Ship Routing and Scheduling Problem for Steel Plants Cluster alongside the Yangtze River

Abstract:

This paper aims to provide an optimal shipping schedule for steel-manufacturing plants cluster alongside the Yangtze River by developing a mixed integer programming formulation. The model takes into account the multi-layer shipping network on the Yangtze River, the dynamic iron ore price and transportation costs, and the timely demands of steel plants over multiple periods. By applying the proposed model to a real-world case, several interesting findings are observed, leading to further useful managerial insights. We also propose an exact solution approach based on a dynamic programming algorithm to a special case of this problem motivated by real-world practice.

Keywords: Ship routing and scheduling optimization, Bulk shipping, Dynamic programming algorithm, Scenario analysis

1. Introduction

China is the largest iron ore importer in the world since 2003 (World Steel Association). In 2016, it accounted for over two thirds of the world's iron ore imports (Review of Maritime Transport, 2017). More than 70% of the iron ore consumed in China is imported from Australia and Brazil. Since the 1980s, many steel-manufacturing plants have been built alongside the Yangtze River (Yang and Wang, 2017), and iron ore has become the most important cargo shipped along the Yangtze River. From 2014 onward, however, the Chinese steel industry suffered from production overcapacity, which led to a significant reduction in the profit rate of the steel plants. In response to this decrease in profit, a wave of steel plant consolidation started (Yang et al., 2017). For example, in 2016 the Baosteel group acquired the Wuhan Steel Group, making it the largest steel corporation in China. Prior to this, Baosteel had already acquired Meishan Steel Plant (Nanjing) and Ningbo Steel Plant in 1998 and 2009, respectively.

The demands of these plants for iron ore are all met by transporting the iron ore along the Yangtze River. Due to a ship's draft limitation, the Yangtze River can be divided into three segments, as shown in Figure 1. Ningbo-Zhoushan port, which is located outside of the Yangtze River, can receive large iron ore ships, such as Capesize ships (100,000 to 400,000 dead weight tonne, dwt). Baosteel Group has its own dedicated iron ore terminal in Ningbo-Zhoushan port. Shanghai port, which is located at the mouth of the Yangtze River, can receive Panamax ships (60,000 to 100,000 dwt). 50,000 dwt ships can sail from Shanghai to Nanjing, but in the upper reaches of the river above Nanjing, only 5,000 dwt ships are allowed to carry iron ore (Yang et al., 2017).

In view of this background, when the logistics manager of Baosteel corporation makes an order and purchase plan, multiple factors need to be taken into consideration: first, the timely demand of each subordinate plant has to be met; second, the purchasing cost, transportation cost and storage cost need to be well balanced; third, the transportation scheme between shipping segments along the Yangtze River should be optimized. To achieve these objectives, in this paper we develop a mixed integer programming formulation to study an integrated transportation and inventory problem that considers the effects of both time and transshipment. Furthermore, we solve a special case motivated by real-world applications by using an optimal algorithm in polynomial time.

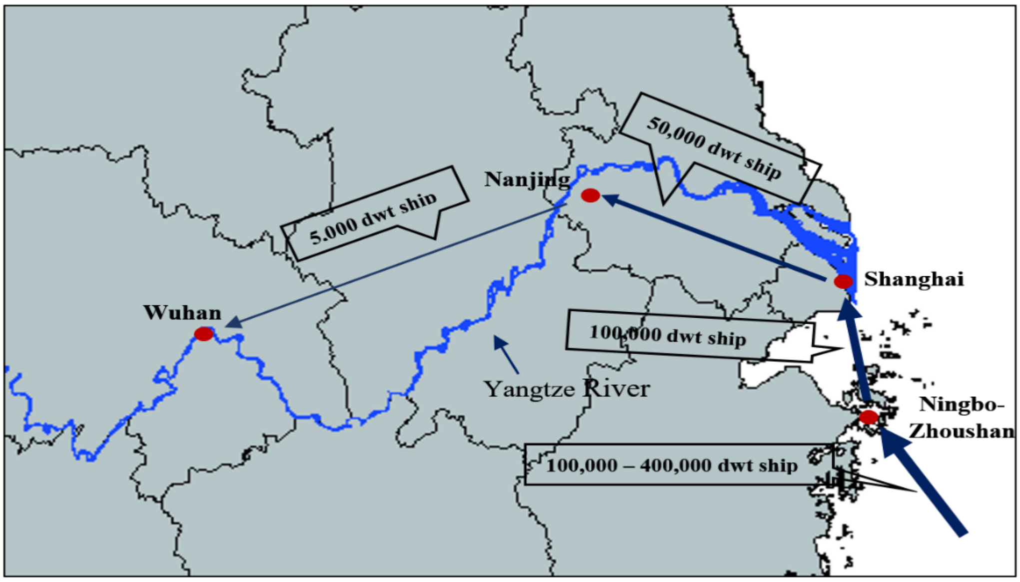

Figure 1. Shipping segments along the Yangtze River

The paper has both practical and theoretic contributions to the literature. Practically, under the background of steel manufacturing integration and Yangtze River Economic Belt strategy implementation, it is of significant importance to study the bulk shipping routing and scheduling problem for a giant steel firm, who integrated the steel manufacturers and owns a cluster of steel plants alongside the Yangtze River. We also propose various scenarios to explore the impact on iron ore trade volume from different source countries as the problem parameter varies with real data. Theoretically, we develop a mathematical model and a polynomial time algorithm to solve the iron ore transportation problem in inland river which has different shipping segments, taking dynamic purchase price and timely transportation over multiple periods into consideration.

This paper is organized into five sections. Section 2 provides a comprehensive literature review. The notation and our proposed programming formulation for the problem are introduced in Section 3. Section 4 presents the data sources, scenario analysis and solution approach performance evaluation and Section 5 presents the conclusion.

2. Literature Review

The Yangtze River shipping and port system have attracted a lot of attentions from shipping research in recent years. Veenstra and Notteboom (2011) investigated the structure and development of the Yangtze River port system, and found the Yangtze River container ports are now going through a regionalization phase, with the process now moving from the lower Yangtze River to further upstream. Wang et al. (2018) explained that the Yangtze River system has evolved into a hierarchical and networking stage since 2000s. Yang et al. (2017) ascertained that a multilayered transshipment port system for iron ore shipping is formed and the iron ore transshipment is moving outward both for sea ports and river ports. Zheng and Yang (2016) found that each region of the Yangtze River (upstream, midstream and downstream) has at least one hub and that each hub is in the same region as its associated feeder ports, which supports the port regionalization trend.

The Shipping Network Design (SND) problem is one of classic tactical problems facing by shipping carriers. There is a rich literature focusing on SND problems but few in bulk shipping in the early stage. Ronen (1983) provided the first survey of researches on ship routing problems. Later in 1993, he further reviewed this problem, as well as ship scheduling problem (Ronen, 1993). Very few studies regarding bulk shipping are included in his review papers. Christiansen et al. (2004) extended the review on ship routing and scheduling problems between 1994 and 2004. They pointed out that little work has been done on bulk ship scheduling problems. Recently, Christiansen et al. (2013) continued to review research on ship routing and scheduling and related problems over the previous decade. Over the last decade, bulk shipping problems have gradually received more and more attention from academia. Generally, the studies in bulk shipping can be divided into two categories. The first category is to maximize profit by increasing quantities of cargos transported (e.g. Brønmo et al., 2007; Sharma and Jana, 2009; Brønmo et al., 2010; Korsvik and Fagerholt, 2010; Korsvik et al., 2011; Norstad et al., 2011; Hemmati et al., 2015; Wen et al., 2016). Among them, Korsvik et al. (2011) developed a large neighborhood search heuristic for a ship routing and scheduling problem with split loads in which each cargo can be transported by more than one ship. Hemmati et al. (2015) proposed a two-phase heuristic to solve a combined cargo routing and inventory management problem consisting of designing routes and schedules for ships. The second category concerns minimizing the cost (e.g., Bilgen and Ozkarahan, 2007; Persson and Göthe-Lundgren, 2005; Hennig et al., 2012; Agra et al., 2013; de Assis and Camponogara, 2016; Yang and Wang, 2017; Christiansen et al., 2017; Arslan and Papageorgiou, 2017). Thereinto, Bilgen and Ozkarahan (2007) considered a blending and shipping problem on a wheat supply chain involving the delivery of bulk products from loading ports to destination ports by different types of vessels. For the problem, the authors proposed a mixed-integer linear programming model to minimize the total cost of blending, loading, transportation, and inventory costs. de Assis and Camponogara (2016) proposed a mixed-integer linear programming formulation for an integrated problem of shuttle tanker scheduling, inventory management, and ship routing. Arslan and Papageorgiou (2017) developed a multi-stage stochastic programming approach to determine suitable fleet size, mix, and deployment strategy for an industrial bulk carrier. In particular, Yang and Wang (2017) is the only study which extended the SND problem to an inland river. To ascertain the potential development pattern of the Yangtze River bulk shipping system, they clarified the main transshipment patterns of bulk shipping, and optimized the bulk-shipping network with an optimization model.

We notice that there is some literature exploring tramp shipping routing and scheduling problems with time windows and dynamic price like our work. Various algorithms have been developed by them to solve the problems. de Armas et al. (2015) proposed a hybridization of a Greedy Randomized Adaptive Search Procedure and a Variable Neighborhood Search for a tramp shipping routing and scheduling problem with time windows. The algorithm is able to achieve the optimal results for many instances, demonstrating its good performance. Lalla-Ruiz et al. (2018) provided a mathematical model and a heuristic algorithm to solve a waterway ship scheduling problem in order to minimize ships' waiting time. In their problem, the aim is to schedule incoming and outgoing ships through different waterways for accessing or leaving the port. Li et al. (2017) solved an integrated production, inventory and delivery problem for a steel manufacturer considering the timely transportation. They proposed a combined column generation and tabu search heuristic algorithm that can find near optimal solutions in a reasonable computational time.

For the problem, based on the unique features of bulk transshipment in Yangtze River, we propose different solution approaches to solve the general problem and its special case. For a general problem which is very challenging, a mathematical model is developed. We also provide an exact approach based on a dynamic programming algorithm to obtain optimal solutions for a special case which is very close to real iron ore shipping network of Yangtze River. The proposed model could significantly improve the efficiency of the iron ore transportation along the Yangtze River based on our simulation using real data. The results also indicate that both solution approaches can solve the proposed problem in reasonable time and either of them has its advantages on certain cases. These solution approaches can be used or further developed to solve similar problems for cargo transshipment in inland river.

3. Problem Description and Formulation

3.1 Problem Description

Consider a steel manufacturer that has several plants. Each plant is located near a port, and hence we use port to refer to a plant. The steel manufacturer needs to coordinate the transportation of iron ore from suppliers to the ports. Let T be the length of the planning horizon consisting of T time periods and one time period is assumed to be one day, because basically, in bulk shipping industry, the ship operation is based on daily hire, $M = \{o_1, o_2, \ldots, o_m\}$ be the set of ports, and $N = \{s_1, s_2, \ldots, s_n\}$ be the set of suppliers from which the steel manufacturer purchases iron ore in the planning horizon, where m and n are the numbers of the ports and suppliers, respectively. The ports $o_1, o_2, \ldots, o_m$ are ordered according to their geographical sequence. For example, referring to Figure 2, o_1, o_2, o_3, and o_4 can be the ports located in Ningbo, Shanghai, Nanjing, and Wuhan, respectively. This problem has two unique characteristics. First, iron ore is transported directly from the suppliers $s_1, s_2, \ldots, s_n$ to port o_1, unloaded and then reloaded onto smaller ships that transport it from port o_1 to the other ports when necessary. Iron ore cannot be transported directly from the suppliers to ports $o_2, \ldots, o_m$ because the drafts of these ports are not sufficient. Second, because of the geographical locations of the ports, iron ore may be transported from port o_j to port o_k, for $j = 1, 2, \ldots, m - 1$, $k = j + 1, 2, \ldots, m$, but cannot be transported from port o_k to port o_j if $k > j$. Figure 2 gives an example to demonstrate the two characteristics of the problem. In the example, there are two suppliers and four ports. In Figure 2, "⟶" indicates that iron ore may be transported from its starting point to its end point.

For each supplier s_i, $i = 1, 2, \ldots, n$, the size of a ship used to transport ore to port o_1 is fixed, and its capacity is denoted by Q_i^0 tonnes (e.g., ships from Australia are of the size 200,000 dwt and ships from Brazil are of the size 400,000 dwt). The corresponding transportation time from supplier s_i to port o_1 is denoted as t_i^0. For each $i = 1, 2, \ldots, n$ and $t = 1, 2, \ldots, T$, let p_{it} denote the price per tonne of iron ore purchased from supplier s_i in period t and c_{it}^0 denote the transportation cost of a shipload of ore from s_i

to port o_1 in period t. Each port o_k, $k = 1, 2, \ldots, m$, has a given initial inventory, I_k^0, in its warehouse at time 0 and is required to have a given inventory of I_k^T at the end of the planning horizon. For each $t = 1, 2, \ldots, T$ and $k = 1, 2, \ldots, m$, let d_{kt} denote the demand (consumption of iron ore) of port o_k at time t (this demand must be met) and h_{kt} denote the holding cost per tonne in the warehouse of port o_k at the end of period t. The capacity of a ship that is used to transport ore from port o_j to port o_k, $j = 1, 2, \ldots,$ $m - 1$, $k = j + 1, 2, \ldots, m$, is Q_{jk}^1 tonnes and the corresponding transportation time is denoted as t_{jk}^1. The transportation cost of a shipload of ore from o_j to o_k is denoted as c_{jkt}^1 for $j = 1, 2, \ldots, m - 1$, $k = j + 1, 2, \ldots, m$, $t = 1, 2, \ldots, T$. Note that in this paper we assume that the transportation times between ports satisfy the triangle inequality, i.e., $t_{jk}^1 + t_{kl}^1 \geq t_{jl}^1$, for $j = 1, 2, \ldots, m - 2$, $k = j + 1, 2, \ldots, m - 1$, $l = k + 1, \ldots, m$.

Our objective is to find the amounts of iron ore transported from suppliers in each period and the amounts of iron ore transported between two ports in each period so as to minimize the total cost of purchasing, transportation, and holding, whilst at same time satisfying all the demands of the ports.

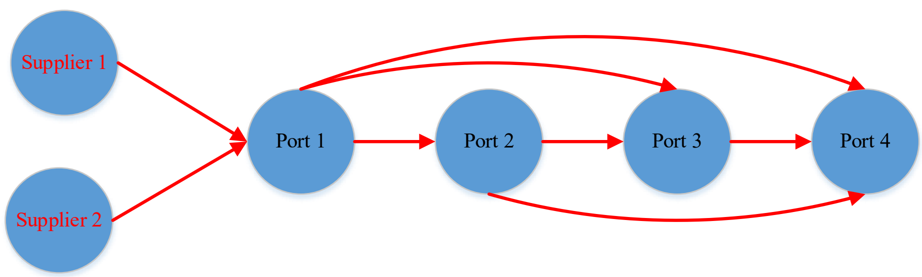

Figure 2. An example to show characteristics of the problem

Note that since, in practice the amounts of iron ore purchased by the ports from the suppliers are in tonnes, in this paper we assume that the parameters of the problems are all integer. Note that for each $k = 1, 2, \ldots, m$, if $I_k^T > 0$, then we can reset $d_{kT} + I_k^T$ to be the demand of port o_k at time T and 0 to be the last inventory of port o_k at time T. By the resetting operation, it is easy to see that the optimal solution of the problem is not changed and hence without loss of generality, we assume that $I_k^T = 0$.

3.2 Solution Approach for General Problem

In this section, we present a mathematical model for the general problem. Let decision variables x_{it}^0 be the amounts of iron ore started to be transported from supplier s_i to port o_1 in period t (the ore will arrive at port o_1 in period $t + t_i^0$), for $i = 1, 2, \ldots, n$ and $t = 1, 2, \ldots, T - t_i^0$, and decision variables y_{it}^0 be the number of ships required to transport the x_{it}^0 amount of ore, i.e., $y_{it}^0 Q_i^0 \geq x_{it}^0$. Let decision variables x_{jkt}^1 be the amounts of iron ore started to be transported from port o_j to port o_k in period t (the ore will arrive at port o_k in period $t + t_{jk}^1$) and decision variables y_{jkt}^1 be the number of ships used to transport the x_{jkt}^1 amounts of iron ore, i.e., $y_{jkt}^1 Q_{jk}^1 \geq x_{jkt}^1$, for $j = 1, 2, \ldots, m - 1$, $k = j + 1, 2, \ldots, m$, $t = 1, 2, \ldots, T - t_{jk}^1$. Finally, let decision variables I_{kt} denote the amounts of iron ore in the warehouse of port o_k at the end of period t, for $k = 1, 2, \ldots, m$, $t = 1, 2, \ldots, T$. For simplicity of notation, we assume that: (i) x_{it}^0 and y_{it}^0 exist with $x_{it}^0 = 0$ and $y_{it}^0 = 0$, for $i = 1, \ldots, n$, $t = T - t_i^0 + 1, \ldots, T$; and (ii) x_{jkt}^1 and y_{jkt}^1 exist with $x_{jkt}^1 = 0$ and $y_{jkt}^1 = 0$, for $j = 1, \ldots, m - 1$, $k = j + 1, \ldots, m$, $t = T - t_{jk}^1 + 1, \ldots, T$. For the problem, we provide an integer program, denoted as IP, to solve it to optimality.

IP $$\text{Min} \sum_{i=1}^{n}\sum_{t=1}^{T} x_{it}^0 p_{it} + \sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}^0 c_{it}^0 + \sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\sum_{t=1}^{T} y_{jkt}^1 c_{jkt}^1 + \sum_{k=1}^{m}\sum_{t=1}^{T} I_{kt} h_{kt} \quad (1)$$

S.T.

$$\sum_{i=1}^{n}\sum_{\tau=1}^{t-t_i^0} x_{i\tau}^0 + I_1^0 = \sum_{\tau=1}^{t} d_{1\tau} + I_{1t} + \sum_{k=2}^{m}\sum_{\tau=1}^{t} x_{1k\tau}^1, \text{ for } t = 1, \ldots, T. \quad (2)$$

$$\sum_{j=1}^{k-1}\sum_{\tau=1}^{t-t_{jk}^1} x_{jk\tau}^1 + I_k^0 = \sum_{\tau=1}^{t} d_{k\tau} + I_{kt} + \sum_{j=k+1}^{m}\sum_{\tau=1}^{t} x_{kj\tau}^1, \text{ for } k = 2, \ldots, m,\ t = 1, \ldots, T. \quad (3)$$

$$y_{it}^0 Q_i^0 \geq x_{it}^0, \text{ for } i = 1, \ldots, n, t = 1, \ldots, T. \quad (4)$$

$$y_{jkt}^1 Q_{jk}^1 \geq x_{jkt}^1, \text{ for } j = 1, \ldots, m - 1, k = j + 1, \ldots, m, t = 1, \ldots, T. \quad (5)$$

$x_{it}^0 \geq 0$ and integer, $y_{it}^0 \geq 0$ and integer, $x_{jkt}^1 \geq 0$ and integer, $y_{jkt}^1 \geq 0$ and integer, $I_{kt} \geq 0$, for $i = 1, \ldots, n$, $j = 1, \ldots, m$, $k = 1, \ldots, m$, $t = 1, \ldots, T$. (6)

In the objective function (1), the first term represents the total purchase cost of iron ore from the suppliers, the second term represents the total transportation cost from the suppliers to port o_1, the third term represents the total transportation cost between the ports, and the last term represents the total holding cost in the warehouses of the ports. Constraints (2) and (3) guarantee the flow balances of the first port and the other ports in each period, respectively. Constraints (4) calculate the number of the ships used to transport the cargo from the suppliers to port o_1 in each period. Constraints (5) calculate the number of the ships used to transport the cargo between the ports in each period. To guarantee the feasibility of the problem before solving it, we first present the following integer program, denoted as Feasibility Integer Program (FIP). To simplify the presentation, let $r = \min\{ t_i^0 \mid i = 1, 2, \ldots, n\}$ be the transportation time from the nearest supplier to port o_1 and $t_{kk}^1 = 0$, for $k = 1, \ldots, m$.

FIP min 0 (7)

S.T.

$$I_1^0 = \sum_{\tau=1}^{t} d_{1\tau} + I_{1t} + \sum_{k=2}^{m}\sum_{\tau=1}^{t} x_{1k\tau}^1, \text{ for } t = 1, \ldots, r. \tag{8}$$

$$\sum_{j=1}^{k-1}\sum_{\tau=1}^{t-t_{jk}^1} x_{jk\tau}^1 + I_k^0 = \sum_{\tau=1}^{t} d_{k\tau} + I_{kt} + \sum_{j=k+1}^{m}\sum_{\tau=1}^{t} x_{kj\tau}^1, \text{ for } k = 2, \ldots, m,\ t = 1, \ldots, \min(r + t_{1k}^1, T). \tag{9}$$

$$x_{jkt}^1 \geq 0 \text{ and integer}, I_{kt} \geq 0, \text{ for } j = 1, \ldots, m, k = 1, \ldots, m, t = 1, \ldots, \min(r + t_{1k}^1, T). \tag{10}$$

In (7), we let the objective value of FIP be 0, since the FIP is given to check the feasibility of the problem. Constraints (8) guarantee the flow balance of port o_1 in each period until iron ore purchased from the nearest supplier arrives at port o_1. Constraints (9) guarantee that the flow balance of each of the other ports in each period until iron ore transported from the nearest supplier arrives at the port (by transshipment at port o_1).

Proposition 1. For the problem, there exists a polynomial-time algorithm to check its feasibility.

Proof. To prove this proposition, we only need to show as follows that in (10) of FIP, $x_{jkt}^1 \geq 0$ and integer can be replaced by $x_{jkt}^1 \geq 0$. We first formulate FIP as a minimum cost flow problem in Ahuja et al. (1993) as follows. Let $G = (A, E)$ be a directed network in which the cost of every arc in E is zero and the capacity of every arc in E is unlimited. Node set A consists of $m + 1$ subsets of nodes. For $k =$

1, …, m, the k-th subset represents the k-th port and consists of the nodes, $(o_k, 1)$, $(o_k, 2)$, …, $(o_k, r + t_{1k}^1)$, where (o_k, t) represents port o_k in period t. Note that without loss of generality we assume that $r + t_{1k}^1 \le T$, and if not, then the k-th subset consists of the nodes, $(o_k, 1)$, $(o_k, 2)$, …, (o_k, T). The $(m + 1)$-th subset consists of a single node, $(0, T)$, which is a dummy node. Let d_{kt} be the demand of node (o_k, t) for $k = 1, …, m$, $t = 1, …, r + t_{1k}^1$ and 0 be the demand of node $(0, T)$. The flow into every node $(o_k, 1)$ is I_k^0, for $k = 1, …, m$. For $k = 1, …, m$, $t = 1, …, r + t_{1k}^1$, every node (o_k, t) is connected from node $(o_k, t - 1)$ with a flow $I_{k,t-1}$, and connected from node $(o_j, t - t_{jk}^1)$ with a flow $x_{jk,t-t_{jk}^1}^1$, for $j = 1, 2, …, k - 1$. Node $(0, T)$ is connected from node $(o_k, r + t_{1k}^1)$ with a flow $I_{k,r+t_{1k}^1}$, for $k = 1, …, m$. The flow coming out of node $(0, T)$ is $\sum_{k=1}^{m} I_k^0 - \sum_{k=1}^{m}\sum_{t=1}^{r+t_{1k}^1} d_{kt}$. This flow problem is to determine the flows $x_{jk,t-t_{jk}^1}^1$ from node $(o_j, t - t_{jk}^1)$ to node (o_k, t), the flows $I_{k,t-1}$ from node $(o_k, t - 1)$ to node (o_k, t), and the flows $I_{k,r+t_{1k}^1}$ from node $(o_k, r + t_{1k}^1)$ to node $(0, T)$, so that the total cost is minimized. To illustrate the above procedure, we give an example as follows. In the problem, there are three ports, five periods, $r = 2$, $t_{12}^1 = 2$, $t_{13}^1 = 3$, and $t_{23}^1 = 2$. By the above procedure, we can construct the directed network in Figure 3.

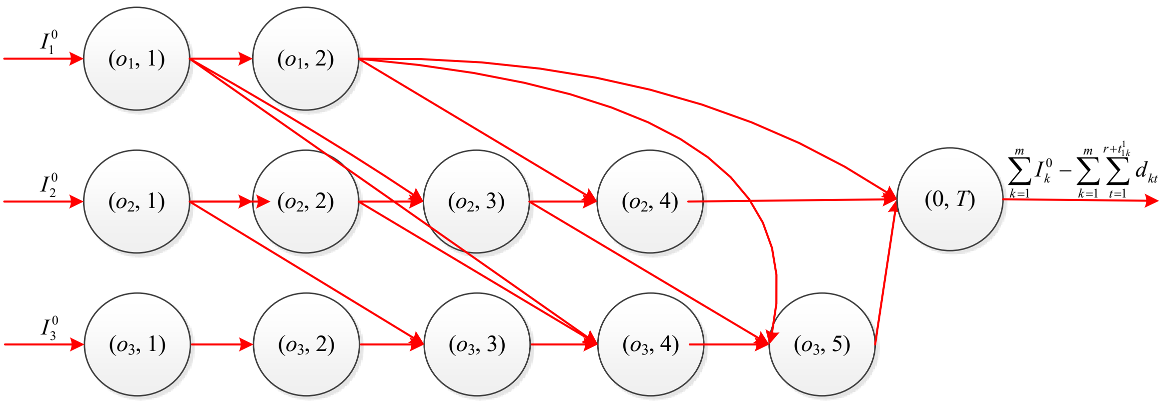

Figure 3. Constructed network for the given example

This minimum cost flow problem always has an integer minimum cost flow according to the integrity property (Theorem 9.10 in Ahuja et al., 1993). Thus, we can adopt an existing polynomial-time algorithm to obtain the optimal solution for the minimum cost flow problem. This implies that there also exists a polynomial-time algorithm to judge the feasibility of the problem considered in this paper. This completes the proof of Proposition 1. □

Proposition 2. If FIP is feasible, then the problem is also feasible. If not, then the problem is infeasible.

Proof. We first prove that if FIP has a feasible solution, then our problem is feasible. For FIP, suppose that x_{jkt}^{1} and I_{kt} form a feasible solution π for $j = 1, \ldots, m$, $k = 1, \ldots, m$, $t = 1, \ldots, \min(r + t_{1k}^{1}, T)$. Based on x_{jkt}^{1} and I_{kt}, we can construct a solution $\bar{\pi}$ for our problem as follows. For $j = 1, \ldots, m$, $k = 1, \ldots, m$, $t = 1, \ldots, \min(r + t_{1k}^{1}, T)$, $\bar{x}_{jkt}^{1} = x_{jkt}^{1}$. For $k = 2, 3, \ldots, m$, $\bar{x}_{1k,r+1}^{1} = \sum_{t=r+t_{1k}^{1}+1}^{T} d_{kt}$. Let $\bar{x}_{11}^{0} = \sum_{k=1}^{m} \sum_{t=r+t_{1k}^{1}+1}^{T} d_{kt}$ and the other decision variables $\bar{x}_{jkt}^{1}$ and $\bar{x}_{it}^{0}$ be 0. Based on $\bar{x}_{it}^{0}$ and $\bar{x}_{jkt}^{1}$, we can calculate the values of decision variables $\bar{y}_{it}^{0}$, $\bar{y}_{jkt}^{1}$, and $\bar{I}_{kt}$. It is easy to see that solution $\bar{\pi}$ is feasible for our problem.

Next, we prove that if FIP is infeasible, then our problem is also infeasible. Since the minimal transportation time from the suppliers to port o_1 is r, for the problem we have that (i) the earliest arrival time of iron ore at port o_1 from the suppliers is $r + 1$, and (ii) the earliest arrival time of iron ore at port o_k from port o_1 is $r + t_{1k}^{1} + 1$. Thus, for our problem we need to determine the flow problem from port o_j to ports $o_{j+1}, \ldots, o_m$ until iron ore purchased from the suppliers arrives at each port, for $j = 1, 2, \ldots, m - 1$. If FIP is infeasible, then we can conclude that iron ore in the warehouses of all the ports at time 0 does not satisfy all demands of the ports before the iron ore purchased from the suppliers arrives at these ports. Hence, the problem is also infeasible. □

From Propositions 1 and 2, we have that it is easy to solve FIP by a linear programming algorithm, and hence it is also easy to check the feasibility of the problem by checking the feasibility of FIP in polynomial time. In the remaining part of this paper, we assume that the problem always has a feasible solution.

Proposition 3. In (6), $x_{jkt}^{1} \geq 0$ and integer, can be replaced by the following inequality: $x_{jkt}^{1} \geq 0$.

Proof. Given the values of x_{it}^{0}, y_{it}^{0} and y_{jkt}^{1}, to prove this proposition, we give the following integer program, denoted as SIP, that is equivalent to IP:

$$\textbf{SIP} \quad \text{Min} \quad \sum_{k=1}^{m} \sum_{t=1}^{T} I_{kt} h_{kt}$$

S.T. Constraints (2) – (5),

$$x^1_{jkt} \geq 0 \text{ and integer}, I_{kt} \geq 0, \text{ for } j = 1, \ldots, m, k = 1, \ldots, m, t = 1, \ldots, T.$$

Similar to the proof of Proposition 1, we model SIP as a minimum cost flow problem as follows. Let $G = (A, E)$ be a directed network. Node set A consists of $m + 1$ subsets of nodes. For $k = 1, \ldots, m$, the k-th subset represents the k-th port and consists of the nodes, $(o_k, 1)$, $(o_k, 2)$, …, (o_k, T), where (o_k, t) represents port o_k in period t. The last subset consists of a single node, $(0, T)$. We associate with each node (o_k, t) a value d_{kt} which indicates its demand for $k = 1, \ldots, m$, $t = 1, \ldots, T$, and node $(0, T)$ value 0 which indicates its demand. The flow into every node $(o_k, 1)$ is I^0_k for $k = 1, \ldots, m$. The flow into every node (o_1, t) is $\sum_{k=1}^{n} x^0_{k,t-t^0_k}$ for $t = 1, \ldots, T$. For $t = 1, \ldots, T$, every node (o_1, t) is connected from node $(o_1, t - 1)$ with a flow $I_{1,t-1}$, unlimited capacity, and cost of $h_{1,t-1}$ per unit. For $k= 2, \ldots, m$, $t = 1, \ldots, T$, every node (o_k, t) is connected from node $(o_k, t - 1)$ with a flow $I_{k,t-1}$, unlimited capacity, and cost of $h_{k,t-1}$ per unit, and is connected from node $(o_j, t - t^1_{jk})$ with a flow $x^1_{jk,t-t^1_{jk}}$, the capacity of $y^1_{jk,t-t^1_{jk}} \cdot Q^1_{jk}$, and 0 unit cost, for $j = 1, 2, \ldots, k - 1$. Node $(0, T)$ is connected from node (o_k, T) with a flow I_{kT}, unlimited capacity, and cost of h_{kt} per unit, for $i = 1, \ldots, m$. The flow coming out of node $(0, T)$ is $\sum_{i=1}^{n}\sum_{t=1}^{T-t^0_i} x^0_{it} + \sum_{k=1}^{m} I^0_k - \sum_{k=1}^{m}\sum_{t=1}^{T} d_{kt}$. This flow problem is to determine the flows $x^1_{jk,t-t^1_{jk}}$ from node $(o_j, t - t^1_{jk})$ to node (o_k, t), the flow amounts $I_{k,t-1}$ from node $(o_k, t - 1)$ to node (o_k, t), and the flows I_{kT} from node (o_k, T) to node $(0, T)$, so that the total cost is minimized. By Theorem 9.10 in Ahuja et al. (1993), we have that this proposition holds. □

3.3 Solution Approach for a Special Case

In reality, the shipping process of iron ore is divided into foreign trade and domestic trade. The Baosteel group has invested heavily to Majishan port in Ningbo-Zhoushan to develop it into a transshipment hub for domestic trade transshipment since 2002. Now, Majishan port is the largest iron ore transshipment center in China and accounts for most of iron ore transshipment for Baosteel Group. If we consider a special case with Majishan port as a transshipment hub in the middle, we can design an exact algorithm to solve it. We assume the ships from the suppliers to port o_1 are fully loaded (i.e., $y^0_{it} Q^0_i = x^0_{it}$ in Eq. (4))

and ships between the ports are also full in order to reduce the transportation costs (i.e., $y^1_{jkt}Q^1_{jk} = x^1_{jkt}$ in Eq. (5)). In practice, the amounts of iron ore purchased from the suppliers are often large, and the cost of chartering a ship for transporting iron ore is often high. Therefore, to reduce transportation costs, the steel manufacturer often adopts a full-shipment strategy. The case represents a case where the directions from port o_1 to the other ports are different, or a case where the operating cost at port o_j for transporting iron ore from port o_j to port o_k is high for $j = 2, \ldots, m-1$, $k = j+1, \ldots, m$. We will provide a dynamic programming algorithm to solve the case to optimality. The structure of this case is shown in Figure 5.

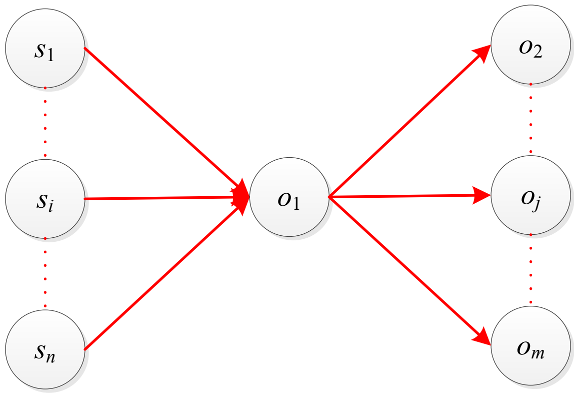

Figure 5. Structure of a case with full shipments and direct transportation from port o_1

To guarantee the feasibility of this case, the following inequalities must be satisfied:

$$\text{(i) } I_1^0 \geq \sum_{t=1}^{r} d_{1t} + \sum_{k=2}^{m}\left\lceil \max\left\{0, \sum_{t=1}^{r+t_{1k}^1} d_{kt} - I_k^0\right\} \Big/ Q_{1k}^1 \right\rceil \cdot Q_{1k}^1 \text{, and (ii) } I_k^0 \geq \sum_{t=1}^{t_{1k}^1} d_{kt} \text{, for } k = 2, \ldots, m.$$

We first focus on the total holding cost at port o_k, for $k = 2, \ldots, m$. For $k = 2, \ldots, m$, $t = 1, \ldots, T$, let l_{kt} be the amount of iron ore that started to be transported from port o_1 to port o_k in period t. Note that l_{kt} is actually the variable x^1_{1kt} defined previously. Therefore, the holding cost at port o_k at the end of period t is $\left(\sum_{\tau=1}^{t} l_{k,\tau-t_{1k}^1} + I_k^0 - \sum_{\tau=1}^{t} d_{k\tau}\right)h_{kt}$. Thus, for $k = 2, \ldots, m$, the total holding cost at port o_k in the planning horizon is

$$\sum_{t=1}^{T}\left(\sum_{\tau=1}^{t} l_{k,\tau-t_{1k}^1} + I_k^0 - \sum_{\tau=1}^{t} d_{k\tau}\right)h_{kt} = I_k^0 \cdot \sum_{t=1}^{T} h_{kt} - \sum_{t=1}^{T} d_{kt} \sum_{\tau=t}^{T} h_{k\tau} + \sum_{t=1}^{T} l_{k,t-t_{1k}^1} \sum_{\tau=t}^{T} h_{k\tau}$$

$$= I_k^0 \cdot \sum_{t=1}^{T} h_{kt} - \sum_{t=1}^{T} d_{kt} \sum_{\tau=t}^{T} h_{k\tau} + \sum_{t=1}^{T-t_{1k}^1} l_{kt} \sum_{\tau=t+t_{1k}^1}^{T} h_{k\tau} = I_k^0 \cdot \sum_{t=1}^{T} h_{kt} - \sum_{t=1}^{T} d_{kt} \sum_{\tau=t}^{T} h_{k\tau} + \sum_{t=1}^{T} l_{kt} \sum_{\tau=t+t_{1k}^1}^{T} h_{k\tau},$$

where the first term and the second term are fixed, and the last equality holds since if $t > T - t_{1k}^1$, then $l_{kt} = 0$.

For $t = 1, \ldots, T$, let z_{it}^0 be the total amount of iron ore from supplier s_i that has already arrived at port o_1 in the first t periods, for $i = 1, \ldots, n$, and z_{kt}^1 be the total amount of iron ore that started to be transported from port o_1 to port o_k in the first t periods, for $k = 2, \ldots, m$, $z_{kt}^1 = \sum_{\tau=1}^{t} l_{k\tau}$. Note that z_{it}^0 and z_{kt}^1 have different meanings: the amount z_{it}^0 refers to the ore that has arrived at port o_1, but z_{kt}^1 refers to the ore that has left port o_1 but may not yet have arrived at port o_k. First, we need to determine all possible values of z_{it}^0 and z_{kt}^1. A lower bound of z_{it}^0 is 0. To obtain an upper bound of z_{it}^0, we focus on a particular supplier s_i and assume that no ore is transported from the other suppliers. Since the transportation time from supplier s_i to port o_1 is t_i^0, the total amount of iron ore from supplier s_i to port o_1 for satisfying the total demand of port o_1 is at most $\sum_{t=t_i^0+1}^{T} d_{1t}$. For $k = 2, \ldots, m$, since the ships from port o_1 to port o_k are full, the total amount of iron ore transported from port o_1 to port o_k for the demands of o_k in the first $t_i^0 + t_{1k}^1$ periods is $\left\lceil \max\left\{0, \sum_{t=1}^{t_i^0+t_{1k}^1} d_{kt} - I_k^0\right\} \Big/ Q_{1k}^1 \right\rceil Q_{1k}^1$. Therefore, the remaining amount of iron ore at port o_k at the end of period $t_i^0 + t_{1k}^1$ is $\left\lceil \max\left\{0, \sum_{t=1}^{t_i^0+t_{1k}^1} d_{kt} - I_k^0\right\} \Big/ Q_{1k}^1 \right\rceil Q_{1k}^1 + I_k^0 - \sum_{t=1}^{t_i^0+t_{1k}^1} d_{kt}$. Hence, the total amount of iron ore transported from port o_1 to port o_k for the demands of port o_k in periods $t_i^0 + t_{1k}^1 + 1, \ldots, T$ is $q_{ik} \cdot Q_{1k}^1$, where

$$q_{ik} = \left\lceil \max\left\{0, \sum_{t=t_i^0+t_{1k}^1+1}^{T} d_{kt} - \left(\left\lceil \max\left\{0, \sum_{t=1}^{t_i^0+t_{1k}^1} d_{kt} - I_k^0\right\} \Big/ Q_{1k}^1 \right\rceil Q_{1k}^1 + I_k^0 - \sum_{t=1}^{t_i^0+t_{1k}^1} d_{kt} \right) \right\} \Big/ Q_{1k}^1 \right\rceil.$$

Thus, the total amount of iron ore from supplier s_i to port o_1 for satisfying the demands of all the ports is at most $e_i^0 \cdot Q_i^0$, i.e., $z_{it}^0 \le e_i^0 \cdot Q_i^0$, for $t = 1, \ldots, T$, where $e_i^0 = \left\lceil \left(\sum_{t=t_i^0+1}^{T} d_{1t} + \sum_{k=2}^{m} \left(q_{ik} \cdot Q_{1k}^1 \right) \right) \Big/ Q_i^0 \right\rceil$.

Thus, we have that for i = 1, …, n, $z_{it}^0 \in H_{it}^0 = \{0\}$ for t = 1, …, t_i^0, and $z_{it}^0 \in H_{it}^0 = \{0, Q_i^0, ..., e_i^0 \cdot Q_i^0\}$ for $t = t_i^0 + 1$, …, T. Since the ships from port o_1 to port o_k are full, the total amount of iron ore from port o_1 to port o_k for the demands of port o_k in the planning horizon is $e_k^1 \cdot Q_{1k}^1$, where $e_k^1 = \left\lceil \max\left\{0, \sum_{t=1}^{T} d_{kt} - I_k^0\right\} \Big/ Q_{1k}^1 \right\rceil$. As a result, we have that $z_{kt}^1 \in H_k^1 = \{0, Q_{1k}^1, ..., e_k^1 \cdot Q_{1k}^1\}$.

Define $f\left(t; z_1^0, ..., z_n^0; z_2^1, ..., z_m^1\right)$ as the minimum objective value of a partial solution where the total amount of iron ore from supplier s_i that has arrived at port o_1 in the first t periods is z_i^0 and the total amount of iron ore that started to be transported from port o_1 to port o_k in the first t periods is z_k^1, for k = 2, …, m. The following algorithm identifies an optimal solution for this case.

Algorithm A1:

Initialization. $f\left(0; 0, ..., 0; 0, ..., 0\right) = \sum_{k=2}^{m} \left(I_k^0 \cdot \sum_{t=1}^{T} h_{kt} - \sum_{t=1}^{T} d_{kt} \sum_{\tau=t}^{T} h_{k\tau} \right)$ and $f\left(0; z_1^0, ..., z_n^0; z_2^1, ..., z_m^1\right) = +\infty$, for any i = 1, …, n such that $z_i^0 > 0$ or for any k = 2, …, m such that $z_k^1 > 0$.

Recursion. For t = 1, …, T, i = 1, …, n, k = 2, …, m, $z_i^0 \in H_{it}^0$, $z_k^1 \in H_k^1$, where z_i^0 and z_k^1 satisfy the following inequalities: (i) $\sum_{i=1}^{n} z_i^0 + I_1^0 \ge \sum_{k=2}^{m} z_k^1 + \sum_{\tau=1}^{t} d_{1\tau}$, (ii) $z_k^1 + I_k^0 \ge \sum_{\tau=1}^{t+t_{1k}^1} d_{k\tau}$, and (iii) if $t < t_i^0 + 1$, then $z_i^0 = 0$,

$$\begin{aligned} f\left(t; z_1^0, ..., z_n^0; z_2^1, ..., z_m^1\right) = \min\{ & f\left(t-1; \bar{z}_1^0, ..., \bar{z}_n^0; \bar{z}_2^1, ..., \bar{z}_m^1\right) + \sum_{i=1}^{n} \left(z_i^0 - \bar{z}_i^0\right) p_{i,t-t_i^0} + \sum_{i=1}^{n} \frac{z_i^0 - \bar{z}_i^0}{Q_i^0} c_{i,t-t_i^0}^0 \\ & + \left(\sum_{i=1}^{n} z_i^0 - \sum_{k=2}^{m} z_k^1 + I_1^0 - \sum_{\tau=1}^{t} d_{1\tau} \right) h_{1t} + \sum_{k=2}^{m} \frac{\left(z_k^1 - \bar{z}_k^1\right)}{Q_{1k}^1} c_{1kt}^1 + \sum_{k=2}^{m} \left(z_k^1 - \bar{z}_k^1\right) \sum_{\tau=t+t_{1k}^1}^{T} h_{k\tau} \mid 0 \le \bar{z}_i^0 \le z_i^0; \bar{z}_i^0 \in H_{it}^0; \\ & 0 \le \bar{z}_k^1 \le z_k^1; \bar{z}_k^1 \in H_k^1; \text{if } t > T - t_{1k}^1, \text{then } \bar{z}_k^1 = z_k^1; \bar{z}_i^0 \text{ and } \bar{z}_k^1 \text{ satisfy inequalities (i) and (ii)}\}. \end{aligned}$$

Output. The optimal solution value is determined as min{ $f\left(T; z_1^0, ..., z_n^0; z_2^1, ..., z_m^1\right) \mid z_k^1$ satisfy the following inequality: $z_k^1 = e_k^1 \cdot Q_{1k}^1$, for k = 2, …, m, and $\sum_{i=1}^{n} z_i^0 Q_i^0 + I_1^0 \ge \sum_{t=1}^{T} d_{1t} + \sum_{k=2}^{m} e_k^1 Q_{1k}^1$ }.

4. Computational Results

In this section, we first perform computational experiments using data collected from actual industrial practices to explore implications on the changes in various factors on iron ore trade. Then, we further evaluate the performance of our formulation using data generated randomly. We adopt CPLEX 12.5 as our commercial optimization solver and run all the instances on a personal computer with Intel Dual Core processors and 8G memory.

4.1 Data Description

In this study, all the data we adopted are collected from industry databases and the problem setting, model assumptions, and specific parameter ranges in our numerical instances are suggested by a senior manager from Haibao Shipping, which was jointly set up by Baosteel Group Corporation (now merged to Baowu Steel Group, the largest steel producer in China) and China Shipping (now merged to China COSCO Shipping) to provide stable transportation services for Baowu Group.

We consider suppliers from three regions, namely Australia (AU), Brazil (BR) and South Africa (SA), as these regions account for more than 70% of the iron ore supply for Baosteel Group. As for the transportation of international iron ore, Capesize ships, which have an average capacity of 180,000 dwt, are the dominant type. It is noted that, when the iron ore ship chartering price was at its peak in 2008, the giant Brazilian mining company, VALE S.A, built a type of super large iron ore ship, called the Valemax, to transport iron ore from Brazil to Asian ports, in order to compete with its rivals in Australia who travel a much shorter distance to their major iron ore consumers in Asia. With a capacity ranging from 380,000 to 400,000 dead weight tonne, Valemax is the largest bulk carrier ever constructed. Now, the Valemax as well as some Very Large Ore Carriers (VLOC: 250,000 to 300,000 dwt) are employed only between Brazil and Asia. To reflect this, we assume two types of unit capacity of a vessel between BR and Asia, these being 200,000 and 400,000 dwt. In the computational results, the full-shipment strategy is adopted. The chartering prices for a Capesize ship and a VLOC ship of 200,000 dwt are collected from the database of Shipping Intelligence Network (https://sin.clarksons.net/). It is noted that the current chartering price of a Capesize ship is very low compared to the price in 2009, due to a serious over-supply of ships in the market. In addition, there are many types of iron ore, each having different prices. The FOB (Free On Board) price of iron ore also fluctuates significantly. In our cases, we use the average FOB price of iron ore in 2016 collected from the Market Index (http://www.marketindex.com.au/). The data related to suppliers is shown in Table 1.

Table 1. Data related to iron ore suppliers

	AU	**SA**	**BR**
Capacity	200,000 dwt	200,000 dwt	200,000/400,000 dwt
Chartering Price	$12,000/day	$12,000/day	$12,000/22,000/day
Transportation Time	20 days	30 days	40 days
Purchase Price	$40/ton	$40/ton	$40/ton

Four key ports along the Yangtze River in China are taken into consideration in this study. They are located in Ningbo-Zhoushan (NB), Shanghai (SH), Nanjing (NJ) and Wuhan (WH), each of which being in different segments along the Yangtze River, and it is noted that the Baosteel Group has steel plants in all four port cities. The production capability and the demands of the four plants are all different. Approximate demand values are collected from an anonymous Baosteel staff member. Currently, the inventory cost is very low. It falls within the scope of 0.2 to 0.4 dollars per tonne per day, and we assume that the cost is the same across all four ports. The quantity at time 0 is assumed to be the correct amount for production in the next time period. The chartering costs of river ships are collected from the Shipping Intelligence Database. Note that in practice, after the iron ore arrives at a port, it may incur a waiting time for unloading onto the port. However, the waiting time of the iron ore on board is very short compared to the transportation time from the suppliers to the ports and between the ports. In particular, the steel plants along Yangtze River we investigated have their dedicated terminals. This also implies that the cost of the iron ore on board is very low relative to the transportation costs. Thus, in this paper we assume that the waiting time and the cost of the iron ore in a ship on board are included in the transportation time and the transportation cost of the iron ore. This means that the waiting time and the cost of the iron ore in a ship on board are not considered separately. The detailed information related to buyers is shown in Table 2.

Although the problem size is still not large, it is applicable and transferable to other similar problems for the following three reasons. First, the problem size is based on the practical operation of Baosteel Corporation in reality. China makes half of the world’s steel and produced 803.8 million tonnes in 2015.

That was almost eight times the output of Japan, the No. 2 producer. Baosteel is the top listed steel maker. Second, inland rivers rarely have more than three shipping segments. In addition, according to Zheng and Yang (2016), very few ships will choose to transship more than three times so as to save load and unload cost. These reasons imply that our problem setting is general enough for most of the inland river transshipment problems. Third, problems that include more suppliers and ports, should be solved within a reasonable time when using a powerful commercial computational server.

Table 2. Data related to iron ore demand

	NB	SH	NJ	WH
Daily Demand (ton)	5,000	25,000	10,000	20,000
Daily inventory cost ($/ton/day)	0.1	0.1	0.1	0.1
Shipping time from NB (days)	0	1	2	5
Ship size price (dwt)	--	50,000	50,000	5,000
Chartering price ($/day)	--	3,000	3,000	800

4.2 Scenario Analysis and Discussion for General Problem

To understand the impact on iron ore trade volume from different source countries due to the changes of environmental variables, for example, ship chartering price, iron ore price, and storage cost, in this study, we propose three scenarios for analysis, each considering different levels of one particular variable, given that all other parameters are fixed. Based on the real-world application discussed in Section 4.1, in this section we assume that the ships from the suppliers to port o_1 and the ships between the ports are full in order to reduce the total transportation cost. In particular, Scenario 1 considers a change in ship chartering price, Scenario 2 assumes that the FOB iron ore price varies, and under Scenario 3, different storage costs of iron ore at the unload ports are taken into account.

It is noted that the trade volume also relates to many other factors, for example, iron ore quality, production capacity, and investment, but in this section we assume that the other factors are fixed. In the iron ore transportation market, the shipping contract can basically be divided into two types. One is a long term contract (Contract of Affreightment). Most of the large ships, for example, VLOC and Valemax, are born with a long contract so as to avoid any financial risk. In contrast, most Capesize ships are in the spot market, under which they continuously need to seek for a Time Contract (TC). In the first scenario, we assume that Brazilian iron ore is transported by a Valemax ship (400,000 dwt) with a fixed monthly cost equal to its loan on mortgage, whereas iron ore from other source countries is transported by Capesize ships (200,000 dwt), that are under Time Contracts. It is noted that the market chartering price of a Capesize ship fluctuates greatly, such changes in chartering price being shown in Figure 4.

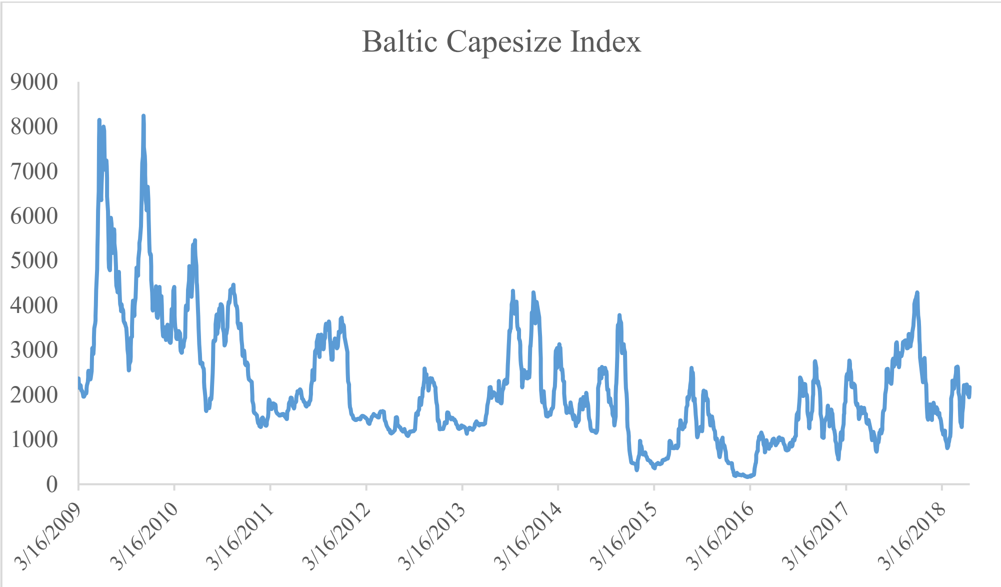

Figure 4. Charting price index of Capesize ship from 2009 to 2018

Source: Lloyd's List.

It is observed that the daily chartering price exceeded 50,000 dollars in 2008, but that in recent years, due to a heavy over-capacity of ships in the market, it has stayed at the bottom of the range, hovering between 5000 and 15,000 dollars per day. In the first scenario, we set the daily chartering cost of a Capesize ship as starting from 12,000 dollars, which is the current market level, up to 48,000 dollars, with an interval every 4,000 dollars.

Table 3 shows the change of trade flow in Scenario 1. It is observed that the Brazilian iron ore is uncompetitive under the current market chartering price of a Capesize ship. When the chartering price of a Capesize ship reaches 32,000 dollars per day, the trade volume from Brazil exceeds that from Australia. When the chartering price exceeds 40,000 dollars, the Brazilian iron ore has absolute competitiveness over Australian iron ore. This result implies that building a Valemax ship is an effective strategy only when the market chartering price of a Capesize ship is much higher than the current price.

Given that the shipping market is very turbulent and the excess supply needs a long time to be digested, building a Valemax ship will not be a wise decision anytime in the next few years.

Table 3. Computational results of Scenario 1

Shipping cost		**Trade volume**		
Valemax	**Capesize**	**AU**	**SA**	**BR**
22,000	12,000	900	0	0
22,000	24,000	900	0	0
22,000	28,000	820	0	80
22,000	32,000	220	0	680
22,000	36,000	220	0	680
22,000	40,000	220	0	680
22,000	44,000	180	0	720
22,000	48,000	140	0	760

Because the shipping price of Brazilian iron ore is much higher than that of Australian iron ore, one possible strategy for the Brazilian iron ore trader is to reduce its iron ore price. It is very important to understand how sensitive the iron ore price is to the trade volume. Under Scenario 2, we consider the effect of iron ore price variance on the changes of trade volume. We consider two cases under this scenario. Case 1 assumes that Australia and South Africa employ Capesize ships but that Brazil uses Valemax ships. Case 2 assumes that all three source countries employ Capesize ships. Table 4 illustrates the results for these two cases.

It is found that the iron ore trade volume from different source countries is very sensitive to iron ore price variation. Although Australia has the absolute advantage if the FOB prices of iron ore remain the same, the Brazilian iron ore will become very competitive if Brazil can reduce its iron ore price by only two dollars per tonne, no matter what ship type is adopted. In addition, South African iron ore will be a good choice if SA offers the same FOB price reduction as Brazil.

Table 4. Computational results of Scenario 2

Iron ore price			**Trade volume**		
AU	**SA**	**BR**	**AU**	**SA**	**BR**
Valemax from Brazil/Capesize ship from Australia and South Africa					
40	40	40	900	0	0
40	40	39	820	0	80
40	40	38	220	0	680
40	40	37	180	0	720
40	39	39	100	800	0
40	39	38	100	800	0
Capesize ship from all three source countries					
40	40	40	900	0	0

40	40	39	900	0	0
40	40	38	140	0	760
40	39	39	120	780	0
40	39	38	100	800	0

Finally, we test the effect from different storage costs on trade volume. It is noted that the storage cost may affect the trade volume only when the size of ship is different from different sources. This is because a larger ship will lead to a higher storage cost. Under Scenario 3, we assume that Valemax and Capesize ships are employed by Brazil and Australia, respectively. In order to obtain a comparative result for analysis, we set the daily chartering price of a Capesize ship as 32,000 dollars per day, and that of a Valemax as 22,000 dollars per day. Then we change the storage cost from 0.4 to 0. The following Table 5 shows the computational results of Scenario 3. The results indicate that a decrease in storage cost can also stimulate an increase in the use of Brazilian iron ore, but only when all other factors between these two countries are comparable.

Table 5. Computational results of Scenario 3

Storage cost	**AU**	**SA**	**BR**
0.4	820	0	80
0.3	740	0	160
0.2	540	0	360
0.1	220	0	680
0	220	0	680

4.3 Performances of Solution Approaches

Based on the data structure given in Section 4.1, we further evaluate the performances of all solution approaches using the data generated randomly. We set the number of the suppliers $n \in \{1, 2, 3, 4\}$, the number of the ports $m \in \{1, 2, 3, 4\}$, the number of periods $T \in \{60, 90, 120, 150, 180\}$, and the other parameters are generated randomly as follows.

(1) Capacity of a ship used to transport iron ore from each supplier to port o_1: Q_i^0 is drawn from the discrete uniform distribution over the interval [200000 dwt, 400000 dwt];

(2) Transportation time from each supplier to port o_1: t_i^0 is drawn from the discrete uniform distribution over the interval [20 days, 40 days];

(3) Price per tonne of iron ore purchased from each supplier: p_{it} is drawn from the discrete uniform distribution over the interval [\$35, \$45];

(4) Transportation cost of a shipload of ore of a ship from each supplier to port o_1: $c_{it}^0 = t_i^0 \cdot \hat{p}_i^0$, where $\hat{p}_i^0$ is the chartering price of a ship from supplier s_i to port o_1 and $\hat{p}_i^0$ is drawn from the discrete uniform distribution over the interval [\$12000/day, \$22000/day];

(5) Capacity of a ship used to transport ore from port o_j to port o_k: Q_{jk}^1 is drawn from the discrete uniform distribution over the interval [10000 dwt, 50000 dwt];

(6) Transportation time from port o_j to port o_k: t_{jk}^1 is drawn from the discrete uniform distribution over the interval [1 days, 7 days];

(7) Transportation cost of a shipload of ore of a ship from port o_j to port o_k: $c_{jkt}^1 = t_{jk}^1 \cdot \hat{p}_{jk}^1$, where $\hat{p}_{jk}^1$ is the chartering price of a ship from port o_j to port o_k and $\hat{p}_{jk}^1$ is drawn from the discrete uniform distribution over the interval [\$800/day, \$3000/day];

(8) Holding cost per tonne in the warehouse of each port at the end of each period: h_{kt} is drawn from the discrete uniform distribution over the interval [\$0.1, \$1.0];

(9) Demand of each port at end of each period: d_{kt} is drawn from the discrete uniform distribution over the interval [500 tonnes, 1000 tonnes].

For each combination of (n, m, T), we generate ten instances randomly and show the computational results of these instances. In Table 6, we report the computational results for the mathematical model solved by CPLEX based on the data generated randomly. In each instance, the computational times of the mathematical model is limited to 600 seconds. For each combination of (n, m, T), the average relative percentage gaps (columns “Ave gap”) between the feasible solutions generated by the mathematical model and the lower bound generated by CPLEX, and the average running times (columns “Ave time”) of the mathematical model are presented in Table 6.

From the computational results in Table 6, it can be seen that the performance of the mathematical model depends heavily on the problem size. For the instances with m = 1 or T = 60, the optimal solutions can be obtained with the mathematical model in very short time. However, for some instances with larger sizes, for example, m = 5 or T = 180, the mathematical model can only find feasible solutions in 600 seconds. The running time of the mathematical model increases as the number of suppliers, the number of ports, and the number of periods increase. Considering that the transshipment times in inland river are decreasing in recent years (Yang et al., 2017), for problem including more suppliers and ports, the computational time should be reasonable with a more powerful commercial computational server.

Table 6. Computational results of the mathematical model within 600 seconds based on the data generated randomly

		$T = 60$		$T = 90$		$T = 120$		$T = 150$		$T = 180$	
n	m	Ave gap	Ave time	Ave gap	Ave time	Ave gap	Ave time	Ave gap	Ave time	Ave gap	Ave time
1	1	0.00	0.1	0.00	0.4	0.00	1.4	0.00	2.7	0.00	11.1
	2	0.00	0.2	0.00	1.7	0.00	10.5	0.34	269.5	4.51	560.8
	3	0.00	0.4	0.00	4.2	0.08	174.6	3.20	478.2	8.39	600.0
	4	0.00	1.1	0.00	42.6	1.10	442.9	7.00	600.0	9.42	600.0
	5	0.00	10.3	0.03	396.8	3.55	600.0	8.35	600.0	11.04	600.0
2	1	0.00	0.1	0.00	0.7	0.00	2.5	0.00	5.5	0.00	18.1
	2	0.00	0.2	0.00	3.3	0.00	68.9	4.47	543.6	11.53	562.0
	3	0.00	1.3	0.00	19.1	1.08	377.1	9.17	600.0	12.40	600.0
	4	0.00	3.0	0.00	98.8	4.95	557.1	10.39	600.0	12.96	600.0
	5	0.00	13.3	0.13	480.4	7.59	600.0	12.25	600.0	14.18	600.0
3	1	0.00	0.1	0.00	0.8	0.00	3.6	0.00	6.1	0.00	30.5
	2	0.00	0.4	0.00	7.3	0.00	132.6	10.50	544.1	14.11	545.7
	3	0.00	2.4	0.00	25.7	6.78	573.0	12.78	570.4	13.37	600.0
	4	0.00	5.8	0.02	292.3	7.52	600.0	11.93	600.0	13.64	600.0
	5	0.00	21.3	2.42	600.0	9.53	600.0	13.35	600.0	13.88	600.0
4	1	0.00	0.2	0.00	1.6	0.00	4.5	0.00	10.7	0.04	215.8
	2	0.00	1.0	0.00	10.9	2.81	379.0	13.11	548.2	15.10	565.4
	3	0.00	4.0	0.00	123.1	8.02	559.7	12.53	583.4	15.55	600.0
	4	0.00	7.9	2.02	437.9	9.13	600.0	12.75	600.0	15.28	600.0
	5	0.00	40.3	3.47	565.9	9.06	600.0	12.85	600.0	14.12	600.0

We further report the performance comparison of Algorithm A1 and the mathematical model based on the data generated randomly for the special case in Table 7. It is worth noticing that Algorithm A1 is based on dynamic programming algorithm, which can either find an optimal solution for the special case of our problem, or cannot even find a feasible solution. Thus we show the average running times (columns "A1") of Algorithm A1 and the average running times (columns "IP") of the mathematical model in Table 7 for each combination of (n, m, T). For each instance, the computational times of Algorithm A1 and the mathematical model are limited to 1,800 seconds because Algorithm A1 needs more time to find the optimal solution rather than a feasible solution for the special problem. Note that "--" represents that the solution approach is not able to find an optimal solution of at least one instance for each combination of (n, m, T).

Table 7. Performance comparison of Algorithm A1 and the mathematical model within 1,800 seconds based on the data generated randomly

		T = 60		T = 90		T = 120		T = 150		T = 180	
n	m	A1	IP	A1	IP	A1	IP	A1	IP	A1	IP
1	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.2	0.0	0.4
	3	0.0	0.0	0.0	0.1	0.0	0.2	0.2	0.4	0.1	0.8
	4	0.0	0.0	0.0	0.1	0.3	0.3	2.1	1.2	19.6	3.0
	5	0.0	0.5	0.1	7.3	2.3	208.4	11.4	--	90.2	--
2	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.2	0.1	0.5
	3	0.0	0.0	0.0	0.1	0.2	0.2	0.5	0.7	1.5	1.4
	4	0.0	0.0	0.3	0.2	1.8	0.5	14.5	4.1	71.1	7.7
	5	0.0	1.5	1.9	20.4	25.9	--	291.5	--	--	--
3	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.5	0.5
	3	0.0	0.0	0.1	0.1	1.2	0.3	2.4	0.9	10.6	1.3
	4	0.0	0.0	1.5	0.2	10.7	0.6	148.4	9.5	482.8	28.0
	5	0.1	2.7	9.0	113.7	324.1	--	--	--	--	--
4	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
	2	0.0	0.0	0.0	0.0	0.0	0.1	0.6	0.3	3.1	0.9
	3	0.0	0.0	0.1	0.1	6.5	0.4	17.9	1.1	76.7	1.2
	4	0.0	0.0	15.1	0.3	56.2	0.7	550.4	12.2	--	20.0
	5	0.1	3.5	64	101.0	--	--	--	--	--	--

In Table 7, the results show that Algorithm A1 is able to find optimal solutions for all the instances within 1,800 seconds except for some instances with $m = 5$ and $T \geqslant 120$. It also shows that the average running time increases as the number of the suppliers, the number of the ports, and the number of the periods increase. In particular, the average running times of the solved instances are less than 600 seconds. For most of the instances, the running times of the mathematical model are not greater than that of Algorithm A1. However, for some instances, Algorithm A1 can find their optimal solutions within a time limit, but the mathematical model cannot. However, we notice that Algorithm A1 does not dominate the mathematical model across the full numerical study.

To further evaluate the performances of the mathematical model and Algorithm A1, we have generated ten instances for each combinations of (n, m, T) with $n = 10$, $m = 5$, and $T = \{120, 150, 180\}$. The computational results show that (i) the average relative percentage gaps between the feasible solutions generated by the mathematical model within time limit of 600 seconds and the lower bound generated by CPLEX increases by 10.32%, 12.86%, to 13.93%, respectively, when the number of periods increases from 120, 150, to 180; and (ii) neither the mathematical model nor Algorithm A1 can find optimal solutions for most of instances for the combinations of (n, m, T) with n = 10, m = 5, and T = {120, 150, 180}, when the time limit is set as 3,600 seconds. These can be generally explained as the difficulty caused by problem sizes. But we noticed that the problem of inland-river shipping rarely reaches this scale in practice.

6. Conclusion

In this paper, we have studied a bulk ship routing and scheduling problem for a cluster of steel plants along the Yangtze River. These plants are all managed by one steel corporation. Due to the unique geographical features of the Yangtze River, the purchased iron ore needs to be imported from suppliers in different regions to these plants via the transshipment ports in the downstream section of the Yangtze River. Our objective is to determine the amounts of iron ore imported from various suppliers to the transshipment port in each period, and also the amounts of iron ore transported between the transshipment port and the other ports in each period by minimizing the purchasing, transportation and inventory cost, where both purchasing price and transportation cost are considered as dynamic over time. A mathematical formulation is developed for this problem and several propositions are proposed for this formulation. We have shown computational results using real data. Several interesting findings have been observed from our discussion of these scenarios, which further lead to some useful managerial insights. In addition, a special case motivated by real-world practice is also studied. We have designed an exact solution approach based on dynamic programming (Algorithm A1) for this case. The performance of Algorithm A1 is evaluated and compared with the mathematical formulation for general case based on the data generated randomly. The findings show that both proposed algorithms are able to find optimal solutions in a reasonable computational time for small- and medium-scale problems and neither of them dominates the other for all instances.

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