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Dynamic programming algorithms for selection of waste disposal ports in cruise shipping

Shuaian Wang[1,2], Lu Zhen[3,*], Dan Zhuge[2]

[1]*The Hong Kong Polytechnic University Shenzhen Research Institute, Nanshan District, Shenzhen, China*

[2]*Department of Logistics & Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Hong Kong*

[3]*School of Management, Shanghai University, Shang Da Road 99, Shanghai 200444, China*

Abstract: The cruise industry has maintained a steady growth in the past 20 years. Due to the large number of cruise passengers and regulations on sea environment protection, determining at which ports to dispose of the waste generated onboard a cruise ship is a key decision to reduce the cost for a cruise company. We address four versions of the problem: the cruise itinerary is either static or dynamic and the amount of waste generated on each voyage leg is either deterministic or stochastic. We propose a polynomial-time solution algorithm for the static deterministic model, and the idea of the algorithm can also be used to solve the static stochastic model and the dynamic deterministic model. Second, we identify the structure of the optimal policy to the dynamic stochastic problem, based on which an efficient dynamic programming algorithm is developed. Extensive numerical experiments derived from problems of real-case scales demonstrate the efficiency of the proposed algorithms.

Keywords: Dynamic programming; cruise ship; water transportation; waste disposal.

1 Introduction

The cruise industry provides a package of recreational activities to passengers. On-board activities include casino gaming, gift shop sales, entertainment arcades, art auctions, photo sales, spa services, bingo games and lottery tickets, enhanced dining experiences in alternative restaurants, video diaries, golf lessons, and snorkel equipment rentals. Moreover, cruise ships will stay at several ports, which are often in different countries, and passengers can have a tour in the port cities. The cruising industry has maintained a steady increase in supply for the past

* Corresponding author: L Zhen. E-mail addresses: wangshuaian@gmail.com (S Wang), lzhen@shu.edu.cn (L Zhen), dan.zhuge@connect.polyu.hk (D Zhuge); Tel: +86-21-66137925; Fax: +86-21-66134284.

20 years. In 2014, the world cruise fleet had 296 ships (Cruise Ship Statistics, 2015), the number of cruise passengers reached a total of 22.04 million, and the global cruise industry generated revenues of 37.1 billion U.S. dollars (Cruise Industry, 2015).

A large cruise ship carries over 6,000 passengers and 1,500 crew members, and therefore the amount of waste generated by a cruise ship is also considerable. This significantly contrasts cargo ships, e.g., bulk ships (Magirou et al., 2015) and container ships (Bell et al., 2013; Psaraftis and Kontovas, 2013, 2014; Ng, 2015), which are usually manned with fewer than 30 crew members. There are mainly five waste streams from cruise ships: sewage, graywater, oily bilge water, solid waste, and hazardous waste (EPA, 2008). Sewage from cruise ships, also known as "black water", generally means human body wastes. Graywater generally means wastewater from sinks, baths, showers, laundry, and galleys. Oily bilge water is the mixture of water, oily fluids and lubricants from the machinery spaces of a cruise ship. Solid waste is the food waste, garbage, refuse, sludge, rubbish, trash, and other discarded materials. Hazardous waste is a type of waste containing hazardous instances. As reported by EPA (2008), every day a large cruise ship may generate 74,000 gallons of sewage, 249,000 gallons of grey water, 5,300 gallons of bilge water, 50 ton of garbage, 12,000 bottles and 12,000 cans, and 10 tons of hazardous waste. Different waste streams are treated differently. Some waste must be off-loaded to shore reception facilities at ports for recycling or disposal, for instance, synthetic ropes, plastic bags, some solid waste that cannot be incinerated, a proportion of oily bilge water, and hazardous waste. Annexes V and VI of The International Convention for the Prevention of Pollution from Ships (MARPOL) of the International Maritime Organization (IMO) have regulated the types of wastes that can be discharged into the sea and have imposed ships to provide facilities for the reception of waste. Most cruise lines have implemented environmental management systems to reduce, select, and manage the waste generated onboard, in order to be in compliance to the MARPOL requirements (Pallis, 2015).

This paper considers the waste that must be landed to the shore reception facilities. A cruise ship has an environmental officer (EO) onboard, a crew member in charge of looking after the waste. The EO reports to the Chief Officer the amount of waste that is onboard and the Chief Officer reports to the shore-side headquarter to decide whether to discharge the waste at the next port of call or not. Cost is a major concern in the choice of waste discharge port. Different ports have quite different prices for waste discharging: some ports, e.g., ports in Cyprus, impose a fixed charge no matter whether a ship actually discharges waste or not (that is equivalent to not cost for waste discharging in the decision process), and other ports can charge up to several thousand dollars. Therefore, it is vital to decide at which ports to discharge waste is the most cost-effective. Given a cruise ship that visits a sequence of ports according to a pre-determined schedule and waste is generated continuously along the voyage and the waste holding tank has limited capacity, we study the problem of determining at which port to dispose of the generated waste so as to minimize the total cost. We identify special structures of the problem and develop dynamic programming based solution approaches to address the problem. Our numerical experiments show a cost reduction of 30% using the proposed methods relative to a greedy waste disposal heuristic.

1.1 Literature review

There are some qualitative works on waste generated onboard cruise ships. Dixon and Hughes (2000) reviewed the current regulations on cruise ship waste management set by IMO, mainly the MARPOL regulations. They also discussed a typical design of waste management systems onboard cruise ships. Johnson (2002) categorized the environmental impacts of cruise tourism, including (i) infrastructure impacts such as ship construction, the creation of cruise passenger terminal facilities and berthing access requirements; (ii) operational impacts involving the use of energy and water and air quality pollution; (iii) distribution impacts associated with tourists' travel and the logistics of supplying a cruise liner with provisions; (iv) use impacts which comprise the cultural impact of wealthy tourists and overcrowding created by large numbers of visitors at one destination; and (v) waste impact. He explored the potential strategies that can be employed by cruise line operators and cruise tourism destinations to manage the impact. He concluded that although the industry was taking a number of belated positive steps, the decision-makers in cruise tourism destinations, such as cruise port operators and port city governments, should work closely with cruise lines to facilitate integrated waste management and sustainable development. Polglaze (2003) highlighted the importance of management of ships' food waste and summarized the estimated rates of food waste generation for merchant ships, passenger ships, and fishing ships, by different sources such as the US National Research Council and International Maritime Organization (IMO). He conducted a survey of six ships and the survey revealed a general upward trend in per capita waste generation rate as a function of crew size. Based on surveys undertaken in 2000 and 2002 regarding the availability of port reception facilities for ship waste in the North Sea area, Carpenter and Macgill (2005) found that most North Sea ports had sufficient reception facilities and already met the EU Directive on reception facilities for ship waste. Butt (2007) studied cruise ship's waste management status in Southampton and investigated the disposal options for ship generated waste and the impacts of the waste on ports. He recommended all cruise ships should pursue a waste reduction strategy and the ports should provide adequate recycling, reduction and re-use facilities for waste. Macpherson (2008) reviewed the environmental, economic and societal impacts of cruise ship tourism. He pointed out that sustainable tourism policies and effective management of the tourism could yield high returns with low risks. Using a life cycle assessment, Zuin et al. (2009) identified and quantified the environmental impacts caused by ships' waste management in the port of Koper, Slovenia. They argued that critical environmental issues are caused by carcinogens substance, inorganic emissions and heavy metals, while the recovery of ship-generated oils is beneficial to reduce the fossil fuel consumption. Klein (2011) measured the impact of cruise tourism with the focus on the perceptions of host communities. Challenges faced by government, communities and the cruise industry were identified and analyzed to give a direction on how tourism can grow in a sustainable and responsible way. Therefore, the existing literature mainly focuses on technological and legal aspect of waste management by cruise ships. To the best of our knowledge, there is no study focusing on how a cruise ship could choose ports to dispose of its waste in the most cost-effective manner.

Our work is also related to literature on dynamic programming applications. Dynamic programming has been applied in many areas, such as inventory management, revenue management, electric vehicle charging, traffic sensor deployment, and appointment scheduling. Berman and Larson (2001) formulated a stochastic dynamic programming model for a vehicle product-delivery problem in which the volume of product required by each customer on a route is random, which is similar with the randomness of the generated waste in this study. The difference is, the problem in Berman and Larson (2001) does not have a nice structure and hence they applied several rules to identify high-quality solutions. Lam et al. (2007) addressed an empty container repositioning problem. As the problem has the curse-of-dimensionality difficulty, they applied an approximate dynamic programming approach. Truong (2015) investigated a dynamic advance scheduling problem in a hospital, where advance scheduling is, when a patient calls, she will immediately be given an appointment in the future or be rejected. Truong (2015) derived a characterization of an optimal policy and based on it, showed that the straightforward dynamic programming formulation with an exponential number of states can be transformed to one with a polynomial number of states. In our problem setting, cruise ships repeat their itineraries (in contrast to an open end planning horizon) and there are "recourse" actions (use of waste disposal vessel in case the waste holding tank is full at sea) when considering random amount of waste generated. These differences enable us to examine special structures of the problem and develop tailored solution approaches.

1.2 Objectives and contributions

The objective of this study is to examine how a cruise company should choose waste disposal ports for a cruise ship to discharge waste at minimum cost. We develop four models for the problem. The four models are categorized according to two dimensions, i.e., the itinerary (either *static* or *dynamic*) and the amount of waste generated on each voyage leg (either *deterministic* or *stochastic*). By combining the two options in each of the two dimensions, four different models are formulated as follows.

(1) The first is a *static deterministic* model. It is static in the sense that the cruise ship repeats its itinerary many times and the same waste disposal ports are used in each repetition. It is deterministic as the amount of waste generated on each voyage leg is assumed to be fixed.

(2) The second one is *static stochastic* as it captures the stochastic nature of the amount of waste generated on each voyage leg. Due to the stochasticity of the amount of waste generated, it is possible that the waste holding tank is full during a voyage; and if this happens, a waste disposal vessel must be called to dispose of the waste at a high cost.

(3) The third model is *dynamic deterministic*: the amount of waste generated on each voyage leg is fixed, and the cruise ship may provide any itineraries (rather than repeating the same itinerary) in a finite planning horizon.

(4) The fourth one is *dynamic stochastic* that captures the stochasticity of the amount of waste generated.

This study is one of the very few attempts on cruise shipping using quantitative approaches. The contribution of the paper is twofold. First, we propose a polynomial-time solution algorithm for the static deterministic model, and the idea of the algorithm can also be used to solve the static stochastic model and the dynamic deterministic model efficiently. Second, we identify the structure of the optimal policy to the dynamic stochastic problem, based on which an efficient dynamic programming algorithm is developed. Extensive numerical experiments based on problems of real-case scales demonstrate the efficiency of the proposed algorithms.

The remainder of the paper is organized as follows. Section 2 presents the static models, including the static deterministic model and the static stochastic model, and solution algorithms. Section 3 presents the dynamic models, including the dynamic deterministic model and the dynamic stochastic model, as well as solution algorithms. Section 4 reports the results of numerical experiments to validate the efficiency of the proposed algorithms. Conclusions are then outlined in Section 5.

2 Static models

Consider a cruise ship that provides a regular cruising itinerary, visiting Ports 1, $2 \ldots n$, and visiting Port 1 again, similar to container liner shipping (Wang et al., 2014). Port 1 is the home port and the other $n-1$ ports are ports of call. The itinerary starts from the home port to pick up passengers and returns to the home port where cruise passengers get off the cruise ship. After that, the itinerary starts again, as shown in Figure 1. For instance, the cruise ship “Quantum of the Seas” operated by Royal Caribbean International (RCI, 2015) visits Shanghai (China) as the home port, then Busan (South Korea), Nagasaki (Japan), and returns to Shanghai in the first rotation; then it starts its second rotation visiting Busan, Nagasaki, and Shanghai again. The same itinerary is repeated many times. The voyage from Port i to Port $i+1$ is called Leg i.

The capacity of the waste holding tank of the ship (waste holding capacity), i.e., the maximal amount of waste that can be stored in the ship, is denoted by V. The cost for disposing of the waste stored in the ship at Port i is denoted by c_i. In the static setting, we need to identify the ports at which the waste is disposed of (discharged). In Sections 2.1 and 2.2, we assume that the same ports are used to dispose of waste in every repetition of the itinerary for the sake of uniformity in managerial decision making; therefore, we only need to take into account one itinerary. Section 2.3 discusses the cases when relaxing this requirement.

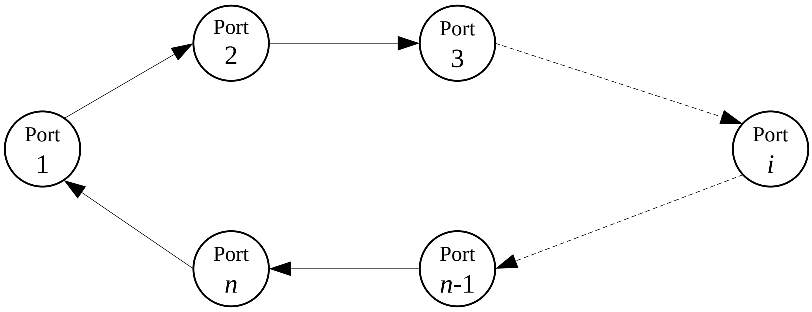

Figure 1: Itinerary in static models

2.1 *Static deterministic model*

In the deterministic model, we assume that the amount of waste generated on Leg i, denoted by q_i, $0 \le q_i \le V$, is deterministic. We assume that $\sum_{i=1}^{n} q_i > V$, because otherwise we only dispose of the waste at Port $i^* \in \arg\min_{i=1,2\cdots n} c_i$.

2.1.1 Mixed-integer linear programming model

The static deterministic problem can be formulated as a mixed-integer linear programming model. The decision variables are: z_i is binary which equals 1 if and only if the waste is disposed of at Port i, and y_i is the amount of waste onboard the ship when the ship just leaves Port i. In the model, we define $y_0 := y_n$ and $q_0 := q_n$. The model is:

[M1]
$$\min \sum_{i=1}^{n} c_i z_i \tag{1}$$

subject to:

$$y_i + q_i \le V, i = 1, 2 \cdots n \tag{2}$$

$$y_i \geq y_{i-1} + q_{i-1} - Vz_i, i = 1, 2 \cdots n \tag{3}$$

$$y_i \geq 0, i = 1, 2 \cdots n. \tag{4}$$

$$z_i \in \{0,1\}, i = 1, 2 \cdots n. \tag{5}$$

Eq. (1) minimizes the total cost of discharging the waste for one itinerary. Constraints (2) impose that when the ship arrives at Port $i+1$, the amount of waste in the ship $y_i + q_i$ should not exceed the capacity of the waste holding tank. Eqs. (3), in which V is actually the "big-M", are a linear form of the following constraints:

$$y_i = \begin{cases} 0, & \text{if } z_i = 1 \\ y_{i-1} + q_{i-1}, & \text{if } z_i = 0 \end{cases}, \ i = 1, 2 \cdots n \tag{6}$$

Eqs. (6) imply when the waste is disposed of at Port i, i.e., $z_i = 1$, then there is no waste in the ship when it leaves Port i; otherwise the amount of waste in the ship when it leaves Port i is equal to the amount when it leaves the previous port (Port $i-1$) plus the amount generated during the voyage from Port $i-1$ to Port i. Constraints (4) define y_i as nonnegative variables and Constraints (5) define z_i as binary variables.

2.1.2 A polynomial-time solution approach

The mixed-integer linear formulation M1 can generally be solved by off-the-shelf mixed-integer linear programming solvers. Nevertheless, we still investigate the properties of the static deterministic problem to gain insights which might be helpful for us to address more complex problems such as the static stochastic one in the next sub-section. We find that M1 has nice properties for us to develop a polynomial-time solution approach.

Consider the example in Figure 2: Figure 2(a) shows the parameters. For instance, the cost for disposing of waste at Port 2 is 2 (unit: 1000 dollars), the amount of waste generated on the leg from Port 2 to Port 3 is $q_2 = 3$, and the holding capacity is $V = 6$.

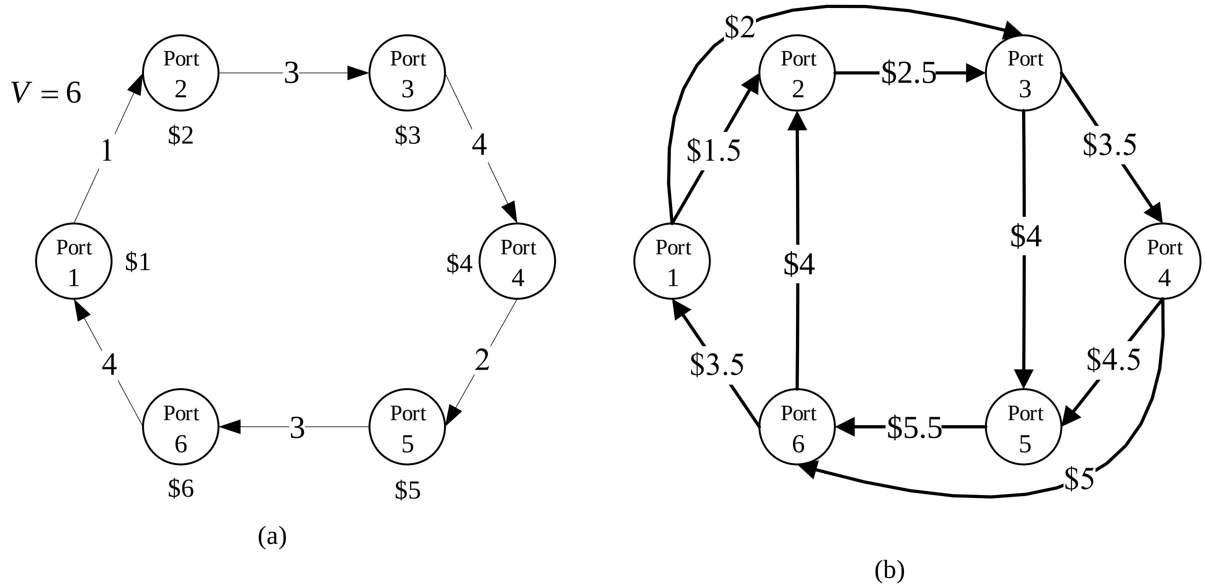

Figure 2: Reformulated network

It occurs to us that if we consider the choice of two adjacent waste disposal ports rather than the choice of each port as in Model M1, the problem will become easier. To this end, we reformulate the network in Figure 2(a) to Figure 2(b). For each Port i, we check for every other Port j whether it is possible to dispose of the waste at i and j, but not any port in between. An arc (i, j), $i = 1, 2 \cdots n, j = 1, 2 \cdots n, j \neq i$, is created if and only if after discharging the waste at Port i, the vessel does not need to discharge waste at any of Ports $i+1, i+2 \cdots j-1$ before discharging at Port j. Mathematically, the set of arcs to create is all of the (i, j)'s that satisfy

$$(i, j) \in \{1, 2 \cdots n\} \times \{1, 2 \cdots n\} \tag{7}$$

$$i \neq j \tag{8}$$

$$\begin{cases} \text{if } i < j,\ \sum_{k=i}^{j-1} q_k \leq V \\ \text{if } i > j,\ \sum_{k=i}^{n} q_k + \sum_{k=1}^{j-1} q_k \leq V. \end{cases} \tag{9}$$

The cost of Arc (i, j), denoted by c_{ij}, is defined as $(c_i + c_j)/2$. We divide by two because each port will be included in two arcs in a solution to the new problem: to find a minimum cost cycle in the reformulated network. Note that not all nodes must be visited in the cycle.

By now the new problem is not much different from M1: both need to determine at which ports to discharge the waste. Therefore, the number of potentially feasible cycles to the new problem can be up to 2^n, which increases exponentially with regard to the size the problem n. However, the new problem has a nice structure that leads to the following theorem.

Theorem 1: The static deterministic problem can be solved in time $O(n^3)$.

Proof: The construction of the reformulated network takes time $O(n^2)$ as we need to check $O(n^2)$ port pairs to see whether an arc between each port pair should be added. The time required for checking a port pair (e.g., from i to j), which involves one summation (sum of the wastes generated on Leg $j-1$ and the wastes generated from port i to port $j-1$) and the comparison of the sum with the holding capacity, is $O(1)$. Then, we need to find the shortest cycle in the reformulated network, which can be completed in time $O(n^3)$ (Lawler, 1976). □

Theorem 1 proves that the static deterministic problem is an easy problem that can be efficiently solved to optimality.

2.2 Static stochastic model

We now consider the case in which the waste generated on a leg is a random variable. Unlike cargo ships whose waste produced is almost constant every day (as the waste is mainly from the ship itself, e.g., oily bilge water), a cruise ship carries up to 5,000 passengers who produce a large amount of waste. As pointed out by Pallis (2015), "[t]he amount and types of waste might vary from one cruise ship to another, yet cruises are generators of the highest amount of garbage. A cruise ship with 3,000 passengers and crew generates about 50 tonnes of solid waste in a single week. An average cruise passenger generates a minimum of one kilogram of solid waste plus two bottles and two cans per day and an average of 50 tonnes of sewage per day. As cruise activities grow, the size of the waste produced during every single cruise […] cannot be ignored." Therefore, the amount of waste generated depends on (i) the number of passengers onboard, and (ii) the amount of waste each passenger produces, which is up to the passenger's consumption of food, beverages, shampoo, newspapers, etc. Véronneau and Roy (2009) stated that "[t]he reality for cruise companies is that when there is a new itinerary or different demographics, a ship's consumption is unpredictable. There is no real statistical model that confirms that the English consume two pounds of roast beef on a daily basis while Spaniards eat more fish." Therefore, the amount of waste generated on a leg is indeed random.

Denote by ξ_i the waste generated on Leg i with cumulative distribution function $F_i(x)$. Similar to Sections 2.1, we assume that in each repetition of the itinerary, the same ports are used to dispose of waste due to ease of management; therefore, we only need to take into account one itinerary. The requirement that the same ports are used to dispose of waste further implies that the choice of waste disposal ports does not change with the actual amount of waste generated.

We assume that ξ_i and ξ_j are independent, $i \neq j$. Define vector $\boldsymbol{\xi} = (\xi_1, \xi_2 \cdots \xi_n)$. If, during the voyage from one port to the next the amount of waste onboard reaches the capacity V, a waste disposal vessel must be called so that the waste can be discharged to the waste disposal vessel. Using the waste disposal vessel incurs a very high cost $\bar{C} > \max\{c_i, i = 1, 2 \cdots n\}$. The static stochastic problem determines the ports at which the waste is disposed of. It is possible that, due to the stochastic nature of the amount of waste generated, the amount of waste onboard reaches the holding capacity during the voyage; then the cost of using waste disposal vessels is incurred. The objective of the static stochastic problem is to minimize the sum of disposing costs at ports and the expected cost of using waste disposal vessels.

Define a vector of decision variables $\mathbf{z} := (z_1, z_2 \cdots z_n)$. The static stochastic problem can be formulated as a two-stage stochastic program. The first-stage model is:

[M2.1]
$$\min \sum_{i=1}^{n} c_i z_i + \mathbb{E}\left[f(\mathbf{z}, \boldsymbol{\xi}) \right] \tag{10}$$

$$z_i \in \{0,1\}, i = 1, 2 \cdots n. \tag{11}$$

In Eq. (10), $\mathbb{E}\left[f(\mathbf{z}, \boldsymbol{\xi}) \right]$ is the expected cost of using waste disposal vessels when the decision is $\mathbf{z}$.

We now calculate $\mathbb{E}\left[f(\mathbf{z}, \boldsymbol{\xi}) \right]$. Define $\mathbb{Z}_+$ as the set of nonnegative integers. For a realization of the amount of waste vector $\boldsymbol{\xi}$, denoted by $\mathbf{q} := (q_1, q_2 \cdots q_n)$, we define decision variable m_i

as the number of times waste disposal vessels are used on Leg i and y_i as the amount of waste in the ship when the ship just leaves Port i. The second stage problem is:

[M2.2] $$f(\mathbf{z},\mathbf{q}) = \min \bar{C}\sum_{i=1}^{n} m_i \quad (12)$$

subject to:

$$y_i + q_i - Vm_i \le V, i = 1, 2 \cdots n \quad (13)$$

$$y_i \ge y_{i-1} + q_{i-1} - Vm_{i-1} - Vz_i, i = 1, 2 \cdots n \quad (14)$$

$$m_i \in \mathbb{Z}_+, i = 1, 2 \cdots n \quad (15)$$

$$y_i \ge 0, i = 1, 2 \cdots n. \quad (16)$$

The objective function (12) minimizes the expected cost of using waste disposal vessels. Constraints (13) calculate how many times waste disposal vessels are used on each leg. Constraints (14) are similar to (6) and use the “big-M” method to define the relation between y_{i-1}, q_{i-1}, and y_i. Constraints (15) define the number of times waste disposal vessels are used on each leg as a nonnegative integer. Constraints (16) enforce that the amount of waste onboard when the ship just leaves a port is nonnegative.

2.2.1 Dynamic programming based on the reformulated network

The model M2.1 embedded with M2.2 seems to be very difficult as it is a two-stage stochastic program with integer variables in the first stage and both integer and continuous variables in the second stage. Moreover, we find that it is challenging to calculate the number of times waste disposal vessels are used on each leg, as this number depends on the amount of waste onboard when the ship just departs from the start port of the leg; however, enlightened by the reformulated network in Figure 2(b), the number of times waste disposal vessels are used on each arc in Figure 2(b) can be determined a priori using Monte-Carlo simulation. Similar to the static deterministic problem, we also use the arcs in Figure 2(b) as the first-stage decisions. There are two differences from the static deterministic problem. First, all of the arcs $(i, j), i \ne j$ that connect a pair of nodes should be considered because with stochastic amount of waste Eqs. (9) are no longer meaningful. Second, the expected cost of Arc (i, j) is no longer just

$(c_i + c_j)/2$, but also includes the expected costs of using waste disposal vessels which can be determined a priori. Hence, this stochastic problem can be formulated as a deterministic one, as shown by the algorithm below.

Algorithm 1: Solving the static stochastic problem

Step 1: The total expected cost for discharging at only one port is

$$\hat{C}^0 = \left(\min_{i=1,2\cdots n} c_i\right) + \bar{C} \times \mathbb{E}\left[\left\lfloor \frac{\sum_{i=1}^{n} \xi_i}{V} \right\rfloor\right]. \tag{17}$$

The first term in the above equation means if the waste is discharged at exactly one port, it should be the one with the lowest cost. The second term is the expected costs of using waste disposal vessels, in which the expectation operator calculates the expected number of times of using waste disposal vessels in an itinerary and $\lfloor x \rfloor$ is the largest integer not greater than x.

Step 2: Construct a new network consisting of n nodes representing the n ports and $n(n-1)$ arcs, i.e., there is an arc connecting any two nodes. The expected cost of arc (i, j) is

$$c_{ij} = \begin{cases} \dfrac{c_i + c_j}{2} + \bar{C} \times \mathbb{E}\left[\left\lfloor \dfrac{\sum_{k=i}^{j-1} \xi_k}{V} \right\rfloor\right], & \text{if } i < j \\ \dfrac{c_i + c_j}{2} + \bar{C} \times \mathbb{E}\left[\left\lfloor \dfrac{\sum_{k=i}^{n} \xi_k + \sum_{k=1}^{j-1} \xi_k}{V} \right\rfloor\right], & \text{if } i > j. \end{cases} \tag{18}$$

Step 3: Find the least cost cycle in the new network in Step 2, denoted by $\hat{C}$, and the lowest expected total cost is

$$\min\left(\hat{C}^0, \hat{C}\right). \tag{19}$$

and the optimal solution can be found accordingly. □

2.2.2 Monte-Carlo simulation

Calculating the exact values of the expectations in Eqs. (17) and (18) could be challenging as they involve multi-dimensional integration. Nevertheless, the expectations can easily be estimated with a high precision using Monte-Carlo simulation. Take the expectation in Eq. (17) as the example, we could generate N realizations of the $\boldsymbol{\xi}$, denoted by $\boldsymbol{\xi}^{(1)}, \boldsymbol{\xi}^{(2)} \cdots \boldsymbol{\xi}^{(N)}$, and according to strong law of large numbers, the expectation can be estimated by

$$\mathbb{E}\left[\left\lceil \frac{\sum_{i=1}^{n} \xi_i}{V} \right\rceil\right] \approx \frac{1}{N} \sum_{k=1}^{N} \left\lceil \frac{\sum_{i=1}^{n} \xi_i^{(k)}}{V} \right\rceil \tag{20}$$

Our numerical tests show that using $N = 5,000$ realizations is accurate enough.

2.3 Relaxation of the same waste disposal ports on different repetitions of an itinerary

This section discusses the cases in which the sets of waste disposal ports on different repetitions of an itinerary can be different.

In the deterministic model, consider an infinite planning horizon. Because disposing the waste will mean a 'clean slate' of the state of the system, it can be shown that there is an optimal solution such that from a certain repetition of the itineraries, the choice of waste disposal ports will be periodic every few repetitions and the period is at most n repetitions. Given this periodic structure, an optimal solution can be identified based on enumeration and the dynamic programming in Section 3.1.

In the stochastic model, if the set of ports at which to dispose of waste is to be determined at the beginning of an infinite planning horizon, then the optimal solution also has the periodic structure and can be obtained in a similar way to the above deterministic case; if whether to dispose of waste at a port is to be determined after observing the amount of waste on board when the ship arrives at the port, then the model in Section 3.2 can be applied.

3 Dynamic models

In the static settings, we assume that the cruise ship provides the same itinerary repeatedly, e.g., Shanghai→Busan→Nagasaki→Shanghai→Busan→Nagasaki→Shanghai, and fixed ports are used to dispose of waste in each itinerary. If we allow different ports to be used to dispose of waste in different repetitions of the itinerary, for example, Busan is used in the first repetition and Shanghai and Nagasaki are used in the second repetition, we then need to select the waste disposal ports dynamically. Moreover, if the ship provides different itineraries in a planning horizon, e.g., the first itinerary is Shanghai→Busan→Nagasaki→Shanghai, and the second itinerary is Shanghai→Jeju→Shanghai, we also need to select the waste disposal ports dynamically.

In the dynamic setting, we consider all of the ports visited by the ship in a planning horizon of e.g. 180 days, defined from Port 1 to Port n, as shown in Figure 3. Note that here n is the total number of ports visited in the planning horizon and is usually much larger than the n in the static setting which means the total number of ports in an itinerary. The decisions are at which ports to discharge waste.

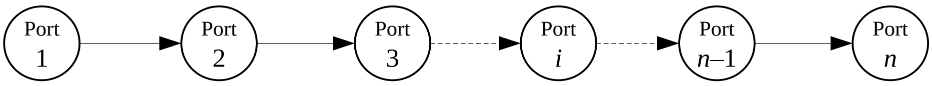

Figure 3: Dynamic setting

3.1 Dynamic deterministic model

If the amount of waste generated on each leg is deterministic, the dynamic deterministic problem can be easily solved using dynamic programming (DP) based on a reformulated network similar to the one in Figure 2(b), and we only need to find the shortest path from a dummy source node to a dummy sink node in the reformulated network.

3.2 Dynamic stochastic model

Now we examine the dynamic stochastic problem. In the dynamic stochastic problem, the amount of waste generated on each leg is a random variable. Moreover, whether the ship will dispose of its waste at a port is not determined a priori, but determined at the time of arrival. Therefore, we need to identify the optimal policy for the dynamic stochastic problem.

The amount of waste generated on Leg $i = 1, 2 \cdots n-1$ is a random variable ξ_i with known distribution functions. We assume that ξ_i has support $[0, V]$ for the purpose of ease of presentation, although our model can easily be revised to accommodate the general case in which the amount of waste generated on a leg may be larger than the holding capacity. Values of ξ_i variables are independent for different indexes of i. Due to stochasticity, it is possible that the disposal tank is full of waste during the voyage and a waste disposal vessel is used to discharge the waste at the cost of $\bar{C}$. Now the decision when the ship arrives at Port $i = 1, 2, 3 \cdots n-1$ is: whether the waste should be disposed of if the amount when the ship arrives at the port is u_i. The value of u_1 is known.

3.2.1 Dynamic programming approach

In the dynamic stochastic problem, the optimal decision to make when the ship arrives at a port only depends on the amount of waste onboard and is independent of the historical information and decisions. Therefore, we could use dynamic programming to solve the problem.

The dynamic programming process has n stages, and the policy decision at state i is whether to discharge the waste at Port i. The state of stage i, denoted by u_i, is the amount of waste onboard when the ship arrives at Port i. The backward reduction procedure for the problem is as follows. Let θ be a binary decision variable where $\theta = 0$ means the waste is not discharged and $\theta = 1$ means the waste is discharged at a port. Define $\Psi_i^*(u_i)$ as the minimum sum of total expected cost incurred during the interval from the arrival at Port i to the arrival at Port n if the amount of waste when the ship arrives at Port i is u_i. The recursive relation is

$$\Psi_i^*(u_i) = \min_{\theta \in \{0,1\}} \left\{ c_i \cdot \theta + \bar{C} \cdot \Pr(u_i - u_i \cdot \theta + \xi_i > V) + \mathbb{E}\left[\Psi_{i+1}^*(\tilde{u}_{i+1}) \right] \right\},\ i = 1, 2, 3 \cdots n-1 \quad (21)$$

subject to

$$\tilde{u}_{i+1} = \begin{cases} u_i - u_i \cdot \theta + \xi_i, & \text{if } u_i - u_i \cdot \theta + \xi_i \le V \\ u_i - u_i \cdot \theta + \xi_i - V, & \text{otherwise} \end{cases} \quad (22)$$

where the ‘tilde’ in $\tilde{u}_{i+1}$ highlights that the state at stage $i+1$ is a random variable, whose value depends on the random variable ξ_i, and the boundary conditions are

$$\Psi_n^*(u_n) = 0, \quad u_n \in [0,V]. \tag{23}$$

In Eqs. (21), the first term $c_i \cdot \theta$ is the waste disposal cost at Port i; the second term $\bar{C} \cdot \Pr(u_i - u_i \cdot \theta + \xi_i > V)$ is the expected cost of using waste disposal vessel on Leg i, where $u_i - u_i \cdot \theta$ is the amount of waste onboard the ship when it just leaves Port i; note that the waste disposal vessel is used at most once as we assume ξ_i has support $[0,V]$; the third term has the expectation operator because the amount of waste onboard when the ship arrives at Port $i+1$ is a random variable. Note that if the random variable ξ_i may take a value larger than V, we only need to slightly change Eqs. (21) and (22). The dynamic stochastic problem aims to find a policy to minimize $\Psi_1^*(u_1)$.

Note that $\Psi_1^*(u_1)$ is neither convex nor concave. For example, suppose that there are only two ports and one leg. The amount of waste generated on the leg is 10, and the waste tank capacity is 20. The waste disposal cost at the port 1 is $2000. Then, the total cost is 0 if the amount of waste onboard when the ship arrives at port 1 is not greater than 10, and is $2000 when the waste onboard is greater than 10.

3.2.2 Structure

Define $\theta_i^*(u_i)$ as the optimal value of θ for Port i given u_i in Eqs. (21), $i = 1,2,3 \cdots n-1$.

Proposition 1: The optimal policy for the dynamic stochastic problem is threshold-based. That is, there exists a vector $(u_i^*, i = 1,2,3 \cdots n-1)$, such that the ship at Port i should dispose of the waste if $u_i \geq u_i^*$ and should not dispose of the waste, i.e., $\theta_i^*(u_i) = 1$ if $u_i \geq u_i^*$ and $\theta_i^*(u_i) = 0$ if $u_i < u_i^*$, $i = 1,2,3 \cdots n-1$.

Proof: We have (i) $\Psi_{i+1}^*(u_{i+1})$ is non-decreasing in u_{i+1}; (ii) $c_i \cdot \theta + \bar{C} \cdot \Pr(u_i - u_i \cdot \theta + \xi_i > V)$ is non-decreasing in u_i for all θ and all i; (iii) $\Pr(\tilde{u}_{i+1} \geq x \mid u_i, \theta)$ is non-decreasing in u_i for all given $x \in [0, V]$, all θ and all i; (iv) $\Psi_n^{*}(u_n)$ is non-decreasing in u_n. Then, based on Proposition 4.7.3 of Puterman (2014), $\theta_i^{*}(u_i)$ is non-decreasing in u_i for all θ and all i. Since θ can either be 0 or 1, there exists a vector $(u_i^{*}, i = 1, 2, 3 \cdots n-1)$ such that $\theta_i^{*}(u_i) = 1$ if $u_i \geq u_i^{*}$ and $\theta_i^{*}(u_i) = 0$ if $u_i < u_i^{*}$, $i = 1, 2, 3 \cdots n-1$. □

The above threshold-based solution structure is similar to some inventory management, revenue management, and electric vehicle routing problems, in which the amount of on-hand inventory, the available capacity, and the amount of electricity stored in the vehicle at each stage are the thresholds, respectively.

Proposition 1 shows that to determine the optimal policy, we only need to determine the vector $(u_i^{*}, i = 1, 2, 3 \cdots n-1)$. Evidently, we should not dispose of the waste if the amount when the ship arrives at a port is 0, and we should always dispose of the waste if the amount is V. We thus must have $0 < u_i^{*} \leq V, i = 1, 2, 3 \cdots n-1$.

Define $\pi := (\pi_i, i = 1, 2, 3 \cdots n-1)$, $0 < \pi_i \leq V, i = 1, 2, 3 \cdots n-1$, as a threshold-based policy: if we use policy π, then the waste will be disposed of at Port i if and only if the amount of waste onboard the ship when it arrives at Port i is greater than or equal to π_i. We further define $\Lambda(\pi)$ as the total expected cost of policy π.

Remark 1: The function $\Lambda(\pi)$ is not convex.

Proof: We prove the remark by constructing a counter-example. Suppose that $n = 3$, $u_1 = 0$, ξ_1 has three possible realizations 0.4, 0.5, and 0.6 with probabilities 0.8, 0.1, and 0.1, respectively; ξ_2 is deterministic and equal to 0.9; $V = 1$, $c_2 = 1$, $\bar{C} = 10$. Then we can calculate that if we never dispose of the waste at Port 2, e.g., $\pi := (V, V)$, then $\Lambda(\pi = (V, V)) = \bar{C} = 10$; if we always dispose of the waste at Port 2, i.e., $\pi := (\varepsilon, \varepsilon)$ where ε is a very small positive number, then $\Lambda(\pi = (\varepsilon, \varepsilon)) = c_2 = 1$. For this simple case, it is easy to see that an optimal policy is $\pi^{*} = (\pi_1^{*}, \pi_2^{*}) = (1, 0.4)$, i.e., we do not dispose of the waste at Port 1 and we dispose of the waste at Port 2 if the amount is at least 0.4. Note that as $u_1 = 0$, any value of $\pi_1^* \in (0,1]$ is optimal; we also note that any $\pi_2^* \in (0, 0.4]$ is optimal. The total expected cost of the optimal policy $\Lambda(\pi^*) = 1$.

Consider two other policies $\pi^{(1)} = (\pi_1^{(1)}, \pi_2^{(1)}) = (1, 0.5)$ and $\pi^{(2)} = (\pi_1^{(2)}, \pi_2^{(2)}) = (1, 0.6)$. We thus have $\Lambda(\pi^{(1)}) = 0.8\overline{C} + 0.2c_2 = 8.2$ and $\Lambda(\pi^{(2)}) = 0.9\overline{C} + 0.1c_2 = 9.1$. Although $\pi^{(1)} = 0.5\pi^* + 0.5\pi^{(2)}$, we have $\Lambda(\pi^{(1)}) > 0.5\Lambda(\pi^*) + 0.5\Lambda(\pi^{(2)})$. □

Remark 2: The function $\Lambda(\pi)$ is not strictly quasi-convex.

Proof: We prove the remark by constructing a counter-example. Consider the example in the proof of Remark 1. The policy $\pi^* = (\pi_1^*, \pi_2^*) = (1, 0.4)$ is optimal with $\Lambda(\pi^*) = 1$. The policy $\hat{\pi} = (1, 0.3)$ is optimal with $\Lambda(\hat{\pi}) = 1$. The policy $\pi^{(1)} = (\pi_1^{(1)}, \pi_2^{(1)}) = (1, 0.5)$ has the total expected cost $\Lambda(\pi^{(1)}) = 0.8\overline{C} + 0.2c_2 = 8.2$. Although $\Lambda(\hat{\pi}) < \Lambda(\pi^{(1)})$ and $\pi^* = 0.5\hat{\pi} + 0.5\pi^{(1)}$, we have $\Lambda(\pi^*) = \min\{\Lambda(\hat{\pi}), \Lambda(\pi^{(1)})\}$. □

Remark 1 and Remark 2 show that the function $\Lambda(\pi)$ may not have a nice structure for us to apply nonlinear optimization techniques to find the optimal threshold-based policy π^* for a general problem. Therefore, we do not take advantage of continuous approximation methods to address the above continuous state dynamic program. Instead, since the system state has only one dimension, we take advantage of Proposition 1 to develop a discretization method, which is elaborated below.

3.2.3 Discretization algorithm

Since u_i is between 0 and V, we can discretize u_i and use dynamic programming over the discretized values. In the DP process, we make a decision at each port (stage) when the amount of waste onboard is a discretized value (state). We can take advantage of Proposition 1 to reduce the search space. The algorithm is elaborated below and we will explain the details later.

Algorithm 2: Solving the dynamic stochastic problem

Step 0: Define $i = n$ and the step size of amount of waste Δ (e.g., 1 cubic meter) (for ease of presentation, we assume that V / Δ is an integer; otherwise we simply reset the capacity to $\lfloor V / \Delta \rfloor \Delta$). Define a set $W := \{0, \Delta, 2\Delta, \cdots, (V / \Delta)\Delta\}$. The set W works as follows: the amount of waste generated on each leg is rounded up to a discrete value in the set W. For instance, if the amount is 2.34Δ, then we consider it as 3Δ. Define the boundary conditions $\Psi_n^*(u_n) = 0$, $u_n \in W$.

Step 1: If $i = 1$, output the optimal policy, and stop. Otherwise set $i \leftarrow i - 1$.

Step 1.1: Set $u_i = 0$ and do not discharge waste at the port and

$$\theta_i^*(u_i = 0) = 0 \tag{24}$$

$$\Psi_i^*(u_i = 0) = \sum_{u_{i+1} \in W \setminus \{0\}} \Pr(u_{i+1} - \Delta < \xi_i \le u_{i+1}) \Psi_{i+1}^*(u_{i+1}) \tag{25}$$

Step 1.2: Set $u_i \leftarrow u_i + \Delta$. Calculate

$$\begin{aligned} \Psi_i(u_i, 0) = & \sum_{u_{i+1} \in \{u_i + \Delta, u_i + 2\Delta, \cdots, (V/\Delta)\Delta\}} \Pr(u_{i+1} - \Delta < u_i + \xi_i \le u_{i+1}) \Psi_{i+1}^*(u_{i+1}) \\ & + \sum_{u_{i+1} \in \{\Delta, 2\Delta, \cdots, u_i\}} \Pr(u_{i+1} + V - \Delta < u_i + \xi_i \le u_{i+1} + V) \left[\bar{C} + \Psi_{i+1}^*(u_{i+1}) \right] \end{aligned} \tag{26}$$

$$\Psi_i(u_i, 1) = c_i + \sum_{u_{i+1} \in W \setminus \{0\}} \Pr(u_{i+1} - \Delta < \xi_i \le u_{i+1}) \Psi_{i+1}^*(u_{i+1}) \tag{27}$$

$$\theta_i^*(u_i) \in \arg\min_{\theta \in \{0,1\}} \Psi_i(u_i, \theta) \tag{28}$$

$$\Psi_i^*(u_i) = \Psi_i(u_i, \theta_i^*(u_i)) \tag{29}$$

If $\theta_i^*(u_i) = 1$, then due to Proposition 1, we have

$$\theta_i^*(x) = 1, \;\; x = u_i + \Delta, u_i + 2\Delta, ..., V \tag{30}$$

$$\Psi_i^*(x) = \Psi_i^*(u_i), \;\; x = u_i + \Delta, u_i + 2\Delta, ..., V \tag{31}$$

and go to Step 1. Otherwise, go to Step 1.2.

and go to Step 1. □

In Eq. (26), when $\xi_i \le V - u_i$, waste disposal vessels are not used and $u_{i+1} \in \{u_i + \Delta, u_i + 2\Delta, \cdots, (V / \Delta)\Delta\}$; when $V - u_i < \xi_i \le V$, a waste disposal vessel is used and

$u_{i+1} \in \{\Delta, 2\Delta, \cdots, u_i\}$. In Eq. (27), since the waste is disposed of at Port i, the amount of waste onboard the ship at Port $i+1$ is equal to the amount of waste generated on Leg i. We consider the amount as u_{i+1} if it is in the interval $(u_{i+1}-\Delta, u_{i+1}]$. When the distribution functions of the random variables ξ_i are known and $\Pr(u_{i+1}-\Delta < \xi_i \le u_{i+1})$ can be computed in time complexity $O(1)$, then the time complexity of Algorithm 2 is $O(n|W|^2) = O(nV^2/\Delta^2)$.

The optimal policy obtained by Algorithm 2 is actually a near-optimal policy and the value of $\Psi_1^*(u_1)$ obtained by Algorithm 2 is an upper bound on the minimum total expected cost due to the discretization of u_i. To obtain a lower bound, we can simply round down the amount of waste generated on each leg to a discrete value in the set W (and treat the capacity as $\lceil V/\Delta \rceil \Delta$ if V/Δ is not an integer). That is, we replace Eqs. (25) to (27) by the equations below:

$$\Psi_i^*(u_i = 0) = \sum_{u_{i+1} \in W \setminus \{(V/\Delta)\Delta\}} \Pr(u_{i+1} \le \xi_i < u_{i+1} + \Delta) \Psi_{i+1}^*(u_{i+1}) \tag{32}$$

$$\Psi_i(u_i, 0) = \begin{cases} \displaystyle\sum_{u_{i+1} \in W \setminus \{0\}} \Pr(u_{i+1} \le u_i + \xi_i < u_{i+1} + \Delta) \Psi_{i+1}^*(u_{i+1}), & u_i = \Delta \\ \displaystyle\sum_{u_{i+1} \in \{u_i, u_i+\Delta, \cdots, (V/\Delta)\Delta\}} \Pr(u_{i+1} \le u_i + \xi_i < u_{i+1} + \Delta) \Psi_{i+1}^*(u_{i+1}) & \\ \displaystyle + \sum_{u_{i+1} \in \{\Delta, 2\Delta, \cdots, u_i - \Delta\}} \Pr(u_{i+1} + V \le u_i + \xi_i < u_{i+1} + V + \Delta) \left[\bar{C} + \Psi_{i+1}^*(u_{i+1})\right], & u_i \in W \setminus \{0, \Delta\} \end{cases} \tag{33}$$

$$\Psi_i(u_i, 1) = c_i + \sum_{u_{i+1} \in W \setminus \{(V/\Delta)\Delta\}} \Pr(u_{i+1} \le \xi_i < u_{i+1} + \Delta) \Psi_{i+1}^*(u_{i+1}) \tag{34}$$

Then the value of $\Psi_1^*(u_1)$ obtained by Algorithm 2 with Eqs. (32) to (34) is a lower bound on the minimum total expected cost of the stochastic dynamic problem.

4 Numerical experiments

Numerical experiments are conducted to evaluate the performance of the proposed models, which are implemented by Visual Studio 2008 on a PC (Intel Core i5, 1.7GHz; Memory, 8G). The parameters are set as follows. The cost of discharging the waste at a port is between \$100 and \$1000 and the cost of using a waste disposal vessel is between \$20,000 and \$50,000, both of which follow the Uniform Distribution (Penco and Di Vaio, 2014). The sailing time between two ports is randomly generated by using Uniform Distribution from one day to five days. The passenger capacity of a cruise ship is an integer uniformly distributed between 2000 and 6000. The amount of waste generated on each leg follows the Truncated Normal Distribution, with the mean value equal to sailing time × number of passengers × 6 liters per passenger per day, the standard deviation equal to $1/12$ of the mean, truncated over the interval [mean – 12,000, mean + 12,000]. The waste holding capacity of a cruise ship is equal to 14 days × number of passengers × 6 liters per passenger per day (Dixon and Hughes, 1999).

The algorithms are very efficient. Computational experiments with randomly generated instances show that: it takes less than 0.01 second to solve a static/dynamic deterministic case with 50 ports; the time required to solve a 50-port instance of the static stochastic problem is less than three minutes.

4.1 Comparing the static deterministic model and the static stochastic model

We now examine what benefit is gained by using the more complex static stochastic model rather than the static deterministic model. To this end, we first clarify how a deterministic model works in a stochastic environment. A modeler or a planner may have an estimate of the amount of waste generated on each leg and then use the deterministic estimate as input to decide at which ports to dispose of waste. If the amount of waste generated turns out to be too large, then it is necessary to use waste disposal vessels. We do not assume that the modeler adopts the expected amount of waste on each leg as input; instead, we consider a number of choices: use the 50th percentile (i.e., the expected value), 60th, 70th, 80th, 90th, and 95th percentiles. For instance, if the amount of waste on a leg is uniformly distributed between 0 and 100, then we solve different static deterministic models assuming the amount of waste is 50, 60, 70, 80, 90, and 95. After the static stochastic model and the static deterministic models are solved, we compare each deterministic model with the stochastic one by generating another 5,000 scenarios of waste. We report in Table 1 the average cost ratio between a deterministic model and the stochastic one over ten instances (Column "Average cost ratio") and the number of instances for which the deterministic and the stochastic model give the same result of at which ports to dispose of waste (Column “#identical solutions”). In case the deterministic and the stochastic model give different decisions of waste disposal ports, we analyze whether the results are significantly different statistically. As we generate 5,000 scenarios of waste to evaluate the decisions by the deterministic and the stochastic model, we use a paired-t test and consider two decisions to be significantly different with a significance level of 0.003 (i.e., when the average difference is larger than three times the standard deviation of the difference). The statistical analysis results are shown in the last two columns of Table 1.

According to the average cost ratio in Table 1, the stochastic model outperforms the deterministic model in the average sense. Moreover, the stochastic model significantly outperforms the deterministic model for most instances, especially for the instances with a large number of ports. No deterministic model significantly outperforms the stochastic model for any instance. In addition, with the increase in the number of ports, it is less likely that the stochastic model and the deterministic model give the same solution. Finally, even if we were to use the deterministic model, we should never use the average amount of waste generated as the input. The numerical results show that using the 90th percentile is much preferable.

Table 1: Comparison of the static deterministic model and the static stochastic model

Number of ports	Number of instances	Deterministic model (percentile)	Average cost ratio	#identical solutions	#solutions with no significant difference	#solutions significantly different
10	10	50%	1760%	2	4	6
		60%	1337%	1	4	6
		70%	1007%	0	3	7
		80%	351%	1	2	8
		90%	192%	0	0	10
		95%	244%	0	0	10
20	10	50%	2457%	0	6	4
		60%	1303%	0	5	5
		70%	1021%	0	5	5
		80%	398%	0	4	6
		90%	334%	0	2	8
		95%	227%	0	2	8
30	10	50%	2046%	0	2	8
		60%	1637%	0	5	5
		70%	747%	0	4	6
		80%	212%	0	3	7
		90%	102%	0	0	10
		95%	264%	0	0	10
40	10	50%	2840%	0	1	9
		60%	2061%	0	3	7
		70%	1107%	0	2	8
		80%	254%	0	3	7
		90%	121%	0	4	6
		95%	240%	0	1	9
50	10	50%	1206%	0	1	9
		60%	1402%	0	0	10
		70%	585%	0	2	8
		80%	482%	0	2	8
		90%	402%	0	1	9
		95%	433%	0	0	10

4.2 Results of the dynamic deterministic model

We investigate the difference between the static deterministic and the dynamic deterministic models. Suppose that an itinerary is to be repeated for ten times. In the static deterministic model, each repetition must use the same disposal ports, whereas in the dynamic deterministic model, different disposal ports may be used in different repetitions, thus providing more flexibility for the decisions. We compute the ratio of the cost using the static deterministic and the dynamic deterministic models and report the results in Table 2. The results show that allowing different disposal ports to be used in different repetitions can reduce up to one third of cost. Therefore, cruise companies should dispose of waste in a flexible manner rather than dispose at a few fixed ports.

Table 2: Comparison between the static and the dynamic deterministic models

Number of ports	Number of instances	Average cost ratio (static/dynamic)
10	100	1.33
20	100	1.17
30	100	1.11
40	100	1.07
50	100	1.08

We further compare the quality of the solutions obtained by the exact DP method and a greedy heuristic for the dynamic deterministic models. The greedy heuristic works as follows: the ship does not discharge waste at a port unless there is no sufficient capacity for the next sailing leg. The total costs of the solutions obtained by the two methods are reported in Table 3. The results show the DP method significantly outperforms the greedy heuristic and this demonstrates the necessity of developing an exact method.

Table 3: Comparison between the exact dynamic programming approach and a greedy heuristic

#Port	#Instance	DP average cost	Greedy heuristic average cost	Gap
10	100	731	977	25.18%
20	100	1406	2020	30.40%
50	100	3472	5289	34.35%
80	100	5535	8408	34.17%
100	100	6880	10654	35.42%

Note: Gap = (the average cost of the greedy heuristic − the average cost of DP) / the average cost of the greedy heuristic

4.3 Computational efficiency of the dynamic stochastic model

Finally, we report the results of the computational performance of the dynamic stochastic model. We carry out six groups of experiments with 50, 60, 70, 80, 90, and 100 ports. Each group has ten instances. The step size of discretization Δ is set to be 1/25 of the capacity of the waste holding tank of the cruise ship. We take advantage of Proposition 1 to accelerate the dynamic programming algorithm. Table 4 reports the average upper bound of the ten instances in each group, the average CPU time required to find the upper bound (i.e., to solve an instance), the average lower bound of the ten instances in each group, the average CPU time required to find the lower bound, and the gap between the upper and lower bounds. We can see that the dynamic stochastic model is much more difficult to solve than the deterministic one. However, even a large instance with 100 ports can be solved in 8 minutes, which is fast enough for practical purposes. Moreover, the optimality gap is less than 0.1%, which demonstrates that the solutions obtained are near-optimal.

Table 4: Computational efficiency of the dynamic stochastic model

#Ports	#Instances	#Discretization	Upper bound		Lower bound		Average gap between the bounds
			Average total cost	Average CPU time (s)	Average total cost	Average CPU time (s)	
50	10	25	3627	231	3625	235	0.06%
60	10	25	4266	263	4264	263	0.05%
70	10	25	4993	294	4989	296	0.08%
80	10	25	5837	365	5833	368	0.07%
90	10	25	6839	394	6836	397	0.04%
100	10	25	7353	461	7349	463	0.05%

5 Conclusions

Due to the large number of cruise passengers and regulations on sea environment protection, determining at which ports to dispose of the waste generated onboard a cruise ship is a key decision to reduce the cost for a cruise company. We have addressed four versions of the problem: the cruise itinerary is either static or dynamic and the amount of waste generated on each voyage leg is either deterministic or stochastic. We have proposed a polynomial-time solution algorithm for the static deterministic model, and the idea of the algorithm can also be used to solve the static stochastic model and the dynamic deterministic model. Second, we have identified the structure of the optimal policy to the dynamic stochastic problem, i.e., a threshold-based optimal policy, based on which an efficient dynamic programming algorithm is developed. Computational results demonstrate that although the static stochastic model and the dynamic stochastic model are much harder than their deterministic counterparts, all of the four models can be efficiently solved for large-scale problem instances. The comparison of the static deterministic model and the static stochastic model shows that considerable cost reductions are achieved by incorporating the randomness in waste generation on each leg. If a static deterministic model must be used, then using the average amount of waste generated on a leg is inferior to using a higher percentile value (e.g., 90th percentile for the truncated normal distribution in our numerical computation). The comparison of the static deterministic model and the dynamic deterministic model shows that allowing different disposal ports to be used in different repetitions of the itinerary can reduce up to one third of cost. Therefore, cruise companies should dispose of waste in a flexible manner rather than dispose at a few fixed ports. Finally, the dynamic stochastic model for a large instance with 100 ports can be solved in 8 minutes with an optimality gap less than 0.1%, which is fast enough for practical purposes.

This study is one of the very few attempts on cruise shipping using quantitative approaches. Although there are many quantitative models for other shipping modes such as tramp shipping (Christiansen et al., 2013) and liner shipping (Meng et al., 2014; Lee and Song, 2017), most literature on cruise shipping is descriptive, with a few exceptions of Maddah et al. (2010), Wang et al. (2017a, b). Nevertheless, cruise shipping has its own characteristics that need to be explored by industrial engineers/operations researchers. Moreover, the cruise market has maintained steady growth in the past 20 years despite of the economic crisis in 2008 and cruising companies have ordered a number of large cruise ships to serve the mass market of cruising. We believe that there are a broad range of research topics in cruise shipping. Hopefully, more quantitative models will be developed for such an emerging area.

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