https://doi.org/10.1016/j.ijpe.2017.07.011

Analysis of three container routing strategies

Lu Zhen[1], Shuaian Wang[2*], Dan Zhuge[2]

[1] *School of Management, Shanghai University, Shang Da Road 99, Shanghai 200444, China*
[2] *Department of Logistics & Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Hong Kong*

Abstract: As containers need several weeks to be delivered to their destinations and the future container shipment demand is uncertain, a myopic strategy for container routing that maximizes the immediate profit may lead to insufficient remaining capacity for transporting more profitable containers in the future. This study conducts a theoretical analysis to compare three container routing strategies, i.e., (i) a widely used myopic strategy, (ii) a non-myopic heuristic routing strategy that incorporates information about the future demand distribution, and (iii) a strategy that assumes future demand is known. Then a practical non-myopic heuristic container routing strategy is developed for practical purposes to increase the profit of liner shipping companies. Extensive numerical experiments show that the proposed non-myopic heuristic container routing strategy brings in an extra of 2% to 3% profit over the myopic strategy.

Keywords: container routing; stochastic demand; liner shipping; myopic strategy; non-myopic heuristic routing strategy.

1. Introduction

Global liner shipping companies transport containerized cargoes between ports to make a profit. Since the financial crisis in 2008, the container shipping market has been at the trough of the market cycle as indicated by overcapacity, low freight rates, and low profit margins. Although the container shipment demand has shown signs of recovery, for instance, the containerized trade volume in 2013 was 160 million TEUs (twenty-foot equivalent units), 17% higher than 2008 (UNCTAD, 2009), the freight rates remain very low. In 2013, the freight rate

*Corresponding author. Email: wangshuaian@gmail.com (S. Wang).

from Shanghai to Northern Europe was only $1084/TEU and from Shanghai to US West Coast was $2033/FEU (forty-foot equivalent unit) (UNCTAD, 2014); by contrast, in the fourth quarter of 2007, the freight rate from Asia to Europe was averaged at $2054/TEU and from Asia to the US was averaged at $3414/FEU ($1707/TEU) (UNCTAD, 2008).

In an effort to deal with low freight rate levels and to leverage some earnings, liner shipping companies have taken measures to improve efficiency and optimize operations to increase revenue and curb costs. For instance, some companies settle agreements to reduce capacity on certain trade lanes and thereby increase freight rate (Bloomberg, 2012). This is possible because the container shipment demand is inelastic to the freight rate which constitutes only a small proportion of the value of the cargoes. Some companies form alliances to expand their service scope and increase service frequency. For instance, the G6 Alliance, formed at the end of 2011 to bring members of the New World Alliance and the Grand Alliance together, expanded cooperation to the Asia–North America East Coast trade lane in May 2013 (UNCTAD, 2014). Efficient repositioning of empty containers can also reduce the operating costs for liner shipping companies (Akyüz and Lee, 2014). In addition, slow-steaming and trim optimization of containerships are adopted by liner shipping companies due to the high bunker cost (Ng, 2014, 2015; Du et al., 2015b).

This study considers an operational-level container routing problem. Here the container routing problem is mainly concerned with the decisions on which container shipment demand to fulfill in case of limited capacity and how to transport the accepted containers in the network. Broadly speaking, the most profitable containers should be accepted considering capacity limitations. Operational and tactical-level container routing problems are different from each other. The tactical-level container routing models are usually part of liner service design models (Zheng et al., 2015a, b; Du et al., 2015a). For instance, a liner company first predicts the container shipment demand for the next three to six months. Then, a shipping network is designed to fulfill the demand at minimum cost (Tran and Haasis, 2015b). In tactical-level container routing models the demand is usually expressed as the volume of containers for each port pair; in other words, the demand is assumed to be the same in different weeks because capturing the volatility of container shipment demand in service design is too difficult. It should be mentioned that the container routing decisions obtained from tactical-level models are not implemented but discarded after the liner services are designed. After the liner services are designed, the operational-level container routing decisions are made based on the revealed demand, and the container routing decisions are implemented. This is the focus of our study.

The operation-level container routing problem is challenging as the operational-level transaction between the customer and the liner shipping company is very complex, including booking, picking up an empty container, and delivery of the loaded container to the export port (Du et al., 2011; Chen et al., 2013; Li et al., 2015). Space reservation in shipping does not require prior payment and is not binding. As a result, cancellations are very common (30% according to Leach, 2011) and no-show is not penalized. Moreover, the number of containers received by a liner shipping company from the spot market is unpredictable. Consequently, the container shipment demand in each week is highly random, as shown in Figure 1. When a liner shipping company makes decisions on which container shipment demand to fulfill and how to transport the accepted containers, the exact future demand for each week is unknown. It takes up to several weeks for a container to be transported from its origin port to its destination port. Then the liner company may accept a less profitable container while having to reject a more profitable one in the future. In this study, the term “non-myopic heuristic container routing” or “non-myopic heuristic routing” is used to represent the operational-level container routing problem considering uncertain demand. Next, a toy example is presented to illustrate the challenges of non-myopic heuristic container routing.

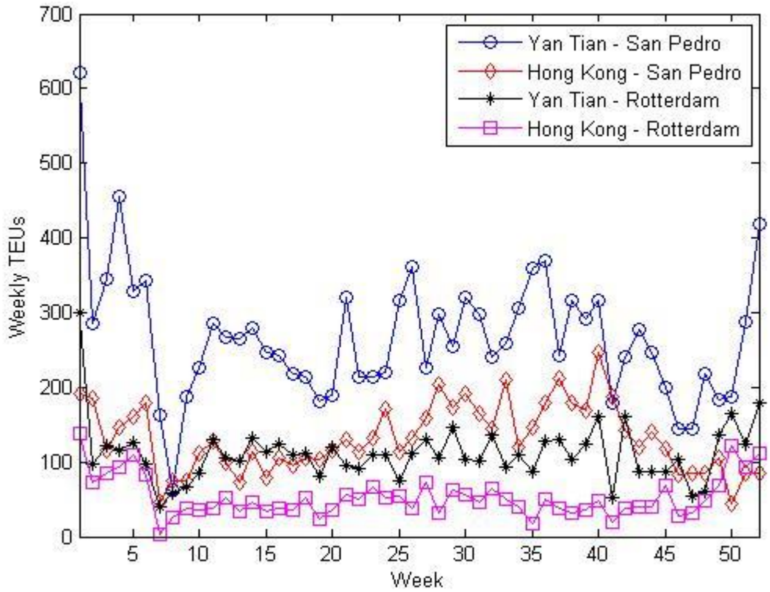

Figure 1: Volume of containers transported in each week between four port pairs by a global liner shipping company (Source: Meng and Wang, 2012)

1.1 An illustrative example

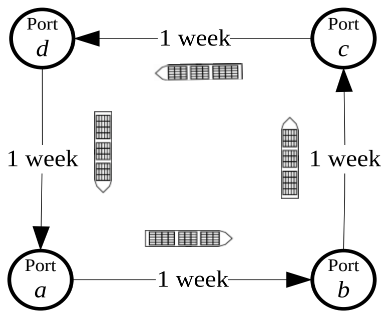

Figure 2: A route with four ports

Example 1: Figure 2 shows a route that includes four ports $a, b, c,$ and d. Four ships, each with a capacity of one container, are deployed on the route to provide a weekly service frequency, and each port is visited on Sunday. The voyage time plus the time spent at ports between two adjacent ports is one week. Suppose that the container shipment demands in different weeks are independently and identically distributed: in each week there are two demand scenarios with the same probability: “low” and “high”; when the demand is low, the numbers of containers for port pairs (a, c) and (b, d) are $q_{ac} = 1, q_{bd} = 0$; when it is high, the numbers are $q_{ac} = 1, q_{bd} = 1$. That is, in each week the probability that there is one container from port a to port c is 1, and the probability that there is one container from b to d is 0.5. Suppose customers require that containers must be delivered to their destinations in two weeks, otherwise the containers are rejected by the shipping company. The profit for delivering one container from a to c is $g_{ac} = \varepsilon \ll 1$, and from$b$ to d is $g_{bd} = 1$. That is, containers from b to d are much more profitable than those from a to c.

At the beginning of each week, the liner shipping company observes the realized demand and then makes the container routing decisions. Note that the company only knows the probability distribution of the demand based on historical data and does not know the exact realizations of the demand in the future weeks.

An intuitive approach for making container routing decisions in each week is the ***myopic strategy***: the decision is made in the manner to maximize the profit of only that particular week without considering the demand in the future weeks. In Example 1, if the myopic strategy is adopted, then if the demand in week 1 is low, we will accept the cargo from a to c. In week 2, if the demand is low again, we will still accept the cargo from a to c. If the demand is high in week 2 (one container from a to c and one from b to d), since the ship that visits port b in week 2 already carries the container from a to c in week 1, it cannot carry the container from b to d any more. As a result, to maximize the profit in week 2, we will still accept the container from a to c and reject the container from b to d. Hence, using the myopic strategy, the container from a to c is always accepted and the one from b to d is always rejected in all of the following weeks. Table 1 shows other examples of the realizations to elaborate the myopic approach. When the myopic strategy is implemented, the expected profit will be only ε per week.

Table 1: Weekly profit of the myopic strategy under different demand patterns

	Weeks	1	2	3	4	Subsequent weeks
Realization 1	*Realized demand*	low	high	high	low	low or high
Demand pattern (L-H-H-L)	*Routing decision*	a to c	a to c	a to c	a to c	a to c
	Profit	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$
Realization 2	*Realized demand*	low	low	low	low	low or high
Demand pattern (L-L-L-L)	*Routing decision*	a to c	a to c	a to c	a to c	a to c
	Profit	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$
Realization 3	*Realized demand*	high	low	low	high	low or high
Demand pattern (H-L-L-H)	*Routing decision*	b to d	a to c	a to c	a to c	a to c
	Profit	**1**	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$
Realization 4	*Realized demand*	high	high	high	low	low or high
Demand pattern (H-H-H-L)	*Routing decision*	b to d	b to d	b to d	a to c	a to c
	Profit	**1**	**1**	**1**	$\boldsymbol{\varepsilon}$	$\boldsymbol{\varepsilon}$

Note: In demand patterns, '*L*' denotes low demand; '*H*' denotes high demand.

From this toy example, the shortcoming of the myopic strategy can be identified: the container from b to d is much more profitable than that from a to c. However, even if the demand is high in a particular week, we still cannot transport the container from b to d because the container from a to c loaded in the previous week already occupies the ship slot. The shortcoming, i.e., the inability to take future decisions into account, is rooted in the conservative nature of the myopic strategy. In essence, the myopic strategy makes container routing decisions in a particular week based on the most conservative assumption that there is no demand in the future weeks. Based on this example, we can easily prove:

Myopic routing paradoxes: Using the myopic routing strategy, (i) when the demand between a port pair increases, it is possible that the total profit decreases; (ii) when the capacity of a leg increases, it is possible that the total profit decreases; (iii) when the freight rate for a port pair increases, it is possible that the total profit decreases. ■

To overcome the shortcoming of the myopic strategy for this toy example, an improved strategy is given: always rejecting the container from a to c and always accepting the container from b to d when the demand is high. The profit in each week using the improved strategy (called a ***non-myopic heuristic routing strategy*** as it accounts for the future random demand) for the four scenarios in Table 1 is shown in Table 2. Since the probability of obtaining a unit demand at a high demand week is 0.5, the expected profit is $1/2$ per week, much higher than that of the myopic strategy.

It should be noted that when we make container routing decisions in a week, the future demand realizations are unknown, although in this example we know it is either low or high. If we had known the exact demand realizations for all of the future weeks, we could make the best container routing decisions using the ***full information strategy***. Evidently, the full information strategy for this example is: (i) if the demand is high, deliver the container from b to d; (ii) if the demand is low, (ii.1) accept the container from a to c if the demand in the subsequent week is also low, and (ii.2) reject the container if the subsequent week has high demand. The expected profit using the full information strategy is $1 \times \frac{1}{2} + \left(\varepsilon \times \frac{1}{4} + 0 \times \frac{1}{4}\right) = \frac{1}{2} + \frac{\varepsilon}{4}$. The profit in each week for the four scenarios using the full information strategy is also shown in Table 2. The full information strategy outperforms the non-myopic heuristic routing strategy: when the demand is low, the non-myopic heuristic routing strategy has to reserve the capacity for potential demand from b to d in the following week; whereas the full information strategy can take advantage of the exact demand information in the subsequent weeks. By continuing the example in Table 1, the comparison of the profits between the three strategies under different demand patterns is summarized in Table 2.

Table 2: Profit comparison of the three strategies under different demand patterns

Strategies \ Weeks		1	2	3	4	Subsequent weeks
Realization 1 *Demand pattern (L-H-H-L)*	*Myopic*	ε	ε	ε	ε	ε
	Stoc_Rou	0	1	1	0	0 if low and 1 if high
	Full_Info	0	1	1	0 if week 5 is high ε if week 5 is low	1 if high, 0 if low and followed by high, ε if low and followed by low
Realization 2 *Demand pattern (L-L-L-L)*	*Myopic*	ε	ε	ε	ε	ε
	Stoc_Rou	0	0	0	0	0 if low and 1 if high
	Full_Info	ε	ε	ε	0 if week 5 is high ε if week 5 is low	1 if high, 0 if low and followed by high, ε if low and followed by low
Realization 3 *Demand pattern (H-L-L-H)*	*Myopic*	1	ε	ε	ε	ε
	Stoc_Rou	1	0	0	1	0 if low and 1 if high
	Full_Info	1	ε	0	1	1 if high, 0 if low and followed by high, ε if low and followed by low
Realization 4 *Demand pattern (H-H-H-L)*	*Myopic*	1	1	1	ε	ε
	Stoc_Rou	1	1	1	0	0 if low and 1 if high
	Full_Info	1	1	1	0 if week 5 is high ε if week 5 is low	1 if high, 0 if low and followed by high, ε if low and followed by low

Notes: (1) In demand patterns, '*L*' denotes low demand; '*H*' denotes high demand. (2) For strategies, '*Myopic*' denotes the myopic strategy; '*Stoc_Rou*' denotes the non-myopic heuristic routing strategy; (3) '*Full_Info*' denotes the full information strategy.

1.2 Objectives and contributions

The objective of our study is to develop a non-myopic heuristic routing strategy for liner shipping companies to increase their profit. The non-myopic heuristic routing strategy outperforms the myopic routing strategy and could bring more profits to the liner shipping companies. The contributions of our study are fourfold:

(1) The worst case ratio of the profit using the myopic routing strategy and the profit using the full information strategy can be proved to approach zero; however, when all containers must be delivered to their destinations in $\check{T}$ weeks, there exists a non-myopic heuristic routing strategy whose profit is at least $1/\check{T}$ of the profit using the full information strategy.

(2) It is proved that, even if all containers must be delivered to their destinations in $\check{T}$ weeks, to make container routing decisions in week t, the possible demands in all of the future weeks should be taken into account, including the demands in weeks after $t + \check{T}$.

(3) It is affirmed that no practical routing strategy guarantees an expected total profit of more than 2/3 of the profit using the full information strategy for all problems.

(4) A non-myopic heuristic routing strategy that combines the myopic strategy and the full information strategy is proposed. Extensive numerical experiments show that the non-myopic heuristic routing strategy considerably outperforms the myopic strategy.

The remainder of the paper is organized as follows. Section 2 reviews relevant literature. Section 3 describes the container routing problem with stochastic demand. Section 4 presents models for the myopic and full information strategy, and then proposes a non-myopic heuristic routing strategy. Section 5 reports the results of numerical experiments. Conclusions are presented in Section 6.

2. Related works

There are three categories of relevant research: operational-level container routing, container slot allocation, and pricing. Readers interested in broader works could refer to Christiansen et al. (2013), Fransoo and Lee (2013), Meng et al. (2014), and Tran and Haasis (2015a).

In the category of research on operational-level container routing, Brouer et al. (2011) considered a planning horizon of many periods and assumed that the container shipment demand in each period is known. In other words, they adopted the full information strategy. Moreover, empty container repositioning was included in their formulations. Song and Dong (2012) investigated a similar problem but captured a more detailed cost structure including container lifting on/off costs at ports, customer demand backlog costs, waiting costs at the transshipment ports for temporarily storing laden containers, empty container inventory costs at ports, and empty container transportation costs.

Ting and Tzeng (2004) and Lu et al. (2010) have addressed a slot allocation planning problem of a container shipping company for satisfying the estimated demands on a liner service. They examined different types of containers (origins and destinations, dry and reefer, and 20-foot and 40-foot) to be transported on a liner service. The demand in a season is fixed and the decision is on how many ship slots on each leg to commit to each type of container. This slot allocation plan is a "pre-allocation" and will be adjusted based on real demand. Hence, the slot allocation planning problem in the two studies is actually the tactical-level container routing problem with a different focus. The slot allocation planning problem focuses on providing guidelines for operational-level booking management while the tactical-level container routing problem aims to assist liner service design. Based on the slot allocation models with the expected demand, Zurheide and Fischer (2012, 2015) examined whether to accept a container booking. They applied simulation approaches to compare several strategies and concluded that the first-come-first-serve strategy is generally the worst.

Dynamic pricing on containers cannot be applied in some regions. For instance, regulations in the United States require that the price needs to be fixed 30 days in advance of the first booking with this announced price. That is why there are few quantitative studies on dynamic pricing of liner shipping. In 2010, Maersk Line introduced a 'Priority Product upgrade' plan (Maersk Line, 2010). In the plan, Maersk Line keeps some slots available until closing and call these slots 'Priority Product upgrade' service. Customers need to pay extra for booking these slots and these slots are priced dynamically. Acciaro (2011) addressed a problem setting similar to the 'Priority Product upgrade' plan. The study considers two types of containers to be shipped from an origin to a destination: standard containers and express containers, the former of which can wait at a port for the next ship and the latter of which must be transported immediately while bringing in higher profit. Acciaro (2011) considered many booking periods. In each period the probabilities that a standard container is requested, an express container is requested or no container is requested, are known. The shipping line determines whether to accept or reject a container in a period to maximize profit.

Different from the above works, our study considers a general container shipping network with multiple ports, multiple origin-destination (OD) pairs, multiple services, and multiple periods. A space-time network model that captures the maximum transit time requirement without path enumeration is applied. The future container shipment demand distribution information, rather than just the expected demand, is captured when making container routing decisions. Theoretical properties and practical algorithms are developed.

3. Problem descriptions

3.1 Backgrounds

This study considers a liner shipping company that operates a set of routes $\mathcal{R}$ for to transport containers over a group of ports denoted by the set $\mathcal{P}$. Each route has a fixed port rotation, and fixed arrival and departure times at each port of call. Figure 3 shows an illustrative example for a liner shipping service network, which includes three routes denoted by $\mathcal{R} = \{1,2,3\}$ and eight ports denoted by $\mathcal{P} = \{\text{NN}, \text{CW}, \text{HK}, \text{SG}, \text{SH}, \text{KH}, \text{NS}, \text{CB}\}$. The itinerary of each route $r \in \mathcal{R}$ forms a loop. Let N_r denote the number of ports of call on a round trip of route r and $p_{r,i} \in \mathcal{P}$ be the physical port corresponding to the i^{th} port of call. One port of call can be defined arbitrarily as the first.

Example 2: Consider the three routes shown in Figure 3. Nan Sha is the first port of call on route 1, Chi Wan, Hong Kong, and Singapore are the second, third, and fourth ports of call, respectively, and the number of ports of call on route 1 is $N_r = 4$.

Let $\mathcal{I}_r = \{1,2, \cdots, N_r\}$ be the set of port calling sequences for ship route r. As a route is a round trip, for brevity we define $p_{r,N_r+1} = p_{r,1}$, meaning that we mean $p_{r,1}$ whenever we refer to p_{r,N_r+1}, and the voyage between two consecutive ports of call $p_{r,i}$ and $p_{r,i+1}$ is called leg i of route r, $i \in \mathcal{I}_r$. Every route has a fixed service schedule. The arrival day in a week at each port of call i for each route r, denoted by $\hat{t}_{r,i}$, is listed in Table 3. In Table 3 we actually define a particular Sunday as day 0, and consider a ship that visits the first port of call on each ship route in week 1 (on day 0, day 1-Mon, day 2-Tue, ⋯, day 6-Sat) and report its arrival time at each port of call.

As the majority of shipping services are weekly, this study assumes each ship route has a weekly service frequency, which means every port of call (not every physical port) on the route will be visited once a week. A weekly service frequency assumption has the following two implications. (i) The round-trip journey time of a route, including the time at sea and the time at port, is an integer number of weeks. (ii) This integer number equals the number of ships deployed on the route. The ships deployed on route r form a string and have the same capacity denoted by E_r (TEUs).

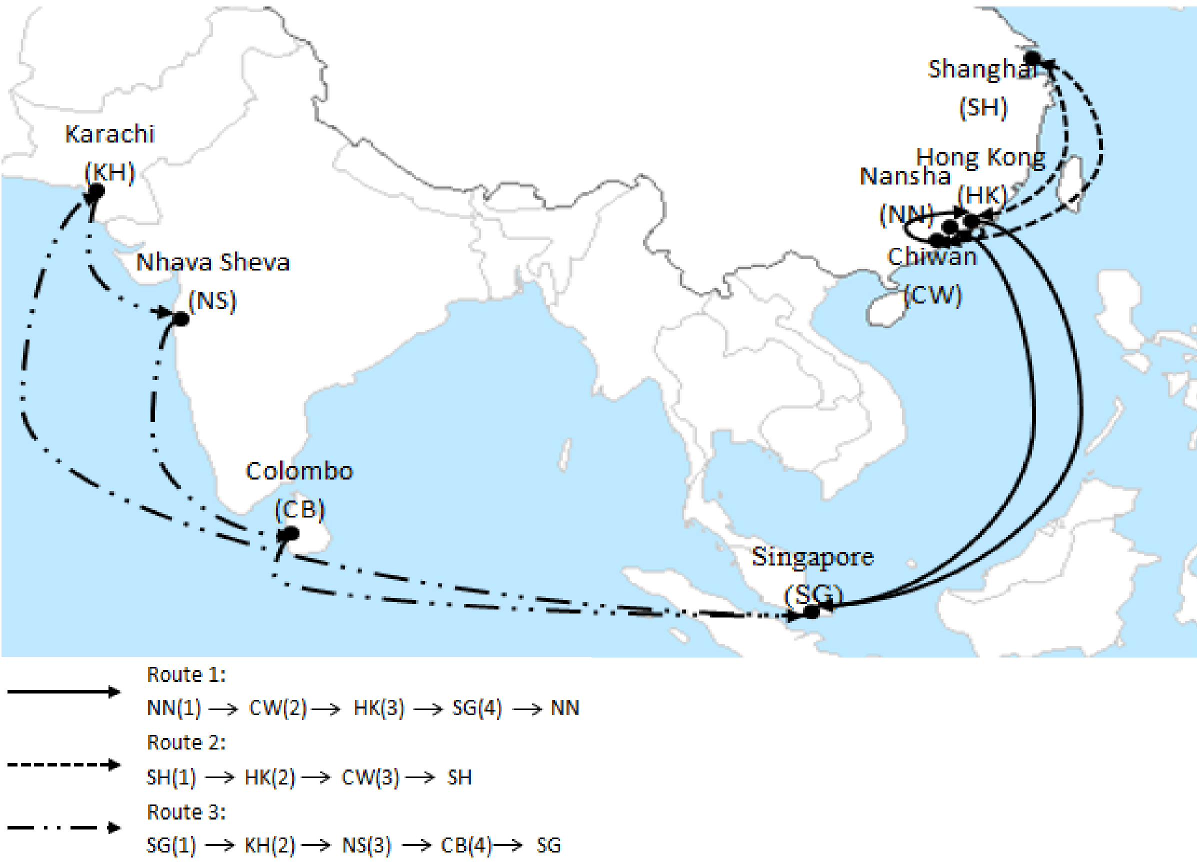

Figure 3: A liner shipping network with three routes

Table 3: Schedules of the three routes

		Route 1			Route 2			Route 3
ID	Port	Arrival day	ID	Port	Arrival day	ID	Port	Arrival day
1	NN	3(Wed)	1	SH	2(Tue)	1	SG	2(Tue)
2	CW	4(Thu)	2	HK	6(Sat)	2	KH	11(Thu)
3	HK	6(Sat)	3	CW	7(Sun)	3	NS	14(Sun)
4	SG	10(Wed)	1	SH	9(Tue)	4	CB	18(Thu)
1	NN	17(Wed)				1	SG	23(Tue)

Note: The departure time does not affect the problem, and hence is not reported in the table

3.2 OD demand and transit time

This study assumes that a shipping line receives some container shipment requests at the beginning of every week. Each shipment request corresponds to an origin-destination (OD) pair of ports. There is a set of OD pairs in the liner shipping network, denoted by $\mathcal{W} \subseteq \{(o,d), o \in \mathcal{P}, d \in \mathcal{P}\}$. In this paper, it is assumed that the liner shipping network has been designed in advance and is used as input. (i) Define g'_{od} as the freight rate (USD/TEU) paid by shippers for OD pair (o,d). The additional cost for transporting one more container at sea is marginal compared with the handling cost. The freight rate for an OD pair is not changed in a planning horizon. Therefore, the container handling cost is considered as the only variable cost. (ii) $\hat{c}_p$, $\tilde{c}_p$, and $\bar{c}_p$ denote loading, discharging, and transshipment cost (USD/TEU) at port $p \in \mathcal{P}$, respectively. (iii) The shipping line has the right to accept or reject a demand partially or fully. The maximum profit g_{od} that can be gained from shipping one TEU for OD pair (o,d) can be calculated as follows:

$$g_{od} = g'_{od} - \hat{c}_o - \tilde{c}_d, (o,d) \in \mathcal{W} \tag{1}$$

We assume that containers are available at the beginning of each week (Sunday) and containers from OD pair (o,d) must be delivered to their destination port d in no longer than $\hat{T}_{od}^{\max}$ days. As a result, if a container is available in week t, it is available on day $7(t-1)$ and must be delivered to its destination no later than day $7(t-1)+\hat{T}_{od}^{\max}$; otherwise, the container has to be rejected.

At the beginning of each week, the shipping line observes the container shipment demand and decides which proportion of the demand to fulfill and how to fulfill it in order to maximize the profit. This study considers a planning horizon of T weeks, i.e., week $1, 2 \cdots T$. The demand in each week is uncertain, but the probability distribution of demand is assumed to be known in advance based on historical data. It should be noted that the probability distribution in this study can be fairly general. The demand in week $t+1$ can depend on the demand in week t; the marginal probability distribution in week $t+1$ also can be different from that in week t. Here the demand is denoted by a vector $\tilde{\boldsymbol{q}} \coloneqq (\tilde{q}^1, \cdots, \tilde{q}^t, \cdots, \tilde{q}^T)$ with $\tilde{q}^t \coloneqq (\tilde{q}^t_{od}, (o,d) \in \mathcal{W})$. A realization of the demand is denoted by $\boldsymbol{q}$. The objective of the decision model proposed in this study should be the maximization of the expected profit during the whole planning horizon under the uncertain demand.

3.3 Space-time network: the basis for model formulation

The liner shipping network $(\mathcal{R}, \mathcal{P}, \mathcal{W}, T, \hat{T}^{\max}_{od})$ can be represented by a space-time network $G = (N, A)$, where N is the set of nodes and A is the set of arcs, to deal with the maximum transit time requirements of containers. The construction of the space-time network is also employed by Wang et al. (2016). To make the paper self-contained, the construction procedure of the space-time network is repeated as follows.

Step 1: Construct a space-axis that corresponds to ports of call (not ports) in the liner shipping network, and a time-axis that represents the arrival time (day) at each port of call. The time-horizon $\hat{N}_T$ (weeks) for the time-axis is calculated as follows:

$$\hat{N}_T = \left\lceil \left(\max_{(o,d)\in\mathcal{W}} \hat{T}^{\max}_{od} \right) / 7 \right\rceil + T - 1 \tag{2}$$

in which $\lceil x \rceil$ is the smallest integer greater than or equal to x. For example, if the maximum threshold transit time for all OD pairs is 30 days and $T = 3$ weeks, the time-horizon should be $\lceil 30/7 \rceil + 2 = 7$ weeks because any container (say, one that is available at the beginning of week 3, i.e., day 14) must be delivered to its destination no later than the end of week 7.

Step 2: The number of ports of call in the liner shipping network is $\sum_{r\in\mathcal{R}} N_r$. For each port of call, we construct $\widehat{N}_T$ nodes in the space-time network. The arrival days of the $\widehat{N}_T$ nodes indicated by the time-axis are: $\hat{t}_{r,i} \bmod 7$, $(\hat{t}_{r,i} \bmod 7) + 7$, $\cdots$, $(\hat{t}_{r,i} \bmod 7) + 7(\widehat{N}_T - 1)$. The total number of nodes in the space-time network is $|N| = \widehat{N}_T \sum_{r\in\mathcal{R}} N_r$. Each node can be denoted by a triplet $(r, i, \hat{t})$, which means the visit at port of call i on route r on day $\hat{t}$.

Step 3: After a ship visits a node $(r, i, \hat{t}) \in N$, it will visit the next port of call on the route at time $\hat{t} + (\hat{t}_{r,i+1} - \hat{t}_{r,i})$. If $\hat{t} + \hat{t}_{r,i+1} - \hat{t}_{r,i} < 7\widehat{N}_T$, it means the node $(r, i+1, \hat{t} + \hat{t}_{r,i+1} - \hat{t}_{r,i})$ is also in the space-time network, and we construct a voyage arc from the node $(r, i, \hat{t})$ to the node $(r, i+1, \hat{t} + \hat{t}_{r,i+1} - \hat{t}_{r,i})$.

Step 4: As some different nodes in the network correspond to the same physical port, containers can be transshipped between these nodes. For each node $(r, i, \hat{t}) \in N$, we check all the other nodes $(r', i', \hat{t}') \in N$. If $p_{r,i} = p_{r',i'}$ and $\hat{t} \le \hat{t}' < \hat{t} + 7$, a transshipment arc from the node $(r, i, \hat{t})$ to the node $(r', i', \hat{t}')$ will be constructed.

Example 2 (continued): The space-time network in Figure 4 is built for the liner shipping network shown in Figure 3 with $\widehat{N}_T = 3$.

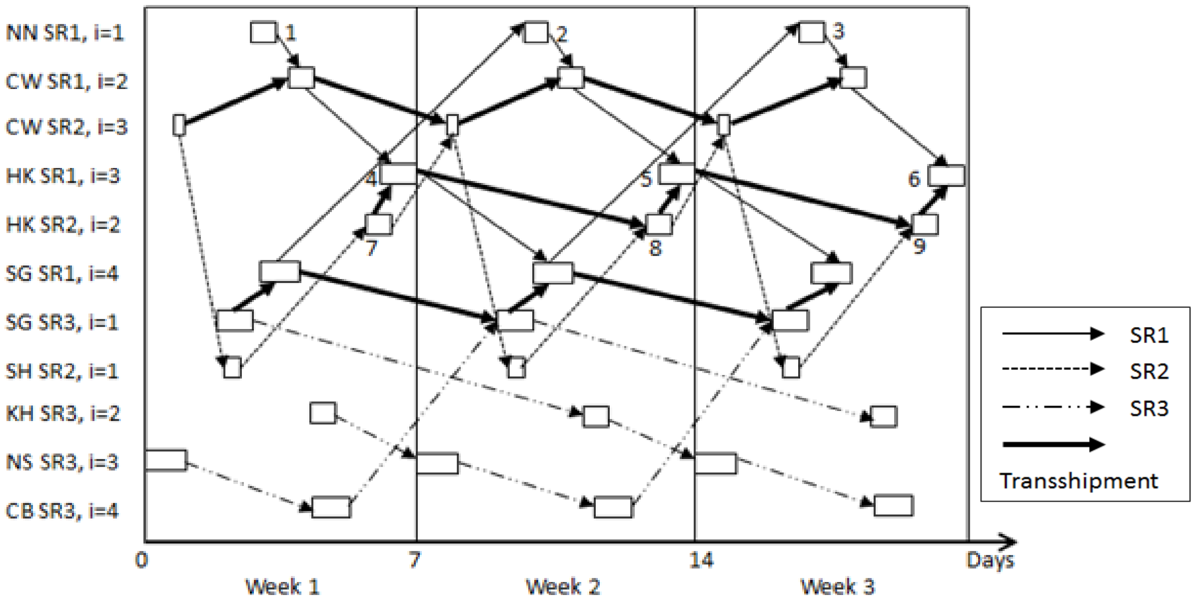

Figure 4: The space-time network for Example 2

Some notations of the nodes and arcs in the space-time network $G = (N, A)$ are defined. They are the basis for the model formulations in the remainder of this paper.

n: node in the space-time network, $n \in N$. In the previous explanation, a node in the network is denoted by a triplet $(r, i, \hat{t})$. Here, the newly defined parameter n contains information about ship route $r_n = r$, port of call $i_n = i$ on the route, the corresponding physical port p_n, and arrival time (the time when node n is visited) $\hat{t}_n = \hat{t}$.

(m, n): arc in the space-time network; $(m, n) \in A$; $m \in N, n \in N$. More specifically, the set of voyage arcs is defined as $\bar{A}$ and the set of transshipment arcs is defined as $\hat{A}$; $A = \bar{A} \cup \hat{A}$. The cost of transporting a container on arc (m, n) is denoted by $c_{m,n}$. In reality, the cost of transporting one more container on a ship is much smaller than the handling cost of a container. So this study assumes $c_{m,n} = 0$ if $(m, n) \in \bar{A}$; and $c_{m,n} = \bar{c}_{p_m}$ if $(m, n) \in \hat{A}$. Here $\bar{c}_{p_m}$ is the transshipment cost at the port p_m. The capacity of voyage arc (m, n), denoted by $s_{m,n}$, equals E_{r_n}, i.e., the capacity of the ship deployed on route r_n. The capacity of transshipment arc is assumed to be infinite. That is, ports can meet all container handling operations required by vessels visiting them.

3.4 Container flows in the space-time network

The core of the problem is to decide the container flows in the space-time network. Recall that this study assumes the shipping line observes the container shipment demand at the beginning of each week. For the containers from OD pair (o, d) that are available at the beginning of week t, we need to specify the set of nodes, from which these containers may be loaded onto ships, and another set of nodes, to which these containers may be discharged from the ships. It is defined that $\widehat{\mathcal{N}}_{od}^{t}$ and $\widetilde{\mathcal{N}}_{od}^{t}$ to denote the above two sets, respectively:

$$\widehat{\mathcal{N}}_{od}^{t} = \{m | p_m = o, 7(t-1) \leq \hat{t}_m \leq 7(t-1) + \hat{T}_{od}^{max}\} \tag{3}$$

$$\widetilde{\mathcal{N}}_{od}^{t} = \{n | p_n = d, 7(t-1) \leq \hat{t}_n \leq 7(t-1) + \hat{T}_{od}^{max}\} \tag{4}$$

For example, consider the containers between the OD pair (NN, HK) in Figure 4 that are available in week 1 and suppose $\hat{T}_{NN,HK}^{\max} = 21$. There exists $\widehat{\mathcal{N}}_{NN,HK}^{1} = \{1, 2, 3\}$ and $\widetilde{\mathcal{N}}_{NN,HK}^{1} = \{4, 5, 6, 7, 8, 9\}$, which are marked by numbers in Figure 4. As long as the origin is in set $\widehat{\mathcal{N}}_{NN,HK}^{1}$ and the destination is in set $\widetilde{\mathcal{N}}_{NN,HK}^{1}$, the maximum transit time requirement is satisfied. It is not possible for containers to flow in the space-time network from some nodes in $\widehat{\mathcal{N}}_{NN,HK}^{1}$ to some nodes in $\widetilde{\mathcal{N}}_{NN,HK}^{1}$ and this does not affect the suggested formulations.

Now, we summarize the container routing problem with stochastic demand. In week 1, a liner company observes the demand in this week and decides which containers to accept and how to transport the accepted containers based on the demand in week 1, the capacity of each arc in the shipping network, and the probability distribution of the demand in the future weeks $t = 2, 3 \cdots T$. The container routing decisions are implemented. In week $t' \geq 2$, the liner company observes the demand in this week and decides which containers to accept and how to transport the accepted containers based on the demand in week t', the remaining capacity of each arc in the shipping network, and the probability distribution of the demand in the future weeks $t = t' + 1, t' + 2 \cdots T$. Then the container routing decisions are implemented. The above process repeats in each week until the end of the T weeks. The objective of the decisions is to maximize the total expected profit over the T weeks.

4. Model formulation

This section presents mathematical models. Only one type of container (TEU) is considered, and there is no long-term contractual demand. In Section 4.1 we list the notations used in the models. Sections 4.2 and 4.3 present the myopic model and the full information model, respectively. Section 4.4 investigates properties of practical routing models and proposes a non-myopic heuristic routing strategy that combines the myopic and full information strategies.

4.1 Notation

Indices

(o, d)	OD pairs of demand
m, n, l	nodes in the space-time network

Sets:

$\mathcal{W}$	set of all OD pairs
A	set of all arcs in the space-time network
$\bar{A}$	set of all voyage arcs in the space-time network
$\hat{A}$	set of all transshipment arcs in the space-time network
N	set of all nodes in the space-time network
$\widehat{\mathcal{N}}_{od}^{t}$	set of origin nodes at which the containers for the demand of the OD pair (o, d) in week t may be loaded onto ships
$\widetilde{\mathcal{N}}_{od}^{t}$	set of destination nodes at which the containers for the demand of the OD pair (o, d) in week t may be discharged from ships

Parameters:

q_{od}^{t}	realized demand of the OD pair (o, d) in week t
g_{od}	maximum profit that can be gained from shipping a container for OD pair (o, d), i.e., revenue minus the loading and discharging costs
$c_{m,n}$	cost of transporting a container on arc $(m, n) \in \hat{A}$; note that $c_{m,n} = 0$ for all voyage arcs in $\bar{A}$
$s_{m,n}$	capacity of arc (m, n); note that $s_{m,n}$ is infinity for all transshipment arcs $\hat{A}$
$s_{m,n}^{t}$	remaining capacity of voyage arc (m, n) when making decisions in week t

Decision variables:

$f_{m,n}^{od,t}$	volume of accepted containers for the demand of the OD pair (o, d) in week t that are transported on the arc (m, n)
y_{od}^{t}	volume of containers for the demand of the OD pair (o, d) in week t that are fulfilled, which implies '$q_{od}^{t} - y_{od}^{t}$' containers are rejected

4.2 Myopic strategy

An intuitive strategy for making container routing decisions in each week is taking the myopic approach: the decision in week t maximizes the profit of only week t without considering the decisions in the future weeks. This approach is natural especially in view of the fact that the future demand is uncertain. In essence, this approach is the most conservative strategy as it makes container routing decisions in week t as if there is no demand in the future weeks.

4.2.1 Mathematical model

The myopic strategy can be derived by solving linear programming models. Specifically, in week t, the decision maker learns the demand in that week and needs to decide how many containers from OD pair to accept and which path to use to transport the accepted containers.

In week t, the decision variables' values solved for weeks $1, 2, \cdots, t-1$ are known parameters. The unreserved capacity of voyage arc (m, n) at the beginning of week t (some capacity of the arc is reserved for containers accepted in previous weeks), denoted by $s_{m,n}^t$, is updated as follows:

$$s_{m,n}^t = s_{m,n} - \sum_{(o,d)\in\mathcal{W}} \sum_{\tau=1}^{t-1} f_{m,n}^{od,\tau*}, (m,n) \in \bar{A}, \tag{5}$$

where $f_{m,n}^{od,\tau*}$ are the optimal container routing decisions obtained in the previous weeks $\tau = 1,2, \cdots, t-1$.

Now the decision problem for week t, with decision variables y_{od}^t and $f_{m,n}^{od,t}$, using the myopic strategy, can be formulated as the following linear programming model:

$$[\mathbb{M}_1(\boldsymbol{q}, t)] \qquad \max \sum_{(o,d)\in\mathcal{W}} g_{od} y_{od}^t - \sum_{(m,n)\in\hat{A}} c_{m,n} \sum_{(o,d)\in\mathcal{W}} f_{m,n}^{od,t} \tag{6}$$

$$s.t. \ \sum_{m:(m,l)\in A} f_{m,l}^{od,t} = \sum_{n:(l,n)\in A} f_{l,n}^{od,t} \qquad \forall l \in N \backslash \tilde{\mathcal{N}}_{od}^t \backslash \hat{\mathcal{N}}_{od}^t; \ \forall (o,d) \in \mathcal{W} \tag{7}$$

$$\sum_{m\in\hat{\mathcal{N}}_{od}^t, l:(m,l)\in A} f_{m,l}^{od,t} = y_{od}^t \qquad \forall (o,d) \in \mathcal{W} \tag{8}$$

$$\sum_{m\in\hat{\mathcal{N}}_{od}^t, l:(l,m)\in A} f_{l,m}^{od,t} = 0 \qquad \forall (o,d) \in \mathcal{W} \tag{9}$$

$$\sum_{n\in\tilde{\mathcal{N}}_{od}^t, l:(l,n)\in A} f_{l,n}^{od,t} = y_{od}^t \qquad \forall (o,d) \in \mathcal{W} \tag{10}$$

$$\sum_{n\in\tilde{\mathcal{N}}_{od}^t, l:(n,l)\in A} f_{n,l}^{od,t} = 0 \qquad \forall (o,d) \in \mathcal{W} \tag{11}$$

$$\sum_{(o,d)\in\mathcal{W}} f_{m,n}^{od,t} \le s_{m,n}^t \qquad \forall (m,n) \in \bar{A} \tag{12}$$

$$y_{od}^t \le q_{od}^t \qquad \forall (o,d) \in \mathcal{W} \tag{13}$$

$$y_{od}^t \ge 0 \qquad \forall (o,d) \in \mathcal{W} \tag{14}$$

$$f_{m,n}^{od,t} \ge 0 \qquad \forall (o,d) \in \mathcal{W}; \forall (m,n) \in A \tag{15}$$

Objective (6) is to maximize the profit of week t. Constraints (7) state that the incoming containers equal the outgoing containers for each node except for the nodes where containers are loaded onto or discharged from ships. Constraints (8) and (9) state that for the nodes where containers are loaded onto ships, the number of outgoing containers equals the number of containers that are transported; and there is no incoming container flow. Similarly, Constraints (10) and (11) state that for the nodes where containers are discharged from ships, the number of incoming containers equals the number of containers that are transported; and there is no outgoing container flow. Constraints (12) guarantee the capacity limitation on each arc. Constraints (13) ensure the volume of the fulfilled containers does not exceed the demand. Constraints (14) and (15) state that the decision variables are nonnegative.

4.2.2 Estimation of the expected profit

Given a problem, in this section we present how to estimate the expected profit with the myopic strategy. The purpose of the estimation is to compare with the profits with other strategies.

Let $F_1(\tilde{\boldsymbol{q}}, t)$ denote the profit in week t using the myopic strategy. $F_1(\tilde{\boldsymbol{q}}, t)$ is random as the demand $\tilde{\boldsymbol{q}}$ is random. Let $F_1(\tilde{\boldsymbol{q}})$ denote the total profit in the T weeks using the myopic strategy, $F_1(\tilde{\boldsymbol{q}}) = \sum_{t=1}^{T} F_1(\tilde{\boldsymbol{q}}, t)$. Clearly, the expected total profit, denoted by $E[F_1(\tilde{\boldsymbol{q}})]$, is finite. When $E[F_1(\tilde{\boldsymbol{q}})]$ were known, it could be used by a liner company to estimate the future profit and thereby make more informed tactical and strategic decisions. Unless $\tilde{\boldsymbol{q}}$ has a limited number of scenarios (limited support), the accurate value of $E[F_1(\tilde{\boldsymbol{q}})]$ is generally very difficult to obtain as it involves multi-dimensional integration where the integrand has no analytical expression. However, we can estimate $E[F_1(\tilde{\boldsymbol{q}})]$ with a fairly high accuracy based on the strong law of large numbers. In particular, as the demand is uncertain, the myopic decision strategy can be evaluated by the average value of the profits under a finite number of demand realizations, which can be generated according to the known probability distribution of the demand. The average value is an unbiased point estimate of $E[F_1(\tilde{\boldsymbol{q}})]$.

In particular, a set of U realizations of demand are denoted by $\boldsymbol{q}^{(1)}, \boldsymbol{q}^{(2)}, \cdots, \boldsymbol{q}^{(u)}, \cdots, \boldsymbol{q}^{(U)}$. The demand from port o to port d in week t of realization u is denoted by $q_{od}^{t,(u)}$. Define $F_1(u, t)$ and $F_1(u)$ as the profit of week t under realization u using the myopic strategy and as the total profit over the T weeks, respectively. Let F_1 denote the average profit

$$F_1 = \frac{1}{U}\sum_{u=1}^{U} F_1(u) = \frac{1}{U}\sum_{u=1}^{U}\sum_{t=1}^{T} F_1(u,t) \tag{16}$$

The strong law of large numbers shows that: as $E[F_1(\tilde{\boldsymbol{q}})]$ is finite, we have

$$\lim_{U\to\infty} F_1 \to E[F_1(\tilde{\boldsymbol{q}})] \quad \text{almost surely.} \tag{17}$$

Here $F_1(u,t)$ is calculated from week 1 to week T: For each realization (u) and a week t, $F_1(u,t)$ is the optimal objective value of the model $[\mathbb{M}_1(\boldsymbol{q}^{(u)},t)]$.

4.3 Full information strategy

Different from the myopic strategy, the full information strategy assumes a shipping line knows the demand for all the T weeks at the beginning of the planning horizon. In reality the liner shipping company does not know the realization in advance; however, examining the full information strategy is helpful to develop effective non-myopic heuristic routing strategies and to obtain effective bounds on non-myopic heuristic routing strategies.

4.3.1 Mathematical model

In the myopic strategy the demand is observed week by week, and hence the container routing decisions are also made week by week in our model. However, with full information, we know the demand for all of the T weeks and hence we can make container routing decisions in the first week. A linear programming model $[\mathbb{M}_2(\boldsymbol{q})]$, with decision variables y_{od}^t and $f_{m,n}^{od,t}$ for $t = 1,\cdots,T$, is developed below for the full information strategy:

$$[\mathbb{M}_2(\boldsymbol{q})] \qquad \max \sum_{t=1}^{T}\left\{\sum_{(o,d)\in\mathcal{W}} g_{od} y_{od}^t - \sum_{(m,n)\in\hat{A}} c_{m,n}\sum_{(o,d)\in\mathcal{W}} f_{m,n}^{od,t}\right\} \tag{18}$$

$$\text{s.t.} \quad \sum_{m:(m,l)\in A_{od}^t} f_{m,l}^{od,t} = \sum_{n:(l,n)\in A_{od}^t} f_{l,n}^{od,t} \quad \forall l \in N\backslash\widetilde{\mathcal{N}}_{od}^t\backslash\widehat{\mathcal{N}}_{od}^t;\ \ \forall(o,d)\in\mathcal{W}; t=1,\cdots,T \tag{19}$$

$$\sum_{m\in\widehat{\mathcal{N}}_{od}^t, l:(m,l)\in A} f_{m,l}^{od,t} = y_{od}^t \qquad \forall(o,d)\in\mathcal{W}; \forall t=1,\cdots,T \tag{20}$$

$$\sum_{m\in\widehat{\mathcal{N}}_{od}^t, l:(l,m)\in A} f_{l,m}^{od,t} = 0 \qquad \forall(o,d)\in\mathcal{W}; \forall t=1,\cdots,T \tag{21}$$

$$\sum_{n\in\widetilde{\mathcal{N}}_{od}^t, l:(l,n)\in A} f_{l,n}^{od,t} = y_{od}^t \qquad \forall(o,d)\in\mathcal{W}; \forall t=1,\cdots,T \tag{22}$$

$$\sum_{n\in\tilde{\mathcal{N}}_{od}^{t},l:(n,l)\in A} f_{n,l}^{od,t} = 0 \qquad \forall(o,d)\in\mathcal{W};\forall t=1,\cdots,T \tag{23}$$

$$\sum_{t=1}^{T}\sum_{(o,d)\in\mathcal{W}} f_{m,n}^{od,t} \le s_{m,n} \qquad \forall(m,n)\in\bar{A} \tag{24}$$

$$y_{od}^{t} \le q_{od}^{t} \qquad \forall(o,d)\in\mathcal{W};\forall t=1,\cdots,T \tag{25}$$

$$y_{od}^{t} \ge 0 \qquad \forall(o,d)\in\mathcal{W};\forall t=1,\cdots,T \tag{26}$$

$$f_{m,n}^{od,t} \ge 0 \qquad \forall(o,d)\in\mathcal{W};\forall(m,n)\in A;\forall t=1,\cdots,T \tag{27}$$

4.3.2 Analysis of the expected profit

Let $F_2(\tilde{\boldsymbol{q}})$ denote the random total profit in the T weeks using the full information strategy, $F_2(u)$ be the profit under scenario (u), i.e., $F_2(u)$ is the optimal objective value of model $[\mathbb{M}_2(\boldsymbol{q}^{(u)})]$, and F_2 denote the average profit using the full information strategy under the U scenarios of demand $\boldsymbol{q}^{(1)},\boldsymbol{q}^{(2)},\cdots,\boldsymbol{q}^{(u)},\cdots,\boldsymbol{q}^{(U)}$. Here $F_2=\frac{1}{U}\sum_{u=1}^{U}F_2(u)$.

Lemma 1: Given the same demand scenario (u), we have $F_2(u)\ge F_1(u)$.

Proof: If the optimal solutions are combined to the T myopic models $[\mathbb{M}_1(\boldsymbol{q}^{(u)},t)]$, then we can obtain a feasible solution to model $[\mathbb{M}_2(\boldsymbol{q}^{(u)})]$. In other words, the optimal container routing decisions from the myopic models are also feasible to model $[\mathbb{M}_2(\boldsymbol{q}^{(u)})]$. Moreover, the resulting total profits are the same. Therefore, the optimal objective function value of $[\mathbb{M}_2(\boldsymbol{q}^{(u)})]$ is not smaller than the sum of the optimal objective function values of the T myopic models $[\mathbb{M}_1(\boldsymbol{q}^{(u)},t)]$. ■

Proposition 1: The expected profit with the full information strategy is not smaller than that with the myopic strategy, i.e., $E[F_2(\tilde{\boldsymbol{q}})]\ge E[F_1(\tilde{\boldsymbol{q}})]$. In particular, $\frac{E[F_2(\tilde{\boldsymbol{q}})]}{E[F_1(\tilde{\boldsymbol{q}})]}$ can be arbitrarily large even if $E[F_1(\tilde{\boldsymbol{q}})]>0$.

Proof: Since $F_2(u)\ge F_1(u)$ for any realization of the demand, the expected profit, i.e., the integration of the profit over the probability distribution of the demand, also satisfies

$E[F_2(\tilde{\boldsymbol{q}})] \geq E[F_1(\tilde{\boldsymbol{q}})]$.

To prove the second part of the proposition, consider Example 1. The expected profit of the myopic strategy $E[F_1(\tilde{\boldsymbol{q}})] = \varepsilon$ and the full information strategy has $E[F_2(\tilde{\boldsymbol{q}})] = \frac{1}{2} + \frac{\varepsilon}{4}$. The conclusion holds when $\varepsilon \to 0^+$. ■

4.4 Non-myopic heuristic routing strategy

The full information strategy is an ideal method but cannot be used in practice because it is difficult to know the exact future demand in advance (unless the ship slots are booked at the beginning of the T weeks and shippers will not be refunded if they cancel their shipment). The myopic strategy is a practical decision policy in reality. However, the demerit of the myopic strategy under uncertain demand is also evident. This study proposes a non-myopic heuristic routing strategy by approximating the advantages of the above two strategies.

The non-myopic heuristic routing strategy is a non-anticipative decision strategy, which means the container routing decisions in week t are made (i) based on the demand realizations in weeks $1, 2 \cdots t$, (ii) the decisions made in weeks $1, 2 \cdots t-1$, and (iii) the probability distribution functions for the demand in the future, but not based on the realizations of the demand in weeks $t+1, t+2 \cdots T$.

4.4.1 Theoretical investigation

The following proposition shows that the full information strategy can be used to evaluate the effectiveness of a non-myopic heuristic routing strategy.

Proposition 2: The expected profit with the full information strategy is an upper bound on that of any feasible practical routing strategy.■

Although the optimal non-myopic routing strategy is easy to identify for Example 1, it may be difficult to obtain an optimal non-myopic routing strategy for larger problem instances. In particular, the optimal decision in week t depends at least on the demand for all the OD pairs in that week and the reserved capacities of all the voyage arcs in week t or later (the reserved capacity on arc (m,n) just before making decisions in week t is $s_{m,n} - s_{m,n}^{t}$). If the demands in different weeks are not independent, the optimal decision in week t also depends on the realizations of the demand in weeks $1, 2 \cdots t-1$.Similarly, the decision space in week t grows exponentially with respect to the number of OD pairs. Thus we cannot expect to develop a method for finding the optimal non-myopic routing strategy for any problem. Note further that, even if $\max_{(o,d)\in W} \hat{T}_{od}^{max} = 14$ days, i.e., all containers must be delivered to destinations within two weeks, the decision in week t may still depend on the demand in weeks after $t+2$, as shown by the example below.

Example 3: Consider the service in Figure 2. There are four OD pairs $(a,c), (b,d), (c,a)$ and (d,b) with unit profits $g_{ac} = \varepsilon^3 \ll 1, g_{bd} = \varepsilon^2, g_{ca} = \varepsilon$ and $g_{db} = 1$, respectively. The demands in the first three weeks are deterministic: one container from a to c in week 1, one from b to d in week 2, and one from c to a in week 3. if in week 4 there is one container from d to b, then we should reject the containers in week 1 and week 3, and accept the containers in week 2 and week 4; if in week 4 there is no demand, then we should accept the containers in week 1 and week 3, and reject the container in week 2.

This example shows that to make container routing decisions in week t, we have to take into account the possible demand in all of the future weeks, even if the containers in week t have already been delivered by that time.

Proposition 3: Suppose all containers must be delivered to destinations in $\check{T} := \left\lceil \max_{(o,d)\in W} \hat{T}_{od}^{\max}/7 \right\rceil$ weeks. Then there exists a non-myopic heuristic routing strategy whose expected profit is at least $1/\check{T}$ of the profit with the full information strategy.

Proof: If we reject all of the demands in weeks $2, 3 \cdots \check{T}, \check{T}+2, \check{T}+3, \cdots 2\check{T}, 2\check{T}+2 \cdots$, then the container routing decisions in weeks $1, \check{T}+1, 2\check{T}+1 \cdots$ do not affect each other. We could use a non-myopic heuristic routing strategy, called strategy 1, to maximize the profits in weeks $1, \check{T}+1, 2\check{T}+1 \cdots$. Strategy 1 is actually the myopic strategy for weeks $1, \check{T}+1, 2\check{T}+1 \cdots$ while rejecting all of the containers in the other weeks. It is easy to see that the profit of strategy 1 in week 1 (weeks $\check{T}+1, 2\check{T}+1 \cdots$ respectively) is at least as high as that of the full information strategy, because full information strategy may not maximize the profit in week 1 (weeks $\check{T}+1, 2\check{T}+1 \cdots$ respectively) for the purpose of reserving capacities for more profitable containers in the future. Akin to F_1 and F_2, the total expected profit of strategy 1 could easily be estimated based on strong law of large numbers.

Similarly, there are strategy 2, which only transports containers in weeks $2, \check{T}+2, 2\check{T}+2 \cdots$, strategy 3, up to strategy $\check{T}$. The sum of the expected profits using the $\check{T}$ strategies is at least as high as the total expected profit of the full information strategy. Consequently, the best one of the $\check{T}$ strategies has an expected profit of at least $1/\check{T}$ of the profit with the full information strategy. ■

Proposition 3 might encourage us to attempt to derive better non-myopic heuristic routing strategies that have a smaller worst-case gap to the full information strategy. Unfortunately, the following proposition implies that such an attempt cannot lead to a non-myopic heuristic routing strategy with an acceptable worst-case gap for real applications.

Proposition 4: No practical routing strategy could guarantee an expected profit of more than $2/3$ of the profit with the full information strategy for all problems.

Proof: Consider Figure 5 which has three routes: route 1 visits ports a and b, route 2 visits ports b and c, and route 3 visits ports b and d. All the three routes visit port b on Sunday and there is no transshipment cost. All the routes have capacity one. Suppose the container shipment demands in different weeks are independent. (i) In odd weeks, there is one container from port a to port c and one from a to d; containers between these two port pairs must be delivered in two weeks and the profit for delivering one container is 0.5. (ii) In even weeks, there is either one container from b to c or one container from b to d with equal probability of 0.5; containers between these two port pairs must be delivered in one week and the profit for delivering one container is 1.

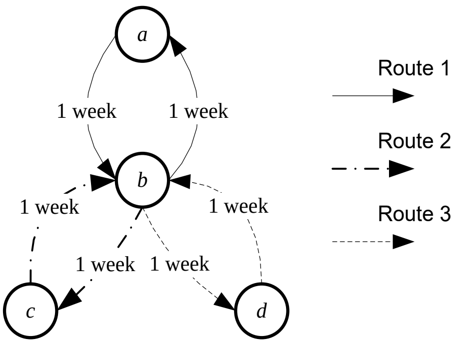

Figure 5: A network showing the limitation of practical routing strategies

Due to the limited capacity on arc (a, b), at most one container can be transported in an odd week. If both containers are rejected in odd weeks, the expected profit per week is $(0 + 1)/2 = \frac{1}{2}$. If we accept a container in odd weeks, then the expected profit per week is $\left[0.5 + \left(\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 1\right)\right]/2 = \frac{1}{2}$. Clearly, both of them are the optimal practical routing strategy with an expected profit per week of $\frac{1}{2}$ and there is no better practical routing strategy. It is easy to see that with full information, we could deliver one container in each odd week and one container in each even week, and the resulting expected profit is $(0.5 + 1)/2 = \frac{3}{4}$. ■

Proposition 4 demonstrates that it is impossible to develop a practical routing strategy that works excellently for all problems. Thus a practical stochastic routing strategy that works well for most problems is proposed in the next subsection.

4.4.2 A non-myopic heuristic routing strategy combining the myopic and full information strategies

Our study develops a non-myopic heuristic routing strategy that combines the myopic strategy and the full information strategy. Such a strategy, although having an inferior worst-case performance than the strategy mentioned in Proposition 3, outperforms the latter in almost all of the instances. The idea of our non-myopic heuristic routing strategy is related to Lagrangian relaxation of the non-anticipativity constraints in multi-stage stochastic programming. In multi-stage stochastic programming, there are usually long-term decisions to be made at stage 1, which are non-anticipative. In our problem, there are no long-term decisions in week 1, however, the constraints are much more complex than those in e.g. inventory and portfolio optimization multi-stage stochastic programming problems. Our non-myopic heuristic routing strategy is also related to nesting booking limits policies in revenue management, where, e.g. in airline, seats allocated to the least profitable booking classes are made available to more profitable classes as well, which is called nesting. Because of the high dimensionality and the complex network structure of our problem, dynamic programming based analyses are not applicable and the non-myopic heuristic routing strategy reserves some capacity for future demand based on possible realizations of future demand.

To appreciate the non-myopic heuristic routing strategy, consider a particular week t'. (i) We observe this week's demand $q_{od}^{t'}, (o,d) \in \mathcal{W}$. (ii) V realizations of demand from week $t'+1$ through to week T based on the distribution function are generated, denoted by $\breve{\boldsymbol{q}}^{(1)}, \breve{\boldsymbol{q}}^{(2)}, \cdots, \breve{\boldsymbol{q}}^{(v)}, \cdots, \breve{\boldsymbol{q}}^{(V)}$. V is a parameter pre-determined by a decision maker. (iii) To simplify the notation, we define the demand in week t' in realization $v = 1, 2 \cdots V$, denoted by $q_{od}^{t',(v)}$, to be equal to the real demand $q_{od}^{t'}$. (iv) For each realization $v = 1, 2 \cdots V$, the full information of demand in weeks t', $t'+1, \cdots, T$ is known. Hence, we solve the following model, which is similar to the full information model [$\mathbb{M}_2(\boldsymbol{q}^{(u)})$]:

$$[\mathbb{M}_3(\breve{\boldsymbol{q}}^{(v)}, t')] \quad \max \sum_{t=t'}^{T} \{\sum_{(o,d)\in\mathcal{W}} g_{od} y_{od}^{t} - \sum_{(m,n)\in\hat{A}} c_{m,n} \sum_{(o,d)\in\mathcal{W}} f_{m,n}^{od,t}\} \tag{28}$$

$$s.t. \ \sum_{m:(m,l)\in A} f_{m,l}^{od,t} = \sum_{n:(l,n)\in A} f_{l,n}^{od,t} \ \ \forall l \in N \backslash \widetilde{\mathcal{N}}_{od}^{t} \backslash \widehat{\mathcal{N}}_{od}^{t}; \ \ \forall (o,d) \in \mathcal{W}; t = t', \cdots, T \tag{29}$$

$$\sum_{m\in\widehat{\mathcal{N}}_{od}^{t},l:(m,l)\in A} f_{m,l}^{od,t} = y_{od}^{t} \qquad \forall(o,d)\in\mathcal{W};\forall t=t',t'+1,\cdots,T \tag{30}$$

$$\sum_{m\in\widehat{\mathcal{N}}_{od}^{t},l:(l,m)\in A} f_{l,m}^{od,t} = 0 \qquad \forall(o,d)\in\mathcal{W};\forall t=t',t'+1,\cdots,T \tag{31}$$

$$\sum_{n\in\widetilde{\mathcal{N}}_{od}^{t},l:(l,n)\in A} f_{l,n}^{od,t} = y_{od}^{t} \qquad \forall(o,d)\in\mathcal{W};\forall t=t',t'+1,\cdots,T \tag{32}$$

$$\sum_{n\in\widetilde{\mathcal{N}}_{od}^{t},l:(n,l)\in A} f_{n,l}^{od,t} = 0 \qquad \forall(o,d)\in\mathcal{W};\forall t=t',t'+1,\cdots,T \tag{33}$$

$$\sum_{t=t'}^{T}\sum_{(o,d)\in\mathcal{W}} f_{m,n}^{od,t} \le s_{m,n}^{t'} \qquad \forall(m,n)\in\bar{A} \tag{34}$$

$$y_{od}^{t} \le q_{od}^{t',(v)} \qquad \forall(o,d)\in\mathcal{W};\forall t=t',t'+1,\cdots,T \tag{35}$$

$$y_{od}^{t} \ge 0 \qquad \forall(o,d)\in\mathcal{W};\forall t=t',t'+1,\cdots,T \tag{36}$$

$$f_{m,n}^{od,t} \ge 0 \qquad \forall(o,d)\in\mathcal{W};\forall(m,n)\in A;\forall t=t',\cdots,T \tag{37}$$

Note that in constraints (34) $s_{m,n}^{t'}$ is the available capacity of arc (m,n) at the beginning of week t'. Let $(y_{od}^{t',(v)*}, f_{m,n}^{od,t',(v)*})$ denote the optimal solution to model $[\mathbb{M}_3(\breve{\boldsymbol{q}}^{(v)},t')]$.

Recall that the myopic solution may accept too much demand in week t', leaving insufficient capacity for future demand. To overcome this deficiency, the non-myopic heuristic routing strategy for week $t'$of realization u solves the following model (note that realization u is different from the V realizations of demand in $[\mathbb{M}_3(\breve{\boldsymbol{q}}^{(v)},t')]$):

$[\mathbb{M}_4(\boldsymbol{q}^{(u)},t')]$ The myopic model $[\mathbb{M}_1(\boldsymbol{q}^{(u)},t')]$ (38)

with the following constraints

$$f_{m,n}^{od,t'} \le \frac{1}{V}\sum_{v=1}^{V} f_{m,n}^{od,t',(v)*} \qquad \forall(o,d)\in\mathcal{W};\forall(m,n)\in\bar{A} \tag{39}$$

As shown above, the non-myopic heuristic routing model $[\mathbb{M}_4(\boldsymbol{q}^{(u)},t')]$ differs from the myopic model $[\mathbb{M}_1(\boldsymbol{q}^{(u)},t')]$ in that the container flow of each OD pair on each arc is bounded by the average flow using the full information strategy. In other words, constraints (39) ensure that some capacity of each arc is reserved for the future demand, and how much capacity to reserve is determined based on the average capacity required by the full information strategy.

The optimal solution to model [$\mathbb{M}_4\left(\boldsymbol{q}^{(u)}, t'\right)$] is implemented in week t'. This policy is similar to the myopic strategy to some extent, but integrates some future information into the model.

Example 1 (continued): We show how the proposed non-myopic heuristic routing strategy works on this example. Suppose that the ship capacity is 3; when the demand is low, the numbers of containers for port pairs (a, c) and (b, d) are $q_{ac} = 3, q_{bd} = 0$; when it is high, the numbers are $q_{ac} = 3, q_{bd} = 3$. Suppose further that the planning horizon is 5 weeks. In week 1 we observe the demand is low. We then generate e.g. $V = 6$ realizations of demand for the following four weeks. Suppose that the realizations are: (1) low-high-low-high, (2) high-low-high-low; (3) low-low-high-high; (4) high-high-low-low; (5) high-low-low-low; (6) high-low-high-high. Then the full information strategy can be used to calculate the flow of containers for port pair (a, c) on arc (a, b) in week 1: $f_{a,b}^{ac,1,(1)*} = 3$, $f_{a,b}^{ac,1,(2)*} = 0$, $f_{a,b}^{ac,1,(3)*} = 3$, $f_{a,b}^{ac,1,(4)*} = 0$, $f_{a,b}^{ac,1,(5)*} = 0$, $f_{a,b}^{ac,1,(6)*} = 0$. According to Eq. (39), the following constraint is added to the myopic model in our non-myopic heuristic routing strategy:

$$f_{a,b}^{ac,1} \leq \frac{1}{6}\sum_{v=1}^{6}(3 + 0 + 3 + 0 + 0 + 0) = 1$$

Therefore, we will transport one container from port a to port c in week 1. Although the proposed non-myopic heuristic routing strategy is worse than the optimal one that does not transport any container, it considerably outperforms the myopic approach that transport one container from a to c.■

The non-myopic heuristic routing strategy is summarized as follows:

Non-myopic heuristic routing strategy

Input: The space-time network, demand related information, planning horizon T.

Step 0: Define parameter V that is used in the non-myopic heuristic routing strategy. Define $t' = 1$. Initialize the voyage arc capacity in the first week as follows:

$$s_{m,n}^{1} = s_{m,n}, \forall (m,n) \in \bar{A} \tag{40}$$

Step 1: At the beginning of week t', observe the demand denoted by $q_{od}^{t'}, (o,d) \in \mathcal{W}$. Randomly generate V realizations of demand from week $t'+1$ through to week T, denoted by $\breve{\boldsymbol{q}}^{(1)}, \breve{\boldsymbol{q}}^{(2)}, \cdots, \breve{\boldsymbol{q}}^{(v)}, \cdots, \breve{\boldsymbol{q}}^{(V)}$, based on the distribution function. Define the demand in week t' in realization $v = 1, 2 \cdots V$, denoted by $q_{od}^{t',(v)}$, to be equal to the real demand $q_{od}^{t'}$. For each realization $v = 1, 2 \cdots V$, solve model [$\mathbb{M}_3\left(\breve{\boldsymbol{q}}^{(v)}, t'\right)$] and obtain the optimal solution denoted by $(y_{od}^{t',(v)*}, f_{m,n}^{od,t',(v)*})$.

Step 2: Solve model [$\mathbb{M}_4\left(\boldsymbol{q}^{(u)}, t'\right)$] and obtain the optimal solution $(y_{od}^{t'*}, f_{m,n}^{od,t'*})$. Implement the solution and update the voyage arc capacities as follows:

$$s_{m,n}^{t'+1} = s_{m,n}^{t'} - \sum_{(o,d)\in\mathcal{W}} f_{m,n}^{od,t'*}, \forall (m,n) \in \bar{A} \tag{41}$$

Step 3: If $t' = T$, stop. Else, set $t' \leftarrow t' + 1$ and go to Step 1. ■

We can evaluate the value of the non-myopic heuristic routing strategy by comparing it with the myopic strategy. Define $F_4(u, t')$ as the optimal objective value to model [$\mathbb{M}_4\left(\boldsymbol{q}^{(u)}, t'\right)$].

Similar with Eq. (5), the expected profit using the non-myopic heuristic routing strategy can be estimated as follows.

$$F_4 = \frac{1}{U} \sum_{u=1}^{U} \sum_{t'=1}^{T} F_4(u, t') \tag{42}$$

The difference between the F_4 and F_1 evaluates the ***value of non-myopic heuristic routing***, which is denoted by Val_StoRtg.

$$Val\_StoRtg = F_4 - F_1 \tag{43}$$

which measures the extra benefit that a shipping line can earn from the stochastic routing strategy. A large value of 'Val_StoRtg' can validate the necessity of the stochastic routing strategy for the problem instance. For the ease of comparison in numerical experiments, the relative difference is defined as

$$Gap_{StoRtg} = \frac{Val\_StoRtg}{F_1} = \frac{F_4 - F_1}{F_1} \tag{44}$$

Similarly, the difference between F_2 and F_4 evaluates the ***value of perfect information***, which is denoted by Val_Info.

$$Val\_Info = F_2 - F_4 \tag{45}$$

Note that strictly speaking, we should replace F_4 by the expected profit of the optimal non-myopic routing strategy. However, as it is impossible to find the optimal non-myopic routing strategy in general, the "value of perfect information" defined in Eq. (45) is actually an upper bound. Eq. (45) measures the maximum amount of money that a shipping line is willing to pay for knowledge value of random demands before making their decision (Avriel and Williams, 1970). A large value of 'Val_Info' means the randomness plays an important role in this decision problem. The relative difference is defined as

$$Gap_{Info} = \frac{Val\_Info}{F_4} = \frac{F_2 - F_4}{F_4} \tag{46}$$

5. Numerical experiments

We conduct extensive numerical experiments that are randomly generated to test the performance of the proposed non-myopic heuristic routing strategy. All of the linear programming models are solved by CPLEX12.5.1 with the technology of C# (VS2008) on a PC (Intel Core i5, 1.70 GHz; Memory, 8G).

The test instances are generated as follows. The ports are randomly selected from major Asian ports. The port rotations on the services are randomly chosen. Two types of ships—5,000-TEU and 6,000-TEU—are deployed in the network to provide weekly services on each route. The freight rate for a container depends on the distance between the origin port and the destination port. The transshipment cost is randomly chosen between US$100/TEU and US$200/TEU. A quarter of all of the port pairs have container shipment demand, and the demand for each OD pair is randomly chosen between 1000TEUs/week and 2000TEUs/week. The maximum transit time for containers is randomly chosen between 7 days and 28 days.

5.1 Fine-tuning of the non-myopic heuristic routing strategy

There is only one parameter V in the non-myopic heuristic routing strategy that needs to be specified. Therefore, we first investigate how to choose a suitable value of V. To this end, we conduct ten groups of experiments. Each group corresponds to different problem settings including shipping networks and OD pairs. All of the groups have five services and a planning horizon of six weeks. For each group, ten realizations of demand (ten instances) for the six weeks are generated. We calculate the total profit of each instance using different values of $V \in \{1, 2, 5, 10, 20\}$. Note that when $V = 1$, $\breve{\boldsymbol{q}}^{(v)}$ for the model [$\mathbb{M}_3\left(\breve{\boldsymbol{q}}^{(v)}, t'\right)$] are not generated in a random manner. Instead, we let $\breve{\boldsymbol{q}}^{(v)}$ be the fixed expected demand. That is, if only one possible future realization of demand in the non-myopic heuristic routing strategy is taken into account, we simply assume that the future demand is equal to the expected value of demand. Table 4 reports the average profit of the ten instances in each group. To simplify the comparison, the table normalizes the profit for $V = 1$ and reports the ratio of the profit for a particular V and the profit for $V = 1$. In general, a larger V leads to a higher profit. However, the profit increase is less significant when V is large and the improvement of the case $V = 20$ over the case $V = 10$ is marginal. Hence, $V = 20$ are chosen in the subsequent experiments.

Table 4: Impact of the parameter $\boldsymbol{V}$ on the performance of the non-myopic heuristic routing strategy

Group	$V = 1$	$V = 2$	$V = 5$	$V = 10$	$V = 20$
1	1	1.0133	1.0208	1.0225	1.0235
2	1	1.0023	1.0062	1.0070	1.0069
3	1	1.0108	1.0144	1.0164	1.0166
4	1	1.0094	1.0150	1.0155	1.0158
5	1	1.0048	1.0093	1.0097	1.0100
6	1	1.0039	1.0072	1.0079	1.0081
7	1	1.0043	1.0085	1.0094	1.0092
8	1	1.0053	1.0086	1.0094	1.0099
9	1	1.0107	1.0125	1.0132	1.0135
10	1	1.0012	1.0065	1.0068	1.0073

5.2 Performance of the non-myopic heuristic routing strategy

Next, we examine how much improvement the non-myopic heuristic routing strategy can bring in over the myopic strategy. Ten groups, each with 20 instances and a planning horizon of eight weeks, are computed. The results are reported in Table 5, in which “Best instance” includes the absolute profit increase (“Improvement”) and relative profit increase (“Gap”) by the non-myopic heuristic routing strategy for the best one of the 20 instances and “Average” reports the average of the 20 instances. In Table 5, the non-myopic heuristic routing strategy is better than or at least as good as the myopic strategy in all of the 200 instances. The relative improvement over the myopic strategy Gap_{StoRtg} is between 1.45% and 3.28%.

Table 5: Comparison between the non-myopic heuristic routing strategy and the myopic strategy

	Best instance		Average over 20 instances	
Group	Improvement	Gap	Improvement	Gap_{StoRtg}
1	2.9×10^6	2.54%	2.4×10^6	2.09%
2	2.2×10^6	1.83%	1.7×10^6	1.45%
3	4.1×10^6	3.51%	3.5×10^6	2.99%
4	2.6×10^6	2.25%	2.1×10^6	1.83%
5	3.0×10^6	2.85%	2.4×10^6	2.27%
6	4.3×10^6	3.83%	3.7×10^6	3.28%
7	2.8×10^6	2.43%	2.3×10^6	2.00%
8	3.3×10^6	2.82%	2.7×10^6	2.30%
9	2.5×10^6	2.11%	2.0×10^6	1.73%
10	3.6×10^6	3.23%	2.9×10^6	2.58%

Table 6 reports the total number of transported containers (TEUs) and the total profit (USD) in the eight weeks over the 20 instances in each group using the three strategies. The profit increase of the non-myopic heuristic routing strategy over the myopic strategy is related to the fact that using non-myopic heuristic routing strategy more containers are transported than using the myopic strategy. The average relative gap between the non-myopic heuristic routing strategy and the full information strategy is only 1.83%, much smaller than the theoretical bound defined in Proposition 4. Note that the relative gap of 1.83% does not mean the non-myopic heuristic routing strategy proposed in our paper is worse than the best possible non-myopic heuristic routing strategy by 1.83%. Instead, it means that there may or may not be better non-myopic heuristic routing strategies, however, none of the strategies would improve over our strategy by more than 1.83% for the test instances.

Table 6: Comparison between the three strategies

Group	Myopic fulfilled demand	Non-myopic heuristic routing fulfilled demand	Myopic profit	Non-myopic heuristic routing profit	Gap_{StoRtg}	Full information profit	Gap_{Info}

1	94,381	96,378	1.146×10^8	1.170×10^8	2.09%	1.191×10^8	1.79%
2	97,245	98,858	1.171×10^8	1.188×10^8	1.45%	1.210×10^8	1.85%
3	99,556	102,249	1.171×10^8	1.206×10^8	2.99%	1.228×10^8	1.82%
4	93,882	95,051	1.149×10^8	1.170×10^8	1.83%	1.192×10^8	1.88%
5	90,519	91,849	1.056×10^8	1.080×10^8	2.27%	1.099×10^8	1.76%
6	94,894	97,448	1.128×10^8	1.165×10^8	3.28%	1.187×10^8	1.89%
7	95,876	97,899	1.150×10^8	1.173×10^8	2.00%	1.195×10^8	1.88%
8	98,046	99,760	1.176×10^8	1.203×10^8	2.30%	1.225×10^8	1.83%
9	96,355	97,989	1.154×10^8	1.174×10^8	1.73%	1.195×10^8	1.79%
10	91,678	94,571	1.123×10^8	1.152×10^8	2.58%	1.173×10^8	1.82%
Average					2.25%		1.83%

A detailed comparison between the non-myopic heuristic routing strategy and the myopic strategy for an instance is reported in Table 7. At the beginning of the planning horizon, the non-myopic heuristic routing strategy transports fewer containers than the myopic strategy to reserve capacity for future containers. Therefore, the profit gained by the non-myopic heuristic routing strategy is less than that of the myopic strategy. Nevertheless, at the end of the planning horizon, the non-myopic heuristic routing strategy has more capacity and hence transports more containers. Overall, the non-myopic heuristic routing strategy gains higher profit than the widely used myopic strategy.

Table 7: Comparison between the non-myopic heuristic routing strategy and the myopic strategy

Week	1	2	3	4	5	6	7	8	Sum
Myopic fulfilled demand	10,421	13,528	12,136	11,885	11,125	12,319	11,961	11,519	94,894
Non-myopic heuristic routing fulfilled demand	10,229	12,254	11,787	11,404	12,400	13,166	12,979	13,229	97,448
Myopic profit (10^7USD)	1.18	1.60	1.44	1.45	1.40	1.47	1.43	1.31	11.28
Non-myopic heuristic routing profit (10^7USD)	1.16	1.48	1.41	1.42	1.54	1.59	1.54	1.51	11.65
Gap_{StoRtg}	−1.69%	−7.50%	−2.08%	−2.07%	10.00%	8.16%	7.69%	15.27%	3.28%

5.3 Impact of the trend of the freight rates

The container freight market is volatile. For instance, the freight rates are likely to rise in the future when the demand increases or the supply decreases. Thus, a group of ten instances with five different trends (the freight rates decrease by 4% each week, decrease by 2%, do not change, increase by 2%, and increase by 4%) are conducted to examine how the change of the freight rates will affect the results. The average fulfilled demand and profit over the ten instances are reported in Table 8. The gap between the non-myopic heuristic routing strategy and the myopic strategy becomes smaller and smaller when the freight rates decrease; or the gap gradually increases with the freight rates growing. This result is intuitive: when the freight rates decrease, the best choice is to transport as many containers as early as possible. Consequently, the non-myopic heuristic routing strategy is more effective in the rapidly developing shipping market.

Table 8: Impact of the trend of the freight rate on the results

Price trend	Myopic fulfilled demand	Non-myopic heuristic fulfilled demand	Myopic profit	Non-myopic heuristic profit	Gap_{StoRtg}
−4%	115,748	120084	1.4998×10^8	1.5548×10^8	3.67%
−2%	115,926	120174	1.5070×10^8	1.5632×10^8	3.73%
0	115,926	120166	1.5118×10^8	1.5703×10^8	3.87%
2%	115,926	120142	1.5167×10^8	1.5773×10^8	4.00%
4%	115,760	120124	1.5198×10^8	1.5842×10^8	4.24%

6. Conclusions and future works

As containers need several weeks to be delivered to their destinations and the future container shipment demand is uncertain, a myopic strategy for container routing that maximizes the immediate profit may lead to insufficient remaining capacity for transporting more profitable containers in the future. Therefore, it is of high interest to develop a non-myopic routing strategy that incorporates information on the future demand distribution for liner shipping companies to increase their profit. We have examined the problem from both the theoretical and practical perspectives. Theoretically, first, we show that in the worst case, the profit ratio of the myopic routing strategy to the full information strategy can approach zero; however, when all containers must be delivered to their destinations in $\check{T}$ weeks, there exists a non-myopic routing strategy whose profit is at least $1/\check{T}$ of the profit using the full information strategy. Second, we prove that, even if all containers must be delivered to their destinations in $\check{T}$ weeks, to make container routing decisions in week t, we still have to take into account the possible demand in all of the future weeks, including the demands in weeks after $t + \check{T}$. Third, we affirm that no practical routing strategy can guarantee an expected profit of more than 2/3 of the profit with the full information strategy for all problems.

Although the theoretically worst relative gap of the profit between the full information strategy and any non-myopic routing strategies is 1/3, it is not in vain to develop practical non-myopic routing strategies that work well for most real problems. Exploiting the problem structure, we have proposed a non-myopic heuristic routing strategy that combines the myopic strategy and the full information strategy. Extensive numerical experiments show that the non-myopic heuristic routing strategy brings in an extra profit of 2% to 3% over the myopic strategy. Moreover, the profit by the non-myopic heuristic routing strategy is less than that of the full information strategy by less than 2% on average.

This study demonstrates the significance of using the proposed non-myopic heuristic routing strategy to increase the profit over the myopic strategy. The novelty of the non-myopic heuristic routing strategy lies in that it elegantly combines the myopic strategy and the full information strategy. Real container routing problems are much more complex than the one discussed in our paper. For instance, there are empty containers to be repositioned and multiple-type containers (dry and reefer, 20-foot and 40-foot, and contractual and spot) to be transported. Moreover, whenever customers request a booking, liner shipping companies must immediately answer whether the booking is accepted or not, rather than accumulate several requests and process them simultaneously. Finally, there is a time gap between when a customer books and when the customer's containers arrive at the origin port for loading. Late delivery and cancellations may occur during this interval. It is possible to take advantage of the idea of the non-myopic heuristic routing strategy to address these practical considerations in future works.

References

Acciaro, M., 2011. Service differentiation in liner shipping: advance booking and express services. *International Journal of Shipping and Transport Logistics* 3(4), 365–383.

Akyüz, M.H., Lee, C.Y., 2014. A mathematical formulation and efficient heuristics for the dynamic container relocation problem. *Naval Research Logistics* 61(2), 101–118.

Avriel, M., Williams, A. C., 1970. The value of information and stochastic programming. *Operations Research* 18 (5), 947–954.

Bloomberg, 2012. Container lines set to repeat mistakes in supply glut: freight. http://www.bloomberg.com/news/articles/2012-06-05/container-lines-set-to-repeat-mistakes-in-supply-glut-freight. Accessed 3 Aug 2015.

Brouer, B.D., Pisinger, D., Spoorendonk, S., 2011. Liner shipping cargo allocation with repositioning of empty containers. *INFOR* 49(2), 109–124.

Chen, G., Govindan, K., Yang, Z., 2013. Managing truck arrivals with time windows to alleviate gate congestion at container terminals. *International Journal of Production Economics* 141(1), 179–188.

Christiansen, M., Fagerholt, K., Nygreen, B., Ronen, D., 2013. Ship routing and scheduling in the new millennium. *European Journal of Operational Research* 228 (3), 467–478.

Du, Y., Chen, Q., Quan, X., Long, L., Fung, R.Y.K., 2011. Berth allocation considering fuel consumption and vessel emissions. *Transportation Research Part E* 47 (6), 1021–1037.

Du, Y., Chen, Q., Lam, J. S. L., Xu, Y., Cao, J. X., 2015a. Modeling the impacts of tides and the virtual arrival policy in berth allocation. *Transportation Science* 49(4), 939–956.

Du, Y., Meng, Q., Wang, Y., 2015b. Budgeting fuel consumption of container ship over round-trip voyage through robust optimization. *Transportation Research Record* 2477, 68–75.

Leach, P.T., 2011. Maersk to charge fees for no-show containers. http://www.joc.com/maritime-news/international-freight-shipping/maersk-charge-fees-no-show-containers_20110623.html. Accessed 13 Aug 2015.

Lee, C. Y., Fransoo, J. C., 2013. The critical role of ocean container transport in global supply chain performance. *Production and Operations Management* 22(2), 253–268.

Li, C., Qi, X., Lee, C. Y., 2015. Disruption recovery for a vessel in liner shipping. *Transportation Science* 49(4), 900–921.

Lu, H. A., Chu, C. W., Che, P. Y., 2010. Seasonal slot allocation planning for a container liner shipping service. *Journal of Marine Science and Technology* 18(1), 84–92.

Maersk Line, 2010. The answer is 'yes'. http://classic.maerskline.com/link/?page=lhp &path=/north_america/nicaragua. Accessed 13 Aug 2015.

Ng, M.W., 2014. Distribution-free vessel deployment for liner shipping. *European Journal of Operational Research* 238 (3), 858–862.

Ng, M.W., 2015. Container vessel fleet deployment for liner shipping with stochastic dependencies in shipping demand. *Transportation Research Part B* 74, 79–87.

Meng, Q., Wang, S., 2012. Liner ship fleet deployment with week-dependent container shipment demand. *European Journal of Operational Research* 222 (2), 241–252.

Meng, Q., Wang, S., Andersson, H., Thun, K., 2014. Containership routing and scheduling in liner shipping: overview and future research directions. *Transportation Science* 48 (2), 265–280.

Song, D.P., Dong, J.X., 2012. Cargo routing and empty container repositioning in multiple shipping service routes. *Transportation Research Part B* 46 (10), 1556–1575.

Talley, W. K., Ng, M.W., 2013. Maritime transport chain choice by carriers, ports and shippers. *International Journal of Production Economics* 142(2), 311–316.

Tran, N.K., Haasis, H.D., 2015a. Literature survey of network optimization in container liner shipping. *Flexible Services and Manufacturing Journal* 27(2-3), 139–179.

Tran, N.K., Haasis, H.D., 2015b. An empirical study of fleet expansion and growth of ship size in container liner shipping. *International Journal of Production Economics* 159, 241–253.

Ting, S. C., Tzeng, G. H., 2004. An optimal containership slot allocation for liner shipping revenue management. *Maritime Policy & Management* 31(3), 199–211.

UNCTAD, 2008. Review of Maritime Transportation: Paper presented at the United Nations Conference on Trade and Development. New York and Geneva. http://unctad.org/en/Docs/rmt2008_en.pdf. Accessed 10 May, 2017.

UNCTAD, 2009. Review of Maritime Transportation: Paper presented at the United Nations Conference on Trade and Development. New York and Geneva. http://www.unctad.org/en/docs/rmt2009_en.pdf. Accessed 20 Mar, 2010.

UNCTAD, 2014. Review of Maritime Transportation: Paper presented at the United Nations Conference on Trade and Development. New York and Geneva. http://unctad.org/en/PublicationsLibrary/rmt2014_en.pdf. Accessed 28 Nov 2014.

Wang, S., Meng, Q., Lee, C.Y., 2016. Liner container assignment model with transit-time-sensitive container shipment demand and its applications. *Transportation Research Part B* 90, 135–155.

Zheng, J., Gao, Z., Yang, D., Sun, Z., 2015a. Network design and capacity exchange for liner alliances with fixed and variable container demands. *Transportation Science* 49 (4), 886 – 899.

Zheng, J., Sun, Z., Gao, Z., 2015b. Empty container exchange among liner carriers.

Transportation Research Part E 83, 158–169.

Zurheide, S., Fischer, K., 2012. A revenue management slot allocation model for liner shipping networks. *Maritime Economics & Logistics* 14(3), 334–361.

Zurheide, S., Fischer, K., 2015. Revenue management methods for the liner shipping industry. *Flexible Services and Manufacturing Journal* 27, 200–223.