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Review on the new development of vibration-based damage identification for civil engineering structures: 2010–2019

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Abstract

Structural damage identification has received considerable attention during the past decades. Although several reviews have been presented, some new developments have emerged in this area, particularly machine learning and artificial intelligence techniques. This article reviews the progress in the area of vibration-based damage identification methods over the past 10 years. These methods are classified in terms of different damage indices and analytical/numerical techniques used with discussions of their advantages and disadvantages. The challenges and future research for vibration-based damage identification are summarised. This review aims to help researchers and practitioners in effectively implementing existing damage detection algorithms and developing more reliable and practical methods for civil engineering structures in the future.

Keywords

Damage identification, vibration method, model updating, review

Content

List of abbreviations

AANN	Auto-associative neural network
ABC	Artificial bee colony
AI	Artificial intelligence
ANN	Artificial neural network
AR	Auto-regressive
ARD	Automatic relevance determination
ARMA	Auto-regressive moving average
ARMAX	Auto-regressive moving average with exogenous input
ARX	Autoregressive with exogenous input
BPNN	Back propagation neural network
CS	Compressive sensing
CMS	Component mode synthesis
DOF	Degree of freedom
EKF	Extended Kalman filter
EM	Expectation–maximization
EMD	Empirical mode decomposition
FD	Fractal dimension
FE	Finite element
FRF	Frequency response function
GA	Genetic algorithm
GS	Gibbs sampling
HHT	Hilbert–Huang transform
IMF	Intrinsic mode function
KPCA	Kernel principal component analysis
Lasso	Least absolute shrinkage and selection operator
MAC	Modal assurance criterion
ML	Machine learning

MSC	Mode shape curvature
MSD	Mahalanobis squared distance
MSE	Modal strain energy
NDT	Non-destructive testing
ODS	Operational deflection shape
OSP	Optimal sensor placement
PCA	Principal component analysis
PDF	Probability density function
PSD	Power spectral density
PSO	Particle swarm optimization
RBF	Radical basis function
RD	Random decrement
RF	Random forest
RSM	Response surface methodology
SBL	Sparse Bayesian learning
SHM	Structural health monitoring
SVD	Singular value decomposition
SVM	Support vector machine
TSD	Traffic speed deflectometer
WT	Wavelet transformation

1. Introduction

Civil engineering structures are exposed to natural and manmade hazards, which may cause structural damage or even collapse. An unpredicted structural failure can be catastrophic not only in terms of life and economic losses but also in terms of the subsequent societal impacts. Therefore, structural damage detection is important, especially in the early damage state, to avoid sudden failures and improve the safety and reliability of structures.

Structural damage identification has been widely explored over the past decades. Rytter [1] classified damage identification into four levels: determination of damage existence, determination of damage location, quantification of damage severity, and finally prediction of the remaining service life of structures, which are referred to as Levels 1 to 4, respectively. Most current studies focus on the first three levels [2].

Structural damage detection methods can be divided into two categories, namely, nondestructive testing (NDT) and vibration-based methods. The former are local methods that cannot easily detect damage located inside structures (e.g. cracks in concrete and/or corrosion of steel bars) or damage enclosed by non-structural components (e.g. decorations of buildings). By contrast, vibration-based damage identification methods examine changes in structural global vibration characteristics and are thus regarded as global methods and have attracted considerable attention during the past decades. In this regard, only vibration-based damage detection methods are reviewed in this paper.

Early vibration-based damage detection methods were reviewed and summarised in several literatures. For example, Doebling et al. [2] and Sohn et al. [3] comprehensively reviewed vibration-based damage detection methods and their applications to various types of structures before 1996 and between 1996 and 2001, respectively. Salawu [4] reviewed damage identification methods using natural frequency changes. Carden and Fanning [5] focused on publications between 1996 and 2003. Fan and Qiao [6] compared several damage detection methods. Hakim and Razak [7] and Chen et al. [8] reviewed the applications of artificial neural networks (ANNs) and Hilbert–Huang transform (HHT) for damage identification over the last 20 years, respectively.

A recent literature survey was conducted by Kong et al. [9], who discussed the prediction of the remaining life of structures and related decision making. Cao et al. [10] provided an overall survey of damping-based damage detection methods. Huang et al. [11] presented a complete review on the recent development of Bayesian inference for structural damage detection and assessment. Weng et al. [12] reviewed dynamic substructuring methods for damage identification of large-scale structures. In terms of the massive collected data, Gordan et al. [13] intensively reviewed the applications of data mining techniques in damage identification and structural health monitoring (SHM) that have been conducted since 2000. Bao et al. [14] reviewed advancements in data science and engineering in SHM. An et al. [15] summarised damage identification methods for bridge structures between 2011 and 2017.

The fast development in information technology, particularly sensing technology, signal processing techniques, machine learning (ML) and artificial intelligence (AI) technologies, advances vibration-based damage identification methods over the past decade. In addition, damage identification under operational and environmental conditions, output-only identification, statistical damage detection, real-time identification for on-line SHM, and optimal sensor placement (OSP), etc., are also important issues. A number of techniques have been developed to deal with these problems.

This work aims to review the latest vibration-based damage detection methods between 2010 and 2019. Although hundreds of papers are published every year, only representative papers are reviewed in Section 2 because of the length limit. The challenges and future trends of the vibration-based damage identification methods are discussed in Section 3, followed by Conclusions.

In Section 2, damage identification methods are classified according to the damage index used and analytical techniques. First, methods using various modal parameters are reviewed in

Subsection 2.1. Different signal processing based damage identification methods are reviewed in Subsection 2.2. Subsection 2.3 discussed finite element model updating methods, including conventional model updating, substrcuturing methods, regularisation techniques, and sparse recovery techniques. The optimisation algorithms such as genet~~r~~ic algorithm (GA) and ANN are reviewed in Subsection 2.4. Subsection 2.5 introduces the statistical time series methods based on AR models and their variants. Subsequently, the cutting-edge ML methods are comprehensively reviewed in Subsection 2.6, which includes supervised, unsupervised and semi-supervised learning. Subsection 2.7 introduces the probabilistic damage identification approaches, particularly the Bayesian methods. Subsection 2.8 addresses damage identification under varying environmental conditions. The methods are divided into two categories, depending on whether the environmental variables are measured or not. Subsection 2.9 reviews existing algorithms with consideration of nonlinear structural behaviours, and Subsection 2.10 introduces the newly developed drive-by damage identification methods. In Subsection 2.11, some techniques that are excluded from the abovementioned categories are introduced, such as RSM and OSP. Finally, Subsection 2.12 focuses on the papers that compared different damage identification methods.

2. Vibration-Based Damage Identification Methods

Vibration-based damage detection methods have been first developed and applied in aerospace and mechanical engineering; the civil engineering community has studied the vibration-based damage detection of bridge structures since the early 1980s [16]. The basic idea of vibration-based damage detection methods is that structural damage may induce changes in vibration characteristics, such as frequencies and mode shapes [2].

Vibration-based damage detection methods can be categorised into three domains based on vibration parameters: time domain, frequency domain and time–frequency domain methods. In time domain methods, time–history responses are used. In frequency domain methods, modal parameters are utilised. Time–frequency domain methods are based on time–frequency analytical tools. In terms of algorithms used, damage detection methods can be classified into nonmodel-based or data-driven and model-based methods.

The performance of damage detection largely relies on the choice of damage sensitive features. Therefore, vibration-based damage detection methods involving different damage indices are first introduced.

2.1 Modal parameter-based methods

With the development of modal analysis technology, the majority of vibration-based methods fall into frequency domains. Modal parameters, such as natural frequencies, mode shapes and their variants, have been commonly used. However, the use of natural frequencies only as damage indices is no longer popular in recent years because they are insensitive to local damage and the number of available frequencies is limited, generally less than 10.

2.1.1 Mode shapes

Yoon et al. [17] applied their previously proposed global fitting method [18] to identify damage in 2D plate-like structures by using the mode shape data only from a damaged structure. In comparison with the gapped smoothing method, which locally fits mode shape curvature (MSC), global fitting involves the use of a generic mode shape form to globally fit mode shapes, thus eliminating the smearing effects and reducing false detection. Zhang et al. [19] approximately extracted mode shapes from the acceleration responses of a passing vehicle with sinusoidal tapping force. Damage location was then determined on the basis of the difference between damaged and intact mode shape squares. Although baseline information is necessary, the proposed approach does not require many preinstalled sensors and solving eigenvector or singular value problems. Feng and Feng [20] extracted a first-order mode shape from the vehicle-induced displacement response, which was utilised as a damage index to determine damage location and quantitatively monitor the damage progression of bridges.

2.1.2 Natural frequencies and mode shapes

Some researchers combined natural frequencies and mode shapes for damage detection. Sun et al. [21] used a normalised uniform load surface curvature, which was estimated from modal flexibility, to locate damage for beam-like structures. The proposed method performed better in identifying single and multiple damage locations than the uniform load surface curvature and MSC methods. However, this method was only applicable to beam-like structures following Bernoulli–Euler beam theory. Zhao and Zhang [22] utilised the changes ~~of~~ in natural frequencies and mode shapes for damage localisation and quantification. The modal assurance criterion (MAC) was used to analyse the sensitivity of mode shapes between different orders, and mode shapes with high sensitivities to damage were employed to calculate the damage index.

Radzieński et al. [23] compared six widely used damage detection methods based on modal parameters, including MSC, coordinate MAC, strain energy damage index, gapped smoothing method, fractal dimension (FD) and wavelet transformation (WT). However, only the generalised FD and WT damage indicators were able to ~~accurately~~ locate damage position accurately in the presence of measurement noise. In this regard, the authors proposed a new damage indicator based on the change in natural frequencies and any one mode shape (measured or modelled). Capecchi et al. [24] ~~integrated~~ combined natural frequencies, mode shapes and MSCs for damage identification in a parabolic arch.

Single crack identification using modal parameters has been intensively studied. However, relatively few studies have addressed multi crack identification problems. Caddemi and Caliò [25] derived a closed form expression of the exact dynamic stiffness matrix of a multi-cracked Euler–Bernoulli beam based on their previous work [26] and then extended it to frame structures. The natural frequencies and mode shapes of undamaged and damaged frames were calculated on the basis of the Wittrick–Williams algorithm and further used for damage identification. Later, Khiem and Tran [27] derived a simplified closed-form expression of the vibration modes of multiple cracked beams. The shifts of natural frequencies and mode shapes were explicitly expressed in terms of crack locations and magnitudes. An iterative procedure was developed to determine not only the position and severity of cracks but also their quantity. Khiem and Toan [28] proposed an explicit expression of natural frequencies in terms of crack positions and sizes for multiple cracked beams; their proposed method differed from the earlier one by including nonlinear terms with respect to the crack magnitude. The non-uniqueness problem in damage detection under a symmetrical boundary condition was overcame by incorporating nonlinear terms.

2.1.3 Damping

In comparison with natural frequencies and mode shapes, damping has been less commonly used for damage identification because of the complexity of its measurement and mechanism. Frizzarin et al. [29] used the nonlinear damping identified from ambient vibration responses to locate damage in concrete structures without any reference to an undamaged baseline. Mustafa et al. [30] proposed an energy-based damping evaluation method for damage localisation. Ay et al. [31] estimated the damage-induced changes in the overall damping behaviour of a free-vibration dynamic system within a statistical framework.

The damping model used for damping estimation is critical for damping-based methods [10]. Most studies have adopted a Rayleigh damping model because of its mathematical simplicity. However, classical Rayleigh damping may be an inappropriate assumption for most civil structures [32]. In this regard, Liu et al. [33] proposed a technique to identify damage to non-classically damped shear buildings. A novel modal identification technique was developed to identify complex modal parameters from vibration measurements under harmonic excitations. The locations and magnitudes of damage with respect to stiffness reduction and damping defect were then simultaneously identified through the sensitivity-based model updating.

Damage identification methods based on modal parameters possess the merit of direct physical interpretation. However, modal identification is susceptible to measurement noise, especially for damage-sensitive high modes. This process may produce some unavoidable errors and render the damage identification results unreliable. In this regard, some researchers directly utilised the measured data, such as frequency response function (FRF), for damage identification.

2.1.4 FRFs and their variants

In contrast to modal data, FRFs are calculated over the entire frequency range and can provide more information about damage. Limongelli [34] proposed a damage detection method by using the difference between the FRF and its spline interpolation. The method was later successfully applied to frame structures under seismic excitation [35] and a single-span reinforced concrete bridge [36].

An operational deflection shape (ODS) is usually defined as the deflection of a structure at a particular frequency [37]. On the basis of their previous work, Zhang et al. [38] used an ODS curvature to locate damage. The ODS curvature of an undamaged structure was approximated by a smooth line under the assumption that an intact structure was homogeneous and uniform to avoid the requirement of a baseline. Damage location was identified by comparing ODS curvatures before and after damage.

The major problem of FRF-based damage detection methods lies in the choice of the optimum frequency range for analysis. Moreover, the FRF requires the measurement of excitation forces and structural responses simultaneously.

As a substitute of the FRF, transmissibility is defined as the relationship between two sets of responses and independent of input excitations. ~~Another motivation of using transmissibility for damage detection relies on the fact that~~ ~~t~~The transmissibility is a local quantity ~~that~~ and ~~suggests~~ is of high sensitivity to damage [39].

Maia et al. [39] utilised the correlations of acceleration response transmissibilities to detect and quantify structural damage. The transmissibility ~~damage indicator~~ was found more sensitive than FRFs in ~~terms of~~ damage detection and quantification. Li et al. [40] reconstructed the auto-spectral density functions by using power spectral density (PSD) transmissibility. Damage identification was then conducted by minimising the difference between the measured and reconstructed PSD functions. Kong et al. [41] used the transmissibility of vehicle responses in a vehicle-bridge coupled system to detect bridge damage.

The aforementioned damage detection methods can be classified as damage index methods and are summarized in Table 1. In the subsequent subsection, different signal processing techniques for damage detection are reviewed.

Table 1. Modal parameter-based damage identification methods

Features	Authors	Damage identification level	Applications	Remarks
Mode shapes	Yoon et al. [17]	1-3	Experimental 1D beam and 2D plate	M
	Zhang et al. [19]	1 and 2	Experimental plywood/plate	B
	Feng and Feng [20]	1 and 2*	Numerical simply supported bridge	Output only
Frequencies and mode shapes	Sun et al. [21]	1 and 2	Numerical beam-like structures	B
	Zhao and Zhang [22]	1-3	Numerical planar truss beam	B, M
	Radzieński et al. [23]	1 and 2	Experimental beam-like structures	B
	Capecchi et al. [24]	1-3	Experimental parabolic arch	M
	Caddemi and Caliò [25]	1-3	Numerical frame	B, M
	Khiem and Tran [27]	1-3	Numerical beam-like structures	M

Damping	Frizzarin et al. [29]	1 and 2	Experimental RC bridge	Output only
	Mustafa et al. [30]	1 and 2	Steel truss bridge (viscous damping)	M
	Ay et al. [31]	1-3	Experimental steel bridge (viscous damping)	B
	Liu et al. [33]	1-3	Numerical shear building (non-classically damping)	M
FRFs and their variants	Limongelli [34]	1 and 2	I40 bridge	B
	Limongelli [35]	1 and 2	Numerical frame under seismic excitation	B
	Dilena et al. [36]	1 and 2*	RC bridge	B
	Zhang et al. [38]	1 and 2	Experimental plywood beam and plate	Output only
	Maia et al. [39]	1-3	Experimental steel beam	B
	Li et al. [40]	1-3	Experimental steel plane frame	M
	Kong et al. [41]	1 and 2	Numerical simply supported beam	M

Note: * — Quantitatively indicate the relative severity of damage;
B — Baseline information is required;
M — Analytical model is required.

2.2 Signal processing-based methods

In order to improve damage sensitivity, dynamic responses should be further processed to extract hidden information. In this regard, a number of signal processing techniques, such as WT, HHT and FD, have been developed and applied for structural damage detection.

Yang and Nagarajaiah [42] combined independent component analysis with WT for output-only damage identification. Structural vibration responses were transformed into a wavelet domain and then fed as mixtures into a blind source separation model, which was examined through independent component analysis. Consequently, the damage information hidden in wavelet-domain signals was clearly revealed by a sharp spike.

B-spline wavelets have been widely applied to signal processing since its introduction by Chui and Wang [43]. Katunin [44] derived the analytical formulation of high-order (e.g. fifth-, sixth- and seventh-order) B-spline wavelets, in addition to the first four B-spline wavelets. The discrete WT with the sixth-order B-spline wavelets was then

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applied to identify damage ~~to~~ in composite beams by evaluating the singularities of detailed coefficients. In another study, Katunin [45] applied the sixth-order B-spline wavelets to detect damage to composite plates. Numerical and experimental results showed that high-order B-spline wavelets could improve the sensitivity and accuracy of damage detection and localisation.

Identification of multiple damage is more challenging than the identification of single damage. Cao et al. [46, 47] applied WT to a curvature mode shape to alleviate noise effect. A Teager energy operator was then implemented to intensify the local singularities of the signal. With this proposed technique, slight and multiple cracks in beams could be detected even under high-noise conditions. Recently, Shahsavari et al. [48] presented a statistical procedure to detect low levels of multiple damage in beams. Continuous WT was first applied to the first mode shape. A damage indicator was then extracted from the wavelet coefficients through PCA. Once damage was detected, a likelihood ratio test was further conducted to determine the likely location.

HHT [49] is a time–frequency analytical technique that is able to process nonlinear and nonstationary signals. Dong et al. [50] used a vector auto-regressive (AR) moving average (ARMA) model, unlike conventional HHT, to represent intrinsic mode functions (IMFs) obtained from the empirical mode decomposition (EMD) of vibration signals. A damage index was defined on the basis of vector ARMA coefficients, which indicated the occurrence and relative severity of damage. Bao et al. [51] developed a multistage output-only identification scheme based on an improved HHT, which was able to provide Levels 1–3 damage identification. The improved HHT could decompose structural response data under ambient excitations and possess~~esed~~ better robustness to noise compared with the traditional HHT. Han et al. [52] used HHT for modal identification and damage detection. Modal parameters were identified through HHT in combination with other techniques, such as random decrement (RD), natural excitation and stochastic subspace identification.

Aied et al. [53] applied ensemble EMD to the acceleration responses of a bridge to a moving load to detect a sudden stiffness change. Experimental results revealed that changes in the stiffness were successfully identified even in the presence of rough profiles, high vehicle speeds and noisy signals.

The FD introduced by Mandelbrot [54] is an effective indicator to characterise irregularities in nonlinear systems. In comparison with WT that needs high ~~-~~spatial resolution in measurements, FD-based methods only require a small number of measured points; as such, they are convenient and effective for online data processing and structural damage detection over the past 10 years.

Li et al. [55] expressed the difference in the angles of sliding windows between two successive points of a displacement mode shape in FD, and then utilised the change in angle for damage localisation. A damage quantification index was developed on the basis of the relationship between the angle and modal strain energy (MSE). Bai et al. [56] extended the previous FD-based methods to a higher mode shape, which was transformed into a new mode shape through affine transformation. The newly generated mode shape preserved the original damage information and eliminated the local extrema that may cause false damage identification. The developed method was then successfully applied to detect damage in beam and plate structures [57].

An intrinsic deficiency of the FD analysis is its susceptibility to measurement noise. To address this problem, Bai et al. [58, 59] applied WT to decompose a mode shape into scale mode shapes before the FD analysis. Damage information and noise were thus separated because they had different scale characteristics.

The aforementioned studies applied the FD analysis to mode shapes. An and Ou [60] directly analysed acceleration data through FD to locate damage in order to avoid modal identification errors. FDs were computed at each measured node. The curvature difference in FDs before and after damage indicated damage location. The proposed method exhibited a high robustness to noise and was still feasible even if the noise level was up to 15%. Li et al. [61] combined the time–frequency analysis and FD to identify seismic damage for shear-type building structures by using the acceleration data only. WT was applied to determine the time–frequency feature, whose FD was then calculated using a box-counting method. Damage-induced nonlinearity was localised by comparing FDs in different stories.

A bridge and moving loads on it is a nonstationary dynamic system. The interaction between the loads and structure is closely related to the ~~actual~~ safety condition of the bridge and thus has been ~~analysed~~ used for damage identification. Hester and González [62] applied WT to the acceleration response of a bridge to a moving vehicle. The wavelet energy content was employed as a damage indicator, which was calculated on the basis of a range of scales rather than a given scale, to improve the sensitivity of the wavelet coefficient to damage. Roveri and Carcaterra [63] used the HHT to identify damage in bridge structures under a travelling load. A single point response was measured and processed through HHT, and the peak of the first instantaneous frequency indicated the damage location. Kunwar et al. [64] adopted HHT to locate damage in a bridge model under transient vibration loads. The relative amplitude of a marginal Hilbert spectrum was used to identify damage location, and the joint time–frequency distribution referred to damage evolution.

Signal processing-based methods are typically nonparametric and only require experimental data from a damaged structure. Although these methods are efficient for practical applications without resorting to a baseline model, they are mostly limited to Level 2 damage identification, that is, damage location. This is because the quantitative relation between the signal and damage severity cannot be established. The following table compares and summarizes the signal processing-based methods that have been reviewed in this subsection.

Table 2. Signal processing-based methods

Authors	Methods/Features	Damage identification level	Applications	Remarks
Yang and Nagarajaiah [42]	WT+BSS/ approximated and detailed components	1, 2, and damage instant	Real seismic-excited building	Output only
Katunin [44]	WT using fifth-order B-spline wavelet/ detail coefficients	1 and 2	Experimental beams	Output only, on-line
Katunin [45]	WT using sixth-order B-spline wavelet/ detail coefficients	1, 2 and shape	Experimental plates	Output only, on-line
Cao et al. [46, 47]	WT+Teager energy operator/ curvature mode shape	1 and 2*	Experimental beams	Output only
Shahsavari et al. [48]	WT+PCA/ WT coefficients	1 and 2	Experimental beams	Output only
Dong et al. [50]	EMD+ARMA/ ARMA coefficients	1*	Benchmark structures	Output only
Bao et al. [51]	HHT/ instantaneous phase and frequency, Hilbert marginal spectrum	1, 2*, and damage instant	Experimental three-storey frame	Output only, on-line
Han et al. [52]	HHT/ instantaneous frequency and energy, Hilbert marginal spectrum	1 and 2*	Experimental 12-story reinforced concrete frame model	B
Aied et al. [53]	EMD/IMF1	Damage instant and duration	Numerical 3D VBI model	Output only
Li et al. [55]	FD/FD-based indices	1-3	Experimental beam	B
Bai et al. [56]	FD+affine transformation/ FD trajectory	1 and 2*	Experimental beam	Output only

Bai et al. [57]	FD+affine transformation/ FD trajectory	1 and 2	Experimental plate	Output only
Bai et al. [58]	FD+WT/ scale FD trajectory	1 and 2*	Experimental beam	Output only
Bai et al. [59]	FD+WT/ scale FD trajectory	Location of delamination	Experimental composite plate	Output only
An and Ou [60]	FD/waveform FD	1 and 2	Experimental beam	B
Li et al. [61]	FD+WT/FDs	1 and 2	Numerical building	Output only
Hester and González [62]	WT/ wavelet energy	1 and 2*	Numerical bridge beam	Output only
Roveri and Carcaterra [63]	HHT/ first instantaneous frequency	1 and 2	Numerical bridge beam	Output only
Kunwar et al. [64]	HHT/ marginal Hilbert spectrum	1 and 2*	Experimental bridge	B

Note: * — Quantitatively indicate the relative severity of damage
VBI — vehicle–bridge interaction
B — Baseline information is required;
M — Analytical model is required.

2.3 Finite element model updating methods

Model updating methods modify model property matrices, such as mass, stiffness and damping matrices, to ensure that the analytical predictions of the updated model resemble experimental data as closely as possible [65]. When undamaged and damaged measurement data are available, changes in structural parameters can be utilised to detect the presence of damage, identify damage location and quantify damage extent (Levels 1–3 damage identification).

2.3.1 Conventional model updating

Early model updating methods are one-step approaches that directly reconstruct the stiffness and mass matrices of an analytical model to reproduce the measured modal data [66, 67]. The main drawback of these methods is that the updated mass and stiffness matrices have minor physical meaning, that is, they cannot be related to the changes in the parameters of the original model. Nowadays, model updating methods are iterative approaches that repeatedly modify the physical parameters of a finite element (FE) model. This approach directly changes matrices and adjusts the physical parameters at an elemental or substructural level. System stiffness and mass matrices are assembled from all elements in a discrete FE model. Therefore, (1) the matrix properties of symmetry, sparseness and positive definiteness are guaranteed; (2) structural connectivity is preserved; and (3) changes in the updated global matrices are represented by changes in the updated parameters [68].

Model updating methods in mathematics are regarded as an optimisation problem that minimises the difference between the measured and predicted responses or referred to as an error function. Similar to data-based methods, measurement data can be FRFs [69], natural frequencies and mode shapes [70], time histories [71, 72], dynamic strain responses [73-77] or a combination of static and modal test data [78].

2.3.2 Substructuring techniques

Substructuring techniques have been developed and employed efficiently in the structural analysis of large-scale structures since the 1960s [79]. In these techniques, a global structure is divided into small manageable substructures, each of which is analysed independently to obtain its designated solution. These solutions are then assembled to recover the solutions of the global structure by imposing constraints at the interfaces. The component mode synthesis (CMS) method is a popular substructuring technique ~~and~~. It can be classified as the free interface, fixed interface and hybrid methods, according to the interface condition of the substructures. Yu et al. [80] proposed a free interface CMS method and applied it to the element-by-element model updating of large-scale structures. Later, Wang et al. [81] improved the free interface CMS method for model updating, in which the residual flexibility attachment matrix was constructed without inverting the stiffness matrix. Liu et al. [82] used the CMS method developed in [83] to update the FE model of a scaled arch bridge model. Papadimitriou and Papadioti [84] proposed a fixed interface CMS method and applied it to damage detection of a highway bridge, in which each substructure had one unknown parameter for updating.

Weng et al. [85, 86] proposed an iterative substructuring method for FE model updating on the basis of Kron's substructuring concept [87]. During model updating, each substructure was independently handled. When a structure was damaged in a local area, some specific substructures were re-analysed and assembled with other unchanged substructures to recover the solutions of the global structure without repeatedly analysing the global structural properties. Later, Weng et al. [88] developed ~~the~~ an inverse substructuring method~~ in an inverse manner~~.~~,~~ ~~where~~ In their study, the substructural flexibility was extracted from the experimental modal data of the global structure and then used as a reference for updating the substructural FE model. Eigenvalues and eigenvectors decomposed from substructural flexibility were employed as damage indicators [89]. Substructural properties were more sensitive to damage than the global ones because damage typically occurred in a local area.

Yuen and Huang [90] developed an improved Bayesian substructure identification approach based on their previous work [91]. The improvement was made by modelling the boundary force as filtered white noise, which imposed extra constraints and thus enhanced the identifiability of the inverse problem.

2.3.3 Regularisation techniques

FE model updating is an inverse problem and typically ill conditioned. Moreover, the number of available measurements is usually less than that of unknown parameters, resulting in an underdetermined problem. To solve the problems, the regularisation technique has been developed in the model updating by including an additional item in the error function, usually a 2-norm item, which leads to a convex error function. This technique is also referred to as Tikhonov regularisation or l_2 regularisation.

Wang and Yang [92] presented a modified Tikhonov regularization in model updating. Since the structural system contains modelling errors and measurement noise, the identified results may diverge after several iterations. In this regard, they imposed limits on the identified parameters based on their physical meanings to circumvent the divergence problem. Li and Law [93] proposed an adaptive Tikhonov regularisation, which forced the stiffness reduction factors of intact elements close to zeros in each iteration. Comparative studies showed that the proposed approach had an obvious advantage over the traditional Tikhonov regularisation, especially when measurements contained ~~large~~ considerable noise. Zhu et al. [94] developed a sensitivity-based model updating approach with the Tikhonov regularisation in a state~~-~~ space domain without the need for input measurements. Damage was identified by minimising the difference between the measured and reconstructed responses based on transmissibility.

The regularisation parameter plays a critical role in ~~all~~ regularisation problems, which controls the trade-off between the data fidelity and solution sparsity~~, and thus may exert a crucial effect on solutions~~. In general, it is an unknown a priori and problem dependent. A number of methods, including discrepancy principle [95, 96], ordinary and generalised cross validations [97], min–max rules [98] and L-curve criterion [99], have been developed to determine the optimal regularisation parameter for inverse problems in mathematics.

In structural damage identification, the optimal regularisation parameter of l_2 regularisation is typically determined using the L-curve criterion [92-94, 100]. This criterion utilised a parametric plot of the solution norm versus the residual norm, and the corner of the curve is regarded as a good choice of the regularisation parameter, which simultaneously satisfies a small solution norm and a small residual norm [99]. Another widely used technique is generalised cross-validation [101]. In this technique, the regularisation parameter is calculated by minimising the overall prediction error based on the leave-one-out rule without any knowledge of the noise variance [97,102].

The Tikhonov regularisation is convenient for implementation, and it has received wide applications in structural damage detection. However, it tends to produce over smooth solutions, that is, the damage identification results are distributed to many structural elements, which are inconsistent with practical situations.

2.3.4 Exploiting structural damage sparsity

Structural damage often occurs at several locations only, especially at the early stage, which is sparser than the large total number of elements in the entire structure. Structural damage sparsity is regarded as essential prior information that can be utilised for more accurate damage identification. The preliminary use of the sparsity for damage detection can date back to earlier years in the 1990s, where sparsity prior information is exploited by the minimum rank model updating method [103-105].

In this connection, structural damage identification can be treated as a sparse recovery problem. Sparse recovery theory, particularly compressive sensing (CS), has recently attracted considerable interest in a wide range of applications [106-108]. The basic idea manifests that an unknown sparse vector can still be accurately recovered when the number of measurements is smaller than the size of the entire vector, provided that the sensing matrix satisfies certain incoherence properties [106]. However, this theory has been introduced to structural damage detection since the 2010s. One possible reason is most CS deals with linear problems where damage detection is generally a nonlinear one.

Sparse recovery theory adopts the l_p $(0 \leq p \leq 1)$ regularisation instead of the l_2 regularisation [109], that is, a p-norm item is added to the error function. For a small item of a vector, the p-norm retains a relatively larger weight than the 2-norm. Therefore, a small item significantly contributes to the error function and tends to be penalised by being pushed to zero, resulting in a sparse solution [110]. Chen et al. [111] and Chartrand [112] demonstrated that l_p $(0 \leq p < 1)$ regularisation can provide a sparser solution by using ~~less~~ fewer measurement data than l_1 regularisation. In addition, the l_p regularisation is robust to noise. However, for $0 \leq p < 1$, the corresponding nonconvex optimisation problem is NP-hard. Solving this NP-hard problem requires a combinatory search and is thus computationally infeasible for large-scale problems. Moreover, a globally optimal solution cannot be obtained for a nonconvex optimisation problem [113]. For these reasons, the l_1 regularisation is more widely used than the l_p regularisation ($0 \leq p < 1$) counterpart.

Bao et al. [114] first introduced the CS technology to structural damage detection. Hernandez [115] expanded the sensitivity-based model updating by using l_1 norm minimisation. Following a similar idea, Zhou et al. [116] developed an l_1 regularisation approach by using the first few frequency data. The proposed technique outperformed the conventional l_2 regularisation through numerical and experimental studies. The effects of the measurement number, damage severity, number of damage and noise level on the damage detection results were numerically investigated. Zhang and Xu [117] compared the Tikhonov regularisation with sparse regularisation by using time history data. The reweighted l_1-norm regularisation technique was adopted to enhance sparsity in the solution. This comparison showed that the proposed sparse regularisation exhibited certain superiority to the Tikhonov regularisation in terms of the identification accuracy and computational efficiency. Hou et al. [118] further extended this technique by using frequencies and mode shapes. Wu and Zhou [119] developed a l_1-regularised one-step model updating approach in which the measured modal data before and after damage was compared directly. In this manner, model updating at an undamaged stage was unnecessary.

Zhang et al. [120] combined the extended Kalman filter (EKF) and l_1 regularisation for damage identification by using free vibration responses. The original unconstrained optimisation problem was transformed into an optimisation problem with the l_1-norm constraint, and a pseudo-measurement technique was utilised to enforce the constraint into each recursive step of EKF. Recently, Huang et al. [121] proposed an improved EKF method based on the l_p regularisation. A novel L-surface approach was used to determine an appropriate p. Numerical and experimental examples showed that the proposed improved EKF method was superior to EKF with Tikhonov and l_1 regularisation methods in terms of identification accuracy and required measurement quantities.

Zhou et al. [122] presented an iteratively reweighted l_1 regularisation algorithm, which closely resembled the l_0 regularisation. The nonconvex l_0 regularisation problem was solved by transforming it into a series of weighted l_1 regularisation problems. An experimental example demonstrated that the proposed algorithm was able to provide sparser damage identification results with higher accuracy than the l_1 regularisation.

Chen et al. [123] employed a weighted strategy and trace the least absolute shrinkage and selection operator (Lasso) for FE model updating by using modal parameters. Different weight coefficients were used to balance information from frequencies and mode shapes. The trace Lasso improved the accuracy and stability of the l_1 regularisation, especially when unknown variables were highly correlated. The ant lion technique was employed to solve the optimisation problem. Later, Chen and Yu [124] combined the optimiser with an improved Nelder–Mead algorithm to improve the local searching ability. A comparison study indicated that the proposed algorithm was more robust and accurate than the ant lion optimiser and required more computational time.

Wang and Lu [125] proposed a new error function that was decoupled from the damage parameters. The new error function, including an l_1 regularisation term, was solved using an alternative~~ng~~ minimisation approach without a sensitivity analysis. Ding et al. [126] proposed a novel error function based on sparse regularisation and Bayesian inference by considering the uncertainty effect and a limited number of measurements. A new heuristic algorithm, namely, the Jaya algorithm, was employed for the optimisation. Comparison results showed that the proposed error function yielded more reliable and accurate identification results than those with either sparse regularisation or Bayesian inference alone.

Numerous methods have been developed to select the regularisation parameter for l_2 regularisation, whereas few methods have been devised for l_p regularisation ($0 \le p \le 1$) [109]. This gap is due to the fact that the former has a closed-form solution, whereas the latter does not. In SHM and damage identification, an appropriate regularisation parameter of an l_1-regularised problem is typically selected on the basis of experience. Mascarenas et al. [127]

heuristically selected the regularisation parameter as a unit. Yang and Nagarajaiah [128] set the regularisation parameter as 0.01 in CS-based modal identification. Yang and Nagarajaiah [129] calculated the regularisation parameter by using $\beta = 1/\sqrt{N}$, where N is the number of the time history sampling points corresponding to the dimension of an unknown vector. Zhang and Xu [117] used the reweighted l_1 regularisation technique to determine the regularisation parameter. Yao et al. [130] showed that the plot of the residual term versus the regularisation term on a linear scale resembled an "L" shape. Afterwards, they selected the regularisation parameter corresponding to the corner of the L curve.

More recently, Hou et al. [131] developed two strategies to select the regularisation parameter of the l_1 regularisation problem. The first method utilized the residual and solution norms of the optimisation problem and ensured that they are both small. The other selection criterion was based on the discrepancy principle, which required that the variance of the discrepancy between the calculated and measured responses was close to the variance of the measurement noise. A range of the regularisation parameter, rather than one single value, could be determined using these two strategies. Wang and Lu [125] selected the optimal regularisation parameter based on the threshold setting method, which was closely related to two threshold parameters determined through numerical studies.

In l_1-regularised damage detection, the sensitivity matrix serves as the sensing matrix and is directly related to sensor locations. Sensor placement is a typical combinatorial problem, and the global optimum is difficult to obtain using conventional techniques. In this regard, Hou et al. [132] developed a GA-based OSP technique such that the columns of the resulting sensitivity matrix have maximum independence. Although the optimal sensor location is generally damage-dependent, the proposed technique worked on the sensitivity matrix in the undamaged state and did not need the prior knowledge of damage location and severity.

Although the use of FE model updating for damage detection has been ~~greatly~~ significantly developed, these methods have some limitations. For example, the performance of these methods largely depends on the accuracy of the analytical FE model. In terms of optimising a large-scale and complex structure, constructing a well-conditioned sensitivity matrix is difficult, and the computational load is heavy. The FE model updating methods reviewed in this subsection have been summarised in the following table.

Table 3. Finite element model updating methods

Method	Feature	Application	Authors
Conventional model updating			
	FRFs	Numerical six-bay truss	Sipple and Sanaye [69]
	Frequencies and mode shapes	Experimental full-scale RC building slice	Moaveni et al. [70]
	Accelerations	Experimental ~~quarter-scale~~ RC bridge	Jafarkhani and Masri [71]
	Accelerations	Experimental steel frame	Li et al. [72]
	Dynamic strain	Experimental steel frame	Li et al. [73-77]
	Static strains, displacements, slopes, frequencies and mode shapes	Benchmark bridge model	Sanayei et al. [78]
Substructuring techniques			
Free interface CMS	Frequencies and mode shapes	Numerical frame	Yu et al. [80]
Free interface CMS	Frequencies and mode shapes	Numerical wing structure and bolted plate	Wang et al. [81]
Free interface CMS	Frequencies and mode shapes	Experimental arch bridge ~~model~~	Liu et al. [82]
Fixed interface CMS	Frequencies and mode shapes	Numerical highway bridge	Papadimitriou and Papadioti [84]
Kron's substructuring method	Frequencies and mode shapes	Numerical frame	Weng et al. [86]
Inverse substructure method	Frequencies and mode shapes	Experimental steel frame and numerical tower	Weng et al. [89]
Bayesian substructure method	Accelerations	Numerical shear building	Yuen and Huang [90]

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Regularisation techniques			
Tikhonov regularisation	Accelerations	Numerical 3D frame	Wang and Yang [92]
Adaptive Tikhonov regularisation	Accelerations	Numerical plane truss	Li and Law [93]
Tikhonov regularisation	Accelerations	Experimental steel beam	Zhu et al. [94]
L_1 regularisation	Frequencies and mode shapes	Numerical truss structure	Bao et al. [114]
L_1 regularisation	Frequencies	Numerical beam and plate	Hernandez [115]
L_1 regularisation	Frequencies	Experimental steel beam	Zhou et al. [116]
Reweighted l_1 regularisation	Accelerations	Experimental steel beam	Zhang and Xu [117]
L_1 regularisation	Frequencies and mode shapes	Experimental steel frame	Hou et al. [118]
L_1 regularisation	Frequencies	Experimental steel beam	Wu and Zhou [119]
L_1 regularisation	Frequencies	Experimental beam and frame	Zhang et al. [120]
L_p regularisation	Accelerations	Experimental steel shear building	Huang et al. [121]
Iteratively reweighted l_1 regularisation	Frequencies and mode shapes	Experimental 3D steel frame	Zhou et al. [122]
Trace Lasso	Frequencies and mode shapes	Experimental steel beam	Chen et al. [123]
Weighted trace Lasso	Frequencies and mode shapes	Experimental steel beam	Chen and Yu [124]
L_1 regularisation	Frequencies and mode shapes	Experimental steel beam	Wang and Lu [125]
L_1 regularisation	Frequencies and mode shapes	Experimental RC bridge	Ding et al. [126]

Note: The methods listed in the table all achieve Levels 1–3 damage identification.

2.4 Optimisation algorithms

Optimisation algorithms have been employed by many researchers for damage detection and can be regarded as an effective alternative to the sensitivity-based FE model updating technique in solving inverse problems. Traditional optimisation methods are usually gradient-based and require a good initial value, thereby limiting their potential applications.

With the development of computational intelligence, a number of optimisation algorithms, such as GA, ANN, particle swarm optimization (PSO) and artificial bee colony (ABC), have been proposed. These algorithms do not rely on specific formulas for optimisation and thus avoid the aforementioned shortcomings. Moreover, these algorithms are effective in dealing with uncertainties and insufficient information, which are typical problems in structural damage detection. ANN is an important ML technique and will be detailed in Subsection 2.6.

GA, ~~which was~~ developed in the 1970s [133], is based on the concept of natural selection and has been utilised for damage detection since the 1990s. The main critical problem of GAs is the ~~large~~ heavy computational ~~effort~~ load due to the high dimension of the search space.

Meruane and Heylen [134] implemented a hybrid real-coded GA to locate and quantify structural damage by using five different parameters, namely, frequency, modal displacement, MAC, MSE and modal flexibility. In comparison with conventional optimisation methods, the proposed approach could reach a more precise solution. Ghodrati Amiri et al. [135] compared a pattern search and GA for damage identification in plates. The numerical study indicated that the GA provided better results than the pattern search in some cases.

Guo and Li [136] combined the evidence theory and PSO for multiple damage identification. First, an information fusion method was applied to detect damage sites by integrating the damage localisation information from MSE and natural frequencies. An improved PSO was then used to determine the damage extent. ~~–~~Chen and Yu [137] combined the PSO algorithm and an improved Nelder–Mead method to maximise the likelihood function in Bayesian inference constructed using natural frequencies and mode shapes. The identification results obtained by searching the local area around the optimum solution found by PSO were more stable and accurate than those obtained by the PSO-based algorithm.

Ding et al. [138] presented a modified ABC algorithm to optimise the objective function by using modal parameters. Two modifications were introduced to improve the convergency~~t~~ rate and local search ability of the ABC algorithm. The numerical study revealed that the damage identification results of the proposed algorithm were more accurate than those of other evolutionary algorithms, such as GA and PSO. Later, they applied the modified ABC algorithm to identify cracks in beams by using natural frequencies only [139].

The aforementioned optimisation methods are compared and summarized in Table 4.

Table 4. Optimisation methods

Method	Strengths	Limitations	Features	Damage identification level	Applications	Authors
GA	A global optimizer	Heavy computation				
Real-coded GA			Frequencies, mode shapes and variants	1-3	Experimental 3D truss	Meruane and Heylen [134]
Binary-coded GA			Frequencies and mode shapes	1-3	Numerical plate	Ghodrati Amiri et al. [135]
PSO	Efficient, a few function evaluations, a few parameters to adjust	Difficult to control the balance between exploration and exploitation				
Mutation PSO			Frequencies and MSE	1-3	Numerical truss	Guo and Li [136]
Hybird PSO			Frequencies and mode shapes	1-3	Phase I IASC-ASCE benchmark problem	Chen and Yu [137]
ABC	Simple structure, high flexibility, good robustness	Improper exploitation for complicated problems, slow convergence rate				
Modified ABC			Frequencies and mode shapes	1-3	Numerical beam and plate	Ding et al. [138]
Improved ABC			Frequencies	1-3	Experimental steel beam	Ding et al. [139]

2.5 Statistical time series methods

Time series-based methods typically fit time series models to the measured time history data and then extract features sensitive to variations caused by damage and insensitive to operational and environmental variations. AR models and their variants, such as autoregressive with exogenous input (ARX) and ARMA models, are commonly used to extract damage-sensitive features based on residual errors or AR model parameters [140]. These methods inherently account for uncertainties and do not depend on the physical models. As such, they are more suitable for automated SHM systems than model updating methods. Statistical time series methods mainly consist of three components: (i) random excitation and/or response signals, (ii) statistical time series model building and (iii) statistical decision making for damage diagnosis.

Mahalanobis squared distance (MSD) is a statistical measure for outlier detection and has received wide applications because of its simplicity and computational efficiency. Mosavi et al. [141] located damage to continuous structures under ambient vibrations by using vector AR models and MSDs. Statistical evaluations were performed on extracted damage features for each individual sensor location. A sensor with a ~~large~~ significant variation was identified as the one closest to the damage location. However, a dense array of accelerometers was required to identify the accurate damage location. Chang and Kim [142] performed multivariate AR analysis and extracted frequencies, mode shapes and damping ratios as damage-sensitive features. The outlier analysis was then conducted on the basis of MSD. A field experimental study on a simply supported steel truss bridge showed that the inclusion of additional parameters in the outlier analysis might lead to more sensitive features.

Although the MSD-based method possesses several distinguishing advantages, it requires multivariate ~~normal~~ training data from the undamaged structure. Previous studies achieved an approximation of multinormal data by increasing the observation-to-variable ratio [143, 144]. However, the number of measurements is limited in many practical applications. To this end, Nguyen et al. [145] proposed a data generation scheme based on Monte Carlo simulation.

Simulation data was added as input parameters to compensate for the data shortage. Thus, the computational stability and reliability of MSD-based damage identification were improved.

Gul and Catbas [146] constructed ARX models by using the free acceleration responses of a structure and developed two different approaches to extract damage-sensitive features. In the first approach, the B-term coefficients of the ARX models were directly used, and numerical results showed that this approach was effective for simple and noise-free models. In the second approach, the ARX model fit ratios were selected as the damage feature to consider the noise effect and model complexity. The difference between fit ratios indicated the relative change in damage severity, although direct quantification was not achieved. Numerical and experimental results showed that the second approach performed successfully under different damage scenarios for complex models and test specimens. Later, they extended this method to the ambient vibration case, in which RD was applied to obtain pseudo-free vibration data from ambient vibration time histories [147].

Yao and Pakzad [148] proposed two new damage features: one was based on the Ljung–Box statistic of the AR model residual and the other on the Cosh spectral distance of the AR model spectrum. The results of Ljung–Box statistic were more accurate than those of the existing algorithms based on the AR model residual variance and coefficient distance. The Cosh spectral distance was less sensitive to changes in excitation sources. Lakshmi et al. [140] conducted a singular spectral analysis to enhance the sensitivity of the damage features derived from the auto-regressive moving average with an exogenous input (ARMAX) model. Shahidi et al. [149] investigated the performance of four damage features derived from single- and multivariate regression models in detecting the timing and location of the structural damage. Sequential normalised likelihood ratio test and two-sample control statistics were adopted to detect the change in two families of damage features.

A critical problem for efficiently employing the AR models for feature extraction is the determination of an appropriate model order. Traditionally, the optimal model order was selected in an *ad hoc* manner. In this regard, Figueiredo et al. [150] presented four techniques based on the Akaike information criterion, partial autocorrelation function, root mean square error and singular value decomposition (SVD) to determine the appropriate model order. An appropriate range of the model order, rather than a unique value, could be determined using the four proposed techniques. A comparative study was carried out to investigate the influence of the model order on damage detection results.

Although the aforementioned methods are successful in identifying the presence and location of damage, they provide limited information about damage severity. Therefore, they belong to Levels 1–~~-~~2 damage identification.

Kalman filtering is a useful technique for time series analysis and state estimation and has received wide applications in signal processing and econometrics. Lei et al. [151] used an extended Kalman estimator to ~~sequentially~~ identify structural parameters and unknown excitation sequentially. The proposed algorithm was extended to damage identification of large-scale structures based on the substructural approach. The inter-connection effect between adjacent substructures was estimated without the measurements of the substructure interface degrees of freedom (DOFs). This feature was a major advantage over previous substructural identification approaches, which require all the responses at the substructure interfaces DOFs.

In a traditional EKF approach, unknown structural parameters were incorporated into an extended state vector. When a large number of unknown parameters and extended state vectors are identified simultaneously, a divergence problem may arise. In this regard, Lei et al. [152] proposed a two-step Kalman filter approach. The structural state vector was first recursively estimated using the traditional Kalman filter technique with the assumed structural parameters, which were then estimated given the state vector. The proposed algorithm reduced the number of estimated parameters in each step, and thus improved estimation convergence. They [153] adopted the approach to detect damage of frame structures in which a beam-column joint was modelled as in Weng et al. [154].

The aforementioned statistical time series methods are compared and summarized in Table 5.

Table 5. Statistical time series methods

	Authors	Methods	Features	Damage identification level	Applications	Remarks
Output only	Mosavi et al. [141]	MSD	AR coefficients	1 and 2	Experimental steel beam	Require multivariate ~~normal~~ training data
	Chang and Kim [141]	MSD	Frequencies and mode shapes	1 and 2	Steel truss bridge	Require multivariate ~~normal~~ training data
	Nguyen et al. [145]	MSD, Monte Carlo simulation	AR vectors	1 and 2	Benchmark building	Compensate for data shortage and multivariate ~~normal~~ data condition
	Gul and Catbas [146]	ARX	ARX coefficients, ARX model fit ratios	1 and 2*	Experimental steel grid structure	Free acceleration responses
	Gul and Catbas [147]	RD	ARX model fit ratios	1 and 2*	Experimental steel grid structure	Ambient acceleration responses
	Yao and Pakzad [148]	Ljung–Box statistic/Cosh spectral distance	AR model residual, AR model spectrum	1 and 2	Experimental truss and bridge slab	Insensitive to changes in excitation sources
	Lakshmi et al. [140]	Singular Spectrum Analysis	ARMAX models	1 and 2	Benchmark bookshelf structure and experimental RC beam	Forced acceleration responses
	Shahidi et al. [149]	Control chart	Coefficients of single-variate regression, collinear regression, ARX and AR	1, 2, and damage instant	Experimental ~~scaled~~ steel frame	Require multivariate ~~normal~~ training data

	Figueiredo et al. [150]	Akaike information criterion and SVD	AR residual errors and model parameters	1	Experimental aluminum frame	Determine optimal AR model order
Input-output	Lei et al. [151]	EKF	Accelerations	1-3	Numerical study on Phase I IASC-ASCE benchmark problem, beam, and truss structure	Use limited input measurements
	Lei et al. [152]	Two-step Kalman filter	Accelerations	1-3	Numerical study on Phase I IASC-ASCE benchmark problem and a ~~30-story~~ shear building; Experimental frame	Reduce the number of estimated parameters
	Lei et al. [153]	Two-step Kalman filter	Accelerations	1-3	Experimental steel frame with joint damage	Reduce the number of estimated parameters

Note: * — Quantitatively indicate the relative severity of damage

2.6 ML methods

Structural damage identification can be treated as a pattern recognition problem, which is divided into three parts: (1) data acquisition, (2) feature extraction and (3) feature classification [16]. Feature extraction aims to fit either a data-driven or a physics-based model to the measured structural response data by using statistical or signal processing techniques. The parameters of these models or model residuals are then selected as damage sensitive features. Finally, with the selected features, the classification algorithm is utilised to determine the presence, location and severity of damage.

In recent years, a number of ML classifiers have been utilised and developed for structural damage identification [155]. These ML algorithms can be broadly divided into supervised, unsupervised and semi-supervised learning modes.

2.6.1 Supervised learning

Most ML algorithms are based on a supervised learning manner, which requires features of both undamaged and damaged states of the structure with their labels to establish a statistical model during the training process [156]. Three commonly used classification methods, i.e., ANN, SVM and RF, will be reviewed in this subsection, and their strengths and limitations are provided in Table 6.

Table 6. Supervised learning methods

Method	Strengths	Limitations
ANN	Self-learning, flexible Suitable for complex and nonlinear problem	Computational expensive Prone to overfitting
SVM	Effective for high dimension data Fault-tolerance Prone to global optimal solution	Computational expensive in large dataset ~~Perform~~ P~~p~~oor performance for noisy datasets with overlapping classes Binary classification algorithm Tricky selection of kernel function
RF	Accurate and stable Computational efficient Can extract variable importance Able to reduce variance	Sensitive to noise and outliers Not easy to interpret Need to tune hyperparameters

The ANN technique is a widely used ML algorithm, which has been introduced to civil engineering since the 1980s [157]. ANNs have drawn considerable attention in SHM and damage identification due to their ability of pattern recognition and error tolerance in establishing a nonlinear relationship between the inputs and outputs. For structural damage identification, the ANN is used to establish a model representing the relationship between features extracted from structural vibration data and structural model parameters through a training process. This trained ANN model is then capable of identifying damage from measurement data [158]. The learning algorithm of ANNs can either be supervised or unsupervised, whilst most of them are supervised, especially in damage detection applications.

Jiang et al. [159] proposed a two-stage approach by combining fuzzy NNs and data fusion techniques. In this approach, structural modal parameters were fed into fuzzy NNs as inputs. The data fusion technique was applied to the outputs from different fuzzy NNs, and a consistent and reliable damage assessment result was obtained. Dackermann et al. [160] identified the member connectivity and mass changes in a frame structure using ANNs. In their study, individual networks were first trained with FRF data at different measurement locations. The outcomes of each network were then fused through a network ensemble to generate final damage conditions. The proposed network ensemble technique was superior to the approach that simply added the FRF data to train the ANN. Xu et al. [161] constructed NNs to locate and quantify joint damage by directly using dynamic displacement responses and excitation information.

Hakim and Razak [162] trained an ANN with natural frequencies and utilised it to quantify damage severity in a steel girder bridge model. Moreover, Hakim and Razak [163] compared an ANN with an adaptive neuro-fuzzy inference system by using the same experimental model. The latter incorporated ANNs and FL systems in a single framework and had benefits of both techniques. Experimental results showed that the damage identification results of the proposed framework were more accurate than those of the ANN.

However, a considerable amount of computational effort is required in the ANN techniques, especially when large DOFs are involved. Therefore, the ANN-based damage identification is generally applicable to small structures with a limited number of DOFs. To this end, Bakhary et al. [158] used a multi-stage ANN method with a substructure technique to detect the damage location and extent. The full structure was divided into substructures, and each substructure was independently analysed to ~~progressively~~ identify damage progressively. In this manner, the size of the ANN models was reduced, and the computational effort was saved.

PCA is a statistical technique for dimension reduction and feature extraction. This method reduces a large set of correlated variables to a low dimension through orthogonal transformation whilst retaining the most relevant information. PCA has been extensively applied to measured structural vibration responses for reduced-order modelling, modal analysis and parameter identification. Worden et al. [164] first applied the PCA in structural damage detection. Some researchers [160, 165-167] applied PCA to reduce the dimension of the FRF data and then utilised the ANN to train the FRF data for damage detection.

To improve damage identification accuracy, Bandara et al. [167] investigated the number of hidden layers and the number of neurons per hidden layer to formulate an optimal architecture of ANN with minimum training and testing errors.

Recently, with the improvement of computing capacity and network architecture, deep learning algorithms evolved from the ANN, e.g., convolutional NNs (CNNs), have been developed rapidly [168]. Abdeljaber et al. [169] proposed adaptive 1D CNNs, which fused feature extraction and classification blocks into a single and compact learning body. Consequently, these NNs could directly learn from the acceleration data measured under known random excitations. Since modal identification was not required, it could be implemented near-real-time and suitable for online SHM. Duan et al. [170] used a CNN technique to detect damage in bridge hangers, in which the Fourier amplitude spectra of the acceleration responses were used as the input. Bao et al. [171] transformed the time series signals into image data. The randomly selected and manually labelled image data were then used to train the deep NNs via the greedy layer-wise training technique and automatically detect anomalies of a cable-stayed bridge.

Support vector machine (SVM) is a supervised learning model that aims to separate two classes of data. It is trained to estimate the boundary between two classes by maximizing the margin and minimizing the misclassification [172]. SVM is becoming increasingly popular these years for damage identification because of its superior ability to solve nonlinear, high-dimensional and small sample problems [173]. In comparison with conventional NNs, SVM overcomes the problems of local minimisation and inadequate statistical capabilities [174].

Kourehli [175] used the first two incomplete mode shapes and natural frequencies as input data to train the SVM. In this approach, a radical basis function (RBF) was chosen as a kernel function. The parameters of the kernel function were determined on the basis of the coupled simulated annealing and standard simplex method. Liu and Jiao [176] used GA to optimise the SVM parameters with ~~the~~ similar input data and kernel function to detect damage of bridges. Numerical studies on a simply supported bridge have demonstrated the feasibility and superiority of the GA-SVM algorithm to RBF networks and back propagation NNs (BPNNs) optimised by GA.

Ghiasi et al. [174] introduced the thin plate spline Littlewood–Paley wavelet kernel function to improve the learning ability of SVM. In this approach, feature vectors, as the input of the SVM, were extracted from the acceleration responses through the wavelet packet decomposition. A social harmony search algorithm was used to determine the parameters of the SVM. In comparison with the SVM based on other combinational and conventional kernels, the proposed kernel achieved an enhanced performance for multiple damage identification.

Gui et al. [177] compared SVM on the basis of three optimisation algorithms, namely, grid search, PSO and GA, to optimise the penalty parameters and parameters of the RBF kernel parameters. Two types of features, namely, the parameters of the AR model and the residual errors of the statistical parameters, were extracted from the time series data. The optimisation-based methods significantly improved the sensitivity, accuracy and effectiveness of the conventional SVM. Using the residual errors achieved a significantly higher accuracy than using the AR type.

Random forest (RF) is an ensemble classifier that consists of a large number of decision trees [178]. The model prediction is obtained through combining the predictors of each individual tree by majority voting. Zhou et al. [179] proposed a damage detection method by RFs and data fusion. In this method, the wavelet packet decomposition was applied to decompose acceleration signals into energy features, which were fused into new energy features through data fusion. The obtained features were then inputted into RFs to classify structural damage. Experimental results showed that the accuracy and stability of the proposed method ~~was~~ were higher than those of RF alone, SVM alone and SVM and data fusion.

The supervised ML techniques require data from undamaged and damaged structures for training purposes. However, data associated with various damage scenarios may be unavailable in practical structures. Most studies have generated training samples from laboratory testing or through numerical FE simulations. Therefore, the efficiency of the supervised learning approaches depends on the model accuracy. For this reason, the development of unsupervised algorithms is of particular interest.

2.6.2 Unsupervised learning

An unsupervised learning algorithm only requires data from the intact state of a structure for training, which belongs to the outlier or novelty detection category. A model is trained by machine learning algorithms based on the data in the undamaged state. The trained model is then used to evaluate the structural condition when new measurement data are available. If the difference between the measured data and those predicted from the model exceeds a threshold, the structure is regarded as a deviation from its normal condition and is probably damaged.

Santos et al. [180] combined two statistical learning methods for online early-damage detection. Multi-layer perceptron NNs were used for the statistical modelling of the structural responses. The unsupervised K-means clustering algorithm was employed to classify the neural networks' estimation errors. These methods were sequentially applied in successive time windows to realize continuous on-line damage identification.

Neves et al. [181] developed a model-free ANN-based approach for damage detection of bridge structures. ANNs were trained with an unsupervised learning approach using accelerations collected on the healthy bridge. The prediction errors of each network were then statistically characterized by a Gaussian process to determine a damage detection threshold. Consequently, the structural condition, namely healthy or damaged, was determined by comparing damage indices with the selected threshold.

Rafiei and Adeli [182] used an unsupervised restricted Boltzmann machine to extract features from the frequency domain of the ambient vibration signals. A structural health index was established for each substructure in terms of a PDF, which measured the similarity between the ambient vibrations of the current state of the structure and those of the healthy one. The larger the difference, the higher the likelihood of damage.

Cha and Wang [183] revised the original density peaks-based fast clustering algorithm to an unsupervised machine learning method to detect and locate structural damage. An intact statistical model was built by using the training points from each sensor in the intact state of the structure. The sensor location corresponding to the novelty point was identified as the location where damage occurred.

Avci and Abdeljaber [184] proposed an unsupervised damage detection algorithm based on self-organizing maps, which is a class of ANNs. In their algorithm, self-organizing maps were used to extract the damage indices from the random acceleration responses of the monitored structure. The summation of the indices indicated the overall condition of the structure, ~~and~~ the value of which could be used to evaluate the damage severity.

Although unsupervised learning approaches are preferred for practical damage detection, most of them are limited to novelty detection, i.e., Level 1 damage identification, and fail to give additional information in terms of the location and severity of damage.

2.6.3 Semi-supervised learning

In practice, the acquisition of fully labelled data for training is infeasible, whilst a small number of labelled data may be available. In such situations, semi-supervised learning can be of great practical value, which falls between unsupervised and supervised learning, using both labelled and unlabelled data for training classifiers. Many researchers have found that using unlabelled data in conjunction with a small amount of labelled data may considerably improve the accuracy of ML algorithms [155]. Rather than pure novelty detection, semi-supervised learning approaches are able to locate and quantify structural damage. However, the applications of semi-supervised ML algorithms for damage identification are very limited in the literature.

Chen et al. [185] combined multi-resolution classification with semi-supervised learning for damage detection of bridge structures. The features were extracted from localised time-frequency sub-bands. The adaptive graph filter classifier was used to classify unlabelled data given previously labelled signals. A weighting algorithm was developed to combine information from both labelled and unlabelled signals to make a global decision. Furthermore, in addition to unlabelled data, the adaptive graph filtering was able to handle mislabelled as well as unseen signals.

Lai and Nagarajaiah [186] developed a semi-supervised algorithm to detect and characterize linear/nonlinear structural damage. The baseline (undamaged) model was established using a sparse identification method based on supervised learning with input-output time history data. Damage was considered as a variation of the restoring force, and thus the damaged system was transformed into an equivalent linear system subjected to external disturbance forces and pseudo-forces. Consequently, the nonlinearity (including presence, type, and extent of damage) was represented by the pseudo-forces and discovered in an unsupervised way without the creation of various damage scenarios.

Rogers et al. [187] used Dirichlet process clustering models for online damage detection based on features ~~from~~ in the high dimensional frequency domain. The algorithm learned clusters of data online without a training phase~~,~~ and then assigned labels to new clusters in a semi-supervised manner. The model with already known states of the structure was continually updated as more data were added. As time progressed, the method learned more states and the robustness of the method increased.

In recent years, ML algorithms are growing rapidly and have received considerable attention in damage identification. However, there are still some challenges and difficulties requiring further research. The training dataset is extremely important for the performance of ML algorithms. Consequently, data selection, data cleaning, data compression, data fusion, data normalization, and data labelling are inevitable to establish the appropriate datasets. The processes are time consuming and labour intensive. Moreover, for structural damage detection, the lack of enough training samples may lead to over-fitting problems, e.g. over-extraction of irrelevant features such as measurement noise [188]. The generalisation ~~ability~~ is another critical problem for ML algorithms. A well trained and validated model may only perform well for a specified type of structures and a ~~certain~~ particular pattern of damage. The following table compares and summarizes the ML methods that have been reviewed in this subsection.

Table 7. ML methods

Authors	Methods	Inputs	Damage identification level	Applications	Remarks
Supervised learning					
Jiang et al. [159]	Fuzzy NN+data fusion	Frequencies and mode shapes	1-3	Numerical shear-type building	

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Dackermann et al. [160]	ANN+PCA+NN ensembles	FRF	1, 2	Experimental frame	Joint damage and mass changes
Xu et al. [161]	NNs	Dynamic displacement	1-3	Experimental steel frame	
Hakim and Razak [162]	ANNs	Frequencies	1 and 3	Numerical steel girder bridge	
Hakim and Razak [163]	Fuzzy NN	Frequencies	1 and 3	Experimental steel girder bridge	
Bakhary et al. [158]	ANN+substructrue technique	Frequencies and mode shapes	1-3	Numerical RC slab and frame	
Li et al. [165]	ANN+PCA+NN ensembles	FRF	1-3	Experimental steel beam	
Samali et al. [166]	ANN+PCA	FRF	1-3	Experimental steel frame	Notch-Type damage
Bandara et al. [167]	ANN +PCA	FRF	1-3	Numerical framed structure	Optimal architecture of ANN
Abdeljaber et al. [169]	1D CNNs	Acceleration under ~~known~~ random excitations	1 and 2	Experimental steel frame of a grandstand simulator	On-line, joint damage
Duan et al. [170]	CNNs	Fourier amplitude spectra of wind-induced acceleration	1-3	Numerical tied arch bridge	Automatic
Bao et al. [171]	DNNs	Image vectors converted from acceleration–	1	Long-span cable-stayed bridge	Automatic, real-time
Kourehli [175]	SVM	Frequencies and mode shapes	1-3	Numerical beam, plane frame, and spring-mass system	RBF kernel function
Liu and Jiao [176]	SVM	Mode shape ratio and frequency rate	1-3	Numerical simply supported bridge	RBF kernel function
Ghiasi et al. [174]	SVM	Wavelet energy spectrum	1 and 2	Numerical Phase I IASC-ASCE benchmark problem and a 120-bar dome	Thin plate spline Littlewood–Paley wavelet kernel function

				truss	
Gui et al. [177]	SVM	AR coefficients and residual errors of the statistical parameters	1 and 2	Experimental benchmark frame	RBF kernel function
Zhou et al. [179]	RF+data fusion	Energy features from acceleration	Damage type and location	Numerical steel benchmark frame and experimental steel shear frame	
Unsupervised learning					
Santos et al. [180]	NNs+K-means clustering	Time-series displacements and rotations	1	Experimental cable-stayed bridge	On-line
Neves et al. [181]	ANNs	Accelerations from ~~a~~ passing vehicle	1	Numerical railway bridge	
Rafiei and Adeli [182]	Deep Boltzmann machine	Frequency domain of ~~the~~ ambient vibration ~~signals~~	1	Experimental RC building	
Cha and Wang [183]	Density peaks-based fast clustering	Crest factor and T-continues WT extracted	1	Experimental steel structure	
Avci and Abdeljaber [184]	Self-organizing maps	Random acceleration responses	1	Phase II IASC-ASCE benchmark problem	
Semi-supervised learning					
Chen et al. [185]	Multi-resolution classification+label propagation+ Adaptive Graph Filter	Localised time-frequency sub-bands	1	Experimental bridge-vehicle dynamic system	
Lai and Nagarajaiah [186]	Sparse identification + pseudo-force	Velocity, acceleration and displacement	1-3	Experimental steel frame, benchmark frame, a base-isolated	Linear/ nonlinear ~~type~~ damage

				building
Rogers et al. [187]	Dirichlet process clustering	FRF and frequencies	1	Benchmark building, Z24 bridge

2.7 Bayesian methods

Civil structures are generally subjected to significant measurement noise and modelling errors, which may lead to incorrect damage identification [189, 190]. For example, the existence of measurement noise may mask subtle structural changes caused by damage. Consequently, deterministic methods may fail once they are applied to practical civil structures. In this regard, many researchers proposed probabilistic damage identification approaches [191]. Amongst these methods, Bayesian inference has attracted considerable attention since the 1990s, which explicitly quantifies the posterior probability of uncertainties based on observations and prior information [192, 193]. Apart from addressing uncertainties, Bayesian methods also provide an efficient way to deal with the ill-posed inverse problem by specifying probability distributions over uncertain parameters; this approach is equivalent to introducing a regularisation term to the optimisation problem [194].

Figueiredo et al. [195] developed a Bayesian pattern recognition approach based on a Markov-chain Monte Carlo method. The Bayesian approach was employed to cluster structural responses into a reduced number of global state conditions by using a finite mixture of Gaussian distributions. Outlier detection was then conducted on the basis of MSD. The applicability of the proposed approach was demonstrated using the data sets from the Z-24 Bridge.

Arangio and Beck [196] used the Bayesian NN for bridge integrity assessment under ambient vibrations. In this method, an optimal network architecture was determined on the basis of Bayesian model class selection. An automatic relevance determination (ARD) method was applied to measure the relative importance of different inputs in NNs and separate relevant variables from redundant ones. Comparison studies indicated that the accuracy of the optimal network model in damage localisation and quantification was better than that of a heuristic-based model. Later, Arangio and Bontempi [197] applied a Bayesian NN to the Tianjin Yonghe Cable-Stayed Bridge and detected cracks at the external portions of both spans and damage at two piers. In this rare case, the accelerations of the deck before and after damage were employed.

Lam et al. [198] detected damage to a railway ballast by using modal parameters within a Bayesian framework. In their study, the ballast under a concrete sleeper was uniformly divided into a number of regions with similar stiffness. The number of divided regions was determined on the basis of the Bayesian model class selection method. The posterior probability density function (PDF) of the ballast stiffness in different regions was approximated by a multivariable Gaussian distribution.

Behmanesh and Moaveni [199] implemented a Bayesian FE model updating to identify damage to a full-scale structure. Damage was simulated by adding concrete blocks onto a bridge deck. The adaptive Metropolis–Hastings algorithm was used to sample the posterior distribution of the updating parameters. Behmanesh et al. [200] investigated the effects of the subset of modes used on the performance of Bayesian FE model updating. The optimal subset of modes in the model updating process was determined using Bayesian model class selection. Damage identification was then conducted for different weight factors, and the final estimation was obtained by averaging all the results via the Bayesian model averaging technique.

Yin et al. [201] developed a probabilistic damage identification method for bolt connections by using incomplete modal parameters. They combined a system mode-based method [202] and a dynamic model reduction method [203] to avoid using the complete mode shapes. The joint posterior PDF of the model and modal parameters was approximated via Gaussian distribution.

In recent years, Sparse Bayesian Learning (SBL), as a supervised learning framework [204-206], has received great attention as a means of deriving sparse solutions in the context of regression and classification [207-209]. The SBL has some similarities to sparse recovery theory that uses l_p ($0 \leq p \leq 1$) regularisation technique. The prior distribution in SBL can induce sparsity in inferred parameters, which functions as the regularisation term in sparse recovery. In SBL, a parameterised prior, that is, the ARD prior, is adopted instead of a fixed prior to incorporate a preference for sparse parameters. An individual hyperparameter is assigned to each unknown parameter, resulting in sparse solutions [204, 206].

SBL has several significant advantages over the deterministic sparse recovery, for example, the l_1 or l_0 regularisation techniques and orthogonal matching pursuit. The SBL closely resembles the l_0 regularisation, which typically results in a sparser solution with higher accuracy than the l_1 regularisation. In SBL, the global minimum is achieved at the maximally sparse solution, which is a desirable property of l_0 regularisation [205]. When the sensing matrix does not satisfy the incoherence criteria, the performance of most existing CS algorithms will degrade, whilst the SBL still retains the excellent ability for sparse recovery [210]. The SBL technique is more general and more flexible than the sparse recovery theory. The latter using the regularisation techniques disregards the relative uncertainties between different variables and requires estimation of the regularisation parameter. However, the hyper-parameters in SBL possess a clear physical meaning that represents the precision of the uncertainties. The hyper-parameters can be updated automatically, thereby avoiding the tricky selection of the regularisation parameter in sparse recovery.

Although the Bayesian probabilistic approach has been introduced and applied to structural damage identification for nearly two decades, SBL has not been utilised and explored for structural damage detection until recently. One main reason is that the modal data are a nonlinear function of the structural damage parameters. Consequently, the integral in the evidence of the Bayesian equation cannot be calculated directly. Analytical and numerical approaches have been developed to tackle this difficulty. The former includes hierarchical modelling and asymptotic techniques (e.g. Laplace's approximation), whilst the latter includes expectation–maximization (EM) technique and sampling techniques.

Bayesian hierarchical modelling is a model written in a hierarchical form, which is particularly useful in dealing with complex and nonlinear problems. Huang and Beck [211] developed a hierarchical SBL method by expanding the nonlinear eigenvalue problem as multiple linear regression functions. The simulated damage was successfully detected with improved accuracy compared with the Bayesian updating method in Yuen et al. [202]. Later, Huang et al. [212] improved the SBL algorithm by eliminating two approximations in the theoretical formulation. The efficiency of the improved algorithm was higher and its performance was better than that of the previous SBL method for a real structure with a significant modelling error.

Multi-task learning is a useful tool to exploit data redundancy between different groups of measurements. To improve the reliability of damage localisation, Huang et al. [213] used a multi-task SBL to fuse the respective strengths of two FD-based damage indices. The linear regression models were employed to model the relationship between a damage localisation vector and two damage indices, which were then incorporated in the likelihood function. Huang et al. [214] simultaneously utilised multiple groups of measurements and proposed two hierarchical Bayesian models for multi-task SBL. In these models, an ARD prior was assigned across multiple tasks to characterise the shared sparseness profile. Unlike the previous multi-task SBL algorithm [209], the prediction error precision parameters were marginalised from hierarchical models to improve the learning robustness and characterise the posterior uncertainty.

Hou et al. [216] proposed an EM-based SBL method for damage detection. An iterative EM technique was employed to tackle a nonlinear eigenvalue problem without performing asymptotic approximation or stochastic simulation. Wang et al. [217] extended SBL via Laplace approximation, in which a complicated integral in the evidence was approximated as a Gaussian PDF. Consequently, damage parameters and hyper-parameters were derived in an analytical form and iteratively solved without sampling.

On the basis of a similar hierarchical SBL model, Huang et al. [218] proposed two Gibbs sampling (GS) algorithms to sample the posterior PDF of uncertain parameters and provide a full treatment of the posterior uncertainty. Laplace's approximation was used to estimate hyper-parameters. Later, they [219] developed a full GS method to characterise the posterior uncertainty of hyper-parameters. The proposed partial and full GS algorithms were applied to the IASC-ASCE Phase II benchmark problem. The full GS algorithm was verified to be more reliable than the partial one for real experimental studies.

Table 8 compares and summarizes the Bayesian methods that have been reviewed in this subsection.

Table 8. Bayesian methods

Authors	Methods	Features	Bayesian inference	Damage detection level	Application
Figueiredo et al. [195]	MSD	~~Natural~~ F~~f~~requencies	MCMC	1	Z-24 Bridge
Arangio and Beck [196]	NN+ARD	Acceleration under ambient excitation	Laplace's approximation	1-3	Numerical long-span suspension bridge
Arangio and Bontempi [197]	NN+ARD	Acceleration under ambient excitation	Laplace's approximation	1	Benchmark cable-stayed bridge
Lam et al. [198]	Bayesian model class selection	Frequencies and mode shapes	Laplace's approximation	1-3	Experimental ballasted track
Behmanesh and Moaveni [199]		Frequencies and mode shapes	Metropolis–Hastings algorithm	1-3	Numerical footbridge
Behmanesh et al. [200]	Bayesian model class selection and ~~Bayesian model~~ averaging	Frequencies and mode shapes	MCMC	1-3	Numerical steel frame

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Yin et al. [201]		Frequencies and mode shapes	Laplace's approximation	1-3	Experimental bolted frame
Huang and Beck [211]	SBL	Frequencies and mode shapes	Hierarchical modelling+ Laplace's approximation	1-3	Numerical shear-building, Phase II IASC-ASCE benchmark problem
Huang et al. [212]	SBL	Frequencies and mode shapes	Hierarchical modelling+ Laplace's approximation	1-3	Phase II IASC-ASCE benchmark problem
Huang et al. [213]	Multi-task SBL	FD-based damage indices	Bayesian linear regression	1, 2	Experimental steel beam, a real cable-stayed bridge
Huang et al. [215]	Multi-task SBL	Frequencies and mode shapes	Hierarchical modelling+ Laplace's approximation	1-3	Phase II IASC-ASCE problem
Hou et al. [216]	SBL	Frequencies and mode shapes	EM	1-3	Experimental cantilever beam
Wang et al. [217]	SBL	Frequencies and mode shapes	Laplace's approximation	1-3	Experimental steel frame
Huang et al. [218]	SBL	Frequencies and mode shapes	Hierarchical modelling+ partial GS+ Laplace's approximation	1-3	Phase II IASC-ASCE benchmark problem
Huang et al. [219]	SBL	Frequencies and mode shapes	Hierarchical modelling+ full GS	1-3	Phase II IASC-ASCE benchmark problem

2.8 Varying temperature conditions

Structural responses vary under the changing operational and environmental conditions, particularly temperature. Temperature variations influence Young's modulus of most construction materials [220] and boundary conditions [221-223]. Thus, such variations cause changes in structural dynamic properties [224, 225]. Xia et al. [224] quantified the effect of temperature on variations in frequencies, mode shapes and damping through a series of experiments on a continuous concrete slab for nearly 2 years. Some studies have found that changes in structural responses due to temperature variations could be more significant than those due to a medium degree of structural damage [4] or under normal operational loads [226]. Consequently, if the temperature effects are not fully understood, then false structural condition identification may occur. Some techniques [227-229] have been developed to reduce the effects of temperature on damage detection or to detect damage under different temperature conditions.

The approaches that consider the effects of temperature on damage identification can be divided into two categories depending on whether or not environmental variables are measured.

2.8.1 Using temperature measurement

When temperature data are available, the correlation between temperature variables and damage features can be established, and the effects of temperature can be removed from damage features.

Deng et al. [230] developed a six-order polynomial regression model to describe the correlations of frequency–temperature and displacement–temperature by using long-term monitoring data. They then classified the measured changes in the structural responses caused by damage and environmental variations by using a control chart. Bao et al. [231] investigated the relation between modal properties and temperature and applied the Dempster–Shafer data fusion technique [232] to identify damage under varying temperature conditions. The accuracy of damage identification results was increased by incorporating temperature variations. Magalhaes et al. [233] studied the time evolution of the modal parameters of an arch bridge for 2 years. The regression analysis complemented with PCA was conducted to eliminate the effects of environmental and operational factors on natural frequencies. The existence of damage was then successfully detected using the control chart.

In practice, correlations between temperature and damage features are complicated and may not be well established through regression analysis. In this regard, Zhou et al. [225, 234] proposed a BPNN-based approach to eliminate the effects of temperature. A total of 770 h monitoring data of natural frequencies and temperatures obtained from the Ting Kau Bridge in Hong Kong and their FE model was used to train and test a BPNN model. Case studies indicated that the approach could detect the occurrence of damage when the change in damage-induced frequency was as small as 1%. Considering the high-computational load of the BPNN, Jin et al. [235] presented NN trained with EKF to detect damage to a highway bridge under severe temperature changes. The correlation analysis between natural frequencies and temperature was conducted on the basis of 1-year monitoring data. The convergence of the proposed method was faster and its results were more accurate than those of traditional BPNN. Numerical results also showed that the proposed method was superior to the multi-linear regression approach.

Methods using temperature data have some practical issues. For example, damage features should be extracted under a wide range of temperature condition. The shortage of temperature data may affect the accuracy of structural damage detection.

2.8.2 Without temperature measurement

A number of methods have been developed to alleviate the need for ~~a~~ direct measurement of temperature variations by using the measured response data only under varying environmental conditions. In these methods, temperature variations are treated as embedded variables. These methods are typically based on the assumption that variations in structural vibration characteristics due to damage behave differently from those due to varying temperature conditions [227]. In this case, ML algorithms and statistical pattern recognition techniques are typically used to derive a robust damage index for outlier analysis.

Kullaa [236] proposed an approach to distinguish three sources of variability, namely, environmental or operational effects, sensor faults and structural damage. Structural damage was global, and sensor faults were local. Accordingly, sensor fault and structural damage were distinguished through sensor isolation. Measurement data under different environmental or operational conditions were included in the training data to consider environmental or operational effects. Damage or sensor fault localisation was detected on the basis of the maximum likelihood ratio.

Shokrani et al. [237] presented a PCA-based approach for damage localisation under varying environmental conditions. During the training stage, the MSC data matrix was formulated across a representative operational period and the statistical characteristics of the operational variations on a curvature were then extracted via PCA transformation. The residual of MSC matrices between inspected and baseline structural states was used as the damage index based on a hypothesis test. Numerical studies indicated that the proposed method was effective in the case of linear or weakly nonlinear situation. Kostić et al. [238] integrated a sensor-clustering-based time-series analysis method with ANNs to compensate for the effects of temperature. A sensor-clustering-based ARX method was applied to the free vibration acceleration data to calculate the damage features. Multilayer ANNs were then trained using the obtained damage features resulting from different temperature scenarios. Differences between the damage features from the time series and ANN analyses were used for damage detection. Numerical results demonstrated that the proposed method could successfully determine the existence, location and relative severity of damage under varying temperature conditions. Fallahian et al. [239] combined couple sparse coding and deep NNs to assess damage by considering uncertainties, such as noise and temperature. The simulated FRF data was first generated from a numerical model, and PCA was applied to decrease the dimension of FRF data and extract the features. The couple sparse coding and deep NN were then individually trained. The outputs were combined ~~by~~ with the weight majority voting method to make a better decision about the healthy state of the structure.

Liang et al. [240] proposed a novel frequency-based technique to eliminate the interference of varying environmental conditions. The non-stationary frequencies sensitive to environmental variation were transformed to a stationary sequence by using the co-integration algorithm. The co-integration residual was then employed as the damage feature, which would display a~~n~~ ~~obvious~~ noticeable jump once damage occurred. Erazo et al. [241] used a Kalman filter to decouple structural damage from temperature variations. According to the properties of the filtering residual, the residual spectral density was approximately constant under global changes caused by environmental variations and ~~greatly~~ significantly affected by local changes caused by structural damage. Therefore, damage location and severity were defined on the basis of the spectral moments of the residual spectral density.

Most previous studies have assumed that environmental effects are linear or piecewise linear [242]. However, the relationship between damage features and unknown environmental factors may be nonlinear in practice. Nonlinear analysis can be conducted in several ways, including auto-associative NN (AANN), kernel PCA (KPCA) and principal curves.

KPCA is a nonlinear PCA. It non-linearly maps the input variables into a high-dimensional linear space through incorporating the kernel functions, where normal PCA can be conducted [243]. Hsu and Loh [244] conducted KPCA by using AANN instead of SVM to locate and quantify structural damage. The element stiffness identified from natural frequencies and mode shapes was used as features to conduct nonlinear PCA. Nguyen et al. [245] successfully applied KPCA to identify damage to a bridge by considering temperature and soil variations. A global nonlinear model that described the relationship of damage-sensitive features with variations in environmental conditions was developed using Gaussian KPCA.

KPCA requires the specification of two parameters, namely, the kernel width and the number of extracted principle components, which may have a profound effect on the algorithm performance. Reynders et al. [242] proposed an improved KPCA to consider the nonlinear environmental and operational effects, in which these two parameters were automatically determined. Experimental results showed that the proposed method successfully identified damage to a three-span pre-stressed concrete bridge, whereas linear PCA could not.

In the algorithms of the second category (e.g. environmental variables are not measured), all sources of environmental variability should be efficiently characterised by the training data sets. Otherwise, these algorithms may be unreliable when they are applied to new data corresponding to the environmental conditions excluded in the training phase. In addition, if changes in the structural dynamic characteristics due to damage are analogous to those due to varying environmental conditions, then the effectiveness of these algorithms cannot be guaranteed. In this circumstance, the first-category algorithms are suggested.

The aforementioned approaches considering the effects of temperature are summarized in Table 9.

Table 9. Damage identification methods considering the effect of temperature

Authors	Methods	Features	Relation	Damage detection level	Applications
Using the measurement of temperature					
Deng et al. [230]	Control chart, online	Frequencies and displacement	Six-order polynomial regression model	1	Numerical suspension bridge
Bao et al. [232]	Dempster–Shafer data fusion	Frequencies and mode shapes	Linear regression model	1 and 2	Experimental steel frame
Magalhaes et al. [234]	Control chart, online	Frequencies	Linear regression model	1	Numerical concrete arch bridge
Zhou et al. [225]	AANN	Frequencies	BPNN correlation model	1	Numerical cable-stayed bridge
Jin et al. [236]	NN+EKF	Frequencies	-	1	Numerical composite bridge
Without direct measurement of temperature					

Kullaa [237]	Likelihood ratio test+ control chart	Accelerations	1	Experimental bridge
Shokrani et al. [238]	PCA	MSC	1 and 2	Numerical spring–mass chain and bridge
Kostic et al. [239]	ARX+ANN	Accelerations	1 and 2*	Numerical footbridge
Fallahian et al. [240]	PCA+ deep NN+ couple sparse coding	FRF	1-3	I-40 bridge
Liang et al. [241]	Co-integration algorithm	Frequencies	1	Real steel truss bridge
Erazo et al. [242]	Kalman filter+ Bayesian whiteness test	PSD residual	1-3	Numerical bridge
Hsu and Loh [245]	KPCA+AANN	Frequencies, mode shapes	1-3	Numerical bridge
Nguyen et al. [246]	KPCA	Frequencies	1	Real bridge
Reynders et al. [245]	Improved KPCA	Frequencies	1	Z-24 bridge

Note: *: Quantitatively indicate the relative severity of damage

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2.9 Nonlinear methods

Most existing vibration-based damage identification methods assume that a structure behaves linearly before and after damage. However, in practice, structures may behave nonlinearly in the undamaged state because of complex joints and interfaces [246]. In these situations, fitting a linear model to the data measured from an intrinsically nonlinear structural system results in the biased estimation of parameters. This circumstance may result in false damage identification. Therefore, structural nonlinearity should be appropriately considered for accurate and reliable damage identification.

The breathing crack or delamination is a typical type of damage that ~~likely~~ may induce~~s~~ nonlinearity to a structure [247]. The Volterra series may be an effective method ~~in~~ of describing nonlinear behaviours. As a generalization of the linear convolution, the Volterra series can separate a system response into linear and nonlinear components [248]. Chatterjee [249] analysed the nonlinear response of a cantilever beam with a breathing crack by using the Volterra series. They also developed a nonlinear dynamic model by utilising high-order FRFs and estimated crack severity based on the first and second harmonic amplitudes. Andreaus and Baragatti [250] exploited the nonlinear features of harmonic forced vibration and determined the crack location and depth by analysing sub- and super-harmonic components in the Fourier spectra and phase portrait distortions. Later, the proposed method was experimentally verified using a steel cantilever beam [251]. Peng et al. [252] applied a nonlinear ARMAX to establish a nonlinear ARX model, from which nonlinear output FRFs and an associated index were determined. Structural damage was then identified by comparing the nonlinear output FRF indices of the inspected structure before and after damage.

Another common type of nonlinearity in structural systems is the yielding of steel members during an earthquake [246]. Chanpheng et al. [253] proposed a nonlinear feature, that is, the degree of nonlinearity, for damage detection due to earthquakes. The degree of nonlinearity was calculated from the data of the ground motion and structural vibration based on the Hilbert transform, which indicated whether damage occurred. Wang et al. [254] proposed an analytical mode decomposition method in combination with the Hilbert transform for structural nonlinearity quantification and damage detection during earthquakes. The measured structural responses were decomposed into IMFs by using the proposed decomposition method, and the instantaneous frequencies were extracted using the Hilbert transform. The instantaneous frequency was integrated over time duration to eliminate the effects due to nonlinearity. On this basis, the degree of nonlinearity index was defined to represent damage severity.

Since many structures behave nonlinearly even in an undamaged state, previous linear methods may not be suitable to handle these initially nonlinear systems. Bornn et al. [255] applied an AR-SVM approach to time-series data for detecting damage to an initially nonlinear system, which was experimentally simulated with a column being suspended from the top floor and a bumper being placed on the second floor. Shiki et al. [256] used a discrete Volterra model to represent the behaviour of a magneto-elastic beam with nonlinearity even in a reference state. Input and output data were utilised to estimate the Volterra kernels. The prediction error of the model, together with hypothesis testing, was used to detect damage to the system during the linear and nonlinear regime of motion. Experimental studies showed that the nonlinear index could detect structural changes in both regimes of motion, while the linear one failed during the nonlinear regime.

Villani et al. [257] presented a stochastic version of the Volterra series to describe the nonlinear behaviour to consider uncertainties. The presence of damage was detected on the basis of the MSD. Numerical results showed that the performance of the present method was better than that of linear analysis and could detect small cracks even in the presence of uncertainties.

Recently, nonlinear FE model updating has been developed to identify structural nonlinear parameters. Ebrahimian et al. [258] and Ebrahimian et al. [259] used an EKF and a batch of Bayesian approaches, respectively, to estimate time-invariant parameters of the nonlinear FE models of frame-type structures by using input excitation and dynamic response of a structure during earthquake events.

However, input excitations to a structure are difficult to be measured accurately in practice. In this regard, Ebrahimian et al. [260] proposed an output-only nonlinear FE model updating method by using the measured time history responses during an earthquake event. The proposed methodology simultaneously estimated the unknown FE model parameters and input excitations based on a sequential maximum likelihood estimation approach and a sequential maximum a posteriori estimation approach, respectively. Astroza et al. [261] used an unscented Kalman filter to solve the nonlinear state-space equation and circumvent the computation of FE response sensitivities with respect to model parameters and input excitations.

Although a number of methods have been developed for nonlinear damage identification, a general model that can characterise the nonlinear behaviour of a structure is still challenging to construct. For example, when high levels of excitation are applied, structural responses may exhibit high-order nonlinearities that are inaccurately approximated using a small number of terms in the Volterra expansion. Nonlinear analysis is computationally intensive and time consuming. Consequently, a nonlinear FE model updating approach is difficult to be applied to practical structures that contain large DOFs. The substructuring approach may be integrated to solve the challenge. The following table compares and summarises the nonlinear methods that have been reviewed in this subsection.

Table 10. Nonlinear methods

Authors	Methods	Features	Damage type and identification level	Applications
Chatterjee [249]	Volterra series	First and second response harmonic amplitudes	Presence and size of breathing crack of cantilever beam ~~under harmonic excitation~~	Numerical spring-mass-damper system
Andreaus and Baragatti [250]	-	Phase portrait distortions, sub- and super-harmonic components and curved shape of the modal line	Location and depth of breathing crack of cantilever beam ~~under harmonic excitation~~	Numerical cantilever beam
Andreaus and Baragatti [251]	-	Phase portrait distortions and sub- and super-harmonic components	Presence and size of breathing crack of cantilever beam ~~under harmonic excitation~~	Experimental steel cantilever beam
Peng et al. [252]	Nonlinear ARMAX	FRFs	Presence and degree of structural nonlinearity	Experimental aluminium plates
Chanpheng et al. [253]		FRFs	Degree of nonlinearity ~~for large civil structures due to earthquake~~	Simulation data
Wang et al. [254]	Analytical mode decomposition+ Hilbert transform	Instantaneous frequency	Nonlinearity quantification ~~under earthquake~~	Experimental building
Bornn et al. [255]	AR-SVM	Residual error of AR-SVM model	Presence of damage for initially nonlinear	Experimental aluminum plates and

			systems	columns connected with bolted joints
Shiki et al. [256]	Discrete-time Volterra series	Prediction error of Volterra model	Presence of damage for initially nonlinear systems	Experimental aluminum beam
Villani et al. [257]	Stochastic Volterra series+ MSD	Volterra kernels coefficients and contribution	Presence of damage for initially nonlinear systems	Experimental aluminum beam
Ebrahimian et al. [258]	EKF	Input-output time histories data under earthquake excitation	Levels 1-3	Numerical steel bridge column and 2D moment resisting steel frame
Ebrahimian et al. [259]	Batch Bayesian approach	Input-output time histories data under earthquake excitation	Levels 1-3	Numerical steel bridge column and 2D moment resisting steel frame
Ebrahimian et al. [260]	Bayesian inference	Output acceleration under earthquake excitation	Levels 1-3	Numerical 3D RC building
Astroza et al. [261]	Unscented Kalman filter	Output acceleration under earthquake excitation	Levels 1-3	Numerical 3D RC frame

2.10 Other methods

In addition to the methods introduced earlier, some other damage identification techniques are reviewed here.

Drive-by (or indirect) damage identification methods have been proposed for bridge structures more than a decade. These methods extract the dynamic properties of the bridge, such as natural frequencies, from the measured responses of a passing vehicle instrumented with sensors [262, 263]. As a vehicle passing over a bridge, ~~there is dynamic interaction between them, and~~ the moving vehicle can be considered as both exciter and receiver due to the bridge-vehicle interaction [264]. The major advantage of the drive-by approach is that it ~~only~~ uses sensors installed on the vehicle only and thus minimises the effect on the normal traffic of the bridge. It is more economical, efficient, and mobile compared with conventional methods. In addition, the vehicle passing over the bridge as a "moving sensor" can provide ~~greater~~ higher spatial ~~information~~ resolution than fixed sensors [264].

Siringoringo and Fujino [265] estimated the first natural frequency of ~~the~~ a bridge ~~through~~ using ~~spectral analysis of~~ the vehicle response ~~by setting the driving velocity constant and employing the prior knowledge of vehicle frequency. Field experiment demonstrated that the first frequency of the bridge was identified with reasonable accuracy under moderate~~as the driving velocity was below 30 km/h. However, ~~one problem with this approach is that~~ the vehicle frequency usually appears as a dominant peak in the spectrum, making ~~it difficult to~~the identification of ~~y~~ the bridge frequency challenging. ~~To~~ In this ~~end~~connection, Yang et al. [266] combined ~~two filters, i.e.,~~ the band-pass filter and singular spectrum analysis~~,~~ to filter out the vehicle frequency from the spectrum and thus enhance the visibility of bridge frequencies. Zhang et al. [267] approximately extracted structural mode shape squares from the acceleration of a passing vehicle with tapping devices. However, acceptable damage identification accuracy was achieved only for the vehicle speed as low as 2 m/s. Oshima et al. [268] estimated bridge mode shapes from the dynamic responses of moving vehicles ~~through a four-step process~~ based on SVD. ~~Numerical~~ The numerical study showed that damage could be detected accurately for vehicle speeds varying from 5 to~~m/s -~~ 15 m/s. Nguyen and Tran [269] applied the WT to the displacement response of a moving vehicle with low speed to determine the existence and location of cracks for beam-like structures. Obrien et al. [270] used EMD to decompose the response measured in a passing vehicle into three components. The damage location was detected using the IMFs corresponding to the pseudo-frequency component.

The vehicle speeds for the aforementioned methods are all lower than the highway speed range, which thus may require temporary bridge and/or lane closures [264]. ~~More recently, i~~In 2004, traffic speed deflectometer (TSD) ~~has been~~was developed ~~as a device~~ for pavement deflection measurements at speeds of up to 80 km/h. OBrien and Keenahan [271] used a TSD vehicle containing two displacement sensors for damage detection for short-medium span bridges. ~~The time-shifted difference in the apparent profile derived from the displacement~~

~~data was employed as the damage indicator.~~ Keenahan and Obrien [272] employed a TSD vehicle containing five displacement sensors. The time-shifted curvature derived from the displacements was selected as the damage indicator. Numerical studies showed that the proposed method was more robust to noise than the algorithm proposed in OBrien and Keenahan [271].

Although the drive-by damage detection methods possess some distinct advantages over the conventional direct methods, they are primarily limited to Levels 1 and 2 damage identification. Moreover, most of these methods require the vehicles’ speed~~s~~ to be slow, which may cause traffic congestion and disruption. Some researchers used the TSD devices to address this problem, while the high cost of the equipment hinders their ~~practical~~ wide applications.

Response surface methodology (RSM), which is a combination of mathematical and statistical techniques, can provide an approximate mathematical model mapping the input parameters of a physical system to its output responses [273]. Linear or second-order polynomial models are typically employed as the fundamental structures of RS models to describe a studied system. In comparison with conventional FE model updating methods, RSM is a good alternative to solve inverse problems without using a sensitivity matrix. This method also requires low modelling and updating efforts.

Fang and Perera [274] used D-optimal designs to establish RS models for screening out non-significant updating parameters, which required few samples for the desired RS modelling. Damage was identified by minimising the discrepancy between reference-state RS and experimental models.

Kim et al. [275] combined global vibration-based and local impedance-based approaches to distinguish two typical damage types, namely, girder damage and tender damage, for pre-stressed concrete girder bridges. First, damage occurrence was detected on the basis of changes in frequency responses. Electromechanical impedance was then used to monitor whether tendon damage occurred. The location and extent of damage were estimated using natural frequencies or mode shapes based on damage type.

Yang et al. [276] proposed a 1D generalised local entropy method to detect cracks in beam structures. Yang et al. [277] further developed 2D generalised local entropy combined with statistical analysis and AI to identify damage in plate-like structures. They then evaluated the damage severities at the identified locations by using the ABC algorithm with the objective function defined by the combination of natural frequencies.

Considering the sparsity of structural damage, some researchers developed CS-based damage identification methods. Yang and Nagarajaiah [129] combined blind feature extraction and sparse representation classification to locate and quantify structural damage. They extracted the modal features of structures by using the unsupervised complexity pursuit algorithm and expressed the test modal feature as a linear combination of the bases of the overcomplete reference feature dictionary. The resulting highly underdetermined linear system was correctly solved via the l_1 minimisation~~ by exploiting the sparsity nature~~. Wang and Hao [278] proposed a CS-based pattern classification algorithm by constructing the feature matrix based on the sparse representation of numerically simulated time domain data. The existence, location and extent of damage were then determined sequentially by solving the l_1 optimisation problem.

The number of available sensors is always limited in practice due to the economic and technological considerations. Given a total number of sensors, the ~~selection of~~ sensor locations ~~is~~should be determined ~~necessary~~ such that the measured data contain useful features of the structure as much as possible. Consequently structural damage can be detected accurately. Although a number of OSP techniques have been developed for decades, most of these techniques are devoted to modal identification. OSP methods for damage identification are still far from the end.

Zhou et al. [279] introduced a sensor placement index in terms of the ratio of two parameters, namely, the contribution of the measurement points to the Fisher information matrix and the damage sensitivity to the measurement noise. Li et al. [280] proposed a two-phase OSP scheme based on the Fisher information matrix. The first phase was to find out the sensor locations that reconstructed accurate responses. In the second phase, the optimal sensor locations were determined based on the sensitivity analysis with respect to the elemental stiffness parameter. Lin et al. [281] employed two objective functions for multi-type sensor placement based on the simultaneous optimization of the response covariance sensitivity and the response independence. Later, they [282] applied the multi-type OSP method for damage detection in a nine-bay 3D frame structure.

The aforementioned sensitivity-based OSP methods calculate the sensitivity analysis with respect to the model parameters of each structural element, which are not applicable for large-scale structures. Guo et al. [283] developed an information-entropy-based OSP method targeting damage detection of large-scale bridges subject to ship collision. The sensor configuration was optimized by a multi-objective optimization algorithm, which simultaneously minimized the information entropy index for each possible ship-bridge collision scenario. The proposed method was applicable in practice to determine the OSP prior to field testing. Beygzadeh et al. [284] proposed an improved GA algorithm for OSP in space structures damage detection. A numerical study showed that the proposed algorithm performed better than the GA in terms of convergence speed and damage detection accuracy.

2.11 Comparative studies

Over the past 10 years, some researchers have compared existing vibration-based damage detection methods in terms of different aspects. Different techniques may behave differently on different types of structures. Comparative studies may give insight on the performance, applicability and feasibility of each technique.

Kopsaftopoulos and Fassois [263] experimentally compared several statistical time series methods via application to a lightweight aluminium truss structure. They assessed two nonparametric methods (i.e. PSD- and FRF-based methods) and four parametric methods (e.g. a model parameter, a residual variance, a residual likelihood function and a residual uncorrelatedness-based method). All the methods were effective for various damage scenarios, but the accuracy of parametric methods was higher than that of nonparametric methods.

Talebinejad et al. [264] evaluated four mode shape-based techniques to identify damage to long-span cable-stayed bridges by using simulated acceleration data. These methods included an enhanced coordinate MAC, a damage index, MSC, and modal flexibility methods. The performance of the damage index and MSC was better than that of the two other methods. When measurement noise was considered, only high-intensity damage could be detected using the damage index and MSC, and none of them could identify multiple damage to the deck.

Saeed et al. [265] compared the performance of single and multiple ANNs and multiple adaptive neuro-fuzzy inference systems in detecting the location and length of a crack in curvilinear beams. Natural frequencies and FRFs were employed as inputs, and PCA was conducted to reduce the size of FRF data. Multiple ANN models produced the lowest average prediction errors. Multiple adaptive neuro-fuzzy inference systems were less sensitive to noise than other classifier models.

Dessi and Camerlengo [266] compared nine damage identification methods based on natural frequencies, MSCs and MSE. The selected techniques were divided into two categories: one required baseline data, whilst the other did not need any reference to the undamaged state. Their performances were evaluated on the basis of the same test of a simply supported Euler–Bernoulli beam. They found that a damage index performed well in identifying the damage location may be not so accurate in estimating the damage severity and vice versa. The position of damage along the beam affected the accuracy of damage identification for all methods.

Figueiredo et al. [267] comparatively studied four ML algorithms, including AANN, factor analysis, MSD and SVD, for damage detection under operational and environmental variabilities. These algorithms relied only on the measured time-series responses and did not require a direct measurement of parameters related to operational and environmental variations. An MSD-based method was proven to be an optimal algorithm in terms of classification performance and computational complexity. Santos et al. [268] presented four kernel-based algorithms, i.e. one-class SVM, support vector data description, KPCA and greedy KPCA. They compared the performance of these algorithms by using a benchmark three-story frame structure. Experimental results showed that the proposed kernel-based algorithms had an optimal performance, especially when a nonlinear temperature–stiffness relationship was present.

3. Challenges and Future Research

Numerous vibration-based damage detection methods have been developed over the past decades. However, their applications in practical civil engineering structures are still immature. Some challenging issues deserve further research to achieve accurate and practical damage identification.

(1) Although some researchers comparatively studied different damage identification methods, no general consensus has been achieved regarding which type of data is a good damage indicator and which identification method is most effective. ASCE organized a benchmark study on damage identification of a lab frame 20 years ago [269, 270]. Benchmark studies on real structures are necessary.
(2) For civil engineering structures, nonlinearity often exists and may be induced by connections or the presence of damage. However, few existing algorithms consider nonlinear structural behaviour in the reference and damaged states. Nonlinear damage identification methods are preferable. Given the heavy computation load, nonlinear

analysis can be combined with substructuring methods to reduce computational effort and improve efficiency when it is applied to large-scale structures.

(3) Structural damage detection inevitably entails uncertainties, such as measurement noise and modelling errors, due to inaccurate physical parameters, non-ideal boundary conditions and structural nonlinear properties. The existence of uncertainties may cause large variations in structural vibration characteristics, which may lead to incorrect damage identification. Therefore, statistical damage identification methods that appropriately consider these uncertainties should be continually developed. Moreover, noise quantification and elimination need to be explored further.

(4) Operational and environmental variations, particularly temperature, cause changes in structural vibration properties. Many field studies have found that varying temperatures may cause more significant changes in the structural vibration properties than those due to damage. Although researchers have proposed various techniques to consider the effects of temperature on damage detection, no consensus and generally effective methods have been developed.

(5) Although vibration-based damage identification methods have been successfully applied to mechanical and aerospace structures, the applications of these methods to practical civil structures are far from maturity due to the complexity and uniqueness of civil structures. Civil engineering structures are typically different because of the variability in the materials and construction processes, the uniqueness of the soil conditions, and environmental conditions. Consequently, it is difficult to extend a well-established method to other structures.

(6) In practice, a large volume of data is generated from an SHM system. The present data processing and damage identification are manually conducted by practitioners, thereby causing inefficiency or ineffectiveness or false identification results. Automatic data processing and condition assessment are lacking. Rapid developing big data and AI techniques may be a direction to achieve this purpose. Many interesting studies in the topic are underway.

(7) In recent years, ML methods have received considerable attention for structural damage identification. However, most of the studies use laboratory or numerical data for training

as real data ~~corresponding to~~in damaged structures are very rare. Damage identification problems have poor data conditions when they are casted into a supervised learning framework. The introduction of semi-supervised and unsupervised algorithms is expected in the future.

(8) Structural damage detection is essentially a multidiscipline area, involving sensor technology, data processing, mathematics, and structural analysis. A successful damage identification method needs ~~the~~ close collaboration among different disciplines.

4. Conclusions

The vibration-based damage detection methods between 2010 and 2019 have been reviewed. The applicability and effectiveness of different available techniques depend on damage type, structural configuration and available data. However, a universal methodology that can identify all damage types of different structures has yet to be developed. In addition, few existing algorithms can predict the remaining service life of structures, which is regarded as Level 4 structural damage detection.

In summary, there is a pressing need to develop more accurate and reliable damage identification methods for practical civil engineering structures by using vibration measurement data. The prognosis of damage to a monitored structure also requires extensive investigations.

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