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Trial-and-error train fare design scheme for addressing boarding/alighting congestion at CBD stations

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Abstract

This study deals with boarding/alighting congestion of congested commuter train stations at central business district (CBD), in which the additional fares are determined to shift an appropriate number of passengers to board/alight at the neighboring uncongested stations on the railway line. A bi-objective model is investigated to minimize both fare increases of the congested stations, while alleviating their boarding/alighting congestion to a certain level simultaneously. The existence of the unique Pareto-optimal solution is proven mathematically in the absence of explicit demand functions. A trial-and-error fare design scheme is proposed to identify the Pareto-optimal solution. An illustrative study demonstrates the effectiveness of the trial-and-error scheme.

Keywords: trial-and-error; boarding/alighting passenger; train station; train fare; unknown demand.

1 Introduction

Train is one of the dominated modes of public transport for passengers' daily commuting between home and workplace in mega cities such as Tokyo, Beijing, Sydney, and Hong Kong. With the development of urbanization, the train service has been continuously extending to outer suburbs and more passengers are attracted to central business district (CBD) where job opportunities are concentrated. For instance, approximately 320,000 commuters in Sydney alight at stations in CBD each day with the average commuting distance 16.5 kilometres (Australian Bureau of Statistics, 2018), from Sydney's Eastern Suburbs, North Shore and Inner South Western regions. In the morning peak period, cumulative passengers over an

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entire train line, from outer suburbs to near suburbs, require boarding or alighting at stations in CBD, such that the boarding/alighting demands at commuter train stations in CBD are excessively high. High demands of boarding/alighting operation cause boarding/alighting congestion in CBD train stations, which is reflected by the overcrowded phenomenon that passengers with limited walking space hamper each other to board or alight on trains in the peak hours. In Sydney's CBD train stations, a large number of station staff are added to work at the platforms to avoid overcrowding while passengers board/alight on trains during the peak (Fairfax Media, 2017). However, this strategy only relieves the crowding phenomenon but cannot essentially solve the boarding/alighting congestion exerted by the excessively high volume of the boarding/alighting passengers.

Boarding/alighting congestion contributes to the other negative impact besides overcrowding phenomenon, that is, the dwell time easily blows out at stations in CBD. Dwell time is defined as the time that a public transport vehicle remains stopped transferring passengers (TRB, 2000). At high-demand train stations, dwell time is determined by the volume of boarding passengers (Puong, 2000), and is the major component of headways at short frequencies (TRB, 2003). High volume of boarding/alighting passengers contributes to the relatively long dwell time at CBD stations. Long dwell time causes delay of this train and subsequent trains. Especially for high frequency trains for daily commuting, even a minor delay can cascade to subsequent trains causing poor performance of the entire system and severely affects passenger service quality. Although dwell times at lower-demand stations outside CBD are much shorter, the subsequent train cannot arrive if the previous train still dwells at the station in CBD, since they utilize the same infrastructure resource. It is also explainable that longer dwell time intensifies the boarding/alighting congestion instead of solving the problem. Although planning a longer dwell time at CBD stations in the train service schedule enables passengers to have sufficient time to board and alight, it will reduce the service frequency[1], thereby decrease the total service capacity. In CBD train stations,

[1] The highest possible service frequency = 1/(dwell time at the busiest station of the line + shortest headway for safety concerns).

boarding/alighting congestion is mostly related to daily commuting in the peak hours, when the train services should be operated at their highest frequency to deal with high travel demands.

Few existing approaches can be used for efficiently tackling the boarding/alighting congestion problem of stations located at CBD. Typically, the capacity of stations can be increased to match the high demands, namely, to increase the number of platforms of CBD train stations. However, this solution is less applicable due to the limited spaces and shortage land resources of CBD. The number of platforms is also restricted by the length of train and constraint of tracks (Whelan and Johnson, 2004), which implies that the entire system should be adjusted accordingly. Increasing the system capacity can only be a long-term solution but not an effective short-term method in consideration of the large financial investment and time to implement. Another method is to reduce the travel demand of passengers commuting by trains: if passengers transfer to other travel mode to work, the boarding/alighting congestion of train stations in CBD will alleviate. However, this solution violates the principle of green commute, since transport by train is more efficient and environmentally friendly than other travel modes, and it imposes burden to other transport modes.

Other approaches refer to the pedestrian traffic management, which is defined as "rational administration of the movement of people to generate adequate behavior in public spaces to improve the use of pedestrian infrastructure" according to Seriani and Fernández (2015). The mostly used strategy of pedestrian traffic management involves changing commuters' travel time from peak hours to off-peak hours. Changing travel time to off-peak hours is feasible for some commuters with flexible working hours, while it is not effective for commuters with a strong restricted "9 to 5" working hours, since the schedule delay costs for such work trips is commonly high. Commuters also tend to refuse travel earlier to avoid peak hours, otherwise they have to not only get up earlier but also tolerate a longer work day. It is even harder for the workplaces to implement flexible working hours since many works need team cooperation, e.g., assembly line production companies. Spreading demands over a longer time period is not applicable for solving boarding/alighting congestion at CBD train stations.

Novel approaches based on demand control and congestion management are required for boarding/alighting congestion at CBD stations. Our data indicates that even for train stations both located in CBD with only hundreds of meters walking distance apart, the congested level can be significantly different. Under this circumstance, a realizable approach is to change commuters' boarding/alighting stations to the nearby less congested stations by fare differentiation. For example, in Sydney, *Wynyard* station (closest station to most offices/companies) an*d Circular Quay* (closest station to Sydney opera house and botanic garden) are both at CBD with only 700 meters walking distance apart. However, during the morning peak hours, *Circular Quay* station is much quieter since the tourist flow and commuting flow have different arrival patterns. As a result, by charging appropriate additional fares to congested stations (or equivalently giving appropriate credits to less congested stations), we can transfer the travel demands of the congested train station to the neighboring uncongested train stations. Compared with the traditional method that spreads peak demand over time, spreading demand to a wider set of stations is more realizable for inflexible daily commuting.

This work is motivated by an actual situation in Sydney. After opening the northwest metro rail line in 2019, more people will take trains to commute from suburbs to the CBD for work. Four sequential stations (at T2 line) in CBD are close to each other: *Central*, *Town Hall*, *Wynyard*, and *Circular Quay* train stations (platforms), where the middle two stations, *Town Hall* and *Wynyard*, are closest to offices/companies and cannot meet the surge demand of boarding and alighting passengers. The current headway between two trains at T2 line is 7 minutes during peak hours, while the frequency of trains cannot be improved due to the long dwell time at the middle two stations. The potential approach is to increase fares to passengers boarding/alighting at these middle two stations, then passengers may spread to the outer two train stations with substantial capacity, which are *Central* and *Circular Quay*. The walking distance among these adjacent stations ranges from 700 meters to 1,100 meters, such that some passengers are willing to switch to neighboring stations. We need to determine the least amounts of fare increases in *Town Hall* and *Wynyard* to shift passengers to *Central* and *Circular Quay*, so that the remaining boarding/alighting passenger volume matches the existing boarding/alighting ability of the platforms.

The fare system in Sydney metros is a typical tap-on tap-off system through a credit card-sized contactless Opal card. The card includes a microchip and internal RFID aerial for its communications with readers. Due to the discount of Opal card compared to Single Tickets, in 2017, over 90% of commuters used Opal card. The microchip enables data to be loaded onto the card, records the journey details and deducts appropriate fare. The fare charged when a passenger taps off, and it is calculated based on a function of zone based distance from entry (tap-on) to exit (tap-off). Stations in CBD are very close to each other and the difference in distance does not affect the overall fare. Boarding/alighting surcharges in certain congested stations at CBD are investigated to alleviate the boarding/alighting congestion.

This paper studies the context of a railway line with two congested stations at CBD[2], in which additional fares are determined to shift an appropriate number of passengers to board/alight at the two neighboring stations on the line with substantial capacity. Considering the public acceptability, the objective is to minimize both fare increases at the two stations in the bi-objective, while alleviating boarding/alighting congestion to a certain level. We prove the existence of the unique Pareto-optimal solution in which both fare increases can be minimized. The challenge of this problem is that the relationship between fare differences between stations and commuters' choices of boarding/alighting stations (referred to as demand functions hereafter) is hard to establish with exact formulation. We therefore aim to propose a trial-and-error scheme in this work considering the absence of demand functions in the real-life application. The trial-and-error scheme guarantees optimal fare increases without requiring the explicit forms of the unknown demand functions. Fare increase can be adjusted in a series of iterations, such that each increase is adjusted based on the observation of the

[2] The situation of one congested station at CBD can easily be addressed by the method proposed in this study.

boarding/alighting passenger volume determined by last fare increase, until achieving the optimal fare increase.

This work fills the literature gap from the following perspective. (i) Most previous studies manage travel demands over time by changing commuters' travel time to off-peak hours, while few study investigates how to manage travel demands over space to a wider set of stations by fare differentiation. (ii) The fare increases are minimized in both stations while reducing boarding/alighting passengers to a certain amount. The existence of the unique Pareto-optimal solution is rigorously proved mathematically with the absence of explicit demand functions. Through a trial-and-error fare design scheme, the optimal fare increases can be identified after a series of iterations. (iii) Instead of tackling congestion problem of one station in most literature, two congested stations are handled simultaneously. Two congested neighboring stations in CBD are commonly seen in real-life, still, more complex traffic situations may be encountered. This study can inspire challengeable future research, comprehensively considering a greater number of congested and uncongested stations based on more complicated rail networks. (iv) To the best of our knowledge, this is the first study that applies a trial-and-error scheme in a bi-objective transportation problem.

This paper is organized as seven sections. Section 2 reviews the related literature. In Section 3, a bi-objective model minimizing both fare increases of the two congested stations is proposed. The trial-and-error scheme designed for this work is explained in Section 4. We then mathematically prove the existence of the unique Pareto-optimal solution in Section 5. Section 6 presents an illustrative example to demonstrate the effectiveness of the proposed model and methodology. Section 7 concludes.

2 Literature review

2.1 Pedestrian traffic management

Boarding/alighting management falls into the category of pedestrian traffic management, while the efficiency of pedestrian traffic management can be measured by dwell time (Seriani and Fernández, 2015). High density of pedestrian flow or inefficiency management contributes to crowding phenomenon. Related literature can be categorized into three streams based on different research approaches, which are pedestrian models, laboratory experiments, and empirical studies.

The first stream is related to pedestrian models. Hughes (2002) developed a continuum theory for the flow of pedestrians, and derived equations leading to subcritical and supercritical regimes of flow. Hoogendoorn and Bovy (2004) developed a route choice and activity scheduling modeling approach to describe pedestrian behavior based on utility maximization. Antonini et al. (2006) proposed a discrete choice model for pedestrian walking behavior, which is further extended by Robin et al. (2009) by specification, estimation and validation. Guo et al. (2012) developed a microscopic pedestrian model with discrete space representation to investigate the route choice of pedestrians. Davidich et al. (2013) proposed a model for waiting pedestrians based on cellular automata (CA) models. Flötteröd and Lämmel (2015) presented a model of stationary bidirectional pedestrian flow. Nikolić et al. (2016) introduced a probabilistic modeling approach which can represent the speed-density relationship of pedestrian traffic. Hänseler et al. (2017) proposed a macroscopic loading model for multi-directional, time-varying and congested pedestrian flows.

The second stream is based on laboratory experiments with controlled environments. Daamen et al. (2008) conducted boarding/alighting experiments to test the effect of the physical environment, flow composition, and prevailing traffic conditions. Fernández et al. (2010) presented dwell time parameters obtained from real-scale experiments with controlled platform height, door width and fare collection method. Fujiyama et al. (2014) conducted experiments to examine the effects of the design factors of the train-platform interface on boarding-dominant pedestrian flows and alighting-dominant pedestrian flows. Liao et al. (2014) investigated the pedestrian flow through wide bottlenecks through a series of laboratory study. Fernández et al. (2015) demonstrated that pedestrian saturation flows exist in public transport doors and investigated the values under different conditions by laboratory experiments. Seriani and Fernández (2015) determined the effect of pedestrian traffic management in the boarding and alighting time of passengers at metro stations by means of simulation and experiments. de Ana Rodríguez et al. (2016) explored the impact of platform edge doors on passengers' boarding and alighting time and passenger behaviors through laboratory experiments under controlled conditions. Holloway et al. (2016) tested the effect of vertical step height on boarding and alighting time of train passengers by designed experiments. Seriani et al. (2017) explored a new precise indicator, the pedestrian level of interaction, on platform conflict areas at metro stations by real-scale laboratory experiments. Seriani and Fujiyama (2018) estimated the required passenger space for alighting at metro stations by performing laboratory experiments.

The third stream is about empirical study by surveys or real observations in existing stations. Evans and Wener (2007) examined the impacts of train density and seat proximity to indices of stress among 139 passenger train commuters. Zhang et al. (2008) studied passenger alighting and boarding movement according to observations in existing stations. Karekla and Tyler (2012) collected actual train operation data by on-site observations to examine the cost and benefits of improving passenger accessibility. Currie et al. (2013) presented an empirical study to examine the impacts of crowding tram stop design on dwell times. Kroes et al. (2014) established value of crowding on public transport services by conducting qualitative research, stated preference experiments, and passenger counts and surveys. Harris et al. (2014) examined the impact of urban rail boarding and alighting factors by analyzing real data at almost 130 locations. Kim et al. (2015) investigated crowding effect on the path choice of metro passengers from Smart Card data. Wang et al. (2016) conducted field research in subway stations to study impacts of train-based and station-based factors on boarding and alighting movement. Li et al. (2016) estimated train dwell time based on track occupation event data at a railway station.

2.2 Travel demand management

The research is also related to the category of demand management, that the solution strategy of this research for solving congestion is to shift demands from congested stations to uncongested stations such that to deliver passengers smoothly. Demand shifting can reduce peak congestion such that remaining demands better match the system capacity. Reviews of managing peak demand for passenger rail have been published by Hale and Charles (2009) and Henn et al. (2010). Many existing studies aim at shifting trips from peak hours to off-peak hours by fare increase or decrease. Douglas et al. (2011) modelled the effect of discounts and surcharges of fare on passenger's choice of peak hour trains. The results showed that the surcharge is effective than discount to reduce the peak hour loads. Currie (2011) investigated who shifted demand from the peak and by how much. More detailed review of spreading peak demand through differential fare policy can be found in Liu and Charles (2013). We notice that existing studies have done many efforts in spreading demand over time, while few discussed how to spread demand over space. In this work, we develop a wider set of peak destinations by differential pricing strategy to spread demand over space. The relationship between demand shifting in time and pricing surcharge is well investigated, but the unknown demand function should be faced if we spread demand over space, since it is still unclear in the literature that how many passengers are willing to shift their destinations and by how much of the fare increase.

The common approach to deal with unknown demand function is trial-and-error method, which has been widely used in pricing congestion studies to deal with road congestion. Vickrey (1993) and Downs (1993) first proposed the trial and error method for unknown demand function, while neither proposed a detailed implementation. Later, trial-and-error implementations in road congestion pricing have been conducted and can be summarized into two categories. Category (i) is to levy tolls such that system optimal (SO) flows can be achieved. Trial-and-error procedures are applied to a single road (Li, 2002; Wang and Yang, 2012), or a congested road network (Yang et al., 2004; Han and Yang, 2009), minimizing the total travel time of all users. Li (2002) proposed a bi-section method to find the optimal congestion toll based on a speed-flow relationship and a value of the generalized cost without demand function along a single road. However, an inaccurate value of generalized value due to unknown demand function may lead to false convergence. Then, Wang and Yang (2012) fixed this issue by providing a modified bi-section method and established the convergence for trial-and-error implementation on the marginal-cost pricing with unknown demand functions, and also emulate it in a revenue manner, but only considered a homogeneous traffic stream along a single road. Yang et al. (2004) proposed a trial-and-error implementation on marginal-cost pricing on a general road network in the absence of demand function, later enhanced by Han and Yang (2009) with a faster convergence. The above-mentioned studies are in category (i) to achieve a system optimum (SO), while another category (ii) is to levy tolls such that the volumes of traffic flow or passengers do not exceed the predetermined threshold. Trial-and-error methods for road congestion implemented in this category is applicable not only without demand functions, but also without demand functions and user's value of time (Meng et al., 2005; Yang et al., 2010). Meng et al. (2005) proposed a trial-and-error procedure, a variation of a gradient projection method for dual formulation of the traffic assignment problem, only requiring observed traffic flow at the entry links to the congested area. This procedure indicates the road-pricing scheme can be considered as a user equilibrium traffic assignment with link capacity constraints. Yang et al. (2010) developed a two-stage pricing scheme with the general variational inequality (VI) based traffic equilibrium model for the iterative trial-and-error adjustment. Zhou et al. (2015) proposed a trial-and-error congestion pricing scheme for networks, not only considering the minimization of the total system cost but also addressing the capacity constraints.

3 Train fare design problem and model

In this section, we first analyze the properties of the demand functions for the purpose of designing a pricing scheme. Although the exact demand functions are absent in this study, a general relationship to describe the response of passengers after fare increases can be formulized. Then, a model is formulated to determine the least amount of fare increases of the congested stations.

3.1 Demand functions

Four stations are considered in our study which are S_0, S_1, S_2 and S_3. Currently the fares for passengers boarding/alighting at the four stations are all the same. It is projected that in the future, a total of q_1 and q_2 passengers (per train) will board or alight at S_1 and S_2 respectively, if the fare structure is unchanged. Suppose the headway requires that at most Q passengers can board or alight at a station [3], whereas $q_1 > Q$ and $q_2 > Q$.

We thus increase the fares for those boarding or alighting at S_1 and S_2 by x and y, respectively, so that some passengers will use S_0 and S_3 (some passengers will switch between S_1 and S_2 if $x \neq y$). The transport authority imposes upper bounds $\bar{x}$ and $\bar{y}$ on the fare increase: $0 \le x \le \bar{x}$, $0 \le y \le \bar{y}$. The general response of the passengers after fare increases of the two congested stations is constructed as Figure 1.

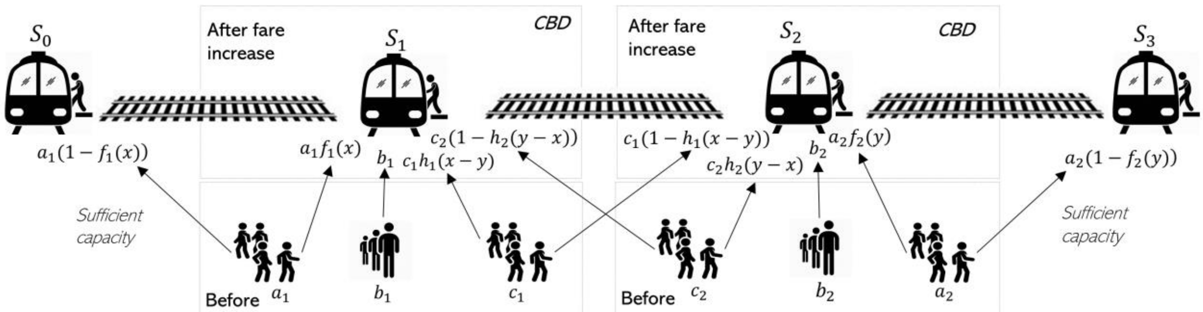

Figure 1. Pricing scheme to decentralize the travel demands.

We classify the q_1 passengers whose ideal (boarding or alighting) station is S_1 into three categories:

(i) The first category, consisting of a_1 passengers (the subscript '1' represents S_1), is those who may switch to S_0 if x is too high.

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[3] The capacity that Q passengers board/alight at a station is related to the maximum dwell time of a train at a station. The maximum dwell time is determined by the designed headway and the headway has to meet the travel demand of the train. For instance, if 300,000 people per hour go to CBD by train in the morning peak, and each train can load 10,000 people, then the headway of train should be 2 minutes (60/(300,000/10,000)).

(ii) The second category, consisting of b_1 passengers, is those who always use S_1.

(iii) The third category, consisting of c_1 passengers, is those who may switch to S_2 if x is higher than y.

By definition,

$$a_1 + b_1 + c_1 = q_1 \tag{1}$$

The percentage of passengers in the first category who will still choose S_1 is a function of x, denoted by $f_1(x)$; $a_1(1 - f_1(x))$ passengers will switch to S_0. We have

$$f_1(0) = 1,\ 0 \le f_1(x) \le 1, x \in [0, \bar{x}]. \tag{2}$$

Assumption 1: $f_1(x)$ is continuously differentiable and strictly decreasing over $[0, \bar{x}]$.

The percentage of passengers in the third category who will still choose S_1 is a function of $x - y$, denoted by $h_1(x - y)$; $c_1(1 - h_1(x - y))$ passengers will switch to S_2. We have

$$\begin{cases} h_1(x-y) = 1, & -\bar{y} \le x - y \le 0 \\ h_1(x-y) \ge 0, & 0 < x - y \le \bar{x}. \end{cases} \tag{3}$$

Assumption 2: $h_1(x - y)$ is continuously differentiable and strictly decreasing over $x - y \in [0, \bar{x}]$.

Similarly, we classify the q_2 passengers whose ideal (boarding or alighting) station is S_2 into three categories:

(i) The first category, consisting of a_2 passengers, is those who may switch to S_3 if y is too high.

(ii) The second category, consisting of b_2 passengers, is those who always use S_2.

(iii) The third category, consisting of c_2 passengers, is those who may switch to S_1 if $y - x$ is too high.

We have

$$a_2 + b_2 + c_2 = q_2 \tag{4}$$

The percentage of passengers in the first category who will still choose S_2 is a function of y, denoted by $f_2(y)$; $a_2(1 - f_2(y))$ passengers will switch to S_3. We have

$$f_2(0) = 1,\ 0 \le f_2(y) \le 1, y \in [0, \bar{y}] \tag{5}$$

Assumption 3: $f_2(y)$ is continuously differentiable and strictly decreasing over $[0, \bar{y}]$.

The percentage of passengers in the third category who will still choose S_2 is a function of $y - x$, denoted by $h_2(y - x)$; $c_2(1 - h_2(y - x))$ passengers will switch to S_1. We have

$$h_2(y-x)\begin{cases} = 1, & -\bar{x} \le y - x \le 0 \\ \ge 0, & 0 < y - x \le \bar{y} \end{cases} \tag{6}$$

Assumption 4: $h_2(y - x)$ is continuously differentiable and strictly decreasing over $y - x \in [0, \bar{y}]$.

The number of passengers using S_1 after the price increase is a function of (x, y), denoted by $X(x, y)$:

$$X(x, y) = a_1 f_1(x) + b_1 + c_1 h_1(x - y) + c_2(1 - h_2(y - x)) \tag{7}$$

The number of passengers using S_2 after the price increase is also a function of (x, y), denoted by $Y(x, y)$:

$$Y(x, y) = c_1(1 - h_1(x - y)) + c_2 h_2(y - x) + b_2 + a_2 f_2(y) \tag{8}$$

We have:

Lemma 1: Both $X(x, y)$ and $Y(x, y)$ are continuous in both x and y. Given x, $X(x, y)$ increases in y and $Y(x, y)$ strictly decreases in y. Given y, $X(x, y)$ strictly decreases in x and $Y(x, y)$ increases in x.

Remark 1: Increasing both x and y by the same amount $\Delta > 0$ will not shift passengers between S_1 and S_2, but will shift passengers from S_1 to S_0 and from S_2 to S_3, so that $X(x + \Delta, y + \Delta) \le X(x, y)$ and $Y(x + \Delta, y + \Delta) \le Y(x, y)$ where $x + \Delta \le \bar{x}$ and $x + \Delta \le \bar{y}$.

The formulation constructed above is very general under the above weak assumptions, since the detailed information is unknown in practice. The only available information is that passengers "may" shift to neighboring train stations after "appropriate" fare surcharge. It is unclear in realistic that "how many" passengers are willing to shift and "how much" the fare should increase. It is intuitive that different types of passengers trigger different selections of station, e.g., kids, tourists, individual commuters or groups, such that in our model, some passengers stick to their original stations (b_1 and b_2) while others (a_1, c_1 and a_2, c_2) may switch to another stations. However, demands also depend on many other factors, i.e., the design and layout of stations, passenger distributions on platforms, crowding levels of train doors, times or distances walking to other stations, etc. All kinds of factors comprehensively make the impact on the demand functions and lead to different decisions of passengers. Therefore, in our formulation, the unknown factors are captured by implicit parameters of generalized functions of x and y, i.e., $f_1(x)$, $h_1(x-y)$, $f_2(y)$, and $h_2(y-x)$, such that the main challenge in this study is to tackle the generalized demand functions with implicit components. Still, the benefit of the generalized function is that, although this study is motived by the specific phenomenon happening in Sydney train station, it is applicable to various situations referring to different stations and locations. The scope is to solve a type of boarding/alighting congestion problem, other than a specific field research on existing stations.

3.2 Model

The transport authority aims to raise the minimum amount of fare to achieve the objective.

[M1]
$$\min_{\substack{x\in[0,\bar{x}]\\ y\in[0,\bar{y}]}} \begin{pmatrix} x \\ y \end{pmatrix} \tag{9}$$

subject to:

$$X(x, y) \le Q \tag{10}$$

$$Y(x, y) \le Q \,. \tag{11}$$

Since [M1] has two objectives, we aim to find Pareto optimal solutions of (x, y). A feasible solution (x', y') is Pareto optimal if and only if there does not exist a feasible solution (x'', y'') such that $x'' \le x'$, $y'' \le y'$, and at least one of the two inequalities is strict.

For model [M1], we have the following properties.

Remark 2: If model [M1] is feasible with a feasible solution (x', y'), then there must be a feasible solution (x, y) satisfying $x = \bar{x}$ or $y = \bar{y}$. The reason is that, if following the logic of Remark 1 and setting $\Delta := \min(\bar{x} - x', \bar{y} - y')$, then $(x, y) := (x' + \Delta, y' + \Delta) = (\bar{x}, y)$ satisfies $X(\bar{x}, y) \le Q$ and $Y(\bar{x}, y) \le Q$ when $\Delta := \bar{x} - x'$, or $(x, y) := (x' + \Delta, y' + \Delta) = (x, \bar{y})$ satisfies $X(x, \bar{y}) \le Q$ and $Y(x, \bar{y}) \le Q$ when $\Delta := \bar{y} - y'$.

Proposition 1: Given an error $\varepsilon > 0$, checking whether [M1] is feasible can be completed in time at most $\lceil \log_2 \bar{x}/\varepsilon \rceil + \lceil \log_2 \bar{y}/\varepsilon \rceil$.

Proof: From Remark 2, if model [M1] is feasible, $(\bar{x}, y)$ or $(x, \bar{y})$ is feasible. We can check feasibility of model [M1] by bi-section search as follows, given an error term $\varepsilon > 0$:

First, we check whether there is a $y \in [0, \bar{y}]$ such that the solution $(\bar{x}, y)$ is feasible. This can be completed in time at most $\lceil \log_2 \bar{y}/\varepsilon \rceil$.

We have the lower and upper bounds of y being $y^{\min} \leftarrow 0$ and $y^{\max} \leftarrow \bar{y}$, respectively. Set $y' \leftarrow (y^{\min} + y^{\max})/2$. Check feasibility of $(\bar{x}, y')$. Note that Lemma 1 shows $X(\bar{x}, y)$ increases in y and $Y(\bar{x}, y)$ strictly decreases in y, situations may occur as follows: (i) If $X(\bar{x}, y') \le \mathrm{Q}$ and $Y(\bar{x}, y') \le \mathrm{Q}$, then [M1] is feasible and stop; (ii) If $X(\bar{x}, y') > \mathrm{Q}$ and $Y(\bar{x}, y') > \mathrm{Q}$; then $(\bar{x}, y)$ cannot be feasible and stop; (iii) If $X(\bar{x}, y') \le \mathrm{Q}$ and $Y(\bar{x}, y') > \mathrm{Q}$; then $y^{\min} \leftarrow y'$ and repeat the above procedure; (iv) If $X(\bar{x}, y') > \mathrm{Q}$ and $Y(\bar{x}, y') \le \mathrm{Q}$; then $y^{\max} \leftarrow y'$ and repeat the above procedure. Stop when $y^{\max} - y^{\min} \le \varepsilon$, then $(\bar{x}, y)$ cannot be feasible.

If a feasible solution has not been found, we continue checking whether $(x, \bar{y})$ is feasible in the same manner, which can be completed in time at most $\lceil \log_2 \bar{x}/\varepsilon \rceil$. Therefore, checking feasibility of model [M1] can be completed totally in times at most $\lceil \log_2 \bar{x}/\varepsilon \rceil + \lceil \log_2 \bar{y}/\varepsilon \rceil$.

In the sequel, we assume that [M1] has at least one feasible solution, which is the case as long as $\bar{x}$ and $\bar{y}$ are sufficiently large.

Proposition 2: In any Pareto-optimal solution (x^*, y^*) we have $x^* > 0$ and $y^* > 0$.

Proof: If $x^* = 0$ then the number of passengers who use S_1 will not decrease because

$$\begin{aligned}
& X(0, y^*) \\
&= a_1 f_1(0) + b_1 + c_1 h_1(0 - y^*) + c_2(1 - h_2(y^* - 0)) \\
&= a_1 + b_1 + c_1 + c_2(1 - h_2(y^* - 0)) \\
&= q_1 + c_2(1 - h_2(y^* - 0)) \\
&> q_1 \\
&> Q
\end{aligned} \tag{12}$$

Hence the solution $(0, y^*)$ is infeasible. A similar argument holds for the case of $y^* = 0$.

4 Existence of a unique Pareto-optimal solution

In this section, we prove the existence of a unique Pareto-optimal solution in the proposed model, in which both fare increases of the two congested stations are the minima. Recall the last section where we assumed that, as long as $\bar{x}$ and $\bar{y}$ are sufficiently large, model [M1] has at least a feasible solution (x, y), where $0 \le x \le \bar{x}$ and $0 \le y \le \bar{y}$. Moreover, all constraints are non-strict inequalities ($\le$) in model [M1], such that the set of feasible solutions should be a bounded closed set. Thus, one or more than one Pareto-optimal solution(s) should exist in the bounded closed set, while in this section, we further proof that a unique Pareto-optimal solution exists. The proof procedure of the unique Pareto-solution is as follow: we first prove that the two inequalities in model [M1] are binding simultaneously for all Pareto-optimal solutions, such that model [M1] can be transferred to model [M2]; we then prove that model [M2] has exactly one feasible solution, which implies that exactly one Pareto-optimal solution exists.

Theorem 1: The two inequalities in [M1] are binding simultaneously for all Pareto-optimal solutions.

Proof: We prove this theorem by contradiction. Without loss of generality, suppose that there exists a Pareto-optimal solution (x^*, y^*) such that the first inequality is unbinding (the second inequality could be binding or unbinding):

$$X(x^*, y^*) = a_1 f_1(x^*) + b_1 + c_1 h_1(x^* - y^*) + c_2(1 - h_2(y^* - x^*)) = Q - \delta < Q \tag{13}$$

$$Y(x^*, y^*) = c_1(1 - h_1(x^* - y^*)) + c_2 h_2(y^* - x^*) + b_2 + a_2 f_2(y^*) \le Q \tag{14}$$

where δ is a positive number.

Case 1: $x^* \le y^*$ (note that $x^* > 0$ due to Proposition 2). We will show below that by decreasing x^* by a small positive amount Δ while not changing y^*, the above two inequalities still hold.

As $x^* \le y^*$, we have $h_1(x^* - y^*) = 1$ and $h_2(y^* - x^*) \le 1$. The first of the above two equations implies,

$$a_1 f_1(x^*) + b_1 + c_1 \le Q - \delta \tag{15}$$

Hence,

$$-a_1(1 - f_1(x^*)) + a_1 + b_1 + c_1 \le Q - \delta \tag{16}$$

That is,

$$-a_1(1 - f_1(x^*)) + q_1 \le Q - \delta \tag{17}$$

As $q_1 > Q$, we have

$$a_1(1 - f_1(x^*)) - \delta > 0 \tag{18}$$

Because $x^* > 0$ due to Proposition 2 and $f_1(x)$ is strictly decreasing, we have $a_1 > \delta$ and

$$a_1 f_1(x^*) + \frac{\delta}{2} < a_1 - \frac{\delta}{2} < a_1 \tag{19}$$

Since $f_1(x)$ is continuous and strictly decreasing, there is a value $\Delta_1 \in (0, x^*]$ such that

$$a_1 f_1(x^* - \Delta_1) = a_1 f_1(x^*) + \frac{\delta}{2} \tag{20}$$

That is, if we reduce x^* by Δ_1, then $\delta / 2$ passengers from the a_1 passengers will use S_1.

Since $h_2(y - x)$ is continuous and strictly decreasing, there is a value $\Delta_2 \in (0, x^*]$ such that

$$c_2(1 - h_2(y^* - x^*)) \geq c_2(1 - h_2(y^* - (x^* - \Delta_2))) - \frac{\delta}{2} \tag{21}$$

That is, if we reduce x^* by Δ_2 while not changing y^*, then at most another $\delta / 2$ passengers from the c_2 passengers will use S1 due to the reduction of fare at S_1.

Define $\Delta := \min(\Delta_1, \Delta_2)$ and we thus have

$$a_1 f_1(x^* - \Delta) \leq a_1 f_1(x^*) + \frac{\delta}{2} \tag{22}$$

$$c_2(1 - h_2(y^* - (x^* - \Delta))) \leq c_2(1 - h_2(y^* - x^*)) + \frac{\delta}{2} \tag{23}$$

Construct a new solution $(x, y) = (x^* - \Delta, y^*)$. Then this new solution is feasible because

$$\begin{aligned}
& X(x^* - \Delta, y^*) \\
&= a_1 f_1(x^* - \Delta) + b_1 + c_1 h_1(x^* - \Delta - y^*) + c_2(1 - h_2(y^* - x^* + \Delta)) \\
&= a_1 f_1(x^* - \Delta) + b_1 + c_1 h_1(x^* - y^*) + c_2(1 - h_2(y^* - x^* + \Delta)) \\
&\leq a_1 f_1(x^*) + \frac{\delta}{2} + b_1 + c_1 h_1(x^* - y^*) + c_2(1 - h_2(y^* - x^*)) + \frac{\delta}{2} \\
&= a_1 f_1(x^*) + b_1 + c_1 h_1(x^* - y^*) + c_2(1 - h_2(y^* - x^*)) + \frac{\delta}{2} + \frac{\delta}{2} \\
&= Q - \delta + \delta \\
&= Q
\end{aligned} \tag{24}$$

and Lemma 1 implies

$$Y(x^* - \Delta, y^*) \leq Y(x^*, y^*) \leq Q \tag{25}$$

This contradicts to the assumption that (x^*, y^*) is Pareto-optimal.

Case 2: $x^* > y^*$. We have $h_2(y^* - x^*) = 1$. Since $f_1(x)$ is continuous and strictly decreasing, there is a value $\Delta_1 \in (0, x^* - y^*]$ such that

$$a_1 f_1(x^* - \Delta_1) \le a_1 f_1(x^*) + \frac{\delta}{2} \tag{26}$$

Since $h_1(x-y)$ is continuous and strictly decreasing, there is a value $\Delta_2 \in (0, x^* - y^*]$ such that

$$c_1 h_1(x^* - \Delta_2 - y^*) \le c_1 h_1(x^* - y^*) + \frac{\delta}{2} \tag{27}$$

Define $\Delta := \min(\Delta_1, \Delta_2)$ and similarly we can prove that solution $(x, y) = (x^* - \Delta, y^*)$ is feasible and dominates (x^*, y^*).

Similarly, we can prove that there does not exist a Pareto-optimal solution (x^*, y^*) such that the second inequality is unbinding.

Theorem 1 means that the model is equivalent to:

[M2] $$\min_{\substack{x \in (0, \bar{x}] \\ y \in (0, \bar{y}]}} \begin{pmatrix} x \\ y \end{pmatrix} \tag{28}$$

subject to:

$$X(x, y) = Q \tag{29}$$

$$Y(x, y) = Q \tag{30}$$

Note that in [M2], we replaced $x \in [0, \bar{x}]$ and $y \in [0, \bar{y}]$ by $x \in (0, \bar{x}]$ and $y \in (0, \bar{y}]$, respectively, due to Proposition 2.

For model [M2], we have the following properties.

Lemma 2: Consider the constraint $Y(x, y) = Q$. If (i) for an $x = x_1$, $0 < x_1 < \bar{x}$, there exists a $y = y_1, 0 < y_1 \le \bar{y}$, such that $Y(x_1, y_1) = Q$ and (ii) for an $x = x_2$, $x_1 < x_2 \le \bar{x}$, there exists a

$y = y_2$, $0 < y_2 \le \bar{y}$, such that $Y(x_2, y_2) = Q$, then for any $x_1 < x' < x_2$ there exists a $0 < y' \le \bar{y}$ such that $Y(x', y') = Q$.

Proof: As given y, $Y(x, y)$ strictly increases in x (Lemma 1), we have

$$Y(x', y_1) > Y(x_1, y_1) = Q \tag{31}$$

$$Y(x', y_2) < Y(x_2, y_2) = Q. \tag{32}$$

The continuity of $Y(x, y)$ in y implies there exists a $y' \in [\min(y_1, y_2), \max(y_1, y_2)]$ such that $Y(x', y') = Q$.

Define l_x and u_x as the lower and upper bounds of possible values of x such that constraint $Y(x, y) = Q$ has a feasible solution of y. Note that l_x and u_x can easily be computed using bisection search: the outer iteration applies bisection search to x as $x \in [0, \bar{x}]$; given x, the inner iteration applies bisection search to find the value of y such that $Y(x, y) = Q$ as $Y(x, y)$ strictly decreases in y.

Lemma 3: Given an $x \in [l_x, u_x]$, as $Y(x, y)$ strictly decreases in y, there is a unique solution of y to equation $Y(x, y) = Q$, which can be found using bisection search, and we denote this unique solution by $\pi(x)$.

Lemma 4: (i) $\pi(x)$ is continuous in x. (ii) $\pi(x)$ strictly increases with x, i.e., for $l_x \le x_1 < x_2 \le u_x$, we have $\pi(x_2) > \pi(x_1)$. (iii) For $l_x \le x_1 < x_2 \le u_x$, we have $\pi(x_2) - \pi(x_1) < x_2 - x_1$.

Proof: (i) This result holds because $Y(x, y)$ is continuous in both x and y. (ii) $Y(x_1, \pi(x_1)) = Q$. It is easy to see that $Y(x_2, \pi(x_1)) > Y(x_1, \pi(x_1)) = Q = Y(x_2, \pi(x_2))$; therefore, $\pi(x_2) > \pi(x_1)$. (iii) $Y(x_1 + (x_2 - x_1), \pi(x_1) + (x_2 - x_1)) < Y(x_1, \pi(x_1)) = Q = Y(x_2, \pi(x_2))$, where the first inequality holds from Remark 1; therefore, $\pi(x_2) < \pi(x_1) + (x_2 - x_1)$.

Lemma 5: There exists at most one point $\hat{x} \in [l_x, u_x]$ such that $\pi(\hat{x}) = \hat{x}$.

Proof: If Lemma 5 does not hold, then point (iii) of Lemma 4 is violated, as seen in Figure 2.

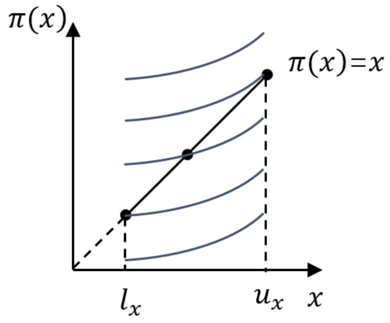


Figure 2. An example of Lemma 5.

Replacing y in $X(x,y)$ by $\pi(x)$, we have a new function

$$\lambda(x) := X(x,\pi(x)) = a_1 f_1(x) + b_1 + c_1 h_1(x-\pi(x)) + c_2(1-h_2(\pi(x)-x)) \tag{33}$$

Theorem 2: $\lambda(x)$ strictly decreases in x.

Proof: for $l_x \le x_1 < x_2 \le u_x$,

$$\begin{aligned}\lambda(x_1)-\lambda(x_2) &= a_1(f_1(x_1)-f_1(x_2)) \\ &+c_1(h_1(x_1-\pi(x_1))-h_1(x_2-\pi(x_2)))-c_2(h_2(\pi(x_1)-x_1)-h_2(\pi(x_2)-x_2))\end{aligned} \tag{34}$$

Lemma 4 implies $x_1-\pi(x_1) < x_2-\pi(x_2)$. Therefore, $f_1(x_1)-f_1(x_2)>0$, $h_1(x_1-\pi(x_1))-h_1(x_2-\pi(x_2))>0$, and $h_2(\pi(x_1)-x_1)-h_2(\pi(x_2)-x_2)<0$. Hence, $\lambda(x_1)-\lambda(x_2)>0$.

Theorem 3: Model [M2] has exactly one feasible solution. This implies that there is exactly one Pareto-optimal solution.

Proof: Theorem 2 implies that there is exactly one solution of x; Lemma 3 implies that given the solution of x, there is exactly one solution of y.

After proving that exactly one Pareto-optimal solution exists, we will propose a trial-and-error method to discover this solution with the absence of demand functions.

5 Trial-and-error scheme with unknown demand functions

In this section, we first describe the main idea of the trial-and-error fare design scheme, and how to apply this method, followed by two detailed algorithms. The main reason to adopt the trial-and-error method is that acquiring the functional forms of f_1, h_1, f_2, and h_2 is difficult in practice, while observing the numbers of passengers using S_1 and S_2 after increasing the prices by (x, y) is practicable, i.e., $X(x, y)$ and $Y(x, y)$. For brevity, we sometimes use X and Y in place of $X(x, y)$ and $Y(x, y)$, respectively. We need to identify a price adjustment mechanism based on each trial and observation accordingly, to eventually identify the Pareto-optimal prices (x^*, y^*). Since model [M2] only has one feasible solution which is exactly the Pareto-optimal solution (Theorem 3), if we find a feasible solution (x, y) satisfying $X(x, y) = Q$ and $Y(x, y) = Q$, it is the Pareto-optimal prices. The trial-and error-method to identify (x^*, y^*) is described as follow.

Initially, we make an arbitrary trial of price increases $(x^{(1)}, y^{(1)})$ of the two congested stations. After implementing $(x^{(1)}, y^{(1)})$, the number of passengers $X^{(1)} := X(x^{(1)}, y^{(1)})$ and $Y^{(1)} := Y(x^{(1)}, y^{(1)})$ can be observed and provide us some information, that certain regions of the domain $(x, y) \in [0, \bar{x}] \times [0, \bar{y}]$ are infeasible of model [M2] since they cannot lead to $X(x, y) = Q$ and $Y(x, y) = Q$. The infeasible regions can be eliminated based on Table 1 and 2. In the bellowing, detailed explanations refer to how to conclude the infeasible regions based on observations.

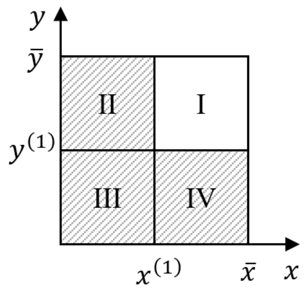
Although the demand functions are still unknown, the number of passengers $X^{(1)}$ and $Y^{(1)}$ of congested stations after fare surcharge $(x^{(1)}, y^{(1)})$ can be observed by field observation. By comparing the realized passenger amount $X^{(1)}$ and $Y^{(1)}$ with the target passenger amount Q, one of the six different comparison results presented in Table 1 and 2 may occur. Taking case (i) of Table 1 and 2 as an example, the observation shows that $X^{(1)} \geq Q$, $Y^{(1)} \geq Q$ and $X^{(1)} + Y^{(1)} > 2Q$. In this case, the next trial $(x^{(2)}, y^{(2)})$ should be taken within region I (by increasing both x and y), while infeasible region II (by decreasing x but increasing y), infeasible region III (by decreasing both x and y) and infeasible region IV (by increasing x but decreasing y) should be eliminated, for the following reasons.

Region II with regard to decreasing x and increasing y leads to $X > X^{(1)}$ (simply based on Lemma 1), adding with the previous result $X^{(1)} > Q$, thus we can conclude that $X > Q$, such that region II is infeasible. Similar reason is for region IV which leads to $Y > Y^{(1)} \geq Q$. For region III, by decreasing both x and y, fewer passengers of category a_1 and a_2 will shift to the neighboring uncongested stations, such that $X + Y > X^{(1)} + Y^{(1)} > 2Q$. It is noticeable that the demand shifting of category c_1 and c_2 between congested stations will not affect $X + Y$, which is the sum of passenger numbers of congested stations.

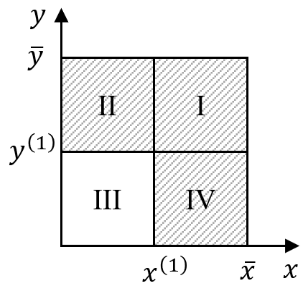
The trial-and-error scheme is to conduct iterations following the above procedures until we identify the Pareto-optimal prices (x^*, y^*) which satisfy $X(x^*, y^*) = Q$ and $Y(x^*, y^*) = Q$.

Table 1. Infeasible regions of the domain.

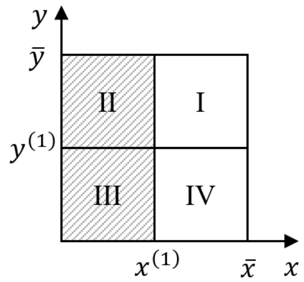
(i) $X^{(1)} \geq Q$, $Y^{(1)} \geq Q$, $X^{(1)} + Y^{(1)} > 2Q$.



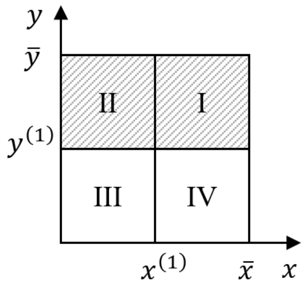
(ii) $X^{(1)} \leq Q$, $Y^{(1)} \leq Q$, $X^{(1)} + Y^{(1)} < 2Q$.



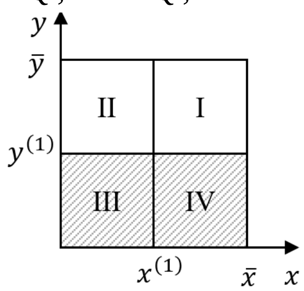
(iii) $X^{(1)} \geq Q$, $Y^{(1)} \leq Q$, $X^{(1)} + Y^{(1)} \geq 2Q$.



(iv) $X^{(1)} \geq Q$, $Y^{(1)} \leq Q$, $X^{(1)} + Y^{(1)} \leq 2Q$.



(v) $X^{(1)} \leq Q$, $Y^{(1)} \geq Q$, $X^{(1)} + Y^{(1)} \geq 2Q$.



(vi) $X^{(1)} \leq Q$, $Y^{(1)} \geq Q$, $X^{(1)} + Y^{(1)} \leq 2Q$.

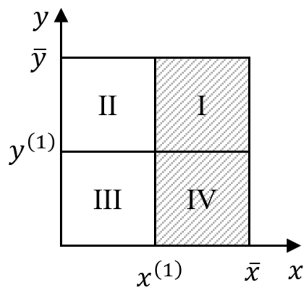


Table 2. Infeasible regions and infeasible reasons.

Case		Infeasible Region	Reason
(i)	$X^{(1)} \geq Q$, $Y^{(1)} \geq Q$, $X^{(1)} + Y^{(1)} > 2Q$.	II	Decreasing x and increasing y will increase X, leading to $X > X^{(1)} \geq Q$.
		III	Decreasing both x and y will increase $X+Y$, leading to $X+Y > X^{(1)} + Y^{(1)} > 2Q$.
		IV	Decreasing y and increasing x will increase Y, leading to $Y > Y^{(1)} \geq Q$.
(ii)	$X^{(1)} \leq Q$, $Y^{(1)} \leq Q$,	I	Increasing both x and y will decrease $X+Y$, leading to $X+Y < X^{(1)} + Y^{(1)} < 2Q$.

	$X^{(1)}+Y^{(1)}<2Q$.	II	Decreasing x and increasing y will decrease Y, leading to $Y<Y^{(1)}\le Q$.
		IV	Decreasing y and increasing x will decrease X, leading to $X<X^{(1)}\le Q$.
(iii)	$X^{(1)}\ge Q$, $Y^{(1)}\le Q$, $X^{(1)}+Y^{(1)}\ge 2Q$, $(X^{(1)}-Q)(Y^{(1)}-Q)\ne 0$.	II	Decreasing x and increasing y will increase X, leading to $X>X^{(1)}\ge Q$
		III	Decreasing both x and y will increase $X+Y$, leading to $X+Y>X^{(1)}+Y^{(1)}\ge 2Q$
(iv)	$X^{(1)}\ge Q$, $Y^{(1)}\le Q$, $X^{(1)}+Y^{(1)}\le 2Q$, $(X^{(1)}-Q)(Y^{(1)}-Q)\ne 0$.	I	Increasing both x and y will decrease $X+Y$, leading to $X+Y<X^{(1)}+Y^{(1)}<2Q$.
		II	Decreasing x and increasing y will increase X, leading to $X>X^{(1)}\ge Q$; and decrease Y, leading to $Y<Y^{(1)}\le Q$.
(v)	$X^{(1)}\le Q$, $Y^{(1)}\ge Q$, $X^{(1)}+Y^{(1)}\ge 2Q$, $(X^{(1)}-Q)(Y^{(1)}-Q)\ne 0$.	III	Decreasing both x and y will increase $X+Y$, leading to $X+Y>X^{(1)}+Y^{(1)}\ge 2Q$
		IV	Decreasing y and increasing x will decrease X, leading to $X<X^{(1)}\le Q$; and increase Y, leading to $Y>Y^{(1)}\ge Q$.
(vi)	$X^{(1)}\le Q$, $Y^{(1)}\ge Q$, $X^{(1)}+Y^{(1)}\le 2Q$, $(X^{(1)}-Q)(Y^{(1)}-Q)\ne 0$.	I	Increasing both x and y will decrease $X+Y$, leading to $X+Y<X^{(1)}+Y^{(1)}<2Q$.
		IV	Decreasing y and increasing x will decrease X, leading to $X<X^{(1)}\le Q$; and increase Y, leading to $Y>Y^{(1)}\ge Q$.
(vii)	$X^{(1)}=Q$, $Y^{(1)}=Q$.	Null	Solution is found.

It can be found that if $(x,y)=(\bar{x}/2,\bar{y}/2)$, then at least half of the area of the domain $[0,\bar{x}]\times[0,\bar{y}]$ can be removed. The remaining domain is still a rectangle. Then we set the price at the center point of the rectangle and continue.

Remark 3: More areas can be removed as Figure 3, which indicates infeasible region for case (iii): $X^{(1)} \geq Q$, $Y^{(1)} \leq Q$, $X^{(1)} + Y^{(1)} \geq 2Q$. we can see that all points in the black triangle (formed by the line $y = x$, while the points do not include the boundary of the triangle on $y = x$) are infeasible, because these points mean $y^{(2)} > y^{(1)}$ and $y^{(2)} - x^{(2)} > y^{(1)} - x^{(1)}$; as a result, fewer passengers will use S_2 and hence we must have $Y^{(2)} < Y^{(1)} < Q$. For the ease of analysis, such exclusion is not considered.

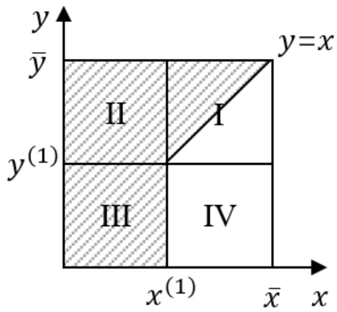


Figure 3. Additional infeasible region for case (iii).

Based on above analyses, a trial-and-error algorithm can be conducted as follow.

Algorithm 1. *A trial-and-error algorithm.*

Step 0: Set the iteration number $\kappa = 0$. Set the domain at $(x, y) \in [x_l^{(1)}, x_u^{(1)}] \times [y_l^{(1)}, y_u^{(1)}] := [0, \bar{x}] \times [0, \bar{y}]$, where $x_l^{(1)}$, $x_u^{(1)}$, $y_l^{(1)}$, and $y_u^{(1)}$ are the lower and upper bounds of x and the lower and upper bounds of y, respectively. Set the error gap $\varepsilon_1 > 0$.

Step 1: Set $\kappa \leftarrow \kappa + 1$. Set the price increase at

$$(x^{(\kappa)}, y^{(\kappa)}) \leftarrow \left(\frac{x_l^{(\kappa)} + x_u^{(\kappa)}}{2}, \frac{y_l^{(\kappa)} + y_u^{(\kappa)}}{2} \right) \quad (35)$$

Step 2: Observe $X^{(\kappa)}$ and $Y^{(\kappa)}$. If $Q - \varepsilon_1 \leq X^{(\kappa)} \leq Q + \varepsilon_1$ and $Q - \varepsilon_1 \leq Y^{(\kappa)} \leq Q + \varepsilon_1$, $(x^{(\kappa)}, y^{(\kappa)})$ is an approximately optimal solution and stop.

Step 3: Remove the infeasible regions of the domain accordingly to the above analysis and define the domain $[x_l^{(\kappa+1)}, x_u^{(\kappa+1)}] \times [y_l^{(\kappa+1)}, y_u^{(\kappa+1)}]$, which is at most half the size of $[x_l^{(\kappa)}, x_u^{(\kappa)}] \times [y_l^{(\kappa)}, y_u^{(\kappa)}]$. Go to Step 1.

Suppose that the unit of price is 1 cent. In practice, the prices can only be an integer number of cents. Algorithm 1 can be slightly modified as:

Algorithm 2. *A trial-and-error algorithm for practical implementation.*

Step 0: Set the iteration number $\kappa = 0$. Set the domain at $(x, y) \in [x_l^{(1)}, x_u^{(1)}] \times [y_l^{(1)}, y_u^{(1)}] := [0, \bar{x}] \times [0, \bar{y}]$. Set the error gap $\varepsilon_1 > 0$.

Step 1: Set $\kappa \leftarrow \kappa + 1$. Set the price increase at

$$(x^{(\kappa)}, y^{(\kappa)}) \leftarrow \left(\left\lceil \frac{x_l^{(\kappa)} + x_u^{(\kappa)}}{2} \right\rceil, \left\lceil \frac{y_l^{(\kappa)} + y_u^{(\kappa)}}{2} \right\rceil \right) \tag{36}$$

where $\lceil a \rceil$ represents the ceiling function of a, i.e., the smallest integer not smaller than a.

Step 2: Observe $X^{(\kappa)}$ and $Y^{(\kappa)}$. If $Q - \varepsilon_1 \le X^{(\kappa)} \le Q + \varepsilon_1$ and $Q - \varepsilon_1 \le Y^{(\kappa)} \le Q + \varepsilon_1$, $(x^{(\kappa)}, y^{(\kappa)})$ is an approximately optimal solution and stop.

Step 3: (i) If $x_u^{(\kappa)} - x_l^{(\kappa)} = 1$ and $y_u^{(\kappa)} - y_l^{(\kappa)} = 1$, $(x^{(\kappa)}, y^{(\kappa)})$ is an approximate optimal solution and stop. (ii) Case 1: If $x_u^{(\kappa)} - x_l^{(\kappa)} = 1$ and $X^{(\kappa)} + Y^{(\kappa)} > 2Q$, define the domain $[x_l^{(\kappa+1)}, x_u^{(\kappa+1)}] \times [y_l^{(\kappa+1)}, y_u^{(\kappa+1)}] = [x_l^{(\kappa)}, x_u^{(\kappa)}] \times [y^{(\kappa)}, y_u^{(\kappa)}]$, and go to Step 1. Case 2: If $x_u^{(\kappa)} - x_l^{(\kappa)} = 1$ and $X^{(\kappa)} + Y^{(\kappa)} < 2Q$, define the domain $[x_l^{(\kappa+1)}, x_u^{(\kappa+1)}] \times [y_l^{(\kappa+1)}, y_u^{(\kappa+1)}] = [x_l^{(\kappa)}, x_u^{(\kappa)}] \times [y_l^{(\kappa)}, y^{(\kappa)}]$, and go to Step 1. Case 3: If $x_u^{(\kappa)} - x_l^{(\kappa)} = 1$ and $2Q - 2\varepsilon_1 < X^{(\kappa)} + Y^{(\kappa)} < 2Q + 2\varepsilon_1$, $(x^{(\kappa)}, y^{(\kappa)})$ is an approximate optimal solution and stop. (iii) Case 1: If $y_u^{(\kappa)} - y_l^{(\kappa)} = 1$ and $X^{(\kappa)} + Y^{(\kappa)} > 2Q$, define the domain $[x_l^{(\kappa+1)}, x_u^{(\kappa+1)}] \times [y_l^{(\kappa+1)}, y_u^{(\kappa+1)}] = [x^{(\kappa)}, x_u^{(\kappa)}] \times [y_l^{(\kappa)}, y_u^{(\kappa)}]$, and go to Step 1. Case 2: If $y_u^{(\kappa)} - y_l^{(\kappa)} = 1$ and $X^{(\kappa)} + Y^{(\kappa)} < 2Q$, define the domain $[x_l^{(\kappa+1)}, x_u^{(\kappa+1)}] \times [y_l^{(\kappa+1)}, y_u^{(\kappa+1)}] = [x_l^{(\kappa)}, x^{(\kappa)}] \times [y_l^{(\kappa)}, y_u^{(\kappa)}]$, and go to Step 1. Case 3: If $y_u^{(\kappa)} - y_l^{(\kappa)} = 1$ and $2Q - 2\varepsilon_1 < X^{(\kappa)} + Y^{(\kappa)} < 2Q + 2\varepsilon_1$, $(x^{(\kappa)}, y^{(\kappa)})$ is an approximate optimal solution and stop. (iv) If $x_u^{(\kappa)} - x_l^{(\kappa)} \ge 2$ and $y_u^{(\kappa)} - y_l^{(\kappa)} \ge 2$, remove the infeasible regions of the domain according to the above analysis (i.e., Table 1 and 2), define the domain $[x_l^{(\kappa+1)}, x_u^{(\kappa+1)}] \times [y_l^{(\kappa+1)}, y_u^{(\kappa+1)}]$, and go to Step 1.

For Algorithm 2, we have the following properties.

Theorem 4: In the worst case, we can find an ε-optimal solution in at most $\lceil \log_2 \bar{x} \rceil + \lceil \log_2 \bar{y} \rceil$ iterations.

Theorem 4 implies, if $\bar{x} = \bar{y} = 1000$ cents (10 dollars), then in the worst case we need 20 iterations.

6 Illustrative example

We test the effectiveness of the proposed trial-and-error schemes using an illustrative example. The test is performed using a PC with Intel® Core™ i7-3770 CPU (3.40 GHz) and 8 GB RAM. Related parameters used in the example are described as follows.

Suppose the dwell time designed for the train line is 15 seconds at each station, on condition that the actual demands of each station match the designed boarding/alighting ability. Concerning that each train has 8 carriages with 3 doors per carriage, and approximately 3 passengers can board or alight per second per door, thus the boarding/alighting ability is considered as at most $Q = 15 \times 8 \times 3 \times 2 = 720$ passengers can board or alight at a station within the designed dwell time. If the actual demand over the designed boarding/alighting ability, the dwell time will not keep fixed and depend on the boarding/alighting passenger volume (Seriani and Fernández, 2015). Over-long dwell time of trains cause poor performance of the entire train system, influencing the headway and leading to inefficiency.

Currently, stations other than S_1 and S_2 are uncongested that still have constant dwell times equaling 15 seconds, while dwell times of the two congested stations S_1 and S_2 located at CBD blow out (over 15 seconds) due to the surge demands of boarding/alighting passengers (q_1 and q_2). In detail, a total of $q_1 = 800$ passengers per train desire to board or alight at S_1 with three categories $a_1 = 200$, $b_1 = 400$, $c_1 = 200$, and a total of $q_2 = 850$ passengers desire to board or alight at S_2 with three categories $a_2 = 200$, $b_2 = 400$, $c_2 = 250$.

Recall model [M2], the objective is to reduce the currently high boarding/alighting demands (q_1 and q_2) of the two congested train stations (S_1 and S_2) by price surcharges (x and y), such that each demand can match the boarding/alighting ability corresponding to Q passengers. If the boarding/alighting passengers are well controlled to Q, the dwell time can be fixed at 15 seconds as designed. The unknown demand functions are used to simulate the responds of passengers after each trial of pricing surcharges, described as follows, where $\bar{x}$ and $\bar{y}$ indicates that the transport authority imposes upper bounds of fare increases as $\bar{x} = \bar{y} = 3$ dollars.

Unknown demand functions:
$f_1(x) = \exp(-x)$, $0 \le f_1(x) \le 1, x \in [0, \bar{x}]$.
$h_1(x-y) = \begin{cases} 1, & -\bar{y} \le x-y \le 0 \\ 1-(x-y)^2/\bar{x}^2, & 0 < x-y \le \bar{x}. \end{cases}$
$f_2(x) = \exp(-0.5x)$, $0 \le f_2(x) \le 1, x \in [0, \bar{x}]$.
$h_2(y-x) = \begin{cases} 1, & -\bar{x} \le y-x \le 0 \\ 1-(y-x)^2/(2\bar{y}^2), & 0 < y-x \le \bar{y}. \end{cases}$

We first apply Algorithm 1 to solve the above example. The error gap is set at $\varepsilon_1 = 1$, indicating once $Q - \varepsilon_1 \le X(x^{(\kappa)}, y^{(\kappa)}) \le Q + \varepsilon_1$ and $Q - \varepsilon_1 \le Y(x^{(\kappa)}, y^{(\kappa)}) \le Q + \varepsilon_1$ are both satisfied in iteration κ, we consider the algorithm has reached the optimum with the Pareto-optimal solutions $x^{(\kappa)}$ and $y^{(\kappa)}$. The trial-and-error procedure are reported in Figure 4 and Table 3.

In Figure 4, the trialed price increases of stations S_1 and S_2 in each iteration κ are presented by lines $x^{(\kappa)}$ and $y^{(\kappa)}$, respectively. After each trial, the realized boarding/alighting passenger volumes of station S_1 and S_2 are observed and presented by lines $X^{(\kappa)}$ and $Y^{(\kappa)}$. Figure 4 presents that both line $X^{(\kappa)}$ and $Y^{(\kappa)}$ are convergent to the target volume Q after 12 iterations, indicated by $Q - \varepsilon_1 \le X^{(12)} \le Q + \varepsilon_1$ and $Q - \varepsilon_1 \le Y^{(12)} \le Q + \varepsilon_1$, with the corresponding fare increase $x^{(12)}$ and $y^{(12)}$ identified as the Pareto-optimal solutions, according to Theorem 3.

In Table 3, more details of the convergence procedure are reported. Columns $x^{(\kappa)}$, $y^{(\kappa)}$, $X^{(\kappa)}$ and $Y^{(\kappa)}$ in Table 3 are consistent with lines $x^{(\kappa)}$, $y^{(\kappa)}$, $X^{(\kappa)}$ and $Y^{(\kappa)}$ in Figure 4, followed by columns $x_l^{(\kappa)}$, $x_u^{(\kappa)}$ and columns $y_l^{(\kappa)}$, $y_u^{(\kappa)}$, which represent the lower bound, upper bound of $x^{(\kappa)}$ and $y^{(\kappa)}$, respectively. The convergence can be reflected by the iteratively smaller feasible regions of $x^{(\kappa)} \in (x_l^{(\kappa)}, x_u^{(\kappa)})$ and $y^{(\kappa)} \in (y_l^{(\kappa)}, y_u^{(\kappa)})$ along with the progress of iterations. In each iteration κ, certain infeasible regions of the domains of $x^{(\kappa)}$ and $y^{(\kappa)}$ can be concluded and eliminated, by observing the realized boarding/alighting passenger volume $X^{(\kappa)}$ and $Y^{(\kappa)}$. The infeasible regions identified by each trial κ is pointed out by the column "Case" in Table 3, consistent with cases illustrated in Table 1 and 2 in previous section.

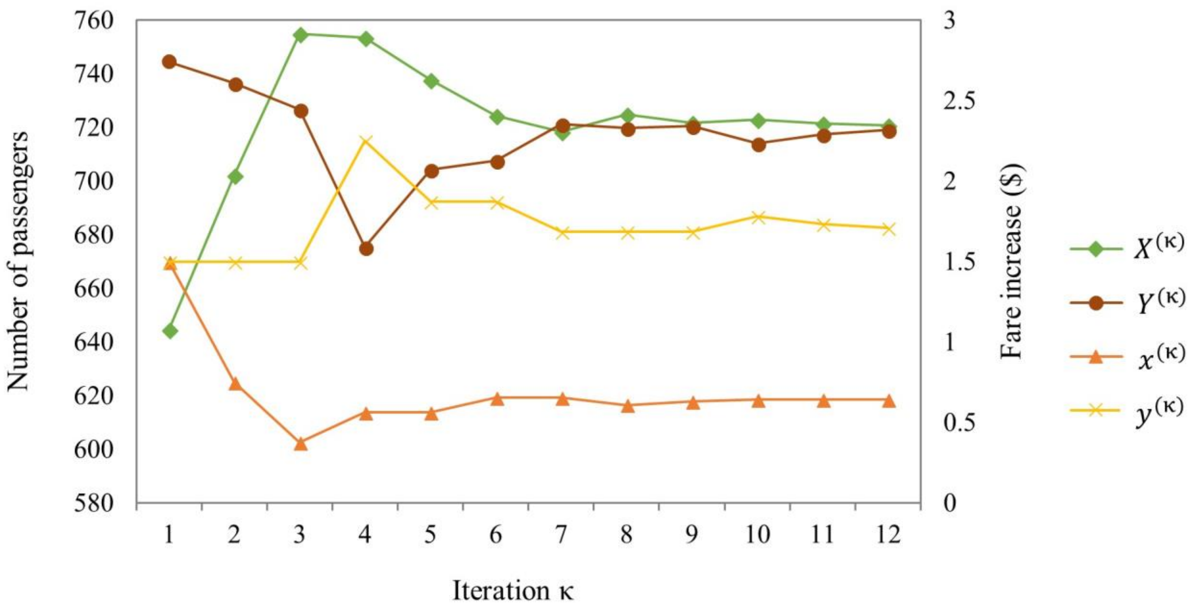


Figure 4. Fare increases and the observed numbers of boarding/alighting passengers using Algorithm 1.

Table 3. Results for Algorithm 1.

κ	$x^{(\kappa)}$	$y^{(\kappa)}$	$X^{(\kappa)}$	$Y^{(\kappa)}$	$x_l^{(\kappa)}$	$x_u^{(\kappa)}$	$y_l^{(\kappa)}$	$y_u^{(\kappa)}$	Case
1	1.5	1.5	644.626	744.473	0	3	0	3	(vi)
2	0.75	1.5	702.286	736.661	0	1.5	0	3	(vi)
3	0.375	1.5	755.036	726.895	0	0.75	0	3	(i)
4	0.5625	2.25	753.507	675.380	0.375	0.75	1.5	3	(iv)
5	0.5625	1.875	737.882	704.395	0.375	0.75	1.5	2.25	(iii)
6	0.65625	1.875	724.389	707.691	0.5625	0.75	1.5	2.25	(iv)
7	0.65625	1.6875	718.529	721.248	0.5625	0.75	1.5	1.875	(vi)
8	0.609375	1.6875	724.882	719.875	0.5625	0.65625	1.5	1.875	(iii)
9	0.6328125	1.6875	721.669	720.569	0.609375	0.65625	1.5	1.875	(i)
10	0.6445313	1.78125	722.928	714.134	0.6328125	0.65625	1.6875	1.875	(iv)
11	0.6445313	1.734375	721.478	717.530	0.6328125	0.65625	1.6875	1.78125	(iv)
12	0.6445313	1.7109375	720.776	719.222	0.6328125	0.65625	1.6875	1.734375	(optimal)

We then apply Algorithm 2 to solve this example, in which the unit of price is set as 0.01 dollar such that more consistent with the actual situation. The error gap is still set as $\varepsilon_1 = 1$. The results, reported in Figure 5 and Table 4, demonstrate that the optimal solution can be identified after 13 iterations by Algorithm 2.

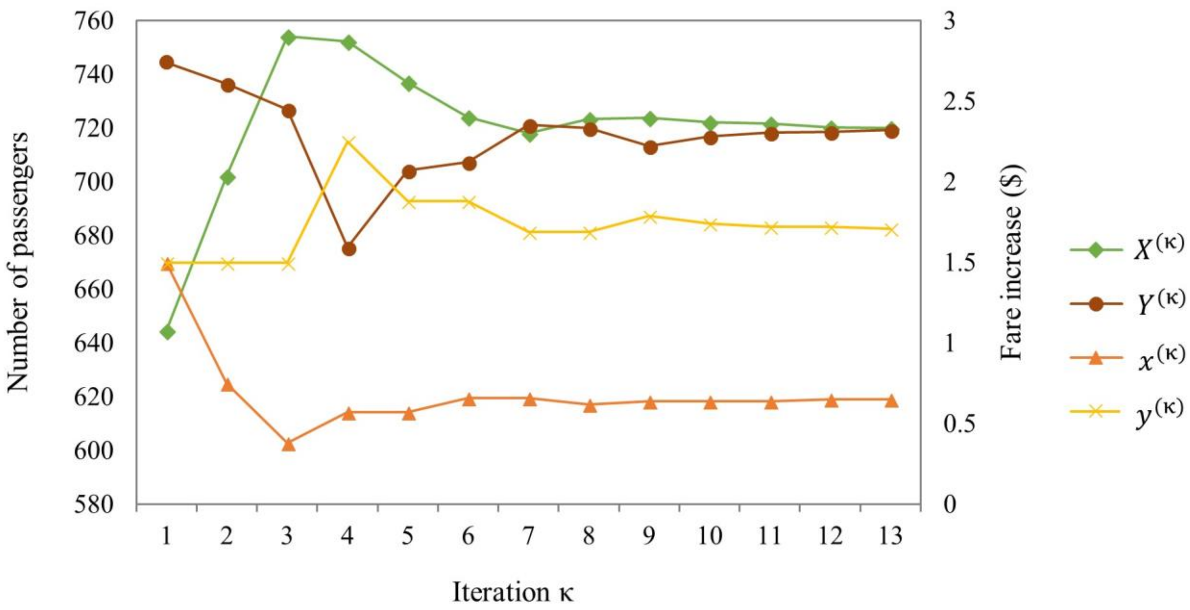


Figure 5. Fare increases and the observed numbers of boarding/alighting passengers using Algorithm 2.

Table 4. Result for Algorithm 2.

κ	$x^{(\kappa)}$	$y^{(\kappa)}$	$X^{(\kappa)}$	$Y^{(\kappa)}$	$x_l^{(\kappa)}$	$x_u^{(\kappa)}$	$y_l^{(\kappa)}$	$y_u^{(\kappa)}$	Case
1	1.5	1.5	644.626	744.473	0	3	0	3	(vi)
2	0.75	1.5	702.286	736.661	0	1.5	0	3	(vi)
3	0.38	1.5	754.195	727.051	0	0.75	0	3	(i)
4	0.57	2.25	752.305	675.730	0.38	0.75	1.5	3	(iv)
5	0.57	1.88	736.940	704.291	0.38	0.75	1.5	2.25	(iii)
6	0.66	1.88	724.042	707.453	0.57	0.75	1.5	2.25	(iv)
7	0.66	1.69	718.105	721.177	0.57	0.75	1.5	1.88	(vi)
8	0.62	1.69	723.490	720.010	0.57	0.66	1.5	1.88	(i)
9	0.64	1.79	723.827	713.353	0.62	0.66	1.69	1.88	(iv)
10	0.64	1.74	722.264	716.985	0.62	0.66	1.69	1.79	(iv)
11	0.64	1.72	721.658	718.432	0.62	0.66	1.69	1.74	(iii)
12	0.65	1.72	720.311	718.731	0.64	0.66	1.69	1.74	(iv)
13	0.65	1.71	720.015	719.451	0.64	0.66	1.69	1.72	(optimal)

Public acceptability is one of the major concerns of fare increase strategy to alleviate congestion, that is the reason why we minimize both fare increases of the two stations in the bi-objective in model [M1]. At the same time, if the fare increases differ significantly between the two stations, which are both congested, both located in CBD and indeed close to each other, it may be difficultly accepted by the public. To develop more acceptable pricing schemes, a new constraint $|x-y| \le \alpha$ can be added in model [M1] to control the fare increases difference between the two stations within α dollars. Sensitivity analyses regarding α are conducted, taking 0.05 dollars as an interval. The other parameter settings are the same as the above-mentioned example.

Figure 6 shows the result of sensitivity analyses of α. Lines $x_{(\alpha)}$, $y_{(\alpha)}$, $X_{(\alpha)}$ and $Y_{(\alpha)}$ in Figure 6 are consistent with columns $x_{(\alpha)}$, $y_{(\alpha)}$, $X_{(\alpha)}$ and $Y_{(\alpha)}$ in Table 5. By solving model [M1] with the newly added constraint $|x-y| \le \alpha$, in which α represents a given level of fare increases differences, the optimal fare increases ($x_{(\alpha)}$ and $y_{(\alpha)}$) and the corresponding boarding/alighting passenger volume ($X_{(\alpha)}$ and $Y_{(\alpha)}$) can be obtained. Added with $|x-y| \le \alpha$, model [M1] cannot transfer to model [M2], such that only inequalities $X_{(\alpha)} \le Q$ and $Y_{(\alpha)} \le Q$ are satisfied.

Figure 7 illustrates the constitutions of $X_{(\alpha)}$ and $Y_{(\alpha)}$ referring to each category, consistent with the details reported by the last four columns of Table 5, recalling that we have classified q_1 (q_2) passengers whose ideal boarding/alighting station is S_1 (S_2) into three categories with a_1, b_1 and c_1 (a_2, b_2 and c_2) passengers. In Table 5, column $a_1 \to X_{(\alpha)}$ ($a_2 \to Y_{(\alpha)}$) refers to how many of a_1 (a_2) passengers still use station S_1 (S_2) after fare increases. Column $b_1 \to X_{(\alpha)}$ ($b_2 \to Y_{(\alpha)}$) is not reported in the Table 5, since all of b_1 (b_2) passengers stick to use station S_1 (S_2) as previous assumptions. Column $c_1 \to X_{(\alpha)}$ is also omitted, since $x_{(\alpha)} \le y_{(\alpha)}$ holds for each α, such that all of c_1 passengers will not switch to S_2 and still use S_1. Column $c_2 \to X_{(\alpha)}$ refers to how many of c_2 passengers switch from S_2 to S_1, while column $c_2 \to Y_{(\alpha)}$ refers to how many of c_2 passengers still use S_2 after increased fares.

Further explanations based on Table 5 are as follows. First, when parameter α is set as zero, both stations have to increase 2.08 dollars, meanwhile the passenger boarding/alighting volume of station S_2 is 720, but that of station S_1 is only 625. It illustrates the inevitable efficiency loss when fairness of public strategy is imposed. To guarantee the same fare increases of S_1 and S_2, fare increase of the less congested station S_1 (station S_1 is less congested than station S_2) is much higher than the reasonable value, leading to underutilization at station S_1. Second, when $x_{(\alpha)}$ decreases from 2.08 to 0.64 dollars, the boarding/alighting passenger volume of station S_1 increases from 625 to 720, that is mostly contributed by the category with a_1 passengers, as shown in column $a_1 \to X_{(\alpha)}$. Third, when $y_{(\alpha)}$ decreases from 2.08 to 1.69 dollars, the boarding/alighting passenger volume of station S_2 maintains on 720, that is mainly because S_2 is more congested than S_1. Finally, an interesting phenomenon is that both surcharges will decline by enlarging α, until $x_{(1.1)} = 0.64$ and $y_{(1.1)} = 1.69$, which are the Pareto-optimal solutions of model [M1] without the newly added constraint $|x - y| \le \alpha$.

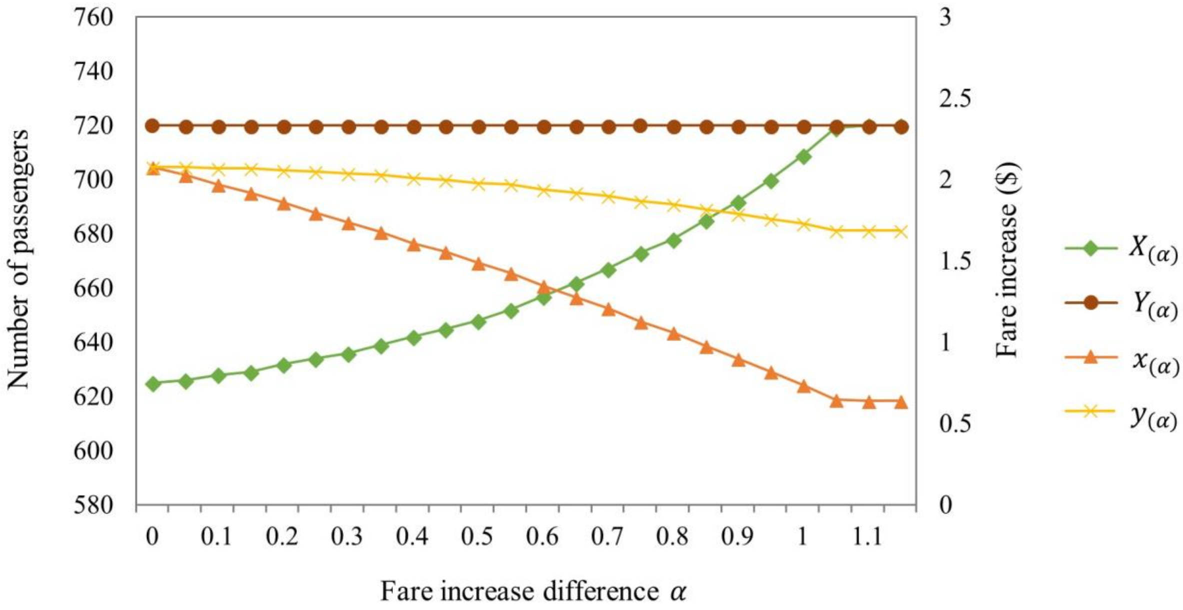


Figure 6. Optimal fare increases and the numbers of boarding/alighting passengers of each fare increase difference α.

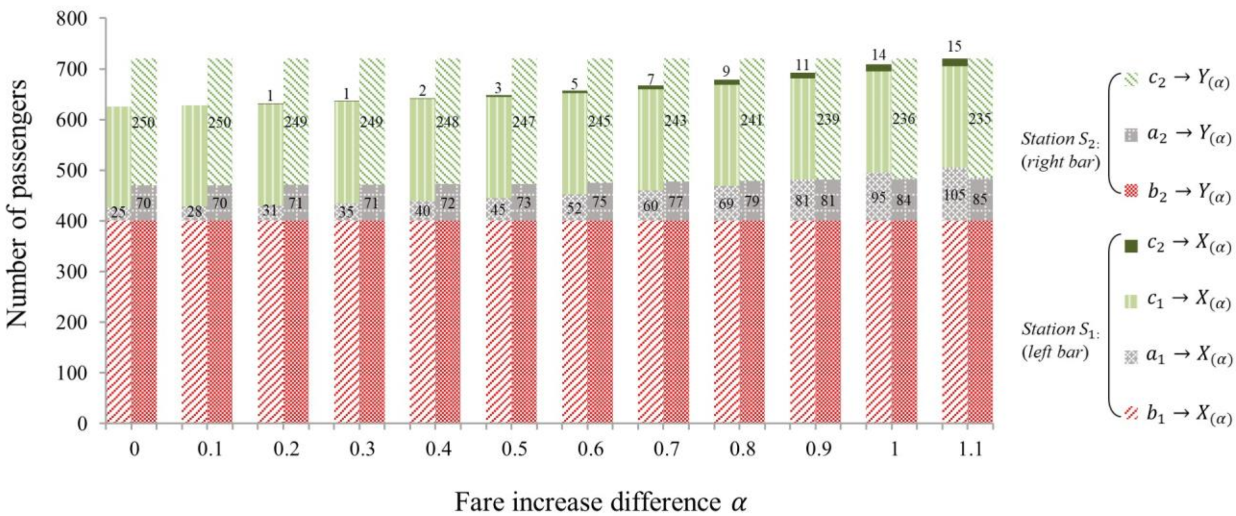


Figure 7. Constitutions of boarding/alighting passengers of each fare increase difference α.

Table 5. Sensitivity analyses of α .

α	$x_{(\alpha)}$	$y_{(\alpha)}$	$X_{(\alpha)}$	$Y_{(\alpha)}$	$a_1 \to X_{(\alpha)}$	$c_2 \to X_{(\alpha)}$	$a_2 \to Y_{(\alpha)}$	$c_2 \to Y_{(\alpha)}$
0.00	2.08	2.08	625	720	25	0	70	250
0.05	2.03	2.08	626	720	26	0	70	250
0.10	1.97	2.07	628	720	28	0	70	250
0.15	1.92	2.07	629	720	29	0	70	250
0.20	1.86	2.06	632	720	31	1	71	249
0.25	1.80	2.05	634	720	33	1	71	249
0.30	1.74	2.04	636	720	35	1	71	249
0.35	1.68	2.03	639	720	37	2	72	248
0.40	1.61	2.01	642	720	40	2	72	248
0.45	1.56	2.00	645	720	42	3	73	247
0.50	1.49	1.98	648	720	45	3	73	247
0.55	1.43	1.97	652	720	48	4	74	246
0.60	1.35	1.94	657	720	52	5	75	245
0.65	1.28	1.92	662	720	56	6	76	244
0.70	1.21	1.90	667	720	60	7	77	243
0.75	1.13	1.87	673	720	65	8	78	242
0.80	1.06	1.85	678	720	69	9	79	241
0.85	0.98	1.82	685	720	75	10	80	240
0.90	0.90	1.79	692	720	81	11	81	239
0.95	0.82	1.76	700	720	88	12	82	238
1.00	0.74	1.73	709	720	95	14	84	236
1.05	0.65	1.69	719	720	104	15	85	235
1.10	0.64	1.69	720	720	105	15	85	235
1.15	0.64	1.69	720	720	105	15	85	235

7 Conclusion and discussion

This study addresses the problem of congested CBD train stations along a subway line having boarding/alighting difficulty due to large volumes of boarding/alighting commuters. With consideration that spreading peak demand over time is difficult for commuters with inflexible working time, we propose a method to spread peak demand to a wider set of train stations. By fare differentiation of congested and uncongested train stations, passengers originally boarding/alighting at the congested stations will shift to the neighboring uncongested train stations. This work is to determine the optimal fare increases of the congested stations to decrease an appropriate number of boarding/alighting passengers. We prove that both fare increases of the stations are minimized in the unique Pareto-optimal solution. A trial-and-error scheme is proposed in this work to deal with unknown demand functions. Each observation in a trial gives information to the next trial of fare design until reaching the optimal fare increases. After a series of iterations, the optimal solution can be achieved and identified. Experiments are conducted to test the effectiveness of the trial-and-error scheme based on a case of two congested stations, and results show that the optimal solution can be obtained after a few trials. If the fare increase difference between the two stations is controlled to be within a certain amount for equity consideration, the fare increases at the two stations will be higher than those in the Pareto-optimal solution and one station may be underutilized.

The trial-and-error implementations are mostly involved in road congestion pricing problem in existing literature, which consists in two categories (i) to achieve a system optimum (SO), and (ii) is to levy tolls such that the volume of traffic flow or passengers does not exceed the predetermined threshold. Our research is analogous to the target in category (ii), that is, to reduce passenger (boarding or alighting) number to below a desirable target level, while it is also a minimization problem as studies in category (i) in that we minimize fare-increases of each congestion train station (two train stations in this work). The number of boarding/alighting passengers of the congested stations can be observed after trial of each train fare increases and the optimal fare increase can be achieved after a series of trial-and-error iterations. Pareto-optimal solution exists in this problem, which is considered as the minimum increased fare of each bus station meanwhile satisfying the boarding/alighting passenger capacity constraints and is different from Pareto-improving congestion pricing discussed by Guo and Yang (2010), in which every user is better off compared with the untolled case.

Although road congestion pricing is similar with this study in that both aim at alleviating overcrowding phenomena, fundamental differences exist. Three basic elements are frequently considered for road congestion pricings—the demand functions, speed-flow relationships and the users' value of times (or generalized cost). Road congestion pricing may consider interaction of different vehicle types and interdependence of demands in different time periods (Xu et al., 2013). The commonly used road congestion pricing scheme is the marginal-cost toll scheme, in which each network user should pay the toll offsetting the congestion effect due to its presence (Yang and Huang, 1998; Yang, 1999). Nevertheless, the scheme of this article is to spread travel demand from the congested boarding stations to the neighboring uncongested stations by boarding/alighting congestion pricing. This study concentrates on train stations, so that complex traffic flow situations or link travel time functions on the road are not considered. It aims at finding a set of minimized fare increases to reduce passenger demands of congested train stations (as origins or destinations) to below a desirable target level. Delays are avoided, and punctuality can be guaranteed if the number of boarding/alighting passengers are well-controlled in the congested train stations.

Certain limitations are involved in this work which need future study. First, this work aims at exploring a generalized method to solve a certain type of boarding/alighting congestion problem, while field research or a real case study based on existing stations is not included. Second, the solvable type of boarding/alighting congestion problem of this study refers to the situation in which the congested and uncongested stations are very close, such that only small influence will exert to passengers' work trips if passenger shift from the congested stations to the uncongested ones. The proposed approach of spreading demand to a wider set of stations is not applicable for stations far from each other. Third, the proposed method cannot deal with extreme cases, in which even the upper bound of the fare surcharges $\bar{x}$ and $\bar{y}$ are implemented, the inequalities of the proposed model still cannot be met. The extreme case may attribute to the high proportion of passengers of categories b_1 and b_2, who always stick to choose the congested stations, or the high proportion of categories c_1 and c_2, who only shift between the two congested stations but not shift to the uncongested ones. Fourth, two congested stations are handled simultaneously in this paper, along with two uncongested stations which are respectively closet to the two congested ones. The situation of one congested station at CBD can easily be addressed by the method proposed in this study. However, future research is expected referring to more complex situations, i.e., a larger number of congested and uncongested stations on the basis of more complicated rail networks.

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