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Pricing Substitutable Products under Consumer Regrets

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Abstract

Since many firms provide consumers with substitutable products which may be launched sequentially over several periods, strategic consumers need to make their purchase-or-wait decisions at the beginning of the first period. However, they may face second period valuation uncertainty when such decisions are made, and decisions made under uncertainty may lead to regrets ex post. Lots of empirical studies show that consumers anticipate these regrets and take anticipated regrets into consideration. This paper studies the impact of valuation uncertainty and consumer anticipated regrets on consumer purchasing behavior, firm's prices and pricing strategy. We consider a firm selling two substitutable products over two periods, one product for one period, in a selling season. The firm announces all its prices at the beginning of the selling season (price commitment) or announces each price at the beginning of each period (dynamic pricing). Besides, a consumer may experience purchase regret from missing a better choice if she chooses to buy in the first period; and, otherwise, wait regret if she chooses to wait. Consumers make decisions based on the firm's prices and strategies, and the anticipated regrets. We find that a firm may need to set a lower price in the first period even when the average valuation in this period is higher. Second, with price commitment, the impact of each type of regret on the optimal second period price can be opposite and depends on the value of the uncertainty and regret parameters. Finally, we show that price commitment dominates dynamic pricing and the value of price commitment depends on the uncertainty and anticipated regret.

Key words: substitutable products; anticipated regret; second period valuation uncertainty; pricing strategy

1 Introduction

Modest product refinement is one of the basic growth strategies of a firm. Many firms launch products which are updated based on the former versions and sell them in different periods. For example, Bloomsbury Publishing PLC published the Harry Potter and the Philosopher's Stone Illustrated Edition on 6 October 2015 (harrypotter.bloomsbury.com). And On 2 February 2016 it announced that it would publish the 20th Anniversary Editions (the Hogwarts House Edition) on 1 June 2017 to celebrate the 20th anniversary of publication of this book (mugglenet.com). If a fan of Harry Potter wants to buy a Harry Potter and the Philosopher's Stone after the announcement, what should she do, buy the Illustrated Edition or wait for the 20th Anniversary Edition? Note that her valuation for the latter may be uncertain as one has no idea about what it would look like before it is published.

Another example comes from the real state industry. Country Garden started to sell its flats in Zhanggong, a district in Ganzhou, China, on 3 June 2017. On 7 Feb 2018, the firm announced that it will provide a new project in Ganxian, another district in the same city. If a customer decides to buy a flat during the Spring Festival in 2018, what should he/she do, purchase now or wait?

Clearly, in the above examples, time plays a role as the products are launched and sold in different periods. Second, these products are substitutable goods. Third, a consumer faces the second period valuation uncertainty when she makes the purchase-or-wait decision in the first period. Such uncertainty may come from personal factors, such as healthy problem, mood and sudden changes in work schedule etc. (see, Shugan and Xie, 2005), and product factors, such as size, color, and quality etc. The uncertainty may lead to uncertain valuations for targets. Moreover, when the uncertainty is realized ex post, decision made under uncertainty may leave a decision-maker with negative emotion, such as regret, which stems from realizing that one's situation is worse than that of the rejected options (Zeelenberg et al., 2000).

Anticipated regret is worth a discussion. On the one hand, anticipated regret exists in our daily life. For example, a decision-maker who gambles may simultaneously purchase different kinds of insurance (Bell, 1982). On the other hand, literature shows that anticipated regret affects personal behavior. Actually, when thinking back, one always regrets and hopes she has chosen differently (Simonson, 1992). Moreover, consumers are able to anticipate negative emotions caused by disappointing results (Baron, 1991). And, in anticipation of regret, one tends to spend more time on decision making (Zeelenberg, 1999). In other words, anticipation of negative emotion affects one's behavior ex ante (Zeelenberg et al., 2000) and one's purchasing behavior (Simonson, 1992). Hence, it is meaningful to study the impact of anticipated regret.

Given the fact that consumers face second period valuation uncertainty at the beginning of the selling season and that anticipated regrets may have an impact on consumers purchasing behavior, this paper focuses on the following research questions:

- What are the impacts of valuation uncertainty on the firm's operation?
- What are the effects of anticipated regret on consumers behavior and firm's prices?
- Which should be the firm's pricing strategy, price commitment or dynamic pricing?

This paper develops a model to answer the above questions. We consider a firm selling two substitutable products with different prices in different periods. There is only one product available in each period. Consumers know that there are two products and they know their valuations for the first product (the product sold in the first period), but they do not know their valuations of the second product. The game between the seller and the consumers begins with the firm choosing one of the pricing strategies: price commitment, meaning that both prices are announced at the beginning of the selling season, or dynamic pricing, meaning that the prices are declared at the beginning of each period. Given the firm's pricing strategy and price(s), consumers need to make their purchase-or-wait decisions at the beginning of the selling season in anticipation of either purchase regrets or wait regrets from missing a better choice.

Our contributions are as follows. First, a firm may set a lower price for the first product. This could happen even when the average valuation of the first product is higher than that of the second one. Strategic waiting helps consumers to share the uncertainty they faced to the firm and thus the marginal revenue of strategic waiting is lower than that of immediate purchase. Therefore, the firm has incentive to lower its price in the first period in order to moderate the uncertainty and mitigate consumers' strategic waiting. Consumers anticipate these regrets and take them into consideration when making the purchase-or-wait decisions.

Our second main finding is that the impact of each type of anticipated regret on the optimal second period price may be opposite and depends on the value of regret and uncertainty parameters. In most cases, purchase regret increases the second period price and wait regret reduces price. However, when the uncertainty is high and consumers anticipate much more purchase regret, the second period price can be decreasing in the purchase regret and increasing in the wait regret. The reason is that high uncertainty and more purchase regret drive consumers to wait. Besides, aversion to purchase regret always harms the firm's profit as it leads consumers to share some of the risks caused by uncertainty to the firm. On the contrary, aversion to wait regret makes the firm better off, which means that the firm should launch more marketing campaigns to evoke potential wait regret. For example, the firm should highlight the fact that the first product will not be offered in the second period.

Finally, price commitment is always better than dynamic pricing since the former mitigate strategic waiting better. Besides, the value of price commitment depends on the uncertainty and anticipated regret.

2 Literature Review

This paper studies the impacts of valuation uncertainty and anticipated regret on consumer behavior and firm's operation. The area closely related to this paper is about the effects of emotions on decision-making. These emotions include regret (e.g., Braun and Muermann, 2004; Nasiry and Popescu, 2012) and disappointment (e.g., Liu and Shum, 2013). Zeelenberg et al. (2000) summarizes the differences between regret and disappointment in detail. It points out that the former results from bad decisions (what one gets is worse than the rejected options), while the latter is caused by disconfirmed expectancies (what one gets is worse than the expectation). According to that research, because of self-blame, regret may be more intense than disappointment. So this paper only accounts for regret.

Bell (1982) and Loomes and Sugden (1982) develop the Regret Theory and explain several types of violation of conventional Expected Utility Theory. Quiggin (1994) makes Regret Theory axiomatic and extends it to general choice sets by defining regret as the disutility of not having chosen the ex post best forgone alternative. Based on these works, regret has received great attention during recent years. Braun and Muermann (2004) incorporates regret into insurance decision-making. Muermann et al. (2006) and Michenaud and Solnik (2008) apply regret to financial decision. Besides, Filiz-Ozbay and Ozbayn (2007) and Engelbrecht-Wiggans and Katok (2008) link regret with auction issues, and Perakis and Roels (2008) extends regret to newsvendor model.

There are only several papers considering the impact of consumer anticipated regret in the operations management area. Nasiry and Popescu (2012) analyzes the effect of anticipated regret in advance selling context. They model action regret (regret due to buying in advance) and inaction regret (regret due to waiting for spot period) into advance selling and find that action regret diminishes the benefits of advance selling while inaction regret makes the firm better off. Diecidue et al. (2012) models buyer's regret and hesitater's regret in a two period advance selling context. This research carefully analyzes the sources of regret. Özer and Zheng (2015) studies the sellers pricing and inventory policies with the effects of anticipated regret and consumers' availability mis-perception. It models high-price regret and stockout regret. This research finds that under the influence of anticipated regret and consumers' availability mis-perception, markdown may be more profitable than everyday low price. All these works focus on a monopoly firm, while Jiang et al. (2016) takes competitive context into account. It models switching regret and repeat-purchase regret, studies the impacts of anticipated regret on competitive firms' profit and product innovation, and finds that the impacts, both on the profit and product innovation, are not monotonic. Chao et al. (2015) is different from above as it studies the effects of the consumers' ability to anticipate the regret (no, full or partial) on the profit of competitive probabilistic selling. Our research is closely related to Jiang et al. (2016) as both of us consider two products, but they study the competition between two firms while we focus on a firm's optimization problem when selling two substitutable products over two periods.

Many studies focus on the interaction between strategic consumers and firms' pricing and operation decisions, such as Besanko and Winston (1990), Su (2007), Tilson and Zheng

(2014), Du et al. (2015), Shum et al. (2016), Dong and Wu (2017)). Besanko and Winston (1990) show that the prices with strategic consumers are always lower than with myopic consumers. Shum et al. (2016) study the effects of cost reduction. They take price commitment, price matching, and dynamic pricing into account, and conclude that the source of cost reduction matters a lot when deciding the firm's pricing strategy. Since strategic behavior hurts the firm, researches provide many suggestions to reduce strategic waiting. Commitment (both price commitment and quantity commitment) is one of the suggestions. For example, Su and Zhang (2008) study the interaction between commitment and strategic behavior and show that commitment can benefit the firm. Besides, some researches (e.g., Aviv and Pazgal (2008) and Dasu and Tong (2010)) show that price commitment outperforms dynamic pricing in reducing strategic waiting. Some (e.g., Cachon and Swinney (2009) and Aflaki et al. (2016)) show the opposite results and think that, under certain conditions, dynamic pricing may be better than price commitment. Aflaki et al. (2016) consider price commitment and dynamic pricing and study the impact of strategic behavior on consumers, firm and society in a model in which consumers are able to choose whether or not to become strategic by exerting costly efforts. Our research differs from these studies as we take anticipated regret into consideration. We contribute to this stream of literature by showing that the value of price commitment is affected by uncertainty and regret parameters.

3 Modeling

This paper considers a monopolistic firm who sells two substitutable products, denoted by product 1 and product 2, to a mass of infinitesimal consumers over a selling season with two periods. There is only one product available in each period. Without loss of generality, we assume that product 1 and product 2 are sold in the first and second periods, respectively.

The firm first decides which pricing strategy to implement: price commitment (superscript c) or dynamic pricing (superscript d). With price commitment, it announces all its prices at the beginning of the selling season; while with dynamic pricing, the prices are declared at the beginning of each period, aiming to maximize the firm's profit-to-go (Aflaki et al., 2016). Dynamic pricing prevails in practice because of its flexibility.

Given the firm's pricing strategy, consumers make their purchase-or-wait decisions at the beginning of the first period. They decide whether to buy product 1 in the first period or wait for product 2 in the second period. Each consumer buys at most one product. This paper assumes that each consumer has a heterogeneous and independent valuation for each product. She faces a second period valuation uncertainty at the beginning of the first period when the purchase-or-wait decision is made. Valuation uncertainty exists because there are lots of uncertainty about product 2 before it is sold in the market. Second period valuation uncertainty will be revealed at the beginning of the second period. After it is realized, those who wait in the first period make their decisions of purchase-or-leave and buy product 2 as long as their valuations are higher than its price.

We assume that consumers' valuation are $v_o + v_1$ and $v_o + v_2$ in first and second periods, respectively. We use v_o to model the same baseline function of these two substitutable products. For example, different versions of Harry Potter and the Philosopher's Stone share the same main content. Besides, we use v_1 and v_2 to model the additional functions of product 1 and 2, respectively. v_1 and v_2 are independent. This assumption is reasonable as we take two products into account. Our model is different from the model in Jiang and Tian (2016), which considers only one product and assumes that consumers can independently obtain different usage valuations in different usage periods. Since only product 1 is available in the market at the beginning of the first period, each consumer knows her own v_1 while v_2 is uncertain at that time. We normalize v_o to zero to make our results more transparent.

Consistent with the behavior literature, we assume v_i $(i = 1, 2)$ follows a uniform distribution on $[0, \overline{v}_i]$ with cumulative distribution function $F_i(\cdot)$ and probability density function $f_i(\cdot)$. Both the firm and consumers know the distribution and the value of $\overline{v}_i$. Let $\overline{v}_2 = \xi \overline{v}_1$. Since the two products are substitutable, the difference between these two products' valuations, $v_o + v_1$ and $v_o + v_2$, is not expected to be much. Therefore, we assume $\xi \in (\frac{1}{2}, 2)$. From the perspective of consumers, ξ captures the magnitude of the second period valuation uncertainty. Valuation uncertainty increases in ξ. If ξ is large enough, a consumer may have a higher valuation on product 2 with the same probability.

Following the Regret Theory (Bell, 1982; Loomes and Sugden, 1982), we assume that consumers are strategic and emotionally rational. Given the firms' prices, they make decisions in order to maximize their expected surplus, which is the sum of economic surplus and emotional surplus (i.e., regret). Following Quiggin (1994) and Braun and Muermann (2004), we define regret as the disutility of not having chosen the ex post best forgone alternative and is proportional to this disutility. In other words, regret is proportional to the difference between one's actual economic surplus and the ex post best economic surplus one could have got in the same state.

Consistent with behavior research in the operations management area, we model two types of regret, that is, purchase regret and wait regret. Purchase regret is triggered when a consumer purchases the first product and finds out that she could have gained more surplus if she has chosen to wait. Wait regret is triggered oppositely. It happens when a consumer has waited in the first period but finds out that she could have been better if she has purchased product 1 in the first period. We assume α and β are the coefficients of purchase regret and wait regret, respectively. $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, since actual surplus seems to be more valuable than counter-factual surplus (Özer and Zheng, 2015).

We assume that the total market size for the two products is deterministic and fixed at 1, and the demands of the first and second periods are D_1 and D_2. Let $D_T = D_1 + D_2$ be the total demand. We also assume the availability of each product in its selling period is guaranteed. This assumption makes sense because the second period valuation uncertainty provides consumers incentives to buy in the first period even in the case where $p_1 > p_2$ and the availability is guaranteed in the second period. Lastly, this paper assumes the marginal costs of both products are normalized to zero.

We discuss price commitment first and then focus on dynamic pricing. We show that dynamic pricing is a special case of, and is always dominated by, price commitment in this paper.

4 Price Commitment

If the seller implements price commitment strategy, then the first period valuation v_1, the seller's first period price p_1 and the second period price p_2 are known to each consumer at the beginning of the first period, while the second period valuation v_2 is uncertain at that time and will be realized at the beginning of the second period. The sequence of events are described as follows:

i. At the beginning of the first period, the firm declares p_1 and p_2;

ii. All consumers arrive, observe v_1 (consumer dependent), p_1 and p_2, and make their decisions of purchase-or-wait;

iii. At the beginning of the second period, v_2 (consumer dependent) is revealed. All waiting consumers make their decisions of purchase-or-leave.

For a particular consumer in the first period, the expected net surplus of purchasing is denoted by S_1, and the expected net surplus of waiting is S_2. S_1 is characterized as follows.

$$S_1 = (v_1 - p_1) - \alpha \int_0^{\overline{v}_2} \left[(v_2 - p_2) - (v_1 - p_1)\right]^+ f_2(v_2)\, \mathrm{d}v_2 \tag{1}$$

Here, $x^+ = \max\{x, 0\}$. The first term in (1) is the economic surplus and the second term is the emotional surplus, i.e., negation of the regret triggered by choosing to buy in the first period. α captures the strength of purchase regret. A consumer experiences purchase regret under the following simultaneous conditions: (i) $v_1 \geq p_1$; otherwise, the consumer prefers waiting; (ii) $v_2 - p_2 > v_1 - p_1$, which makes product 2 more attractive; and (iii) product 2 is available, which is assured in this paper. S_2 is characterized similarly and formulated as follows.

$$S_2 = \int_0^{\overline{v}_2} \left\{ \left[(v_2 - p_2) - \beta\big((v_1 - p_1) - (v_2 - p_2)\big)^+\right] \phi(v_2) + \left[-\beta(v_1 - p_1)^+\right](1 - \phi(v_2)) \right\} f_2(v_2)\, \mathrm{d}v_2 \tag{2}$$

Here, $\phi(v_2)$ takes the value 1 if $v_2 \geq p_2$, and takes the value 0 if $v_2 < p_2$. S_2 is formulated based on whether a consumer's second period valuation is higher than the price. If product 2 is affordable, one gets positive economic surplus and the corresponding wait regret if she could be better by choosing the other option. However, those who find that product 2 is too expensive to buy may also experience wait regret if the product 1 is affordable.

Let $\Delta S = S_1 - S_2$ be the consumer's differential surplus, then,

$$\Delta S = \frac{1}{2\overline{v}_2} \left\{ -(\alpha - \beta)(v_1 - p_1)^2 + 2\left[(1 + \alpha)\overline{v}_2 - (\alpha - \beta)p_2\right](v_1 - p_1) - (1 + \alpha)(\overline{v}_2 - p_2)^2 \right\} \tag{3}$$

Let v_1^o denotes the cutoff value, which is the value of v_1 when $\Delta S = 0$. v_1^o plays as a threshold rule, i.e., for any given prices and regret parameters, consumers purchase product 1 if and only if $v_1 \geq v_1^o$. v_1^o is solved in the following lemma:

Lemma 1. *(i). If $\alpha = \beta$, then $v_1^o = \min\{\frac{(\overline{v}_2-p_2)^2}{2\overline{v}_2} + p_1, \overline{v}_1\}$. That is, if $\overline{v}_2 \leq \overline{v}_1 - p_1 + p_2 + \sqrt{(\overline{v}_1 - p_1)(\overline{v}_1 - p_1 + 2p_2)}$, then $v_1^o = \frac{(\overline{v}_2-p_2)^2}{2\overline{v}_2} + p_1$; otherwise, $v_1^o = \overline{v}_1$.*
(ii). If $\alpha \neq \beta$, then $v_1^o = \min\{\frac{(1+\alpha)\overline{v}_2-(\alpha-\beta)p_2-\sqrt{(1+\beta)\left[(1+\alpha)\overline{v}_2^2-(\alpha-\beta){p_2}^2\right]}}{\alpha-\beta} + p_1, \overline{v}_1\}$. That is, if $\overline{v}_2 \leq \overline{v}_1 - p_1 + p_2 + \sqrt{\frac{1+\beta}{1+\alpha}(\overline{v}_1 - p_1)(\overline{v}_1 - p_1 + 2p_2)}$, then
$v_1^o = \frac{(1+\alpha)\overline{v}_2-(\alpha-\beta)p_2-\sqrt{(1+\beta)\left[(1+\alpha)\overline{v}_2^2-(\alpha-\beta){p_2}^2\right]}}{\alpha-\beta} + p_1$; otherwise, $v_1^o = \overline{v}_1$.

All proofs can be found in the Appendix. According to Lemma 1, if $\alpha \neq \beta$, anticipated regrets make sense. Besides, if the upper bound of the second period valuation is large enough, all consumers prefer waiting as they set the maximum of the first period valuation as their cutoff value. This is because the uncertainty that buyers faced in the first period increases in $\overline{v}_2$. Therefore, if $\overline{v}_2$ is large enough, they prefer waiting in order to avoid the possible economic loss and the counter-factual emotions. The following proposition shows how the cutoff value is affected by prices, uncertainty and anticipated regrets.

Proposition 1. *If $v_1^o < \overline{v}_1$, then (i) $\frac{\partial v_1^o}{\partial p_1} > 0$, $\frac{\partial v_1^o}{\partial p_2} < 0$, $\frac{\partial v_1^o}{\partial \xi} > 0$; and (ii) if $\alpha \neq \beta$, then $\frac{\partial v_1^o}{\partial \alpha} > 0$ and $\frac{\partial v_1^o}{\partial \beta} < 0$.*

Proposition 1 describes the impacts of prices (p_1 and p_2), uncertainty (ξ) and behavioral parameters (α and β) on the cutoff value v_1^o. Since v_1^o plays as a threshold role in the first period, and a higher price always decrease the attractiveness of a product, the cutoff value v_1^o decreases in p_1 and increases in p_2.

According to Proposition 1, v_1^o is increasing in the uncertainty parameter ξ. Under second period valuation uncertainty, purchase behavior in the first period is associated with the probability of worse than waiting, which causes buyers' inertia in the first period. Given prices, this probability increases in ξ. Therefore, v_1^o increases in ξ. Besides, if $\alpha \neq \beta$, regret parameters make sense. Aversion to purchase regret drives consumers to wait and thus v_1^o increases. By contrast, aversion to wait regret lures more shoppers to purchase in the first period and thus v_1^o decreases.

v_1^o determines the firm's demands. Then $D_1 = \frac{\overline{v}_1 - v_1^o}{\overline{v}_1}$, $D_2 = \frac{v_1^o}{\overline{v}_1}\frac{\overline{v}_2 - p_2}{\overline{v}_2}$ and $D_T = D_1 + D_2 = \frac{1}{\overline{v}_1}(\overline{v}_1 - v_1^o \frac{p_2}{\overline{v}_2})$. Actually, D_T consists of three parts,"pure first period" consumers (whose valuation of product 1 belongs to $(v_1^o, \overline{v}_1)$), "first-second period" consumes (whose valuations $v_1 \in (p_1, v_1^o)$ and $v_2 \in (p_2, \overline{v}_2)$), and "pure second period" consumers (whose valuations $v_1 \in (0, p_1)$ and $v_2 \in (p_2, \overline{v}_2)$). "Pure first period" consumers form D_1, and the last two parts form D_2. "First-second period" consumers refer to those who strategically choose to wait and at the same time find that product 2 is affordable. By contrast, it is always better for "pure second period" consumers to wait. We define the demand of "first-second period" consumers as "switching demand" of product 2, which is denoted by $D_{2s} = \frac{v_1^o - p_1}{\overline{v}_1}\frac{\overline{v}_2 - p_2}{\overline{v}_2}$; whereas the demand of "pure second period" consumers as "original demand" of product 2, which is denoted by $D_{2o} = \frac{p_1}{\overline{v}_1}\frac{\overline{v}_2 - p_2}{\overline{v}_2}$. D_{2s} captures consumers' switching behavior while D_{2o} is independent of strategic waiting. The following proposition summarizes the properties of demand.

Proposition 2. *If $v_1^o < \overline{v}_1$, then*

Effect of Prices. (i) D_T is decreasing in p_1. (ii) There exists a threshold $\dot{p}_2$ such that if $p_2 > \dot{p}_2 > p_1$, then D_T is increasing in p_2; otherwise, D_T is decreasing in p_2.

Effect of Uncertainty. (iii) There exists a threshold $\dot{\xi}$ such that if $p_2 > p_1$ and $\xi > \dot{\xi}$, then D_T is decreasing in ξ; otherwise, D_T is increasing in ξ.

Effect of Anticipated Regrets. (iv) If $\alpha \neq \beta$, then D_T is decreasing in α and increasing in β.

Proposition 2 shows that the impacts of prices, uncertainty and emotional parameters on D_T are complex. According to the first part of Proposition 2, D_T is decreasing in p_1. A higher first period price reduces the attractiveness of product 1 and gives consumers more incentives to wait. However, purchasing behavior in the second period depends on the condition that valuation is larger than price, thus the lost sales in the first period may not be captured by product 2. In other words, the increase in D_{2o} is less than the decrease in D_1. Besides, D_{2s} is independent of p_1. Consequently, the total demand decreases.

Generally, an increase in p_2 makes product 2 too expensive and thus leads the retailer to lose more demands than the lost sales in the first period that are not captured by product 2. Therefore, the total demand decreases. However, opposite phenomenon can be observed if p_2 is large enough. In this case, the demand of product 2 is already small since most consumers purchase in the first period. If p_2 increases, consumers have less incentives to wait and more consumers will buy product 1, thus D_1 increases. At the same time, the decrease in D_2 is small or even unobservable. As a result, D_T increases.

According to the third part of Proposition 2, D_T could be decreasing in ξ under some conditions. Actually, if $p_2 > p_1$ and $\xi > \dot{\xi}$, most consumers choose to wait in the first period, in this case, a decrease in D_1 outweighs an increase in D_2 and the total demand decreases in ξ.

The last part in Proposition 2 justifies the significance of taking consumers' anticipated regret into consideration. The total demand decreases in purchase regret and increases in wait regret, which makes our research different from Özer and Zheng (2015) whose model shows that the total demand is independent of the regret parameters. Purchase regret leads to inertia and thus hurts the total demand. The reason is that waiting consumers may leave the market without buying any products if they realize low valuation of product 2. By contrast, wait regret raises the total demand since it drives consumers to buy, rather than to wait, in the first period.

In order to focus on the case of $v_1^o < \overline{v}_1$ since we consider two products, our further analyses assume that $\xi \in (\frac{1}{2}, 2)$. Based on the above discussion, the firm's profit maximization problem is formulated as follows.

$$\max_{p_1,p_2} \Pi(p_1, p_2) = p_1 D_1 + p_2 D_2 \tag{4}$$

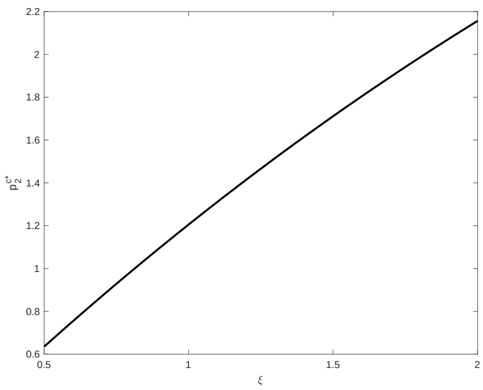
4.1 Case of $\alpha = \beta$

According to Lemma 1, if $\alpha = \beta$, the purchase regret offsets the wait regret and anticipated regrets have no effect on consumer behavior. That is, consumers make their purchase-or-wait decisions as if there is no anticipated regret. However, the case of $\alpha = \beta$ plays as a benchmark. It captures the impact of valuation uncertainty. Let p_1^{c*} and p_2^{c*} be the optimal prices for product 1 and product 2, respectively. p_1^{c*} and p_2^{c*} are determined by the following lemma.

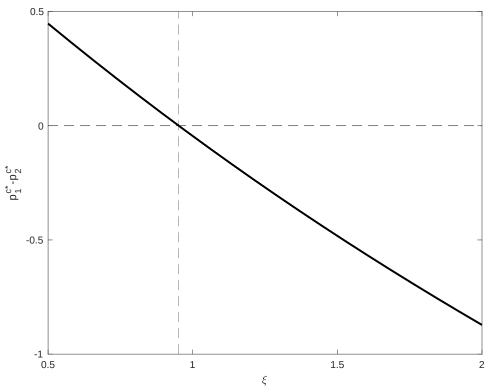
Lemma 2. *If $\alpha = \beta$, then $p_1^{c*} = \frac{\overline{v}_1\overline{v}_2 - \frac{1}{2}\overline{v}_2^2 + 2\overline{v}_2 p_2^{c*} - \frac{3}{2}(p_2^{c*})^2}{2\overline{v}_2}$ and*
$p_2^{c*} = 2\sqrt{\frac{6+\xi}{3\xi}}\cos(\frac{1}{3}\arccos(-\frac{6}{6+\xi}\sqrt{\frac{3\xi}{6+\xi}}) - \frac{2\pi}{3})\overline{v}_2$.

Some properties of the optimal prices are described in Proposition 3.

Proposition 3. *(i) p_1^{c*} is increasing in p_2^{c*};*
(ii) p_2^{c*} is increasing in ξ;
(iii) $p_1^{c*} < p_2^{c*}$ if and only if $\xi > -2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2 \approx 0.9519$;
(iv) With optimal prices, the total demand increases in ξ.



(a)



(b)

Figure 1: Price p_2^{c*}, and price difference $p_1^{c*} - p_2^{c*}$ as a function of ξ ($\alpha = \beta$).

The first part of Proposition 3 shows that the optimal prices have a positive correlation. This is intuitive since these two products are substitutable. Besides, increasing ξ leads to a higher second price because ξ increases both D_2 and the upper bound value $\overline{v}_2$.

If $-2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2 < \xi < 1$, then one may expect a higher price of product 1 since the average valuation of product 1 is larger than that of product 2. Interestingly, the third part of Proposition 3 shows that $p_1^{c*} < p_2^{c*}$ when $\xi > -2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2$. Note that ξ denotes the uncertainty consumers faced in the first period. If ξ is large enough, consumers prefer waiting. However, as mentioned above, waiting consumers may leave the market without any products if they realize low valuations for product 2. In this case, the firm has incentive to lower p_1 in order to lure consumers to purchase in the first period, and thus increases D_1. Since the marginal revenue of purchase in the first period

(p_1^{c*}) is larger than the effective marginal revenue of wait $(p_2^{c*}\frac{\overline{v}_2-p_2^{c*}}{\overline{v}_2})$, the increase in D_1 will more than offset the loss in both p_1 and D_2 and therefore the firm gains more profit. In other words, strategic waiting helps consumers to share the risk caused by the uncertainty to the firm. By lowering its price of product 1, the firm moderates the uncertainty faced by the consumers and mitigates strategic waiting, and thus becomes better off. Therefore, $p_1^{c*} < p_2^{c*}$ when $-2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2 < \xi < 1$.

Notice that, ξ affects D_2 in two ways. On the one hand, if ξ increases, switching demand increases. However, as mentioned above, these strategic waiting may not be captured by product 2. So the increase in D_{2s} is less than the decrease in D_1. In this case, the total demand decreases. On the other hand, if ξ increases, the original demand D_{2o} increases. Therefore, the impact of ξ on the total demand depends on the loss in strategic waiting consumers and the increase in the original demand. If the firm prices its products according to the optimal rule, the total demand increases in ξ.

4.2 Case of $\alpha \neq \beta$

If $\alpha \neq \beta$, anticipated regrets make sense. Therefore, the case of $\alpha \neq \beta$ captures both the effects of valuation uncertainty and consumers' anticipated regret. Similar to the analysis of the case of $\alpha = \beta$, we first show the following lemma, which describes the uniqueness of the optimal prices.

Lemma 3. *If $\alpha \neq \beta$, then* $p_1^{c*} = \frac{1}{2}\left\{\overline{v}_1 + p_2^{c*} - \frac{(p_2^{c*})^2}{\overline{v}_2} - \frac{(1+\alpha)\overline{v}_2-(\alpha-\beta)p_2^{c*}-\sqrt{(1+\beta)\left[(1+\alpha)\overline{v}_2^2-(\alpha-\beta)(p_2^{c*})^2\right]}}{\alpha-\beta}\right\}$, *and there exists a unique optimal solution p_2^{c*} that maximizes $\Pi(p_1,p_2)$ and solves* $2\overline{v}_1\overline{v}_2 - 2\overline{v}_1p_2 - 2{p_2}^2 + 2\frac{{p_2}^3}{\overline{v}_2} - 2p_2n + (\overline{v}_2n - \overline{v}_1\overline{v}_2 + \overline{v}_2p_2 - {p_2}^2)p_2\sqrt{\frac{1+\beta}{t}} = 0$.

Here, $t = (1+\alpha)\overline{v}_2^2 - (\alpha-\beta){p_2}^2$, and $n = \frac{(1+\alpha)\overline{v}_2-(\alpha-\beta)p_2-\sqrt{(1+\beta)t}}{\alpha-\beta}$. Similar to the case of $\alpha = \beta$, some properties are summarized as follows.

Proposition 4. *(i) p_1^{c*} is increasing in p_2^{c*};*

(ii) Generally, p_2^{c*} is increasing in the purchase regret coefficient α and decreasing in

the wait regret coefficient β. However, if ξ is large enough, then,

- *given β, there exists a threshold $\hat{\alpha}$ such that if $\alpha > \hat{\alpha}$, then p_2^{c*} is decreasing in α;*
- *given α, there exists a threshold $\hat{\beta}$ such that if $\beta < \hat{\beta}$, p_2^{c*} is increasing in β.*

(iii) With optimal prices, the total demand increases in ξ.

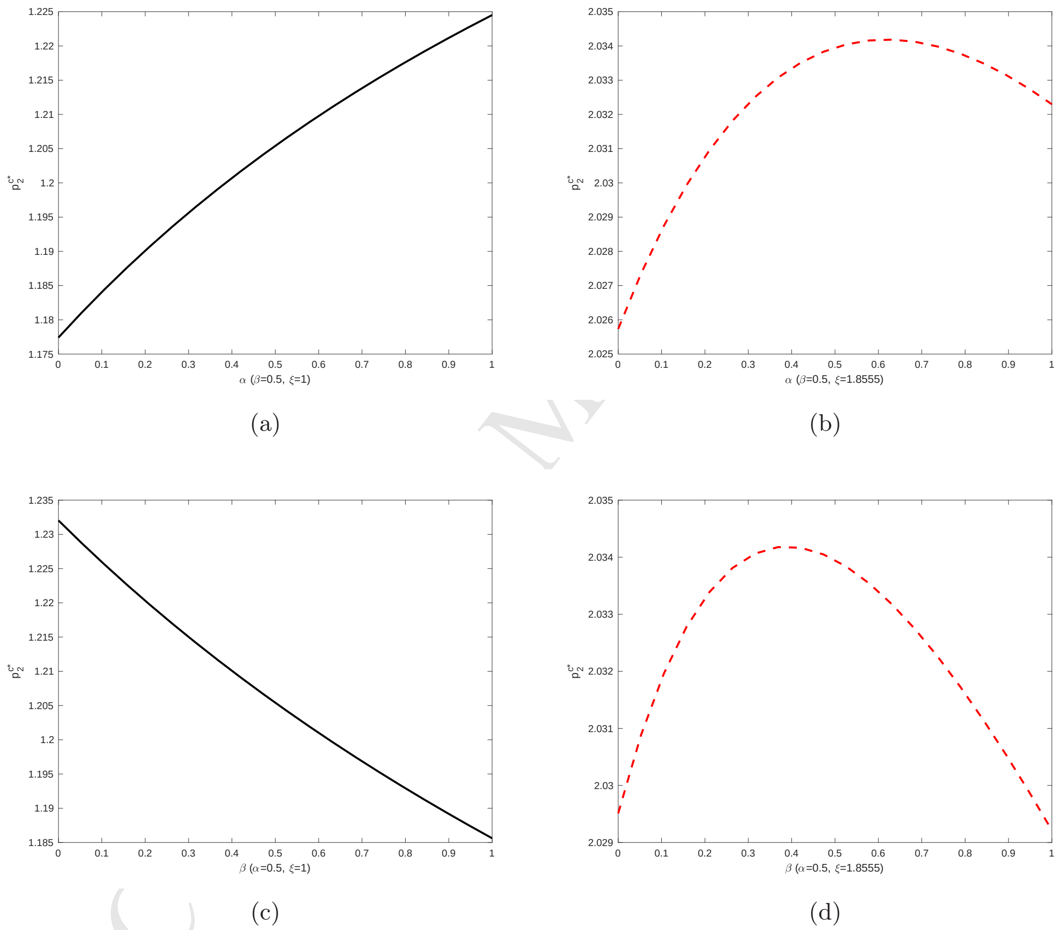


Figure 2: Price p_2^{c*} as a function of α or β ($\alpha \neq \beta$).

The first and the last part of Proposition 4 are the same as those in the case of $\alpha = \beta$, and their explanations can be found accordingly. Here, we focus on the impacts of α and β on the optimal second period price p_2^{c*}, which are shown in the second part of Proposition 3. It shows that, depending on the value of ξ, each of the regret parameters may have opposite effects on the optimal second period price.

Above all, the uncertainty ξ and the regret parameters α and β affect consumers' decisions. Uncertainty ξ and aversion to purchase regret lead consumers to wait whereas aversion to wait regret results in immediate purchase. Besides, strategic waiting helps consumers to share the valuation uncertainty they faced to the firm.

If the risk of uncertainty is moderate, switching demand D_{2s} and original demand D_{2o} is small. Most consumers prefer purchasing in the first period. In this case, the optimal second period price is increasing in α and decreasing in β. Given ξ, any increase in α or decrease in β lures consumers to move from the first period to the second one. Thus, D_1 decreases and D_{2s} increases. However, as mentioned above, product 2 may not capture the lost sales in the former period, and the effective marginal revenue of wait is less than that of purchase. Then, as a result, the firm's profit suffers from any increase in α or decrease in β. Therefore, the seller has incentive to mitigate the strategic waiting behavior. He manages to achieve such objective by increasing p_2 since a higher price in the second period lowers the opportunity cost of purchasing product 1. In this way, the benefit of increased prices and D_1 is more than the decreases in D_2 and the firm becomes better off.

By contrast, if ξ is large enough and $\alpha >> \beta$ such that consumers prefer waiting at the beginning of the selling season, p_2 can be decreasing in α and increasing in β. Note that ξ captures the valuation uncertainty. If ξ is large enough, then both switching demand D_{2s} and original demand D_{2o} are large while D_1 is small. Most consumers prefer waiting. If there is any increase in α or decrease in β. D_1 decreases and D_{2s} increases. With the same logic as above, the firm's profit decreases. But in this case, lower p_2 is required in order to attract more consumers to buy product 2 since D_2 is large enough that, although the effective marginal of wait is still less than that of purchasing product 1, the increase in D_{2o} offset such drawbacks. In other words, if there is any increase in α or decrease in β, the firm needs to lower its price in the second period in order to attract more demands.

Proposition 4 is depicted graphically in Figure 2.

5 Dynamic Pricing

Dynamic pricing prevails in both practice and research. With this pricing strategy, only v_1 and p_1 are known at the beginning of the first period, while both v_2 and p_2 are unknown at that time and will be realized at the beginning of the second period. As assumed in the previous model, consumers' valuations are uniformly distributed. The sequence of events is described as follows:

i. At the beginning of the first period, the firm declares p_1;

ii. All consumers arrive, observe v_1 (consumer dependent) and p_1, and make their decisions of purchase-or-wait;

iii. At the beginning of the second period, the firm chooses p_2 to maximize its profit-to-go;

iv. v_2 (consumer dependent) is revealed. Based on v_2 and p_2, all waiting consumers make their decisions of purchase-or-leave.

The firm's optimization problem is formulated as follows.

$$\begin{aligned} &\max_{p_1,p_2} \Pi(p_1,p_2) = p_1 D_1 + p_2 D_2 \\ \text{s.t.} \quad &p_2 = arg\max_x D_2' x \end{aligned} \tag{5}$$

Here, $D_2' = \frac{1}{\overline{v}_1 \overline{v}_2} v_1^o(\overline{v}_2 - x)$. The difference between price commitment and dynamic pricing is that the former chooses both its first and second period prices at the beginning of the first period; whereas the latter announces its second period price at the beginning of the second period, which means the firm chooses its second period price in order to maximize its expected profit in the second period. This explains why there is a condition that $p_2 = arg\max_x D_2' x$ in Equation (5). Actually, this condition reflects both the sequence of events and the main idea of backward induction under dynamic pricing. Let p_1^{d*} and p_2^{d*} denote the optimal prices of product 1 and product 2 in the dynamic pricing model. Using backward induction, the analysis begins with the firm's decision of p_2. It is straightforward to find that $p_2^{d*} = \frac{1}{2}\overline{v}_2$. This result is interesting as it is independent of the cutoff value v_1^o. Actually, from the perspective of strategic consumers, the price in the second period can be found out ex ante. Thus, the dynamic pricing can be seen as a special case of the price commitment and in this special case, $p_2^{c*} = \frac{1}{2}\overline{v}_2$. The comparison of the price commitment and dynamic pricing is summarized in Proposition 5.

Proposition 5. *$p_1^{c*} > p_1^{d*}$, $p_2^{c*} > p_2^{d*}$ and $\Pi^c > \Pi^d$.*

Actually, according to Aflaki et al. (2016), from the perspective of mathematics, the profit maximization problem with price commitment is a relaxation of that with dynamic pricing. Therefore, price commitment dominates dynamic pricing. In specific words, because of the fact that strategic consumers are able to figure out the possible second period price, dynamic pricing reduces to a special case of price commitment and is always dominated by the latter. In the next section, we further show that, although price commitment dominates dynamic pricing, the relative value of price commitment is affected by the uncertainty and behavioral parameters.

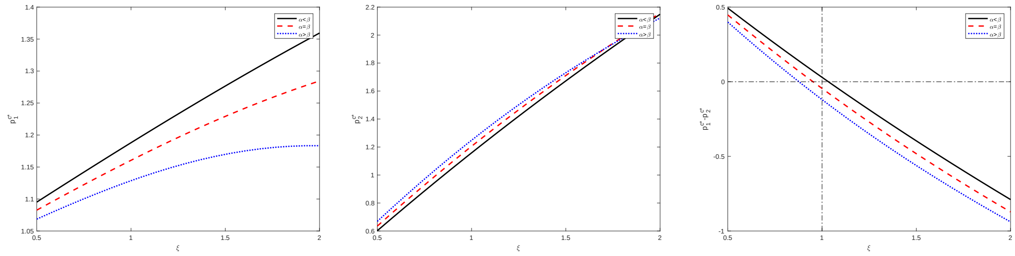
6 Numerical Study

By numerical study, this section furthers our analysis to the effects of uncertainty and anticipated regrets on the firm's prices and profit and on the relative value of price commitment. In this section, we assume $\overline{v}_1 = 2$. Besides, $\alpha > \beta$ means $\alpha = 0.99$ and $\beta = 0.01$, $\alpha = \beta$ means $\alpha = \beta = 0.5$, and $\alpha < \beta$ means $\alpha = 0.01$ and $\beta = 0.99$.

Figure (3) summarizes the effects of the uncertainty and behavioral parameters on the firms prices. According to this figure, we have three observations. First, prices are increasing in the uncertainty parameter ξ. The logic is that, although increasing ξ results in more strategic waiting and may cause lost sales in the first period that may not be captured by product 2, it also increases the upper-bound of the second period valuation and increases the original demand of product 2. Thus the firm can gain more profit by rising its prices.

Second, although we mentioned in Proposition 4 that each type of the anticipated regrets has opposite effect on the optimal second period price, here we find that the effects of anticipated regrets on the optimal first period price are monotonic. In other words, the optimal first period price is increasing in the wait regret and decreasing in the purchase regret.

Third, the difference between the two prices depends on the value of both the uncertainty parameter ξ and the emotional parameters α and β. If $\alpha > \beta$, the firm is more likely to set a higher price in the first period, since aversion to wait regret lures consumers to buy in the first period and increases D_1.



(a) (b) (c)

Figure 3: p_1^{c*}, p_2^{c*} and $p_1^{c*} - p_2^{c*}$ as a function of ξ.

The following figure (Figure (4)) depicts the impacts of ξ, α and β on the expected maximum profit. First, the profit is increasing in ξ. This is intuitive since the optimal prices are increasing in ξ, and, as shown in Proposition 3 (iv) and 4 (iv), if the firm prices according to the optimal rule, the total demand increases in ξ. Second, compared with the case where there is no behavioral effects, purchase regret hurts while wait regret benefits the firm. The reason is that purchase regret causes strategic waiting and helps consumers to share some of the risks caused by uncertainty to the firm. On the contrary, wait regret lures consumers to purchase in the first period. Third, the effects of anticipated regrets on the profit are monotony.

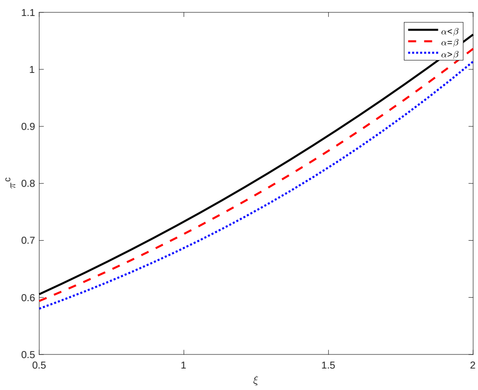


Figure 4: Profit (Π^c) as a function of ξ.

Although we have showed in Section 5 that price commitment dominates dynamic pricing, we are interested in the impacts of the behavioral parameters β on the relative value of price commitment. Therefore, we compare the profits under these two pricing strategies. Notice that the biggest difference between these two pricing strategies is how they choose the optimal second period price. Therefore, the fact we find that the value of price commitment depends on the value of the uncertainty parameter ξ and the regret parameters α and β can be explained through the parameters' impact on the optimal second period price. For example, when the ξ is close to 1, the value of price commitment hits its maximum since the optimal second period price under commitment is far away from $\frac{1}{2}\overline{v}_2$, which is the optimal second period price under dynamic pricing. Besides, the effects of the emotional parameters follow the same pattern as their effects on the optimal second period price, and the explanation follows the same logic as that behinds Proposition 4.

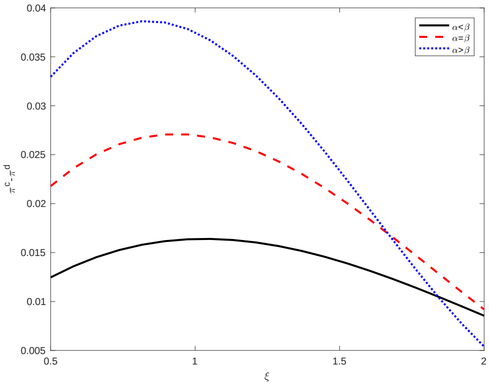


Figure 5: The value of PC ($\Pi^c - \Pi^d$) as a function of ξ.

7 Conclusion

This paper studies the impact of the second period valuation uncertainty and consumers anticipated regret on consumers behavior and firm's prices and pricing strategy. We consider a firm selling two substitutable products to rationally emotional consumers over two periods. There is only one product available in each period. Consumers face the second period valuation uncertainty, and make their purchase-or-wait decisions at the beginning of the first period. The uncertainty may come from personal factors or the product factors, and each action may result in the corresponding regrets. Consumers can anticipate the regrets and also take them into consideration.

We find that the prices of the two substitutable products are positively correlated, which seems like a good news to the seller. However, it is worth to notice that the price of product 1 can be lower than that of product 2 even when the average valuation of the former is larger than that of the latter. The reason is that the firm needs to lower its first period price to moderate the uncertainty faced by the consumers and thus mitigates strategic waiting since the effective marginal revenue of strategic waiting is less than that of immediate purchase.

Our research also shows that the impacts of each kind of anticipated regrets on the firm's prices can be opposite and depends on the value of the uncertainty. The optimal second period price increases in purchase regret and decreases in wait regret when the uncertainty is moderate. The impacts can turn to the opposite if the uncertainty is large enough and consumers anticipate much more aversion to purchase regret. Besides, attention should be paid to the fact that although purchase regret increases the second period price in most cases, it harms the firm's total demand and profit. The reason is that purchase regret results in strategic waiting and thus leads consumers to share some of the risks caused by uncertainty to the firm. This result justifies the existence of return policy in our daily life, which mitigates the negative impact of purchase regret. On the contrary, wait regret benefits the firm. So it is wise for the firm to carry out some marketing campaigns to evoke consumers' aversion to wait regret.

Finally, because of consumers' strategic behavior to figure out the possible second period price under dynamic pricing, dynamic pricing is a special case of price commitment and is dominated by the optimal price commitment. And the value of price commitment is affected by the uncertainty and anticipated regrets.

To the best of our knowledge, this paper is the first in the literature to study the interaction between second period valuation uncertainty and anticipated regret. With our pioneer work, there are some research directions valuable for future studies. First, the assumption that the second period valuation follows a uniform distribution can be changed to a general distribution, or other distributions. Second, the assumption that the upper-bound value is commonly known to both the consumers and the firm can also be relaxed. Third, in our model, the second period valuation uncertainty exists because the consumers have only limited information for the product which is not available presently. It will be interesting to study the effects of information sharing between the firm and the consumers on anticipated regret and consumer behavior.

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Appendix. Proofs

Proof of Lemma 1. According to the definition, v_1^o is determined by $\Delta S = 0$. That is, $(\alpha-\beta)(v_1^o-p_1)^2 - 2\big[(\alpha-\beta)(\overline{v}_2-p_2)+(1+\beta)\overline{v}_2\big](v_1^o-p_1)+(1+\alpha)(\overline{v}_2-p_2)^2 = 0$. Thus,

i) if $\alpha=\beta$, $v_1^o = \frac{(\overline{v}_2-p_2)^2}{2\overline{v}_2}+p_1$;

ii) if $\alpha\neq\beta$, $v_1^o = \frac{(1+\alpha)\overline{v}_2-(\alpha-\beta)p_2\pm\sqrt{(1+\beta)\left[(1+\alpha)\overline{v}_2^2-(\alpha-\beta){p_2}^2\right]}}{\alpha-\beta}+p_1$.

Given p_2, there are three conditions that v_1^o needs to satisfy: $v_1^o \geq p_1$, and the equality holds when $\overline{v}_2 = p_2$; $v_1^o - p_1 < \overline{v}_2 - p_2$; and $v_1^o \leq \overline{v}_1$. Then Lemma 1 is obtained.

□

Proof of Proposition 1. If $\alpha = \beta$,

$$\frac{\partial v_1^o}{\partial \xi} = \frac{\overline{v}_2^2 - p_2^2}{2\overline{v}_2^2}\overline{v}_1 > 0$$

$$\frac{\partial v_1^o}{\partial p_1} = 1 > 0$$

$$\frac{\partial v_1^o}{\partial p_2} = -\frac{\overline{v}_2 - p_2}{\overline{v}_2} < 0$$

If $\alpha \neq \beta$, let $t = (1+\alpha)\overline{v}_2^2 - (\alpha - \beta){p_2}^2$,

$$\frac{\partial v_1^o}{\partial \xi} = -\frac{(1+\alpha)(\sqrt{1+\beta}\,\overline{v}_2 - \sqrt{t})}{(\alpha-\beta)\sqrt{t}}\overline{v}_1 > 0$$

$$\frac{\partial v_1^o}{\partial p_1} = 1 > 0$$

$$\frac{\partial v_1^o}{\partial p_2} = \frac{\sqrt{1+\beta}\,p_2 - \sqrt{t}}{\sqrt{t}} < 0$$

$$\frac{\partial v_1^o}{\partial \alpha} = \sqrt{\frac{1+\beta}{t}}\frac{(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)\overline{v}_2^2 - \frac{1}{2}(\alpha-\beta)p_2^2 - \overline{v}_2\sqrt{(1+\beta)t}}{(\alpha-\beta)^2} > 0$$

$$\frac{\partial v_1^o}{\partial \beta} = \frac{(1+\alpha)\overline{v}_2\sqrt{(1+\beta)t} - (1+\alpha)(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)\overline{v}_2^2 + \frac{1}{2}(1+\alpha)(\alpha-\beta)p_2^2}{\sqrt{(1+\beta)t}(\alpha-\beta)^2} < 0$$

□

Proof of Proposition 2. The proof of $\frac{\partial D_T}{\partial p_1}$, $\frac{\partial D_T}{\partial \alpha}$ and $\frac{\partial D_T}{\partial \beta}$ are obvious based on the proof of Proposition 1.

If $\alpha = \beta$, then we have

$$\frac{\partial D_T}{\partial \xi} = -\frac{(\overline{v}_1\xi(p_2 - p_1) - p_2^2)p_2}{\overline{v}_1^3\xi^3} \tag{6}$$

In this case, $\dot{\xi} = \frac{p_2^2}{\overline{v}_1(p_2-p_1)}$.

If $\alpha \neq \beta$, then we have

$$\frac{\partial D_T}{\partial \xi} = -\frac{p_2}{\overline{v}_1^2 \xi^2}((p_2 - p_1) - \sqrt{1+\beta}\frac{p_2^2}{\sqrt{t}}) \tag{7}$$

In this case, $\dot{\xi} = \sqrt{\frac{\frac{(1+\beta)p_2^4}{(p_2-p_1)^2} + (\alpha-\beta)p_2^2}{(1+\alpha)\overline{v}_1^2}}$.

Similarly, if $\alpha = \beta$, then we have

$$\frac{\partial D_T}{\partial p_2} = -\frac{1}{\overline{v}_1 \overline{v}_2} \frac{(\overline{v}_2 - p_2)(\overline{v}_2 - 3p_2) + 2\overline{v}_2 p_1}{2\overline{v}_2}$$

If $\alpha \neq \beta$,

$$\frac{\partial D_T}{\partial p_2} = -\frac{1}{\overline{v}_1 \overline{v}_2}(\frac{\sqrt{1+\beta}p_2^2}{\sqrt{t}} + \frac{(1+\alpha)\overline{v}_2 - (\alpha-\beta)p_2 - \sqrt{(1+\beta)t}}{\alpha - \beta} + (p_1 - p_2))$$

In both case, we can show that there exists $\dot{p}_2$ such that if $p_2 > \dot{p}_2$, $\frac{\partial D_T}{\partial p_2} > 0$. □

Proof of Lemma 2. Given p_2,

$$\frac{\partial \Pi(p_1, p_2)}{\partial p_1} = \frac{1}{\overline{v}_1 \overline{v}_2}\left[\overline{v}_1 \overline{v}_2 - \overline{v}_2 v_1^o + p_2(\overline{v}_2 - p_2) - \overline{v}_2 p_1\right]$$

$$\frac{\partial^2 \Pi(p_1, p_2)}{\partial {p_1}^2} = -\frac{2}{\overline{v}_1} < 0$$

Thus $\Pi(p_1, p_2)$ is concave in p_1.

$$\frac{\partial \Pi(p_1, p_2)}{\partial p_1}|_{p_1=0} = \frac{1}{\overline{v}_1 \overline{v}_2}\left[\overline{v}_1 \overline{v}_2 - \overline{v}_2 v_1^o + p_2(\overline{v}_2 - p_2)\right] > 0$$

$$\frac{\partial \Pi(p_1, p_2)}{\partial p_1}|_{p_1=\overline{v}_1} < 0$$

There is an optimal $p_1^{c*} \in (0, \overline{v}_1)$, such that $\frac{\partial \Pi(p_1,p_2)}{\partial p_1} = 0$. Thus, $p_1^{c*} = \frac{\overline{v}_1\overline{v}_2 - \frac{1}{2}\overline{v}_2^2 + 2\overline{v}_2 p_2 - \frac{3}{2}{p_2}^2}{2\overline{v}_2}$ and $v_1^o = \frac{(\overline{v}_2 - p_2)^2}{2\overline{v}_2} + p_1$. Besides, it is not difficult to check that under our assumption of ξ, $v_1^o < \overline{v}_1$.

Let $H = \overline{v}_2 p_1(\overline{v}_1 - v_1^o) + p_2 v_1^o(\overline{v}_2 - p_2)$, which is proportional to the total profit, then, by substituting $p_1^{c*} = \frac{\overline{v}_1\overline{v}_2 - \frac{1}{2}\overline{v}_2^2 + 2\overline{v}_2 p_2 - \frac{3}{2}{p_2}^2}{2\overline{v}_2}$ into H and taking the derivative, we have the following equations:

$$\frac{\mathrm{d}H}{\mathrm{d}p_2} = \frac{1}{4\overline{v}_2}({p_2}^3 - \overline{v}_2^2 p_2 - 6\overline{v}_1\overline{v}_2 p_2 + 4\overline{v}_1\overline{v}_2^2) \tag{8}$$

$$\frac{\mathrm{d}H}{\mathrm{d}p_2}|_{p_2=0} = \frac{1}{4\overline{v}_2}(4\overline{v}_1\overline{v}_2^2) > 0$$

$$\frac{\mathrm{d}H}{\mathrm{d}p_2}|_{p_2=\overline{v}_2} = \frac{1}{4\overline{v}_2}(-2\overline{v}_1\overline{v}_2^2) < 0$$

It is easy to check that $\frac{\mathrm{d}^2H}{\mathrm{d}{p_2}^2} < 0$. Therefore, given p_1, the profit is concave in p_2 and there exists an unique $p_2^{c*} \in (0, \overline{v}_2)$ such that $\frac{\mathrm{d}H}{\mathrm{d}p_2} = 0$.

Since $\frac{\mathrm{d}H}{\mathrm{d}p_2} = 0$ is a depressed monic cubic equation, its solution is:

$$t_k = 2\sqrt{\frac{(\overline{v}_2 + 6\overline{v}_1)\overline{v}_2}{3}}\cos(\frac{1}{3}\arccos(-\frac{6\overline{v}_1\overline{v}_2}{\overline{v}_2 + 6\overline{v}_1}\sqrt{\frac{3}{(\overline{v}_2 + 6\overline{v}_1)\overline{v}_2}}) - k\frac{2\pi}{3}).$$

Here, $k = 0, 1, 2$. Since $0 < p_2^{c*} < \overline{v}_2$, the optimal solutions are $p_1^{c*} = \frac{\overline{v}_1\overline{v}_2 - \frac{1}{2}\overline{v}_2^2 + 2\overline{v}_2 p_2^{c*} - \frac{3}{2}{p_2^{c*}}^2}{2\overline{v}_2}$, and $p_2^{c*} = 2\sqrt{\frac{6+\xi}{3\xi}}\cos(\frac{1}{3}\arccos(-\frac{6}{6+\xi}\sqrt{\frac{3\xi}{6+\xi}}) - \frac{2\pi}{3})\overline{v}_2$.

□

Proof of Proposition 3. (i) It is easy to show that $\frac{\partial^2\Pi(p_1,p_2)}{\partial p_1\partial p_2} = \frac{1}{\overline{v}_1\overline{v}_2}(2\overline{v}_2 - 3p_2)$ and $\frac{\mathrm{d}H}{\mathrm{d}p_2}|_{p_2=\frac{2}{3}\overline{v}_2} < 0$, which means $p_2^{c*} < \frac{2}{3}\overline{v}_2$. Hence, $\frac{\partial^2\Pi(p_1,p_2)}{\partial p_1\partial p_2} > 0$ and p_1^{c*} is increasing in p_2^{c*};

(ii) According to the Implicit Function Theorem, $\frac{\partial p_2^{c*}}{\partial\xi} = -\frac{\partial(\frac{\mathrm{d}H}{\mathrm{d}p_2})}{\partial\xi}/\frac{\mathrm{d}^2H}{\mathrm{d}p_2^2}$, then one can find that $\frac{\partial p_2^{c*}}{\partial\xi} > 0$

(iii) $p_1 - p_2 = \frac{1}{4\overline{v}_2}(2\overline{v}_1\overline{v}_2 - \overline{v}_2^2 - 3{p_2}^2)$. If ${p_2}^2 < \frac{2\overline{v}_1\overline{v}_2 - \overline{v}_2^2}{3}$, $p_1^{c*} > p_2^{c*}$, otherwise $p_1^{c*} \le p_2^{c*}$. With the help of replacing p_2 in (8) with $\sqrt{\frac{2\overline{v}_1\overline{v}_2 - \overline{v}_2^2}{3}}$, we can get the conclusion that if $27{\overline{v}_1}^2\overline{v}_2 - 32{\overline{v}_1}^3 + \overline{v}_2^3 + 6\overline{v}_1\overline{v}_2^2 \ge 0$, $p_1^{c*} \le p_2^{c*}$; otherwise $p_1^{c*} > p_2^{c*}$. Notice that $27{\overline{v}_1}^2\overline{v}_2 - 32{\overline{v}_1}^3 + \overline{v}_2^3 + 6\overline{v}_1\overline{v}_2^2$ is increasing in ξ. Solving $27{\overline{v}_1}^2\overline{v}_2 - 32{\overline{v}_1}^3 + \overline{v}_2^3 + 6\overline{v}_1\overline{v}_2^2 = 0$, we have $\xi = -2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2 \approx 0.9519$. Therefore, $p_1^{c*} < p_2^{c*}$ if and only if $\xi > -2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2$.

(iv) Based on equations (6) and (7), with the optimal prices, it can be shown that $\frac{\partial D_T}{\partial\xi} > 0$.

□

Proof of Lemma 3. $\Pi(p_1, p_2)$ is concave in p_1, then,

$$p_1 = \frac{1}{2}\Big\{\overline{v}_1 + p_2 - \frac{p_2^2}{\overline{v}_2} - \frac{(1+\alpha)\overline{v}_2 - (\alpha-\beta)p_2 - \sqrt{(1+\beta)\big[(1+\alpha)\overline{v}_2^2 - (\alpha-\beta)p_2^2\big]}}{\alpha-\beta}\Big\} \quad (9)$$

Let $n = \frac{(1+\alpha)\overline{v}_2 - (\alpha-\beta)p_2 - \sqrt{(1+\beta)t}}{\alpha-\beta}$, then $p_1 = \frac{1}{2}(\overline{v}_1 - n + p_2 - \frac{{p_2}^2}{\overline{v}_2})$, and $v_1^o = \frac{1}{2}(\overline{v}_1 + n + p_2 - \frac{{p_2}^2}{\overline{v}_2})$. Here, we first show that $v_1^o < \overline{v}_1$, and then continue our proof.

$$v_1^o - \overline{v}_1 = \frac{1}{2}(-\overline{v}_1 + n + p_2 - \frac{{p_2}^2}{\overline{v}_2}) \propto (\overline{v}_2^2 - p_2^2) - \overline{v}_1\overline{v}_2\frac{\sqrt{t} + \overline{v}_2\sqrt{1+\beta}}{\sqrt{t}}$$

It can be shown that, if $\alpha < \beta$, $\frac{\sqrt{t}+\overline{v}_2\sqrt{1+\beta}}{\sqrt{t}} \geq 2$, thus $\overline{v}_1\frac{\sqrt{t}+\overline{v}_2\sqrt{1+\beta}}{\sqrt{t}} > \overline{v}_2^2$ and $v_1^o < \overline{v}_1$. If $\alpha > \beta$, we can prove $(\overline{v}_2^2 - p_2^2) < \overline{v}_1\overline{v}_2\frac{\sqrt{t}+\overline{v}_2\sqrt{1+\beta}}{\sqrt{t}}$ by contradiction. We assume that $(\overline{v}_2^2 - p_2^2) > \overline{v}_1\overline{v}_2\frac{\sqrt{t}+\overline{v}_2\sqrt{1+\beta}}{\sqrt{t}}$. In this case, $v_1^o = \overline{v}_1$ and $D_1 = 0$, $D_2 = \frac{\overline{v}_2 - p_2}{\overline{v}_2}$. By solving the firm's maximization problem, we get $p_2^* = \frac{1}{2}\overline{v}_2$. However, in this case, $(\overline{v}_2^2 - p_2^2) < \overline{v}_1\overline{v}_2\frac{\sqrt{t}+\overline{v}_2\sqrt{1+\beta}}{\sqrt{t}}$. Hence, $v_1^o < \overline{v}_1$ when $\alpha > \beta$.

Based on the above analysis, we get the formulation of H as follows:

$$H = \frac{1}{2}\Big[\frac{1}{2}{\overline{v}_1}^2\overline{v}_2 - \overline{v}_1\overline{v}_2 n + \overline{v}_1\overline{v}_2 p_2 - \overline{v}_1{p_2}^2 + \frac{1}{2}\overline{v}_2(n + p_2 - \frac{{p_2}^2}{\overline{v}_2})^2\Big]$$

$$\frac{\mathrm{d}n}{\mathrm{d}p_2} = -1 + p_2\sqrt{\frac{1+\beta}{t}}$$

$$\frac{\mathrm{d}(2H)}{\mathrm{d}p_2} = 2\overline{v}_1\overline{v}_2 - 2\overline{v}_1 p_2 - 2{p_2}^2 + 2\frac{{p_2}^3}{\overline{v}_2} - 2p_2 n + (\overline{v}_2 n - \overline{v}_1\overline{v}_2 + \overline{v}_2 p_2 - {p_2}^2)p_2\sqrt{\frac{1+\beta}{t}} \quad (10)$$

It can be shown that when $\alpha \to \beta$, this function reduces to that in the case of $\alpha = \beta$.

$$\frac{\mathrm{d}(2H)}{\mathrm{d}p_2}|_{p_2=0} = 2\overline{v}_1\overline{v}_2 > 0$$

$$\frac{\mathrm{d}(2H)}{\mathrm{d}p_2}|_{p_2=\overline{v}_2} = -\overline{v}_1\overline{v}_2 < 0$$

Based on the assumption that $\xi \in (\frac{1}{2}, 2)$, the following proof shows that $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} < 0$

and thus p_2^{c*} is unique when $\alpha \neq \beta$.

$$\begin{aligned}
\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} &= -2\overline{v}_1 - 4p_2 + 6\frac{{p_2}^2}{\overline{v}_2} - 2n - 2p_2\frac{\mathrm{d}n}{\mathrm{d}p_2} \\
&\quad + (\overline{v}_2\frac{\mathrm{d}n}{\mathrm{d}p_2} + \overline{v}_2 - 2p_2)p_2\sqrt{\frac{1+\beta}{t}} \\
&\quad + (\overline{v}_2 n - \overline{v}_1\overline{v}_2 + \overline{v}_2 p_2 - {p_2}^2)\sqrt{\frac{1+\beta}{t}} \\
&\quad + (\overline{v}_2 n - \overline{v}_1\overline{v}_2 + \overline{v}_2 p_2 - {p_2}^2)p_2\frac{(\alpha-\beta)p_2}{t}\sqrt{\frac{1+\beta}{t}} \\
&= -2\overline{v}_1 + 6\frac{{p_2}^2}{\overline{v}_2} - \frac{(3+2\alpha+\beta)\overline{v}_2}{\alpha-\beta} \\
&\quad + \Big[\frac{2t}{\alpha-\beta} - 4{p_2}^2 + \frac{1+\alpha}{\alpha-\beta}\overline{v}_2^2 - \frac{1+\alpha}{t}\overline{v}_1\overline{v}_2^3\Big]\sqrt{\frac{1+\beta}{t}}
\end{aligned}$$

We can check that $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}|_{p_2=0} < 0$, and $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}|_{p_2=\overline{v}_2} < 0$. Besides, $t = (1+\alpha)\overline{v}_2^2 - (\alpha-\beta){p_2}^2$ implies that ${p_2}^2 = \frac{(1+\alpha)\overline{v}_2^2 - t}{\alpha-\beta}$.

Let $u = \sqrt{t} = \sqrt{(1+\alpha)\overline{v}_2^2 - (\alpha-\beta){p_2}^2}$, which lies between $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$. Then we have,

$$\begin{aligned}
u^3\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} &= -\frac{6}{\overline{v}_2(\alpha-\beta)}u^5 + \frac{6\sqrt{1+\beta}}{\alpha-\beta}u^4 + \big[-2\overline{v}_1 + \frac{(3+4\alpha-\beta)\overline{v}_2}{\alpha-\beta}\big]u^3 \\
&\quad - \frac{3(1+\alpha)\sqrt{1+\beta}\overline{v}_2^2}{\alpha-\beta}u^2 - (1+\alpha)\sqrt{1+\beta}\overline{v}_1\overline{v}_2^3
\end{aligned}$$

Let $I = u^3\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}$.

$$\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} = -\frac{30}{\overline{v}_2}u^3 + 24\sqrt{1+\beta}u^2 + 3\big[-2(\alpha-\beta)\overline{v}_1 + (3+4\alpha-\beta)\overline{v}_2\big]u - 6(1+\alpha)\sqrt{1+\beta}\overline{v}_2^2$$

On the one hand, when $u = \sqrt{1+\alpha}\overline{v}_2$,

$$\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} = 3\big[6(1+\alpha)^{\frac{1}{2}}(1+\beta)^{\frac{1}{2}}\overline{v}_2 - 2(\alpha-\beta)\overline{v}_1 + (-7-6\alpha-\beta)\overline{v}_2\big](1+\alpha)^{\frac{1}{2}}\overline{v}_2$$

Let $a = (1+\alpha)^{\frac{1}{2}}$, $b = (1+\beta)^{\frac{1}{2}}$, and $\gamma = \frac{1}{\xi}$, then,

$$\begin{aligned}
&\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} = 3\big[6ab - 2(a^2-b^2)\gamma + (-6a^2-b^2)\big](1+\alpha)^{\frac{1}{2}}\overline{v}_2^2 \\
&= -6(\gamma+3)(a - \frac{6-\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b)(a - \frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b)(1+\alpha)^{\frac{1}{2}}\overline{v}_2^2
\end{aligned}$$

This implies that,

$$\frac{\mathrm{d}I}{\mathrm{d}u} = -6u(\gamma+3)(a-\frac{6-\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b)(a-\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b)\frac{a}{(a^2-b^2)}\overline{v}_2^2$$

Note that $(a-\frac{6-\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b)>0$, and that $0<(a-\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b)<1$.

Therefore, we have $\frac{\mathrm{d}I}{\mathrm{d}u}<0$ when $\alpha<\big[\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}\big]^2(1+\beta)-1$ or $\alpha>\beta$ and $\frac{\mathrm{d}I}{\mathrm{d}u}>0$ when $\big[\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}\big]^2(1+\beta)-1<\alpha<\beta$.

On the other hand, when $u=\sqrt{1+\beta}\overline{v}_2$,

$$\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} = 3\big[-2(\alpha-\beta)\overline{v}_1+(-1+2\alpha-3\beta)\overline{v}_2\big]\sqrt{1+\beta}\overline{v}_2$$

If $\alpha>\beta$, then

$$\begin{aligned}\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} &< 3\big[-(\alpha-\beta)\overline{v}_2+(-1+2\alpha-3\beta)\overline{v}_2\big]\sqrt{1+\beta}\overline{v}_2\\ &= 3(-1+\alpha-2\beta)\sqrt{1+\beta}\overline{v}_2^2\\ &< 0\end{aligned}$$

Therefore, $\frac{\mathrm{d}I}{\mathrm{d}u}<0$.

If $\alpha<\beta$, then

$$\begin{aligned}\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} &< 3\big[-4(\alpha-\beta)\overline{v}_2+(-1+2\alpha-3\beta)\overline{v}_2\big]\sqrt{1+\beta}\overline{v}_2\\ &= 3(-1-2\alpha+\beta)\sqrt{1+\beta}\overline{v}_2^2\\ &< 0\end{aligned}$$

Therefore, $\frac{\mathrm{d}I}{\mathrm{d}u}>0$.

$$\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big) = -\frac{90}{\overline{v}_2}u^2+48(1+\beta)^{\frac{1}{2}}u+3\big[-2(\alpha-\beta)\overline{v}_1+(3+4\alpha-\beta)\overline{v}_2\big]$$

$\Delta_2 = 2304(1+\beta)+12\frac{90}{\overline{v}_2}\big[-2(\alpha-\beta)\overline{v}_1+(3+4\alpha-\beta)\overline{v}_2\big]$

Note that $-2(\alpha-\beta)\overline{v}_1+(3+4\alpha-\beta)\overline{v}_2 > -2(\alpha-\beta)\overline{v}_1+(1.5+2\alpha-0.5\beta)\overline{v}_1 = (1.5+1.5\beta)\overline{v}_1 > 0$. Thus, $\Delta_2>0$. Hence, there are two roots θ_1 and θ_2, where $\theta_1 =$

$\frac{48\sqrt{1+\beta}-\sqrt{\Delta_2}}{180}\overline{v}_2 < 0$ and $\theta_2 = \frac{48\sqrt{1+\beta}+\sqrt{\Delta_2}}{180}\overline{v}_2 > 0$

$$\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\alpha}\overline{v}_2} = -90(1+\alpha)\overline{v}_2+48\sqrt{1+\beta}\sqrt{1+\alpha}\overline{v}_2+3\big[-2(\alpha-\beta)\overline{v}_1+(3+4\alpha-\beta)\overline{v}_2\big]$$

If $\alpha > \beta$, then

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\alpha}\overline{v}_2} &< -90(1+\alpha)\overline{v}_2 + 48\sqrt{1+\alpha}\sqrt{1+\alpha}\overline{v}_2 + 3(3+4\alpha-\beta)\overline{v}_2\\ &= 3(-11-10\alpha-\beta)\overline{v}_2\\ &< 0\end{aligned}$$

If $\alpha < \beta$, then

$$\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\alpha}\overline{v}_2} < (-90\sqrt{1+\alpha}+48\sqrt{1+\beta})\sqrt{1+\alpha}\overline{v}_2+3\big[-4(\alpha-\beta)\overline{v}_2+(3+4\alpha-\beta)\overline{v}_2\big]$$

Note that $-90\sqrt{1+\alpha}+48\sqrt{1+\beta} \leq -90+48\sqrt{2} < 0$. So,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\alpha}\overline{v}_2} &< (-90\sqrt{1+\alpha}+48\sqrt{1+\beta})\overline{v}_2 + 9(1+\beta)\overline{v}_2\\ &\leq (-72+48\sqrt{2})\overline{v}_2\\ &< 0\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\beta}\overline{v}_2} &= -90(1+\beta)\overline{v}_2 + 48(1+\beta)\overline{v}_2 + 3\big[-2(\alpha-\beta)\overline{v}_1 + (3+4\alpha-\beta)\overline{v}_2\big]\\ &= 3\big[-2(\alpha-\beta)\overline{v}_1 + (-11+4\alpha-15\beta)\overline{v}_2\big]\\ &\leq 3\big[2\overline{v}_1 + (-11+4\alpha-15\beta)\overline{v}_2\big]\\ &< 3\big[4\overline{v}_2 + (-11+4\alpha-15\beta)\overline{v}_2\big]\\ &= 3(-7+4\alpha-15\beta)\overline{v}_2\\ &< 0\end{aligned}$$

This implies that both $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$ are greater than θ_2.

Therefore, $\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big) < 0$ for all u between $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$.

When $\alpha > \big[\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}\big]^2(1+\beta)-1$, it is easy to find that $\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} < 0$ for all u between $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$. Hence, if $\alpha < \beta$, then $\frac{\mathrm{d}I}{\mathrm{d}u} > 0$ for all u and if $\alpha > \beta$, then $\frac{\mathrm{d}I}{\mathrm{d}u} < 0$ for all u. Therefore, I is monotonic and $I < 0$. Since $I = u^3\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}$, thus

$\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}$ is monotonic and $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} < 0$.

When $\alpha < [\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}]^2(1+\beta)-1$, it is also easy to find that $I < 0$ although it is not monotonic.

To sum up, $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} < 0$ for all u. This completes the proof.

□

Proof of Proposition 4. (i) If $\alpha \neq \beta$, replace p_2 in (10) with $\frac{1}{2}\overline{v}_2$ we get,

$$\frac{\mathrm{d}2H}{\mathrm{d}p_2}\Big|_{p_2=\frac{1}{2}\overline{v}_2} = (1-\frac{1}{2}\sqrt{\frac{1+\beta}{1+\frac{3}{4}\alpha+\frac{1}{4}\beta}})[\overline{v}_1\overline{v}_2-\frac{1}{4}\overline{v}_2^2-\frac{(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)-\sqrt{(1+\beta)(1+\frac{3}{4}\alpha+\frac{1}{4}\beta)}}{\alpha-\beta}\overline{v}_2^2]$$

$$1-\frac{1}{2}\sqrt{\frac{1+\beta}{1+\frac{3}{4}\alpha+\frac{1}{4}\beta}} = 1-\sqrt{\frac{1+\beta}{4+3\alpha+\beta}} > 0$$

$$\begin{aligned}
&1-\Big[\frac{1}{4}+\frac{(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)-\sqrt{(1+\beta)(1+\frac{3}{4}\alpha+\frac{1}{4}\beta)}}{\alpha-\beta}\Big]\xi \\
&> 1-2\Big[\frac{1}{4}+\frac{(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)-\sqrt{(1+\beta)(1+\frac{3}{4}\alpha+\frac{1}{4}\beta)}}{\alpha-\beta}\Big] \\
&= \frac{1}{2}-\frac{1+\alpha}{2+\alpha+\beta+\sqrt{(1+\beta)(4+3\alpha+\beta)}} \\
&> 0
\end{aligned}$$

Therefore, $\frac{\mathrm{d}2H}{\mathrm{d}p_2}\Big|_{p_2=\frac{1}{2}\overline{v}_2} > 0$, $p_2^{c*} > \frac{1}{2}\overline{v}_2$.

Based on (9), differentiating p_1 w.r.t. p_2, we get

$$\frac{\mathrm{d}(2p_1^{c*})}{\mathrm{d}p_2^{c*}} = 2-\frac{2}{\overline{v}_2}p_2-p_2\sqrt{\frac{1+\beta}{t}}$$

We have the same results as that in the case of $\alpha = \beta$ by showing that $\frac{\mathrm{d}(2p_1^{c*})}{\mathrm{d}p_2^{c*}} > 0$.

Let $Z = \overline{v}_2\frac{\mathrm{d}2p_1^{c*}}{\mathrm{d}p_2^{c*}} = 2(\overline{v}_2-p_2)-\overline{v}_2p_2\sqrt{\frac{1+\beta}{t}}$

As $\frac{\mathrm{d}Z}{\mathrm{d}p_2} = -2-\overline{v}_2\sqrt{\frac{1+\beta}{t}}\frac{(1+\alpha)\overline{v}_2^2}{t} < 0$, Z is decreasing in p_2. Besides, $Z|_{p_2=0} = 2\overline{v}_2 > 0$, and $Z|_{p_2=\overline{v}_2} = -\overline{v}_2 < 0$. Hence, there is ${p_2}^Z \in (0,\overline{v}_2)$ such that $Z|_{p_2={p_2}^Z} = 0$.

The following results will be useful in the later proof.

$$Z|_{p_2=\frac{1}{2}\overline{v}_2} = \overline{v}_2-\frac{1}{2}\overline{v}_2\sqrt{\frac{1+\beta}{1+\frac{3}{4}\alpha+\frac{1}{4}\beta}} > 0$$

$$Z|_{p_2=\frac{2}{3}\overline{v}_2} = \frac{2}{3}\Big(1 - \sqrt{\frac{1+\beta}{1+\frac{5}{9}\alpha+\frac{4}{9}\beta}}\Big)\overline{v}_2$$

When $\alpha > \beta$, $Z|_{p_2=\frac{2}{3}\overline{v}_2} > 0$; while when $\alpha < \beta$, $Z|_{p_2=\frac{2}{3}\overline{v}_2} < 0$.

Putting ${p_2}^Z$ into (10), we get

$$\begin{aligned}\frac{\mathrm{d}2H}{\mathrm{d}p_2}|_{p_2={p_2}^Z} &= -6{p_2}^{Z2} + \frac{4{p_2}^{Z3}}{\overline{v}_2} + 2(\overline{v}_2 - 2{p_2}^Z)\Big[\frac{(1+\alpha)\overline{v}_2}{\alpha-\beta} - {p_2}^Z - \frac{(1+\beta)\overline{v}_2{p_2}^Z}{2(\alpha-\beta)(\overline{v}_2-{p_2}^Z)}\Big] + 2\overline{v}_2{p_2}^Z \\ &= \frac{(1+\beta)\overline{v}_2{p_2}^Z(\overline{v}_2-2{p_2}^Z)(3{p_2}^Z-2\overline{v}_2)}{2(\alpha-\beta)(\overline{v}_2-{p_2}^Z)^2} \\ &< 0\end{aligned}$$

Therefore, $\frac{\mathrm{d}2H}{\mathrm{d}p_2}|_{Z=0} < 0$, which means $Z|_{p_2^{c*}} > 0$ since both $\frac{\mathrm{d}2H}{\mathrm{d}p_2}$ and Z are decreasing in p_2 when $p_2 \in (0, \overline{v}_2)$.

Notice that $Z = \overline{v}_2\frac{\mathrm{d}2p_1^{c*}}{\mathrm{d}p_2^{c*}}$, thus $\frac{\mathrm{d}2p_1^{c*}}{\mathrm{d}p_2^{c*}} > 0$. This completes the proof.

(ii) According to (10),

$$\frac{\partial p_2^{c*}}{\partial\alpha} = -\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha}\Big/\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}$$

As $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} < 0$, and $\frac{\partial p_2}{\partial\alpha}\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha} > 0$, we then focus on $\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha}$.

By the same logic of Lemma 3, one gets the following:

$$\frac{2(\alpha-\beta)^2u^3}{\sqrt{1+\beta}p_2}\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha} = -3u^4+6\sqrt{1+\beta}\overline{v}_2u^3+\big[(\alpha-\beta)\overline{v}_1-3(1+\beta)\overline{v}_2\big]\overline{v}_2u^2-(\alpha-\beta)(1+\beta)\overline{v}_1\overline{v}_2^3$$

Notice that u lies between $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$. Let $J = \frac{2(\alpha-\beta)^2u^3}{\sqrt{1+\beta}p_2}\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha}$. $J > 0$ means that the best second period price p_2^{c*} is increasing in α and vice versa.

$$\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u} = -12u^2 + 18\sqrt{1+\beta}\overline{v}_2u + 2\big[(\alpha-\beta)\overline{v}_1 - 3(1+\beta)\overline{v}_2\big]\overline{v}_2$$

If $16\alpha < 13\beta - 3$ and $\frac{1}{2} < \xi < \frac{8(\beta-\alpha)}{3(1+\beta)}$, $\Delta_{\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}} < 0$, $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u} < 0$, otherwise $\Delta_{\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}} \geq 0$.

Notice that if $\alpha > \beta$, then $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}|_{u=0} < 0$ and $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}|_{u=\sqrt{1+\beta}\overline{v}_2} > 0$; If $\alpha < \beta$, then $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}|_{u=\sqrt{1+\beta}\overline{v}_2} < 0$.

Let u_1 and u_2 be the two roots of $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u} = 0$ when $\Delta_{\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}} \geq 0$ and $u_1 < u_2$, then,

$$u_1 = \frac{9\sqrt{1+\beta}\overline{v}_2 - \sqrt{3\big[3(1+\beta)\overline{v}_2 + 8(\alpha-\beta)\overline{v}_1\big]\overline{v}_2}}{12}$$

$$u_2 = \frac{9\sqrt{1+\beta}\overline{v}_2 + \sqrt{3\left[3(1+\beta)\overline{v}_2 + 8(\alpha-\beta)\overline{v}_1\right]\overline{v}_2}}{12}$$

The analysis on the relative relationship between u_1, u_2, $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$ results in the following table:

No.	$\sqrt{1+\alpha}$	ξ	$\Delta_{\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}}$	$u_1 - \sqrt{1+\alpha}\overline{v}_2$	$u_2 - \sqrt{1+\alpha}\overline{v}_2$
1		$(\frac{1}{2}, \delta_1)$	$-$	N/A	N/A
2		δ_1	0	$+$	$+$
3	$[1, \frac{3}{4}\sqrt{1+\beta})$	(δ_1, δ_2)	$+$	$+$	$+$
4		δ_2	$+$	0	$+$
5		$(\delta_2, 2)$	$+$	$-$	$+$
6		$(\frac{1}{2}, \delta_1)$	$-$	N/A	N/A
7	$\frac{3}{4}\sqrt{1+\beta}$	δ_1	0	0	0
8		$(\delta_1, 2)$	$+$	$-$	$+$
9		$(\frac{1}{2}, \delta_1)$	$-$	N/A	N/A
10		δ_1	0	$-$	$-$
11	$(\frac{3}{4}\sqrt{1+\beta}, \frac{\sqrt{13}}{4}\sqrt{1+\beta})$	(δ_1, δ_2)	$+$	$-$	$-$
12		δ_2	$+$	$-$	0
13		$(\delta_2, 2)$	$+$	$-$	$+$
14		$(\frac{1}{2}, \delta_2)$	$+$	$-$	$-$
15	$[\frac{\sqrt{13}}{4}\sqrt{1+\beta}, \sqrt{1+\beta})$	δ_2	$+$	$-$	0
16		$(\delta_2, 2)$	$+$	$-$	$+$
17		$(\frac{1}{2}, \delta_2)$	$+$	$-$	$+$
18	$(\sqrt{1+\beta}, \frac{5}{4}\sqrt{1+\beta})$	δ_2	$+$	$-$	0
19		$(\delta_2, 2)$	$+$	$-$	$-$
20	$[\frac{5}{4}\sqrt{1+\beta}, \sqrt{2}]$	$(\frac{1}{2}, 2)$	$+$	$-$	$-$

Here, $\delta_1 = \frac{8(\beta-\alpha)}{3(1+\beta)}$ and $\delta_2 = \frac{\sqrt{1+\alpha}+\sqrt{1+\beta}}{3(2\sqrt{1+\alpha}-\sqrt{1+\beta})}$. $\delta_1 < \delta_2$. The table helps to define the sign of $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}$ and thus imply the shape of J. Finally, by analyzing $J|_{u=u_1}$, $J|_{u=\sqrt{1+\alpha}\overline{v}_2}$ and $J|_{u=\sqrt{1+\beta}\overline{v}_2}$, we can complete the proof.

According to the definition of J, $J|_{u=\sqrt{1+\beta}\overline{v}_2} = 0$, and $J|_{u=\sqrt{1+\alpha}\overline{v}_2} = (\sqrt{1+\alpha} - \sqrt{1+\beta})^2\left[(\sqrt{1+\alpha}+\sqrt{1+\beta})^2\overline{v}_1 - 3(1+\alpha)\overline{v}_2\right]\overline{v}_2^3$.

Let $\delta_3 = \frac{(\sqrt{1+\alpha}+\sqrt{1+\beta})^2}{3(1+\alpha)}$, if $\xi < \delta_3$, $J|_{u=\sqrt{1+\alpha}\overline{v}_2} > 0$; otherwise, $J|_{u=\sqrt{1+\alpha}\overline{v}_2} \leq 0$. Note that $\delta_2 < \delta_3 < 2$.

So, only in cases 5, 8, 13, 16, 19 and 20, can we find $J|_{u=\sqrt{1+\alpha}\overline{v}_2} < 0$. In this case, for cases 5, 8, 13 and 16, there is a $u' \in (\sqrt{1+\alpha}\overline{v}_2, u_2)$ such that if $u \in (\sqrt{1+\alpha}\overline{v}_2, u')$, $J < 0$, otherwise $J \geq 0$. Similarly, for cases 19 and 20, there is a $u'' \in (u_2, \sqrt{1+\alpha}\overline{v}_2)$ such that if $u \in (u'', \sqrt{1+\alpha}\overline{v}_2)$, $J < 0$; otherwise $J \geq 0$.

Then, we focus on case 3. We need to focus on $J|_{u=u_1}$. Notice that, by using factoring, we can get:

$$
\begin{aligned}
J|_{u=u_1} = & \frac{1}{8}\left[8(\alpha-\beta)\overline{v}_1 + 3(1+\beta)\overline{v}_2\right]\sqrt{1+\beta}\overline{v}_2^2 u_1 \\
& - (\alpha-\beta)(1+\beta)\overline{v}_1\overline{v}_2^3 \\
& + \frac{1}{24}\left[(\alpha-\beta)\overline{v}_1 - 3(1+\beta)\overline{v}_2\right]\left[2(\alpha-\beta)\overline{v}_1 + 3(1+\beta)\overline{v}_2\right]\overline{v}_2^2
\end{aligned}
$$

By factorization, one can show that $J|_{u=u_1} > 0$. So, in case 3, $J \geq 0$. Note that $J > 0$ means that the best second period price p_2^{c*} is increasing in α and vice versa. Therefore, we confirm our results that only in cases 5, 8, 13, 16, 19 and 20, can we find $J|_{u=\sqrt{1+\alpha}\overline{v}_2} < 0$.

However, notice that, given α, β, $\overline{v}_1$ and $\overline{v}_2$, p_2^{c*} is a fixed number rather than an independent variable u discussed above. But we can show that $J < 0$ still exists. For example, the following table shows some examples in cases 16, 19 and 20 in which $J < 0$.

No.	α	β	$\overline{v}_1$	ξ	p_2^{c*}	J
16	0.3515	0.5	2	1.9990	2.1570	-0.0490
19	0.3936	0.1	2	1.7990	2.0595	-0.5304
20	0.8520	0.1	2	1.7406	1.9576	-3.7643

□

Let $\hat{\alpha}$ be the solution of $J = 0$ in cases 5, 8, 13, 16, 19 and 20. Then, we can conclude that, if ξ is large enough, then there exits a $\hat{\alpha}$ such that if $\alpha > \hat{\alpha}$, the optimal second price p_2^{c*} is decreasing in α.

Lastly, note that the analysis of the impact of β on p_2^{c*} is similar to the above. Actually, $\frac{\partial p_2^{c*}}{\partial \beta}$ always has exactly opposite direction as $\frac{\partial p_2^{c*}}{\partial \alpha}$. Here we complete the proof of Proposition 4.

Proof of Proposition 5. Based on the fact that $p_2^{c*} > \frac{1}{2}\overline{v}_2$ and that p_1^{c*} increases in p_2^{c*}, Proposition 5 is obvious.

□