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Ship type decision considering empty container repositioning and foldable containers

Abstract: This paper addresses a problem of ship type decision considering empty container repositioning and foldable containers, which determines the capacity of ships deployed in a given shipping route at a tactical level and empty container repositioning between ports at an operational level. It considers the use of foldable containers and aims to find under what conditions, a shipping liner needs to use the foldable containers. To solve the problem, we formulate a network flow model with a revised network simplex algorithm, based on which an exact solution approach is designed to determine the optimal ship type.

Keywords: Empty container repositioning; foldable containers; ship fleet deployment; network simplex algorithm.

1. Introduction

A shipping liner normally operates weekly-serviced ship routes with fixed schedules to transport containers. Given a shipping route, a shipping liner deploys a fleet of container ships for the operation over a planning horizon, e.g., six months. One of the critical decisions for the shipping liner on the fleet is ship type decision, which determines the capacity of container ships of the fleet deployed on the shipping route. Empirically, the shipping liner deploys a suitable fleet type of ships on each route based on the laden container transportation over the planning horizon, which guarantees that the deployed ships have the capacity to accommodate all the laden containers in all the voyages. Under this circumstance, the shipping liner would not deploy a ship fleet with a larger capacity as it increases the fixed operation cost for maintaining the fleet. It is reasonable for the shipping liner to make such decision only considering the laden container transportation. However, if we further consider the empty container repositioning on the route, the ship type decision can be more complicated.

The empty container repositioning originates from the imbalance of container flow between different regions in liner shipping routes. Take the trans-Pacific trade lane for example: according to UNCTAD (2016), in 2015, the annual container flow from Asia to North America (i.e., the eastbound) was around 15.8 million twenty-foot equivalent units (TEUs), and the container flow in the opposite westbound direction was 7.4 million TEUs, which generated the imbalance of container flow for 8.4 million TEUs. This imbalance contributes to tremendous empty container accumulation in import-dominant areas (North America) and the serious empty container shortage in export-dominant areas (Asia). This leads to a critical problem on the empty container availability for the laden container transportation consignment in those export-dominant areas or ports. In those export-dominant ports, the arriving laden containers from incoming ships become empty and are stored in depots after devanning, which can only fulfill part of the empty container requirement for the sake of their export-dominant characters. As a result, the empty container repositioning from the surplus ports (i.e., import-dominant ports) to the deficit ports (i.e., export-dominant ports) becomes necessary. However, the empty containers repositioned between the ports occupy the capacity of the container ships traversing the corresponding voyages in the shipping route. Henceforth, the ship type decision is no longer straightforward when considering the empty container repositioning.

Figure 1: Four foldable containers and a standard container. *Source*: Shintani et al. (2010)

Storing empty containers in the depots and repositioning empty containers among the ports inevitably incur storage cost and repositioning cost for the shipping liner, respectively (Lee and Yu, 2012). To reduce the costs, the usage of foldable containers is an effective method. The idea of foldable containers is not so new, and several container companies have developed foldable containers, such as Fallpac AB and Holland Container Innovation. Those foldable containers have equivalent storage capacity and size as standard containers and a foldable container only occupies one-quarter storage space of a standard container in folded status, as shown in Figure 1 (See Appendix A for the specification comparison between a standard container and a foldable container). After becoming empty, the foldable containers will be in folded status for the storage in the depots or for the repositioning to other ports. As four foldable empty containers in the folded status equal one standard empty container, it saves 75% storage space by using foldable containers, which leads to significant decrease in the repositioning cost and the storage cost. However, using the foldable containers could incur additional costs for the shipping liner. Firstly, the purchasing fee or long-term leasing cost of the foldable containers is higher than that of the standard containers. Secondly, folding and unfolding processes involve labor cost in the ports for the empty container repositioning. Therefore, there is a trade-off by using foldable containers between reducing the storage cost and the repositioning cost and incurring the additional costs.

Currently, foldable containers are not widely used in the liner shipping industry and stakeholders of the industry are trying to make the foldable containers prevalent. Here, we summary two practical concerns that may impede the usage of foldable containers at present. Firstly, maintaining a foldable container fleet needs a considerable investment at the first phrase, as the building cost of a foldable container is double as that of a standard container (Goh et al., 2016). Considering the shipping market is experiencing a depression (UNCTAD, 2016), the majority of shipping lines may not have enough funding to replace the standard containers in their container fleet with foldable containers. Secondly, folding and unfolding activities in container terminals incur additional labor operations. Henceforth, container terminals and shipping lines need to negotiate a comprehensive agreement on maintaining the operations and training technicians, which may not be achieved in the moment. Although these concerns can exist in practices, the usage of foldable containers is still promising in the near future. We take Holland Container Innovations (HCI), the manufacturer of 4FOLD foldable container, as a typical example to illustrate industry trends of using foldable containers. Holland Container Innovations (2017a) reported that some major shipping lines (e.g., APL, Samudera Indonesia and Seatrade) have used 4FOLD foldable containers in their shipping routes, and an increasing number of shipping lines have signed the contracts with HCI to promote the usage of the foldable containers, such as Emirates Shipping Line (Word Cargo News, 2017). Meanwhile, HCI is providing the folding training programs for some container terminals around the world in the preparation for using foldable containers, such as Tetris Container Terminal in Moscow, Ljubljana Container Terminal in Slovenia and Qingdao Shitengkeyun Depot.

Motivated by the above problem justifications and industry trends, our study aims to solve a problem of ship type decision considering the empty container repositioning and foldable containers, in order to minimize the total cost that occurs in a given planning horizon for the shipping route. The problem focuses on related decisions in both tactical and operational levels. In the tactical level, it first determines the ship type of the container ship fleet (denoted as ship type decision), which decides the capacity (in TEUs) of container ships deployed in the shipping route. Then, the problem determines the number of foldable and standard containers leased (or kept) in the ports initially for the usage of the planning horizon, which is a container fleet sizing in essential (denoted as long-term container leasing). In the operational level, upon each weekly service, if there are empty containers surplus in some ports, the problem decides the number of empty containers that the visiting ship should reposition to other deficit ports. In case of the empty container deficit, the shipping liner can lease empty containers in origin ports and return them in destination ports (denoted as short-term container leasing) to fulfill the transportation consignments. Here, we summarize the empty container repositioning and the container fleet sizing as the empty container allocation.

If the empty container repositioning is not involved, the ship type decision is to guarantee that the deployed ships have the capacity to accommodate all the laden container transportation and a ship fleet with a larger capacity will not be an option. However, involving empty container repositioning complicates the ship fleet deployment. The empty container repositioning provides the shipping liner with the motivation to deploy a ship fleet with a larger capacity. Although it will raise the fixed operation cost, it gives the shipping liner more flexibility and capacity to reposition empty containers among ports. Meanwhile, the container fleet sizing intertwines with the ship fleet deployment and the empty container repositioning. The container fleet sizing determines the total number of containers flowing in the planning horizon. As those containers would be either in the depots or on the ships, the effects on the ship type decision are inescapable. Normally, the larger the container fleet, the more the empty containers repositioned.

Based on above analysis, this paper presents an explorative study on the problem of ship type decision considering the empty container repositioning and foldable containers. In the study, we find that given the ship type with a specific capacity, the problem transfers to a nonstandard minimum cost flow problem, which solves the container fleet sizing and the empty container repositioning. For the nonstandard minimum cost flow problem, we build a network flow model by constructing a network for the flow of empty containers. Due to the usage of standard and foldable containers, the network inevitably has some parallel arcs sharing the same capacity restriction such that one cannot apply some standard network algorithms (e.g., the network simplex algorithm (Ahuja et al., 1993)) to solve it. To tackle the above issue, we propose a revised network simplex algorithm (an exact algorithm) by introducing dynamic capacity restrictions and revising the pivot operation of the standard algorithm. Based on the reduced costs derived from the revised network simplex algorithm, we propose a solution approach to determine the optimal ship type. By applying the solution approach, we conduct extensive experiments to find insights on the ship type decision and the foldable container usage.

The remainder of this paper is as follows: Section 2 reviews some related works. Section 3 elaborates the background information and decisions on the problem. Section 4 presents a network flow model given a ship capacity. Section 5 elaborates the developed solution approach that embeds a revised network simplex algorithm. Section 6 shows some experiments for finding insights on the ship type decision and the foldable container usage. The last section presents some conclusions.

2. Literature review

There have been numerous studies related to the ship fleet deployment and the empty container repositioning problems. For the ship fleet deployment, to the best of our knowledge, Perakis and Jaramillo (1991) was the first study to address it, in which they built integer linear programming models for the problem. Thereafter, there were generally two types of studies on the problem. One type assumes that the container shipment demand is deterministic (Gelareh and Meng, 2010; Brouer et al., 2013; Plum et al., 2013). For instance, Gelareh and Meng (2010) developed a mixed integer nonlinear programming model for a short-term fleet deployment problem, in which the optimal vessel speeds for different vessel types on different routes are considered. The other type of the studies relaxed the deterministic demand assumption and treated the container shipment demand in a stochastic manner (Meng and Wang, 2012; Ng, 2014; Ng, 2015). Meng and Wang (2012) addressed a practical ship fleet problem under the background of week-dependent container shipment demand. Their study generated practical container routes considering transit time constraints by using space-time network approach. For more works on the ship fleet deployment, one can refer to Ng (2016), in which it elaborated a class of fleet deployment models.

For the empty container repositioning or empty container allocation, Crainic et al. (1993) introduced two dynamic formulations for empty container allocation in single and multi-commodity cases, which provided a general framework to formulate this class of problems. Cheung and Chen (1998) considered a dynamic empty container allocation problem, which helped to determine the number of containers leased to fulfill the demands of customers over time. The management of importing and exporting empty containers in a port was analyzed by Li et al. (2004) based on the multi-stage inventory theory, and Markov decision processes were proposed for the problem. Li et al. (2007) extended the previous study to a multi-port application with a proposed heuristic algorithm for the problem. The empty container repositioning problem for general shipping service routes was formulated by Song and Dong (2010) based on container flow balancing mechanism. Menh and Wang (2011) embedded the empty container repositioning into the liner shipping service network design. They verified that the network considering empty container could cut down the network cost significantly. Song and Dong (2011) combined the laden container routing problem and empty container repositioning problem. With fixed vessel schedules and shipping service network, their study attempted to minimize the sum of all container related costs in routing and repositioning processes.

However, the majority of these related works treated the ship fleet deployment and the empty container repositioning as individual parts, i.e. they did not consider the two decisions in their studies simultaneously. As it shows in the introduction that the two decisions interact with each other in real-world operations, one may obtain a local optimum rather than the global optimum for the shipping service only considering a single part of the two decisions.

On the other hand, few kinds of literature have studied the empty container repositioning considering the usage of foldable containers. Konings (2005) has addressed the economic and logistical viability of using foldable containers, which enhanced the confidence to use foldable containers in sea transport. Shintani et al. (2010) investigated the impact of using foldable containers in hinterland transport, which showed that the foldable containers substantially save on the repositioning cost compared to the standard containers. Moon et al. (2013) was almost the first to model the usage of standard containers and foldable containers in empty container repositioning, and proposed a heuristic algorithm to solve their problem. Based on Moon et al. (2013), Myung and Moon (2014) found that the previous problem transfers to a minimum cost flow if they do not consider the capacity restrictions when repositioning empty containers. In our study, we recognize the trouble that when considering standard containers and foldable containers, after involving the capacity restrictions (which is compulsory, as our problem needs to determine the ship capacity), we will obtain a nonstandard minimum cost flow. Fortunately, we design a revised network simplex algorithm that can easily tackle the trouble.

Compared with the above literature, our study incorporates the empty container allocation (including the container fleet sizing and the empty container repositioning) into the ship type decision in order to obtain the global optimal solution for a shipping route over a planning horizon. It further considers the usage of foldable containers, which aims to see whether the shipping liner should apply the foldable containers in the shipping service. Given a ship type for considering capacity restrictions, it overcomes the trouble faced by Myung and Moon (2014) by designing a revised network simplex algorithm. In all, we can tackle all the above decisions by an integrated solution approach.

3. Problem description

Our problem focuses on ship type decision considering the empty container repositioning and foldable containers for a given shipping service route. A fleet of container ships with certain capacity is to be determined for weekly serving the ports along the shipping route. The incoming container ship transports weekly laden containers originating from the ports to destination ports. In each port, the empty container availability is critical for the shipping liner to meet the laden container transportation. Generally, there are three empty container supplies to fulfill laden container transportation. The economic supply is the arriving containers with fully loaded goods from the incoming ship. Once these fully loaded containers arrive at the ports, and are delivered to consignees for unloading, the containers become empty and are stored in depots for the laden consignment. The second supply is to reposition the empty containers from surplus ports to deficit ports along the shipping route, which occupy the capacity of the container ships among voyage legs. However, if the stored and repositioned empty containers cannot meet the weekly laden container transportation, the shipping liner has to lease empty containers from container companies by short-term container leasing, which is the third supply for the empty containers. The short-term container leasing is on an O-D pair basis (See Section 3.2), which is different from the long-term container leasing that is on a planning horizon basis (See Section 3.1).

Figure 2: A transpacific shipping service route operated by CMA CGM. *Source*: CMA CGM (2017)

In this study, the given shipping route has a fixed port rotation, shown by an illustrative example in Figure

2. The itinerary of this route forms a loop: we can arbitrarily deem that this itinerary starts at Shanghai and ends at Shanghai. Let $p \in P$ represent the index of the ports on a round trip for the route. Then, for this route, we can define Shanghai as Port 1, Ningbo as Port 2, Pusan as Port 3, Los Angeles as Port 4 and Oakland as Port 5. Based on the given route, the shipping liner has a set of O–D (Origin-Destination) port pairs D. The laden container transportation arises on those pairs in each week. We represent (o, d) as the index for O–D port pairs, where $o \in P, d \in P$. Note that we study the shipping routes that have no butterfly ports in the routes, i.e., the route can visit each port at most once.

For the given shipping service route, the shipping liner normally offers services for the ports on a weekly basis. In other words, there is a week interval for each port to be visited by one round trip and its next round trip. Let e represent the index of round trips, and E as the set of round trips for one planning horizon. The weekly laden container transportation consignments for each port accumulate between the visiting times of two adjacent round trips and are fulfilled by the latter round trip. Here, we denote $d_{od,e}$ as the number of laden containers accumulated in Port o between the time that the port is visited by the $(e-1)^{st}$ round trip and the time that the port is visited by the e^{th} round trip, and will be transported to Port d by the e^{th} round trip. Here, note that we use the number of round trips rather than the number of weeks to represent the planning horizon, and the time interval between two round trips for a port is one week.

The number of ships deployed in the route is pre-determined. The number of ships corresponds to the number of weeks needed for the round trip in the weekly shipping service, i.e., if a round trip for the route needs N weeks, there must be N ships deployed in the route such that the weekly shipping services can be guaranteed. Here notice that the n^{th} ship ($n \in \{1,2,\dots,N\}$) is assigned for the $\{n^{th}, (n+N)^{th}, (n+2N)^{th}, (n+3N)^{th}, \dots\}$ round trips.

3.1 Tactical level planning

In the tactical level, the problem involves two decisions. The first one is the ship type decision. The ship type decision is mainly on choosing the ship type to deploy with a certain capacity, measured by TEUs. Fulfilling the laden container transportation on the shipping service route is the basic criteria for the deployment, which means the deployed ship type must have the capacity to carry all the laden containers from origin ports to destination ports. However, there is a trade-off on whether to deploy the ships with a larger capacity or not. If the deployed ship type has a larger capacity, the fixed operation cost for the ship fleet is higher, but the larger capacity means more empty containers can be repositioned from surplus ports to deficit ports, which could save the high cost spent on short-term container leasing in those deficit ports.

The second decision at the tactical level is the container fleet sizing (or we say the long-term container leasing), which aims to determine the number of foldable empty containers and standard empty containers that are leased in the ports along the shipping route initially. All those leased empty containers construct a container fleet for the usage of the planning horizon to serve the laden container demands. Both foldable containers and standard containers are available for the long-term leasing.

3.2 Operational level planning

Once a port is visited by a round trip, the shipping liner should make some operational level decisions. To fulfill the weekly laden container transportations, the numbers of foldable containers and standard containers in the depot need to be allocated for the laden container transportation. However, if the stored empty containers are insufficient for the transportation consignment, the shipping liner has to lease empty containers from container leasing companies by short-term leasing. The leased containers return to the container leasing companies at destination ports, and the repositioning of the containers, as well as container repairs and maintenance, are the duties of the container leasing companies. The short-term leasing follows the so-called Master Lease Agreement for shipping containers (Wolff et al., 2007; Container Auction, 2017). If there are empty containers surplus after fulfilling the transportation consignment, the shipping liner needs to decide on whether to reposition those empty containers to other deficit ports, and how many empty containers to be repositioned by considering the remaining capacity of the ship when visiting the port by the round trip.

In summary, our problem aims to solve the ship type decision and the container fleet sizing in the tactical level, and allocate the empty containers among ports in the operational level. In the tactical level, the ship type decision determines the capacity (in TEUs) of the container ships deployed for the fleet. The container fleet sizing determines the number of long-term leasing foldable containers and the number of long-term leasing standard containers in each port. In the operation level, foldable empty containers and standard empty containers are allocated in each port to fulfill the laden container transportation consignment, the short-term leasing containers are involved if there is empty container deficit, and the empty container repositioning is to be determined if there is empty container surplus.

3.3 Assumptions

Before addressing the model for our problem, we clarify some underlying assumptions:

(i) In the beginning, all the ships depart from the first port in the shipping route, and they depart one by one with one-week interval (i.e., the first ship departs in the first week; the second ship departs in the second week, and so on).

Assumption (i) indicates that when a shipping liner starts to operate a weekly shipping service, it will mass a ship fleet at a homeport (i.e., the first port), and dispatch ships with a one-week interval to form the weekly service pattern.

(ii) For each port, the weekly laden container transportation consignments that accumulate between two adjacent round trips must be fulfilled by the latter round trip, i.e., those laden containers must be loaded into the incoming ship by the latter round trip.

Under assumption (ii), the accumulated consignments must be transported to destination ports by the incoming ship; otherwise, they will not arrive at customers on time.

(iii) When laden containers arrive at destination ports, it will take several weeks for devanning (i.e., the process that the containers are delivered to consignees for unpacking and returned to the ports). After the container devanning process, the laden containers become empty containers for next transportation consignments.

Assumption (iii) shows when laden containers arrive at destination ports, the containers cannot be used immediately for next consignments, as they carry cargo inside. It takes time for delivering the laden containers to customers and unloading the cargo. It is worthwhile to mention that shipping liner companies normally would pose a required time for the devanning process, upon which the consignees should return empty containers (Hanh, 2003; Shipping and Freight Resource, 2017). The required time for returning empty containers (i.e., the devanning process) vary among shipping liner companies, such as, COSCO requires ten days for returning (COSCO, 2017), OOCL allows five days for returning (OOCL, 2017). Supposing a shipping liner company sets two weeks for the required time, in reality, it may happen that consignees return empty containers earlier than two weeks (e.g., five days or ten days) in a stochastic manner. Here, to facilitate the exposition of our study, we assume all those empty containers are available for next transportation consignments after the required time for the devanning process (e.g., two weeks in the above case), even though some empty containers may be returned earlier.

The objective of our problem is to minimize the total cost over one planning horizon, including the fixed operation cost, the long-term container leasing cost, the short-term container leasing cost, the repositioning cost, and the storage cost.

4. Model formulation

In essence, our problem has a latent network structure when the capacity of the ship fleet is determined. Given the ship capacity, we can transfer the problem to *a nonstandard minimum cost flow problem* by formulating a network flow (NF) model, rather than a mathematical programming (MP) model. An MP model normally needs some commercial solvers to optimize, such as CPLEX and Gurobi, which are not desirable for many shipping liner companies. However, several network algorithms can easily solve an NF model to optimality, for example, the *network simplex algorithm*. Therefore, in this section, we construct an NF model by building a network for the problem given the ship capacity. In the next section, we will design a solution approach with a revised network simplex algorithm to solve the NF model and optimize the ship capacity.

In the section, we first introduce some notations for sets and input parameters. Then, to avoid confusion, we will build a sub-network for the problem when only considering using standard containers, which leads to a preliminary NF model (denoted as $\boldsymbol{PNF}$). In the next, we further build a sub-network for foldable containers. By incorporating it to the sub-network for standard containers, we obtain a whole network for the NF model (denoted as $\boldsymbol{TNF}$) that solves the problem given a ship capacity. However, the incorporation would impose a trouble that two parallel arcs share a specific capacity restriction, simultaneously. We will tackle this trouble when designing the revised network simplex algorithm in the next section.

4.1 Notations

***Indices and sets*:**

p: Index of ports,

P: Set of all the ports in the shipping service route,

(o, d): Index of O–D port pairs, where $o \in P, d \in P$,

D: Set of O–D port pairs and $D \coloneqq \{(o, d) \in P \times P | o \neq d\}$,

v: Index of ship types,

V: Set of ship types for the shipping service route,

e: Index of round trips,

E: Set of all the round trips in the planning horizon,

***Input parameters*:**

N: Number of ships deployed in the shipping service route,

M: Number of foldable containers that can be folded into one standard container,

C^v: Fixed operation cost in the planning horizon when the ships in Type v are deployed in the shipping service route, $v \in V$,

K^v: Container capacity of the ships in Type v with the unit of TEUs, $v \in V$,

w_p: Number of weeks that are needed for the devanning process in Port p, $p \in P$,

$d_{od,e}$: Number of laden container transportation consignments from Port o to Port d that should be transported by the e^{th} round trip, $(o, d) \in D, e \in E$,

s_p^S: Unit weekly storage cost of a standard container in Port p, $p \in P$,

s_p^F: Unit weekly storage cost of a foldable container in Port p, $p \in P$,

r_{od}^S: Unit repositioning cost of a standard container from Port o to Port d, including container loading cost (a_o^S) and unloading cost (b_d^S), $(o, d) \in D$,

r_{od}^F: Unit repositioning cost of a foldable container from Port o to Port d, including container loading cost (a_o^F) and unloading cost (b_d^F), folding cost (A_o) and unfolding cost (B_d), $(o, d) \in D$,

l_{od}: Unit short-term leasing cost of a standard container from Port o to Port d, $(o, d) \in D$,

L_p^S: Long-term leasing cost of a standard container for the planning horizon usage in Port p, $p \in P$,

L_p^F: Long-term leasing cost of a foldable container for the planning horizon usage in Port p, $p \in P$.

Here, notice that "from Port o to Port d by the e^{th} round trip" means that the containers are loaded from Port o to the ship when the port is visited by the e^{th} round trip, and those containers will be transported to Port d. If $o < d$, the containers will arrive at Port d by the e^{th} round trip; If $o > d$, the containers will arrive at Port d by the $(e + N)^{th}$ round trip, as the ship would return to Port 1 and restart a new round trip.

4.2 A preliminary NF model for standard containers

Given the type of ships deployed has the capacity in K TEUs, we only allow using standard containers to transport goods in this subsection. The sub-network built for standard containers will firstly embed the decisions on the container fleet sizing (i.e., the long-term container leasing) and the empty container repositioning, as the decisions are also the same for foldable containers. Then, we extend the sub-network to consider the short-term container leasing, which only involves the standard containers.

4.2.1 Long-term containers leasing and empty container repositioning

Before constructing the sub-network for standard containers, we divide r_{od}^S (i.e., unit repositioning cost of a standard container from Port o to Port d) into two parts, i.e., the loading cost in Port o (denoted as a_o^S) and the unloading cost in Port d (denoted as b_d^S), where $r_{od}^S = a_o^S + b_d^S$ as suggested in the parameter definition.

Notice that the imbalance of laden container flow leads to the empty container repositioning among the ports. As all laden container transportation consignments (i.e., $d_{od,e}$) have to be fulfilled by the e^{th} round trip, we could analyze the empty containers deficit or surplus in each port on each round trip caused by the imbalance of laden container flow. Here, we denote $Q_{p,e}$ as the number of empty containers deficit or surplus in Port p when visited by the e^{th} round trip, calculated by Eq. (1) and Eq. (2).

$$Q_{p,e} = -\sum_{o=p,(o,d)\in D} d_{od,e} \qquad \forall p \in P, e = 1, \tag{1}$$

$$Q_{p,e} = \sum_{d=p,o<d,(o,d)\in D} d_{od,e-w_p} + \sum_{d=p,o>d,(o,d)\in D} d_{od,e-N-w_p} - \sum_{o=p,(o,d)\in D} d_{od,e} \qquad \forall p \in P, e \in E/\{1\}, \tag{2}$$

If $Q_{p,e} > 0$ ($Q_{p,e} \leq 0$), it indicates that there are $Q_{p,e}$ ($-Q_{p,e}$) number of empty containers surplus (deficit) in Port p when visited by the e^{th} round trip. The long-term containers leasing generates empty containers, and the empty container repositioning induces the empty container flow between the deficit ports (i.e., the demand ports) and the surplus ports (i.e., the supply ports). Henceforth, we can construct a flow network for the empty containers.

The sub-network contains $|P| + 2 \cdot |E| \cdot |P| + 2$ nodes. Among them, the $|P|$ nodes define the flow conversion for the long-term container leasing in Port p at the beginning of the planning horizon, labelled as $Lterm^S\_p$. The $2 \cdot |E| \cdot |P|$ nodes are categorized into $|E| \cdot |P|$ groups of two kinds of nodes. One kind of nodes in a group denotes the flow conservation in the ship for the e^{th} round trip after visiting Port p, labeled as $Ship^S\_p\_e$. The other kind of nodes denotes the flow conservation in Port p after visited by the e^{th} round trip, labeled as $Port^S\_p\_e$. Two additional dummy nodes represent the source node (labelled as $Source$) and the sink node (labelled as $Sink$) respectively. To facilitate the understanding of the sub-network construction, we give a shipping service route for the example through this section. The route is: Singapore (1) → Hong Kong (2) → Xiamen (3) → Singapore. A round trip for the route needs two weeks, which means two liner ships are deployed with the capacity as K. A planning horizon includes four round trips. Then, Figure 3 illustrates an example for the sub-network construction.

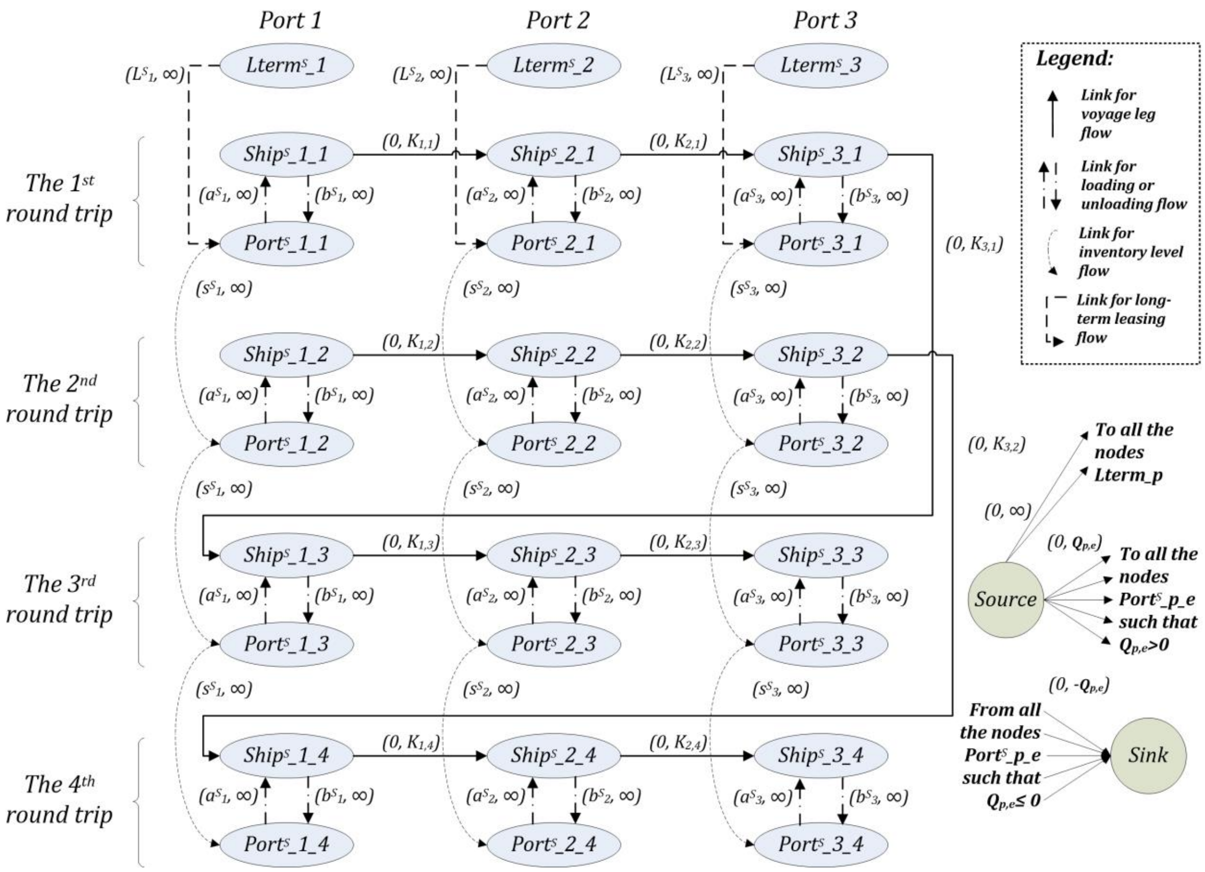

Figure 3: A sub-network for standard containers without considering the short-term container leasing

In the sub-network network for standard containers, the arcs between nodes correspond to the decision variables of the problem.

- The arc from $Lterm^S\_p$ to $Port^S\_p\_1$ represents the long-term container leasing activity in Port p. The amount of flow on the arc denotes the number of empty containers leased for the planning horizon usage. The cost for the arc is L_p^S. The capacity on the arc is ∞.
- The arc from Node $Ship^S\_p\_e$ to $Ship^S\_(p+1)\_e$ represents the voyage leg from Port p to Port $p+1$ by the ship for the e^{th} round trip. The amount of the flow on the arc denotes the number of empty containers carried on the ship in the voyage leg. Here note that: (i) The voyage leg from Port $|P|$ to Port 1 by the same ship for the e^{th} round trip is the arc from Node $Ship\_|P|\_e$ to $Ship\_1\_(e+N)$. (ii) The cost for the arc is zero, as we have decomposed the repositioning cost. (iii) As the flows of laden containers are pre-determined, the remaining capacity in each voyage leg (on each arc) is confirmed, denoted as $K_{p,e}$ (the remaining capacity in the voyage leg from Port p to Port $p+1$ by the ship for the e^{th} round trip), which can be calculated as follows.

$$K_{p,e} = K - \sum_{o\le p, d>p, (o,d)\in D} d_{od,e} - \sum_{d\ge p+1, o>d, (o,d)\in D} d_{od,e-N} \quad \forall p \in P, e \in E, \tag{3}$$

- The arc from Node $Port^S\_p\_e$ to $Ship^S\_\_p\_e$ (resp., Node $Ship^S\_p\_e$ to $Port^S\_p\_e$) represents the empty container loading process from Port p to the ship (resp., unloading process from the ship to Port

p) for the e^{th} round trip. The amount of the flow on the arc denotes the number of empty containers loaded to the ship (resp., unloaded to the port), and the cost for the arc is a_p^S (resp., b_p^S), i.e., the container loading cost at Port p (resp., the container unloading cost at Port p). The capacity on the arc is ∞.

- The arc from Node $Port^S\_p\_e$ to $Port^S\_p\_(e+1)$ represents the empty container inventory (i.e., the empty containers left) in Port p after visited by the e^{th} round trip. The amount of the flow on the arc denotes the number of empty containers left in the port after visited by the e^{th} round trip (i.e., the inventory level $\delta_{p,e}^S$). The cost for the arc is s_p. The capacity on the arc is ∞.

With the two additional dummy nodes, i.e., Node $Source$ and Node $Sink$, we firstly construct the dummy arcs from Node $Source$ to all the nodes $Lterm^S\_p$ with zero cost coefficient and infinity capacity. Then, we build the dummy arcs from Node $Source$ to all the nodes $Port^S\_p\_e$ such that $Q_{p,e} > 0$ (resp., from all the nodes $Port\_p\_e$ such that $Q_{p,e} \leq 0$ to Node $Sink$). The costs for all the arcs are zero. The capacity on the arc from Node $Source$ to Node $Port^S\_p\_e$ is $Q_{p,e}$ (resp., from Node $Port^S\_p\_e$ to $Sink$ is $-Q_{p,e}$).

4.2.2 Short-term container leasing

We extend the previous sub-network to consider the short-term container leasing. Referring to the concept of short-term container leasing in Section 3, if empty containers in origin ports are scarce, the shipping liner has to lease empty containers in the ports for laden container transportation consignments, and return those containers at destination ports after unpacking the containers. In essence, the short-term container leasing has no effects on laden containers, as all transportation consignments must be fulfilled. However, it affects the empty container demand in the origin ports and the empty container supply in the destination ports. Specifically, assuming that when Port o is visited by the e^{th} round trip, due to the dearth of empty containers in the port, the shipping liner has to lease some empty containers (say $\gamma_{od,e}$) to transport the goods in laden containers to Port d. The short-term container leasing fulfills $\gamma_{od,e}$ empty container demand in Port o, but $\gamma_{od,e}$ empty containers are excluded in the empty container supply in Port d as those leased empty containers have to be returned to container leasing companies. Therefore, the empty container demand in Port o by the e^{th} round trip decreases by $\gamma_{od,e}$, and the empty container supply in Port d by the $(e+w_d)^{th}$ (if $d > o$) or $(e+N+w_d)^{th}$ (if $d < o$) round trip decreases by $\gamma_{od,e}$, virtually.

Based on the above analysis, if we further consider the short-term container leasing, the previous sub-network without considering the short-term container leasing (i.e., Figure 3) should be modified as follows (see Figure 4 for the following modifications): (i) We add a pairwise node for each node $Port^S\_p\_e$, denoted as $BPort^S\_p\_e$. Here, Node $BPort^S\_p\_e$ (resp., Node $Port^S\_p\_e$) shows the empty container flow before (resp., after) using the short-term container leasing. (ii) We disconnect the dummy arcs from Node

$Source$ to all the nodes $Port^S\_p\_e$ and from all the nodes $Port^S\_p\_e$ to Node $Sink$. (iii) We construct the dummy arcs from Node $Source$ to all the nodes $BPort^S\_p\_e$ such that $Q_{p,e} > 0$ (resp., from all the nodes $BPort^S\_p\_e$ to Node $Sink$ such that $Q_{p,e} \leq 0$). The costs for all the arcs are zero. The capacity on the arc from Node $Source$ to $BPort^S\_p\_e$ is $Q_{p,e}$ (resp., from Node $BPort^S\_p\_e$ to $Sink$ is $-Q_{p,e}$). (iv) If $Q_{p,e} > 0$ ($Q_{p,e} \leq 0$), we construct the dummy arc from Node $BPort^S\_p\_e$ to $Port^S\_p\_e$ (resp., from Node $Port^S\_p\_e$ to $BPort^S\_p\_e$) with arc capacity as ∞ and arc cost as zero. (v) For two nodes $BPort^S\_p\_e$, there exist an origin node $BPort^S\_o\_e$ and a destination node $BPort^S\_d\_e^1$ (if $d > o$, $e^1 = e + w_d$; if $d < o$, $e^1 = e + N + w_d$). When there is empty container deficit in Node $BPort^S\_o\_e$ (i.e., $Q_{o,e} \leq 0$), the short-term container leasing is possible in the node, and the arc from Node $BPort^S\_d\_e^1$ to $BPort^S\_o\_e$ for the short-term container leasing is constructed. Notice that for the voyage direction, the laden containers flow from Node $BPort^S\_o\_e$ to $BPort^S\_d\_e^1$, but here, the empty containers flow in the opposite direction. The capacity for the arc is $d_{od,e}$ as the maximum empty container demand from Port o to d is the laden container demand $d_{od,e}$. The cost on the arc is l_{od} (i.e., the short-term leasing cost).

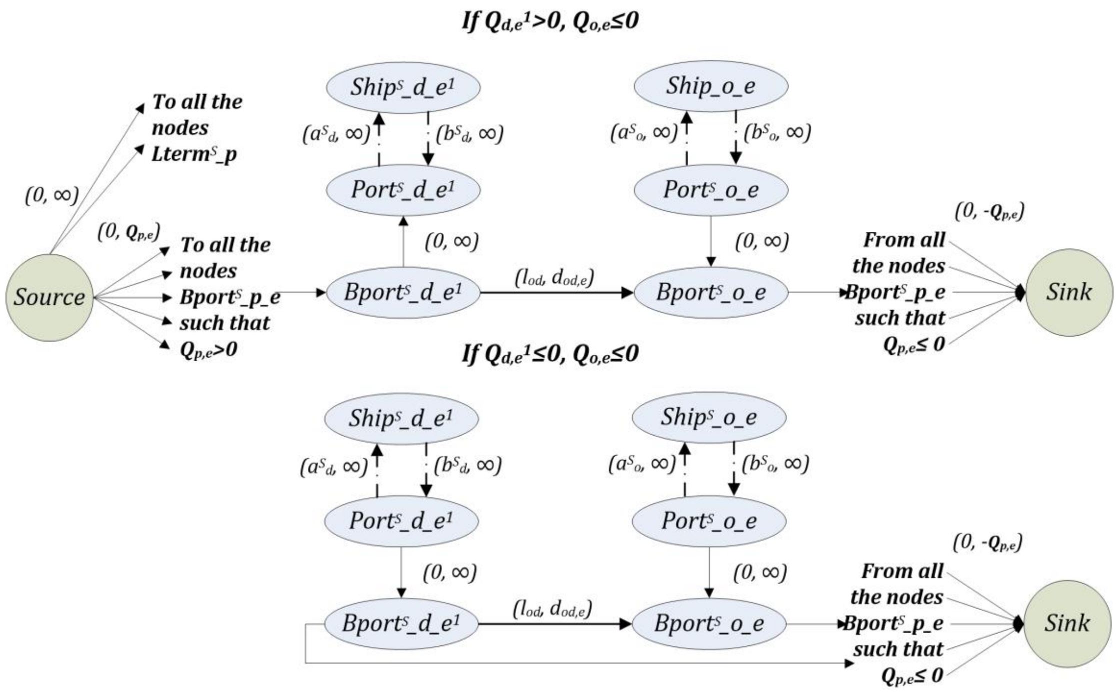

Figure 4: The sub-network defined for the preliminary NF model

It is worthwhile to mention that in this subsection, we refer the short-term container leasing to the Master Lease Agreement mentioned in Section 3.2, in which empty containers are leased and the cost is charged on an O-D port pair basis. In real-world operations, the shipping liner company and container leasing company could have a lease agreement that is on a round trip basis. Under the agreement, the shipping liner company can lease a certain number of empty containers at Port p, and this number of empty containers must be returned at Port p after several weeks (or round trips). If this is the agreement applied between some shipping liner companies and container leasing containers, we can make the following modifications to the network. Firstly, we define g_p as the number of weeks that the shipping liner company needs to return leased empty containers at Port p, and define q_p as the unit cost of leasing empty containers for g_p weeks at Port p. Then, for modifying the network, we connect the arcs from $Port^S\_p\_(e+q_p)$ to $Port^S\_p\_e$ with arc capacity as ∞ and arc cost as g_p. Note that although some empty containers (say ξ empty containers) are leased by the e^{th} round trip and are returned by the $(e+q_p)^{th}$ round trip at Port p, the empty container flow in the network is in the opposite direction. This is due to that the empty container supply by the $(e+q_p)^{th}$ round trip (resp., the e^{th} round trip) at Port p will decrease by (resp., increase by) ξ empty containers for the returning process (resp., the leasing process).

Until now, we have constructed a whole sub-network for the problem when only considering using standard containers, which leads to the preliminary NF model ($\boldsymbol{PNF}$ model). For $\boldsymbol{PNF}$ model, we need to fulfill Node $Sink$ with total empty container demand $\sum_{p\in P,e\in E}(-Q_{p,e})^+$ by originating from Node $Source$. The goal is to minimize the total cost through the sub-network, which is a standard minimum cost flow problem.

4.3 The NF model for both standard containers and foldable containers

In this subsection, we incorporate the foldable containers into $\boldsymbol{PNF}$ model for the NF model that solves our problem given the ship capacity K ($\boldsymbol{TNF}$ model). Firstly, we construct a sub-network for foldable containers. Except from the short-term container leasing, the foldable containers also have the long-term container leasing and the empty container repositioning. Henceforth, the sub-network for foldable containers is similar to the sub-network shown for the standard containers in Figure 3. Figure 5 shows an example for the sub-network for foldable containers, in which the nodes $Ship^F\_p\_e$, $Port^F\_p\_e$ and $Lterm^F\_p$ correspond to the nodes $Ship^S\_p\_e$, $Port^S\_p\_e$ and $Lterm^S\_p$ in Figure 3, respectively. Note that the costs on the arcs from Node $Port^F\_p\_e$ to $Ship^F\_\_p\_e$ and from Node $Ship^F\_p\_e$ to $Port^F\_p\_e$ are slightly different from that of standard containers, as there are additional folding and unfolding costs (A_o and B_d) for foldable containers.

To embed the sub-network for the foldable containers into $\boldsymbol{PNF}$ model defined in Section 4.2, we need to build some connections as shown in Figure 6: (i) we add a pairwise dummy node for nodes $BPort^S\_p\_e$ and $Port^F\_p\_e$, denoted as $TPort\_p\_e$. Here, Node $TPort\_p\_e$ accumulates the empty container surplus or deficit for standard containers and foldable containers; (ii) we disconnect all dummy arcs from Node $Source$ to all other nodes or from all other nodes to Node $Sink$; (iii) we construct the dummy arcs from Node $Source$ to all the nodes $TPort\_p\_e$ such that $Q_{p,e}>0$ (resp., from all the nodes $TPort\_p\_e$ to Node $Sink$ such that $Q_{p,e}\le 0$). The costs for all the arcs are zero. The capacity on the arc from Node $Source$ to $TPort\_p\_e$ is $Q_{p,e}$ (resp., from Node $BPort^S\_p\_e$ to $Sink$ is $-Q_{p,e}$); (iv) if $Q_{p,e}>0$, we construct the dummy arcs from Node $TPort\_p\_e$ to $BPort^S\_p\_e$ and Node $TPort\_p\_e$ to $Port^F\_p\_e$, to split empty container flow to the sub-network of standard containers and foldable containers, respectively. (v) if $Q_{p,e} < 0$, we construct the dummy arcs from Node $BPort^S\_p\_e$ to $TPort\_p\_e$ and Node $Port^F\_p\_e$ to $TPort\_p\_e$ to accumulate empty container flow from the sub-network of standard containers and foldable containers, respectively. Note that $TPort\_p\_e$ serve as the node to allocate the empty container demand to standard containers by node $BPort^S\_p\_e$ and foldable containers by node $Port^F\_p\_e$.

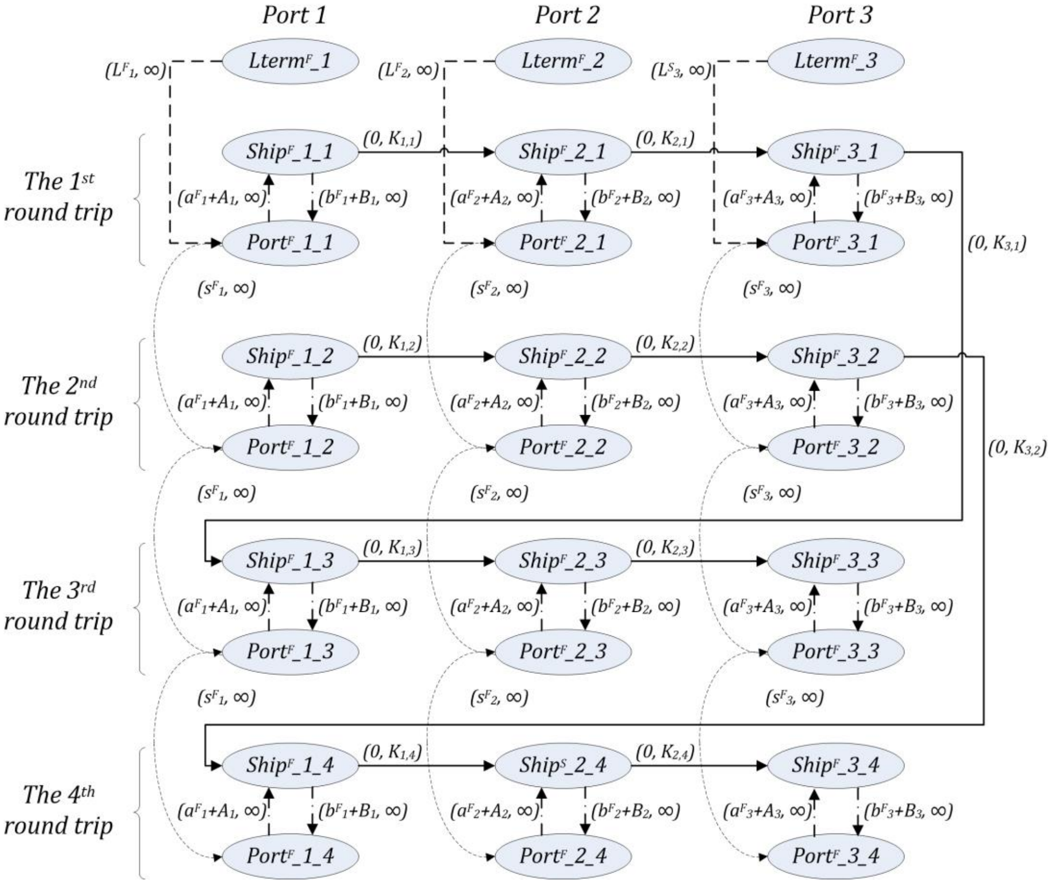

Figure 5: A sub-network for foldable containers

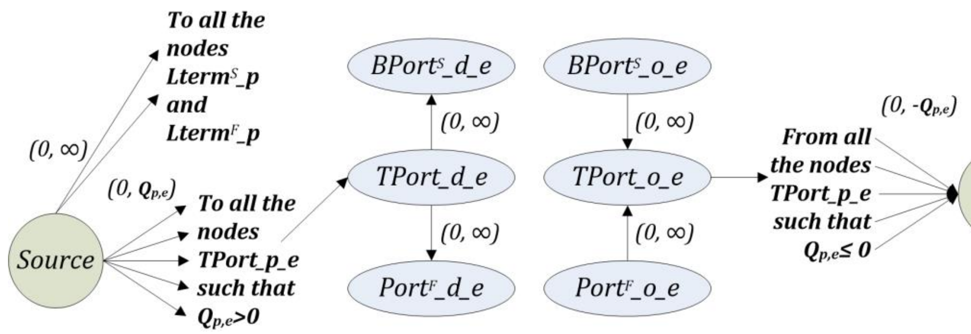

Figure 6: Connections between standard containers and foldable containers in $\boldsymbol{TNF}$ model

As we have constructed the whole flow network for our problem, it is worthwhile to show the total number of nodes involved in the network by referring Figure 3 to Figure 6. Figure 3 defines the subnetwork for standard containers and it contains $|P| + 2|P||E|$ nodes; Figure 4 introduces $BPort^S\_p\_e$ nodes for considering the short-term container leasing and it adds $|P||E|$ nodes to the network; Figure 5 defines the subnetwork for foldable containers and it adds $|P| + 2|P||E|$ nodes to the network; Figure 6 introduces $TPort\_p\_e$ nodes for combining two subnetworks and it adds $|P||E|$ nodes to the network. In all, the network has $|P| + 6|P||E| + 2$ after adding the dummy $Source$ and $Sink$ nodes, which is a polynomial function to the input parameters $|P|$ and $|E|$.

4.3.1 Capacity restriction sharing

The above incorporation process of the two sub-networks for $\boldsymbol{TNF}$ model induces a trouble that makes our problem a nonstandard minimum cost flow problem. That is two parallel arcs sharing a specific capacity restriction, simultaneously. In Figure 3, the arc from Node $Ship^S\_p\_e$ to $Ship^S\_(p+1)\_e$ carries the flow of standard empty containers on the voyage leg from Port p to Port $p+1$ by the ship for the e^{th} round trip. In Figure 5, the arc from Node $Ship^F\_p\_e$ to $Ship^F\_(p+1)\_e$ carries the flow of foldable empty containers on the same voyage leg. Henceforth, the two parallel arcs share the same capacity restriction $K_{p,e}$, as they require repositioning empty containers on the same ship at the same voyage. More importantly, the flows on the parallel arcs occupy different units of the capacity, due to the feature of foldable containers. By supposing $x^S_{p,e}$ and $x^F_{p,e}$ are the flow on the two parallel arcs, we need to enforce that:

$$0 \le x^S_{p,e} + \frac{x^F_{p,e}}{M} \le K_{p,e} \qquad \forall p \in P, e \in E, \qquad (4)$$

for the capacity restriction sharing. This characteristic leads to the trouble that we cannot directly transfer our problem given the ship capacity to a standard minimum cost flow problem. Alternatively, to avoid the trouble, we introduce two dynamic capacity restrictions for the two parallel arcs, denoted as $\mu^S_{p,e}$ and $\mu^F_{p,e}$, respectively. Based on $\mu^S_{p,e}$ and $\mu^F_{p,e}$, we separate the sharing capacity restriction $K_{p,e}$ to two individual ones such that:

$$0 \le x^S_{p,e} \le \mu^S_{p,e} \qquad \forall p \in P, e \in E, \qquad (5)$$

$$0 \le x^F_{p,e} \le \mu^F_{p,e} \qquad \forall p \in P, e \in E. \qquad (6)$$

Meanwhile, the two individual capacity restrictions must hold that:

$$\mu^S_{p,e} + \frac{\mu^F_{p,e}}{M} = K_{p,e} \qquad \forall p \in P, e \in E. \qquad (7)$$

Note $\mu^S_{p,e}$ and $\mu^F_{p,e}$ keep dynamically changed in the algorithm developed in the next section, so long as EQ.(7) holds. Here, we can initialize any values for $\mu^S_{p,e}$ and $\mu^F_{p,e}$ without considering that the values will lead to no feasible solutions for the $\boldsymbol{TNF}$ model. This is due to that the short-term container leasing of standard containers can always treat the empty container imbalance between deficit nodes and surplus nodes, as suggested in Section 4.2.2. With the separated capacity restrictions, we can solve the $\boldsymbol{TNF}$ model by several network algorithms.

5. Solution approach

The solution approach for our problem is an iterative procedure. In this section, we firstly design a revised network simplex algorithm to solve the $\boldsymbol{TNF}$ model given a ship capacity. Then, for the solution approach, we initialize at $K = K^1$ that is the capacity of Type 1 ship (assuming to be the smallest ship type) for running the algorithm. Based on the information from the previous running of the algorithm, we select another ship capacity K^v, given which it again invokes the algorithm to derive the minimum flow cost for empty containers, denoted as σ^v. The summation of the fixed operation cost C^v and σ^v is the total minimum cost when using ships in Type v for the fleet. By comparing those costs, we can obtain the optimal solution for our problem as well as the total optimal cost. In this section, we also formulate a mixed-integer linear programming model (MILP) for the problem, which can be solved directly by CPLEX solver for verifying the optimality of the proposed solution approach.

5.1 A revised network simplex algorithm

The network simplex algorithm is perhaps the most powerful algorithm to solve the minimum cost flow problem. The *Cycle Free Property*, *Spanning Tree Property* and *Minimum Cost Flow Optimality Conditions* are the major principles supporting the algorithm. One can refer to Chapter 11 of the book by Ahuja et al. (1993) for detailed descriptions of those properties and the algorithm. Here, we will briefly introduce the properties and the algorithm, and propose a revised network simplex algorithm for solving our $\boldsymbol{TNF}$ model. Note that to present the algorithm in a general way and avoid confusions, we use the set $\mathcal{A}$ to denote the set of all arcs in the network constructed in Section 4. Arc $(i,j) \in \mathcal{A}$ denotes a specific arc from node i to node j in the network with a cost coefficient c_{ij} and a capacity restriction μ_{ij}.

(i) *Spanning Tree Property*: If a minimum cost flow problem is bounded from below by a feasible region, the problem will always have an optimal spanning tree solution. A spanning tree solution splits the arc set $\mathcal{A}$ of the network into three parts. (a) $\boldsymbol{T}$, the spanning tree arc set, in which the flow on the arcs are unbounding. (b) $\boldsymbol{L}$, a non-tree arc set, in which the flow on the arcs equals zero. (c) $\boldsymbol{U}$, a non-tree arc set, in which the flow on the arcs equals the capacity restrictions. The triple $(\boldsymbol{T}, \boldsymbol{L}, \boldsymbol{U})$ is the so-called spanning tree structure.

(ii) *Minimum Cost Flow Optimality Conditions*: A feasible spanning tree $(\boldsymbol{T}, \boldsymbol{L}, \boldsymbol{U})$ is the optimal solution if the *arc reduced costs* c_{ij}^{π} satisfy the following conditions. (a) For all $(i,j) \in \boldsymbol{T}$, $c_{ij}^{\pi} = 0$; (b) For all $(i,j) \in \boldsymbol{L}$, $c_{ij}^{\pi} \geq 0$; (c) For all $(i,j) \in \boldsymbol{U}$, $c_{ij}^{\pi} \leq 0$. Given a spanning tree structure, c_{ij}^{π} is derived by using the cost coefficients on arcs and defined node potentials for nodes: it first assigns a node potential $\pi(1) = 0$ for the root node 1. Then by holding,

$$c_{ij}^{\pi} = c_{ij} - \pi(i) + \pi(j) = 0 \qquad \forall (i,j) \in \boldsymbol{T}, \tag{8}$$

it obtains all the nodes potentials $\pi(i)$ in the network, based on which c_{ij}^{π} of the non-tree arcs is derived by $c_{ij}^{\pi} = c_{ij} - \pi(i) + \pi(j), \forall (i,j) \in \boldsymbol{L}, (i,j) \in \boldsymbol{U}$. Note that c_{ij}^{π} has a similar meaning with the reduced cost in the *primal simplex algorithm* for the *linear programming problem*, which indicates the cost changed in the objective if we increase one more unit flow on the arc (i, j).

(iii) The network simplex algorithm generally maintains a feasible spanning tree structure in each iteration and moves from one structure to another one until reaching the optimality. (a) Given a $(\boldsymbol{T}, \boldsymbol{L}, \boldsymbol{U})$, the algorithm checks the optimality conditions. If all the conditions are satisfied, the algorithm stops, otherwise selects the most violation arc (by finding the maximum $|c_{ij}^{\pi}|$) from the arc set $\mathcal{V} = \{(i,j) \in \boldsymbol{L} \cup \boldsymbol{U}\colon\ if\ \ (i,j) \in \boldsymbol{L}, c_{ij}^{\pi} < 0;\ \ if\ \ (i,j) \in \boldsymbol{U}, c_{ij}^{\pi} > 0\}$. (b) It adds the violation arc (called the *entering arc*) into the current spanning tree structure, by which it obtains a negative cost cycle. That means increasing the flow on the forward direction of the cycle will decrease the current objective of the total flow cost. (c) It augments the maximum possible flow on the negative cost cycle until the flow of one arc in the cycle reaches zero or its capacity restriction (called the *leaving arc*). (d) It replaces the *leaving arc* with the *entering arc* for a new feasible spanning tree structure $(\boldsymbol{T}', \boldsymbol{L}', \boldsymbol{U}')$. The algorithm repeats the above procedure (a)-(d) until finding the optimal spanning tree structure $(\boldsymbol{T}^*, \boldsymbol{L}^*, \boldsymbol{U}^*)$.

5.1.1 Revisions in the pivot operation

The Part (c) in the above procedures of the network simplex algorithm is the *pivot operation*. Considering that our network constructed in Section 4 has dynamic capacity restrictions on parallel arcs (see Section 4.3.1), we revise the pivot operation of the standard network simplex algorithm for a revised network simplex algorithm to solve our $\boldsymbol{TNF}$ model. Here, we will firstly elaborate the standard pivot operation and then propose our revisions.

Supposing that the entering arc is (k, l), the combination of (k, l) and the spanning tree $\boldsymbol{T}$ forms a negative cost cycle $\boldsymbol{W}$, known as the *pivot cycle*. The cycle $\boldsymbol{W}$ has a direction that is the same as (k, l) if $(k, l) \in \boldsymbol{L}$, and is opposite to (k, l) if $(k, l) \in \boldsymbol{U}$. For all arcs in the cycle, the arcs following the cycle direction belong to a *forward arc* set $\overline{\boldsymbol{W}}$, and the arcs following the opposite cycle direction belong to a *backward arc* set $\underline{\boldsymbol{W}}$. Figure 7 shows an example for the cycle, in which arc $(3, 4)$ is the entering arc, arcs $(5, 0)$, $(0, 1)$, $(2, 3)$ belong to the set $\overline{\boldsymbol{W}}$, and arcs $(5, 4)$, $(2, 1)$ belong to the set $\underline{\boldsymbol{W}}$.

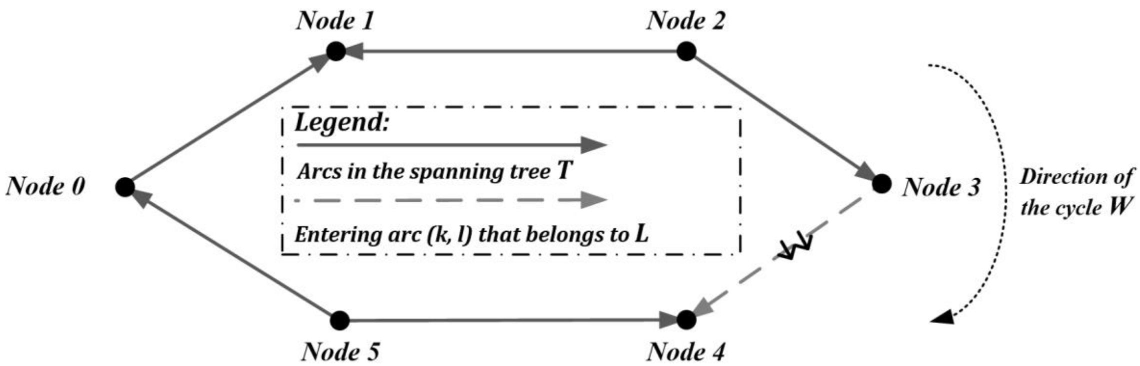

Figure 7: An example of the pivot cycle $\boldsymbol{W}$

Supposing that x_{ij} is the existing flow on arc (i,j), the maximum flow change on each arc in the cycle is denoted by δ_{ij}, calculated by:

$$\delta_{ij} = \begin{cases} \mu_{ij} - x_{ij}, if \ \ (i,j) \in \overline{\boldsymbol{W}} \\ x_{ij} \qquad\quad , if \ \ (i,j) \in \underline{\boldsymbol{W}} \end{cases} \tag{9}$$

In the standard pivot operation, it will augment $\delta^* = \min\{\delta_{ij}: (i,j) \in \boldsymbol{W}\}$ amount of flow to guarantee the feasibility. Here, we start to revise the pivot operation to capture the dynamic capacity restrictions under two principles: (i) we need to make δ^* as large as possible. This is due to $\boldsymbol{W}$ is a negative cost cycle such that the more flow augmented on the cycle, the larger decreasing on the total flow cost. (ii) We must guarantee the feasibility when augmenting flow.

To facilitate our description of revisions for the pivot operation, we define $\epsilon_{p,e}^S$ and $\epsilon_{p,e}^F$ as the indices for two parallel arcs mentioned in Section 4.3.1 rather than still use (i,j) to denote the arcs. Then, we have two parallel arc sets $\mathcal{G}^S$ and $\mathcal{G}^F$, where $\epsilon_{p,e}^S \in \mathcal{G}^S \subset \mathcal{A}$ and $\epsilon_{p,e}^F \in \mathcal{G}^F \subset \mathcal{A}$. Supposing that the current allocated capacity for two parallel arcs are $\mu_{p,e}^S$ and $\mu_{p,e}^F$ satisfying EQ.(5)-(7), we will dynamically change the capacity allocation in the revised pivot operation to reach the optimality and guarantee the feasibility. Here, we suppose that $x_{p,e}^S$ and $x_{p,e}^F$ are existing flow on the two parallel arcs.

In the revised pivot operation, we need to check each pair of parallel arcs on the cycle $\boldsymbol{W}$. Basically, we need to distinguish three situations for one of two parallel arcs, i.e., (i) the arc belongs to the forward arc set $\overline{\boldsymbol{W}}$; (ii) the arc belongs to the backward arc set $\underline{\boldsymbol{W}}$; (iii) the arc does not belong to the cycle $\boldsymbol{W}$. Then, for an integration of two parallel arcs, there are nine scenarios, for each of which we have different ways to change the capacity restrictions for the parallel arcs.

Scenario 1: if $\epsilon_{p,e}^S \in \overline{\boldsymbol{W}}$ and $\epsilon_{p,e}^F \in \overline{\boldsymbol{W}}$, this is the scenario that needs the most attention, as we need to care about their capacity restrictions simultaneously based on EQ.(8). To make δ^* larger, we need to change the capacity restrictions by solving the following optimization problem, where $\mu_{p,e}^S$ and $\mu_{p,e}^F$ are decision variables.

$$[\boldsymbol{M1}] \quad \max\{\min\{\mu_{p,e}^S - x_{p,e}^S, \mu_{p,e}^F - x_{p,e}^F\}\} \tag{10}$$

subject to:

$$\mu_{p,e}^S + \frac{\mu_{p,e}^F}{M} = K_{p,e} \tag{11}$$

$$\mu_{p,e}^S \geq x_{p,e}^S \tag{12}$$

$$\mu_{p,e}^F \geq x_{p,e}^F \tag{13}$$

where objective (9) is equivalent to $\max\{(\mu_{p,e}^S - x_{p,e}^S) \times (\mu_{p,e}^F - x_{p,e}^F)\}$ holding constraints (12) and (13). Then, we can easily obtain closed-form solutions for the $\boldsymbol{M1}$ such that ${\mu_{p,e}^S}^* = \frac{MK_{p,e}+Mx_{p,e}^S-x_{p,e}^F}{2M}$ and ${\mu_{p,e}^F}^* = \frac{MK_{p,e}-Mx_{p,e}^S+x_{p,e}^F}{2}$.

Scenario 2: if $\epsilon_{p,e}^{S} \in \overline{\boldsymbol{W}}$ and $\epsilon_{p,e}^{F} \in \underline{\boldsymbol{W}}$, as the capacity restriction is not important for arc $\epsilon_{p,e}^{F}$ based on EQ.(8), we adjust ${\mu_{p,e}^{F}}^* = x_{p,e}^{F}$ and ${\mu_{p,e}^{S}}^* = K_{p,e} - \frac{{\mu_{p,e}^{F}}^*}{M} = K_{p,e} - \frac{x_{p,e}^{F}}{M}$ such that the capacity restriction on arc $\epsilon_{p,e}^{S}$ is maximum. Note that in this scenario, we need to pay more attention on the flow after the revised pivot operation. Supposing that we argument δ^* on the cycle, and $\epsilon_{p,e}^{S}$ is the leaving arc such that ${\mu_{p,e}^{S}}^* - x_{p,e}^{S} - \delta^* = 0$, we can further argument more flow after the revised pivot operation. This is due to the flow on arc $\epsilon_{p,e}^{F}$ will decrease by δ^*, under which condition we can further decrease its capacity to ${\mu_{p,e}^{F}}^* = x_{p,e}^{F} - \delta^*$ and increase the capacity of arc $\epsilon_{p,e}^{S}$ to ${\mu_{p,e}^{S}}^* = K_{p,e} - \frac{{\mu_{p,e}^{F}}^*}{M} = K_{p,e} - \frac{x_{p,e}^{F} - \delta^*}{M}$. We repeat the above process until $\epsilon_{p,e}^{S}$ is not the leaving arc. This leads to the following proposition:

Proposition 1*: if $\epsilon_{p,e}^{S} \in \overline{\boldsymbol{W}}$ and $\epsilon_{p,e}^{F} \in \underline{\boldsymbol{W}}$, the arc $\epsilon_{p,e}^{S}$ cannot be the leaving arc.*

Scenario 3: if $\epsilon_{p,e}^{S} \in \overline{\boldsymbol{W}}$ and $\epsilon_{p,e}^{F} \notin \boldsymbol{W}$, as the arc $\epsilon_{p,e}^{F}$ is not in the cycle, its flow will not change during the revised pivot operation. Then, we adjust ${\mu_{p,e}^{F}}^* = x_{p,e}^{F}$ and ${\mu_{p,e}^{S}}^* = K_{p,e} - \frac{{\mu_{p,e}^{F}}^*}{M} = K_{p,e} - \frac{x_{p,e}^{F}}{M}$ such that the capacity restriction on arc $\epsilon_{p,e}^{S}$ is maximum.

Scenario 4: if $\epsilon_{p,e}^{S} \in \underline{\boldsymbol{W}}$ and $\epsilon_{p,e}^{F} \in \overline{\boldsymbol{W}}$, the operation is similar to **Scenario 2**, which is not repeated here. We can also obtain a proposition that is similar to Proposition 1:

Proposition 2*: if $\epsilon_{p,e}^{S} \in \underline{\boldsymbol{W}}$ and $\epsilon_{p,e}^{F} \in \overline{\boldsymbol{W}}$, the arc $\epsilon_{p,e}^{F}$ cannot be the leaving arc.*

Scenario 5: if $\epsilon_{p,e}^{S} \in \underline{\boldsymbol{W}}$ and $\epsilon_{p,e}^{F} \in \underline{\boldsymbol{W}}$, we do not change the capacity restrictions.

Scenario 6: if $\epsilon_{p,e}^{S} \in \underline{\boldsymbol{W}}$ and $\epsilon_{p,e}^{F} \notin \boldsymbol{W}$, we do not change the capacity restrictions.

Scenario 7: if $\epsilon_{p,e}^{S} \notin \boldsymbol{W}$ and $\epsilon_{p,e}^{F} \in \overline{\boldsymbol{W}}$, the operation is similar to **Scenario 3**, which is not repeated here.

Scenario 8: if $\epsilon_{p,e}^{S} \notin \boldsymbol{W}$ and $\epsilon_{p,e}^{F} \in \underline{\boldsymbol{W}}$, we do not change the capacity restrictions.

Scenario 9: if $\epsilon_{p,e}^{S} \notin \boldsymbol{W}$ and $\epsilon_{p,e}^{F} \notin \boldsymbol{W}$, we do not change the capacity restrictions.

Until now, we obtain a revised network simplex algorithm, which embeds the above-revised pivot operation. The intuition behind the revised network simplex algorithm is that the capacity restrictions are only involved in the pivot operation when determining a leaving arc. Thus, we can dynamically change the capacity allocation for two parallel arcs in the pivot operation, by holding $\mu_{p,e}^{S} + \frac{\mu_{p,e}^{F}}{M} = K_{p,e}$ to guarantee the feasibility and by maximizing δ^* to reach the optimality. Note that the revised network simplex algorithm is applicable to any minimum cost flow problems with sharing a capacity restriction among arcs.

5.1.2 A further revision of the network

In the network constructed in Section 4, there are some pairs of nodes having two arcs in the opposite directions, such as the two opposite arcs between node $Ship^S\_p\_e$ and $Port^S\_p\_e$ shown in Figure 3. When applying the revised network simplex algorithm, it may cause trouble in the revised pivot operation on increasing/decreasing flow between two nodes. To avoid the trouble, we introduce some dummy nodes to ensure that between any two nodes, there is at most one connected arc. Figure 8 shows an example when there are two opposite arcs between node i and node j. By adding two dummy node i^1 and node j^1 and reconnecting arcs, we can deal with the above trouble.

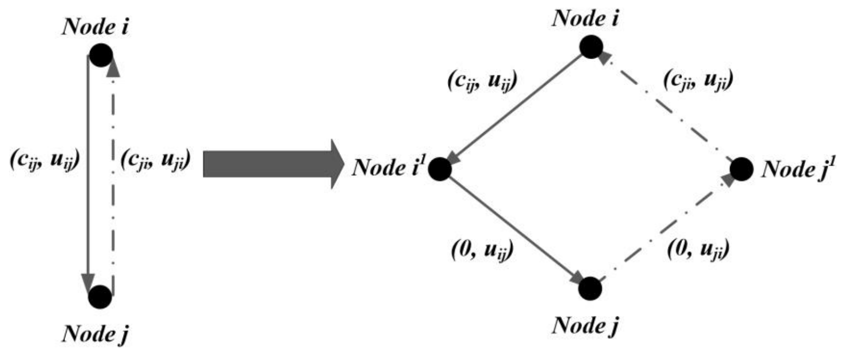

Figure 8: Adding dummy nodes to the network

5.2 Ship type decision by reduced costs

Given a ship type v with the capacity K^v and fixed operation cost C^v, we can invoke the above revised network simplex algorithm to derive the minimum flow cost σ^v for the empty containers. A straightforward way to find the optimal ship type is to invoke the algorithm to derive σ^v for all $v \in V$. Then, we can get the optimal ship type by $\min\{C^v + \sigma^v : \forall v \in V\}$. This way is applicable to the practice, as the candidate set V for ship types is normally limited (in computational experiments, there are four ship types for selection). However, in this subsection, we introduce a reduced cost based bound to exclude some ship types from the optimal one.

We rank the set of ship types with an increasing container capacity order such that $K^1 < K^2 < \cdots < K^{|V|}$ and $C^1 < C^2 < \cdots < C^{|V|}$. Here, K^1 is the smallest capacity that should be able to carry all laden containers. We initialize the ship capacity at $K = K^1$ and run the revised network simplex algorithm. Supposing that it obtains the optimal spanning tree solution $(\boldsymbol{T^1}, \boldsymbol{L^1}, \boldsymbol{U^1})$, the solution has reduced costs c_{ij}^{π} satisfying the Minimum Cost Flow Optimality Conditions. Starting from the ship capacity K^1, if we increase the ship capacity to K^2, the fixed operation cost increases by $C^2 - C^1$. However, the minimum flow cost may decrease as we relax the ship capacity from K^1 to K^2. Therefore, when increasing ship capacity, there is a trade-off between the increasing of fixed operation cost and the decreasing of minimum flow cost. As the increasing of the fixed operation cost is pre-determined ($C^2 - C^1$), we need to judge whether the decreasing of minimum flow cost can compensate the increasing operation cost $C^2 - C^1$.

For notational convenience, we denote $\Delta_{1,2} = \sigma^1 - \sigma^2$ as the decreasing of minimum flow cost by changing Type 1 ship to Type 2 ship. Here, we can use the reduced costs c_{ij}^{π} at ship capacity $K = K^1$ to derive an upper bound for $\Delta_{1,2}$. Recall that $c_{ij}^{\pi} \leq 0, \forall (i,j) \in \boldsymbol{U}$ actually means if the capacity of arc (i,j) increases by one unit, the minimum flow cost has the potential to decrease by $|c_{ij}^{\pi}|$. Considering a minimum cost flow problem is equivalent to $\min\{\sum_{(i,j)\in \boldsymbol{L}} c_{ij}^{\pi} x_{ij} - \sum_{(i,j)\in \boldsymbol{U}} |c_{ij}^{\pi}| x_{ij}\}$ (Ahuja et al., 1993), $\Delta_{1,2}$ has the upper bound $\sum_{(i,j)\in \boldsymbol{U}} |c_{ij}^{\pi}| (K^2 - K^1)$ if we do not have the parallel arcs and all arcs $(i,j) \in \boldsymbol{U}$ are restricted by the ship capacity. However, under the situation that our problem has parallel arcs that sharing capacity restrictions, the increased ship capacity $K^2 - K^1$ may be used by repositioning standard containers or foldable containers. Thus, we propose Procedure 1 to derive the upper bound for $\Delta_{1,2}$ (denoted by $\Delta_{1,2}^U$), in which $c_{\epsilon_{p,e}^S}^{\pi}$ and $c_{\epsilon_{p,e}^F}^{\pi}$ denote the reduced costs for the parallel arcs. Note that if we increase one unit ship capacity, the capacity on the parallel arcs for foldable containers can increase by M units. After obtaining the upper bound $\Delta_{1,2}^U$, we can compare it with $C^2 - C^1$. If $\Delta_{1,2}^U \leq C^2 - C^1$, it is unnecessary to further invoke the revised network simplex algorithm to derive σ^2 for Type 2 ship, as the $\Delta_{1,2}$ is impossible to compensate $C^2 - C^1$. We will repeat the Procedure 1 by increasing ship capacity to K^v until we find a ship type v such that $\Delta_{1,v}^U \geq C^v - C^1$, and then invoke the revised network simplex to derive the optimal spanning tree solution $(\boldsymbol{T^v}, \boldsymbol{L^v}, \boldsymbol{U^v})$. Starting from the ship capacity K^v, we also use the above method to exclude some ship types, until exploring the ship capacity $K^{|V|}$.

Procedure 1. *Deriving the upper bound for* $\Delta_{1,2}$

Input: *The reduced cost* c_{ij}^{π} *for the spanning tree* (T^1, L^1, U^1)
Output: *The upper bound* $\Delta_{1,2}^U$
Initialize $\Delta_{1,2}^U \leftarrow 0$
for all parallel arc $\epsilon_{p,e}^S \in \mathcal{G}^S$ **and** $\epsilon_{p,e}^F \in \mathcal{G}^F$ **do**
  if $\epsilon_{p,e}^S \in \boldsymbol{U}$ **and** $\epsilon_{p,e}^F \notin \boldsymbol{U}$ **then**
    $\Delta_{1,2}^U \leftarrow \Delta_{1,2}^U + (K^2 - K^1)|c_{\epsilon_{p,e}^S}^{\pi}|$
  else if $\epsilon_{p,e}^S \notin \boldsymbol{U}$ **and** $\epsilon_{p,e}^F \in \boldsymbol{U}$ **then**
    $\Delta_{1,2}^U \leftarrow \Delta_{1,2}^U + (K^2 - K^1)M|c_{\epsilon_{p,e}^F}^{\pi}|$
  else if $\epsilon_{p,e}^S \in \boldsymbol{U}$ **and** $\epsilon_{p,e}^F \in \boldsymbol{U}$ **then**
    $\Delta_{1,2}^U \leftarrow \Delta_{1,2}^U + (K^2 - K^1)\max\{|c_{\epsilon_{p,e}^S}^{\pi}|, M|c_{\epsilon_{p,e}^F}^{\pi}|\}$
  end if
end for

To facilitate the understanding, we show an example to exclude ship types. Supposing that we have invoked the algorithm to derive the reduced costs for the ship capacity at $K = K^1$, we derive $\Delta_{1,2}^U = 120$ by Procedure 1. Then, if $C^2 - C^1 = 200 \geq \Delta_{1,2}^U$, it is unnecessary to invoke the algorithm for the ship capacity at $K = K^2$. In the next, we replace K^2 with K^3 in Procedure 1. If we have $\Delta_{1,3}^U = 300$ and $C^3 - C^1 = 250$, we need to invoke the algorithm for the ship capacity at $K = K^3$.

Based on the upper bound derived by reduced costs, we can skip some ship types when invoking the revised network simplex algorithm. In the end, we compare the total costs of all those ship types explored by the algorithm for finding the optimal ship type.

In summary for the proposed solution approach, we have made the following improvements compared with the traditional network simplex algorithm. (i) Our network flow model has some parallel arcs sharing the same capacity restrictions, which are inevitable due to the co-existence of foldable and standard containers in the network (cf. Section 4.3.1). As the capacity restrictions tackled in the traditional network simplex algorithm must be unchanged with given values, we propose a tailored (or revised) network simplex algorithm for the problem. In the revised one, we introduce dynamic capacity restrictions for the parallel arcs and revise the pivot operations to guarantee the optimality (cf. Section 5.1.1). (ii) Our problem needs to determine the ship capacity, which further decides capacity restrictions on the arcs of the network. To achieve the goal, we use the reduced costs from our tailored algorithm to make the ship type decision (cf. Section 5.2). (iii) Using a network simplex algorithm to solve a minimum cost flow problem requires the construction of a network flow model as the prior task. Our solution approach elaborates a procedure to construct a flow network (cf. Section 4) for the empty container allocation.

5.3 An MILP model

The proposed solution approach is built on the network flow model constructed in Section 4. In this subsection, we formulate an MILP model for our research problem that can be solved by CPLEX solver directly. In the computational experiment section, we will use the results of the MILP model to verify the optimality of the proposed solution approach. Here, we first introduce some decision variables and then provide the model formulation.

***Decision variables*:**

ε_v: binary, set to one if the ships in Type v are deployed, otherwise zero, $v \in V$;

ζ_p^S: integer, number of long-term leasing standard containers in Port $p, p \in P$;

ζ_p^F: integer, number of long-term leasing foldable containers in Port $p, p \in P$;

$\alpha_{od,e}^S$: integer, number of empty standard containers used to satisfy the laden container transportation consignments from Port o to Port d by the e^{th} round trip, $(o,d) \in D, e \in E$;

$\alpha_{od,e}^F$: integer, number of empty foldable containers used to satisfy the laden container transportation consignments from Port o to Port d by the e^{th} round trip, $(o,d) \in D, e \in E$;

$\beta_{od,e}^S$: integer, number of empty standard containers repositioned from Port o to Port d by the e^{th} round trip, $(o,d) \in D, e \in E$;

$\beta_{od,e}^F$: integer, number of empty foldable containers repositioned from Port o to Port d by the e^{th} round trip, $(o,d) \in D, e \in E$;

$\gamma_{od,e}$: integer, number of short-term empty containers leased in Port o for the e^{th} round trip and will returned in Port d, $(o,d) \in D, e \in E$;

$\delta_{p,e}^S$: integer, inventory level of empty standard containers (i.e., the empty standard containers left) at Port p after visited by the e^{th} round trip, $p \in P, e \in E$;

$\delta_{p,e}^F$: integer, inventory level of empty foldable containers (i.e., the empty foldable containers left) at Port

p after visited by the e^{th} round trip, $p \in P, e \in E$;

$\eta_{p,e}$: integer, number of containers carried on the container ship of the e^{th} round trip after it visits Port p, $p \in P, e \in E$;

Mathematical model:

$$[\boldsymbol{M2}] \quad \min \sum_{v \in V} C_v \varepsilon_v + \sum_{p \in P} (L_p^S \zeta_p^S + L_p^F \zeta_p^F) + \sum_{e \in E} \sum_{(o,d) \in D} l_{od} \gamma_{od,e} + \sum_{e \in E} \sum_{(o,d) \in D} (r_{od}^S \beta_{od,e}^S + r_{od}^F \beta_{od,e}^F) + \sum_{e \in E} \sum_{p \in P} (s_p^S \delta_{p,e}^S + s_p^F \delta_{p,e}^F) \tag{14}$$

subject to:

$$\sum_{v \in V} \varepsilon_v = 1 \tag{15}$$

$$\alpha_{od,e}^S + \alpha_{od,e}^F + \gamma_{od,e} = d_{od,e} \qquad \forall (o,d) \in D, e \in E, \tag{16}$$

$$\zeta_p^S + \sum_{d=p,o<d,(o,d) \in D} \beta_{od,e}^S = \sum_{o=p,(o,d) \in D} (\alpha_{od,e}^S + \beta_{od,e}^S) + \delta_{p,e}^S \qquad \forall p \in P, e = 1, \tag{17}$$

$$\zeta_p^F + \sum_{d=p,o<d,(o,d) \in D} \beta_{od,e}^F = \sum_{o=p,(o,d) \in D} (\alpha_{od,e}^F + \beta_{od,e}^F) + \delta_{p,e}^F \qquad \forall p \in P, e = 1, \tag{18}$$

$$\delta_{p,e-1}^S + \sum_{d=p,o<d,(o,d) \in D} \beta_{od,e}^S + \sum_{d=p,o>d,(o,d) \in D} \beta_{od,e-N}^S + \sum_{d=p,o<d,(o,d) \in D} \alpha_{od,e-w_p}^S + \sum_{d=p,o>d,(o,d) \in D} \alpha_{od,e-N-w_p}^S = \sum_{o=p,(o,d) \in D} (\alpha_{od,e}^S + \beta_{od,e}^S) + \delta_{p,e}^S \qquad \forall p \in P, e \in E \backslash \{1\}, \tag{19}$$

$$\delta_{p,e-1}^F + \sum_{d=p,o<d,(o,d) \in D} \beta_{od,e}^F + \sum_{d=p,o>d,(o,d) \in D} \beta_{od,e-N}^F + \sum_{d=p,o<d,(o,d) \in D} \alpha_{od,e-w_p}^F + \sum_{d=p,o>d,(o,d) \in D} \alpha_{od,e-N-w_p}^F = \sum_{o=p,(o,d) \in D} (\alpha_{od,e}^F + \beta_{od,e}^F) + \delta_{p,e}^F \qquad \forall p \in P, e \in E \backslash \{1\}, \tag{20}$$

$$\eta_{p,e} = \sum_{o \le p, d > p,(o,d) \in D} (d_{od,e} + \beta_{od,e}^S + \beta_{od,e}^F / M) + \sum_{d \ge p+1, o > d,(o,d) \in D} (d_{od,e-N} + \beta_{od,e-N}^S + \beta_{od,e-N}^F / M) \qquad \forall p \in P / \{P\}, e \in E, \tag{21}$$

$$\eta_{p,e} = \sum_{o > d,(o,d) \in D} (d_{od,e} + \beta_{od,e}^S + \beta_{od,e}^F / M) \qquad \forall p = |P|, e \in E, \tag{22}$$

$$\eta_{p,e} \le \sum_{v \in V} K_v \varepsilon_v \qquad \forall p \in P, e \in E, \tag{23}$$

$$\varepsilon_v \in \{0,1\} \qquad \forall v \in V, \tag{24}$$

$$\zeta_p^S, \zeta_p^F \ge 0 \qquad \forall p \in P, \tag{25}$$

$$\alpha_{od,e}^S, \alpha_{od,e}^F, \beta_{od,e}^S, \beta_{od,e}^F, \gamma_{od,e} \ge 0 \qquad \forall (o,d) \in D, e \in E, \tag{26}$$

$$\delta_{p,e}^S, \delta_{p,e}^F, \eta_{p,e} \ge 0 \qquad \forall p \in P, e \in E. \tag{27}$$

Here, note that $\alpha_{od,e}^S := 0$, $\beta_{od,e}^S := 0$, $\alpha_{od,e}^F := 0$ and $\beta_{od,e}^F := 0$ when $e < 1$.

In the above model, Objective (14) minimizes the total cost, including the fixed operation cost, the long-term leasing cost, the short-term leasing cost, the repositioning cost, and the storage cost. Constraint (15) guarantees that only one type of ships can be selected. Constraints (16) enforce that the laden container transportation consignments in each port by each round trip must be fulfilled by using available empty standard containers, foldable containers or short-term leasing containers in the port. Constraints (17) provide the inventory equations for empty standard containers in each port after visited by the 1^{st} round trip. The left sides of the equations list the number of leased empty containers in each port at the beginning (i.e., ζ_p^S) and the empty containers arrived in each port by the 1^{st} round trip (i.e., $\sum_{d=p,o<d,(o,d) \in D} \beta_{od,e}^S$). The right

sides of the equations show the inventory level of empty containers in each port after the 1^{st} round trip (i.e., $\delta_{p,e}^{S}$) and the number of empty containers flowing out of the port by the 1^{st} round trip (i.e., $\sum_{o=p,(o,d)\in D}(\alpha_{od,e}^{S}+\beta_{od,e}^{S})$). Constraints (18) are similar as Constraints (17), which are the inventory equations for empty foldable containers by the 1^{st} round trip. Constraints (19) and Constraints (20) list the inventory equations in Port p after visited by the e^{th} round trip. The left-sides of the constraints also show the empty container supply, including the empty containers left in the port after visited by $(e-1)^{th}$ round trip (i.e., the inventory level in Port p after visited by $(e-1)^{th}$ round trip), the arrived repositioning empty containers, and the arrived devanning laden containers. Here, notice that: (i) if Port p is the destination port d, and the origin port $o<d$ (the origin port $o>d$), the containers transported from Port o to Port d by the e^{th} round trip will arrive at the Port d by the e^{th} round trip (the $(e+N)^{th}$ round trip). (ii) When laden containers arrive at the destination ports, it will take w_p weeks for the devanning of the containers, and become available empty containers for the next consignment in the ports. The right sides of Constraints (19) and Constraints (20) are the same as that of Constraints (17) and Constraints (18) respectively. Constraints (21) and Constraints (22) calculate the number of containers carried in the deployed ship on the e^{th} round trip after visiting Port p. Constraints (23) enforce that the number of containers carried in the ships cannot exceed the capacity of the type of the deployed ships. Constraints (24-27) define the decision variables.

6. Computational Experiment

In this section, based on three real-world shipping service routes, we conduct extensive computational experiments to find insights on the ship type decision and the foldable container usage by using our proposed solution approach. We run the experiments by a PC equipped with 3.30GHz of Intel Core i5 CPU and 16GB of RAM. For all the test instances in the experiments, the planning horizon is half a year, i.e., 26 weeks. As the shipping service is on a weekly basis, the total round trips are 26 (i.e., $|E|$=26).

6.1 Test instances on three real-world shipping service routes

To generate test instances for the experiments, we select three real-world shipping service routes operated by CMA CGM shipping liner, which is labeled as BOHAI, LIBERTY2 and AANAANLCMA. Figure 9 depicts the three real-world shipping service routes. (i) The port rotation of the route BOHAI is: Lianyungang (1) → Shanghai (2) → Ningbo (3) → Los Angeles (4) → Oakland (5) → Lianyungang; the rotation time is 42 days, and 6 ships are deployed. (ii) The port rotation of the route LIBERTY2 is: Antwerp (1) → Bremerhaven (2) → Rotterdam (3) → Le Havre (4) → New York (5) → Norfolk (6) → Charleston (7) → Antwerp; the rotation time is 28 days, and 4 ships are deployed. (iii) The port rotation of the route AANAANLCMA is: Yokohama (1) → Osaka (2) → Pusan (3) → Shanghai (4) → Ningbo (5) → Kaohsiung (6) → Melbourne (7) → Sydney (8) → Brisbane (9) → Yokohama; the rotation time is 42 days, and 6 ships are deployed. In fact, the three shipping service routes are the representatives of three trade routes: Asia-

North America trade route, North Europe-North America trade route, and Australia-Far East trade route. The three routes have different degrees of the imbalance of laden container flow.

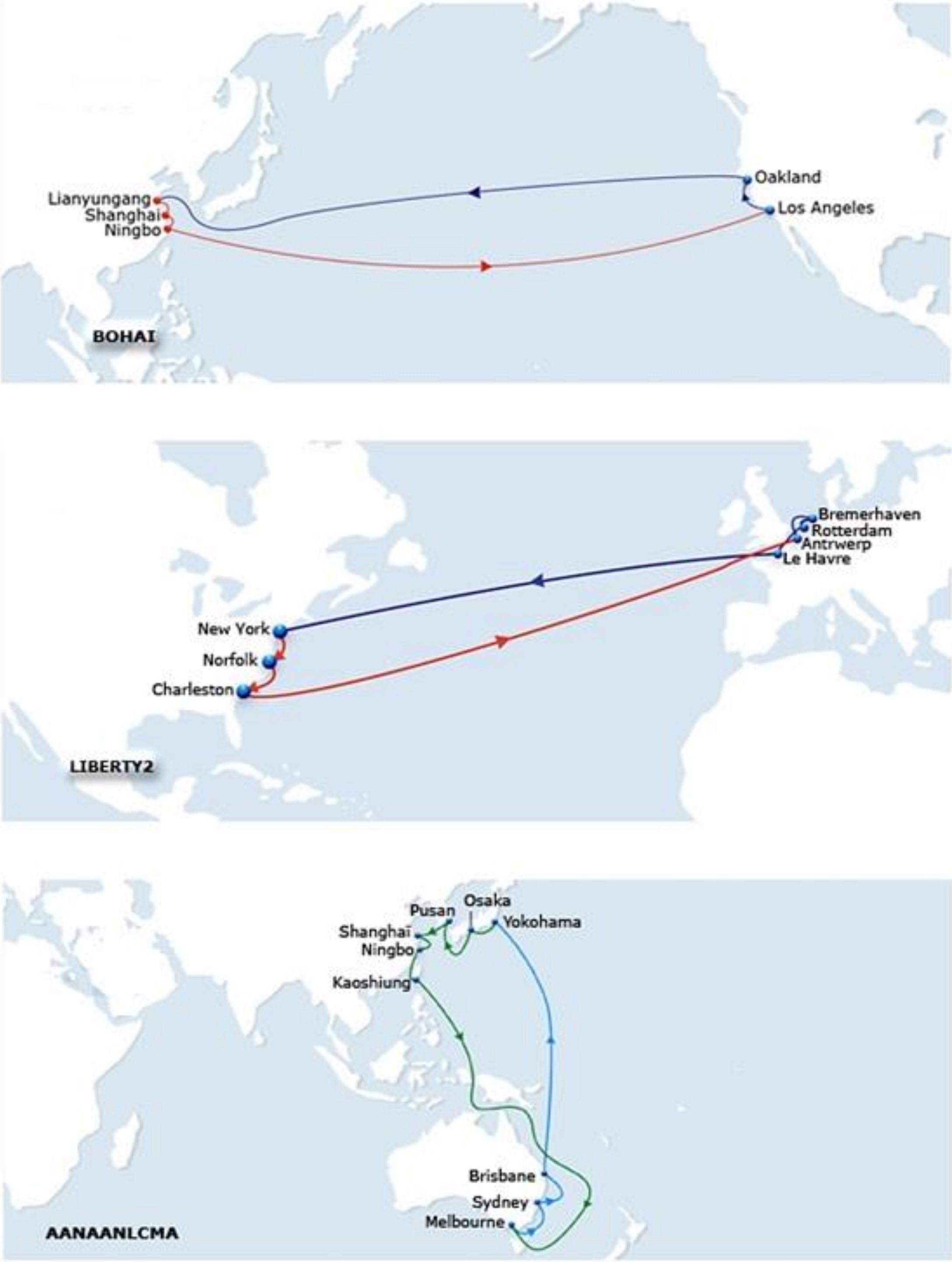

Figure 9: Three selected shipping services routes operated by CMA CGM. *Source*: CMA CGM (2017)

According to Word Shipping Council (2013), we estimate that the imbalance ratio of laden container flow for Asia-North America trade route is about 1.99 (i.e., Eastbound / Westbound), the imbalance ratio for North Europe-North America trade route is about 1.27 (i.e., Westbound / Eastbound), and the imbalance ratio for Australia-Far East trade route is about 1.73 (i.e., Southbound / Northbound). In fact, different imbalance ratios lead to different degrees of the necessity for the empty container repositioning. Based on the imbalance ratios, the weekly laden container transportation consignments are randomly generated for each selected shipping service route as follows: (i) For the route BOHAI, the transportation consignments from an Asian port to a North American port follow the uniform distribution $U(600, 800)$, and the transportation consignments from a North American port to an Asian port follow the uniform distribution $U(300, 400)$. (ii) For the route LIBERTY2, the transportation consignments from a North Europe port to a North American port follow the uniform distribution $U(400,500)$, and the transportation consignments from a North American port to a North Europe port follow the uniform distribution $U(300, 400)$. (iii) For the route AANAANLCMA, the transportation consignments from an Asian port to an Australian port follow the uniform distribution $U(500, 700)$, and the transportation consignments from an Australian port to an Asian port follow the uniform distribution $U(300, 400)$. For all the three routes, there are no transportation consignments within the same region.

Table 1: Data for relevant container costs

Relevant container costs	Per foldable container	Per standard container
Weekly storage cost	US$10	US$40
Loading or unloading cost	US$13	US$50
Long-term leasing cost	US$960	US$480
Folding and unfolding cost	US$20	

Table 2: Candidate ship type and fixed operation cost

Ship route	Ship type (TEUs) and fixed operating costs (million US$)								
	4,400	5,000	5,400	5,800	6,200	11,000	11,400	11,800	12,000
BOHAI	14.07	15.00	15.21	15.32					
LIBERTY2		8.62	9.53	10.20	10.48				
AANAANLCMA						22.23	23.00	23.44	23.52

By referring to Moon and Hong (2016), and Konings (2005), we set all the relevant costs for foldable and standard empty containers, which are shown in Table 1. Here, we assume that all the costs are the same for different ports, and four foldable containers can be folded as one standard container. The short-term leasing cost is charged based on the travel time between origin ports and destination ports. According to Moon and Hong (2016), the unit short-term leasing cost is set as US$170/week, for example, if the travel time between an origin port and a destination port is two weeks, the short-term leasing cost is US$340. For the three selected shipping service routes, the candidate ship types and the fixed operation cost for the whole planning horizon by each ship type are presented in Table 2 (Meng and Wang, 2012).

6.2 Optimality check for the proposed solution approach

In Section 5, we propose the solution approach and formulate the MILP model for the problem. Here, we apply the two methods to solve problem instances of three shipping routes. The results of using the proposed solution approach and using the MILP model by CPLEX solver are given in Table 3. As can be seen, both methods derive the same optimal solution for the problem, which verifies the optimality of the proposed solution approach. With respect to the computation time, both methods outperform each other in some instances and in average, using the proposed solution is slightly faster than using the MILP model by CPLEX solver. Since CPLEX solver is a commercial solver and does not outperform the proposed method, the proposed solution embedded with the revised network simplex algorithm is more desirable for shipping liner companies, as it does not invoke any MILP solvers.

Table 3: Comparing the proposed solution approach with the MILP model by CPLEX solver

Instance		**The solution approach**		**The MILP model**		**Time ratio**
Shipping route	Instance ID	Z(million US$)	CPU time (seconds)	Z(million US$)	CPU time (seconds)	
BOHAI	B_3_1	36.32	3	36.32	3	1.00
	B_3_2	37.45	4	37.45	6	0.67
	B_3_3	38.64	2	38.64	3	0.67
	B_3_4	37.71	3	37.71	6	0.50
	B_3_5	38.06	3	38.06	3	1.00
LIBERTY2	L_3_6	33.78	11	33.78	15	0.72
	L_3_7	33.45	24	33.45	21	1.14
	L_3_8	32.97	12	32.97	15	0.77
	L_3_9	33.63	24	33.63	20	1.21
	L_3_10	33.85	21	33.85	17	1.24
AANAANLCMA	A_3_11	83.74	58	83.74	59	0.99
	A_3_12	82.67	55	82.67	53	1.04
	A_3_13	83.31	70	83.31	74	0.94
	A_3_14	82.78	97	82.78	83	1.16
	A_3_15	82.46	83	82.46	95	0.87
					Average:	0.93

***Note**: (i) "Z(million US$)" represents the objective values derived by the two methods with the unit of million US$. (ii) "CPU time" shows the computation time with the unit of seconds. (iii) "Time ratio" is the CPU time of the solution approach divided by the CPU time of the MILP model by CPLEX solver.*

6.3 Comparison with a model that only considers laden container transportation

One of the major motivations of this paper is to incorporate the empty container allocation into the ship type decision for the ship fleet deployment. Here, we conduct experiments to compare our proposed model with the model that does not consider the empty containers on the problem. For the traditional model, we decide the ship type by only considering the laden containers, by which we can calculate the container flows for all the voyages. Then, the ship type that has the minimum capacity to accommodate all the container flows is determined as the selected ship type. Assuming that the capacity and the fixed operation cost for the selected ship type is K^1 and C^1, we run the revised network simplex algorithm only to find the minimum flow cost for empty containers, which determines the total cost $C^1 + \sigma^1$ for the traditional model.

Table 4: Comparing the proposed model and the traditional model without empty container allocation

Instance		**The proposed model**		**The traditional model**		**Gap**
Shipping route	Instance ID	$Z1$(million US$)	Ship fleet	$Z2$(million US$)	Ship fleet	
BOHAI	B_4_1	38.13	5,400	38.39	5,000	0.68%
	B_4_2	37.68	5,000	38.00	4,400	0.86%
	B_4_3	37.10	5,000	37.37	4,400	0.74%
	B_4_4	37.72	5,400	37.72	5,400	0.00%
	B_4_5	38.38	5,800	38.74	5,400	0.93%
Imbalance ratio of laden container flow: 1.99; # of ports: 5					Average:	0.64%
LIBERTY2	L_4_6	33.13	5,800	33.28	5,400	0.46%
	L_4_7	34.04	6,200	34.21	5,800	0.51%
	L_4_8	33.61	5,800	33.77	5,400	0.49%
	L_4_9	33.26	5,400	33.26	5,400	0.00%
	L_4_10	33.13	5,400	33.13	5,400	0.00%
Imbalance ratio of laden container flow: 1.27; # of ports: 7					Average:	0.29%
AANAANLCMA	A_4_11	84.26	12,000	85.53	11,400	1.51%
	A_4_12	83.14	11,400	84.37	11,000	1.48%
	A_4_13	82.93	11,400	83.93	11,000	1.21%
	A_4_14	83.96	11,800	85.09	11,400	1.35%
	A_4_15	83.16	11,400	84.08	11,000	1.11%
Imbalance ratio of laden container flow: 1.73; # of ports: 9					Average:	1.33%

Note*: (i) "$Z1$(million US$)" and $Z2$(million US$) represent the objective values of the two models with the unit of million US$. (ii) "Ship fleet" shows the capacity (in TEUs) of the selected ship type. (iii) "Gap" shows the difference on the total costs by the two models, which is calculated by* $(Z2 - Z1)/Z1$.

The comparison between the proposed model and the traditional model is listed in Table 4, where "Ship fleet" shows the capacity of the selected ship fleet types by the two models, and "Gap" shows the difference in the total costs by the two models. In the majority of the instances in Table 4, the selected ship fleet types are different by the two models. More importantly, the ship fleet capacity selected by the proposed model is no less than the ship fleet capacity selected by the traditional model, which is consistent with our previous discussion, i.e., considering the empty containers gives the shipping liner motivation to deploy larger container ships. However, the impact of considering the empty container allocation varies among the three shipping routes. For the route AANAANLCMA, which does not consider the empty container allocation, the total cost rises by near 1.33% on average; but for the route LIBERTY2, the total cost rises by near 0.29% on average. Here, note that if the selected ship fleet types by the two models are the same, the total costs are the same, because we also optimize empty container related decisions in the traditional model.

The above-mentioned phenomenon attributes to the different trade routes that the three shipping routes belong. The route AANAANLCMA is one shipping route of Australia-Far East trade route, and the route LIBERTY2 is one shipping route of North Europe-North America trade route. As a result, the route AANAANLCMA has a higher imbalance ratio of laden container flow than the route LIBERTY2, which makes the empty container repositioning more necessary for the route AANAANLCMA. Thus, for the route AANAANLCMA, it is more important to consider empty containers in the ship type decision, which is verified by the high total cost gaps in “Gap”. For the route BOHAI, although the imbalance of laden container flow for the route is significant, the total cost gap is not so obvious compared with the route AANAANLCMA, which is due to that only five ports of call are involved in the route BOHAI. Based on the above analysis, we can conclude that considering the empty container allocation is critical for the ship fleet deployment, especially for the shipping routes that have high imbalance ratios of laden container flow, and the shipping routes that traverse many ports of call.

6.4 Performance evaluation on using foldable containers

Although researchers have proved that the economic and logistical viability of using foldable container, the foldable containers still are not prevalent among the shipping services. In this section, we aim to investigate how much cost the shipping liner can save if the foldable containers are truly used in shipping services. In this subsection, we conduct some experiments on comparing the proposed model (i.e., $\boldsymbol{TNF}$ model and ship type decision) with the model that does not use the foldable containers (i.e., $\boldsymbol{PNF}$ model and ship type decision). Note that we still use the solution approach proposed in Section 5 to derive the optimal container flow and the optimal ship type for the two cases. The results are reported in Table 5, where “Container fleet” shows the total number of containers used for the complete planning horizon, and “Gap” shows the difference on the total costs by the two models.

In Table 5, the container fleet (i.e., the total number of containers) using foldable containers is no less than the container fleet that does not use foldable containers. This result suggests that using foldable containers motivates the shipping liner to enlarge its container fleet, which makes it more powerful to handle the laden container demands. Meanwhile, there is nearly no advantage on cost reduction for the route LIBERTY2 by using foldable containers as the total cost gaps are less than 0.10%, and using foldable containers has the biggest impact on the route AANAANLCMA among the three routes. The results are in line with the results of Table 4. However, the impacts of using foldable containers on the total cost for all the three routes are not so significant as all the total cost gaps are less than 0.70%, which implies that under the current cost settings, the shipping liner does not have a strong incentive on cost reduction to use the foldable containers. This may be the reason why the foldable containers are still not prevalent among the shipping services. To find strong incentives for using the foldable containers for the shipping liner, we will analyze the effects of cost settings on the foldable container usage in Section 6.5. Meanwhile, Table 5 shows the foldable container usage could affect the ship type decision. In two instances of the route BOHAI (Instance B_6_2 and B_6_3) and all instances of the route AANAANLCMA except Instance A_6_13, after using foldable containers, it will choose a smaller ship fleet. It may attribute to that after using foldable containers the empty container repositioning will occupy less storage space on ships when repositioning foldable containers. Thus, a smaller ship fleet may be enough for carrying all laden and empty containers.

Table 5: Comparison between the proposed model and the model that does not use the foldable containers

Instance		**With foldable containers**			**Without foldable containers**			**Gap**
Shipping route	Instance ID	Z(million US$)	Ship fleet	Container fleet	Z(million US$)	Ship fleet	Container fleet	
BOHAI	B_5_1	37.88	5,000	38,474	37.98	5,000	38,451	0.27%
	B_5_2	38.12	5,400	38,825	38.27	5,800	38,367	0.40%
	B_5_3	37.88	5,000	38,367	38.01	5,400	38,290	0.34%
	B_5_4	37.96	5,400	38,605	38.06	5,400	38,460	0.26%
	B_5_5	38.16	5,800	38,880	38.26	5,800	38,429	0.28%
							Average:	0.31%
LIBERTY2	L_5_6	33.13	5,400	42,492	33.15	5,400	42,485	0.05%
	L_5_7	33.23	5,800	42,555	33.25	5,800	42,555	0.03%
	L_5_8	33.24	5,800	42,494	33.25	5,800	42,494	0.04%
	L_5_9	33.22	5,800	42,621	33.25	5,800	42,603	0.08%
	L_5_10	33.05	5,400	42,348	33.06	5,400	42,348	0.04%
							Average:	0.05%
AANAANLCMA	A_5_11	82.84	11,400	99,516	83.30	11,800	99,316	0.56%
	A_5_12	83.12	11,800	100,110	83.63	12,000	100,013	0.62%
	A_5_13	82.80	11,400	99,597	83.27	11,400	99,325	0.56%
	A_5_14	82.71	11,000	99,548	83.23	11,400	99,361	0.63%
	A_5_15	83.25	11,800	99,733	83.69	12,000	99,645	0.53%
							Average:	0.58%

***Note**: (i) "$Z1$(million US$)" and $Z2$(million US$) represent the objective values with the unit of million US$. (ii) "Ship fleet" shows the capacity (in TEUs) of the selected ship type. (iii) "Container fleet" shows the total number of empty containers used for the whole planning horizon, including the total number of the empty containers owned initially and the long-term leasing empty containers. (iv) "Gap" shows the difference on the total costs by the two models, which is calculated by $(Z2 - Z1)/Z1$.*

6.5 Effects of cost settings on the foldable container usage

In this section, our goal is to find under which conditions, the shipping liner would use foldable containers in large scale. Compared with standard containers, foldable containers have higher long-term leasing cost and extra folding and unfolding cost. Therefore, we test the effects of the long-term leasing cost and the folding and unfolding cost on the foldable container usage in the section. We define a ratio $\rho = \sum_{p \in P} \zeta_p^F / (\sum_{p \in P} \zeta_p^F + \sum_{p \in P} \zeta_p^S)$ to show the usage of foldable containers, where ζ_p^S and ζ_p^F are the number of standard containers and foldable containers used for the long-term container leasing. The bigger the ratio is the more foldable containers are leased for the usage of the planning horizon.

Figure 10 illustrates the effect of the long-term leasing cost on the foldable container usage, where the y-axis shows the ratio ρ, and x-axis indicates the long-term leasing cost of a foldable container. In Figure 10, we keep the long-term leasing cost of a standard container unchanged (i.e., US$480), and increase the long-term leasing cost of a foldable container from US528 to US$1200. As can be seen, all three curves for the three shipping routes descend fast when the long-term leasing cost increases, and a formal proof (See

Appendix B) can verify the non-increasing trend of the foldable container usage. Under the current cost setting, i.e., the long-term leasing cost of a foldable container is US$960, the foldable container usage is in low ratio. If the cost reduces to US$768, the shipping liner can have equal usage on both standard containers and foldable containers. However, if the cost is beyond US$1056, there is no need to consider the usage of foldable containers. Therefore, we can summarize that the foldable container usage is highly dependent on the long-term leasing cost, and reducing the long-term leasing cost could be an effective way to make the foldable containers become prevalent.

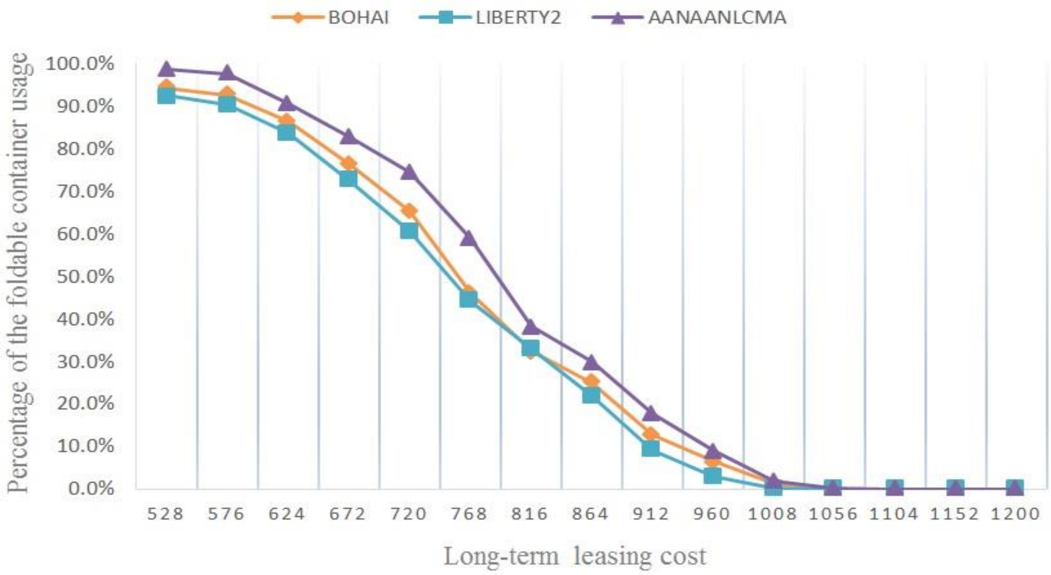

Figure 10: Effect of the long-term leasing cost on the foldable container usage

Under the different long-term leasing cost of foldable containers, we also compare the average total cost gap between the case with using foldable containers and the case without using foldable containers, the results of which are listed in Table 6. As can be seen, when the long-term leasing cost decreases, using foldable containers saves the total cost more significantly. If the long-term leasing cost drops to US$528, the total saving reaches 27.51%. However, if the long-term leasing cost is beyond US$1104, it makes no difference between the case with using foldable containers and the case without using foldable containers.

Table 6: Total cost saving after using foldable containers under different long-term leasing cost

long-term leasing cost	528	576	624	672	720	768	816
Total cost saving	27.51%	22.26%	17.04%	12.49%	8.56%	5.44%	3.19%
long-term leasing cost	864	912	960	1008	1056	1104	1152
Total cost saving	1.83%	0.86%	0.26%	0.11%	0.02%	0.00%	0.00%

Figure 11 shows the effect of the folding and unfolding cost on the foldable container usage, where the y-axis shows the ratio ρ, and x-axis indicates the folding and unfolding cost of a foldable container (US$20 in the current cost setting). Here, we increase the folding and unfolding cost of a foldable container from US10 to US$30. In the figure, the three curves of three shipping service routes are almost flat, and the foldable container usage maintains at around 8.5% for the route AANAANLCMA, around 6.4% of the route

BOHAI, and around 3.0% for the route LIBERTY2. Thus, we can conclude that the foldable container usage is not sensitive to the folding and unfolding cost, and the reduction of the folding and unfolding cost will not spur the foldable container usage significantly.

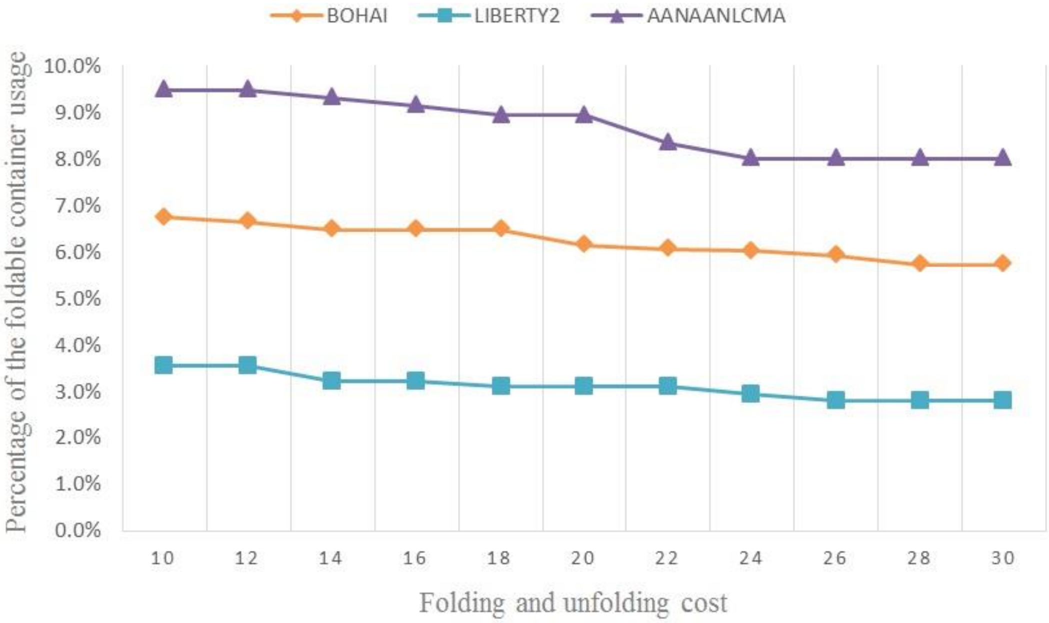

Figure 11: Effect of the folding and unfolding cost on the foldable container usage

In summary, the foldable container usage is highly dependent on the long-term leasing cost, but it is not sensitive to the folding and unfolding cost in the current cost settings. The reason why the usage of foldable containers shows an insensitive reaction to the folding and unfolding cost may attribute to the dominant role of the long-term leasing cost on using foldable containers. According to Figure 10, when the long-term leasing cost of foldable containers is US$960, the percentage of the foldable container usage is low for the three shipping routes (below 10%), indicating that using standard containers is much more cost-effective than using foldable containers. Under the case, the decreasing of the folding and unfolding cost may not be a comparatively strong incentive for the shipping lines to use foldable containers.

6.6 Analysis of cost-dependent sensitivity

The previous subsection shows that when the long-term leasing cost is high, the foldable container usage has a low sensitivity in response to the folding and unfolding cost. In this subsection, we aim to explore the relationship between the foldable container usage's sensitivity to the folding and unfolding cost and the long-term leasing cost. To investigate whether the long-term leasing cost affects the sensitivity to the folding and unfolding, we focus on the route AANAANLCMA and conduct sensitivity analysis for the folding and unfolding under different long-term leasing cost. Given a long-term leasing cost, in order to measure the sensitivity in the same metric, we define a sensitivity ratio $\sigma = \frac{\rho_1 - \rho_0}{\rho_0}$, where ρ_0 is the percentage of foldable containers used under US$20 folding and unfolding cost (the baseline setting), and ρ_1 is the percentage under other costs, such as US$10 and US$30. In the experiments, we set the long-term leasing cost to US$960, US$860, US$760, US$660 and US$560, under each of which, we change the folding and unfolding cost to detect the sensitivity.

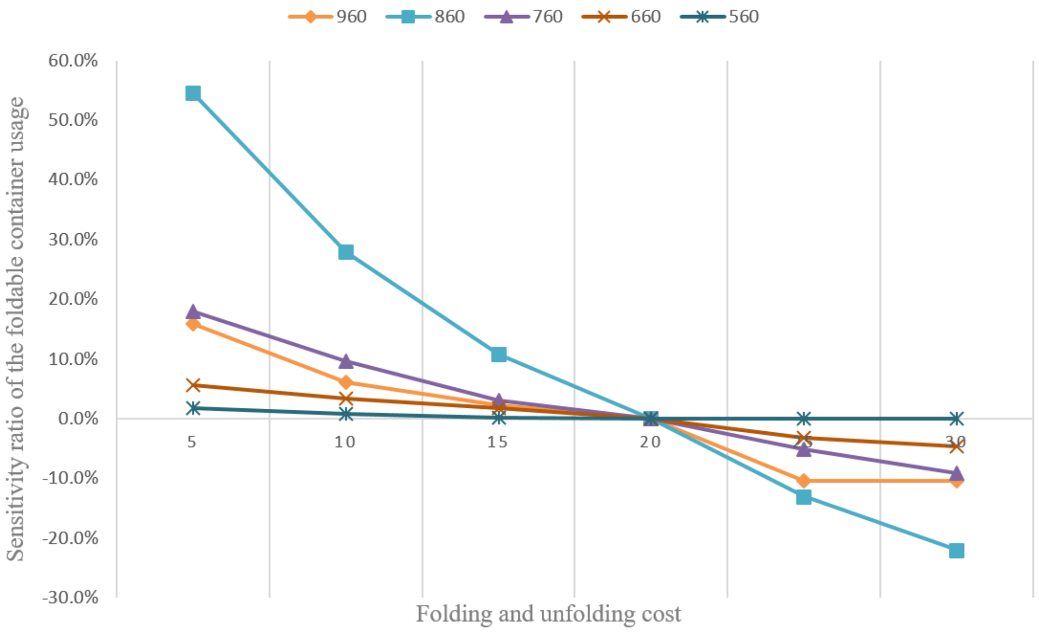

Figure 12: Long-term leasing cost dependent sensitivity to the folding and unfolding cost

Figure 12 shows the relationship between the sensitivity to the folding and unfolding cost and the long-term leasing cost, where the y-axis shows the sensitivity ratio σ, and x-axis indicates the folding and unfolding cost. See the highest "blue" point for an example to depict the figure, which shows that, given the long-term leasing cost as US$860, when the folding and unfolding cost decreases from US$20 to US$5, the percentage of foldable container usage will increase by 54%. Each line in the figure corresponds to the sensitivity under each long-term leasing cost, and a steeper line means the foldable container usage is more sensitive to the folding and unfolding cost. In general, Figure 12 illustrates that the sensitivity to the folding and unfolding cost depends on the long-term leasing cost. More specifically, when the long-term leasing cost decreases from US$960 to US$860 (resp. decreases from US$860 to US$560), the line becomes steeper (resp. smoother) such that the foldable container usage becomes more sensitive (resp. less sensitive) to the folding and unfolding cost.

An intuitive explanation for the phenomenon is that when the long-term leasing cost drops to a certain level (say US$860), using foldable containers becomes nearly the same cost-effective as using standard containers. At that level, the changes in the folding and unfolding cost may have an evident effectiveness towards the foldable container usage. However, if the long-term leasing cost further reduces to a low level (say US$560), using foldable containers is much more cost-effective than using standard containers (cf. Figure 10, approximately 97.8% foldable container usage). Henceforth, decreasing the folding and unfolding cost may have a limited incentive to increase the foldable container usage. Based on the phenomenon, we can obtain some managerial insights for shipping lines when popularizing the usage of foldable containers. (i) If container leasing companies charge a high price for the long-term leasing of foldable containers, it may not be economic to use foldable containers considering some fixed operation costs incurred for using the foldable containers. (ii) If container leasing companies charge a moderate price, the shipping lines may make efforts to cut down the folding and unfolding cost by negotiating with port terminals, which can lead to a profitable result. (iii) If container leasing companies charge a low price, it is cost-effective to use foldable containers and the bargaining motivation of shipping lines on the folding and unfolding cost with port terminals might not be strong, as the cost reduction leads to a tiny benefit.

6.7 Sensitivity analysis on the number of weeks for the devanning process

In Section 4.1, there is an input parameter w_p showing the number of weeks allowed for the devanning process. As discussed in Section 3.3, this parameter indicates the required time for consignees to return empty containers. In essential, the parameter constructs a trade-off between shipping liner companies and customers (or consignees). If w_p becomes larger, the customers will have more flexibility to deal with the cargo carried in containers, but this would increase the opportunity cost for shipping liner companies, as they need empty containers as soon as possible to fulfill next transportation consignments. Here, to investigate the opportunity cost, we conduct a sensitivity analysis on the parameter w_p, the results of which are given in Table 7.

Table 7: Sensitivity analysis on the number of weeks allowed for returning empty containers

No. of weeks	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Total cost	70.93	75.53	85.48	96.13	107.35	118.80	129.98	140.81
Slope	4.60	9.95	10.65	11.22	11.45	11.18	10.83	NaN

Note: *(i) "Total cost" with the unit of million US$ denotes the total cost in average of ten random generated instances. (ii) "Slope" shows the increasing rate at a specific number of weeks, for example, at "0" weeks, the rate is* $(75.53 - 70.93)/(1 - 0) = 4.60$. *(iii) "0" week for the devanning process is unrealistic in the real-world operations. The "0" week setting only serves as a benchmark in the sensitivity analysis.*

In the table, when the number of weeks allowed for the devanning process increases, the total cost grows significantly, which suggests the high opportunity cost for allowing more devanning time. Meanwhile, the slope of total cost increases before reaching "4" weeks and decreases after "4" weeks, which reveals that the total cost is increasing convex in the number of weeks first and then becomes increasing concave. More importantly, given "0" weeks as the benchmark, we can see that from "0" week to "1" week, the total cost increases by 4.60 million US$, which is far less than other increasing rates (e.g., 9.95 million US$ from "1" week to "2" weeks). Here, we can derive a managerial insight for shipping liner companies: Allowing one week for the devanning process is a better choice for shipping companies, which is the choice by OOCL (OOCL, 2017). Giving more weeks for the devanning process offers more flexibility for customers such that shipping liner companies may charge higher freight fee for compensating the opportunity cost. However, the loss may outweigh the benefit, as the total cost increases in a convex manner at the beginning based on the sensitivity analysis.

7. Conclusion

This paper makes an explorative study on the ship type decision considering the empty container repositioning and the foldable containers. Different from traditional research works on the ship fleet deployment, our study incorporates both the laden container transportation and the empty container repositioning into ship type decision in order to achieve the global optimum for a shipping service route over a whole planning horizon. Meanwhile, as researchers have shown the economic and logistical viability of foldable containers, the problem also considers the use of foldable containers, which aims to find under what conditions, the shipping liner needs to use the foldable containers in its liner shipping services.

In this study, we find that given the ship type with a certain capacity, the problem transfers to a nonstandard minimum cost flow. Henceforth, we build a network flow model for the problem by constructing a network. When considering standard containers and foldable containers, a trouble arises in the network construction that is some parallel arcs share the same capacity restriction. To overcome this trouble, we design a revised network simplex algorithm that changes the standard pivot operation. The algorithm is applicable to any minimum cost flow problem with sharing capacity restrictions. Based on the algorithm, we develop a solution approach by using reduced costs for excluding some ship type, which can find the optimal ship type in the end. By using the solution approach, we conducted extensive numerical experiments to find some managerial implications on the ship fleet deployment and the foldable container usage.

Some useful managerial implications of this study are summarized from three perspectives. (i) *Ship type decision*: when deciding the ship type deployed in a ship fleet, only involving laden container transportation leads to sub-optimal solutions, as the possible empty container repositioning affects the decision. After including foldable containers, a smaller ship type can be expected to deploy, because foldable containers have the storage space advantage in the empty container repositioning. (ii) *Foldable container usage*: under the current cost setting, it is not cost-effective for shipping lines to use foldable containers, as the long-term leasing cost is high. However, if the long-term leasing cost cuts down, using foldable containers is encouraged, as foldable container usage is highly dependent on the long-term leasing cost. The foldable container usage's sensitivity to the folding and unfolding cost depends on the long-term leasing cost. With different long-term leasing costs, different efforts can be made to reduce the folding and unfolding cost. For example, if container leasing companies charge a moderate price for long-term leasing, the shipping lines may devote much efforts to cut down folding and unfolding cost, which can lead to a profitable result. (iii) *Container devanning time*: It is better to allow one week time for the container devanning process. Although allowing more weeks for the devanning process brings more flexibility for customers such that shipping lines may obtain some benefits, the opportunity cost can outweigh the benefits, as the opportunity cost increases significantly after the one-week setting.

Appendix A: Specification comparison between standard containers and foldable containers

Table A.1 shows major specifications of standard containers and foldable containers, which are almost the same. The specifications of standard containers are collected from APL (2017) and the specifications of foldable containers are collected from Holland Container Innovations (2017b).

Table A.1: Specifications of foldable and standard containers

Description	Standard containers	Foldable containers
Cubic capacity	67.7 cubic meters	72.9 cubic meters
Maximum payload	26,760 kg	26,600 kg
Gross weight	30,480 kg	32500 kg
External length	12.192 m	12.192 m
External width	2.438 m	2.438 m
External height	2.591 m	2.896 m
Internal length	12.032 m	12.012 m
Internal width	2.352 m	2.324 m
Internal height	2.392 m	2.615 m
Door opening width	2.340 m	2.172 m
Door opening height	2.280 m	2.508 m
Bundle (4 into 1) height	—	2.896 m

Appendix B: Proof for the results of Figure 10

In formal, we can prove the non-increasing trend by using a simplified mathematical model. Assuming X represents the vector for the number of standard containers in ports and Y represents the vector for the number of foldable containers in ports. Then, $\rho = \frac{Ye^T}{Xe^T+Ye^T}$ shows the percentage of foldable container usage, where $e = \{1, \ldots, 1\}$ and e^T is the transposition of e. Given the defined vector variables X and Y, we can use the following simplified standard model to represent the formulation of our problem.

$$Min\ \ C_1X + C_2Y$$

$$s.t.\quad A_1X + A_2Y = B$$

$$X, Y \geq 0$$

where all coefficient matrixes or vectors (C_1, C_2, A_1, A_2, B) are positive. In the next, we can derive:

$$X = A_1^{-1}B - A_1^{-1}A_2Y$$

By substituting it to the objective, we have,

$$Min\ \ C_1A_1^{-1}B - C_1A_1^{-1}A_2Y + C_2Y$$

Based on which, if the cost coefficient C_2 for foldable containers increase, Y will decrease. As a result, X will increase. As we have $\frac{1}{\rho} = \frac{Xe^T+Ye^T}{Ye^T} = \frac{Xe^T}{Ye^T} + 1$, the increasing of cost coefficient C_2 will lead to the increasing of $\frac{1}{\rho}$, that is the decreasing of ρ. Such a proof verifies the result shown in Figure 10.

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