Fuzzy logic based approach for ship-bridge collision alert system

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Abstract: This paper proposes a fuzzy logic based approach for ship-bridge collision alert by considering ship particulars, bridge parameters and natural environment. It can be implemented in the decision support system for safe navigation or be included in the process of autonomous navigation. The ship-bridge collision conditions are analysed by considering the ship location, ship trajectory direction, ship distance to the bridge and ship speed. These factors, together with the natural environment, are treated as the input variables. The IF-THEN rules are then established after fuzzification of the input variables and are used for fuzzy inference to derive the ship-bridge collision risk. From the result analysis, the proposed approach can be used for improvement of the ship handling in the bridge waterway area.

Key words: maritime safety; fuzzy logic; ship-bridge collision; possibility of collision risk

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Notation

 L_{bridge} The clearance breath between two bridge piers

 W_{bridge} The width of the bridge deck

 θ_{course} Ship course

 R_{ρ} Safety domain of the ship between the bridge $(\rho = a, b, c)$

 $L_{O\psi}$ Vertical distance from the bridge $(\psi = A, B, C, D)$

 θ_{trace} Ship trajectory deviates from the Y coordinate

 $\theta_{\psi^-}^{\rho\pm}$ The direction that the ship will collide with the left pier when locating at $(\pm \rho, -\psi)$

 $\theta_{\psi^+}^{\rho\pm}$ The direction that the ship will collide with the right pier when locating at $(\pm \rho, -\psi)$

 $W_{\rm max}$ The maximum width of navigable design ships

 $L_{\rm max}$ The maximum length of navigable design ships

 L_{ship} The length of ship passes under bridge

 α Transform coefficient between ship passes under bridge and navigable design ships

V Ship speed

1 Introduction

Bridges build across a waterway (e.g. river) provide a convenient way for transport from one side to the other. Hence, many bridges have been constructed along inland traffic waterways. The Hong Kong–Zhuhai–Macau Bridge (HKZMB), which is a bridge–tunnel system consists of a series of three cable-stayed bridges and one undersea tunnel, has been completed on 14 November 2017. In the Yangtze River, up to 2014, there are more than 85 constructed bridges from Shanghai to Yibin, and this number is still increasing fast in the recent years. Moreover, in the Wuhan, Nanjin and Chongqing waterways, the bridges are constructed with a high density. Specifically, there are 13 bridges (including 2 being constructed) in a range of 50 km in Wuhan (as shown in Figure 1), 6 bridges in a range of 35 km in Nanjin and 11 bridges in the downtown of Chongqing.

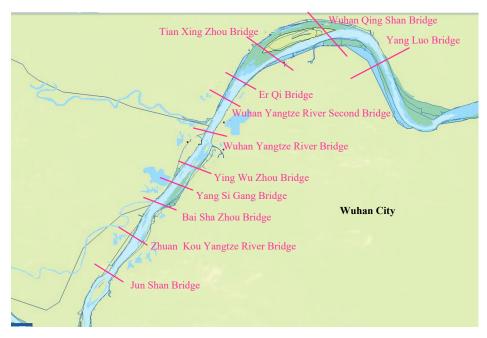


Fig. 1 Eleven Yangtze River bridges (already constructed) in Wuhan City

Although the bridge provides convenience for transport, the bridge across a waterway reduces the navigable width for ships. A bridge pier is an obstacle for ship navigation, and it poses the risk of ship-bridge collision. In 1980, the Sunshine Skyway Bridge was collided by a ship and this accident has caused 35 fatalities. In 2007, the ship collision with Jiujiang Bridge in Guangdong has caused eight fatalities. Moreover, many similar fatal accidents have occurred. One database was developed by the working group 19 of PIANC (Permanent International Association of Navigation Congress), and 151 collision accidents have been recorded in this database (Van Manen, 2001). In China, Dai et al. (2002) collected 213 accident data of ship collision with bridges from 1959 to June of 2002. In fact, the ship collision with bridges is a frequently occurring accident, take the Wuhan Yangtze River Bridge for example, around one hundred collision accidents have been collected from 1959 to 2008 (Gong, 2008). Moreover, many such accidents have occurred though not all of them were recorded owing to their minor consequences.

In order to mitigate the ship-bridge collision risk, several studies have been carried out in the literature. Some studies used Automatic Identification System (AIS) data to discover the abnormal behaviours (Zhang et al., 2016; Mou et al., 2010), and Xiao et al. (2015) analysed the spatial, speed and course distribution in the

Sutong Bridge waterway area in the downstream of Yangtze River using AIS data. Pedersen (2002) proposed a collision risk for fixed offshore structures in order to enhance the safety structure in the design phase, and traffic flow efficiency is also an important decision parameter in the bridge design process (Jensen et al., 2013). Moreover, other studies focused on the consequence analysis of ship-bridge collision. Zhang et al. (2014) analysed the ship-bridge collision process and used finite element analysis to analyse the influences on the running safety of the moving light rail train.

It can be summarised that previous studies focused on the risk analysis using historical data or mitigating the risk by improving the bridge structure (Wang et al., 2008; Hu et al., 2005). However, three problems may be identified in the literature. (1) Few studies focused on defining the risk factors for ship-bridge collision alert system, which requires the risk factors to be collected in real time and easy to implement. The real-time risk assessment is significant to discover the abnormal behaviours and to take countermeasures to avoid collision. This is proved by the previous studies on the ship-ship collision (Zhang et al., 2015; Perera et al., 2013; Bukhari et al., 2013) and it is effective for ship-ship collision risk mitigation (Goerlandt et al., 2015). (2) The risk factors for ship collisions are derived from the historical data, including the ship behaviours from traffic data (Xiao et al., 2015; Silveira et al., 2013; Weng et al., 2012) or causation factors from the historical data (Dai et al., 2002; Wu et al., 2018a; Yip et al., 2015; Eleftheria et al., 2016; Qu et al., 2011). However, though the factors of the natural environment can be derived by using these methods, some factors such as the distance or deviation is hard to directly derive from the historical data. Moreover, the risk factors derived from historical data are hard to be applied to different scenarios because the quantification of these factors can only be used for the specific ship and specific bridge. (3) As previous studies on ship-bridge collision were conducted mainly from the perspective of bridge engineering design, risk control measures are implemented to provide effective protection from potential damage of bridge from ships. Impacts on ships during ship-bridge collisions have not been well addressed yet. Bridges are not designed to reduce fatalities on-board ships in collision situations.

In order to address these three problems, the motivation of this paper is twofold. The first one is to define the risk factors by analysing the critical condition of ship-bridge collision. The second motivation is to introduce the fuzzy logic based method to integrate the risk factors to evaluate the ship-bridge collision in real-time. The remainder of this paper is organized as follows. Section 2 presents the fuzzy logic based three-layer ship-bridge collision risk assessment model, where the collision condition is analysed to derive the influencing factors of ship particulars and bridge parameters, moreover, the natural environment is also considered. Section 3 gives a case study for ship passing under bridge, and an improved trajectory for the ship passing under bridge is also proposed. Discussion is carried out in Section 4, and conclusions are drawn in Section 5.

2 Development of real-time ship-bridge collision methodology

2.1 Establish a fuzzy logic based ship-bridge collision risk model

Fuzzy logic is a widely used method within risk analysis (Pam et al., 2013; Luo and Shin 2016; Mentes et al., 2015) and decision-making (Wu et al., 2016; Perera et al., 2014; Krohling and Campanharo 2011) within maritime transportation. This method utilizes degrees of truth as a mathematical model of vagueness and use linguistic variables to express the input factors, which is especially useful for ship-bridge collision risk assessment because this risk is impacted by multiple factors, and assessment on one factor is often uncertain, imprecise or vague (Pam et al., 2013). Moreover, the result, which is defuzzified from the linguistic variables, is described by crisp values. This is intuitive for the decision-maker to take countermeasures to mitigate the ship-bridge collision risk.

As shown in Figure 2, the fuzzy inference system (FIS) consists of a fuzzification interface, a fuzzy rule base, a fuzzy inference engine, and finally a defuzzification interface (Wu et al., 2016; Wu et al., 2018b). The process of a FIS is as follows: 1) The crisp input is converted to fuzzy by using fuzzification method. 2) The fuzzy rule base is constructed by using a number of fuzzy IF–THEN rules. 3) The fuzzy inference engine performs

the inference operations on the IF-THEN rules. 4) The defuzzification interface transforms the fuzzy results of the inference into a crisp output.

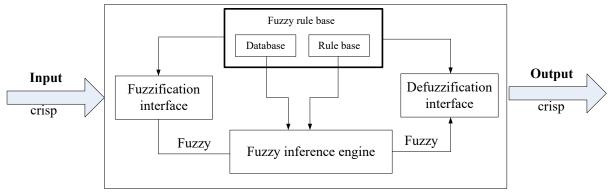


Fig. 2 Generic fuzzy inference system

By introducing FIS, a three-layer framework, which includes input layer, fuzzy inference layer and output layer, is developed to assess the collision risk between ship and bridge. The developed framework is shown in Figure 3 and the detailed description of three layers are as follows. Note that when developing the FIS for collision risk assessment, there are several FIS modules and they will be descried in Subsection 2.4 in detail.

The first layer is the crisp input layer. As this paper focuses on the collision alert of ship-bridge, the input data should be collected in real time. Two types of input data are used in this paper, which are critical condition and natural environment. Specifically, the former type includes horizon distance and vertical distance from the bridge, heading of the ship and the ship speed; the latter type includes wind speed, sea state, visibility and daytime/night-time. The reason why these factors are chosen will be given in Subsections 2.2 and 2.3.

The second layer is the fuzzy inference layer. In this layer, the input factors are fuzzified by using linguistic variables, and then, fuzzy rule base is established by using the IF-THEN scheme, finally, the different fuzzy logic boxes for ship-bridge collision risk assessment are developed, and the linguistic values can be derived. Note that when establishing the IF-THEN rules, the ship behaviours using AIS and domain knowledge are introduced.

The third layer is the output layer. The ship-bridge collision risk can be obtained after defuzzification using centre of gravity method, and the decision-maker can use this result (i.e. crisp value) to take effective actions for collision avoidance between ship and bridge.

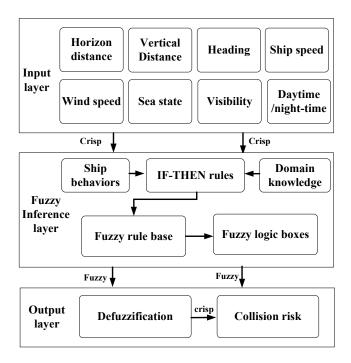


Fig. 3 Fuzzy logic based ship-bridge collision risk model

2.2 Identify the influencing factors by analysing collision conditions

In order to obtain the influencing factors of ship-bridge collision, the critical condition of ship-bridge collision should be first analysed. Although the speed distribution, course distribution, distance can be derived from the historical data of AIS (Xiao et al., 2015), if they are treated as influencing factors, the ship particulars and bridge parameters cannot be taken into consideration. Therefore, similar with the ship-ship collision scenarios (Goerlandt et al., 2012; Perera et al., 2012), the collision conditions should be carried out to achieve a comprehensive, real-time and easy to complement method.

Suppose there are two bridge piers, the clearance breath between them is defined as L_{bridge} , while the width of the bridge deck is defined as W_{bridge} . Define the middle of the clearance breath as the origin of coordinates, where the X coordinate is along the bridge axis, and Y coordinate is a point's vertical distance from the origin. The ship intends to pass the bridge is located at (X_0, Y_0) with the speed V and course θ_{course} .

Similar with the ship domain used for ship-ship collision (Szlapczynski and Szlapczynska 2017; Wang et al., 2009), the safety domain of the ship between the bridge is introduced and divided into three circular sections

with radius $R_{\rho}(\rho = a,b,c)$. Moreover, the vertical distance from the bridge is defined $L_{O\psi}(\psi = A,B,C,D)$. Define the ship heading (θ_{muce}) as the ship trajectory deviates from the Y coordinate, in order to facilitate the fuzzy modelling process, the clockwise is defined as "+" and the counter clockwise is defined as "-", therefore, θ_{muce} should satisfy the equation $-\pi \le \theta_{muce} \le \pi$. Note that the θ_{muce} is different from θ_{course} . The θ_{muce} is the θ_{course} considering the effects of both wind and current, and this can be deduced if the wind and current can be obtained, the detailed relationship can be found in Zhang et al. (2017). It can be seen that if the θ_{muce} deviates from the Y coordinate a lot, the ship will a large possibility to collide against the bridge. Assume the ship is located at $(\pm \rho, -\psi)$, define the direction that the ship will collide with the left pier as $\theta_{\psi^-}^{\rho\pm}$ if keep the current direction, and the direction that the ship will collide with the right pier as $\theta_{\psi^+}^{\rho\pm}$. Note that the ship is simplified as a dot in this paper; this is because this paper intends to assess the collision risk from a relatively far distance from the bridge. Otherwise, it will have not enough time to take response actions by both the ship herself and maritime authorities, and the result will be meaningfulness. All these definitions are shown in Figure 4.

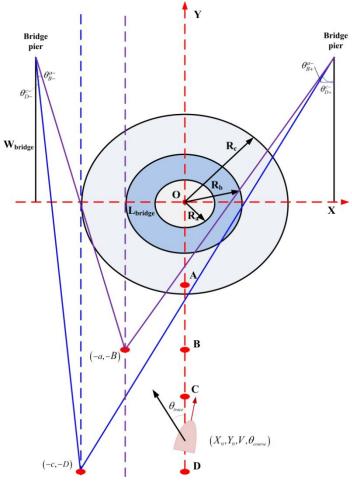


Fig. 4 Ship-bridge collision conditions

By analysing the collision conditions, ship heading, vertical distance and horizon distance are defined as influencing factors. Compared with using the risk factors defined by ship behaviour, two advantages can be discovered by analysing the collision conditions. First, when using historical data to define the risk factors, it is hard to consider the ship particulars and bridge parameters, while by introducing this method, both the ship particulars and the bridge parameters can be considered. Second, the influencing factors defined in this paper are generic and can be introduced in different scenarios, while the influencing factors defined by ship behaviour are specific for the specific bridge, and they are hard to be introduced to other scenarios (e.g. different bridges).

2.3 Fuzzify the risk factors and risk index

As this paper focuses on the real-time risk assessment of ship-bridge collision, the input factors should be easily obtained for the ship herself or for the maritime authorities. From the above analysis, this can be divided

into two types, which are critical condition and natural environment. The former type includes the horizon and vertical distance to the bridge, heading of the ship, and the speed of the ship. The latter type includes the sea state, wind speed, visibility and daytime /night-time, which is similar with the ship-ship collision risk analyse (Balmatet al., 2009).

When utilizing the FIS, the first step is to define the fuzzy set for the collision risk. Define *collision risk* as the fuzzy set in the output space Y. Similar with the previous studies (Pietrzykowski 2008; Gucma and Pietrzykowski 2006), an evaluation interval <0, 1> is introduced, the 0 value denotes a low level of ship-bridge collision risk, while the value 1 denotes the ship collides with the bridge, and higher than 0.6 is defined as unacceptable possibility of collision. Four linguistic variables, which are low, medium, high, very high, are used for fuzzy inference. They are defined as follows.

$$low = (0,0,0.25)$$
; $meduim = (0,0.25,0.50)$; $high = (0.25,0.50,0.75)$; $veryhigh = (0.50,0.75,1.00)$

The next step is to fuzzify the influencing factors using linguistic variables. Commonly, more than two and fewer than seven linguistic variables are utilized when fuzzifying the influencing factors. This is because if more than seven linguistic variables are used, too many fuzzy reasoning rules are required to be established (Jeon et al., 2016; Sahin and Yip 2017). However, if there are only a few variables, the result may not be convincing because it is hard to distinguish them (Wu et al., 2017a; Fu et al., 2016). Moreover, the triangular membership function, which is widely used in practice, is introduced in this paper to describe the linguistic variables. Note that trapezoidal membership function is also utilized to describe the terminal linguistic variables. Specifically, take the linguistic variables (i.e. very good, good, low and poor) for "visibility" as an example, if more than 6nm is believed to be very good, the trapezoidal membership function is used to ensure the degree of membership is one. Similarly, another terminal linguistic variable, poor, is also described by trapezoidal membership function to ensure the visibility lower than 0 is totally poor. The other variables (i.e. good, low) is described by triangular membership function to ensure the degree of membership for fuzzy sets will be changed if the visibility changes.

2.3.1 Fuzzify the critical condition

(1) Distance between ship and bridge in the X direction (horizon distance, R_{ρ}). As shown in Figure 4, it can be easily deduced that when the horizon distance increases, the ship will have a larger possibility to cause collision against the bridge. This is similar with the ship domain used for ship-ship collision, when the ship safety domain increases there is a larger possibility that a close passing will be classified as a collision, and the linguistic values are shown in Table 2. Note that the distance from the left is defined as "–".

Moreover, the values of these radiuses are determined by the width of maximum navigable design ships $(W_{\rm max})$. For the one-way traffic of ship navigation, they are defined as the R_a =0.5×1.1× $W_{\rm max}$, R_b =0.5×1.5× $W_{\rm max}$, R_c =0.5×1.8× $W_{\rm max}$. The 1.5 times of the $W_{\rm max}$ is defined as medium risk because the clearance breath between piers of bridge is defined as 1.5 times of the navigable channel width with the same maximum navigable design ships for bridge design in China. The low risk and high risk is defined less and more than the medium. Specifically, the low risk is defined as 1.1 times, and the high risk is defined as 1.8 times. Similarly, for the two-lane traffic of ship navigation, they are defined as the R_a =0.5×2× $W_{\rm max}$, R_b =0.5×2.8× $W_{\rm max}$, R_c =0.5×3.5× $W_{\rm max}$.

In practice, the ship that passes under bridge will not be with the same width of $W_{\rm max}$, and it is often less than or equal to $W_{\rm max}$. Define the ship width as $W_{\it ship}$, the radius of this specific ship can be transformed by using the coefficient (α), which is defined as $\alpha = \frac{L_{\rm max}}{L_{\it ship}}$, where the $L_{\rm max}$ and $L_{\it ship}$ are the maximum length of navigable design ships and the ship passes under bridge, respectively. By introducing this coefficient, the radiuses are defined as $R_a = 0.5 \times 1.1 \times \alpha \times W_{\it ship}$, $R_b = 0.5 \times 1.5 \times \alpha \times W_{\it ship}$, and $R_b = 0.5 \times 1.8 \times \alpha \times W_{\it ship}$, respectively.

(2) Distance to the bridge in the Y direction (vertical distance, $L_{O\psi}$). As shown in Figure 4, the closer the distance to the bridge in the Y direction is, the higher possibility of the collision situation is. This is because if the ship approaches to the bridge a lot, the ship will not have enough time to adjust the course or speed to avoid collision with the bridge, and the linguistic values are shown in Table 2.

 L_{OA} is defined as the reverse stopping distance, which is the distance that the ship stops the ship by using full astern. L_{OB} is defined as inertial stopping distance, which is the distance that the ship stops by using stop engine. L_{OD} is defined as the distance from the closest waypoint to the bridge. L_{OC} is defined as the 0.8 times of L_{OD} , that is because it is hard to make the location and course well adjusted when just passing the waypoint but will make the location and course well adjusted after a long distance (0.2 times of L_{OD}).

(3) Heading of the ship (θ_{trace}) . It can be seen from Figure 4 that the ship will not collide with the bridge only if the heading satisfies the equation $-\theta_{-\psi}^{-\rho} \prec \theta_{trace} \prec \theta_{+\psi}^{-\rho}$, where $\theta_{-\psi}^{-\rho} = \arctan \frac{L_{bridge} / 2 + (-R_{\rho})}{W_{bridge} + L_{O\psi}}$ and $\theta_{+\psi}^{-\rho} = \arctan \frac{L_{bridge} / 2 + R_{\rho}}{W_{bridge} + L_{O\psi}}$. From this definition, it can be found that $\theta_{+\psi}^{-c} \prec \theta_{+\psi}^{-b} \prec \theta_{+\psi}^{-a}$ and $\theta_{-\psi}^{-c} \prec \theta_{-\psi}^{-b} \prec \theta_{-\psi}^{-a}$, and the linguistic values are shown in Table 2.

(4) Speed of the ship (V). If the ship speed is too low, congestion will exist in this bridge waterway area and finally cause collision accidents. However, if the ship speed is too high, it will not have enough time to adjust the course especially when the ship deviates from the bridge in the horizon direction a lot. In practice, this is determined by the historical data and the outbound and inbound will be different. Define the speed much lower or much higher than the average speed is at high risk, while the average speed is low risk and the detailed parameters will be defined in the application part.

Table 2 Fuzzify the variables of critical conditions

Critical conditions	Low Medium		High	Very high
$R_{\rho}(\mathrm{m})$			$ \left(R_b, R_c, L_{bridge} / 2, L_{bridge} / 2 \right) $ $ \left(-L_{bridge} / 2, -L_{bridge} / 2, -R_c, -R_b \right) $	-
$L_{O\psi}(\mathbf{m})$	$\left(L_{OC},L_{OD},\infty,\infty ight)$	$\left(L_{OB},L_{OC},L_{OD}\right)$	$\left(L_{OA},L_{OB},L_{OC}\right)$	$\left(0,0,L_{OA},L_{OB}\right)$
$ heta_{trace}$ (°)	$\left(-\theta_{-\psi}^{-b},-\theta_{-\psi}^{-a},\theta_{+\psi}^{-a},\theta_{+\psi}^{-b}\right)$	$\begin{pmatrix} \theta_{+\psi}^{-a}, \theta_{+\psi}^{-b}, \theta_{+\psi}^{-c} \end{pmatrix} \\ \begin{pmatrix} -\theta_{-\psi}^{-c}, -\theta_{-\psi}^{-b}, -\theta_{-\psi}^{-a} \end{pmatrix}$	$\left(heta_{\scriptscriptstyle+\psi}^{\scriptscriptstyle-b}, heta_{\scriptscriptstyle+\psi}^{\scriptscriptstyle-c},\pi,\pi ight) \ \left(-\pi,-\pi,- heta_{\scriptscriptstyle-\psi}^{\scriptscriptstyle-c},- heta_{\scriptscriptstyle-\psi}^{\scriptscriptstyle-b} ight)$	-

2.3.2 Fuzzify the natural environment

From the previous studies (Wu et al., 2015; Li et al., 2014; Knapp and Van de Velden 2011), the natural environment is a significant factor for ship-ship collision. Similarly, this factor also caused 24% of ship collision with bridges (Dai et al., 2002). In order to achieve a comprehensive evaluation of collision risk with bridges, the natural environment is also considered in this paper.

- (1) Sea state. The sea state is defined between 0 and 9 by using the Douglas Sea Scale, which also uses the linguistic variables to describe the sea state. In this paper, four linguistic variables, which are calm, choppy, rough, and very tough, are introduced for fuzzification. For the linguistic variables of calm and very rough, the rectangular membership function is introduced, and the triangular membership function is introduced for the linguistic variables of choppy and rough. Note that in order to simplify modelling process, both the sea state and wind speed given in this paper has transformed to cross direction. Therefore, the influence of wind direction and sea direction has been considered by using this simplified method and will not be further discussed.
- (2) Wind speed. The wind speed is defined between 0 and 9 by using Beaufort Scale. In this paper, four linguistic variables, which are calm, breeze, fresh breeze, and strong, are introduced for fuzzification. For the linguistic variables of calm and strong, the rectangular membership function is introduced, and the triangular membership function is introduced for the linguistic variables of breeze and fresh breeze.
- (3) Visibility. The visibility is another important factor for ship-bridge collision. From the historical data, majority of the accidents were caused by lack of visibility. As the visibility less than 1000m or 0.5nm is defined as low visibility, this is defined as very high risk. Moreover, the visibility more than 6nm is easy to detect the targets and is able to have time to avoid collision, this situation is defined as low risk. These two linguistic variables are described by using triangular membership functions, and the fuzzified result is similar with the previous studies and shown in Table 3.
- (4) Night-time/daytime. The historical accident data shows that the proportion of ship-bridge collision occurred in daytime to that in night-time is 2:3. However, this cannot reflect that the night-time is high risk but

can only reflect that the risk at night-time is higher than the risk at daytime. Therefore, the daytime is defined as low risk and the risk at night-time is defined as medium in this paper.

Natural environment	Low	Medium	High	Very high
Sea state	Calm	Choppy	Rough	Very rough
(Douglas Sea Scale)	(0,0,1,3)	(1,3,5)	(3,5,7)	(5,7,9,9)
Wind speed	Calm	Breeze	Fresh breeze	Strong
(Beaufort scale)	(0,0,2,3.5)	(2,3.5,5)	(3.5,5,6)	(5,6,9,9)
Visibility	Very good	Good	Low	Poor
(nm)	(3,5,6,6)	(1,3,5)	(0.5, 1, 2)	(0,0,0.5,1)
Night-time/daytime	Daytime	Night-time	/	/

Table 3 Fuzzify the variables of natural environment

2.4 Establish the fuzzy rule base for ship-bridge collision

After fuzzification of the influencing factors, the fuzzy logic boxes are developed for collision risk assessment. The multiple-input and single output fuzzy logic box is introduced in this paper. In this process, if there are too many input variables, it will be very hard to develop the fuzzy rule base. Therefore, less than three input variables are considered in this paper, and six fuzzy logic boxes are established, which are shown in Figure 5. Specially, they are fuzzy logic boxes for ship condition, critical condition, weather, meteorological condition, natural environment and collision risk. It can be seen from this figure that the collision risk can be divided into two types, one is owing to the critical condition, and another is caused by natural environment.

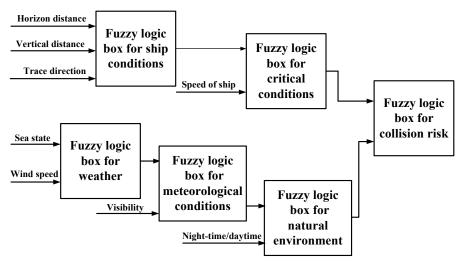


Fig. 5 Fuzzy logic boxes for ship-bridge collision risk assessment

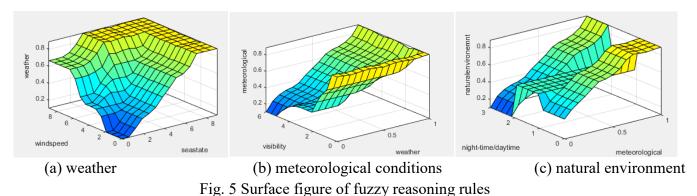
Fuzzy rule base is a significant component of the fuzzy logic box. As the fuzzy logic box of ship condition is essential for the dynamic risk assessment for the ship-bridge collision, this fuzzy rule base is described in detail

and the result is shown in Table 4. As mentioned earlier, the fuzzification of the θ_{trace} are the same in different $L_{O\psi}$ and R_{ρ} . However, the reasoning rules are different. Specifically, when the R_{ρ} is $(\pm R_b, \pm R_a, 0)$ and θ_{trace} is $(-\theta_{B^-}^{a\pm}, -\theta_{A^-}^{a\pm}, -\pi)$ or $(\theta_{B^-}^{a\pm}, \theta_{A^-}^{a\pm}, \pi)$, the ship condition is high when $L_{O\psi}$ is $(-L_{OB}, -L_{OA}, 0)$, this is because if the ship the location is very close to the bridge in both horizon and vertical direction, the collision risk increases, but the ship will not collide with the bridge under the condition that the θ_{trace} satisfies $-\theta_{A-}^{a\pm} \prec \theta_{trace} \prec \theta_{A+}^{a\pm}$ when $R_{\rho} \prec R_{a}$. Similarly, it can be discovered that when the θ_{trace} is $(-\theta_{\scriptscriptstyle D}^{\scriptscriptstyle a\pm},-\theta_{\scriptscriptstyle C}^{\scriptscriptstyle a\pm},-\theta_{\scriptscriptstyle B-}^{\scriptscriptstyle a\pm})$ or $(\theta_{\scriptscriptstyle D}^{\scriptscriptstyle a\pm},\theta_{\scriptscriptstyle C}^{\scriptscriptstyle a\pm},\theta_{\scriptscriptstyle B-}^{\scriptscriptstyle a\pm})$, the ship will not collide with the bridge under the condition that the θ_{trace} satisfies $-\theta_{B^-}^{a\pm} \prec \theta_{trace} \prec \theta_{B^+}^{a\pm}$ when $R_{\rho} \prec R_a$ and $L_{O\psi} \prec L_{OB}$.As the $\theta_{A+}^{a\pm} \succ \theta_{B+}^{a\pm} \succ \theta_{C+}^{a\pm} \succ \theta_{D+}^{a\pm}$, the ship will not collide with the bridge when $L_{OB} \prec L_{O\psi}$. Moreover, when the θ_{trace} is $(0, -\theta_{\scriptscriptstyle D^-}^{\scriptscriptstyle a^\pm}, -\theta_{\scriptscriptstyle C^-}^{\scriptscriptstyle a^\pm})$ or $(0, \theta_{\scriptscriptstyle D^+}^{\scriptscriptstyle a^\pm}, \theta_{\scriptscriptstyle C^+}^{\scriptscriptstyle a^\pm})$, the ship will not collide with the bridge when $L_{\scriptscriptstyle OC} \prec L_{\scriptscriptstyle O\psi}$. Similarly, the fuzzy reasoning rules when the R_{ρ} is $(\pm R_a, \pm R_b, \pm R_c)$ and $(\pm R_b, \pm R_c, \pm \infty)$ are established.

Table 4 Fuzzy rule base for logic box of ship condition							
$R_{ ho}$	$ heta_{ extit{trace}}$	$L_{O\psi}$	Ship condition	$L_{O\!\psi}$	Ship condition		
	$(- heta_{\scriptscriptstyle B-}^{\scriptscriptstyle a\pm},- heta_{\scriptscriptstyle A-}^{\scriptscriptstyle a\pm},-\pi) \ (heta_{\scriptscriptstyle B+}^{\scriptscriptstyle a\pm}, heta_{\scriptscriptstyle A+}^{\scriptscriptstyle a\pm},\pi)$	$(-L_{OC}, -L_{OB}, -L_{OA})$ $(-L_{OD}, -L_{OC}, -L_{OB})$ $(-\infty, -L_{OD}, -L_{OC})$	Low	$(-L_{\scriptscriptstyle OB}, -L_{\scriptscriptstyle OA}, 0)$	High		
$(\pm R_b, \pm R_a, 0)$	$egin{aligned} \left(- heta_{C-}^{\mathtt{a}\pm},- heta_{B-}^{\mathtt{a}\pm},- heta_{A-}^{\mathtt{a}\pm} ight) \ \left(heta_{C+}^{\mathtt{a}\pm}, heta_{B+}^{\mathtt{a}\pm}, heta_{A+}^{\mathtt{a}\pm} ight) \end{aligned}$	$(-L_{OC}, -L_{OB}, -L_{OA})$ $(-L_{OD}, -L_{OC}, -L_{OB})$ $(-\infty, -L_{OD}, -L_{OC})$	Low	$(-L_{\scriptscriptstyle OB}, -L_{\scriptscriptstyle OA}, 0)$	Very high		
	$\begin{pmatrix} (-\theta_{\scriptscriptstyle D-}^{\scriptscriptstyle {\rm a}\pm}, -\theta_{\scriptscriptstyle C-}^{\scriptscriptstyle {\rm a}\pm}, -\theta_{\scriptscriptstyle B-}^{\scriptscriptstyle {\rm a}\pm}) \\ (\theta_{\scriptscriptstyle D+}^{\scriptscriptstyle {\rm a}\pm}, \theta_{\scriptscriptstyle C+}^{\scriptscriptstyle {\rm a}\pm}, \theta_{\scriptscriptstyle B+}^{\scriptscriptstyle {\rm a}\pm}) \end{pmatrix}$	$(-L_{OD}, -L_{OC}, -L_{OB})$ $(-\infty, -L_{OD}, -L_{OC})$	Low	$egin{aligned} (-L_{_{OC}},-L_{_{OB}},-L_{_{OA}}) \ (-L_{_{OB}},-L_{_{OA}},0) \end{aligned}$	Very high		
	$egin{aligned} (0,- heta_{D^-}^{_{a\pm}},- heta_{C^-}^{^{a\pm}}) \ (0, heta_{D^+}^{^{a\pm}}, heta_{C^+}^{^{a\pm}}) \end{aligned}$	$(-\infty, -L_{OD}, -L_{OC})$	Low	$egin{align} (-L_{OB}, -L_{OA}, 0) \ (-L_{OC}, -L_{OB}, -L_{OA}) \ (-L_{OD}, -L_{OC}, -L_{OB}) \ \end{pmatrix}$	Very High		
	$(- heta_{\scriptscriptstyle B-}^{\scriptscriptstyle b\pm},- heta_{\scriptscriptstyle A-}^{\scriptscriptstyle b\pm},-\pi) \ (heta_{\scriptscriptstyle B+}^{\scriptscriptstyle b\pm}, heta_{\scriptscriptstyle A+}^{\scriptscriptstyle b\pm},\pi)$	$(-L_{OC}, -L_{OB}, -L_{OA})$ $(-L_{OD}, -L_{OC}, -L_{OB})$ $(-\infty, -L_{OD}, -L_{OC})$	Low	$(-L_{OB},-L_{OA},0)$	High		
$(\pm R_a, \pm R_b, \pm R_c)$	$(-\theta_{C-}^{b\pm},-\theta_{B-}^{b\pm},-\theta_{A-}^{b\pm})\\ (\theta_{C+}^{b\pm},\theta_{B+}^{b\pm},\theta_{A+}^{b\pm})$	$(-L_{OC}, -L_{OB}, -L_{OA})$ $(-L_{OD}, -L_{OC}, -L_{OB})$ $(-\infty, -L_{OD}, -L_{OC})$	Low	$(-L_{OB}, -L_{OA}, 0)$	Very high		
	$(\theta_{\scriptscriptstyle D-}^{\scriptscriptstyle b\pm}, -\theta_{\scriptscriptstyle C-}^{\scriptscriptstyle b\pm}, -\theta_{\scriptscriptstyle B-}^{\scriptscriptstyle b\pm}) \\ (\theta_{\scriptscriptstyle D-}^{\scriptscriptstyle b\pm}, \theta_{\scriptscriptstyle C-}^{\scriptscriptstyle b\pm}, \theta_{\scriptscriptstyle B-}^{\scriptscriptstyle b\pm})$	$(-L_{OD}, -L_{OC}, -L_{OB})$ $(-\infty, -L_{OD}, -L_{OC})$	Low	$egin{aligned} (-L_{OC}, -L_{OB}, -L_{OA}) \ (-L_{OB}, -L_{OA}, 0) \end{aligned}$	Very high		
	$(0,- heta_{D^-}^{b\pm},- heta_{C^-}^{b\pm}) \ (0, heta_{D^+}^{b\pm}, heta_{C^+}^{b\pm})$	$(-\infty, -L_{OD}, -L_{OC})$	Low	$(-L_{OB}, -L_{OA}, 0)$ $(-L_{OC}, -L_{OB}, -L_{OA})$ $(-L_{OD}, -L_{OC}, -L_{OB})$	Very High		

	$(- heta_{\scriptscriptstyle B^-}^{\scriptscriptstyle c\pm},- heta_{\scriptscriptstyle A^-}^{\scriptscriptstyle c\pm},-\pi) \ (heta_{\scriptscriptstyle B^-}^{\scriptscriptstyle c\pm}, heta_{\scriptscriptstyle A^-}^{\scriptscriptstyle c\pm},\pi)$	$(-L_{OC}, -L_{OB}, -L_{OA})$ $(-L_{OD}, -L_{OC}, -L_{OB})$ $(-\infty, -L_{OD}, -L_{OC})$	low	$(-L_{\scriptscriptstyle OB}, -L_{\scriptscriptstyle OA}, 0)$	High
$(\pm R_b, \pm R_c, \pm \infty)$	$(-\theta_{\scriptscriptstyle C-}^{\scriptscriptstyle c\pm},-\theta_{\scriptscriptstyle B-}^{\scriptscriptstyle c\pm},-\theta_{\scriptscriptstyle A-}^{\scriptscriptstyle c\pm})\\ (\theta_{\scriptscriptstyle C+}^{\scriptscriptstyle c\pm},\theta_{\scriptscriptstyle B+}^{\scriptscriptstyle c\pm},\theta_{\scriptscriptstyle A+}^{\scriptscriptstyle c\pm})$	$(-L_{OC}, -L_{OB}, -L_{OA})$ $(-L_{OD}, -L_{OC}, -L_{OB})$ $(-\infty, -L_{OD}, -L_{OC})$	Low	$(-L_{OB},-L_{OA},0)$	Very high
	$(-\theta_{\scriptscriptstyle D-}^{\scriptscriptstyle c\pm},-\theta_{\scriptscriptstyle C-}^{\scriptscriptstyle c\pm},-\theta_{\scriptscriptstyle B-}^{\scriptscriptstyle c\pm})\\ (\theta_{\scriptscriptstyle D+}^{\scriptscriptstyle c\pm},\theta_{\scriptscriptstyle C+}^{\scriptscriptstyle c\pm},\theta_{\scriptscriptstyle B+}^{\scriptscriptstyle c\pm})$	$(-L_{OD}, -L_{OC}, -L_{OB})$ $(-\infty, -L_{OD}, -L_{OC})$	Low	$(-L_{OC}, -L_{OB}, -L_{OA}) \ (-L_{OB}, -L_{OA}, 0)$	Very high
	$(0,- heta_{D-}^{c\pm},- heta_{C-}^{c\pm}) \ (0, heta_{D+}^{c\pm}, heta_{C+}^{c\pm})$	$(-\infty, -L_{OD}, -L_{OC})$	Low	$(-L_{OB}, -L_{OA}, 0)$ $(-L_{OC}, -L_{OB}, -L_{OA})$ $(-L_{OD}, -L_{OC}, -L_{OB})$	Very High

Another type of fuzzy rule base is much easier, and they are established by using the traditional IF-THEN schemes. As shown in Figure 6, the surface figures of fuzzy reasoning rules are established. Specifically, Fig. 6(a) represents the fuzzy reasoning rules for weather; Fig. 6(b) represents the fuzzy reasoning rules for meteorological conditions; Fig. 6(c) represents the fuzzy reasoning rules for natural environment.



rig. 5 Surface rigure of ruzzy reasoning re

2.5Derive the ship-bridge collision risk using fuzzy inference

After establish the fuzzy rule base, the fuzzy inference engine should be developed to construct the relationship between input and output variables. There are two widely used methods in this process, which are Mamdani-type and Sugeno-type. In this paper, the former type, which uses a Min–Max fuzzy inference method, is adopted.

Moreover, as the output (ship-bridge collision risk) is linguistic values after inference using Min–Max method, the defuzzification method should be introduced to derive the crisp values of the ship-bridge collision risk. In this paper, the common and useful method, namely the centre of gravity method, is utilized.

3Application of the proposed model for ship-bridge collision warning

3.1 Scenario description of ship passing bridge

At12:00 February 2018, a ship passed under the Wuhan Yangtze River Bridge. The ship is 90m length. The clearance breath of Wuhan Yangtze River Bridge is 128m for one-way traffic, and the length of the maximum navigable designed ships is 110m with the ship width 16m. At that time, the wind speed was at Beaufort Scale 3, the sea state was Douglas Sea Scale 3 and the visibility was more than 5nm, the detailed information is shown in Table 5.

Table 5 Detailed information of ship's particulars and natural environment

1	1
clearance breath of Wuhan Yangtze River Bridge	Traffic flow for bridge
128 m	One-way traffic
Length of maximum navigable	Width of maximum navigable
designed ships	designed ships
110m	16m
Ship length	Wind speed
90 m	Beaufort Scale 3
Sea state	Visibility
Douglas Sea Scale 3	more than 5nm

The ship went inbound and eight points have been selected with the interval distance 0.2km from the bridge. The speed of the ship (SOG) decreased from 5kn to 3.3kn when the ship approached to the bridge. There were two significant course changes of the heading (HDG) in the bridge passing, and the horizon distance has decreased from 88m to 6m. The ship finally passed the bridge safely and the detailed information in this process is shown in Table 6.

Table 6 Detailed information of ship during bridge passing

#	Distance(km)	SOG (kn)	HDG(°)	Location deviation(m)
1	1.5	5	216	88
2	1.3	4.6	216	68
3	1.1	4	216	48
4	0.9	3.8	211	32
5	0.7	3.6	211	24
6	0.5	3.4	211	12
7	0.3	3.3	209	6
8	0.1	3.3	209	6

3.2 Fuzzify the input variables based on critical conditions analysis

From the information of the ship and the bridge, the following information can be obtained. The coefficient is $\alpha = \frac{L_{\max}}{L_{ship}} = \frac{110}{90} = 1.22$. Hence, the horizon distance R_{ρ} with high, medium and low risk are $R_a = 0.5 \times 1.1 \times 1.22 \times 16 = 10.7$, $R_b = 0.5 \times 1.5 \times 1.22 \times 16 = 14.6$ and $R_c = 0.5 \times 1.8 \times 1.22 \times 16 = 17.6$, respectively. Moreover, the vertical distance to the bridge $L_{O\psi}$ with very high, high, medium and low risk are $L_{OA} = 6 \times L = 540$, $L_{OB} = 8 \times L = 720$, $L_{OD} = 1500$ and $L_{OC} = 0.8 \times 1500 = 1200$, respectively. This is because from the experience of manoeuvring ships, the reverse stopping distance L_{OA} is around six times of ship length, inertial stopping distance L_{OB} is around eight times of ship length, L_{OD} is defined as 1,500, which is the distance from the closest waypoint to the bridge, and L_{OC} is defined as the 0.8 times of L_{OD} .

Moreover, the heading of the ship θ_{trace} can be obtained using the associated equations. The θ_{trace} when the ship location is within $(-R_a, L_{OA})$ are $\theta_{-A}^{-a} = \arctan \frac{128/2 + (-10.7)}{30 + 540} = 5.3^{\circ}$ and $\theta_{+A}^{-a} = 7.3^{\circ}$, respectively; the θ_{trace} when the ship location is within $(-R_a, L_{AB})$ are $\theta_{-B}^{-a} = 4.1^{\circ}$ and $\theta_{+B}^{-a} = 5.7^{\circ}$, respectively. Similarly, the others can also be derived as follows. Specifically,

$$\begin{split} &\theta_{-C}^{-a}=2.5^{\circ}\,,\;\theta_{+C}^{-a}=3.5^{\circ}\,,\;\theta_{-D}^{-a}=2.0^{\circ}\,,\theta_{+D}^{-a}=2.8^{\circ}\,;\\ &\theta_{-A}^{-b}=5.0^{\circ}\,,\;\theta_{+A}^{-b}=7.9^{\circ}\,,\;\theta_{-B}^{-b}=3.8^{\circ}\,,\;\theta_{+B}^{-b}=6.0^{\circ}\,,\theta_{-C}^{-b}=2.3^{\circ}\,,\;\theta_{+C}^{-b}=3.7^{\circ}\,,\;\theta_{-D}^{-b}=1.8^{\circ}\,,\theta_{+D}^{-b}=2.9^{\circ}\,;\\ &\theta_{-A}^{-c}=4.7^{\circ}\,,\;\theta_{+A}^{-c}=8.1^{\circ}\,,\;\theta_{-B}^{-c}=3.5^{\circ}\,,\;\theta_{+B}^{-c}=6.2^{\circ}\,,\theta_{-C}^{-c}=2.2^{\circ}\,,\;\theta_{+C}^{-c}=3.8^{\circ}\,,\;\theta_{-D}^{-c}=1.7^{\circ}\,,\theta_{+D}^{-c}=3.1^{\circ}\,. \end{split}$$

Another significant factor is the ship speed, in order to obtain the average speed of the ship passing the bridge, the AIS data have been collected in 2015. The data have been collected with longitude between 114°12′N and 114°32′N, and longitude between 30°28′E and 30°42′E, and the data in the Wuhan Yangtze River is utilized in this paper. Specifically, there were 8,649 upstream ships and 9,682 for the downstream. The ship speed distribution is analysed and the results are shown in Figure 7. From this historical data, it can be seen that the

majority of the ship speeds are between 2.5kn and 5.5kn for the inbound ships, while the majority of the ship speed is between 4kn and 10kn for the outbound ships.

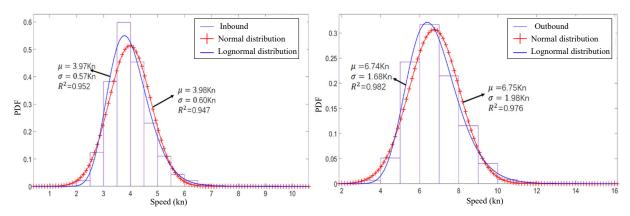


Fig. 7 SOG distribution in the Wuhan Yangtze River Bridge

Based on the above analysis of the ship passing the Wuhan Yangtze River Bridge, the critical condition can be fuzzified as shown in Table 7. The output variables are also fuzzified by using the standard triangular membership function, the mapping values are also given in this table. In this paper, the output variables of the fuzzy logic boxes, including ship condition, critical condition, weather, metrological condition, natural environment and collision risk, are using this membership function. Note that the fuzzification of the θ_{max} is different for the different horizon distances (R_p), and the fuzzy rule base is also established by using the different mapping values though the linguistic variables are the same. Another thing should be noted is that the fuzzification of the ship speed is also different. As can be seen from Figure 7, for the inbound ships, the average speed is around 4kn with standard deviation is around 0.6kn, therefore, the interval number which uses the average speed considering the standard deviation is treated as low risk. Moreover, the ship speed lower or higher than this value is treated as medium and much lower or higher than this value is treated as high. Similarly, the ship speed for outbound ships can be also established. It can be seen from this table and Figure 7 that the average speed for outbound is higher than upstream, this is because the outbound ships can use the water level differences while the inbound ships have to overcome this phenomenon.

Table 7 Mapping value of the critical condition and output variables

Critical condition Low		Medium	High	Very high
$R_{ ho}$ (m)	(0,0,10.6,14.6)	(10.6,14.6,17.6)	(14.6,17.6,40,40)	-
$L_{O\psi}\left(\mathbf{m} ight)$	(0,0,540,720)	(540,720,1200)	(720,1200,1500)	(1200,1500,1800,1800)
θ_{trace} for $-R_a$ (°)	(-2.5,-2.0,2.8,3.5)	(2.8,3.5,5.7) and (-4.1,-2.5,-2.0)	(3.5,5.7,7.5) and (-5.3,-4.1,-2.5)	(5.7,7.5,10,10) and (-10,-10,-5.3,-4.1)
θ_{trace} for $-R_b$ (°)	(-2.3,-1.8,2.9,3.7)	(2.9,3.7,6.0) and (-3.8,-2.3,-1.8)	(3.7,6.0,7.9) and (-5.0,-3.8,-2.3)	(6.0,7.9,10,10) and (-10,-10,-5.0,-3.8)
θ_{trace} for $-R_c$ (°)	(-2.2,-1.7,3.1,3.8)	(3.1,3.8,6.2) and (-3.5,-2.2,-1.7)	(3.8,6.2,8.1) and (-4.7,-3.5,-2.2)	(6.2,8.1,10,10) and (-10,-10,-4.7,-3.5)
V_{speed} upstream (kn)	(3,4,5)	(4,5,6) and (2,3,4)	(5,6,8,8) and (0,0,2,2.5)	-
V_{speed} downstream (kn)	(5,6.5,8)	(6.5,8,9.5) and (3.5,5.25,7)	(8,10.5,14,14) and (0,0,3.5,5)	-
Output	(0,0,0.33)	(0,0.33,0.67)	(0.33, 0.67, 1.00)	(0.67,1.00,1.00)

After establishing the mapping value of the natural environment in Table 3 and critical condition in Table 6, the linguistic values for the fuzzy sets of each output fuzzy logic box in this scenario can be derived. As the natural environment is not changing during the bridge passing process, the linguistic values also did not change, and they can be described as follows. As the sea state is 3 using Douglas Sea Scale, this can be fuzzified as (*choppy*,1.0); wind speed is fuzzified as (*clam*,0.33;*breeze*,0.67); visibility is fuzzified as (*very good*,1.0) and daytime/night-time is fuzzified as (*daytime*,1.0). Different from the natural environment, the critical condition is changing during the process of bridge passing, therefore, the linguistic values of this fuzzy set are also changing and the linguistic values are shown in Table 8. Note that as the normal direction of the bridge is 208°, the heading is the normal direction minus the course of the ship.

Table 8 Linguistic values of critical condition

#	$R_{ ho}$ (m)	$L_{O\psi}$ (m)	$ heta_{ extit{trace}}$ (°)	V_{speed} (kn)
1	(high, 1.0)	(low,1.0)	(high, 0.05; very high, 0.95)	(medium, 1.0)
2	(high, 1.0)	(low, 0.33; medium, 0.67)	(high, 0.05; very high, 0.95)	(low, 0.4; medium, 0.6)
3	(high, 1.0)	(medium, 0.79; high, 0.21)	(high, 0.05; very high, 0.95)	(low,1.0)
4	(high, 1.0)	(medium, 0.37; high, 0.63)	(low, 0.86; medium, 0.14)	(low, 0.8; medium, 0.2)
5	(high, 1.0)	(medium, 0.04; high, 0.96)	(low, 0.86; medium, 0.14)	(low, 0.6; medium, 0.4)
6	(low, 0.67; medium, 0.33)	(very high,1.0)	(low, 0.87; medium, 0.13)	(low, 0.4; medium, 0.6)
7	(low,1.0)	(very high, 1.0)	(low,1.0)	(low, 0.3; medium, 0.7)
8	(low,1.0)	(very high,1.0)	(low,1.0)	(low, 0.3; medium, 0.7)

It can be seen from Table 8 that when the ship navigates close to the bridge, the risk of deviation is decreasing, and the risk of heading is also decreasing. However, the risk of ship speed decreases first but then increases a little. The reason is that if the ship speed decreases to a certain speed that the ship can handle it well, the collision risk will decrease because the ship will have enough time to make decisions for collision avoidance. However, the risk level increases when the ship speed keeps decreasing, this is because the low speed may be caused by the by propulsion system failure (Wu et al., 2016).

3.3 Ship-bridge collision risk result using fuzzy inference

After fuzzifying the input variables and establishing the fuzzy rule base, the ship-bridge collision risk can be inferred by using Mamdani inference system, which uses the Min-Max compositional rule. The inference process is carried out as follows. In order to illustrate the process of fuzzy inference by using Min-Max compositional rule, the ship condition is used as an example in this paper.

First, the fuzzy <and> operator is used to obtain the linguistic values of the output variables for each fuzzy logic box. Take the #2 waypoint as an example, four reasoning rules have been activated, which are shown in Table 9. By introducing the Min method, the result for critical condition can be derived. It can be seen from this table that the linguistic value of ship condition is *very high* for all four rules. This is because when the ship deviates more from the bridge waterway area, the collision risk will be higher at this waypoint.

Table 9 Mamdani inference process using fuzzy < and > operator (ship condition of #2 waypoint	Table 9 Mamdani infer	ence process using	; fuzzy < and >	> operator (ship co	ondition of #2 waypoint)
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		~ .			• 1
Activated rule	$R_{\rho}(\mathbf{m})$	$L_{O\psi}(\mathbf{m})$	$ heta_{ extit{trace}}$ (°)	Min	Ship condition
#151	1.00 high	0.33 <i>low</i>	0.05 high	0.05	Very high
#152	1.00 high	0.33 <i>low</i>	0.95 very high	0.33	Very high
#143	1.00 high	0.67 medium	0.05 high	0.05	Very high
#144	1.00 high	0.67 medium	0.95 very high	0.67	Very high

Second, the output variable (i.e. ship condition) is aggregated by using the Max method as follows.

$$ship\ conditions = \begin{cases} \max(0.05, 0.33, 0.05, 0.67), very\ high; \max(0, 0, 0, 0), high; \\ \max(0, 0, 0, 0), medium; \max(0, 0, 0, 0), low \end{cases}$$

Therefore, the inferred result of ship condition is $ship\ conditions = \begin{cases} 0.67, very\ high; 0, high; \\ 0, medium; 0, low \end{cases}$. The result can be interpreted as that the risk of ship condition is very high at a possibility of 0.67.

Last, the defuzzification is carried out by using the centre of gravity method, the results are shown in Table 10.

Ship condition Critical condition Weather Meteorological condition Natural environment Collision risk 1 0.89 0.75 0.33 0.33 0.50 0.67 0.88 0.73 0.33 0.33 0.50 0.67 3 0.89 0.74 0.330.33 0.50 0.67 0.67 0.67 0.33 0.33 0.50 0.67 5 0.50 0.67 0.67 0.33 0.33 0.67 6 0.12 0.27 0.33 0.33 0.50 0.50 0.140.50 0.50 0.27 0.33 0.33 0.14 0.27 0.33 0.33 0.50 0.50

Table 10 Defuzzified result of the ship-bridge collision risk

From Table 10, it can be seen that the results of weather, meteorological condition and natural environment are the same in the process of ship passing under bridge. This is because these results are influenced by the natural data (i.e. wind speed, visibility, daytime/night-time, sea state), which are at good conditions and stays the same during the ship passing under the bridge.

However, the risk of the ship conditions and critical conditions are changing during the whole ship bridge passing process. This is because the values are also changing during this process. Specifically, the risk of ship condition waypoint decreased to 0.88 #3 waypoint but increases to 0.89 in #3 waypoint. This is reasonable owing to the following reasons: 1) The horizon distance is close to the bridge #1 waypoint, which makes the collision risk high. 2) The collision risk decreases because the horizon distance decreases in #2 waypoint. 3) The risk increases in #3 waypoint because the ship is getting closer to the bridge waterway area, and the ship won't have enough to take actions at this waypoint if an expected event occurs.

Moreover, the ship-bridge collision risk decreased from 0.67 (waypoint 5#) to 0.50 (waypoint 6#). In practice, this ship has successfully passed the bridge, which means the collision risk is reasonable. However,

from further analysis, the risk decreased to 0.50 when the ship is 500m to the bridge. This would be very dangerous because if an expected event occurs, the ship is too close to stop even by using engine full astern. Therefore, although the ship has successfully passed the bridge, the manoeuvring of ship should be improved and this is discussed in the following subsection.

3.4 Comparison of the improved handling of passing bridge

From the above analysis, the ship-bridge collision risk keeps at a high level until changing the course at #6 waypoint. However, at that time, the ship cannot take effective actions to avoid collision if the ship is not under control. Therefore, improved handing of passing bridge should be carried out to decrease the risk level. Although the handing of ships should consider the manoeuvrability of the ship, this paper will not further discuss this but will only give the changed result at these waypoints. The changed trajectory and original trajectory of the ship is shown in Figure 8 during the bridge passing. The changed parameters of the ships are as follows.

- i) The ship takes a sharp alteration of course at the #1 waypoint by using HDG 220°, and changes HDG to 209° at #4 waypoint.
- ii) The deviation suddenly decreased from 88m to 45m at #2 waypoint, to 20m at #3 waypoint, and to 6m at #4point.
 - iii) The ship speed keeps at 3.8kn at #4waypoint.

From Figure 8, it can be seen that the modified trajectory changes more than the original trajectory at #1 waypoint and then, it changes the course at #4 waypoint. In the modified trajectory, the ship keeps the same course from #4 waypoint and it finally safely passed the bridge.

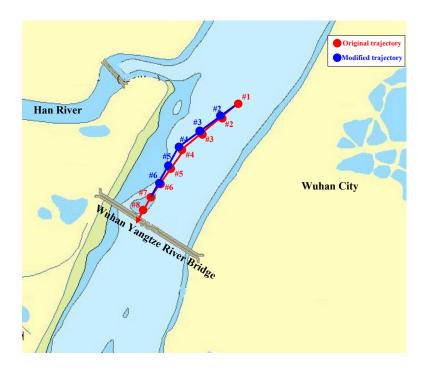


Fig. 8 Trajectory of ship passing the Wuhan Yangtze River Bridge

In this modified trajectory, the risk of R_{ρ} decreases from high to low and the risk of θ_{trace} decreases from very high to low at the #4 waypoint. Moreover, the risk of ship speed remains the same because the ship speed did not change after passing the #4 waypoint. After fuzzifying these input variables, the output variables (i.e. ship conditions, critical conditions, collision risk) can be derived by using the FIS. From Table 11, it can be seen that the results of ship condition, critical condition and collision remain the same after passing #4 waypoint, and the collision risk decreases from 0.67 to 0.50 low at this waypoint. This means the ship-bridge collision risk is low at this waypoint. Compared with the original trajectory, this modified trajectory is better because it will have sufficient time to take actions for ship-bridge collision avoidance even if the ship is not under control or in danger caused by human error (Wu et al., 2017b; Trucco et al., 2008; Wang et al., 2019) in the bridge waterway area.

Table 11 Ship-bridge collision risk for modified trajectory

#	R_{ρ} (m)	$ heta_{trace}$ (°)	V_{speed} (kn)	Ship condition	Critical condition	Collision risk
2	(high, 1.0)	(very high, 1.0)	(low, 0.4;medium, 0.6)	0.88	0.73	0.67
3	(high, 1.0)	(very high, 1.0)	(low, 1.0)	0.89	0.75	0.67

4	(low, 1.0)	(low, 1.0)	(low, 0.8;medium, 0.2)	0.14	0.28	0.50
5	(low, 1.0)	(low, 1.0)	(low, 0.8;medium, 0.2)	0.14	0.28	0.50
6	(low, 1.0)	(low, 1.0)	(low, 0.8;medium, 0.2)	0.14	0.28	0.50

4 Discussion: evaluation of the ship-bridge collision risk analysis

4.1 Evaluation: definition, method and criteria

In risk modelling, the first thing should be determined is how to understand and describe risk. Aven (2010) has summarized that there are eight typical definitions, a widely accepted definition is that risk is equal to a triplet and answers three questions, which are what can wrong, how likely is it and what is the damage. In practice, the triplet is incomplete as two issues should be discussed. First, the background knowledge is often used in the modelling process, however, knowledge is not equally available to understand the interaction among the parts of system (Montewka et al., 2014) and will finally cause uncertainty. Second, it is unavoidable to make simplifying assumptions this will also cause uncertainty. Moreover, the results derived from the risk modelling should be interpretable (Aven 2011), otherwise, it will be meaningless for the decision-maker.

Validity focuses on the qestation whether the risk analysis can be used to describe the specific concepts and for the intended objective, whereas the evaluation is to control the quality of the objective. Similar with the study with Goerlandt and Montewka (2015), two aspects are used for this validation. The first aspect is to validate the plausibility of the model functions, including:

- i) to convoy an augmentation based on available evidence.
- ii) to provide a basis for communication.
- iii) to serve as an aid to thinking.

The second aspect is to use the relevant criteria. The detailed methods and criteria used in this paper are as follows.

(1) Evaluation of the risk model. It is intuitive to use the historical data for validation of the proposed model. However, as the uncertainty exists, the model may not be generic but only can be used for the specific scenario.

Therefore, it is better to use some evaluation methods in this process. Face validity is a subjective and heuristic interpretation of whether the model has well reflected the ship-bridge collision risk. Moreover, the specific tests can be performed on the developed ship-bridge collision risk model. Quantitative feature test uses a number of test conditions to discover the response of the ship-bridge collision risk and to understand whether it is reasonable in the real scenario by domain experts. Concurrent validity text is to compare the elements in the proposed model with other existing models for a similar purpose.

- (2) Evaluation of the risk analysis. In the previous study, four criteria, which are proposed by Aven and Heide (2009) and Rosqvist and Tuominen (2004), are used in this paper for validation.
 - i) The degree to which the uncertainty assessments are complete (V1).
 - ii) The degree to which the bias assessments are complete (V2).
- iii) The degree to which the assigned subjective probabilities adequately describe the accessor's uncertainties (output variables in fuzzy logic box) of the unknown quantities (V3).
 - iv) The degree to which the analysis addresses the right quantities (V4).

From these definitions of the four criteria, V1 and V2 concerns whether the assumptions and knowledge used can well address the uncertainty, and the limitations when applied to other waterways. V3 concerns whether the output values are reasonable by introducing the subjective judgments, while V4 relates the interpretation of the ship-bridge collision risk, and whether or how this result can be used for practice.

4.2 Application of evaluation method for ship-bridge collision risk analysis

Although there are several accident records on the ship-bridge collision accidents, this type of risk can be treated as relatively small probabilities and high large consequences (e.g. the Sunshine Skyway Bridge accident). Therefore, when describing ship-bridge collision risk, this paper focuses on the collision alert of accident occurrence, while the consequences are ignored. However, in practice, when establishing the ship-bridge collision risk, the accident occurrence should be described by background knowledge, specifically, in this paper, when

analysing collision conditions to identify the influencing factors in Subsection 2.2 and establish the fuzzy rule base in Subsection 2.4, this kind of knowledge is used. Moreover, when analysing the critical conditions, this paper simply assumes that the ship is a dot, which is reasonable as the ship is relatively far from the bridge from the collision alert perspective.

(1) Risk model evaluation tests

From face validity, it can be seen that this proposed model can well reflects the ship-bridge collision risk. The ship location, ship trajectory direction, ship distance to the bridge, ship speed and environmental factors are all considered from a generic perspective, and the linguistic variables used for measure risk seem reasonable. The ship conditions are derived by analysing the collision conditions, which will influence the collision risk to some extent. Ship speed will impact the ship manoeuvrability and the somehow related to the ship control especially in the critical situation. The natural environmental factors are related to the human (crews) behaviours and may cause human error in the harsh natural conditions, which will finally influence the collision risk.

The qualitative features test is performed by changing the input variables (influencing factors) of the fuzzy logic-based model. Take the #5 waypoint in Table 6 as the baseline, the changed values of the input variables, corresponding response and plausibility assessment are given in Table 12. From this analysis, it can be seen that the results indicate that the proposed fuzzy logic-based ship-bridge collision risk model qualitatively follows the expected values. Specifically, when the ship conditions changes better, the collision risk decreases. Moreover, the collision risk increases when the natural environment changes bad, and in other scenarios, the collision risk remains the same because the ship conditions are too harsh to make the ship safely pass the bridge.

Table 12 Results from the qualitative features test for the fuzzy logic model

Variable	Value	Risk	Plausibility assessment	
Baseline	#5waypoint	0.67	-	
	48m	n 0.67 Remains the same but with high level as the deviation is too m		
R_{ρ} (m)	12m	0.50	Risk decreases as the deviation decrease and it is safe to pass the bridge.	

$L_{O\psi}$ (m)	900m	0.67	The deviation and heading makes the ship cannot safely pass the bridge.	
	300m	0.67	The deviation and heading makes the ship cannot safely pass the bridge.	
$ heta_{trace}$ (°)	7	0.67	The ship cannot safely pass the bridge.	
	1	0.50	The ship can safely pass the bridge when the heading is close to the direction of the bridge.	
V _{speed} (kn)	6	0.67	The ship cannot safely pass the bridge owing to the ship conditions.	
	2	0.67	The ship cannot safely pass the bridge owing to the ship conditions.	
Sea state	7	0.87	The harsh natural environment makes the collision risk increases.	
	1	0.67	The ship cannot safely pass the bridge owing to the ship conditions.	
Wind speed	6	0.87	The harsh natural environment makes the collision risk increases.	
	1	0.67	The ship cannot safely pass the bridge owing to the ship conditions.	
Visibility	1	0.87	The harsh natural environment makes the collision risk increases.	
	10	0.67	The ship cannot safely pass the bridge owing to the ship conditions.	
Night- time/daytime	Night-time	0.88	The harsh natural environment makes the collision risk increases.	

Concurrent validity is carried out by comparing the risk factors and the modelling process with the existing works focusing on the similar problems. Two existing models which focusing on the ship-ship collision risk modelling are considered in this paper, the comparison results of the concurrent validity are shown in Table 13. It can be seen that the proposed risk model is similar with the study of Perera et al. (2011) though this paper focuses on the ship-bridge collision while that study focused on the ship-ship collision. While using ship behaviours (e.g. heading) to derive risk factors is another solution to define risk factors and this is similar with the study of Mou et al. (2010). In fact, the ship speed is derived by using the historical data in this paper. However, note that although the ship behaviour is derived from the historical data that the ship has safely passed the bridge, there are two deficiencies by using historical data to define membership functions. First, although the ship has safely passed the bridge, it does not mean that the manoeuvring of the ship is perfect. Specifically, if the ship has not successfully changed the course at #6 waypoint owing to human errors or mechanical failure, the ship will have a large possibility to cause collision. This is because the ship will not have enough time to take response actions as the ship is too close to the bridge. Second, it is hard to consider these influencing factors because ship behaviour is derived from the statistical data. This is because the ship behaviour is influenced by the ship particulars and the bridge parameters, and the ship behaviour derived from AIS data cannot consider these parameters.

Table 13 Comparison results of concurrent validity

Items	Mou et al. 2010	Perera et al. 2011	Proposed risk model
Collision type	Ship-ship collision	Ship-ship collision	Ship-bridge collision
Collision risk	Possibility and consequence	Possibility of collision	possibility of collision
Risk factors	Ship particulars, navigational environmental factors	Ship particulars, navigational environmental factors	Ship particulars, bridge parameters, navigational environmental factors
Methods to define risk levels	Historical AIS data	Analysis and expert judgements	Analysis and expert judgements
Modelling method	Regression analysis and other existing models	Fuzzy logic	Fuzzy logic
Model application	For the specific waterway	Generic model	Generic model

(2) Evaluation of the risk analysis

The criteria V1 and V2 are performed in two ways by introducing fuzzy logic method. The first way is to analyse the collision conditions and to define the different risk levels for the different scenarios (R_{ρ} , $L_{O_{\nu}}$ and θ_{vace}). The second way is to use linguistic variables to define the variables with the features of fuzziness or vagueness. However, there is no guarantee that all uncertainties are well addressed in this risk analysis process. It is sure that the linguistic variables can easily distinguish the differences between the two adjacent variables, and when introducing the evidence from the ship passing bridge in reality, this risk analysis model is useful to address the uncertainty caused by the collision conditions and the subjective judgements in the fuzzifying process. Criterion V3 is difficult to verify because each fuzzy logic box cannot be individually verified, that means the subjective fuzzified variables and fuzzy rule base cannot be verified. However, from the final collision risk, this model well meets this V3 criterion. The criterion V4 is also met as this model uses the previous studies (i.e. natural environment factors) and analytical method (collision conditions) to fuzzify the variables fuzzy logic boxes developed in Fig. 4, and the final collision risk can be directly used for collision alert and also modification of the ship trajectory in Subsection 3.4.

5 Conclusions

The main contribution of this paper is to propose a fuzzy logic based risk assessment method for ship-bridge collision considering ship particulars, bridge parameters and natural environment. Specifically, the three-layer framework, including input layer, fuzzy inference layer and output layer, is established, and the collision risk is divided into two parts (i.e. critical conditions and natural environment). Critical conditions are analysed by considering the location, heading, and ship speed. By introducing the ship-bridge collision conditions, the ship particulars and bridge parameters are both considered and fuzzified. From comparison with the modified trajectory, this proposed method can be utilized for improvement of the ship handling in the waterway area around bridge piers.

The proposed method can be applied for the collision risk assessment in different scenarios, including one-way/ two-way traffic bridge, different ship lengths and different bridge parameters in different natural environment (i.e. sea state, wind speed, visibility and daytime/night-time). However, in the future work, the collision avoidance module can be carried out, together with the collision assessment module proposed in this paper for the ship-bridge collision avoidance system. Moreover, the ship-ship collision should also be considered to derive a comprehensive result for maritime safety in the bridge waterway area. Moreover, as this paper only use some specific scenarios for validation, the further analysis should be carried out by applying this model to the real ship-bridge collision alert in the future.

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