

# Application of probabilistic assessment for optimal prediction in active noise control algorithms

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## Abstract

This study explores a modified active noise control (ANC) system using a Bayesian inference approach as a pre-processing method. The key aspect of low-frequency noise attenuation is investigated with the existing control algorithms, the conventional filtered-x least mean square (FxLMS) algorithm and a new convex structure via an FxLMS/F algorithm (C-FxLMS/F), that combine Bayesian inference with a dynamic linear model (DLM). The combination of a Bayesian approach and a DLM comprises the statistic strategy and a descriptive time series, which is conducive to raw signal pre-processing and concurrently generating a predicted signal as a reference signal. For signal processing, pretreatment enables the determination of the noise characteristics of the operating machine and its feedback to the control system. This is an important input to enable the time domain control algorithm to prevent environmental disturbance and time-delay effects. In addition, the use of active control theory mainly relies on the response time of secondary source generation. The predicted signals based on prior observational information and Bayesian inference afford an alternative to the normal costs of the secondary path, such as those associated with electro-acoustic signal conversion and computation efforts in the control algorithm. In this work, the combination of a Bayesian approach and an FxLMS algorithm is studied via a case study. To explore more applicability, the combination of a C-FxLMS/F algorithm with Bayesian inference is also investigated, and a convergence analysis is presented. The in-situ measurement data obtained from a construction site acoustic apparatus is used for analysis. The simulation results are presented via two illustrative cases. In addition, a comparison for three different signal forms under the effect of Bayesian inference is also discussed. It is found that a Bayesian inference approach based on DLM is workable in the ANC system, and the convergence performance is superior to that of an ANC

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system without Bayesian inference. This suggests that to implement such a system for signal control, it is better to enhance the final system performance in the time-domain field of ANC algorithms. This pre-processing system based on a characteristic strategy and having a low computational loss is needed not only to reduce the time-delay compromise, but also to prevent the sudden disturbance of the reference signal.

*Keywords:* Active Noise Control, Bayesian Inference, Dynamic Linear Model, Pre-processing System, Active Control Algorithms

## 1. Introduction

Active noise control (ANC) is an effective technique for noise reduction or cancellation. With the development of digital signal processing systems, noise signals can be sampled and processed, which has led to a growth in ANC techniques. A brief review of algorithms that have been developed and applied to active noise control techniques is given in the following.

Adaptive filters [1] were utilized to adjust weighting coefficients and minimize the error in signals. The least-mean-square (LMS) theory is the most common form of error signal processing. With the development of digital signal processor chips enabled the low-cost implementation, ANC system was encouraged widespread application [2, 3]. The standard LMS algorithm [4] was the original template for the processable adaptive algorithm, but there was a significant cost incurred by introduction of a secondary path. To manage this, a filtered-x LMS (FxLMS) algorithm [2, 3] was proposed to enable estimation of the secondary path for compensation. A time-domain FxLMS algorithm offered easier implementation and higher robustness, but was limited by its convergence speed and sensitivity to environmental disturbance [5]. In addition, a frequency-domain FxLMS algorithm for active sound quality control [6] and a hybrid ANC system time and frequency domain FxLMS algorithm [7] were proposed.

A control algorithm is the kernel component of ANC systems, and has thus been the subject of much research. A novel normalized frequency-domain block FxLMS (NFB-FxLMS) algorithm [8] was proposed. The required calculation load was decreased by processing signals block-by-block instead of point-by-point. A time-frequency-domain FxLMS algorithm was proven [9] to have lower computational complexity. To investigate the effort weighting of time-domain versus frequency-domain techniques, the effort-weighting parameters were analyzed by an error index comparison [10]. The optimum solution for multiple reference of ANC was also given [11] in terms of frequency responses. With application of discrete wavelet transformation, a DWT-FFT-FxLMS algorithm [12] was then proposed for the enhancement of convergence. Aiming at the impulse noise, some new algorithms were proposed, including a convex mixture of norms algorithm [13] and the modification of conventional algorithms [14-19]. With the presence of a filtered-x LMS/fourth (FxLMS/F) algorithm [20, 21] and the idea of a convex structure of two adaptive filters [22-24], a convex combination filter via the FxLMS/F algorithm (C-FxLMS/F) [25] was developed. Based on the consideration of nonlinear characteristics, it led to further research using adaptive multichannel controllers [26]. Recently, a novel nonlinear ANC system [27] that used a nonlinear primary path was investigated. Utilizing the functional link artificial neural network, FsLMS [28] was investigated under strong disturbance conditions. In addition, modified methods were

proposed for nonlinear ANC in which both a linear secondary path and a nonlinear secondary path were illustrated [19, 29].

Many studies of the control algorithm problem have focused on the development of complicated algorithms, which enhance the control algorithm performance but introduce computational burdens. Moreover, such intricate systems have inherently less utility in terms of their cost and feasibility of implementation. Hence, the FxLMS algorithm is still widely used as a balance between computational accuracy and feasibility considerations. It has been used for snore noise-control experiments via controller setup in headboard [30]. Besides, a partition placed at the middle of bed was proposed for active snore sound control [31].

Associated challenges are the loudness of construction machines and the difficulty of implementing noise-control measures. In most engineering situations requiring noise insulation, deployment of barriers/enclosures is a common solution, because of the easier implementation and lower cost of these measures. Whilst barrier insulation provides fairly good high-frequency noise control, in machine operations low frequencies are dominant, and passive noise control has limited efficacy in these situations. We attempted to utilize an ANC technique with a simple algorithm as this was convenient to implement. To maintain the stability and feasibility in the online process, a preprocessing procedure utilizing a Bayesian inference-based DLM was used for raw-signal filtering before the ANC system. Bayesian forecasting is one of the most widely used approaches in statistical analysis. It has been adopted in multiple disciplines, such as econometrics, structural dynamics, and aerospace engineering [32].

To simplify the overall ANC system, Bayesian inference acts as a pre-processing filter based on statistical characteristics and the control algorithm retains an LMS foundation. In a broadband time-varying feed-forward control system, Bayesian inference enables to provide a fuzzy reference signal. Noise signal processing contains many parametric uncertainties. A statistical model can be used for signal parameter identification, and this method possesses great potential on this aspect. The basic element of a Bayesian updating model includes prior information, a likelihood function, and posterior information. It has been widely adopted for decision rules used to classify data patterns and a way to infer unknown parameters from known measurements. A DLM is a mathematical tool for time series analysis that can be used to describe a routine way of viewing a context that changes with time. In the literature, Bayesian inference was adopted to estimate frequency periodic information in a narrowband ANC system [33]. Prediction information contributed a lot to enhance the performance for a specific periodic noise signal. A Bayesian estimation approach [34] was adopted to recast the problem of online secondary path modeling

in the form of a statistical inverse problem. A novel approach to active noise control based on Bayesian estimation theory [35] was investigated utilizing the statistical property for the optimization of parameters. Various structures and theories of dynamic models that incorporate Bayesian forecasting have also been reviewed in Refs. [36, 37].

Due to the versatility and applicability of Bayesian methods, they have many other applications in mechanical and structural engineering. For instance, Bayesian forecasting has been combined with DLM for gas turbine performance detection [38]. Moreover, condition assessments for the structural responses of high-speed trains and cable-stayed bridges were conducted with a combination of time series analysis and Bayesian inference [39-42]. More recently, the authors conducted a pilot study for the inference of noise data [43], the present work is an extension to provide a holistic analysis to the probabilistic assessment for adaptive prediction in ANC algorithms. Making use of a Bayesian approach, another advantage is a pre-processing system can be treated as a filter. As is well known, an ANC system will struggle to achieve broadband frequency noise attenuation. However, decisions made by a pre-processing system while the ANC system is operating prevent interference from cumbersome disturbances from the environment, thereby acting as a filter to narrow down the frequency bandwidth. Meanwhile, the inference from noise model parameters can be identified with a “short-time” prediction.

In real-life applications, time-delay effects introduced by a multiple signal conversion in the secondary path of ANC algorithms such as analog-to-digital and digital-to-analog signal transitions are important factors that can directly affect the control algorithm performance. Many investigations have focused on the convergence speed and robust of control algorithms, but the complicated signal conversion in the secondary path of ANC algorithms is ignored. Hence, the use of data prediction based on a statistical strategy is a promising way to enhance the efficiency of signal processing.

In this work, the prediction signal is a “*K-step estimation*” of the observation in a Bayesian approach. A time series model was adopted to simulate the observational signal characteristic as it enabled modeling of the dynamic behavior. The combination of the Bayesian approach and a time series model is a characteristic strategy for providing the reference signal for an ANC system ahead of time. To investigate the pre-processing system, a reference signal was acquired from in-situ measurement on a construction site, which contained environmental disturbance. The derivations of Bayesian inference for this signal, based on a DLM, are presented in Section 2. The preprocess method is presented in Section 3. Simulations were performed using a conventional control algorithm FxLMS and a new convex structure C-FxLMS/F algorithm [25]. In addition, three types of reference signals (i.e., (i) Gaussian noise; (ii) sinusoidal and

Gaussian noise; and (iii) in-situ measurement data) were used for comparison. The convergence performance and complexity discussion are discussed in Section 4. Finally, key findings of this work are summarized in Section 5.

## 2. Methodologies

The proposed method is presented in the subsequent sections. For ANC algorithms, time-delay effects, including controller computation and secondary source response, can restrict the control performance of adaptive filters. To provide early signal processing in an active algorithm, Bayesian forecasting is utilized to predict the next few steps to reduce the time consumption of the complicated secondary path. Furthermore, Bayesian inference is incorporated with time series analysis to express the variation in the observation signal with time. The combination of Bayesian inference and a DLM can be considered as a pre-processing system to appropriately modify the complicated signal before it enters the active control algorithm, which partly reduced the time delay effects.

### 2.1. Dynamic linear model

DLMs form the basis of time series analysis methods. To describe a time series such as an observational signal, the general state-space function is utilized to express the current status using mathematical elements, e.g. values, gradients, curvature, etc. It focuses on statements about the future development of a time series that are conditional on existing information. Thus, mathematical and statistical representation is the connection that provides communication between the forecaster, model and decision makers.

According to the information available in time cycle  $t-1$ , the guess value in time  $t$  for the parameter can be calculated through the model function. For the general normal DLM, it can be characterized by a quadruple [36-40, 43] as follows:

$$\{F_t, G_t, V_t, W_t\} \quad (1)$$

where  $F_t$  is a known  $(p \times r)$  matrix that acts as a regression matrix of known values of independent variables,  $G_t$  is the known  $(p \times p)$  system or state evolution transfer matrix, and  $V_t$  and  $W_t$  are the observational variance matrix and evolution variance, respectively.

The equations of the model can be expressed as [36-43]:

$$\text{Observation Equation: } Y_t = F_t^T \cdot \theta_t + v_t, \quad v_t \sim N[0, V_t] \quad (2)$$

$$\text{System Equation: } \theta_t = G_t \cdot \theta_{t-1} + w_t, \quad w_t \sim N[0, W_t]$$

$$\begin{aligned} \theta_t &= \begin{pmatrix} \alpha_t \\ \beta_t \end{pmatrix} \quad F_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad G_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \\ W_t &= \begin{pmatrix} \sigma_{level}^2 & 0 \\ 0 & \sigma_{trend}^2 \end{pmatrix} \quad V_t = \sigma_{observation}^2 \end{aligned} \quad (3)$$

where the guess vector  $\theta_t$  contains two elements,  $\alpha_t$  and  $\beta_t$ , which denote the current level of parameter value and gradient, respectively.  $v_t$  and  $w_t$  are two independent Gaussian random vectors with mean zero and unknown covariance matrices  $V_t$  and  $W_t$ . The evolution error-free case is assumed with  $W_t = 0$  for all  $t$  so that  $\theta_t = G_t \cdot \theta_{t-1}$ .

## 2.2. Bayesian forecasting

Bayesian forecasting proceeds according to the known information as its principle of management. Routine forecasting from a model is used directly, while the occurrence of non-routine events alters the parameters of the model to reflect the circumstances. Based on Bayes's rule [32, 37], the prior density provides concise and coherent transferred information to the posterior probability of a model parameter. With a time-varying observed process, this can be formed using a probability density function based on the assumption of Gaussian distribution. It is noted that Bayesian method is available for all distributional assumptions.

The observational distribution with density can be described as [37]:

$$p(Y_t | \mu_t, D_{t-1}) \quad (4)$$

and the prior for  $\mu_t$  given  $D_{t-1}$  has the density:

$$p(\mu_t | D_{t-1}) \quad (5)$$

According to Bayes' theorem [32, 38], the posterior for  $\mu_t$  can be obtained:

$$\begin{aligned} p(\mu_t | D_t) &= p(\mu_t | D_{t-1}, Y_t) \\ &= \frac{p(\mu_t | D_{t-1}) p(Y_t | \mu_t, D_{t-1})}{p(Y_t | D_{t-1})} \end{aligned} \quad (6)$$

The notations of the guess vector  $\theta_t$  are given by [37-40, 43], and the key Eq. (7), below:

$$(\theta_t | D_t) \sim N[m_t, C_t] \quad \text{with } D_t = Y_1, Y_2, \dots, Y_t \quad (7)$$

where  $\{D_t = Y_1, Y_2, \dots, Y_t\}$  are the observations. On the basis of these, Bayes' rule provides a formula for the probability estimation of the next time step, which represents the state space vector  $\theta_t$  obeying Gaussian distribution.  $\{D_t = Y_1, Y_2, \dots, Y_t\}$  expresses the known information at the time cycle  $t$ , and  $m_t$  and  $C_t$  are the mean value and variance, respectively.

The Bayesian updating approach is utilized for signal filtering and short-term forecasting of noise signal inference. It is worth noting that the noise signals processed must be relatively stable and regular construction noise patterns, such as automobile engines or turbine motors. The combination of Bayesian inference and a DLM can provide a “*forecast-observation-analysis*” cycle. The logic of Bayesian inference equations is presented next, in Equations (8)–(11); the derivations of these can be referred to Refs. [36, 37].

Given the state space function at time ( $t$ ):

$$(\theta_t | D_t) \sim N[m_t, C_t] \quad (8)$$

where  $\{D_t = Y_1, Y_2, \dots, Y_t\}$  denotes the state of knowledge at time  $t$ . Prior information on the state space function for the next time step ( $t+1$ ) is summarized as follows:

$$(\theta_{t+1} | D_t) \sim N[a_{t+1}, R_{t+1}] \quad (9)$$

According to the prior information and observation equation (Eq. (2)),  $Y_{t+1}$  can be obtained as below:

$$(Y_{t+1} | D_t) \sim N[f_{t+1}, Q_{t+1}] \quad (10)$$

where the means and variances are given by [37]:

$$\begin{aligned} a_{t+1} &= G_{t+1} \cdot m_t, & f_{t+1} &= F_{t+1}^T \cdot a_{t+1}, \\ R_{t+1} &= G_{t+1} C_t G_{t+1}^T + W_{t+1}, & Q_{t+1} &= F_{t+1}^T R_{t+1} F_{t+1} + V_{t+1} \end{aligned} \quad (11)$$

If a forecast of the next  $K$  time steps is of interest, the  $K$ -step forecasting equations are given by [37].

For a constant observational factor  $F_t = F = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and evolution factor  $G_t = G = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ , forecasting several steps ahead requires the prior information to be projected into the future through repeated application of the system equation. Given the prior for time ( $t+1$ ), the implied prior for time ( $t+2$ ) from the same standpoint, with no additional information, is  $p(\theta_{t+1} | D_t)$ . This prior is obtained by applying Eqs. (12) and (13) below:

$$(\theta_{t+K} | D_t) \sim N[a_t(K), R_t(K)] \quad (12)$$

$$(Y_{t+K} | D_t) \sim N[f_t(K), Q_t(K)] \quad (13)$$

with the following parameters:

$$a_t(K) = G^{K-1} \cdot a_{t+1}, \quad f_t(K) = F_{t+K}^T \cdot a_t(K), \quad (14)$$



$$R_t(K) = G^{K-1}R_{t+1}(G^{K-1})' + \sum_{j=2}^K G^{K-j}W_{t+j}(G^{K-j})',$$

$$Q_t = F_{t+K}^T R_t(K) F_{t+K} + V_{t+K}$$

The state vector  $\theta_t$ , which comprises  $n$  elements, is extracted to enable identification of the time series. Accordingly, the mean response function  $\mu_t$  affords knowledge of the state vector. The  $\mu_t$  can be expressed as below [36]:

$$\mu_t = T \cdot \theta_t \quad (15)$$

$$T = \begin{pmatrix} F_t^T \\ F_t^T \cdot G_t \\ \dots \\ F_t^T \cdot G_t^{n-1} \end{pmatrix}, \quad \theta_t = T^{-1} \cdot \mu_t \quad (16)$$

where the matrix  $T$  must have a full rank  $n$  to satisfy the requirements of observability in model design.

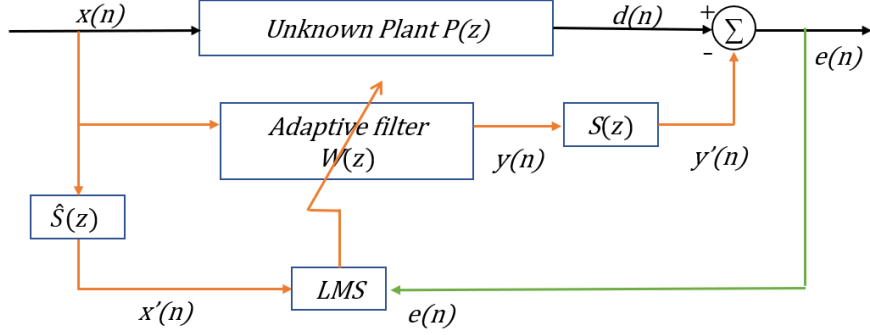
Considering the observability and the forecasting function, for a time domain DLM, the mean response function  $\mu_{t+K}$  defines the implied form of the time series in model design where the  $K$  value is the step ahead index. With the state vector and evolution error  $\theta_t = \begin{pmatrix} \alpha_t \\ \beta_t \end{pmatrix}$  and  $w_t = \begin{pmatrix} w_{t1} \\ w_{t2} \end{pmatrix}$ , respectively, we have

$$Y_t = \alpha_t + \nu_t, \quad \alpha_t = \alpha_{t-1} + \beta_{t-1} + w_{t1}, \quad \beta_t = \beta_{t-1} + w_{t2} \quad (17)$$

The expectation of state space at time cycle  $t-1$  is given as  $E[\theta_t|D_t] = m_t$ . With  $m_t = \begin{pmatrix} m_{t1} \\ m_{t2} \end{pmatrix}$ , the forecasting function is  $f_t(K) = (m_{t1} + Km_{t2})$ .

### 2.3. Active control algorithms

The block diagram of an adaptive active single-channel feed-forward ANC system is shown in Fig. 1 [2]. The primary path is denoted by the transfer function  $P(z)$ , and the secondary path is  $S(z)$ .  $W(z)$  is the ANC controller weight coefficient, which is updated through the process. Variable  $y(n)$  is the adaptive filter response and  $e(n)$  is the residual error.  $\hat{S}(z)$  is the estimation of the secondary path that can be obtained by adaptive filtering using either offline or online modeling.



**Fig. 1.** Block diagram of a conventional FxLMS algorithm.

The adaptive filter output signal  $y(n)$  is given by

$$y(n) = w^T(n)x(n) \quad (18)$$

where  $x(n) = [x(n), x(n-1), \dots, x(n-L+1)]^T$  is the reference signal vector and  $w(n) = [w_0(n), w_1(n), \dots, w_{L-1}(n)]^T$  is the weight vector of the adaptive filter. The expression for the residual error  $e(n)$  is computed as follows:

$$e(n) = d(n) - y'(n) \quad (19)$$

where  $d(n) = x(n) * p(n)$  represents the primary noise signal,  $y'(n) = y(n) * s(n)$  is the secondary cancelling signal, and  $s(n)$  is the impulse response of  $S(z)$ . The symbol “\*” denotes the discrete convolution operator. The corresponding weighting equation can be updated using Eq. (20):

$$w(n+1) = w(n) + \mu x'(n)e(n) \quad (20)$$

where  $\mu$  is the fixed step-size value.

To improve the noise reduction performance of a single-channel feed-forward ANC system, the filtered-x least-mean square/fourth (FxLMS/F) algorithm was introduced [20, 21]. Convex combination structures based on an LMS algorithm [44, 45] were developed to improve the filter performance. After this, a combination of a convex filter structure and an FxLMS/F algorithm was developed to solve the complicated parameter-setting problem and improve the convergence rate [25].

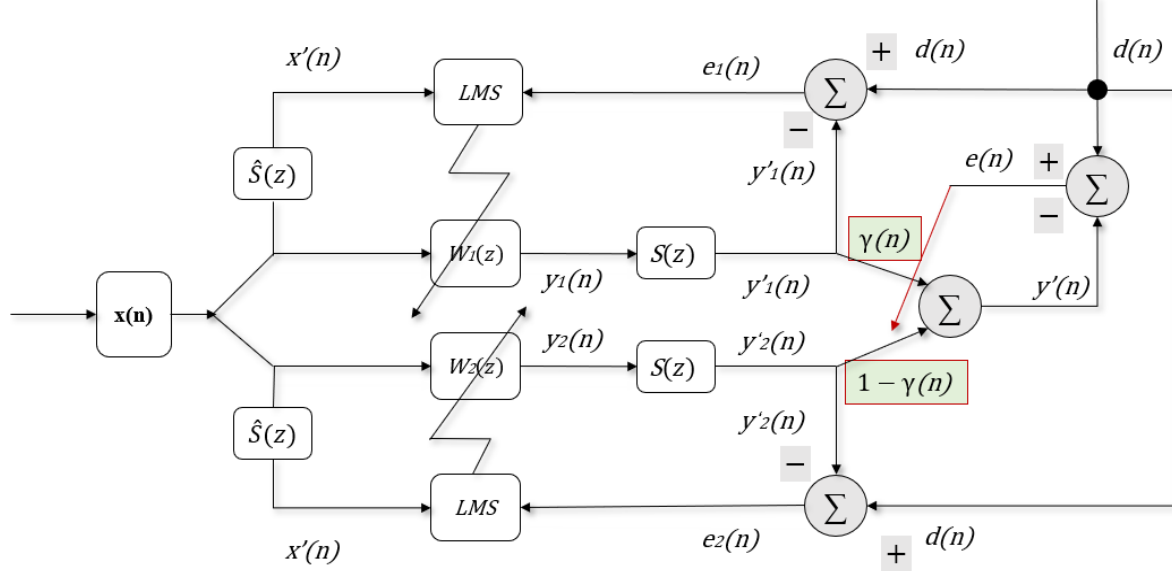
In general, the cost function of the LMS/F algorithm is given by [20, 21]

$$J(n) = \frac{1}{2}e^2(n) - \frac{1}{2}\phi \ln(e^2(n) + \phi) \quad (21)$$

where  $\phi$  is a positive threshold parameter that controls the convergence speed and the noise reduction performance. The updated equation for FxLMS/F is different from the FxLMS algorithm, as shown below:

$$\begin{aligned}
w(n+1) &= w(n) + \mu \frac{e^3(n)}{e^2(n) + \phi} [x(n) * s(n)] \\
&= w(n) + \frac{\mu}{1 + \phi/e^2(n)} e(n)x'(n)
\end{aligned} \tag{22}$$

The block diagram for the convex combination with an FxLMS/F algorithm is shown in Fig. 2 [25]. The convex combination is utilized for improving the convergence rate with two adaptive filters.



**Fig. 2.** Block diagram of a convex combination of adaptive filters based on an FxLMS/F algorithm.

The output of the overall combined filter is given by the following equation:

$$y'(n) = \gamma(n)y'_1(n) + (1 - \gamma(n))y'_2(n) \tag{23}$$

where  $\gamma(n)$  is a mixing coefficient ranging from 0 to 1. This coefficient is used to control the combination of two adaptive filters at each signal iteration, and its explicit form is shown via a sigmoid activation function [44]:

$$\gamma(n) = \frac{1}{1 + e^{-A(n)}} \tag{24}$$

where  $A(n)$  is an inner adaptation according to the gradient descent method. It ranges from  $[-\sigma^+, \sigma^+]$  where  $\sigma^+$  is a parameter to be determined by the control algorithm. The explicit form is given by [23, 46]:

$$\begin{aligned}
A(n+1) &= A(n) - \mu_a \frac{\partial J(n)}{\partial A(n)} \\
&= A(n) + \mu_a \left[ \frac{e^3(n)}{e^2(n) + \phi} \right] (y'_1(n) - y'_2(n)\gamma(n)(1 - \gamma(n)))
\end{aligned} \tag{25}$$

Then, the updated equation for the convex filter structure combined with an FxLMS/F algorithm can be obtained as below:

$$w(n) = \gamma(n)w_1(n) + (1 - \gamma(n))w_2(n) \tag{26}$$

In the convex filter structure combined with an FxLMS/F algorithm (i.e., C-FxLMS/F), the filter performance for active control revealed enhancement in three case studies in terms of Gaussian input signal, sinusoidal wave input signal and impulse noise with symmetric alpha distribution signal [25]. In terms of signal processing, the time domain algorithm contained instabilities when under unpredictable disturbance. The proposed system with pretreatment procedures and Bayesian inference processed via a C-FxLMS/F algorithm is illustrated in the next section. The reference signal was derived from the in-situ measurement data.

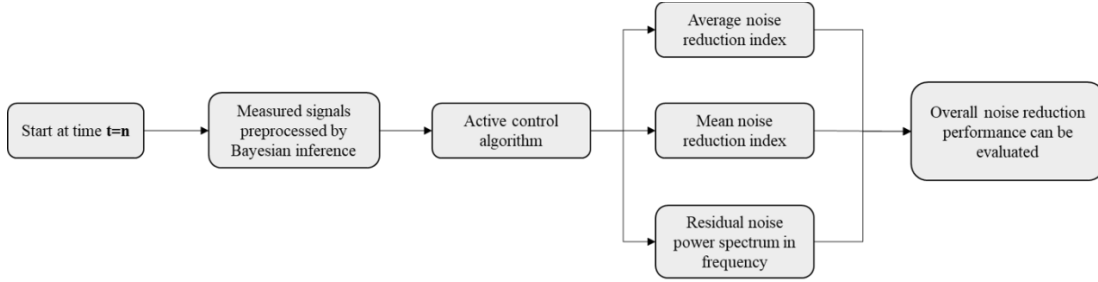
A pretreatment procedure is used to enable reference signal processing. An ANC system is utilized as a secondary source to highlight the desired signal, which means that the response time of the path is an important factor. The response time of the secondary path is also considered as a time-delay effect in an ANC system, which directly affects the converged performance of a control system as it requires complicated computation and conversion of signal form. A pre-processing system based on a characteristic strategy and having a low computational loss is needed not only to reduce the time-delay compromise, but also to prevent the sudden disturbance of the reference signal.

### 3. Proposed ANC system with pretreatment process

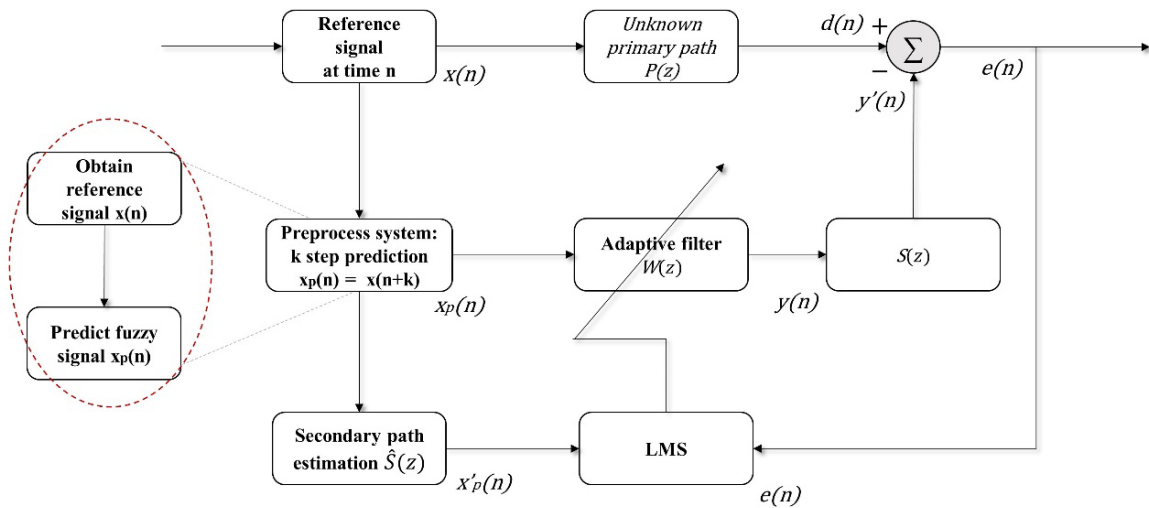
In this section, the use of the Bayesian inference approach based on DLM as the pre-processing procedure before the ANC structure is described, as a way to solve the abovementioned problems. Given that the reference signal is acquired from an in-situ microphone measurement, the observation signal was modeled with DLM. K-step prediction data can be calculated ahead of the current time. The inference signal replaced the reference signal in the ANC system. In this study, we attempted to pre-process the inference signal based on the measurement data and treated it as a reference signal for adaptive prediction

in ANC algorithms. In Eq. (2), the observation (measurement) equation contains the state parameter ( $\theta_t$ ) and the unknown variable ( $v_t$ ). To investigate the dynamic change of the whole dataset, we assumed that the random noise (i.e.,  $v_t$ ) still obeys a normal distribution. With a smaller value of  $v_t$ , the estimation of the posterior parameters relies more heavily on the observation data. The posterior parameters can be updated via the inference loop given in Eqs. (8)–(11) as time goes on.

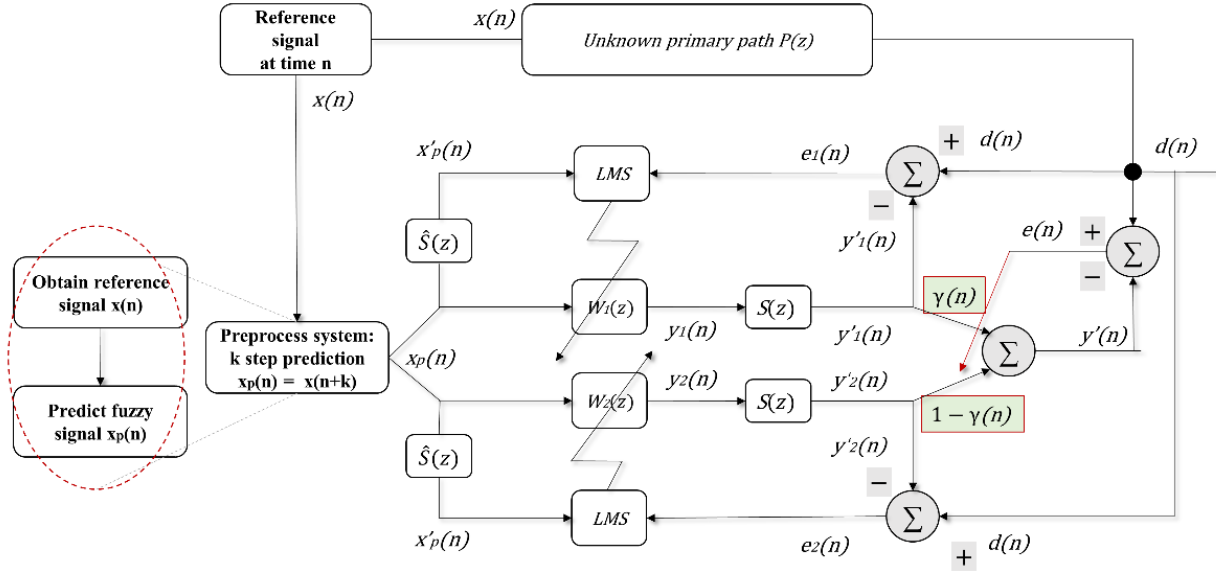
To investigate the performance of noise reduction, a simple diagram is presented for illustration in Fig. 3. To determine the learning process of this control algorithm, both average noise reduction (ANR) [25] and mean noise reduction (MNR) indices are used. Furthermore, the performance of noise reduction can be evaluated by the power spectrum of noise residual [47, 48]. The block diagrams of the active control algorithms (conventional FxLMS algorithm and the C-FxLMS/F algorithm) are illustrated in Figs. 4 and 5, respectively.



**Fig. 3** Evaluation of noise reduction performance



**Fig. 4.** Block diagram of a conventional single channel feedforward FxLMS algorithm with Bayesian inference based on DLM as a pre-processing procedure for signal processing.



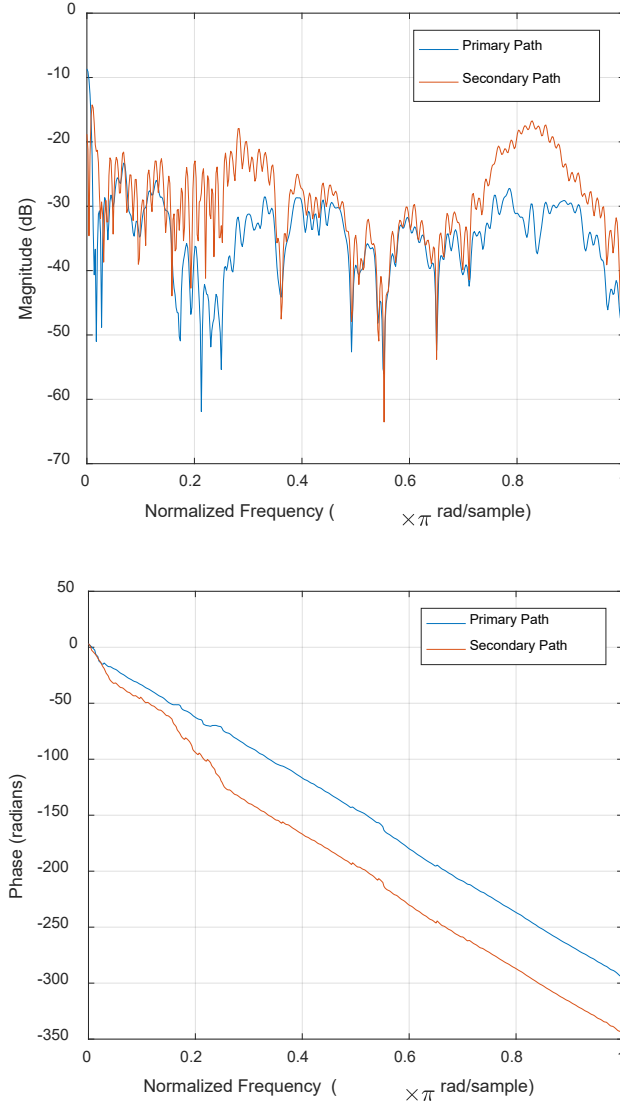
**Fig. 5.** Block diagram of a convex structure with Bayesian inference based on DLM as a pre-processing procedure for signal processing.

The measured noise signals from in-situ measurement of construction machines were processed by the proposed technique as shown in Fig. 6. It contains two processing loops, Bayesian inference loop for online measured data preprocessing and secondary source generation loop. In the pre-processing procedure, a second-order DLM was formulated to model the noise signal series. Then, the Bayesian forecasting approach was applied to conduct a K-step prediction prior to the next K-step observation as the reference signal for the ANC system, with the aim of reducing the time-delay effects of secondary source production. The noise signal acquired from in-situ measurement of machinery such as concrete mixers and road crushers were used as the primary source. The results of various simulations are conducted to validate the effectiveness of the proposed method for the primary noise attenuation. The performance evaluations were carried out using the following averaged noise reduction index [25]

$$ANR(n) = 20 \log \left( \frac{A_e(n)}{A_d(n)} \right) \text{ dB} \quad (27)$$

where  $A_e(n) = \kappa A_e(n-1) + (1-\kappa)|e(n)|$  and  $A_d(n) = \kappa A_d(n-1) + (1-\kappa)|d(n)|$ . The initial conditions were set as  $A_e(n) = 0$ ,  $A_d(n) = 0$  and  $\kappa = 0.999$  is the forgetting factor.





**Fig. 7.** Frequency responses of acoustic paths used in computer simulations

## 4. Illustrative case studies

### 4.1. Case 1: Conventional FxLMS algorithm with Bayesian inference

In this study, the reference data were obtained from in-situ noise measurement (Fig. 8(a)). The sampling rate of the data was 44,100 Hz. The sound pressure signals were stored by a recording tape and re-acquired using an analog-to-digital card in the in-house laboratory. The amplitude of digital signals represents the voltage of output signals. The processing signals below, i.e., Bayesian inference, are based on these raw signals that are plotted in Fig. 8(b). The various simulation parameters used in the case were

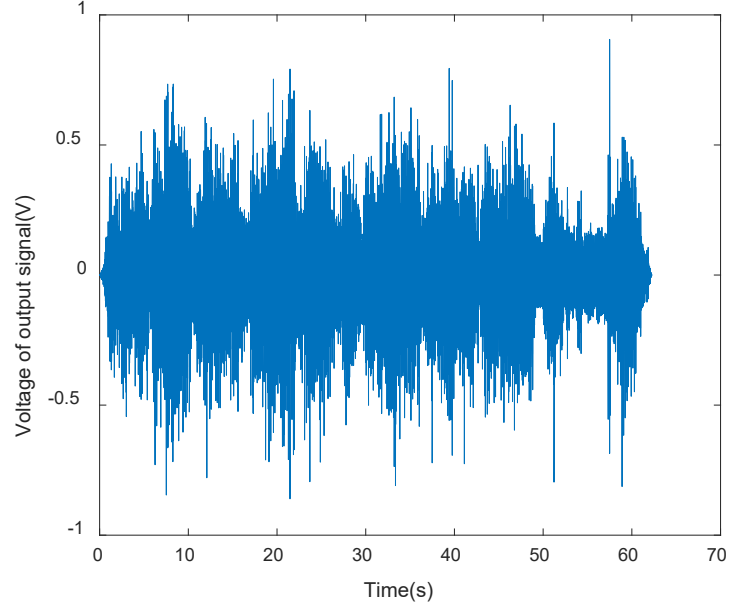


the conventional FxLMS algorithm with  $\mu = 0.01$ . The Bayesian inference approach was applied to conduct K-step predictions prior to the current time. In the DLM, the parameter setting was as below:

$$F_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad G_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad K=5. \quad (29)$$



(a)



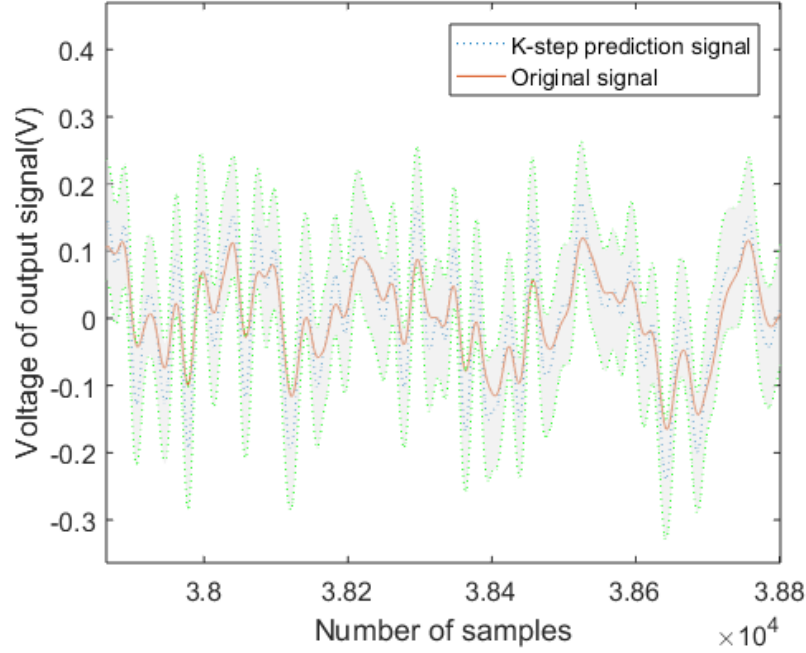
(b)

**Fig. 8.** (a) In-situ noise measurement; and (b) raw data.

The grey area in Fig. 9 depicts the predicted distribution at each iteration when  $K = 5$ . This means that according to the current observational data, the next five steps of data can be obtained via a time series model. The predicted information was presented as a distribution with a mean value and standard deviations. All of the original signals can be covered within the grey area at a 90% confidence interval. Next, the fuzzy prediction data were obtained as a reference signal for ANC control algorithm processing.

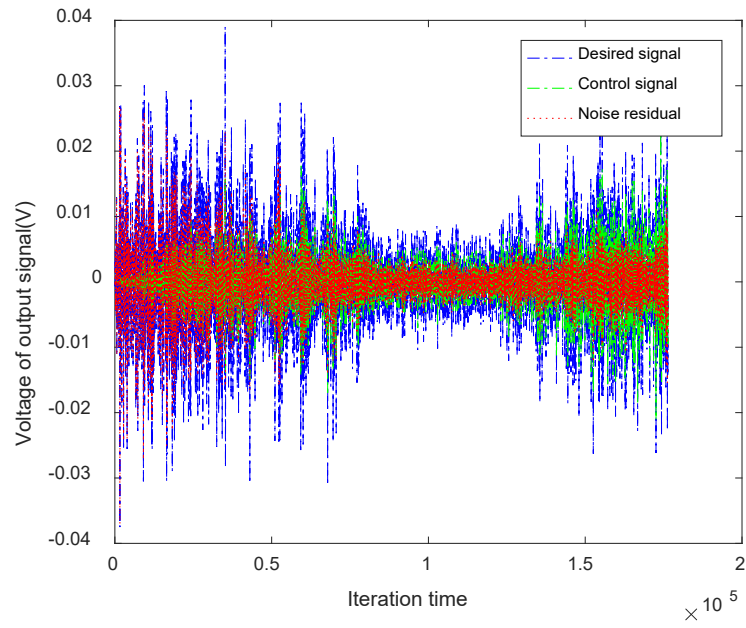
In an active control system, the reference signal is important as a reference in the secondary control path. However, most research has focused on the enhancement of the control algorithm and ignored the reference signal impact. The fuzzy prediction signal may not be identical to the observational data, but this will not affect ANC system performance. Moreover, the advantage of Bayesian inference is obvious: it considers the “*time-delay*” effect on the secondary path. In Fig. 9, the blue line is almost coincident with the middle of the grey area, which means that when  $K = 5$  the prediction data are almost the same as the

observational data. With the increase in  $K$ , the prediction data contain greater uncertainty. Moreover, greater computational effort would be required in the pre-processing step.

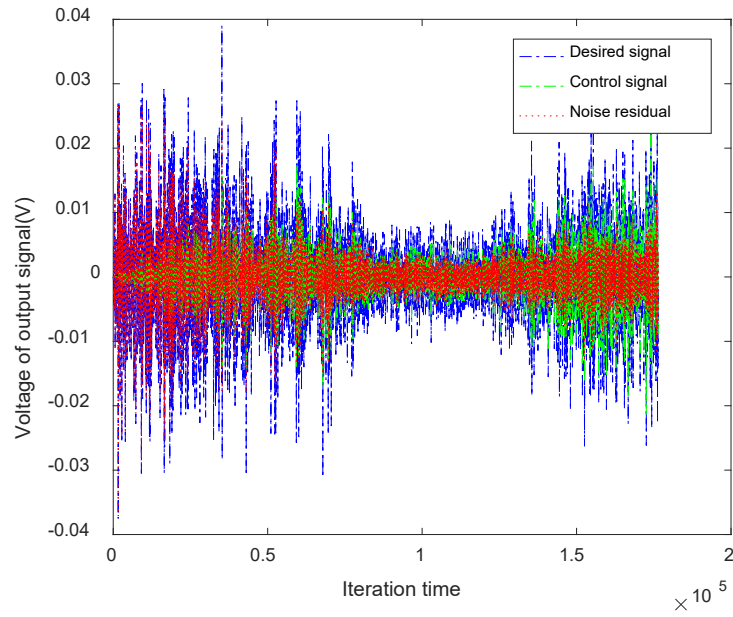


**Fig. 9.** Comparison of the inference and observation signals (Inference signal is denoted by a distribution with  $K=5$ ).

The time-domain signal attenuation results with and without the use of the Bayesian inference pretreatment procedure are presented in Fig. 10. From time-domain analysis, it is clear that both systems can be converged. The convergence curves via the ANR index learning process with and without Bayesian inference are presented in Fig. 11, and the MNR learning curves are shown in Fig. 12; from these data it can be observed that systems using Bayesian inference achieved a higher convergence speed. Unless specified in the context, the term “*without prediction*” refers to the original signals measured from the construction site.

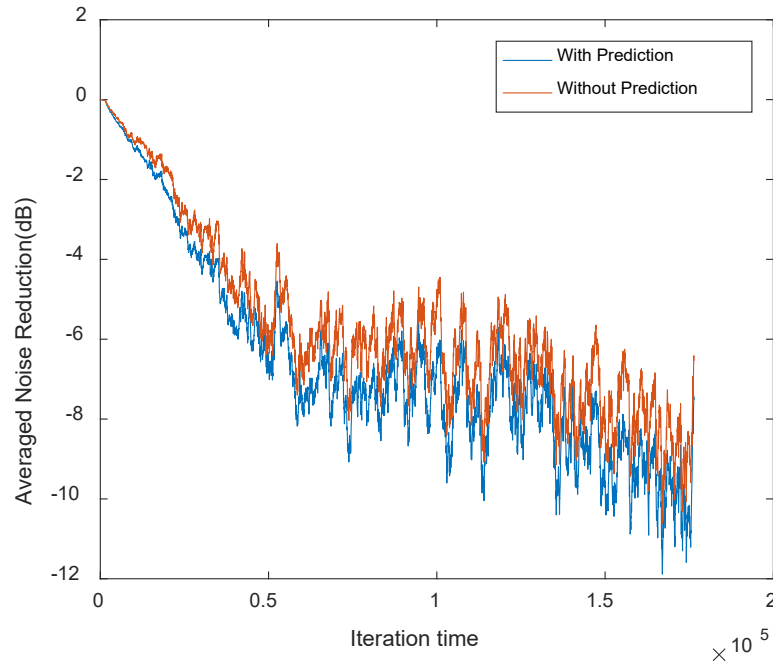


(a)

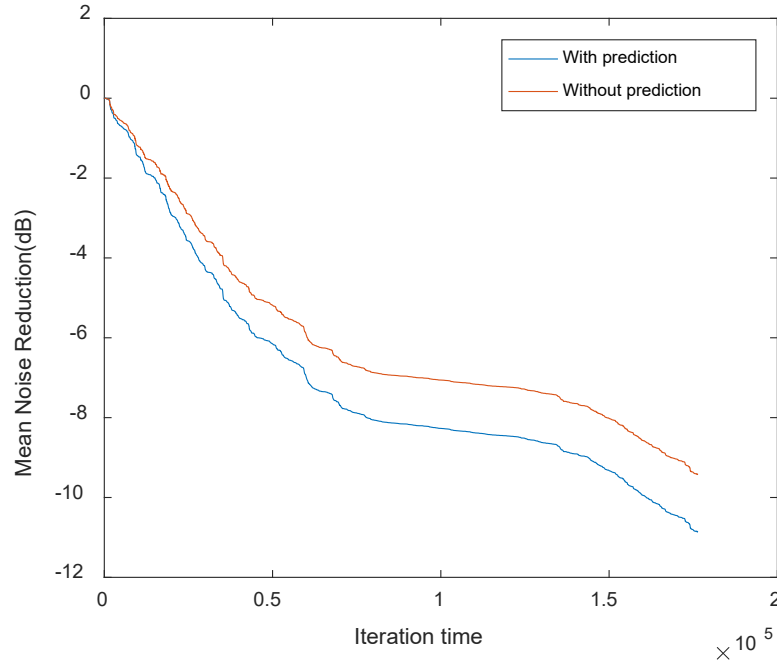


(b)

**Fig. 10.** Time domain signals (a) with and (b) without Bayesian inference: Desired signal (blue line); Control signal (green line); Residual signal (red line).

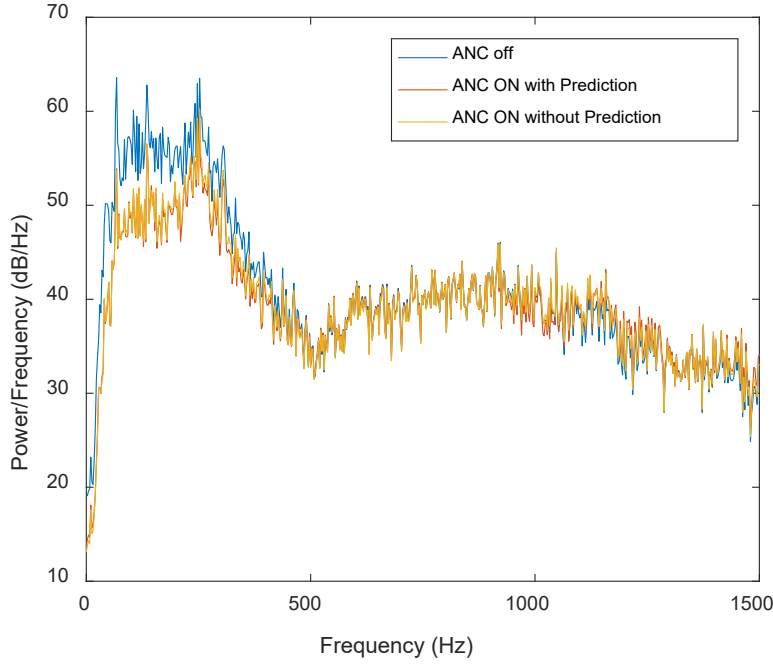


**Fig. 11.** Evolution of learning curves for processing in-situ measurement data based on the ANR index.



**Fig. 12.** Evolution of learning curves for processing in-situ measurement data based on the MNR index.

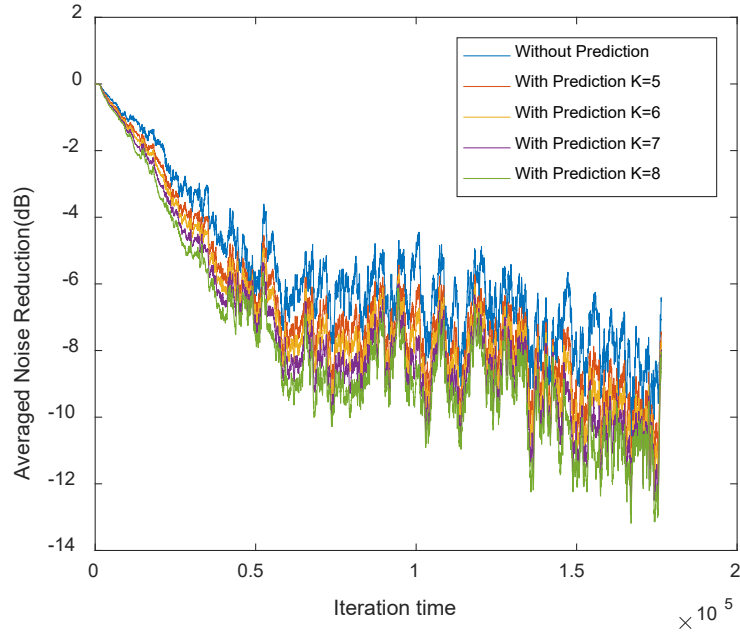
The same noise reduction performance with and without Bayesian inference is shown in Fig. 13. The unit of the time-domain signals was transformed as a sound pressure (Pa). Compared with the reference pressure,  $P_{ref} = 2 \times 10^{-5} Pa$ , the signals were converted to a frequency domain via Welch's power spectral density estimation. The orange line representing data acquired using Bayesian inference and the yellow line data representing data acquired without Bayesian inference are coincident. The fact that our proposed system achieved a similar noise reduction performance as the conventional algorithm under the same parameter setting revealed obvious advantages can be realized. Primarily, it means that time-delay effects on the secondary path can be considered in the control algorithm; in addition, determining the convergence speed with the prediction data as a reference signal can give better results.



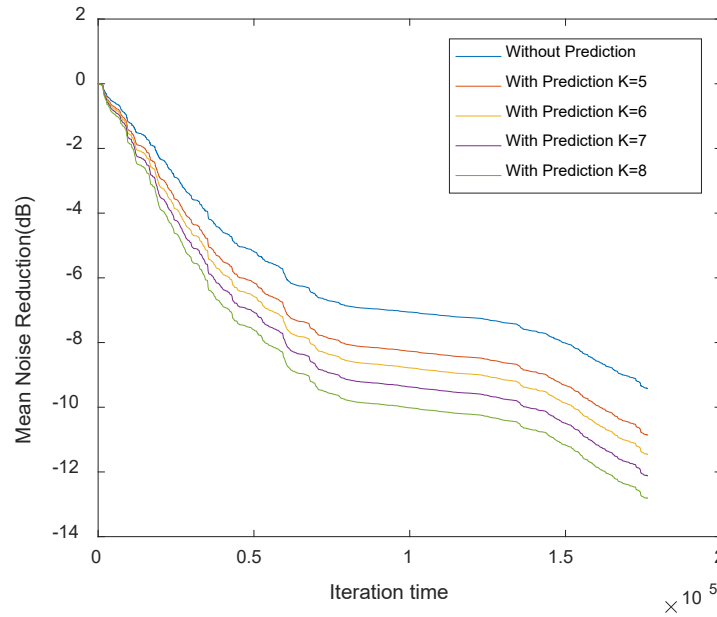
**Fig. 13.** Performance of the proposed ANC system (conventional FxLMS algorithm) with or without Bayesian approach as a preprocess procedure.

Figures 14–16 present the K value impact of the proposed ANC system. With K value increasing from five to eight steps, the convergence performance is better shown in the ANR index and the MNR index. Fig. 16 shows the noise attenuation performance in the frequency spectrum. It is obvious that the K value has less effect on the final noise reduction. In this figure, we also observe that the results with and without the use of the Bayesian inference pretreatment procedure are almost identical, thereby verifying the correctness of the proposed pretreatment procedure. By comparing the data without ANC processing

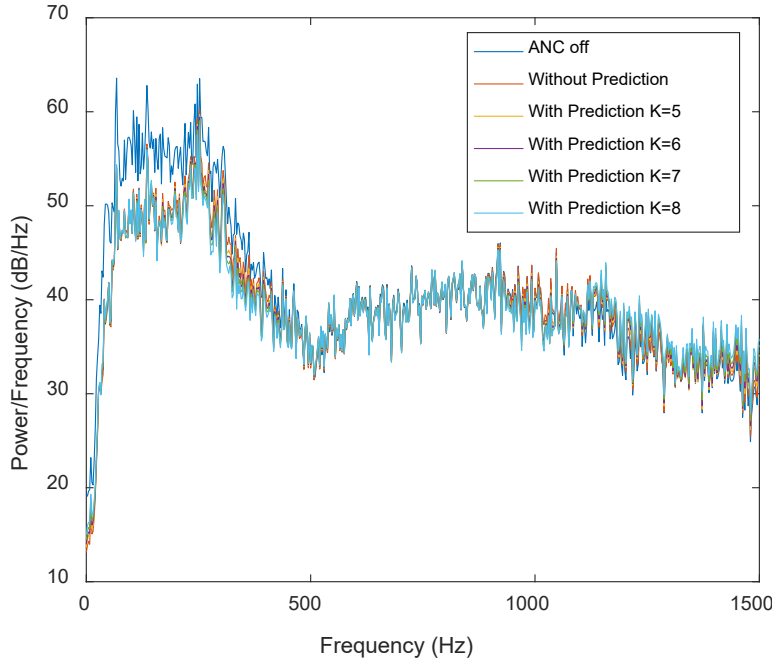
(i.e., “ANC off”), the performance of the ANC system (with and without prediction) is dominant at the low-frequency region ( $\approx$  below 500 Hz), this is mainly due to the effect of the FxLMS algorithm.



**Fig. 14.** Evolution of learning curves for processing in-situ measurement data based on the ANR index.



**Fig. 15.** Evolution of learning curves for processing in-situ measurement data based on the MNR index.



**Fig. 16.** Performance of the proposed ANC system (conventional FxLMS algorithm combined with Bayesian approach) with various K-step values.

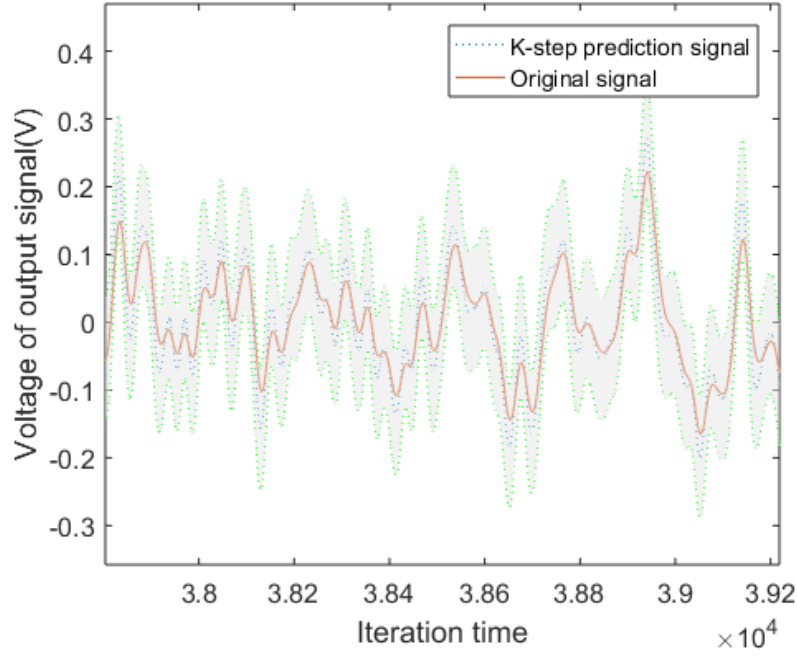
#### 4.2. Case 2: C-FxLMS/F algorithm with Bayesian inference

In a time domain system, a conventional FxLMS algorithm contains distinct restrictions, and thus many researchers have investigated the time-domain control algorithm modification. The C-FxLMS/F algorithm was proposed [25]; it utilized two adaptive filters for convex structure. The FxLMS/F algorithm was adopted as the cost function to reduce the limitations of the FxLMS algorithm. To investigate the practical effects of the Bayesian approach in this case, the C-FxLMS/F algorithm was utilized as a control algorithm. The reference signals were from in-situ measurement noise, as shown in Fig. 8. The various simulation parameters used in the case were:

$$\mu_a = 10, \mu_1 = 0.05, \mu_2 = 0.003, \Phi = 0.0001, \mu_a = 10, \lambda_0 = 0, A_0 = 0, \sigma^+ = 4 \quad (30)$$

The Bayesian inference approach was applied to conduct K-step prediction information prior to the study. In the dynamic linear model, the parameter settings were the same as presented in Eq. (29).

The grey area in Fig. 17 shows the predicted distribution at each iteration where K is 5. By comparing the orange line and grey area, it can be seen that all the original signal can be covered by the grey area at a 90% confidence interval. The fuzzy prediction data were thus used as a reference signal for the ANC control-algorithm processing.

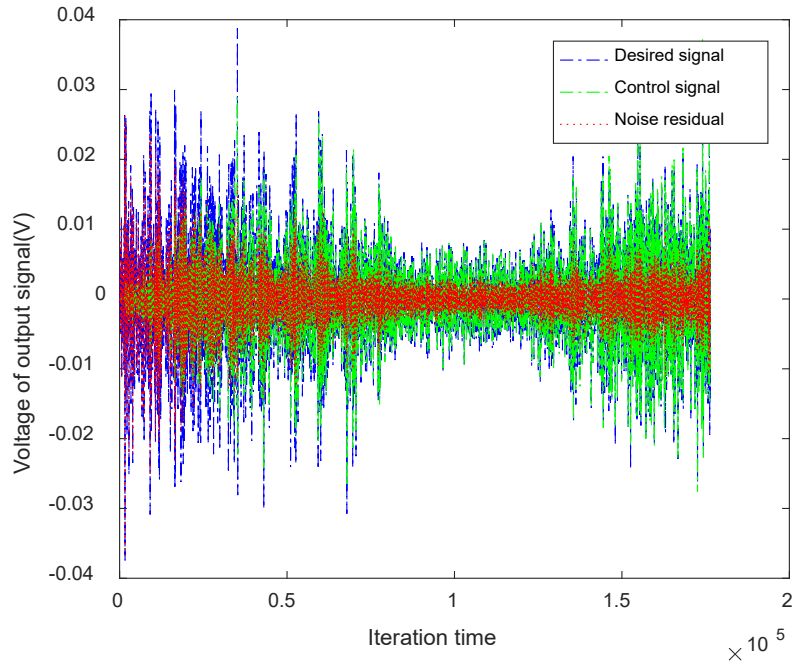


**Fig. 17.** Comparison of the inference and measurement signals  
(Inference signal is denoted by a distribution with  $K=5$ ).

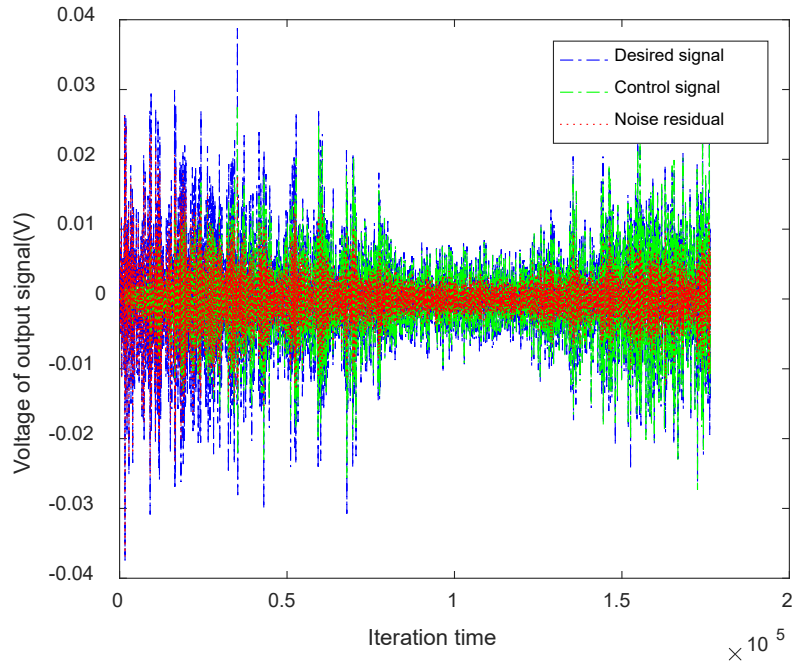
The time-domain signal attenuation results, with and without a Bayesian inference pretreatment procedure, are presented in Fig. 18. From time-domain analysis, it is obvious that both systems can be converged. The convergence curves via the ANR learning process with and without Bayesian inference are presented in Fig. 19, and the MNR learning curves are shown in Fig. 20; this reveals that the system treated with Bayesian inference can achieve a higher convergence speed. The same noise reduction achievement with and without Bayesian inference is shown in Fig. 21.

To evaluate the parameter effects in the ANC system, the comparison results are shown in Figs. 22–24. It can be seen that with increasing  $K$  value, the convergence performance is faster, as illustrated in the ANR and MNR indexes. The noise reduction performance in the frequency spectrum coincides with the performance of the ANC system without Bayesian inference, as shown in Fig. 24. In this figure, the results with and without the use of the Bayesian inference pretreatment procedure are almost identical to each other, this shows again the correctness of the proposed pretreatment procedure. By comparing the data without ANC processing (i.e., “ANC off”), the performance of the ANC system (with and without prediction) is still pronounced at the low-frequency region ( $\approx$  below 500 Hz), this is mainly due to the effect of the C-FxLMS/F algorithm.



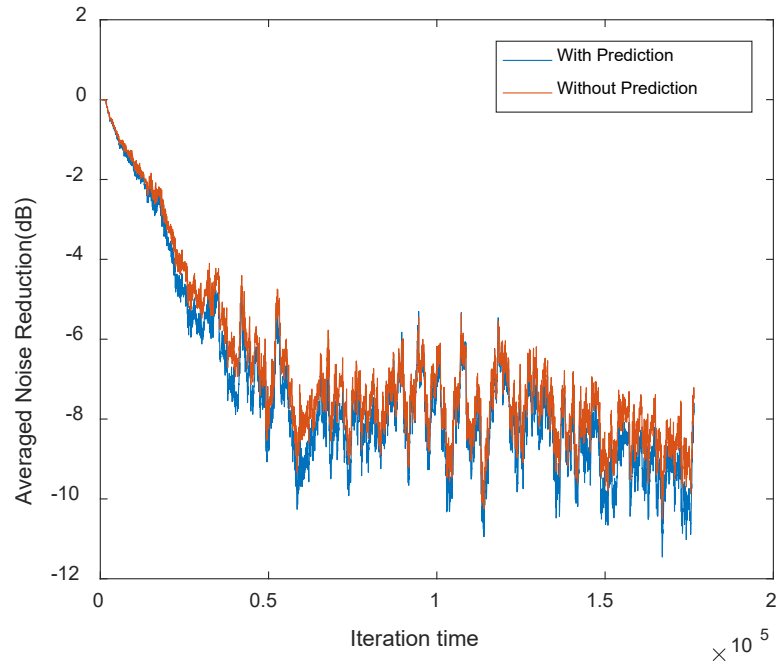


(a)

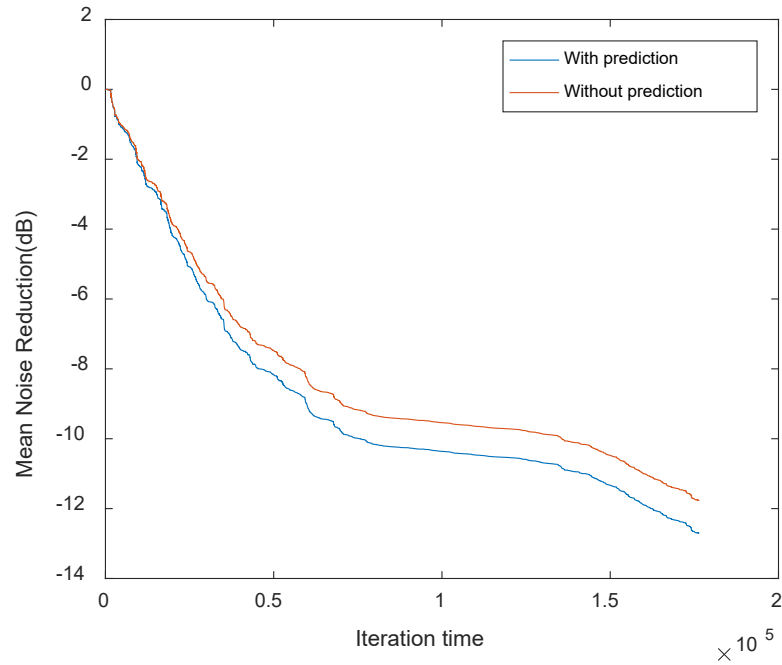


(b)

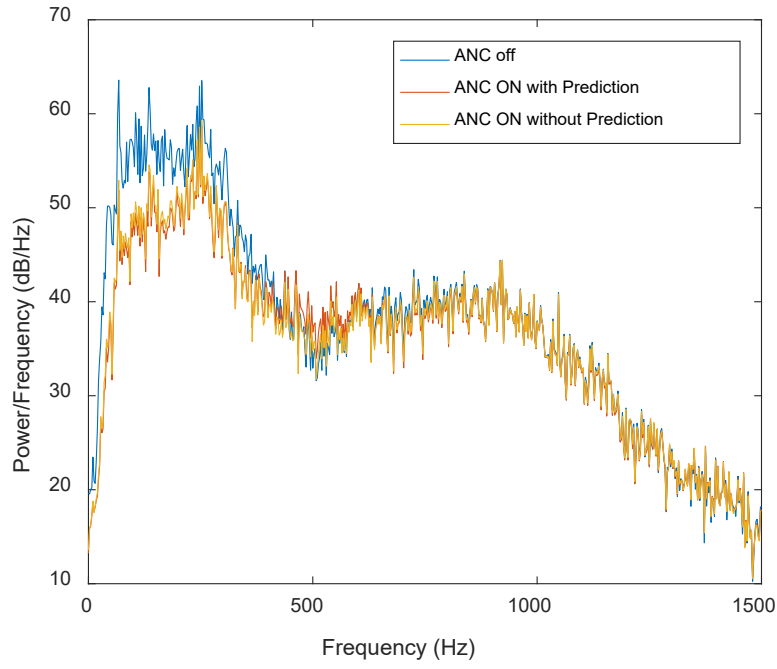
**Fig. 18.** Time domain signals (a) with and (b) without Bayesian inference: Desired signal (blue line); Control signal (green line); Residual signal (red line).



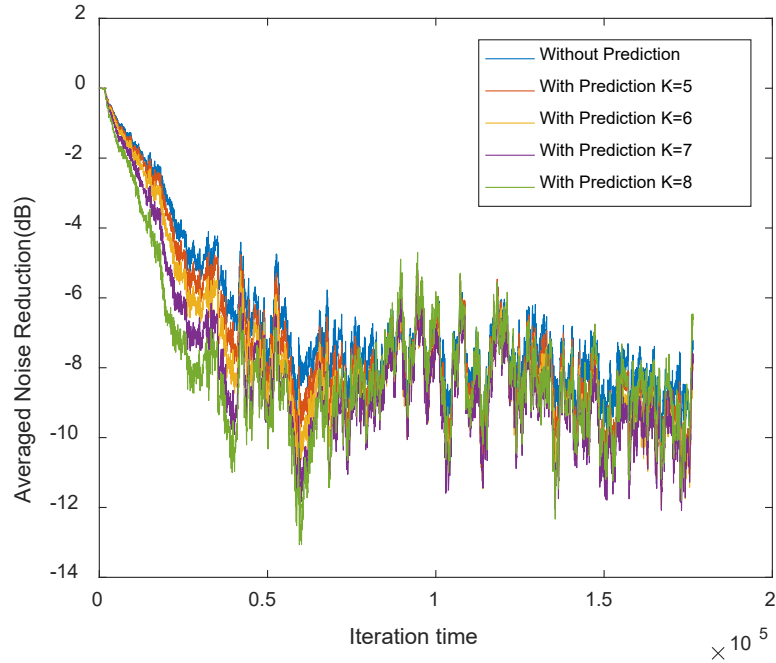
**Fig. 19.** Evolution of learning curves for processing in-situ measurement data based on the ANR index.



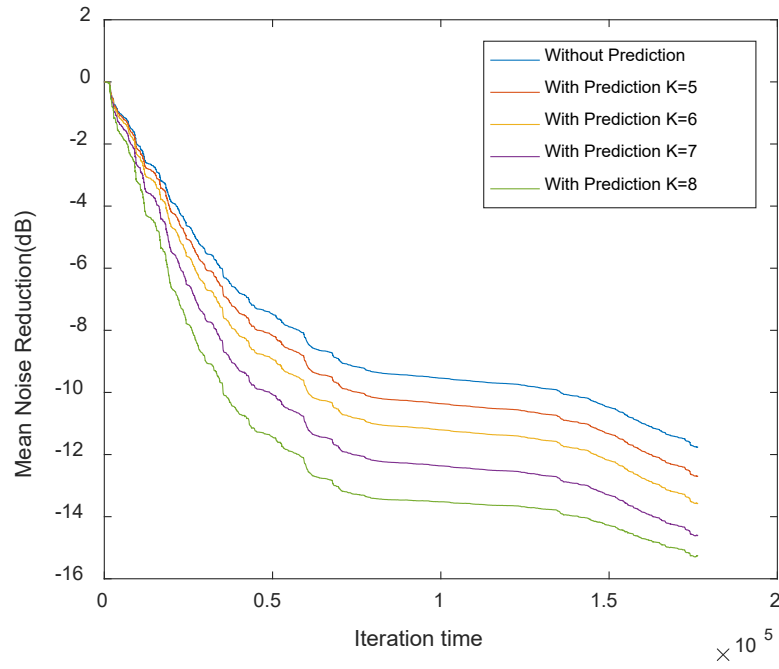
**Fig. 20.** Evolution of learning curves for processing in-situ measurement data based on the MNR index.



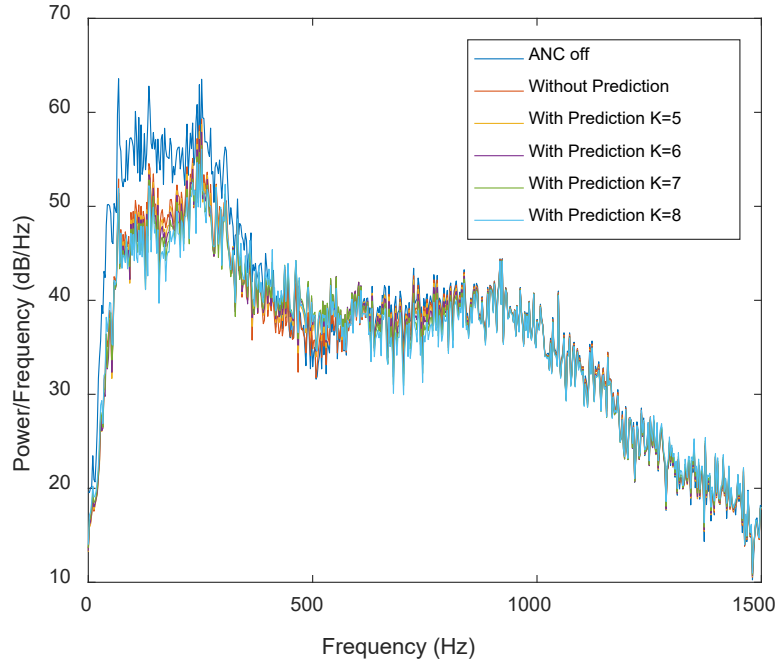
**Fig. 21.** Performance of the proposed ANC system (C-FxLMS/F algorithm) with or without Bayesian approach as a preprocess procedure.



**Fig. 22.** Evolution of learning curves for processing in-situ measurement data based on the ANR index.



**Fig. 23.** Evolution of learning curves for processing in-situ measurement data based on the MNR index.



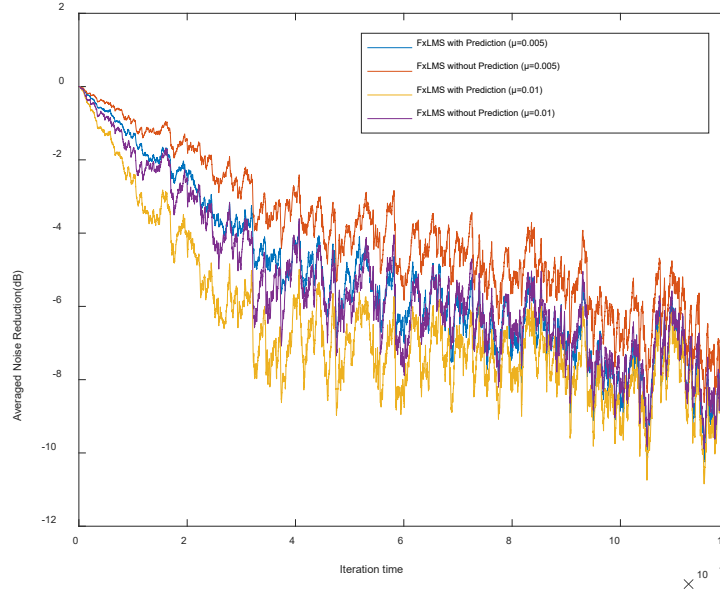
**Fig. 24.** Performance of the proposed ANC system (C-FxLMS/F algorithm combined Bayesian approach) with various K-step values.

### 4.3 Complexity analysis of various signal forms

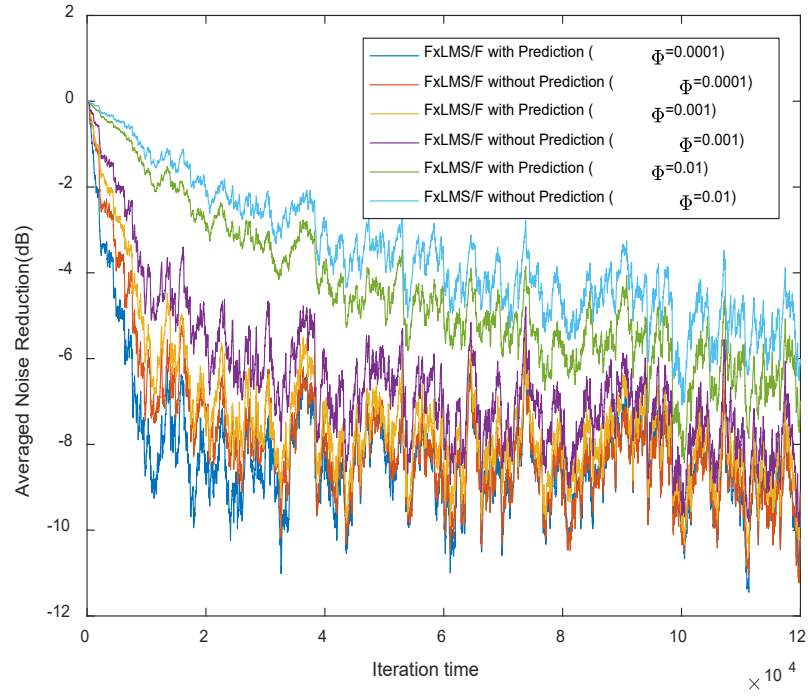
To evaluate the effectiveness of this pre-processing process, three signal forms (i.e., (i) Gaussian noise; (ii) sinusoidal and Gaussian noise; and (iii) in-situ measurement data) are also tested for comparison. In Figs. 25–27, the convergence speed is examined by the ANR index. Only a Gaussian noise input was fed into an ANC system with and without Bayesian inference. The results show that the convergence performance is better with the presence of this pre-processing system. Figs. 28–30 further present a good convergence performance under the effect of both sinusoidal and Gaussian noise signals. In this case, the reference noise signal  $x(n)$  is a sinusoidal wave of 500 Hz and the sampling rate is 8000 Hz as follows:

$$x(n) = \sin\left(\frac{2\pi \times 500 \times n}{8000}\right) + v(n) \quad (31)$$

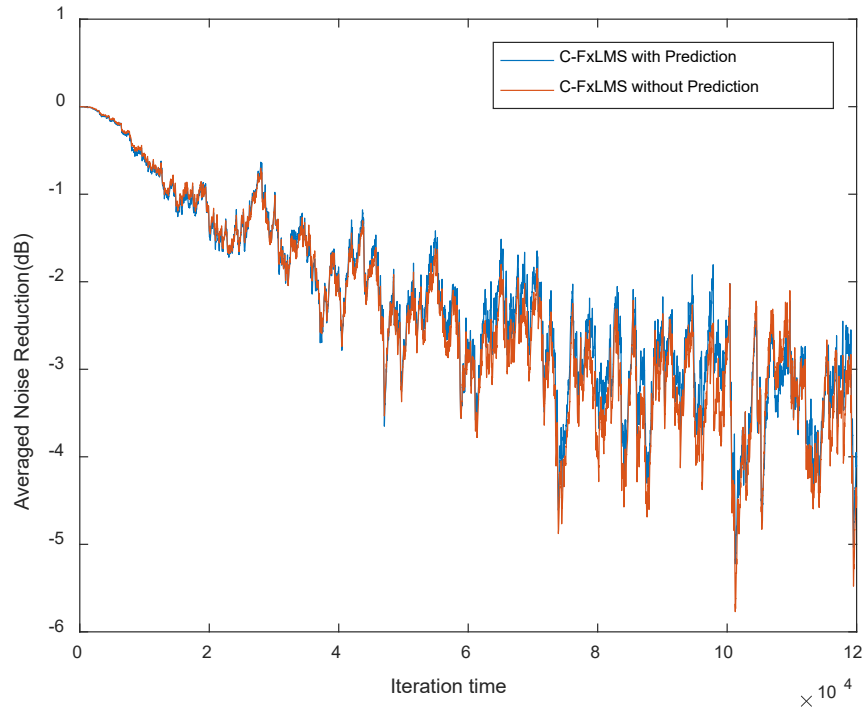
where  $v(n)$  represents a Gaussian noise. In Figs. 27 and 30, the C-FxLMS/F algorithm with and without the pre-processing system presents a similar convergence performance. In Fig. 31, using the in-situ measurement noise input, we observe that the results with Bayesian inference are better than that of those without such a process for the FxLMS, FxLMS/F and C-FxLMS/F algorithms.



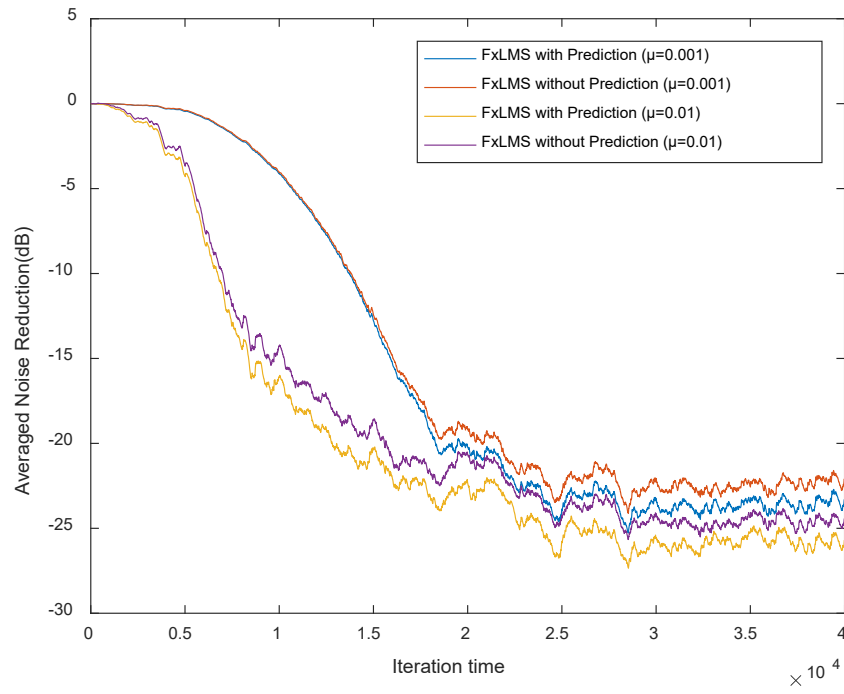
**Fig. 25.** Evolution of learning curves for a Gaussian signal input (FxLMS algorithm with various step-size)



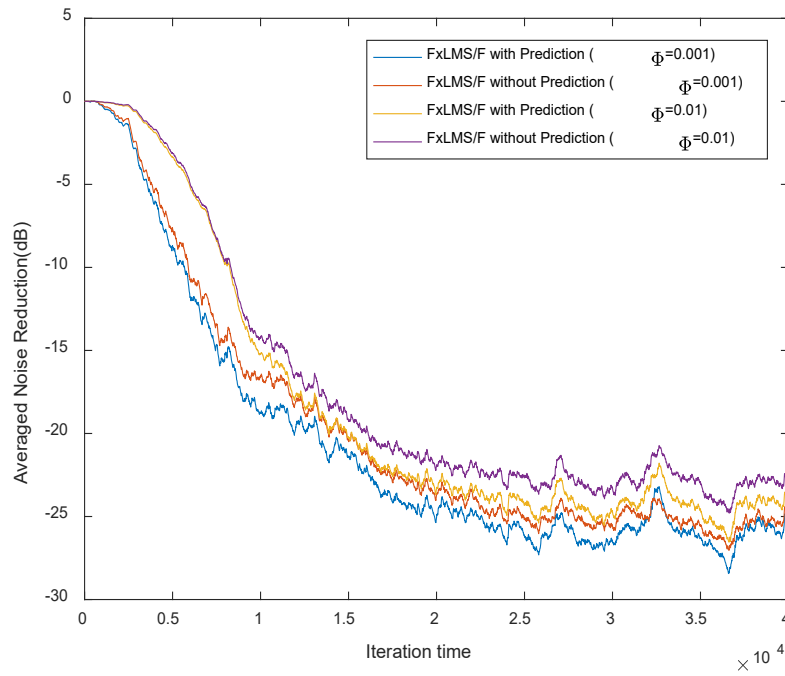
**Fig. 26.** Evolution of learning curves for a Gaussian signal input (FxLMS/F algorithm)



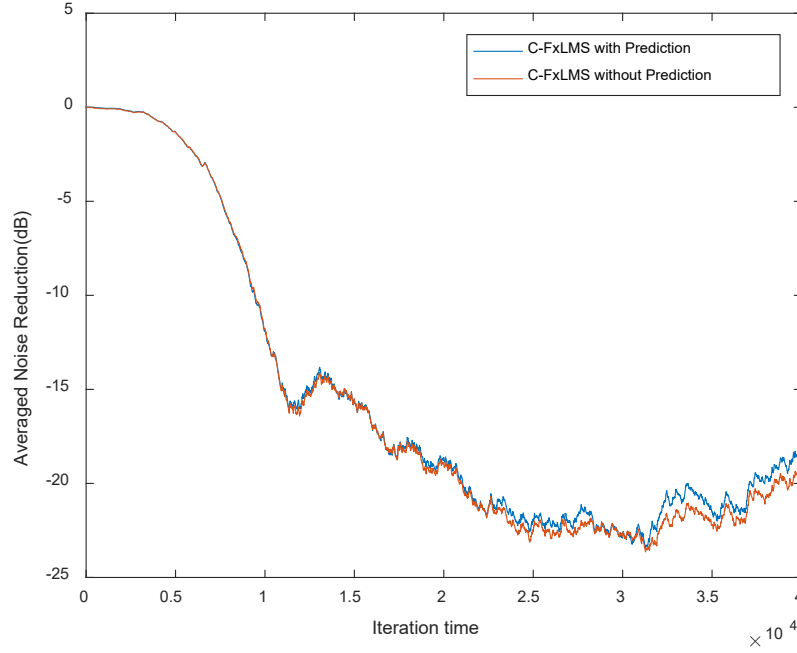
**Fig. 27.** Evolution of learning curves for a Gaussian signal input (C-FxLMS/F algorithm)



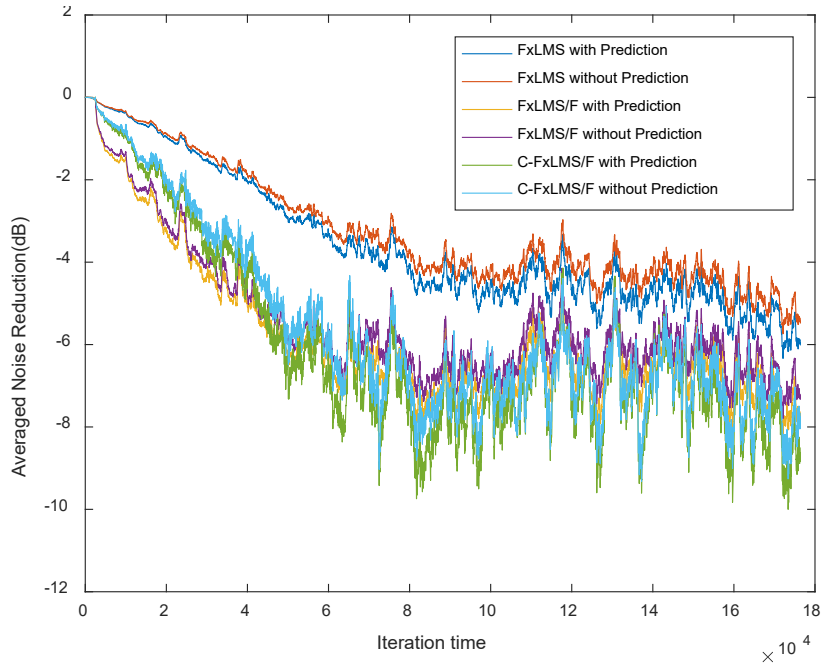
**Fig. 28.** Evolution of learning curves for a combined sinusoidal and Gaussian signal (FxLMS algorithm)



**Fig. 29.** Evolution of learning curves for a combined sinusoidal and Gaussian signal (FxLMS/F algorithm)



**Fig. 30.** Evolution of learning curves for a combined sinusoidal and Gaussian signal (C-FxLMS/F algorithm)



**Fig. 31.** Evolution of learning curves for in-situ measurement noise (FxLMS, FxLMS/F and C-FxLMS/F algorithm with an optimal parameter)



## 5. Conclusions

Bayesian inference based on a DLM approach was adopted in this work as a means of raw signal pretreatment in the ANC system. The major findings of this work are summarized as follows:

- This pre-processing system leads to a higher convergence speed and the coincidence of the noise reduction results with in-situ measurement data. A pretreatment process combined with the conventional FxLMS algorithm and a new C-FxLMS/F algorithm was investigated for its applicability to various time-domain control algorithms.
- To enhance the control performance, the cost of the secondary path was considered from various aspects, such as secondary path estimation and online modeling of the secondary path. In this study, a Bayesian approach can provide a time-step compensation for utilizing the prediction function of Bayesian theory. The combination of prior information and updated observation leads to an updated fuzzy estimation for the future values. The estimation was in some steps dependent on the value of  $K$  ahead of the then-current time, which can reduce the time-delay effects in the control algorithm.
- The updating characteristic “*forecast-observation-analysis*” loop is advantageous for the implementation of signal processing for an ANC system. The simulation results demonstrated that the proposed modified ANC system achieved better convergence performance than the ANC system without pretreatment. The final noise reduction level comparison of the effects of the prediction process showed that it is consistent. This means that the pretreatment processing does not affect the final reduction performance, but in fact confers the advantages described above.
- Bayesian forecasting based on DLM performs as a pre-processing filter due to statistical characteristics. This method requires less parameter identification that is an advantage for many engineering applications. Besides, it is also suitable for different types of noise signals, i.e., (i) Gaussian noise; (ii) sinusoidal and Gaussian noise; and (iii) in-situ measurement data, as investigated herein.

The present approach, as a pretreatment process, can be coupled for various ANC algorithms. Although it is prominent to improve the time-delay effect in an ANC system, the pre-processing system is valid for a short-time period information inference. Further studies will focus on extending the time period of this pre-processing system. Besides, the computational effort will be increased when the Bayesian-based approach is incorporated with an ANC system. However, this problem may be fixed by designing an *ad-hoc* intelligent controller that can reduce the computational complexity with the use of

Bayesian inference. Therefore, increasing the computational complexity can be compensated to achieve a better adaptation system in real-engineering environments.

**Acknowledgement**

The work described in this paper was fully supported by the Environment and Conservation Fund of the Hong Kong Special Administrative Region (Project Title: Smart Noise Barriers/Enclosures for Dual Active and Passive Control of Construction Noise (Project No.: ECF Project 83/2017)).

## References:

- [1] B. Widrow, S.D. Stearns, Adaptive signal processing, Prentice-Hall, Inc., 1985.
- [2] S.M. Kuo, Adaptive active noise control systems: algorithms and digital signal processing (DSP) implementations, SPIE's 1995 Symposium on OE/Aerospace Sensing and Dual Use Photonics, SPIE, 1995, pp. 27.
- [3] S.M. Kuo, D.R. Morgan, Active noise control: a tutorial review, Proceedings of the IEEE, 87 (1999) 943-973.
- [4] B. Widrow, S.D. Stearns, Adaptive signal processing, Prentice-Hall, Englewood Cliffs, N.J., 1985.
- [5] S. Hosur, A.H. Tewfik, Wavelet transform domain adaptive FIR filtering, IEEE Transactions on Signal Processing, 45 (1997) 617-630.
- [6] S.M. Kuo, R.K. Yenduri, A. Gupta, Frequency-domain delayless active sound quality control algorithm, Journal of Sound and Vibration, 318 (2008) 715-724.
- [7] T. Padhi, M. Chandra, A. Kar, M. Swamy, A new hybrid active noise control system with convex combination of time and frequency domain filtered-x LMS algorithms, Circuits, Systems, and Signal Processing, 37 (2018) 3275-3294.
- [8] S. Zhang, Y.S. Wang, H. Guo, C. Yang, X.L. Wang, N.N. Liu, A normalized frequency-domain block filtered-x LMS algorithm for active vehicle interior noise control, Mechanical Systems and Signal Processing, 120 (2019) 150-165.
- [9] X.L. Tang, C.M. Lee, Time-frequency-domain filtered-x LMS algorithm for active noise control, Journal of Sound and Vibration, 331 (2012) 5002-5011.
- [10] E. Friot, Time-domain versus frequency-domain effort weighting in active noise control design, The Journal of the Acoustical Society of America, 141 (2017) EL11-EL15.
- [11] Y. Tu, C.R. Fuller, Multiple reference feedforward active noise control part I: analysis and simulation of behavior, Journal of Sound and Vibration, 233 (2000) 745-759.
- [12] Z. Qiu, C.M. Lee, Z.H. Xu, L.N. Sui, A multi-resolution filtered-x LMS algorithm based on discrete wavelet transform for active noise control, Mechanical Systems and Signal Processing, 66-67 (2016) 458-469.
- [13] P. Song, H. Zhao, Filtered-x generalized mixed norm (FXGMN) algorithm for active noise control, Mechanical Systems and Signal Processing, 107 (2018) 93-104.
- [14] M. Tahir Akhtar, W. Mitsuhashi, Improving performance of FxLMS algorithm for active noise control of impulsive noise, Journal of Sound and Vibration, 327 (2009) 647-656.
- [15] X. Sun, S.M. Kuo, G. Meng, Adaptive algorithm for active control of impulsive noise, Journal of Sound and Vibration, 291 (2006) 516-522.
- [16] G. Sun, M. Li, T. Lim, A family of threshold based robust adaptive algorithms for active impulsive noise control, Applied Acoustics, 97 (2015).
- [17] Y.L. Zhou, Y.X. Yin, Q.Z. Zhang, An optimal repetitive control algorithm for periodic impulsive noise attenuation in a non-minimum phase ANC system, Applied Acoustics, 74 (2013) 1175-1181.
- [18] M.T. Akhtar, W. Mitsuhashi, Improving robustness of filtered-x least mean p-power algorithm for active attenuation of standard symmetric- $\alpha$ -stable impulsive noise, Applied Acoustics, 72 (2011) 688-694.
- [19] N. Kurian, K. Patel, N. George, Robust active noise control: An information theoretic learning approach, Applied Acoustics, 117 (2016).
- [20] L. Shao-Jen, J.G. Harris, Combined LMS/F algorithm, Electronics Letters, 33 (1997) 467-468.
- [21] Y. Li, Y. Wang, T. Jiang, Norm-adaption penalized least mean square/fourth algorithm for sparse channel estimation, Signal Processing, 128 (2016) 243-251.

- [22] M. Ferrer, A. Gonzalez, M.d. Diego, G. Pinero, Convex Combination Filtered-X Algorithms for Active Noise Control Systems, *IEEE Transactions on Audio, Speech, and Language Processing*, 21 (2013) 156-167.
- [23] A.M. Al Omour, A. Zidouri, N. Iqbal, A. Zerguine, Filtered-X Least Mean Fourth (FXLMF) and Leaky FXLMF adaptive algorithms, *EURASIP Journal on Advances in Signal Processing*, 2016 (2016) 39.
- [24] N.V. George, A. Gonzalez, Convex combination of nonlinear adaptive filters for active noise control, *Applied Acoustics*, 76 (2014) 157-161.
- [25] P. Song, H. Zhao, Filtered-x least mean square/fourth (FXLMS/F) algorithm for active noise control, *Mechanical Systems and Signal Processing*, 120 (2019) 69-82.
- [26] G.L. Sicuranza, A. Carini, Nonlinear active noise control, 2004 12th European Signal Processing Conference, 2004, pp. 1887-1890.
- [27] B. Chen, S. Yu, Y. Yu, R. Guo, Nonlinear active noise control system based on correlated EMD and Chebyshev filter, *Mechanical Systems and Signal Processing*, 130 (2019) 74-86.
- [28] N.V. George, G. Panda, A robust filtered-s LMS algorithm for nonlinear active noise control, *Applied Acoustics*, 73 (2012) 836-841.
- [29] H. Zhao, X. Zeng, Z. He, S. Yu, B. Chen, Improved functional link artificial neural network via convex combination for nonlinear active noise control, *Applied Soft Computing*, 42 (2016) 351-359.
- [30] S.M. Kuo, R. Gireddy, Real-time experiment of snore active noise control, 2007 IEEE International Conference on Control Applications, 2007, pp. 1342-1346.
- [31] X. Zhang, X. Qiu, Performance of a snoring noise control system based on an active partition, *Applied Acoustics*, 116 (2017) 283-290.
- [32] K.V. Yuen, *Bayesian Methods for Structural Dynamics and Civil Engineering*, 2010.
- [33] R. Han, M. Wu, F. Liu, H. Sun, J. Yang, A narrowband active noise control system with a frequency estimator based on Bayesian inference, *Journal of Sound and Vibration*, 455 (2019) 299-311.
- [34] I.T. Ardekani, J.P. Kaipio, A. Nasiri, H. Sharifzadeh, W.H. Abdulla, A Statistical Inverse Problem Approach to Online Secondary Path Modeling in Active Noise Control, *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, 24 (2016) 54-64.
- [35] I.T. Ardekani, X. Zhang, H. Sharifzadeh, J. Kaipio, Maximum a posteriori adjustment of adaptive transversal filters in active noise control, 2017 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC), 2017, pp. 118-123.
- [36] M. West, J. Harrison, *Bayesian forecasting and dynamic models* (2nd ed.), Springer-Verlag, 1997.
- [37] M.W. Andy Pole, Jeff Harrison, *Applied Bayesian Forecasting and Time Series Analysis*, 1994
- [38] S.S. Holger Lipowsky, Michael Bauer, Klaus-Juergen Schmidt, Application of Bayesian Forecasting to Change Detection and Prognosis of Gas Turbine Performance, *Journal of Engineering for Gas Turbine and Power*, 132 (2010) 031602.
- [39] Y.W. Wang, Ni, Y.Q., Wang, X., Detection of performance deterioration of high-speed train wheels based on Bayesian dynamic model, *Proceedings of the 11th International Workshop on Structural Health Monitoring*, (12-14 September 2017).
- [40] L.H. Zhang;, Y.W. Wang;, Y.Q. Ni, S.K. Lai, Online condition assessment of high-speed trains based on Bayesian forecasting approach and time series analysis, *Smart structures and systems*, 21 (2018) 705-713.
- [41] Y.W. Wang, Y.Q. Ni, Bayesian dynamic forecasting of structural strain response using structural health monitoring data, *Structural Control and Health Monitoring*, 27 (2020) e2575.

- [42] Y.W. Wang, Y.Q. Ni, X. Wang, Real-time defect detection of high-speed train wheels by using Bayesian forecasting and dynamic model, *Mechanical Systems and Signal Processing*, 139 (2020) 106654.
- [43] S.K. Lai, Y.T. Zhang, Real-time prediction of noise signals for active control using a Bayesian inference approach, *Inter-Noise 2019 Congress*, Madrid, Spain, 16 – 19 June 2019, (2019).
- [44] J. Arenas-García, M. Martínez-Ramón, Á. Navia-Vázquez, A.R. Figueiras-Vidal, Plant identification via adaptive combination of transversal filters, *Signal Processing*, 86 (2006) 2430-2438.
- [45] J. Ni, F. Li, Adaptive combination of subband adaptive filters for acoustic echo cancellation, *IEEE Transactions on Consumer Electronics*, 56 (2010) 1549-1555.
- [46] N.V. George, G. Panda, On the development of adaptive hybrid active noise control system for effective mitigation of nonlinear noise, *Signal Processing*, 92 (2012) 509-516.
- [47] M. Bouchard, S. Quednau, Multichannel recursive-least-square algorithms and fast-transversal-filter algorithms for active noise control and sound reproduction systems, *IEEE Transactions on Speech and Audio Processing*, 8 (2000) 606-618.
- [48] M. Bouchard, F. Yu, Inverse structure for active noise control and combined active noise control/sound reproduction systems, *IEEE Transactions on Speech and Audio Processing*, 9 (2001) 141-151.