

Canal effects on a liner hub location problem

Abstract:

The Panama Canal and the Suez Canal are two key canals around the world, both of which have a great number of connections with other ports because of various liner shipping service routes. Liner carriers can facilitate container transshipment operations by opening hub ports around the canal. This paper aims to investigate canal effects on hub location in liner shipping. In order to describe canal effects, besides its geographic location, this paper mainly considers three aspects: canal toll, canal congestion, and ship capacity limitation when passing through a canal. Several binary linear programming models are mainly developed for our hub location problem considering canal effects. Numerical experiments are carried out to account for the effectiveness of our proposed models.

Keywords: OR in maritime industry; Hub location; Liner shipping; Canal effects

1 Introduction

As two key canals around the world, the Panama Canal connects the Atlantic Ocean with the Pacific Ocean, and the Suez Canal connects the Mediterranean Sea with the Red Sea. In practice, these two canals have a great number of connections with other ports by liner shipping service routes. From this perspective, the two canals are akin to hub ports, which are mainly used to transship containers among different liner shipping service routes. Hence, liner carriers can facilitate container transshipment operations by opening hub ports around the canal. For instance, the port of Balboa connecting to the Panama Canal is one of the most famous hub ports around the world. In other words, a canal may have a significant impact on hub location in liner shipping. This motivates our paper. Especially, we aim to find some important factors used to explain why it is beneficial to establish hub ports around a canal, leading to a hub location problem considering canal effects.

From the carrier perspective, the hub location problem is concerned with locating hub facilities and allocating demand nodes to hubs in order to efficiently route the traffic between origin-destination pairs. Goldman (1969) is the first paper investigating the hub location problem.

Later, many researchers have proposed many different hub location problems and models (Alumur and Kara, 2008): the p -hub median problem, the hub location problem with fixed costs, the p -hub center problem and the hub covering problems. The objective of the p -hub median problem is to minimize the total transportation cost required to serve a given set of flows, given demand nodes, flow between origin-destination pairs and the number of hubs (p) to be opened. The p -hub median problem ignores the fixed costs of opening facilities, which are considered in the hub location problem with fixed costs. The p -hub center problem is a minimax problem that focuses on the minimization of a maximum service or cost measure between origin-destination pairs. The hub covering problem includes the hub set covering problem and the maximal hub covering problem. The hub set covering problem is to locate hubs to cover all demand such that the cost of opening hub facilities is minimized. The maximal hub covering problem maximizes the demand covered with a given number of hubs to be opened. In order to minimize the transportation cost for transporting containers in liner shipping, as well as considering the hub facility (e.g., berth) construction costs, this paper mainly considers the hub location problem with fixed costs. In practice, certain liner shipping companies have their own berths at their major hub ports. For more details on different hub location problems and models, please refer to the review papers: Alumur and Kara (2008), Campbell and O’Kelly (2012) and Farahani et al. (2013).

Related to liner shipping, there are only a few studies on the hub location problem, which is referred to as the liner hub location problem in this paper. Gelareh et al. (2010) proposed a liner hub location problem in a competitive environment, solved by using a Lagrangian relaxation based solution method. Based on a two-stage approach, Sun and Zheng (2016) aimed to find the potential hub locations in a global shipping network. In the first stage, a concave cost multicommodity network flow model is solved to obtain the container flows. In the second stage, an aggregated hubbing probability is evaluated to indicate the potential hub locations. Recently, Zheng et al. (2018) investigated a global liner hub location problem via community structure, which is used to split the global liner hub location problem into some independent subproblems associated with various communities. The location of hub ports within any single community can be independently and efficiently solved. As proved in Zheng et al. (2018), a community structure based optimal hub location solution can be obtained for a global shipping network, by independently solving liner hub location problem for each community. Furthermore, there are some works carried out to investigate the liner hub location problem integrated with ship routing

and fleet deployment (Gelareh and Pisinger, 2011; Gelareh et al., 2013; Zheng et al., 2014, 2015). Gelareh and Pisinger (2011) analyzed the impact of fleet deployment on liner hub-and-spoke (H&S) shipping network design by using a pre-determined discount factor to reflect economies of scale in ship size. Gelareh et al. (2013) proposed a mixed integer linear programming model to study a joint decision of hub location, ship routing and fleet deployment. Based on a two-phase approach, Zheng et al. (2014, 2015) investigated the liner H&S network design problem, where the phase I determines hub location and feeder allocation, and ship routing and fleet deployment are studied in phase II.

Recently, there are some studies related to the canal. Martinez et al. (2016) investigated the impact of the Panama Canal's expansion on the routing of Asia imports into the United States. A two-stage approach is used to address the considered problem. In the first stage, an ocean freight charges estimation model is presented. In the second stage, a logit model is used to estimate the choice between the East Coast Route and the West Coast Route. By using a logit model, Wang et al. (2018) empirically studied how the opening of the Northern Sea Route will influence the Suez Canal Route based on a survey. In these two studies, the route choice behavior is mainly investigated by using a logit model, in order to balance the trade-off between the transit time and the transportation cost for different routes.

To the best of our knowledge, our work is the first study that introduces canal effects into the hub location problem. As mentioned before, a canal may have a significant impact on hub location in liner shipping. In order to describe canal effects, besides its geographic location, this paper mainly considers three aspects: canal toll, canal congestion, and ship capacity limitation when passing through a canal. These three aspects will be introduced in Section 2. Our main work is to investigate how these three aspects affect the location of hub ports around a canal.

The rest of this paper is organized as follows. Section 2 gives notations and assumptions on our liner hub location problem considering canal effects. Section 3 presents two binary linear programming models. Section 4 carries out the numerical experiments to account for the effectiveness of our proposed models. Finally, a summary is given in Section 5.

2 Notations and assumptions

2.1 Canal, community and waterways

Let \mathcal{N} denote a set of ports. These ports are classified into hub ports and feeder ports. This paper investigates the location of hub ports in a region covering a canal. For the sake of easy presentation, the considered region is regarded as a community, where the hub location problem is investigated. Figure 1 shows the ports located in a community covering the Panama Canal. As shown in the inset of Figure 1, Balboa and Manzanillo are two famous hub ports located at two endpoints of the Panama Canal.

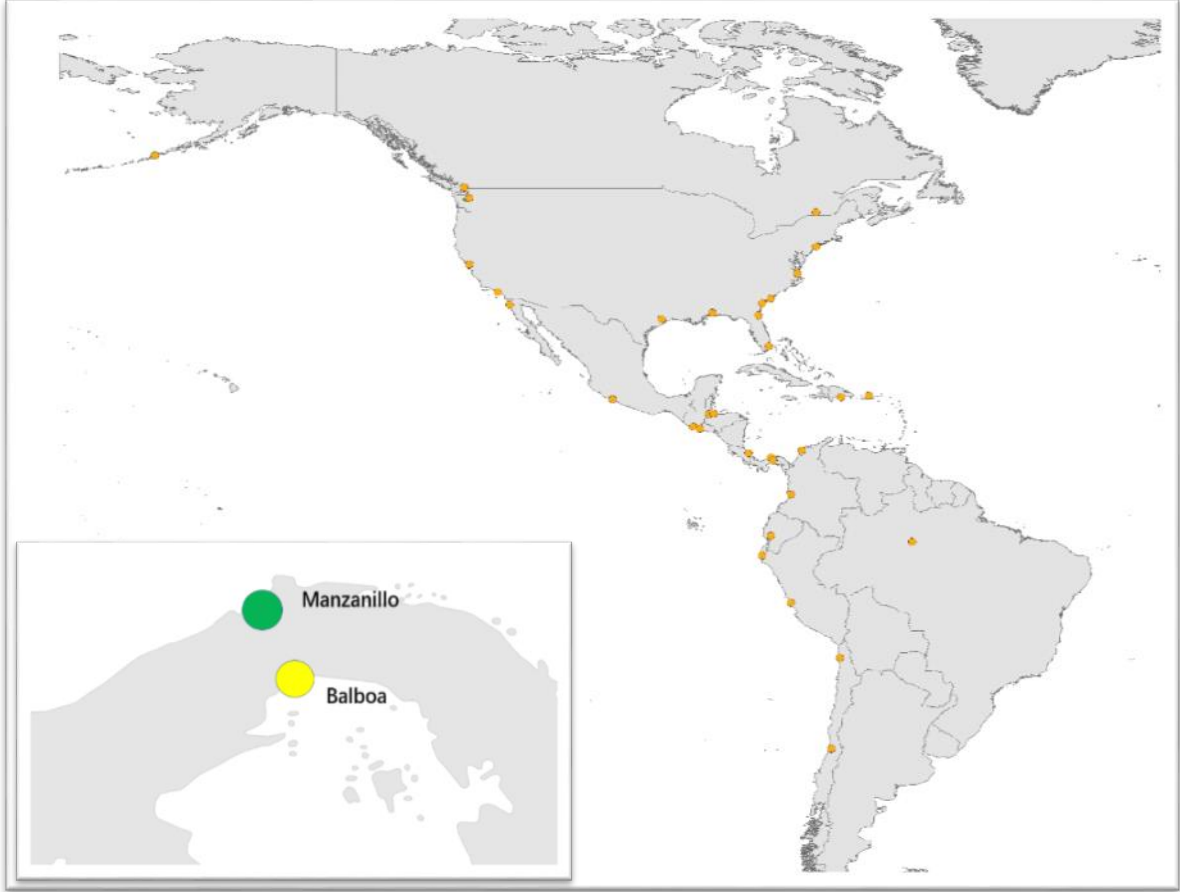


Fig. 1 Ports located in a community covering the Panama Canal.

In maritime transportation, ships sail along waterways. As shown in Sun and Zheng (2016), container flows are often concentrated on some major waterways, which are called main waterways in this paper. Following Zheng et al. (2018), it is reasonable that liner carriers can benefit from economies of scale for shipping containers on the main waterways. For simplicity, we assume that there is only one main waterway within the considered community. This assumption can be relaxed by considering multiple main waterways, as shown later.

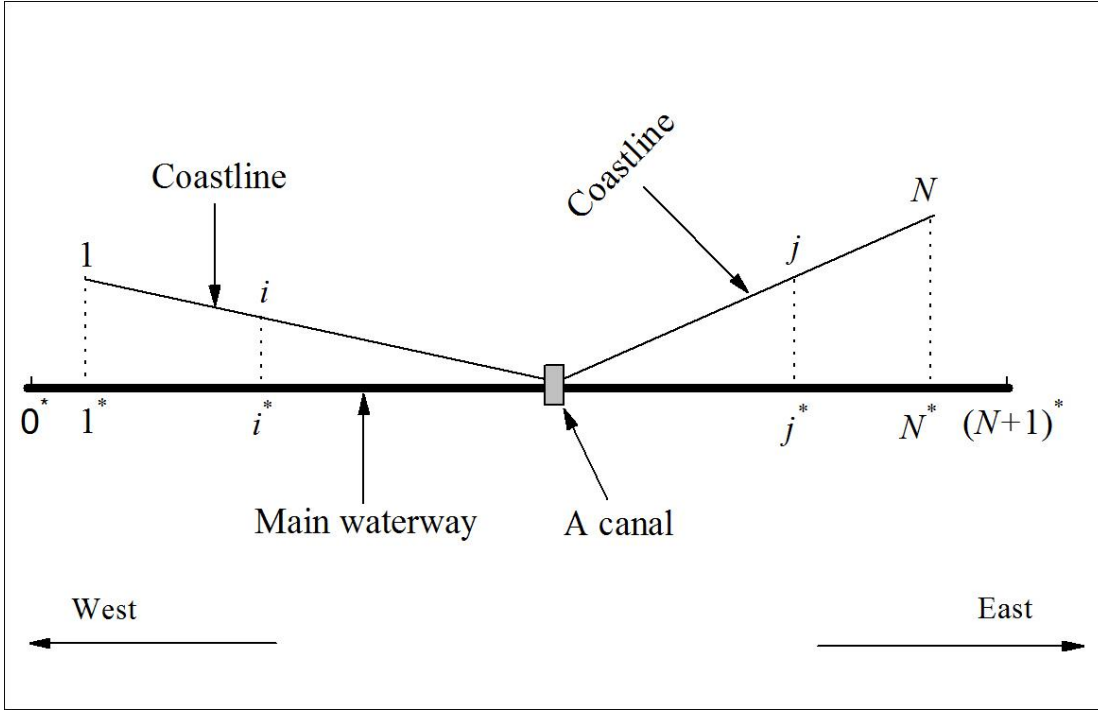


Fig. 2 Illustration of container routing within a community.

2.2 Container routing and cost structure

This paper aims to determine the location of hub ports within a community, while the global container demands are considered. Different from the conventional hub location problem, where cargoes (or containers) are transported from origin nodes to destination nodes (Alumur and Kara, 2008), we only need to determine container routing within the considered community, following Zheng et al. (2018). The community illustrated in Figure 2 consists of $N = |\mathcal{N}|$ ports and a canal. Any port i is projected onto the main waterway, and its projection is indexed by node i^* , via which port i is connected to the main waterway. Let 0^* and $(|\mathcal{N}| + 1)^*$ be two dummy nodes, which denote two endpoints of the main waterway within the community. For any port i , we consider two aggregated container demands (denoted by Q_i^1 and Q_i^2) to be shipped for two directions (i.e., eastbound and westbound). For eastbound (westbound) demand of port i , i.e., Q_i^1 (Q_i^2), we consider container routing between port i and the dummy node $(|\mathcal{N}| + 1)^*$ (0^*) and routed via a hub port. In other words, containers are consolidated and transshipped at hub ports

in order to benefit from economies of scale.

When ships pass through a canal, some generalized costs (time, canal toll, etc.) can be induced. In practice, many ships are waiting for passing through some canals (e.g., the Panama Canal and the Suez Canal) every day and since there is a limited capacity of a canal, traffic congestion often occurs at the canal. As shown in Martinez et al. (2016), it takes an average of 8-10 hours for a container ship to pass through the Panama Canal, with an additional average waiting time of 26-28 hours. When passing through a canal, a canal fee dependent on ship size (or capacity) is often charged. Due to the capacity limitation of a canal, the ship capacity is limited when passing through the canal. Before the Panama Canal expansion, the maximum size of ship that can pass through the canal is restricted to a capacity of approximately 5000 twenty-foot equivalent unit (TEU) containers. In September 2007, work on the Panama Canal expansion project was begun. This expansion project was completed on June 26, 2016 (<http://micanaldepanama.com/expansion/>). After expansion, deeper water capacity along the canal allows ships up to 13000 TEUs to pass through the Panama Canal. Similarly, the original Suez Canal was a single-lane waterway, and then construction was launched to expand and widen the waterway channel in August 2014 (<https://www.caironews.net/news/224460353/new-suez-canal-project-proposed-by-egypt-to-boost-trade>). In order to generally describe canal effects on our liner hub location problem, this paper mainly considers three aspects: canal toll, canal congestion, and ship capacity limitation when passing through a canal, shown as follows.

On the cost for transporting containers, this paper considers the transportation cost along the waterway and some generalized costs induced when passing through a canal. For the transportation cost, we mainly consider the bunker cost, which is the major component of ship operating cost. For a particular ship with a fixed sailing speed, its bunker cost is proportional to the sailing distance. Since the decision making of our liner hub location problem is at a strategic level, sailing speed optimization is not studied here. Hence, it is reasonable to assume that the transportation cost is proportional to the oceanic distance. Let c_{ij} denote the cost for transporting one TEU container between ports (or nodes) i and j .

$$c_{ij} = c_{unit} \times Dis_{ij} \quad (1)$$

where Dis_{ij} is the oceanic distance between ports (or nodes) i and j . In order to determine the coefficient c_{unit} ($\$/TEU \times n.mile$), this paper mainly considers the bunker cost of a particular

ship type such as $Cap = 5000$ (TEUs). Namely,

$$c_{unit} = \frac{Q_{fuel} \times P_{fuel}}{V_{eco} \times 24 \times Cap} \quad (2)$$

where Q_{fuel} (ton/d) denotes daily fuel consumption at economic speed V_{eco} and P_{fuel} ($\$/ton$) is an average fuel price. Imagine that ship is full loaded, $Q_{fuel} \times P_{fuel}$ is the daily bunker cost while transporting Cap containers and the sailing distance per day is $V_{eco} \times 24$. Following Wang and Meng (2012), $Q_{fuel} = 51$, $V_{eco} = 17$ and $P_{fuel} = 330$ are used, and then the coefficient c_{unit} is calculated as 0.00825.

For canal charges, we simply consider the canal toll c^{canal} . In practice, with the increase of ship size, the average canal toll per container will become smaller, according to the provided data shown in Brouer et al. (2014). Namely, there exist scale economies on ship size for the canal toll. Generally, large or mega ships often sail along the main waterways. Hence we assume that the canal toll has a discount factor β ($0 < \beta \leq 1$) when passing through the canal along the main waterway. For simplicity, $\beta=0.5$ is used in our numerical experiments.

For canal congestion, we consider an extra waiting cost when passing through a canal. For simplicity, we consider a value of time per container per hour, indexed by c^{time} . In 2011, the Maersk Line declared that if containers are delayed for 1 to 3 days, \$100 per container will be paid to the shipper as a deferred compensation. Similarly, $c^{time} = \$100 / 24$ per container per hour is used in this paper. Let t_{wait} denote the number of hours used for passing through a canal, including the waiting time.

As mentioned before, ship capacity is limited when passing through a canal. It is reasonable that economies of scale for shipping containers will be affected because of ship capacity limitation when passing through a canal. An influence factor α ($\alpha \geq 1$) is introduced to express this effect, similar to a transportation discount factor used in the conventional hub location problem (Alumur and Kara, 2008).

When feeder port i is allocated to hub port j , we consider two costs (denoted by w_{ij}^1 and w_{ij}^2) on transporting containers associated with feeder port i for two shipping directions (eastbound and westbound, respectively), both of which can be calculated as follows:

$$w_{ij}^1 = \left(c_{ij} + \sigma \times c_{jj^*} \right) \times Q_i^1 + \sigma \times \left[\alpha \times \theta_{j^*0^*} + \left(1 - \theta_{j^*0^*} \right) \right] \times c_{j^*0^*} \times Q_i^1 + c^{canal} \times Q_i^1 \times \left(\theta_{ij} + \beta \times \theta_{j^*0^*} \right) + c^{time} \times t_{wait} \times Q_i^1 \times \left(\theta_{ij} + \theta_{j^*0^*} \right) \quad (3)$$

$$w_{ij}^2 = \left(c_{ij} + \sigma \times c_{jj^*} \right) \times Q_i^2 + \sigma \times \left[\alpha \times \theta_{j^* (|\mathcal{N}|+1)^*} + \left(1 - \theta_{j^* (|\mathcal{N}|+1)^*} \right) \right] \times c_{j^* (|\mathcal{N}|+1)^*} \times Q_i^2 + c^{canal} \times Q_i^2 \times \left(\theta_{ij} + \beta \times \theta_{j^* (|\mathcal{N}|+1)^*} \right) + c^{time} \times t_{wait} \times Q_i^2 \times \left(\theta_{ij} + \theta_{j^* (|\mathcal{N}|+1)^*} \right) \quad (4)$$

where σ ($0 < \sigma \leq 1$) is a transportation discount factor, following the conventional hub location problem. Let parameter θ_{ij} equal 1 if the canal should be passed when moving between ports (or nodes) i and j , and 0 otherwise. Clearly, the two costs (w_{ij}^1 and w_{ij}^2) for transporting containers associated with feeder port i allocated to hub port j are composed of the following terms: the transportation cost between feeder port i and its associated hub port j ($c_{ij} \times Q_i^1$ or $c_{ij} \times Q_i^2$), the transportation cost between hub port j and its projection j^* ($\sigma \times c_{jj^*} \times Q_i^1$ or $\sigma \times c_{jj^*} \times Q_i^2$), the transportation cost between j^* and 0^* ($\sigma \times \left[\alpha \times \theta_{j^*0^*} + \left(1 - \theta_{j^*0^*} \right) \right] \times c_{j^*0^*} \times Q_i^1$) or the transportation cost between j^* and $(|\mathcal{N}|+1)^*$ ($\sigma \times \left[\alpha \times \theta_{j^* (|\mathcal{N}|+1)^*} + \left(1 - \theta_{j^* (|\mathcal{N}|+1)^*} \right) \right] \times c_{j^* (|\mathcal{N}|+1)^*} \times Q_i^2$), and the canal toll ($c^{canal} \times Q_i^1 \times \left(\theta_{ij} + \beta \times \theta_{j^*0^*} \right)$ or $c^{canal} \times Q_i^2 \times \left(\theta_{ij} + \beta \times \theta_{j^* (|\mathcal{N}|+1)^*} \right)$) and the waiting cost ($c^{time} \times t_{wait} \times Q_i^1 \times \left(\theta_{ij} + \theta_{j^*0^*} \right)$ or $c^{time} \times t_{wait} \times Q_i^2 \times \left(\theta_{ij} + \theta_{j^* (|\mathcal{N}|+1)^*} \right)$) induced when passing through a canal. Note that a canal may be passed twice when transporting containers of certain feeder ports.

In practice, some liner shipping companies have their own berths (i.e., hub facilities) at their major hub ports. Hence our hub location model also considers the hub facility construction costs. For any port j , we determine its hub facility construction cost F_j per week. For simplicity, we mainly consider the investment cost c_j^{invest} . We assume that hub facilities can be successively operated for τ years. Let r_j denote the local discount rate at port j . Then, the hub facility construction cost can be calculated as follows.

$$F_j \times 52 \times (1 + r_j)^\tau + F_j \times 52 \times (1 + r_j)^{\tau-1} + \dots + F_j \times 52 = F_j \times 52 \times \frac{(1 + r_j)^\tau - 1}{r_j} = (1 + r_j)^\tau \times c_j^{invest} \quad (5)$$

where 52 is the number of weeks per year. The left side of Eq. (5) denotes the sum of the chance cost of the identical construction cost per year over τ years. The right side of Eq. (5) is the chance cost of the investment cost after τ years. Then we have

$$F_j = \frac{r_j \times (1+r_j)^\tau}{[(1+r_j)^\tau - 1]} \times c_j^{invest} / 52 \quad (6)$$

According to the berth construction investment cost on port of Ningbo (<https://zj.zjol.com.cn/news/698333.html>), its hub facility construction cost can be calculated. For simplicity, parameters $\tau=30$ and $r_j=0.05$ are used in this paper. For the rest ports, we assume that the hub facility construction costs are proportional to that of Ningbo port. The proportional coefficients for different ports are determined with respect to their inland economic levels mainly expressed by using the average local hotel prices, which are inquired by using Google maps.

3 Model

This section proposes linear programming models for the single allocation liner hub location problem, where containers from (or to) a feeder port must be transshipped at one hub, and the multiple allocation liner hub location problem, where at most two hub ports (one for eastbound containers and the other for westbound containers) are used to transship containers from (or to) a feeder port.

3.1 Single allocation

The decision variables for the single allocation liner hub location problem can be defined as follows:

- z_j : A binary variable which takes value 1 if port j is selected to be a hub port, and 0 otherwise;
- z_{ij} : A binary variable which takes value 1 if feeder port i is allocated to hub port j , and 0 otherwise.

The single allocation liner hub location problem can be formulated by the following binary linear programming model:

$$\min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} (w_{ij}^1 + w_{ij}^2) \times z_{ij} + \sum_{j \in \mathcal{N}} F_j \times z_j \quad (7)$$

subject to

$$\sum_{j \in \mathcal{N}} z_{ij} = 1, \forall i \in \mathcal{N}; \quad (8)$$

$$z_{ij} \leq z_j, \forall i, j \in \mathcal{N}; \quad (9)$$

$$\sum_{j \in \mathcal{N}} z_j = p; \quad (10)$$

$$z_j, z_{ij} \in \{0, 1\}, \forall i, j \in \mathcal{N}. \quad (11)$$

where p is the number of hub ports to be established. Constraints (8) mean that any port should be allocated to one hub port. Constraints (9) state that any feeder port can only be allocated to a hub port. Constraint (10) shows that p hub ports will be established. Constraints (11) define the domain of the decision variables.

3.2 Multiple allocation

The decision variables for the multiple allocation liner hub location problem can be defined as follows:

z_j : A binary variable which takes value 1 if port j is selected to be a hub port, and 0 otherwise;

z_{ij}^1 : A binary variable which takes value 1 if Q_i^1 containers of feeder port i are routed via hub port j , and 0 otherwise;

z_{ij}^2 : A binary variable which takes value 1 if Q_i^2 containers of feeder port i are routed via hub port j , and 0 otherwise.

The multiple allocation liner hub location problem can be formulated by the following binary linear programming model:

$$\min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} (w_{ij}^1 \times z_{ij}^1 + w_{ij}^2 \times z_{ij}^2) + \sum_{j \in \mathcal{N}} F_j \times z_j \quad (12)$$

subject to

$$(10),$$

$$\sum_{j \in \mathcal{N}} z_{ij}^1 = 1, \text{ if } Q_i^1 > 0, \forall i \in \mathcal{N}; \quad (13)$$

$$\sum_{j \in \mathcal{N}} z_{ij}^2 = 1, \text{ if } Q_i^2 > 0, \forall i \in \mathcal{N}; \quad (14)$$

$$z_{ij}^1 \leq z_j, \forall i, j \in \mathcal{N}; \quad (15)$$

$$z_{ij}^2 \leq z_j, \forall i, j \in \mathcal{N}; \quad (16)$$

$$z_j, z_{ij}^1, z_{ij}^2 \in \{0, 1\}, \forall i, j \in \mathcal{N}. \quad (17)$$

Constraints (13) and (14) mean that any port should be allocated to one hub port in order to transport its associated containers for each of two shipping directions. Constraints (15) and (16) state that containers of any feeder port should be routed via a hub port. Constraints (17) define the domain of the decision variables.

4 Numerical experiments

In this section, we provide the numerical results in order to discuss the effectiveness of our models. The origin-destination container demand is provided by a liner shipping company. According to different trade lanes including Trans-Pacific, Trans-Atlantic and Asia-Europe, the two container demands associated with any port i (Q_i^1 and Q_i^2) can be simply derived. The container demands and distance information associated with each port are shown in the Appendix. The proposed mathematical programming models are solved by CPLEX, which runs on a 3.2 GHz Dual Core desktop PC with the Windows 7 operating system and 4 GB of RAM. Our models can be efficiently solved within 1 second.

In order to measure canal effects on our liner hub location problem, the impacts of canal toll (c^{canal}), canal congestion (t_{wait}) and ship capacity limitation when passing through a canal (α) are mainly discussed. In the following, we mainly consider two instances: a community covering the Panama Canal and a community covering the Suez Canal. Moreover, an extended instance is studied by considering multiple main waterways within a single community.

4.1 The case of Suez Canal

In this subsection, we aim to show the results of hub location around the Suez Canal. The community covering the Suez Canal is shown in Figure 3.

Firstly, we give the hub location solutions for different numbers of hub ports to be opened under different allocation schemes, as shown in Table 1. Following Martinez et al. (2016), $t_{wait} = 35$ is used here. Following Brouer et al. (2014), $c^{canal} = 72$ is used. In addition, $\alpha = 1.5$ are typically used. Clearly, the hub location results are identical for single allocation and multiple allocation. As shown in Figure 3, Damietta is located at the north side of the Suez Canal and Sokhna is located at the south side of the Suez Canal). From Table 1, we can find that both ports

are always chosen to be hub ports in different cases. Our models also imply that it is reasonable to open hub ports around a canal.

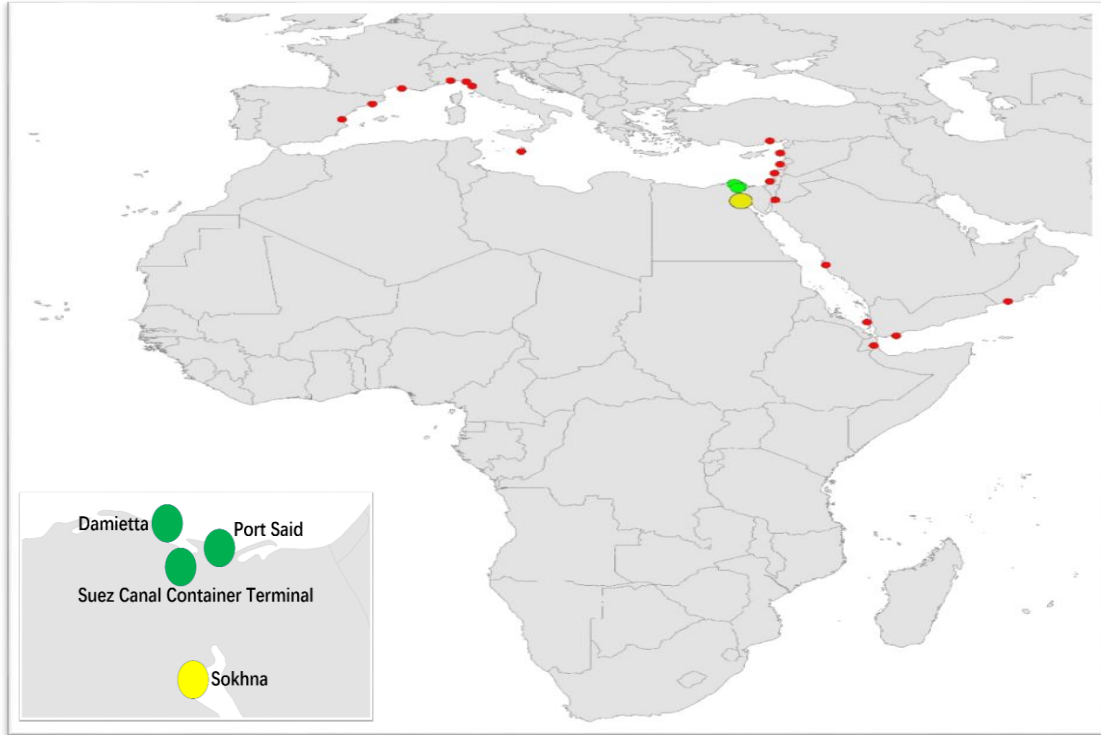


Fig. 3 Ports located in a community covering the Suez Canal.

Table 1 Hub location solutions for the community covering the Suez Canal.

p	Single allocation	Multiple allocation
2	Damietta, Sokhna	Damietta, Sokhna
3	Damietta, Sokhna, Mersin	Damietta, Sokhna, Mersin
4	Damietta, Sokhna, Mersin, Suez Canal Container Terminal	Damietta, Sokhna, Mersin, Suez Canal Container Terminal

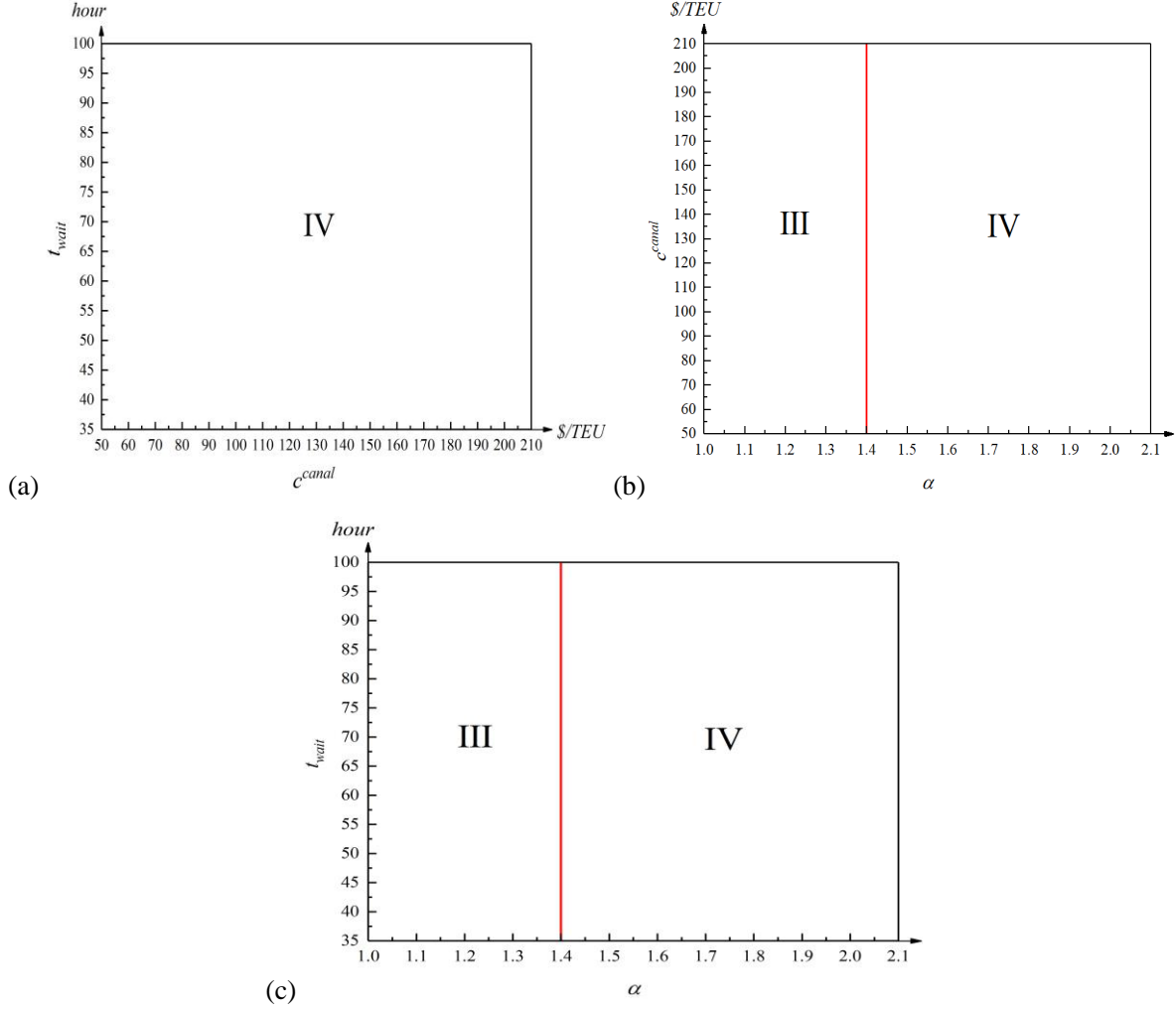


Fig. 4 The phase diagram for the community covering the Suez Canal.

Next, we explore the effect of the Suez Canal on hub location around the canal. For simplicity, we only show the results for the single allocation liner hub location problem. As shown in the inset of Figure 3, there are three ports (Damietta, Port Said and Suez Canal Container Terminal) located at the north side of the Suez Canal and there is one port (Sokhna) located at the south side of the Suez Canal. Here we discuss whether hub ports should be opened at the two sides of the Suez Canal, by using phase diagram in different spaces via three parameters (c^{canal} , t_{wait} and α), as shown in Figure 4. Here, the phase diagram can be generally divided into four regions. In region I, none of the four ports (Damietta, Port Said, Suez Canal Container Terminal and Sokhna) is a hub port. In region II, Sokhna is a hub port while none of the three ports (Damietta, Port Said and Suez Canal Container Terminal) is a hub port. In region

III, at least one of the three ports (Damietta, Port Said and Suez Canal Container Terminal) is a hub port while Sokhna is not a hub port. In region IV, at least one of the three ports (Damietta, Port Said and Suez Canal Container Terminal) is a hub port and Sokhna is also a hub port.

In Figure 4, $p = 2$ is adopted. Figure 4(a) shows the phase diagram in (t_{wait}, c^{canal}) space, where $\alpha = 1.5$ is used, and only one region (region IV) is discovered. Namely, hub ports are always opened at the two sides of the Suez Canal in this case. Figure 4(b) shows the phase diagram in (c^{canal}, α) space, where $t_{wait} = 35$ is used. Figure 4(c) shows the phase diagram in (t_{wait}, α) space, where $c^{canal} = 72$ is used. Both Figures 4(b) and 4(c) show that Sokhna is not chosen to be a hub port when $1 \leq \alpha \leq 1.4$. Namely, it is not economic to open a hub port at the south side of the Suez Canal in this case. From Figure 4, we can infer that parameter α has a bigger impact on hub location around a canal, as compared with parameters t_{wait} and c^{canal} . This phenomenon is partly supported by a simple theoretical analysis as follows.

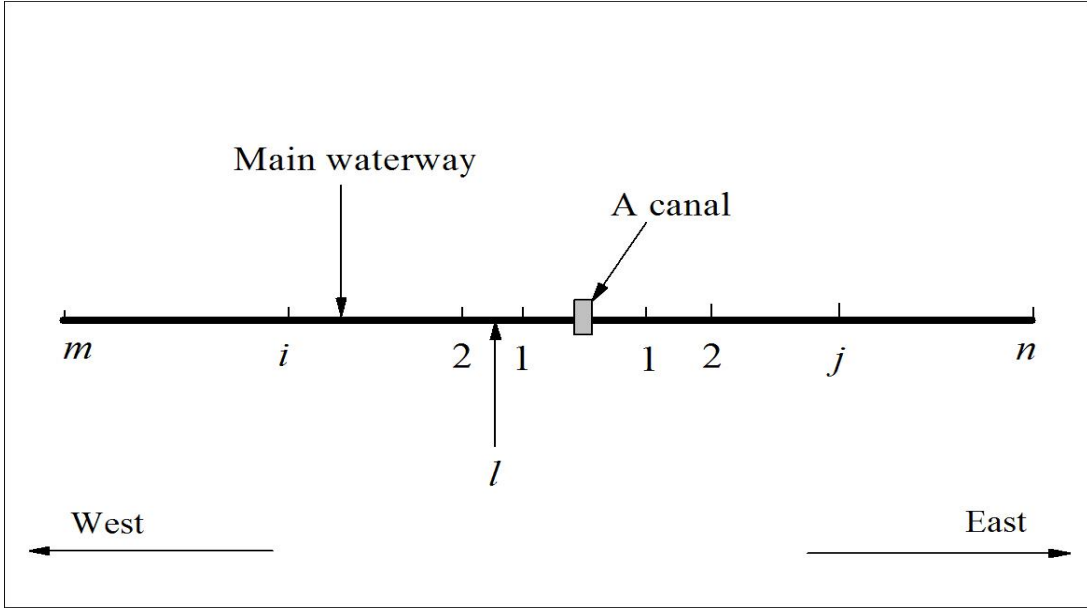


Fig. 5 An abstract community.

In order to provide a theoretical analysis on hub location around a canal, here we consider an abstract community shown in Figure 5. This community consists of $(n + m + 2)$ ports located along the coastline overlapped by the main waterway, which simplifies the community covering

the Suez Canal shown in Figure 3. In order to simplify our theoretical analysis, here we further make the following assumptions.

- (a) The length of the canal is ignored.
- (b) There are two hub ports to be opened for this community, and these two hub ports are located at the two sides of the canal.
- (c) Any feeder port can be allocated to a hub port located at the same side of the canal.
- (d) Ports are equally distributed along the coastline, and there are two ports located at two endpoints of the canal.
- (e) Various ports have identical container demands to be transported for two different shipping directions (eastbound and westbound).
- (f) There is no container demand between any two ports within the community.

Note that the above six assumptions are only considered in our theoretical analysis. In order to proceed, let l denote the distance between any two adjacent ports, except for the two ports connected to the canal. Let Q denote the weekly number of containers associated with any single port, to be transported for any one of the two shipping directions. As shown in Figure 5, let $TC(i)$ denote the total cost for transporting containers of ports located at the west side of the canal when port i is selected to be a hub port. The total cost can be calculated as follows:

$$\begin{aligned} \frac{TC(i)}{c_{unit} \times Q} = & \frac{1+i}{2} \times i \times l \times 2 + \frac{1+(m-i)}{2} \times (m-i) \times l \times 2 \\ & + \sigma \times (m+1) \times (m-i) \times l + \sigma \times \alpha \times (m+1) \times (n+i) \times l \\ & + \frac{1}{c_{unit}} \left[\left(\beta \times c^{canal} + c^{time} \times t_{wait} \right) \times (m+1) \right] \end{aligned} \quad (18)$$

The total cost includes three terms: the cost spent on transporting containers between the hub port and its feeder ports, the cost spent on transporting containers along the main waterway for two shipping directions, and the extra cost (canal toll and the waiting cost) induced when passing through a canal. Clearly, the canal toll and the waiting cost do not affect the hub location solution in this case. This is mainly because of our third assumption. Hence, these two costs can be omitted from Eq. (18) in order for simple presentation. Then we have,

$$\begin{aligned} \frac{TC(i)}{l \times c_{unit} \times Q} = & 2 \times i^2 - \left[2 \times m + \sigma \times (1-\alpha) \times (m+1) \right] \times i \\ & + m^2 + m + \sigma \times (m+1) \times (m + \alpha \times n) \end{aligned} \quad (19)$$

If $\frac{2 \times m + \sigma \times (1 - \alpha) \times (m + 1)}{4} \leq 0.5$ is satisfied, namely,

$$\alpha \geq 1 + \frac{2 \times (m - 1)}{\sigma \times (m + 1)}, \quad (20)$$

and then the optimal hub port for the west side of the canal is located at $i^{\text{opt}} = 0$. As the increase of α , Eq. (20) can be satisfied more easily for a fixed m , and then ports connecting to a canal are apt to be hub ports. Obviously, the theoretical analysis is consistent with the numerical analysis shown in Figures 4(b) and 4(c). Similar results can be obtained for the east side of the canal.

4.2 The case of Panama Canal

In this subsection, we aim to show the results of hub location around the Panama Canal. The community covering the Panama Canal is shown in Figure 1. Similarly, we mainly explore the effect of the Panama Canal on hub location around the canal. For simplicity, we only show the results for the single allocation liner hub location problem.

In practice, Balboa and Manzanillo are two famous hub ports located at two endpoints of the Panama Canal. Here we discuss whether Balboa and Manzanillo should be hub ports or not, by using the phase diagram, as shown in Figure 6, where $p = 4$ is adopted. Clearly, the phase diagram can be divided into four regions. In region I, none of these two ports (Balboa and Manzanillo) is a hub port. In region II, Manzanillo is a hub port while Balboa is not a hub port. In region III, Balboa is a hub port while Manzanillo is not a hub port. In region IV, both Balboa and Manzanillo are hub ports.

Figure 6(a) shows the phase diagram in $(t_{\text{wait}}, c^{\text{canal}})$ space, where $\alpha = 2$ is used, and two regions (region II and region IV) are categorized. In this case, Balboa is not selected to be a hub port when $c^{\text{canal}} \geq 60$, while Manzanillo is always chosen to be a hub port. Figure 6(b) shows the phase diagram in $(c^{\text{canal}}, \alpha)$ space, where $t_{\text{wait}} = 35$ is used, and three regions (region I, region II and region IV) are categorized. Figure 6(c) shows the phase diagram in $(t_{\text{wait}}, \alpha)$ space, where $c^{\text{canal}} = 72$ is used, and two regions (region I and region II) are categorized. Both Figures 6(b) and 6(c) show that none of Balboa and Manzanillo is suitable to be a hub port when $1.1 \leq \alpha \leq 1.7$. In $(c^{\text{canal}}, \alpha)$ space, both Balboa and Manzanillo are apt to be hub ports when $c^{\text{canal}} \leq 70$ and $\alpha \geq 1.4$.

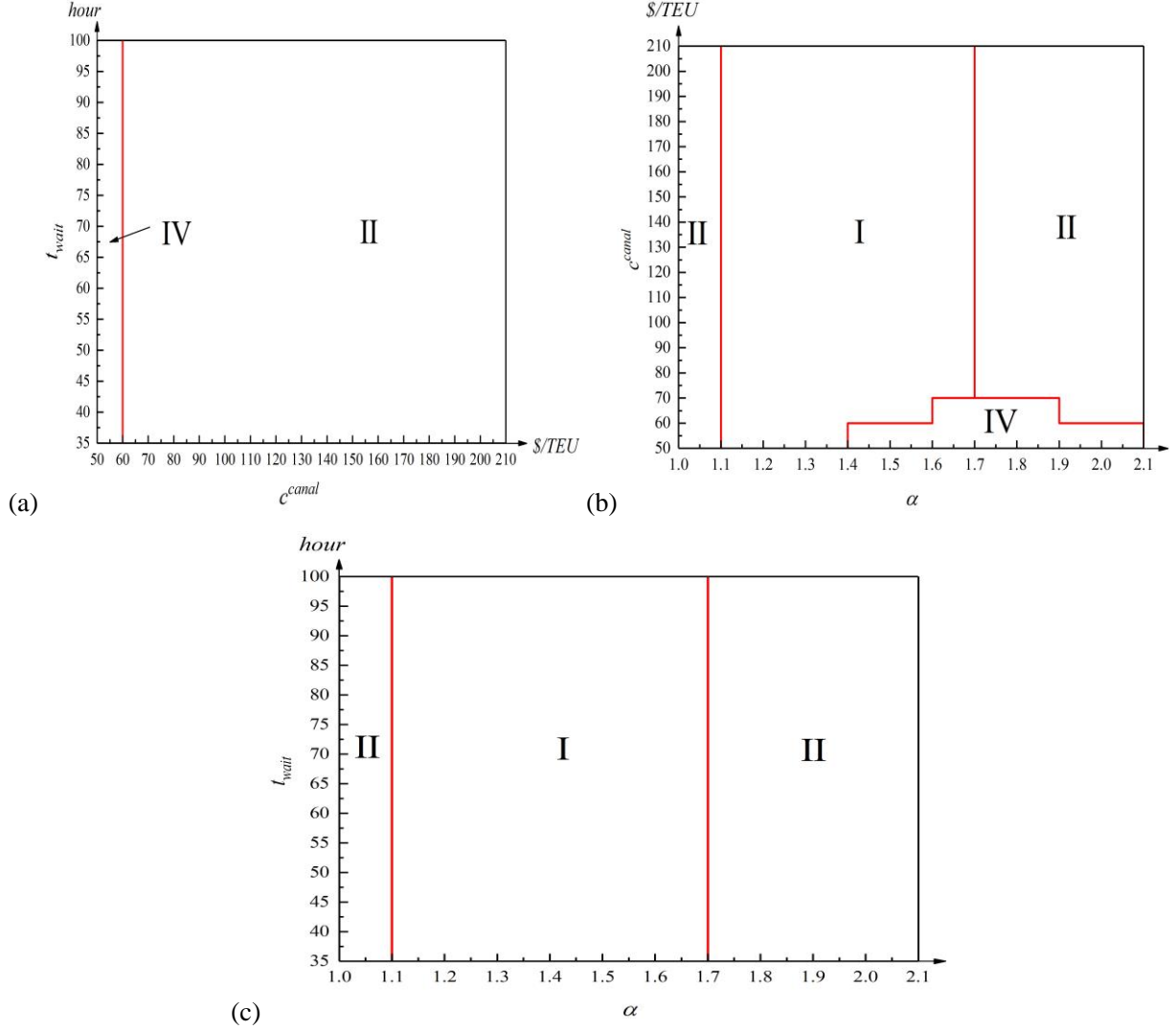


Fig. 6 The phase diagram for the community covering the Panama Canal.

4.3 Different demand scenarios

This subsection aims to show the results of hub location around a canal under different demand scenarios. Two scenarios (Scenario I and Scenario II) are mainly considered. In Scenario I, the container demand of each port is decreased by 20% with probability p_1 . In Scenario II, the container demand of each port is increased by 20% with probability p_2 . The results for $p_1 = p_2 = 0.5$ are typically shown in Figure 7. Figures 7(a) and 7(b) show the results for the community covering the Suez Canal in Scenario I and Scenario II, respectively, while Figures 7(c) and 7(d) show these results for the Panama Canal.

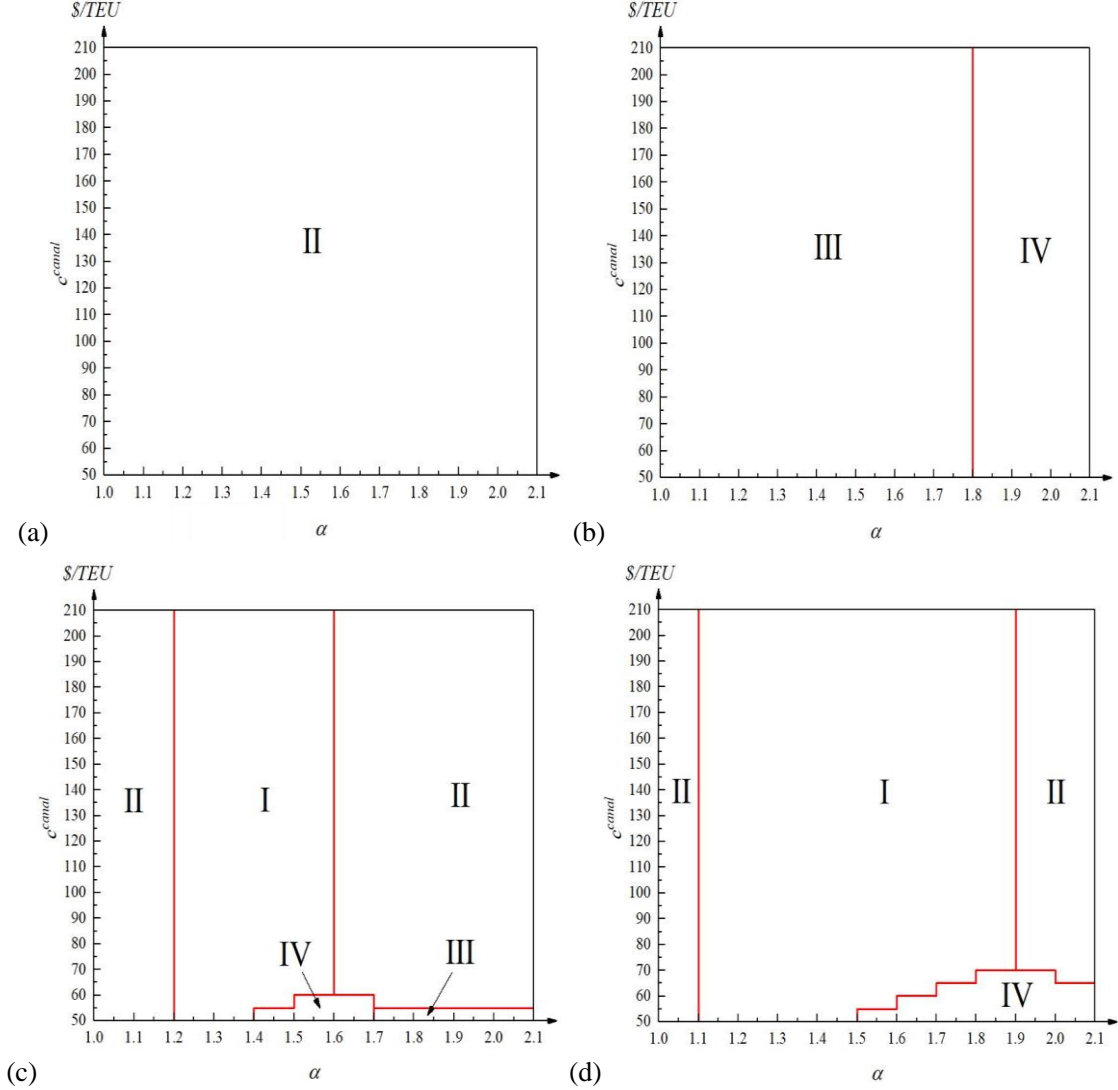


Fig. 7 The phase diagram under different demand scenarios.

As shown in Figures 7(a) and 7(b), the phase diagram for the community covering the Suez Canal in Scenario I is quite different from that shown in Figure 4(b), while the phase diagram in Scenario II is similar to that shown in Figure 4(b). This is because the container demand distribution at the north side of the Suez Canal is affected in Scenario I, especially for the three ports (Damietta, Port Said and Suez Canal Container Terminal) near the Suez Canal. As a result, none of these three ports is chosen to be a hub port in this case. From Figures 7(c) and 7(d), the phase diagrams for the community covering the Panama Canal in the above two scenarios are similar to that shown in Figure 6(b). This may be because of different port distributions leading to different demand distributions, which are quite different between the two

communities covering the Suez Canal and the Panama Canal. Actually, ports are relatively evenly distributed around the two sides of the Panama Canal, as compared with that for the Suez Canal.

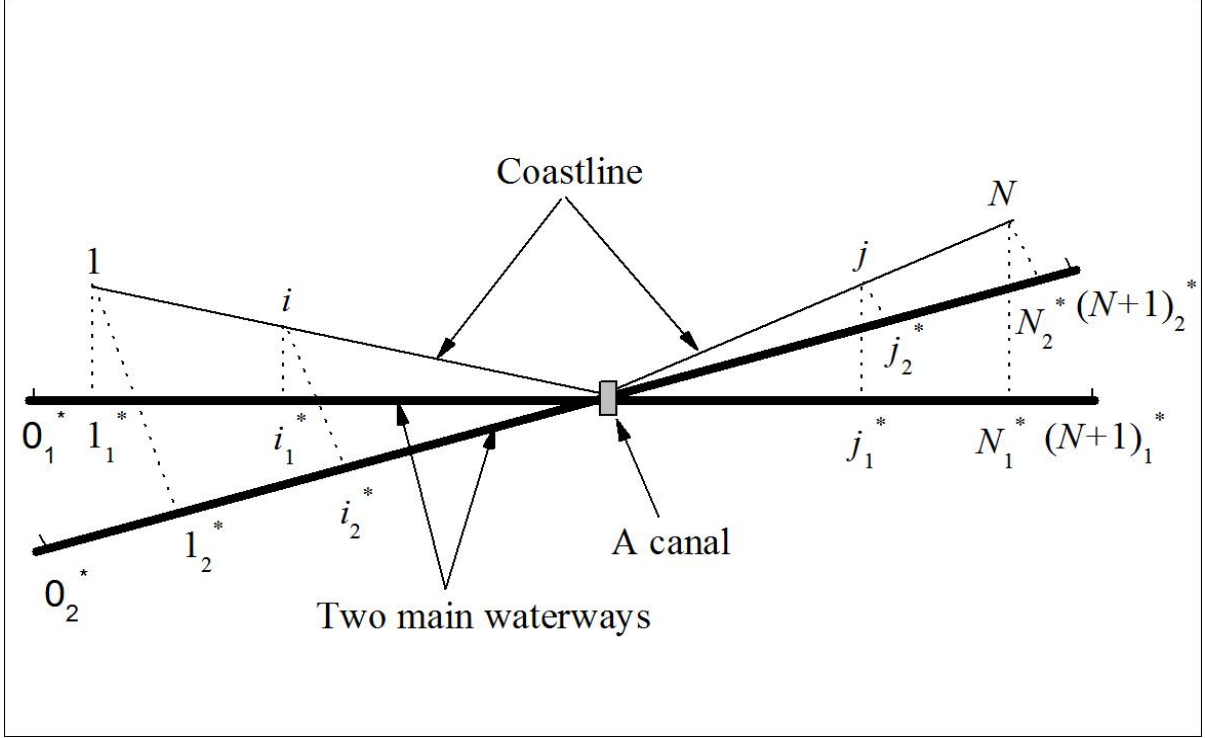


Fig. 8 An illustration of two main waterways in a single community.

4.4 An extension with multiple main waterways

In this subsection, we relax the assumption that there is only one main waterway in a single community. Namely, multiple main waterways are considered here, as illustrated in Figure 8. Obviously, one main waterway is proper for the community covering the Suez Canal, as shown in Figure 3. Hence, we consider multiple main waterways for the community covering the Panama Canal. Clearly, we should also address the selection of main waterways when routing containers from origin ports to destination ports, which is called main waterway choice problem. Let \mathcal{Q} denote the set of container demands, and let q_{rs} ($\forall q_{rs} \in \mathcal{Q}$) be the weekly number of containers transported from port r to port s , i.e., OD pair $\langle r, s \rangle$. For any OD pair $\langle r, s \rangle$, its origin port r and destination port s are generally located at two different communities, respectively. In order to proceed, let H_k denote the number of main waterways in community k . Let \mathcal{N}_k denote the set of ports in community k . Let \mathcal{K} denote the set of communities. For

any port i ($\forall i \in \mathcal{N}_k$), its projection on main waterway h ($\forall h=1, \dots, H_k$) is i_h^* , as illustrated in Figure 8. Two dummy nodes at the end of main waterway h are represented by 0_h^* and $(|\mathcal{N}_k|+1)_h^*$, respectively. In order to simplify our problem with multiple main waterways, we address the main waterway choice problem, before formulating our liner hub location problem. Generally, the main waterway choice problem is mainly dependent on the geographic locations of the origin ports and the destination ports. It is reasonable to assume that the nearest main waterways are selected in order to minimize the total transportation distance from the origin ports to the destination ports. Let π_{hg}^{rs} be a decision variable, which equals 1 if main waterways h and g are chosen for origin port r and destination port s , respectively, and 0 otherwise. Let ψ_{rs}^1 and ψ_{rs}^2 represent two transportation directions for OD pair $\langle r, s \rangle$. If the transportation direction is westbound, $\psi_{rs}^1=1$ and $\psi_{rs}^2=0$, otherwise $\psi_{rs}^1=0$ and $\psi_{rs}^2=1$. Then the main waterway choice problem can be formulated as follows:

$$\begin{aligned} \min \quad & \sum_{(r,s) \in \mathcal{Q}} \sum_{g=1}^{H_{k(s)}} \sum_{h=1}^{H_{k(r)}} q_{rs} \times \pi_{hg}^{rs} \times [Dis_{rr_h^*}^{rs} + Dis_{ss_g^*}^{rs} \\ & + \left(Dis_{r_h^* 0_h^*}^{rs} + Dis_{0_h^* (|\mathcal{N}_{k(s)}|+1)_h^*}^{rs} + Dis_{(|\mathcal{N}_{k(s)}|+1)_h^* s_g^*}^{rs} \right) \times \psi_{rs}^1 + \left(Dis_{i_h^* (|\mathcal{N}_{k(r)}|+1)_h^*}^{rs} + Dis_{(|\mathcal{N}_{k(r)}|+1)_h^* 0_g^*}^{rs} + Dis_{0_g^* s_g^*}^{rs} \right) \times \psi_{rs}^2] \end{aligned} \quad (21)$$

subject to

$$\sum_{g=1}^{H_{k(s)}} \sum_{h=1}^{H_{k(r)}} \pi_{hg}^{rs} = 1, \forall (r, s) \in \mathcal{Q}; \quad (22)$$

$$\pi_{hg}^{rs} \in \{0, 1\}, \forall g=1, \dots, H_{k(s)}, \forall h=1, \dots, H_{k(r)}, \forall (r, s) \in \mathcal{Q}. \quad (23)$$

where $k(r)$ and $k(s)$ denote the communities containing port r and s , respectively. The objective function considers transportation distances at different segments including two possible transportation directions (i.e., eastbound and westbound) for each OD pair, each term of which is illustrated in Figure 9. For any OD pair $\langle r, s \rangle$, constraints (22) mean that two main waterways will be selected for origin port r and destination port s , respectively.

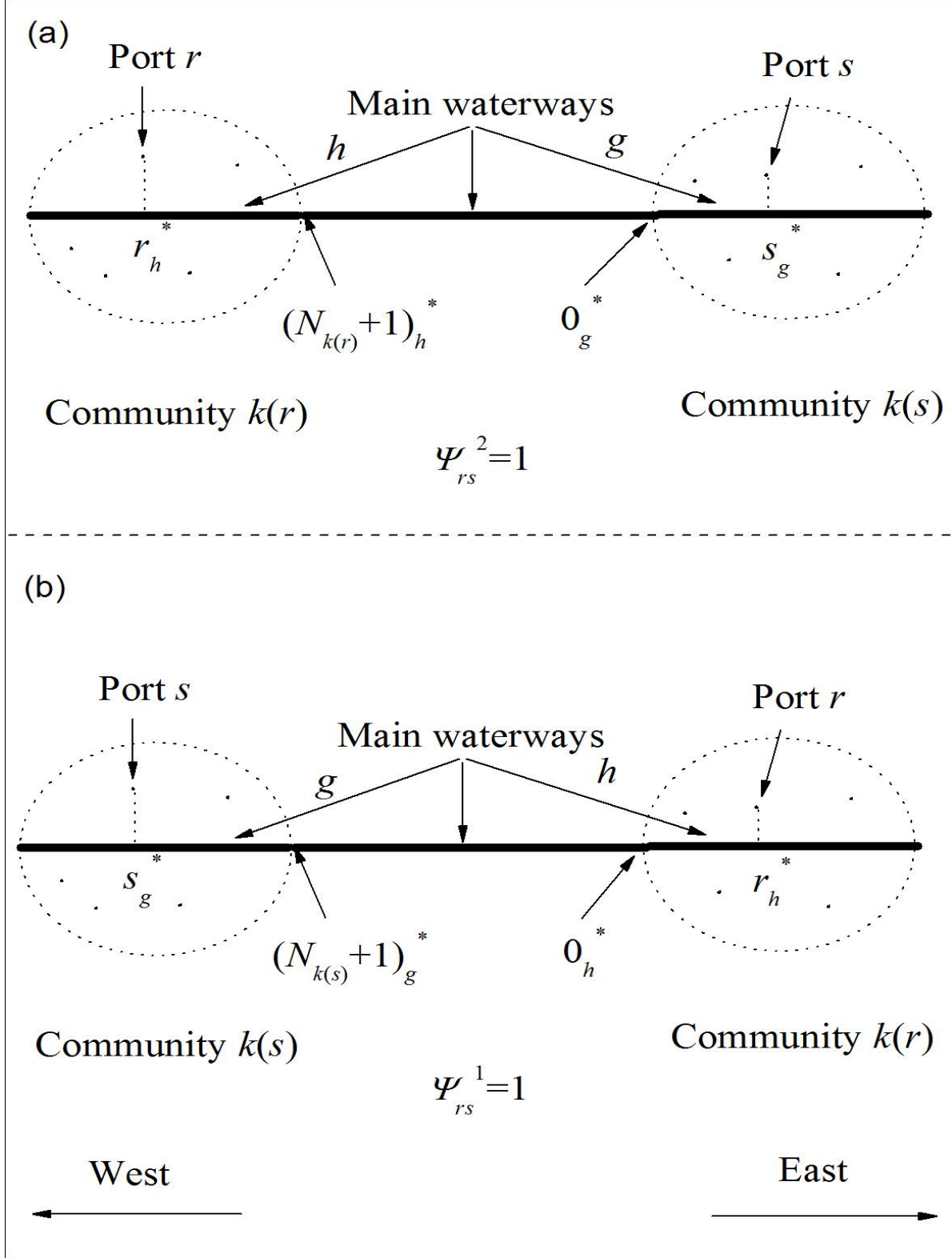


Fig. 9 An illustration of the objective function of the main waterway choice problem with respect to OD pair $\langle r, s \rangle$ in two cases: (a) eastbound and (b) westbound.

By considering the selection of main waterways, the two costs associated with any main waterway h for two shipping directions calculated by Eqs. (3) and (4) can be rewritten as follows:

$$w_{ij}^{1,h} = \left(c_{ij} + \sigma \times c_{jj_h^*} \right) + \sigma \times \left[\alpha \times \theta_{j_h^* 0_h^*} + \left(1 - \theta_{j_h^* 0_h^*} \right) \right] \times c_{j_h^* 0_h^*} + c^{canal} \times \left(\theta_{ij} + \beta \times \theta_{j_h^* 0_h^*} \right) + c^{time} \times t_{wait} \times \left(\theta_{ij} + \theta_{j_h^* 0_h^*} \right) \quad (24)$$

$$w_{ij}^{2,h} = \left(c_{ij} + \sigma \times c_{jj_h^*} \right) + \sigma \times \left[\alpha \times \theta_{j_h^* (|\mathcal{N}_{r(i)}|+1)_h^*} + \left(1 - \theta_{j_h^* (|\mathcal{N}_{r(i)}|+1)_h^*} \right) \right] \times c_{j_h^* (|\mathcal{N}_{r(i)}|+1)_h^*} + c^{canal} \times \left(\theta_{ij} + \beta \times \theta_{j_h^* (|\mathcal{N}_{r(i)}|+1)_h^*} \right) + c^{time} \times t_{wait} \times \left(\theta_{ij} + \theta_{j_h^* (|\mathcal{N}_{r(i)}|+1)_h^*} \right) \quad (25)$$

Note that the aggregated demands (Q_i^1 and Q_i^2) in Eqs. (3) and (4) are also omitted in the above two equations.

Let $\bar{\pi} = \left\{ \bar{\pi}_{hg}^{rs} \right\}$ denote the optimal solution of the main waterway choice problem. By considering single allocation scheme, our liner hub location problem with multiple main waterways can be formulated as follows:

$$\begin{aligned} \min \quad & \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_k} F_i \times z_i + \sum_{(r,s) \in \mathcal{Q}} \sum_{g=1}^{H_{k(s)}} \sum_{h=1}^{H_{k(r)}} \bar{\pi}_{hg}^{rs} \times q_{rs} \times [\\ & \sum_{i \in \mathcal{N}_{k(r)}} z_{ri} \times (w_{ri}^{1,h} \times \psi_{rs}^1 + w_{ri}^{2,h} \times \psi_{rs}^2) + \sum_{j \in \mathcal{N}_{k(s)}} z_{sj} \times (w_{sj}^{1,g} \times \psi_{rs}^2 + w_{sj}^{2,g} \times \psi_{rs}^1) \\ & + \sigma \times (c_{0_h^* (|\mathcal{N}_{k(s)}|+1)_g^*} \times \psi_{rs}^1 + c_{(|\mathcal{N}_{k(r)}|+1)_h^* 0_g^*} \times \psi_{rs}^2)] \end{aligned} \quad (26)$$

subject to

$$\sum_{j \in \mathcal{N}_k} z_{ij} = 1, \forall i \in \mathcal{N}_k, \forall k \in \mathcal{K}; \quad (27)$$

$$z_{ij} \leq z_j, \forall i, j \in \mathcal{N}_k, \forall k \in \mathcal{K}; \quad (28)$$

$$\sum_{j \in \mathcal{N}_k} z_j = p_k, \forall k \in \mathcal{K}; \quad (29)$$

$$z_j, z_{ij} \in \{0, 1\}, \forall i, j \in \mathcal{N}_k, \forall k \in \mathcal{K}. \quad (30)$$

where p_k is the number of hub ports to be established for community k . The objective function (26) includes three terms: the hub construction cost, the transportation costs within two communities associated with the origin port and the destination port for each OD pair, and the

transportation cost between these two communities. Constraints (27) mean that any port should be allocated to one hub port. Constraints (28) state that any feeder port can only be allocated to a hub port. Constraints (29) show that p_k hub ports will be established for any community k . Constraints (30) define the domain of the decision variables.

For the last term of the objective function (26), the transportation cost between two communities associated with origin port r and destination port s $\sigma \times \sum_{g=1}^{H_{k(s)}} \sum_{h=1}^{H_{k(r)}} \bar{\pi}_{hg}^{-rs} \times \left(c_{0_h^* (|\mathcal{N}_{k(s)}|+1)_g^*} \times \psi_{rs}^1 + c_{(|\mathcal{N}_{k(r)}|+1)_h^* 0_g^*} \times \psi_{rs}^2 \right)$ for each OD pair is fixed, and then it can be omitted. Next, we will rearrange the above objective function by considering various transportation costs associated with different feeder ports. Let w_{ij} denote the transportation cost on shipping containers of feeder port i in community $k(i)$ when feeder port i is allocated to hub port j ($j \in \mathcal{N}_{k(i)}$), then we have

$$\begin{aligned} w_{ij} = & \sum_{(i,s) \in \mathcal{Q}} \sum_{g=1}^{H_{k(s)}} \sum_{h=1}^{H_{k(i)}} \left[q_{is} \times \bar{\pi}_{hg}^{-is} \times \left(w_{ij}^{1,h} \times \psi_{is}^1 + w_{ij}^{2,h} \times \psi_{is}^2 \right) \right] \\ & + \sum_{(r,i) \in \mathcal{Q}} \sum_{g=1}^{H_{k(i)}} \sum_{h=1}^{H_{k(r)}} \left[q_{ri} \times \bar{\pi}_{hg}^{-ri} \times \left(w_{ij}^{1,g} \times \psi_{ri}^2 + w_{ij}^{2,g} \times \psi_{ri}^1 \right) \right] \end{aligned} \quad (31)$$

Then the objective function (26) can be rewritten as follows:

$$\min \sum_{k \in \mathcal{K}} \left(\sum_{i \in \mathcal{N}_k} F_i \times z_i + \sum_{i \in \mathcal{N}_k} \sum_{j \in \mathcal{N}_k} w_{ij} \times z_{ij} \right) \quad (32)$$

According to Eq. (32), the liner hub location problem considering multiple communities can be split into many independent subproblems, each of which is associated with a single community. As mentioned before, we focus on the community covering the Panama Canal, as shown in Figure 10, where two main waterways are simply considered and the two main waterways are partly overlapped. In addition, two main waterways are considered for the community in Asia.



Fig. 10 Ports in the community covering the Panama Canal with multiple main waterways.

Figure 11 shows the results by using the phase diagrams in different spaces. Figure 11(a) shows the phase diagram in (t_{wait}, c^{canal}) space, where $\alpha = 2$ is used, and one region (region III) is categorized. Figure 11(b) shows the phase diagram in (c^{canal}, α) space, where $t_{wait} = 35$ is used, and two regions (region III and region IV) are categorized. Figure 11(c) shows the phase diagram in (t_{wait}, α) space, where $c^{canal} = 72$ is used, and two regions (region III and region IV) are categorized. As shown in Figures 11(b) and 11(c), both Balboa and Manzanillo are apt to be hub ports when $\alpha < 1.4$. While for $\alpha \geq 1.4$, Balboa is always selected to be a hub port, and Manzanillo is always not chosen to be a hub port. The phase diagrams shown in Figure 11 are different from those shown in Figure 6. This is because it has an obvious impact on Balboa by introducing the new main waterway, as shown in Figure 10. In this case, the associated

containers are prone to be transshipped at Balboa for many feeder ports, especially for feeder ports located along the East Coast of Latin America.

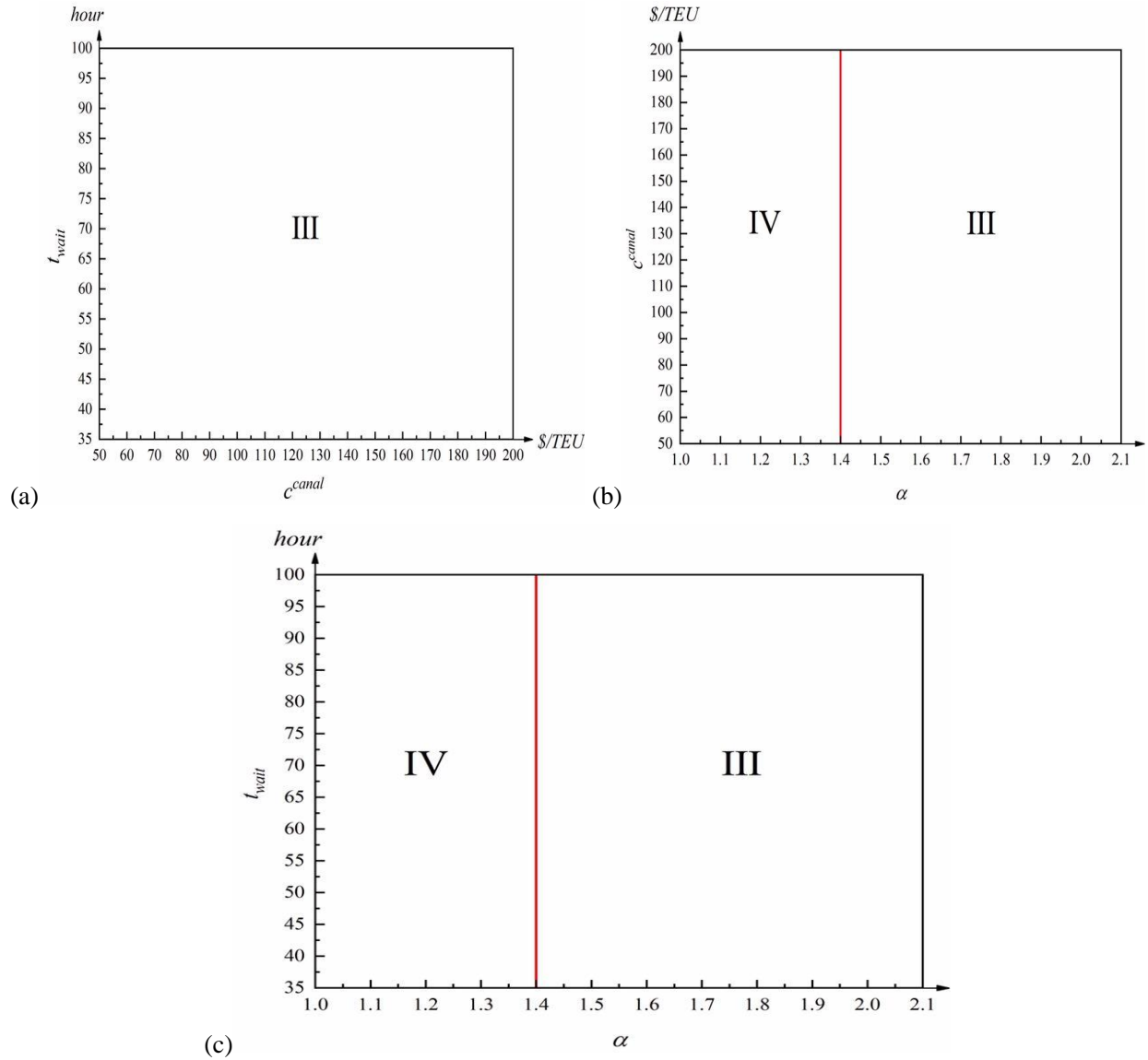


Fig. 11 The phase diagram with multiple main waterways for the community covering the Panama Canal.

5 Summary

This paper introduced canal effects into a liner hub location problem and presented several binary linear programming models, both of which can be efficiently solved by using CPLEX. In order to describe canal effects, we mainly consider three aspects: canal toll (c^{canal}), canal congestion (t_{wait}) and ship capacity limitation when passing through a canal (α). Based on an

abstract community, a simple theoretical analysis on hub location around a canal is also developed. Numerical results are mainly given by using the phase diagram. We can infer that parameter α has a bigger impact on hub location around a canal, as compared with parameters t_{wait} and c^{canal} . Such phenomenon can be partly supported by the theoretical analysis. This insight to some extent justifies the recent canal expansion projects to widen or add a new lane waterway for the Panama Canal and the Suez Canal, respectively.

Acknowledgments

This paper is partly supported by National Natural Science Foundation of China (71871036, 71501021, 71871130, 71831002, 71431001).

Appendix

For each port, its associated container demands and distances to the two endpoints of the main waterway are shown in Table 2.

Table 2 Container demands and distances associated with each port.

Port	Westbound containers	Eastbound containers	$Dis_{ii^*} + Dis_{i^*0^*}$	$Dis_{ii^*} + Dis_{i^*(N+1)^*}$
Marsaxlokk Malta	49	0	906.659	3132.53
Salalah	58	284	4241.58	266.19
Beirut	15	0	2397.17	2064.97
Latakia	0	38	2400.63	2191.94
Valencia	67	265	172.551	4147.94
Aqabah	393	1478	2121.17	2029.06
Suez Canal Container Terminal	254	221	1975.18	2047.02
Fos Sur Mer	0	225	635.149	4050.6
Hodeidah	132	863	3204.65	838.865
Mersin	987	341	2333.1	2339.67
Damietta	15	69	1931.67	2063.99
Genoa	344	768	923.995	3856.86
Haifa	18	101	2209.08	2116.98
Aden	297	716	3482.3	620.485
Djibouti	115	739	3372.48	711.547
Port Said	148	16	1991.23	2003.42
Sokhna	76	5233	2050.93	2061.06
Jeddah	742	4577	2784.15	1346.51
La Spezia	5	0	965.972	3792.27
Barcelona	20	660	423.583	4062.03
Ensenada	430	0	6287.87	2161.97
Montreal	0	905	7349.55	1630.39
Puerto Cortes	0	235	6119.66	2805.09
Houston	0	2056	6929.83	3063.29
Dutch Harbor	344	5	3360.23	5025.79
Balboa	184	0	2494.77	5815.23

Puerto Quetzal	230	0	4695.96	3873.53
San Antonio	135	3	8560.23	4693.69
Lazaro Cardenas	684	7	5310.21	3041.9
Jacksonville	165	98	8165.97	999.39
Portsmouth	2441	821	5471.79	3059.69
San Pedro	39024	609	70.4865	8367.59
Puerto Limon	310	12	6207.14	2340.6
Callao	292	25	9353.96	1237.81
Savannah	3106	1022	6471.82	2605.29
Cartagena	477	25	6284.86	2418.32
Manaus	396	0	9047.78	1881.2
Charleston	2249	1037	7720.29	1221.43
Miami	756	160	8126.56	932.732
Acajutla	702	0	7822.14	761.439
San Juan	778	6	6530.78	2664.24
Mobile	0	268	3984.28	5945.75
Vancouver	3288	139	8893.67	596.559
Oakland	8095	512	5433.72	2956.48
New York / New Jersey	5304	3322	6344.67	2157.29
Puerto Barrios	0	18	8278.8	601.405
Buenaventura	109	0	3064.08	5699.11
Rio Haina	437	0	6327.22	2460.68
Manzanillo	3912	14	7058.35	1238.85
Paita	96	2	7178.48	2570.82
Seattle	13230	268	5085.2	4275.06
Guayaquil	125	45	5890.92	3500.71

References

- Alumur, S., Kara, B.Y., 2008. Network hub location problems: the state of the art. *European Journal of Operational Research* 190(1), 1-21.
- Brouer, B.D., Alvarez, J.F., Plum, C.E.M., Pisinger, D., Sigurd, M.M., 2014. A base integer programming model and benchmark suite for liner-shipping network design. *Transportation Science* 48(2), 281-312.
- Campbell, J.F., O’Kelly, M.E., 2012. Twenty-five years of hub location research. *Transportation Science* 46(2), 153-169.
- Farahani, R.Z., Hekmatfar, M., Arabani, A.B., Nikbakhsh, E., 2013. Hub location problems: A review of models, classification, solution techniques, and applications. *Computers & Industrial Engineering* 64, 1096-1109.
- Gelareh, S., Maculan, N., Mahey, P., Monemi, R.N., 2013. Hub-and-spoke network design and fleet deployment for string planning of liner shipping. *Applied Mathematical Modelling* 37(5), 3307-3321.

- Gelareh, S., Nickel, S., Pisinger, D., 2010. Liner shipping hub network design in a competitive environment. *Transportation Research Part E* 46(6), 991-1004.
- Gelareh, S., Pisinger, D., 2011. Fleet deployment, network design and hub location of liner shipping companies. *Transportation Research Part E* 47(6), 947-964.
- Goldman, A.J., 1969. Optimal location of centers in a network. *Transportation Science* 3, 352-360.
- Martinez, C., Steven, A.B., Dresner, M., 2016. East Coast vs. West Coast: The impact of the Panama Canal's expansion on the routing of Asia imports into the United States. *Transportation Research Part E* 91, 274-289.
- Panama Canal Association, 2014. <https://www.pancanal.com/eng/>. March 5, 2014.
- Sun, Z., Zheng, J., 2016. Finding potential hub locations for liner shipping. *Transportation Research Part B* 93, 750-761.
- Wang, S., Meng, Q., 2012. Sailing speed optimization for container ships in a liner shipping network. *Transportation Research Part E* 48, 701-714.
- Wang, H., Zhang, Y., Meng, Q., 2018. How will the opening of the Northern Sea Route influence the Suez Canal Route? An empirical analysis with discrete choice models. *Transportation Research Part A* 107, 75-89.
- Zheng, J., Meng, Q., Sun, Z., 2014. Impact analysis of maritime cabotage legislations on liner hub-and-spoke shipping network design. *European Journal of Operational Research* 234, 874-884.
- Zheng, J., Meng, Q., Sun, Z., 2015. Liner hub-and-spoke shipping network design. *Transportation Research Part E* 75, 32-48.
- Zheng, J., Qi, J., Sun, Z., Li, F., 2018. Community structure based global hub location problem in liner shipping. *Transportation Research Part E*, 118, 1-19.