container Lines (CSCL) and Mediterranean Shipping Company (MSC) deployed container ships that could carry over 19,000 TEUs. On the other hand, shipping companies have formed alliances to further lower the operating costs, such as the 2M Alliance, G6 Alliance, Ocean Three Alliance, and CKYHE Alliance, and started a new round of merging, such as the pending acquisition of Neptune Orient Lines by CMA CGM and the potential merging of CSCL and China Ocean Shipping (Group) Company (COSCO).

Existing planning models for liner services of a shipping company usually consider a shipping network consisting of services with weekly frequencies. The weekly cargo demand between two ports is given and unchanged in the planning period. The begin-of-period and end-of-period effects are not considered, which is equivalent to assuming that the services and the given weekly cargo demand will persist for an infinite number of weeks. The planning models are mainly concerned with the following questions. (i) Container routing (Bell et al., 2013; Liu et al., 2014; Lu et al., 2016): what is the minimum cost for fulfilling the cargo demand in a given network? Note that in a liner network there are many possible routes to transport containers from their origin ports to their destination ports. They can be transported on one service, or transported on more than one service by transshipment at the common port on the services. Container routing is used for analyzing the profitability of the liner network and providing information on how to improve the network. (ii) Network design (Zheng et al., 2014): for example, in Fig. 1 should Kwangyang be visited on CC1, or excluded and connected to Pusan by a feeder service? (iii) Fleet deployment (Fagerholt et al., 2009; Meng and Wang, 2011; Ng, 2014, 2015): what is the suitable size of ship to deploy on each service? It should be mentioned that container routing is a sub-problem in network design and fleet deployment, because when the network is changed or different ships are deployed, containers should be transported in a different manner.

Although not mentioned, the existing studies have actually assumed that how the containers are transported in a network (the container delivery pattern) is not changed every week because the services are the same and the cargo demand is also the same every week. We refer to Christiansen et al. (2013) and Meng et al. (2014) for comprehensive reviews on relevant studies. We notice that because containerships operate on a 24/7 basis, the container routing decision in one week may affect the decisions in the following weeks. For example, we define Sunday as the first day of a week. On the service in Fig. 1, if a full shipload of containers is transported on a ship from Kwangyang to Los Angeles in a particular week, then when the ship visits Pusan on Sunday in the next week, it could not carry any container from Pusan to Oakland. That is, the number of containers transported from Kwangyang to Los Angeles in one week affects the ship capacity to carry containers from Pusan to Oakland in the next week, whereas the number of containers transported from Kwangyang to Los Angeles in one week does not affect the capacity to carry containers from Pusan to Oakland in the same week because these two batches of containers are transported on different ships. If the container delivery pattern in a week is not affected by the patterns in previous weeks, then of course the container delivery patterns in different week should be the same because each week the same decision problem (same network and demand) is faced and the same optimal solution minimizes the total cost. As the above example shows that the container delivery pattern in a week is affected by the patterns in previous weeks, a natural question is whether there is still an optimal container routing decision with the same container delivery pattern every week. If the answer is yes, then practitioners can be confident to use the existing models; if the answer is no, then a careful examination of the existing models is deserved. The objective of this paper is to answer this fundamental question related to nearly all liner shipping planning models.

The remainder of the paper is organized as follows. Section 2 describes the container routing problem. Section 3 formulates a model with an identical container delivery pattern in each week, and a model with general container delivery patterns where the patterns may be different in different weeks. Section 4 examines whether the model with an identical container delivery pattern can obtain the same optimal solution as the model with general patterns. Section 5 presents illustrative examples. Section 6 summarizes the findings of the study.

2 Container routing problem

2.1 Liner network

A liner shipping company operates a set of service routes (SRs, services, or routes) $\mathcal{R}$ to transport containers between a group of ports denoted by the set $\mathcal{P}$. Each service has a fixed port rotation visited by a string of homogeneous ships and provides a weekly frequency. An illustrative liner network is shown in Fig. 2, including three routes denoted by $\mathcal{R} = \{1, 2, 3\}$. Let N_r represent the number of ports of call on a rotation of route r and $p_{ri} \in \mathcal{P}$ be the physical port corresponding to its i^{th} port of call. We can arbitrarily define one port of call as the first one, e.g., in Fig. 2 Kaohsiung is defined to be the first port of call on SR1, Surabaya and Singapore are the second and third ports of call, respectively, and the number of ports of call on SR1 $N_{r_1} = 3$. It should be mentioned that although Singapore is visited twice during a round trip of SR2, these two calls can easily be differentiated by using the port calling sequence to refer to a port of call. Let $\mathcal{I}_r = \{1, 2, \cdots, N_r\}$ be the set of all the ports of call on route r. Defining $p_{r,N_r+1} = p_{r1}$, the voyage from port of call p_{ri} to $p_{r,i+1}$ on route r is called leg i of the route, $i \in \mathcal{I}_r$.

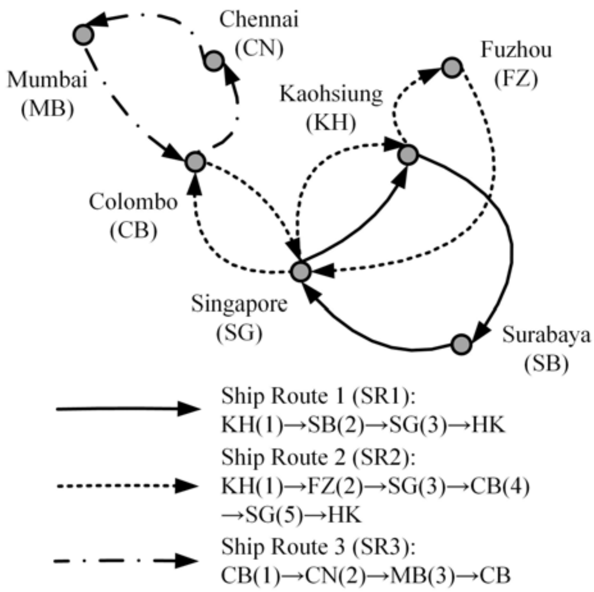

Fig. 2 A liner network consisting of three service routes

2.2 Weekly frequency and ship deployment

Each route has a fixed service schedule, i.e., the arrival days at each port of call in a week, in order to provide consistent weekly shipping services. Table 1 shows the schedules of the three routes. The rotation time of ship route 1 (SR1) (i.e., the total time a ship spends from arriving at the 1st port of call, visiting all the ports of call on the route, and returning to the 1st port of call) is 2 weeks. Hence, two ships must be deployed on SR1 to maintain a weekly frequency. Similarly, four ships are deployed on ship route 2 (SR2) and one ship is deployed on SR3. The container capacity of a ship deployed on route r is E_r (TEUs).

Table 1 Schedules of the three routes

		SR1			SR2			SR3
ID	Port	Arrival time (h)	ID	Port	Arrival time (h)	ID	Port	Arrival time (h)
1	KH	10 (10:00 Sun)	1	KH	0 (00:00 Sun)	1	CB	0 (00:00 Sun)
2	SB	188 (20:00 Sun)	2	FZ	66 (18:00 Tue)	2	CN	60 (12:00 Tue)
3	SG	218 (02:00 Tue)	3	SG	238 (22:00 Tue)	3	MB	130 (10:00 Fri)
1	KH	346 (10:00 Sun)	4	CB	386 (02:00 Tue)	1	CB	168 (00:00 Sun)
			5	SG	514 (10:00 Sun)			
			1	KH	672 (00:00 Sun)			

2.3 Container delivery

To evaluate the quality of the liner network $\mathcal{R}$, the shipping line needs to estimate the future cargo demand and analyze the minimum total cost for fulfilling the demand. Let $\mathcal{W} \in \mathcal{R} \times \mathcal{R}$ be the set of origin-destination (OD) pairs with positive demand. The demand of $(o,d) \in \mathcal{W}$ is denoted by $q^{od} > 0$ (TEUs/week). The q^{od} containers are available to transport at the beginning of each week and must be loaded onto a ship within one week. Some containers may be dropped and the penalty cost for one unfulfilled TEU from (o,d) is $c^{od} > 0$ ($/TEU).

To transport the containers, the liner shipping company designs a set of itineraries represented by $\mathcal{H}^{od}$ for each $(o,d) \in \mathcal{W}$. Let $\mathcal{H} = \bigcup_{(o,d)\in\mathcal{W}} \mathcal{H}^{od}$. An itinerary $h \in \mathcal{H}$ contains all information on how containers are transported. For example, a possible itinerary in Fig. 2 for transporting containers from FZ to CN is defined as:

+FZ-SR2(2,4)+CB-SR3(1,2)+CN (1)

where containers are loaded at Fuzhou port onto ships deployed on SR2, and transported to Colombo port. At Colombo, these containers are discharged and reloaded onto ships deployed on SR3, and subsequently delivered to Chennai port and discharged. Therefore, these containers originate from Fuzhou, are destined for Chennai, and transshipped at Colombo. The numbers 2 and 4 in the parentheses of "SR2(2,4)" stand for the ports of call indices of the load port and discharge port on this route, respectively. That is, containers are loaded at the 2nd port of call (FZ) and discharged at the 4th port of call (CB) on SR2. These additional notations can avoid the ambiguity caused by double calls at the same port. For example, "+FZ-SR2(2,3)+SG" and "+FZ-SR2(2,5)+SG" refer to different itineraries.

An itinerary consists of legs. For instance, the itinerary in (1) consists of legs 2 and 3 of SR2 and leg 1 of SR3. Different itineraries may contain the same leg. We hence define a binary parameter δ_{ri}^{h} which equals 1 if itinerary h contains leg i of route r, and 0 otherwise. We further let c_h ($/TEU) be the total handling cost for one TEU transported on itinerary $h \in \mathcal{H}$. For instance, c_h for the itinerary in (1) is the sum of loading cost at FZ, transshipment cost at CB, and discharge cost at CN.

Given sets $\mathcal{R}$, $\mathcal{P}$, $\mathcal{W}$ and $\mathcal{H}$, and parameters q^{od}, c^{od}, δ_{ri}^{h} and c_h, the container routing problem aims to determine the optimal weekly container delivery pattern, i.e., the number of containers to transport on each itinerary, to minimize the sum of handling cost and penalty cost.

3 Planning models with identical and general weekly container delivery patterns

3.1 Model with an identical container delivery pattern

The existing studies assume implicitly that the container delivery patterns in different weeks are identical. Define y_h (TEUs/week) as the decision variable that represents the number of containers transported on itinerary $h \in \mathcal{H}$. Denote by $\mathbb{Z}_+$ the set of nonnegative integers. The container routing model is:

[Identical-Integer] $$\min_{y_h} \sum_{h \in \mathcal{H}} c_h y_h + \sum_{(o,d) \in \mathcal{W}} c^{od} \left(q^{od} - \sum_{h \in \mathcal{H}^{od}} y_h \right) \tag{2}$$

subject to:

$$\sum_{h \in \mathcal{H}} \delta_{ri}^{h} y_h \le E_r, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{3}$$

$$\sum_{h \in \mathcal{H}^{od}} y_h \le q^{od}, \forall (o,d) \in \mathcal{W} \tag{4}$$

$$y_h \in \mathbb{Z}_+, \forall h \in \mathcal{H} \tag{5}$$

The objective function (2) minimizes the sum of container handling cost and penalty cost for dropping demand. Constraints (3) impose the ship capacity constraint on each leg of each route, where the left-hand side is the total number of containers transported on leg i of route r calculated from the container delivery pattern $(y_h, h \in \mathcal{H})$. Constraints (4) enforce that the number of containers delivered for each OD pair cannot exceed the demand. Constraints (5) define that the number of containers delivered on each itinerary is a nonnegative integer.

The cargo demand q^{od} cannot be estimated without error, and rounding down e.g. $y_h = 84.3$ to $y_h = 84$ is generally acceptable in terms of precision. Hence, most studies formulate y_h as a continuous variable, leading to a model named [Identical-Continuous], which is the same as [Identical-Integer] except that constraints (5) are replaced by

$$y_h \ge 0, \forall h \in \mathcal{H} \tag{6}$$

We let $OBJ(\bullet)$ represent the optimal objective value of a model. It is easy to see that

$$OBJ(\text{Identical-Continuous}) \leq OBJ(\text{Identical-Integer}) \tag{7}$$

The inequality in Eq. (7) can be strict, i.e., [Identical-Continuous] has no optimal solution that is integral, as will be shown by Example 1 in Section 5. Wang (2013) showed that in practical cases the difference between the two optimal objective values is usually less than 0.01%.

3.2 Model with general container delivery patterns

It can be seen that, although not mentioned, all the above two models—[Identical-Integer] and [Identical-Continuous]—have implicitly assumed that the container delivery pattern $(y_h, h \in \mathcal{H})$ is identical every week. To the best of our knowledge, there is no study examining whether the optimality of container routing can be guaranteed after imposing the implicit constraint of the identical container delivery pattern every week. We therefore develop a model for container routing with general container delivery patterns, where the patterns can be different in different weeks.

In this section, we formulate general container delivery patterns, i.e., the container delivery pattern in one week may be different from another week. Note that the sets of itineraries for both the case with identical delivery patterns and the case with general delivery patterns are identical for the following reason. When delivering a container from its origin to its destination, not all itineraries that connect the origin to the destination are feasible. In practice, a company will set a maximum allowable transit time. For example, the maximum allowable transit time for a container to be delivered from Norfolk to Rotterdam is smaller than 60 days because a customer will be very unhappy if the delivery time is so long. Given the maximum allowable transit time, the set of itineraries is finite and the same for both the case with identical delivery patterns and the case with general delivery patterns.

To formulate general container delivery patterns, we need to build a space-time network. The details of how to build the space-time network is presented in Wang et al. (2016). To make the paper self-contained, we briefly repeat the key procedure. We stress that the construction of the space-time network is a contribution of Wang et al. (2016), but not a contribution of this research. An example of the space-time network for the liner network in Fig. 2 is shown in Fig. 3 with a total of four weeks of the shipping services (168, 336, 504 are the number of hours in one, two, three weeks, respectively). The liner shipping network in Fig. 2 has a total of $N_{r_1} + N_{r_2} + N_{r_3} = 3 + 5 + 3 = 11$ ports of call. Hence, the space-axis is divided into 11 segments, each representing a port of call. We plot four copies for each port of call reflecting four weeks, leading to 44 nodes in the space-time network. Fig. 3 shows the IDs of a few nodes. Because the rotation time of SR1 is two weeks, two ships (Ship 1 and Ship 2) are deployed on SR1. Similarly, four ships (Ships 3 to 6) and one ship (Ship 7) are deployed on SR2 and SR3, respectively. Containers may be transshipped at ports visited more than once in a week, including Kaohsiung, Singapore, and Colombo. The thick solid lines in Fig. 3 represent transshipment operations. The liner services in any two weeks are identical due to the weekly frequency.

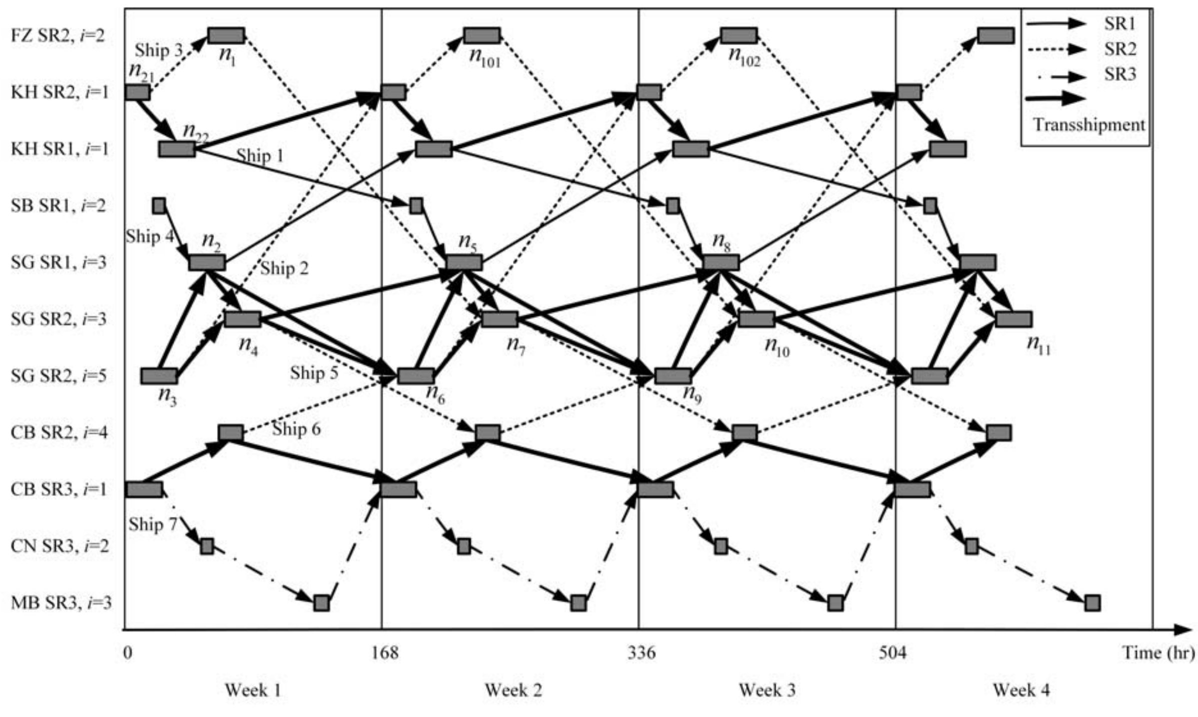

Fig. 3 Space-time network model of liner services

The voyages from node n_{011} to n_{062}, from n_{012} to n_{063}, and from n_{013} to n_{064} all represent the 2nd leg of SR2. To differentiate them, we could add a time index t that represents the week in which the tail node of a leg is visited. Under such a definition, the voyage from node

n_{011} to n_{062} is the 2[nd] leg of SR2 in week 1, from n_{012} to n_{063} is the leg in week 2, and from n_{013} to n_{064} is the leg in week 3.

Similarly, itineraries are also identified by the week in which the origin port is visited. Consider the cargo demand from FZ to CN in week 1, that is, the containers that are available at FZ at the beginning of week 1 and must be loaded onto a ship before the end of week 1. The shipping line can transport the containers on the itinerary in (1), which corresponds to visiting nodes n_{011}, n_{062}, n_{083}, n_{094} and n_{104}, sequentially. We call this itinerary an itinerary in week 1. It uses leg 2 of SR2 in week 1, leg 3 of SR2 in week 2, and leg 1 of SR3 in week 4.

Containers in different weeks may use the same leg. Consider the following itinerary for the demand of (SG, CB):

$$\text{+SG-SR2(3,4)+CB} \tag{8}$$

Containers for (SG, CB) in week 2 transported on the itinerary visit nodes n_{062} and n_{083}. Therefore, they are transported on the same leg in the same week, namely, leg 3 of SR2 in week 2, and thereby on the same ship as the containers for (FZ, CN) in week 1. By contrast, containers for (SG, CB) in week 1 are not transported on the same ship as containers for (FZ, CN) in week 1.

Assume that the longest itinerary in $\mathcal{H}$ is Γ weeks, that is, a container will arrive at its destination by the end of week $\Gamma+1$ if it is loaded onto a ship in week 1. Hence, an itinerary in week t may contain legs in weeks $t+\tau$, $\tau = 0,1,2\cdots\Gamma$. We thereby define a binary parameter $\delta_{ri}^{h\tau}$ which equals 1 if and only if itinerary h in week t contains leg i of ship route r in week $t+\tau$, $\tau = 0,1,2\cdots\Gamma$. Note that an itinerary contains a leg at most once. Hence,

$$\sum_{\tau=0}^{\Gamma} \delta_{ri}^{h\tau} = \delta_{ri}^{h}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall h \in \mathcal{H} \tag{9}$$

If we consider a total of T weeks, and let y_h^t be the number of containers transported on itinerary h in week t, the minimum average total cost per week when T approaches infinity can be calculated by:

$$\text{[General-Integer]} \quad \lim_{T\to+\infty} \min_{y_h^t} \frac{1}{T} \sum_{t=1}^{T} \left[\sum_{h\in\mathcal{H}} c_h y_h^t + \sum_{(o,d)\in\mathcal{W}} c^{od} \left(q^{od} - \sum_{h\in\mathcal{H}^{od}} y_h^t \right) \right] \tag{10}$$

subject to:

$$\sum_{\tau=0}^{\Gamma}\sum_{h\in\mathcal{H}}\delta_{ri}^{h\tau}y_h^{t-\tau}\le E_r,\forall r\in\mathcal{R},\forall i\in\mathcal{I}_r,\forall t=1,2\cdots T \tag{11}$$

$$\sum_{h\in\mathcal{H}^{od}}y_h^t\le q^{od},\forall(o,d)\in\mathcal{W},\forall t=1,2\cdots T \tag{12}$$

$$y_h^t\in\mathbb{Z}_+,\forall h\in\mathcal{H},\forall t=1,2\cdots T \tag{13}$$

where in Eq. (11) we define $y_h^t=0$ if $t\le 0$.

Regarding the above model, we have the following proposition:

Proposition 1: The limit in Eq. (10) exists.

Proof: Define

$$\bar{C}(T):=\min_{y_h^t}\frac{1}{T}\sum_{t=1}^{T}\left[\sum_{h\in\mathcal{H}}c_h y_h^t+\sum_{(o,d)\in\mathcal{W}}c^{od}\left(q^{od}-\sum_{h\in\mathcal{H}^{od}}y_h^t\right)\right]$$

Given $T_1<T_2$, in the optimal solution to the above model with $T=T_2$ we have the container delivery patterns for T_2 consecutive weeks. From these patterns we can identify a total of T_2-T_1 possibilities of container delivery patterns for T_1 consecutive weeks. Any of the T_2-T_1 possibilities is a feasible solution to the above model with $T=T_1$, and the best possibility's average cost per week is not higher than that of the container delivery patterns for T_2 consecutive weeks. Therefore, $\bar{C}(T_1)\le\bar{C}(T_2)$.

Since $\bar{C}(T)$ is monotonically increasing and has an upper bound $\sum_{(o,d)\in\mathcal{W}}c^{od}q^{od}$ that is obtained by dropping all of the containers, the limit in Eq. (10) exists. □

The model where the numbers of containers transported are formulated as continuous variables, represented by [General-Continuous], can be formulated similarly by relaxing the integrality constraints in (13):

$$y_h^t\ge 0,\forall h\in\mathcal{H},\forall t=1,2\cdots T \tag{14}$$

Similar to Eq. (7), we have

$$OBJ\,(\text{General-Continuous})\le OBJ\,(\text{General-Integer}) \tag{15}$$

4 Comparison between identical and general container delivery patterns

Let $(y_h^*, h \in \mathcal{H})$ be an optimal solution to model [Identical-Integer] (or [Identical-Continuous]), then $(y_h^t = y_h^*, h \in \mathcal{H}, t = 1, 2 \cdots T)$ is a feasible solution to [General-Integer] (or [General-Continuous]), and the resulting objective value is the same as the optimal objective value of [Identical-Integer] (or [Identical-Continuous]). Therefore,

Lemma 1: The model [General-Integer] (or [General-Continuous]) is more general than [Identical-Integer] (or [Identical-Continuous]) in that:

$$OBJ(\text{General-Integer}) \leq OBJ(\text{Identical-Integer}) \tag{16}$$

$$OBJ(\text{General-Continuous}) \leq OBJ(\text{Identical-Continuous}) \tag{17}$$

Lemma 1 is actually the result that in [General-Integer] (or [General-Continuous]), we relax the identical container delivery pattern in [Identical-Integer] (or [Identical-Continuous]) by allowing it to vary with weeks.

4.1 Comparison between [Identical-Integer] and [General-Integer]

When the number of containers transported on an itinerary is formulated as an integer variable, the two models [Identical-Integer] and [General-Integer] may give different results, as will be shown by Example 2 in Section 5:

Theorem 1: The optimal objective value of model [General-Integer] may be strictly smaller than that of model [Identical-Integer]. That is, the inequality in Eq. (16) may be strict.

4.2 Comparison between [General-Integer] and [General-Continuous]

As will be shown by Example 3 in Section 5, we have:

Theorem 2: The optimal objective value of model [General-Continuous] may be strictly smaller than that of model [General-Integer]. That is, the inequality in Eq. (15) may be strict.

4.3 Comparison between [Identical-Continuous] and [General-Continuous]

When the number of containers transported on an itinerary is formulated as a continuous variable, whether the two models [Identical-Continuous] and [General-Continuous] could have different optimal objective values is more challenging to examine. Let $(y_h^{t*}, h \in \mathcal{H}, t = 1,2\cdots T)$ be an optimal solution to model [General-Continuous] for a given T. It may be the case that when T approaches infinity, $(y_h^{t*}, h \in \mathcal{H}, t = 1,2\cdots T)$ has a repetitive weekly container delivery pattern every N weeks after a sufficiently large number of weeks. That is, there exists a $T_1 \geq 1$ such that $y_h^{t*} = y_h^{t+N,*}, h \in \mathcal{H}$, for any $t \geq T_1$. For instance, the weekly container delivery pattern in the solution to [General-Integer] in Example 2 is repetitive with $T_1 = 1$ and $N = 2$. We have

Lemma 2: When T approaches infinity, if $(y_h^{t*}, h \in \mathcal{H}, t = 1,2\cdots T)$ has a repetitive weekly container delivery pattern every N weeks after a given number of T_1 weeks, then the optimal objective value of [Identical-Continuous] is not greater than that of [General-Continuous].

Proof: As T approaches infinity in [General-Continuous] and y_h^{t*} is finite, we have

$$\lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T_1} \left[\sum_{h \in \mathcal{H}} c_h y_h^{t*} + \sum_{(o,d) \in \mathcal{W}} c^{od} \left(q^{od} - \sum_{h \in \mathcal{H}^{od}} y_h^{t*} \right) \right] = 0 .$$

We can thus ignore the costs in the first $T_1 - 1$ weeks without changing the optimal objective value. Then the optimal objective value of [General-Continuous] is

$$OBJ(\text{General-Continuous}) = \frac{1}{N} \sum_{t=T_1}^{T_1+N-1} \left[\sum_{h \in \mathcal{H}} c_h y_h^{t*} + \sum_{(o,d) \in \mathcal{W}} c^{od} \left(q^{od} - \sum_{h \in \mathcal{H}^{od}} y_h^{t*} \right) \right] \quad (18)$$

Now we define a solution $(\bar{y}_h, h \in \mathcal{H})$ to [Identical-Continuous]:

$$\bar{y}_h = \frac{1}{N} \sum_{t=T_1}^{T_1+N-1} y_h^{t*} = \frac{1}{N} \sum_{t=T_1+N}^{T_1+2N-1} y_h^{t*}, \forall h \in \mathcal{H} \quad (19)$$

It is easy to see that if the solution $(\bar{y}_h, h \in \mathcal{H})$ is feasible, then the resulting objective value of [Identical-Continuous] is the same as Eq. (18). Evidently, $(\bar{y}_h, h \in \mathcal{H})$ satisfies constraints (6). We now prove that it also satisfies constraints (4) and (3).

As $(y_h^{t*}, h \in \mathcal{H}, t = 1,2\cdots T)$ is a feasible solution to [General-Continuous], constraints (12) imply

$$\sum_{h \in \mathcal{H}^{od}} y_h^{t*} \leq q^{od}, \forall (o,d) \in \mathcal{W}, \forall t = t = T_1 + N, T_1 + N + 1 \cdots T_1 + 2N - 1 \quad (20)$$

Adding up the N inequalities according to OD pairs, we have

$$\sum_{t=T_1+N}^{T_1+2N-1} \sum_{h \in \mathcal{H}^{od}} y_h^{t*} \le N \bullet q^{od}, \forall (o,d) \in \mathcal{W} \tag{21}$$

Exchanging the two summations, we have

$$\sum_{h \in \mathcal{H}^{od}} \frac{\sum_{t=T_1+N}^{T_1+2N-1} y_h^{t*}}{N} \le q^{od}, \forall (o,d) \in \mathcal{W} \tag{22}$$

Hence, $(\bar{y}_h, h \in \mathcal{H})$ defined by (19) satisfies constraints (4).

Similar to constraints (12), constraints (11) can be written as

$$\sum_{\tau=0}^{\Gamma} \sum_{h \in \mathcal{H}} \delta_{ri}^{h\tau} y_h^{t-\tau,*} \le E_r, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall t = T_1+N, T_1+N+1 \cdots T_1+2N-1 \tag{23}$$

Adding up the N inequalities according to legs, we have

$$\sum_{t=T_1+N}^{T_1+2N-1} \sum_{\tau=0}^{\Gamma} \sum_{h \in \mathcal{H}} \delta_{ri}^{h\tau} y_h^{t-\tau,*} \le N \bullet E_r, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{24}$$

Exchanging the three summations, we have

$$\sum_{h \in \mathcal{H}} \sum_{\tau=0}^{\Gamma} \delta_{ri}^{h\tau} \frac{\sum_{t=T_1+N}^{T_1+2N-1} y_h^{t-\tau,*}}{N} \le E_r, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{25}$$

That is,

$$\sum_{h \in \mathcal{H}} \sum_{\tau=0}^{\Gamma} \delta_{ri}^{h\tau} \bar{y}_h \le E_r, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{26}$$

Eqs. (9) and (26) imply $(\bar{y}_h, h \in \mathcal{H})$ defined by (19) satisfies constraints (3). □

It is possible that when T approaches infinity, no repetitive weekly container delivery pattern exists in an optimal solution to model [General-Continuous]. For example, consider the liner network in Fig. 2 with the following parameters: $\mathcal{W} = \{(\text{SG, HK})\}$, $q^{\text{SG, HK}} = 1$, $c_h = 0$, $c^{od} = 1$ for all $(o,d) \in \mathcal{W}$, and $E_{r_1} = E_{r_2} = E_{r_3} = 1$. A possible optimal solution $(y_h^{t*}, h \in \mathcal{H}, t = 1, 2 \cdots T)$ to [General-Continuous] is:

$$\begin{bmatrix} t & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \cdots \\ SR1 & 0.9 & 0.9 & 0.99 & 0.9 & 0.99 & 0.999 & 0.9 & 0.99 & 0.999 & 0.9999 & \cdots \\ SR2 & 0.1 & 0.1 & 0.01 & 0.1 & 0.01 & 0.001 & 0.1 & 0.01 & 0.001 & 0.0001 & \cdots \end{bmatrix} \quad (27)$$

where each column represents a week, the second row is the number of containers transported on SR1 and the third row is the number of containers transported on SR1. □

Lemma 3: When T approaches infinity, if the optimal solution $(y_h^{t*}, h \in \mathcal{H}, t = 1,2\cdots T)$ to model [General-Continuous] has no repetitive weekly container delivery pattern, then for any small positive number $\varepsilon > 0$, we can find a solution with a repetitive weekly container delivery pattern, whose objective value is greater than OBJ(General-Continuous) by at most $\varepsilon \bullet |\mathcal{H}| \bullet \max\{c^{od}, (o,d) \in \mathcal{W}\}$.

Proof: We define the a new solution $(\tilde{y}_h^t, h \in \mathcal{H}, t = 1,2\cdots T)$ as follows:

$$\tilde{y}_h^t := \varepsilon \bullet \left\lfloor \frac{y_h^{t*}}{\varepsilon} \right\rfloor, \forall h \in \mathcal{H}, \forall t = 1,2\cdots T \quad (28)$$

where $\lfloor x \rfloor$ means the largest integer not greater than x. For example, if $\varepsilon = 0.01$ and $(y_h^{t*}, h \in \mathcal{H}, t = 1,2\cdots T)$ is defined in Eq. (27), then $(\tilde{y}_h^t, h \in \mathcal{H}, t = 1,2\cdots T)$ is

$$\begin{bmatrix} t & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \cdots \\ SR1 & 0.9 & 0.9 & 0.99 & 0.9 & 0.99 & 0.99 & 0.9 & 0.99 & 0.99 & 0.99 & \cdots \\ SR2 & 0.1 & 0.1 & 0.01 & 0.1 & 0.01 & 0 & 0.1 & 0.01 & 0 & 0 & \cdots \end{bmatrix} \quad (29)$$

As $\tilde{y}_h^t$ is nonnegative and not greater than y_h^{t*}, all the constraints (11)–(12) and (14) are satisfied. Hence, $(\tilde{y}_h^t, h \in \mathcal{H}, t = 1,2\cdots T)$ is feasible to [General-Continuous]. As at most ε TEUs are removed from each itinerary, there are $|\mathcal{H}|$ itineraries in total, and the penalty cost for one TEU is at most $\max\{c^{od}, (o,d) \in \mathcal{W}\}$, the objective value of $(\tilde{y}_h^t, h \in \mathcal{H}, t = 1,2\cdots T)$ is greater than that of $(y_h^{t*}, h \in \mathcal{H}, t = 1,2\cdots T)$ by at most $\varepsilon \bullet |\mathcal{H}| \bullet \max\{c^{od}, (o,d) \in \mathcal{W}\}$. □

We define

$$Q := \max\left\{ \left\lfloor q^{od} / \varepsilon \right\rfloor, (o,d) \in \mathcal{W} \right\} \quad (30)$$

Since $\tilde{y}_h^t \le y_h^{t*} \le \max\{q^{od},(o,d)\in \mathcal{W}\}$, $\tilde{y}_h^t$ equals ε multiplied by an integer between 0 and Q. Now we look at another model named [General-Discretized], which is [General-Continuous] plus the following constraints:

$$y_h^t = \varepsilon \times \text{an integer between 0 and } Q, \forall h \in \mathcal{H}, \forall t = 1,2\cdots T \tag{31}$$

Hence, $(\tilde{y}_h^t, h \in \mathcal{H}, t = 1,2\cdots T)$ is a feasible solution to [General-Discretized], and we only need to prove that [General-Discretized] has an optimal solution with a repetitive weekly container delivery pattern as any feasible solution to [General-Discretized] is also feasible to [General-Continuous]. In [General-Discretized], the possible values of $(y_h^t, h \in \mathcal{H})$ is finite for any $t = 1,2\cdots T$, which is less than $|\mathcal{H}|^{Q+1}$. Moreover, the possible values of container delivery patterns for $\Gamma+1$ consecutive weeks $(y_h^{t+\tau}, h \in \mathcal{H}, \tau = 0,1,2\cdots\Gamma)$ is less than $(|\mathcal{H}|^{Q+1})^{\Gamma+1}$. Note that for a given week $\hat{T}$, the feasibility of the container delivery patterns in weeks after $\hat{T}+\Gamma$, i.e., $(y_h^{t*}, h \in \mathcal{H}, t = \hat{T}+\Gamma+1, \hat{T}+\Gamma+2\cdots T)$ is only related to the container delivery patterns in weeks between $\hat{T}$ and $\hat{T}+\Gamma$ (inclusive), i.e., $(y_h^{t*}, h \in \mathcal{H}, t = \hat{T}, \hat{T}+1\cdots\hat{T}+\Gamma)$, and is not related to the container delivery patterns in weeks before $\hat{T}$, i.e., $(y_h^{t*}, h \in \mathcal{H}, t = 1,2\cdots\hat{T}-1)$. We thereby define the container delivery patterns for $\Gamma+1$ consecutive weeks as a super-pattern. The feasibility of a super-pattern in a solution thus only depends on its previous super-pattern in the solution. This forms an optimality condition of the problem. As the number of different super-patterns does not exceed $(|\mathcal{H}|^{Q+1})^{\Gamma+1}$, we can enumerate all possible permutations where a super-pattern appears only once. For example, assuming that there are only three feasible super-patterns A, B and C, then there are 15 possible permutations without repeating a super-pattern:

$$\begin{array}{lll} A & B & C \\ AB & BA & CA \\ AC & BC & CB \\ ABC & BAC & CAB \\ ACB & BCA & CBA \end{array} \tag{32}$$

We need to check whether a permutation can be repeated, i.e., whether repeating the permutation is feasible to [General-Discretized]. For example, to check whether ACB can be repeated, we can let $T = 4(\Gamma+1)$, and check whether constraints (11) hold for the solution ACBA (in fact, we should check the solution ACBACBACBACB… However, because the feasibility of a super-pattern only depends on its previous super-pattern, we only need to append a super-pattern "A" to ACB, and that is why T is 4 times $\Gamma+1$). For all feasible permutations, we can calculate the average total cost per week and an optimal solution can be obtained by repeating the minimum cost permutation. This completes the proof of the lemma.

We use Example 2 and Eq. (29) to illustrate Lemma 3. For simplicity, we assume that $\Gamma=0$, i.e., a super-pattern consists of only one week's container delivery pattern. We further assume that there are only three super-patterns:

$$A=\begin{pmatrix}0.9\\0.1\end{pmatrix}, B=\begin{pmatrix}0.99\\0.01\end{pmatrix}, C=\begin{pmatrix}0.99\\0\end{pmatrix} \tag{33}$$

We can see that all the permutations in Eq. (32) are feasible to constraints (11). The total cost of repeating the permutation "A", i.e., AAAAAA…, is 0, as no container is rejected; the total cost of repeating the permutation "AB", i.e., ABABABAB…, is 0; the total cost of repeating the permutation "ACB", i.e., ACBACBACBACB…, is 0.01/3 as in super-pattern "C" 0.01 TEU is unfulfilled. After completing the calculation for all the permutations, we find an optimal solution ABABABAB…, which has a repetitive container delivery pattern AB.

Let us look at Lemma 3 from another viewpoint. Suppose that we have an optimal solution represented by super-patterns A, B, C and D as follows:

$$\underbrace{A}_{1}BBC\underbrace{\underbrace{A}_{5}DCABDACCCDDDACBD\cdots}_{\text{optimal after super-pattern "A"}} \tag{34}$$

Then the optimality condition implies that whenever super-pattern "A" appears, the partial solution below that follows "A" is optimal as this partial solution appears after the 1st "A" in the optimal solution (34):

$$BBCADCABDACCCDDDACBD\cdots \tag{35}$$

Hence, we can replace the partial solution after the "A" in the 5th position of Eq. (34) by Eq. (35), and obtain another optimal solution:

$$\underbrace{A}_{1}BBC\underbrace{A}_{5}\underbrace{BBC\underbrace{A}_{9}DCABDACCCDDDACBD\cdots\cdots}_{\text{new components of the optimal solution}} \tag{36}$$

We can further replace the partial solution after the "A" in the 9th position of Eq. (36) by Eq. (35). Repeating the above process, we obtain an optimal solution with a repetitive container delivery pattern ABBC:

$$\underbrace{A}_{1}BBC\underbrace{A}_{5}BBC\underbrace{A}_{9}BBCABBCABBCABBCABBC\cdots \tag{37}$$

If we let ε approach 0 in Lemma 3, we have

Lemma 4: When T approaches infinity, there exists an optimal solution $(y_h^{t*}, h \in \mathcal{H}, t = 1,2\cdots T)$ to model [General-Continuous] with a repetitive weekly container delivery pattern.

Lemma 1, Lemma 2 and Lemma 4 indicate:

Theorem 3: The optimal objective value of [Identical-Continuous] is the same as that of [General-Continuous]:

$$OBJ(\text{General-Continuous}) = OBJ(\text{Identical-Continuous}) \tag{38}$$

Eqs. (15), (16) and (38) imply that the difference between $OBJ(\text{Identical-Integer})$ and $OBJ(\text{General-Integer})$ is not greater than that between $OBJ(\text{Identical-Integer})$ and $OBJ(\text{Identical-Continuous})$. As Wang (2013) showed that in practical cases the difference between $OBJ(\text{Identical-Integer})$ and $OBJ(\text{Identical-Continuous})$ is usually less than 0.01%, we can conclude that the difference between $OBJ(\text{Identical-Integer})$ and $OBJ(\text{General-Integer})$ is also usually less than 0.01%.

5 Examples

The relations between the four models are summarized in Fig. 4. In what follows, we will present examples to show all of the three "≤" in Fig. 4 can be strict.

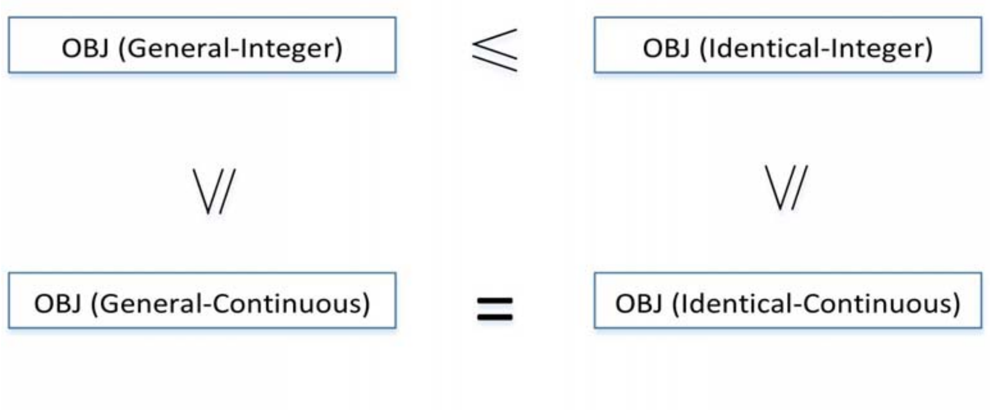

Fig. 4 Relations between the four models

5.1 An example of $OBJ\,(\text{Identical-Continuous}) < OBJ\,(\text{Identical-Integer})$

Example 1: Consider ship route 3 in Fig. 2 with the following parameters: $\mathcal{W} = \{(\text{CB, CC}), (\text{CN, CB}), (\text{CC, CN})\}$, $q^{\text{CB, CC}} = q^{\text{CN, CB}} = q^{\text{CC, CN}} = 1$, $c_h = 0$, $c^{od} = 1$ for all $(o, d) \in \mathcal{W}$, and $E_{r_3} = 1$. It is easy to see that there is only one itinerary for each OD pair.

In model [Identical-Integer], only 1 TEU can be transported each week, and the optimal objective value is 2 because two TEUs are rejected every week. In model [Identical-Continuous], the optimal solution is 0.5 TEU transported for (CB, CC), 0.5 TEU transported for (CC, CN), and 0.5 TEU transported for (CN, CB) in week 2. Therefore, on average 1.5 TEUs are transported every week. The resulting objective value is 1.5. □

5.2 An example of $OBJ\,(\text{General-Integer}) < OBJ\,(\text{Identical-Integer})$

Example 2: Consider the same parameter setting as Example 1. In model [Identical-Integer], only 1 TEU can be transported each week, and the optimal objective value is 2 because two TEUs are rejected every week. In model [General-Integer], in an odd week, one container from CB to CC and another container from CC to CN can be transported; in an even week, one container from CN to CB can be transported. Therefore, on average 1.5 TEUs are transported every week. The resulting objective value is 1.5. □

5.3 An example of OBJ(General-Continuous) < OBJ(General-Integer)

Example 3: Consider the network in Fig. 2. It has six ports and two ship routes. Exactly one ship is deployed on each ship route. The arrival day at each port is also shown in Fig. 2. $\mathcal{W} = \{(1, 4), (1, 6), (3, 6)\}$, $q^{14} = q^{16} = q^{36} = 1$, $c_h = 0$, $c^{od} = 1$ for all $(o, d) \in \mathcal{W}$, and $E_{r_1} = E_{r_2} = 1$. It is easy to see that there is only one itinerary for OD pairs (1, 4) and (3, 6) and there are two itineraries for OD pair (1, 6), one with transshipment and the other without transshipment.

In model [General-Integer], only 1 TEU can be transported each week, and the optimal objective value is 2 because two TEUs are rejected every week. In model [General-Continuous], the optimal solution is 0.5 TEU transported for (1, 4), 0.5 TEU transported for (3, 6), and 0.5 TEU transported for (1, 6) on the itinerary with transshipment. Therefore, on average 1.5 TEUs are transported every week. The resulting objective value is 1.5. □

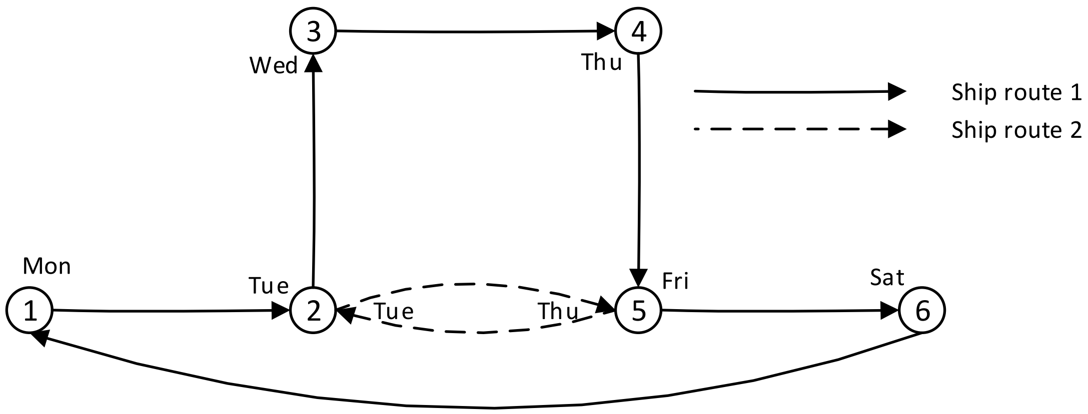

Fig. 5 Relation between model [General-Integer] and model [General-Continuous]

5.4 Cost discount in future weeks

To be consistent with the literature, we have assumed that the cost in different weeks are equally important for the shipping company. If there is a discount factor $0 < \rho < 1$ for the cost in a later week, then it is possible that

$$OBJ(\text{General-Continuous}) < OBJ(\text{Identical-Continuous}) \quad (39)$$

It is even possible that

$$OBJ\,(\text{General-Integer}) < OBJ\,(\text{Identical-Continuous}) \tag{40}$$

as shown by the example below.

Example 4: Consider the same parameter setting as Example 1 with discount factor $0 < \rho < 1$. In model [Identical-Continuous], 1.5 TEUs are transported every week and a cost of 1.5 is incurred each week. In model [General-Integer], in an odd week, one container from CB to CC and another container from CC to CN is transported; in an even week, one container from CN to CB is transported. Therefore, the costs in weeks 1, 2, 3, 4… are 1, 2, 1, 2… and after discounting, the net present value of the average cost is smaller than that of [Identical-Continuous]. □

6 Conclusions

Liner shipping planning models generally assume that the shipping services are unchanged in different weeks and the demand between each port pair is also unchanged in different weeks. For such a setting, this study has addressed a fundamental question related to nearly all liner shipping planning models: whether the implicit assumption of identical container delivery pattern every week is valid. We have rigorously proved that when the number of containers transported is formulated as a continuous variable, the model with an identical container delivery pattern is equivalent to the model with general container delivery patterns that can be different in different weeks. In particular, the two models have the same optimal objective value, and an optimal solution for one model can easily be derived from an optimal solution for the other model. When the number of containers transported is formulated as an integer variable, the model with an identical container delivery pattern is not equivalent to the model with general container delivery patterns. In detail, the optimal objective value of the former (i.e., minimum total cost) may be strictly larger than that of the latter. However, the difference between the two optimal objective values is negligible for practical applications. In sum, this study has proved that little, if not nothing, is lost by assuming an identical container delivery pattern in liner shipping planning models.

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