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Optimal timing of the seismic vulnerability reduction measures using real options analysis

Vahid Asghari[1], Hamed Kashani[2,*], Ph.D., Shu-Chien Hsu[3], Ph.D.

[1]Ph. D. Candidate, Hong Kong Polytechnic University, Hong Kong, formerly M. Sc. student, Sharif University of Technology, Iran, E-mail: vahid.asghari@connect.polyu.hk

[2,*] Corresponding Author; Assistant professor, Center for Infrastructure Sustainabilty and Resilience Research, Department of Civil Engineering, Sharif University of Technology, Iran, E-mail: hamed.kashani@sharif.edu

[3]Associate Professor, Hong Kong Polytechnic University, Hong Kong, E-mail: mark.hsu@polyu.edu.hk

Abstracts: The uninterrupted performance of the infrastructure systems of a community is essential to its welfare and prosperity. Infrastructure assets are continuously subject to degradation, which can compromise their performance and increase their seismic vulnerability. Measures such as seismic retrofits should be adopted to ensure the uninterrupted performance of infrastructure assets. Proper timing of these interventions can reduce the total costs incurred by the communities. To this end, an appropriate investment valuation method is needed. This study proposes an investment valuation framework that addresses this need. At the core of this framework is a real options model that determines the optimal time to implement seismic vulnerability reduction measures considering various uncertainties about the state of the asset, the probable future hazards, and the costs incurred by the community. The main contribution of this study to the body of knowledge is the creation of a framework based on real options analysis that is tailored to the context of investment decision-making and timing of seismic vulnerability reduction measures. Practitioners can use this framework to plan the implementation of seismic vulnerability reduction measures, avoid over- or under-investment in these interventions, enhance their flexibility in using limited available financial resources, and achieve resilience enhancement objectives.

Keywords: Real options, optimal timing, asset management, seismic retrofit**,** resilience enhancement

Introduction

Communities rely heavily on various infrastructure systems (e.g., water, energy, communication, and transport networks) to continue their social and economic activities. The adequate and uninterrupted performance of infrastructure systems is essential to the growth and prosperity of each community (Aktan et al. 2016). Several factors can affect the state of the infrastructure systems and limit their functionality. Aging and fatigue due to a variety of factors such as continuous loading and environmental agents (i.e., wind, solar radiation, precipitation, and chemicals) gradually change the state of critical infrastructure assets, deteriorate their conditions, and increase their vulnerability in the face of natural disasters such as earthquakes. The earthquake-induced damage to the degraded infrastructure assets can disrupt their functionality. The loss of infrastructure systems functionality can lead to significant consequences (i.e., social and economic costs). Approximately $3 trillion is recorded as the economic loss of natural disasters in the last 20 years (United Nations Office for Disaster Risk Reduction (UNISDR) 2019). Interventions such as the seismic retrofit of assets can reduce their vulnerability in the face of earthquakes and reduce the incurred losses (Tapia and Padgett 2016).

To avoid over- and under-investment of limited available resources, asset managers should develop the optimal plan that simultaneously minimizes the vulnerability of assets in the face of probable future disasters, their post-disaster service-interruption, and the whole-life cycle costs (WLC) incurred by the community. Otherwise, the outcomes of the planned interventions will be suboptimal and may lead to unnecessary costs for the community (Ellingham and Fawcett 2007). Past research studies (e.g., Han et al. 2017; Tabandeh and Gardoni 2015) have investigated various aspects of retrofit projects. Past studies have proposed several techniques and methods to devise optimal investment plans for infrastructure assets retrofit and rehabilitation. For instance, Frangopol *et al.* (1997) assessed the role of degradation the WLC of bridges under various circumstances. They attempted to minimize the WLC while maintaining the lifetime reliability of the structure at an acceptable level. Using bridges on a highway in Italy and a hospital consisting of 11 buildings, Nuti and Vanzi (2003) proposed a methodology to decide about retrofitting the assets given the pre- and post-intervention rates of exceeding limit states annually. Dong *et al.* (2014) proposed the application of genetic algorithms to determine the optimal retrofit plan of bridge networks before an earthquake. Taking a similar approach, Mondoro *et al.* (2017) developed a risk-based method for managing coastal systems facing hurricanes. Talebiyan and Mahsuli (2018) prioritized the seismic retrofit of 114 masonry buildings based on the extent of reduction in the regional loss per dollar spent on the retrofit of each building.

The literature analysis on infrastructure retrofits and rehabilitation planning shows that the existing models are subject to a critical limitation. The existing models do not consider the flexibilities that asset managers can incorporate into their capital planning to cope with the evolving uncertainties. Examples of these uncertainties are the future state of the assets, the magnitude and time of probable future hazards, the damage that the asset may sustain in the face of a hazard, and the costs associated with retrofitting and repairing infrastructure assets. Flexibility in capital planning, specifically in retrofit and rehabilitation projects, can take various forms, such as the deferral of the investment (Dixit and Pindyck 1994; Guthrie 2013). Incorporating this flexibility in a project provides the asset managers with the opportunity to delay the investment, observe the way the uncertainties resolve, and invest the limited available resources only when the situation becomes favorable. When it comes to the planning of investments in vulnerability reduction measures (e.g., seismic retrofits), asset managers need appropriate investment valuation models capable of assessing such flexible strategies designed to address the underlying uncertainties.

In recent decades, real options (RO) analysis has been proposed as an alternative to the conventional investment valuation methods. Real options analysis is a practical approach to investment valuation that can address the challenges that underlie an economic environment characterized by a significant degree of uncertainty. It can provide managers and decision-makers the necessary tools to handle flexibility in capital planning by mapping the uncertainties and decisions over time (Triantis and Bensoussan 2001). It can help them overcome the undesirable consequences of decision-making under large uncertainties, including the probable loss of profit (Aven and Ford 2004). There has been a growing trend in various industries toward incorporating flexibility in capital planning and using RO analysis to optimize investment plans. For instance, the application of RO analysis has been reported in production modularization (Xu et al. 2012), land use management (Regan et al. 2015), mining (Evatt et al. 2012), information technology (Yong and Lawrence Sanders 2002), energy (Cover and Thomas 2012; Kashani et al. 2015), agriculture (Sanderson et al. 2016), and retail (Ashuri et al. 2008).

Like the industries mentioned above, infrastructure asset management and construction industries can be characterized by high degrees of uncertainty. Several past studies (Ford et al. 2002; Pereira et al. 2006; Siripongvakin and Athigakunagorn 2020; Taneja et al. 2010) have investigated the application of RO analysis to the management of infrastructure assets or construction projects. For instance, Ford *et al.* (2002) described several advantages of using real options in construction projects by providing managers with 1) a better understanding of project uncertainties, 2) more flexibility through evaluation of projects scenarios, 3) enhanced strategies and comprehensive managerial decisions, and 4) competitiveness to catch latent values in projects. Savvidis *et al.* (2019) evaluated the application of RO to the timing of investment in the seismic upgrade of seaports considering the growth of the demand over time. They considered the upgrade costs, repair costs, and costs associated with the downtime after the earthquake to determine the optimal time to retrofit seaport facilities. Nevertheless, they did not consider the state and functionality of infrastructure assets and the associated uncertainties in determining the optimal investment plan.

The full potential of real options analysis in vulnerability reduction of infrastructure asset management is yet to be realized. To address the limitations mentioned above, this study puts forward a framework that determines the optimal timing of seismic retrofit investments by considering the uncertainty about the state of the asset, future hazards, retrofit and repair costs, and the costs that users incur due to the earthquake-induced damage to a facility. At the core of this framework is a real options model that uses a multinomial lattice as the decision tree and compares the value of the immediate implementation of a retrofit against its deferment to reduce the WLC.

This study contributes to the body of knowledge in infrastructure asset management by proposing a real options framework that can be used for the optimal timing of investment in seismic retrofits of infrastructure assets by incorporating the uncertainty associated with 1) the state of an asset and its response in the face of probable future hazards, 2) the time and magnitude of hazards, and 3) the costs incurred by the community (e.g., the earthquake-induced economic losses and retrofit costs). Using the proposed framework, asset managers can determine the optimal time for implementing seismic vulnerability reduction measures, reduce the associated risks with their portfolio of assets under a limited available budget, and avoid under- or over-allocation of budget to seismic retrofits. As a result, the application of the proposed framework can reduce the costs that communities incur for maintaining and operating infrastructure assets. The proposed framework can provide asset managers with an effective retrofit decision-making scheme that enhances their flexibility in spending their limited available budget and prevents the misappropriation of this budget by limiting its expenditure to the projects for which the benefits exceed the costs. To highlight the capabilities of the proposed framework, it is applied to the retrofit timing of a bridge in INSURER City, a virtual community developed at the Center for Infrastructure Sustainability and Resilience Research (INSURER). The analysis results demonstrate that using the proposed framework to plan seismic retrofits yields the least expected WLC. A sensitivity analysis was conducted as part of this example, the results of which underline the impact of the asset degradation rate and the earthquake occurrence rate on the optimal implementation time of seismic retrofits.

This paper is structured as follows. In the following section, the proposed framework is discussed in detail. Next, an illustrative application, which features the optimal timing of the seismic retrofit of a bridge in INSURER City, is presented. The ensuing insights are subsequently discussed. Finally, the conclusions, limitations, and contributions of this research and directions for further research are presented.

Methodology

Fig. 1 demonstrates the structure of the proposed framework that comprises several interdependent modules. These modules contain models that characterize the variables used in this analysis. An extensive review of the relevant literature was conducted to ensure that choice of models used in this framework is consistent with the existing literature. The structure of the proposed framework is modular and flexible, which facilitates the updating or replacement of a model as the knowledge in the corresponding area evolves, and more sophisticated models become available. The flexible structure of the proposed framework provides the opportunity to extend its boundaries. This structure also enhances scalability making the proposed framework applicable to problems with varying scales in different contexts.

The main component of the proposed framework is a real options model that determines the optimal time to invest in vulnerability reduction measures (i.e., seismic retrofits) using the inputs received from several other models. The real options analysis (ROA) model yields an exercise boundary that, for each given year, characterizes the state of the asset beyond which the immediate retrofit of the asset is preferred to its deferment.

Asset module

The asset module configures and determines the state of an asset and its replacement cost over its life span. This module consists of two main sub-models. The first sub-model, called the state model, determines the state of the asset. The state of an asset can change due to degradation, hazards, or human interventions. The second sub-model, which is called the asset replacement cost model, estimates the replacement cost of an asset over its life span using statistical stochastic processes. In the following, these sub-models are discussed in more detail.

State model

The state of an asset characterizes its service level and functionality at any given time (e.g., before hazard occurrence, after the occurrence of the hazard, and after repair or recovery activities). According to HAZUS – MH2.1 (FEMA-NIBS: Federal Emergency Management Agency (FEMA) by the National Institute of Building Sciences 2003), it is also the principal determinant of costs associated with the operation and maintenance of the asset, its need for a retrofit intervention, and its response to probable hazards that may necessitate a post-hazard recovery action.

Past studies (e.g., Aboura et al. 2008; Cesare et al. 1992) have introduced schemes to rate the state of a bridge based on the subjective analysis of practitioners. The rating scheme used in this study is based on the rating scheme proposed by HAZUS – MH2.1. It expresses the condition of a given asset using one of the five pre-defined states. In this system, shown in Table 1, the first state corresponds to a new and intact asset while the fifth state corresponds to a severely deteriorated or damaged asset.

Degradation, retrofit, hazards, and recovery actions can change the state of an asset over time. Eq. (1) provides the abstract form of the state of an after one decision interval

$$S_{t+1} = S_t + \Delta S_{t+1,d} + \Delta S_{t+1,rt} + \Delta S_{t+1,rc} + \Delta S_{t+1,h} \quad (1)$$

Where S_{t+1} and S_t are the states at time t and $t+1$, $\Delta S_{t+1,j}$ is the expected change in the state of the asset due to phenomenon j (d: degradation, rt: retrofit, rc: recovery, and h: hazard response) in the time interval between t and $t+1$. Degradation, retrofit model, response, and recovery model are described in the following.

Degradation alters the state of an asset over time. The future state of an asset subject to degradation is uncertain since one cannot accurately predict the quality and extent of changes caused by degradation. Golabi *et al.* (1982) first proposed using Markov chain models to characterize the changes in the state of a given asset due to degradation. Numerous studies (e.g., Cesare et al. 1992; Morcous et al. 2002) have applied Markov chain models to characterize the degradation process of assets. Consistent with the past studies, this study uses a first-order Markov Chain to characterize the probable state of an asset that is subject to degradation. A first-order Markov chain model is a stochastic process with a finite number of states in which S_{t+1}(i.e., the state at $t+1$) solely depends on S_t (Ross 2010). Also, it is assumed that there is a fixed probability of transition from one state to another, $\Pr(S_{t+1}|S_t)$.

To determine the post-earthquake state of an asset, there is a need for appropriate models that characterize the probable future hazards in terms of their occurrence time and intensity, their impact on the infrastructure assets, and the consequent reduction in the functionality of assets. Dong *et al.* (2014) adopted a simple frequency-based approach to characterize the probable future hazard scenarios. Talebiyan and Mahsuli (2018) generated earthquake samples based on several parameters such as the fault characteristics and the distance between the location of structures and the epicenter of the earthquake. This process is called scenario sampling, a particular implementation of Monte Carlo sampling, in which magnitude and occurrence time of earthquakes are sampled in a timespan (Mahsuli et al. 2019; Rahimi and Mahsuli 2019).

The proposed framework employs the well-known Poisson point process to model the frequency or temporal distribution of the earthquakes that a seismic source can generate. A Poisson process is a continuous stochastic process in which the number of occurrences in time follows a Poisson distribution. Several past studies have used this model (see, e.g., Ashtari Jafari 2010; Wang et al. 2012; Weichert 1980) to simulate the occurrence of rare events (e.g., earthquakes).

Given the mean occurrence rate for the earthquakes from a specific source, the probability of exactly *n* events from the source during a time interval of duration *t* is:

$$P_N(n|\lambda) = \frac{e^{-\lambda t}(\lambda.t)^n}{n!}, t > 0; n\ integer \geq 0 \tag{2}$$

where λ is the occurrence rate (i.e., average numbers of events per unit of time). It can be shown that the time between two consecutive events follows an Exponential distribution (i.e., $f(t) = \lambda.e^{-\lambda t}$). The inputs needed for the occurrence model are estimated as follows. Assuming that only ground motions with M > m are of interest and that occurrence of earthquakes with magnitudes greater than m follow a Poisson process, the mean occurrence rate, λ is estimated from data as:

$$\lambda = \frac{n_{h|M>m}}{t_2 - t_1} \tag{3}$$

Where $n_{h|M>m}$ is the number of earthquakes with $M > m$ in (t_1, t_2). Alternatively, λ can be determined subjectively. Random event occurrence times can be generated by generating the realizations of t (i.e., the time between two consecutive events) using parameter λ. Note that since *t* is a continuous variable, earthquake events can occur at any time during the study period.

The generated realizations of t are then discretized by assigning the events that occur at any time within the T_i and T_{i+1} interval to time T_i .

Existing risk-analysis approaches often take a "hazard curve" as a starting point. In the context of seismic risk, a hazard curve displays the probability of exceeding values of a site-specific ground shaking intensity, such as S_a. The underlying models of hazard curves characterize earthquake magnitude and location and the propagation of rupture energy. Hazard location models aim at predicting the earthquake location. In this study, the location model proposed by Mahsuli and Haukass (2013) is used. This location model takes random variables as input and generate earthquake location as output. Each realization of the random variables is associated with one earthquake location. Once the coordinates of this location are known, the distance to the site of the asset can be computed.

A truncated exponential distribution is often used to characterize the magnitude frequency statistics of earthquakes. The lower bound of magnitude distribution, m_{min}, is chosen to reflect the minimum magnitude that can cause damage and loss and, hence, must be considered in the risk mitigation analysis. The magnitude distribution is truncated at an upper-bound value m_{max} to take into account the saturation of the magnitude scale and the fact that a given zone cannot generate magnitudes above m_{max}. Therefore, the probability density function is truncated and normalized (McGuire 2004).

An earthquake intensity model primarily uses the characteristics of the earthquake and the path of shock wave propagation to predict the site-specific ground shaking parameters. A variety of models and intensity measures exist in the literature. To generate the data, first, several realizations of earthquake magnitudes and distances are randomly generated. For each magnitude and distance pair, the ground shaking parameter (e.g., S_a) at the location of each asset is computed. To this end, several models, including those proposed by Atkinson and Boore (2003) and Boore and Atkinson (2008), can be used. For each asset, the ground shaking parameters are the inputs to the capacity spectrum method (Freeman 1998). The peak displacement and acceleration responses from the capacity spectrum method are used as inputs to the HAZUS fragility and loss functions (FEMA-NIBS: Federal Emergency Management Agency (FEMA) by the National Institute of Building Sciences 2003) to determine the damage sustained by the asset. HAZUS – MH2.1 (FEMA-NIBS: Federal Emergency Management Agency (FEMA) by the National Institute of Building Sciences 2003) proposes a set of procedures to characterize the response of the infrastructure assets to an earthquake. These procedures involve applying fragility curves, which are a group of cumulative distribution functions that return the probability of exceedance of a given damage state as a function of a ground motion intensity measure (e.g., peak ground acceleration or spectral acceleration). The probability of exceedance of each damage state given an earthquake intensity is calculated using Eq. (4)

$$P_{S \geq s_i|IM} = \Phi\left\{\frac{1}{\beta_{\mathrm{S_i}}}\ln\left(\frac{IM}{m_{S_i}}\right)\right\} \tag{4}$$

where $P_{S \geq s_i|IM}$ is the probability of exceedance of the state of the asset from the damage state s_i, Φ is the standard normal cumulative distribution function, IM is the ground motion intensity measure, m_{S_i} is the state-dependent median value of ground motion intensity with damage state i, $\beta_i(s_t)$ is the dispersion factor (i.e., the state-dependent standard deviation of ground motion intensity) of damage state i. The abovementioned models are implemented in Rtx software (Mahsuli and Haukaas 2013), which was is used in this study.

This approach omits the uncertain rate at which the seismic performance of the asset declines over time. As time passes, the state of an asset changes. The change in the asset state changes the probability of exceedance from a given damage state in response to the probable future hazards. Therefore, there is a need for state-dependent fragility curves (i.e., fragility curves that can determine the response of the asset to probable future hazards considering their states after degradation). Dong *et al.* (2014) used time-variant fragility curves presented by Ghosh and Padgett (2009) to consider seismic performance deterioration due to corrosion and find the conditional probability of damage states in response to a specific ground motion intensity. An approach inspired by the HAZUS methodology was used to find the response of a degraded asset, rather an intact one, in the face of hazards. In this approach, when an asset is in the S_j state ($j \neq 1$), the probabilities of exceedance from S_i $\left(S_i > S_j\right)$ will be normalized so the sum of all probabilities remain 1. Therefore, the probability of exceedance of the state of a degraded asset from the damage state s_i is determined using Eq. (5)

$$P_{S \geq S_i | IM, S_j} = \Phi\left\{\frac{1}{\beta_{S_i}} \ln\left(\frac{IM}{m_{S_i}}\right)\right\} / \Phi\left\{\frac{1}{\beta_{S_i}} \ln\left(\frac{IM}{m_{S_j}}\right)\right\}, \; S_i \geq S_j \quad (5)$$

where S_j is the state of the asset before the occurrence of an earthquake. Fig. 2 demonstrates examples of fragility curves used in this study.

Similar to other state models, change in the state of the asset in response to the earthquakes is calculated using Eq. (6)

$$\Delta S_{t+1,h} = f\left(S_t, \left(m_i(s), \beta_i(s)\right), IM\right) \quad (6)$$

where S_t is the state of the asset at step t, $\left(\mathrm{m_i(s)}, \beta_\mathrm{i}(\mathrm{s})\right)$ is the set of state-dependent median and dispersion ground motion intensities, IM is the ground motion intensity.

The impact of a set of retrofit actions on the state of an asset is characterized using the retrofit model. It is assumed that the retrofit directly improves the state of a system. Two types of retrofit are considered in this study: 1) compulsory retrofit, 2) planned retrofit. Compulsory retrofits are implemented when the state of an asset degrades to the level that renders its functionality unacceptable. Planned retrofits are implemented at a pre-set time in the future to maintain the functionality of the system above a minimum desired level and reduce its expected life cycle costs. Eq. (7) demonstrates the general form of the model that describes the changes in the state of an asset following the implementation of a retrofit action

$$\Delta S_{t+1,rt} = f(S_t) \tag{7}$$

where $\Delta S_{t+1,rt}$ is the expected change in the state of the asset due to retrofit in the time interval between t and $t+1$, and S_t is the state of the asset at step t.

Recovery comprises a set of actions needed to restore an asset to the acceptable service level in the aftermath of a hazard such as an earthquake. In the proposed framework, the extent of the recovery actions is determined by the post-earthquake state of the asset and the recovery lag. The post-earthquake state of the asset describes the state of the asset immediately after an earthquake. Asset managers can start recovery operations immediately after the occurrence of hazards. Alternatively, they can delay the initiation of recovery efforts. The recovery lag, Γ characterize this delay. Several factors, such as the criticality of the asset, the availability of funds, and logistical issues, can determine the recovery lag of a given asset. Eq. (8) demonstrates the general form of the model that describes the changes in the state of an asset following the implementation of a recovery action

$$\Delta S_{t+1,rc} = f(S_t, \Gamma) \tag{8}$$

where $\Delta S_{t+1,rc}$ is the expected change in the state of the asset due to the recovery efforts in the time interval between t and $t+1$, S_t is the state of the asset at step t, and Γ is the anticipated time between the occurrence of hazard and the commencement of recovery operations.

Asset replacement cost model

Consistent with the commonly used approach prescribed by HAZUS – MH2.1 (FEMA-NIBS: Federal Emergency Management Agency (FEMA) by the National Institute of Building Sciences 2003), the proposed model characterizes the earthquake-induced losses incurred by the community as a function of the replacement value of an asset and its post-earthquake state. The asset replacement cost is the sum of all costs to replace an asset with an identical asset. The future replacement costs are subject to uncertainty. This uncertainty about the future asset replacement costs stems from various factors such as the changes in the dynamics of supply and demand of goods and services in the market or the general inflation. To characterize the uncertain movements of costs over time caused by various economic factors such as uncertain market supply and demand, statistical models such as the Wiener process and its variations can be used. The Weiner process has been widely used in finance and economics to model asset price and value movements over time (See, e.g., (Brennan and Schwartz 1976; Capasso et al. 2020; George and George 2018; Hirsa and Neftci 2013; Kim et al. 2017; Kim and Lee 2018; Pindyck 1993; Ross 2010). Past studies (see, e.g., Ilbeigi et al. 2014 ; Ashuri et al. 2012) have used the Weiner process to model the movement of asset replacement, construction, or retrofit costs. The Weiner process is a stochastic process that can effectively model continuously changing market phenomena (e.g., price movements) dominated by ordinary events in contrast to extreme events that may occur very infrequently (Hirsa and Neftci 2013). The Wiener process is the natural choice for modeling the asset replacement costs since, due to market forces, they continuously move over time and have unpredictable increments. Accordingly, in this study, it is assumed that the temporal variations of the asset replacement cost follow a Wiener process. Wiener process with drift can be defined by two parameters: 1) drift rate, η, that determines the trend, 2) standard deviation, σ, that represents the volatility in the replacement costs of the asset. Eq. (9) is used to characterize the asset replacement costs over time:

$$v(t) = v_0 + \eta t + \sigma W_t \tag{9}$$

where $v(t)$ is the replacement costs at time t, η is the drift ratio of the Wiener process, σ is the standard deviation of the predicted cost, W_t is the Wiener process, and v_0 is the initial replacement cost at $t = 0$. It should be noted that the Wiener process is a Markovian process, which means that past events do not affect future events (Ross 2010). Also, deviations from the trend are statistically independent. They follow a normal distribution with a zero mean and a standard deviation that equals volatility in square root of time. The drift rate and standard deviation used in Eq. (9) can be calibrated by historical indices such as construction cost index (CCI), building and construction index, and cost catalogs.

Life cycle cost module

Life cycle costing (LCC) involves analyzing the cash flows of an investment over a study period by including all the direct and indirect costs (Ellingham and Fawcett 2007). The net present value of the mentioned cash flow is the whole-life cost (WLC) of the asset. In the proposed model, the LCC module calculated the whole-life cost of various retrofit plans by considering the costs of owning and operating an asset during its life cycle under various probable scenarios. The retrofit plan with the minimum expected lifecycle cost is the optimal plan, which gives a proposed timing for retrofit of the asset.

Fig. 3 demonstrates a sample scenario of the event that an asset may experience over its life. In this sample, the asset degrades, experiences several earthquakes, and undergoes retrofit and recovery actions during its operational life. Starting in an intact state, S_1, the asset degrades gradually over time and reaches a moderate state, S_3, in approximately 20 years. The response of the asset to the earthquakes that occur at this point leads to no damage or loss. Next, the asset undergoes a planned retrofit and returns to the state S_1. In its 32nd year of service, a relatively strong earthquake occurs. Due to extensive earthquake-induced damage, the state of the asset changes to S_4. Recovery actions take place immediately after the earthquake and restore the state of the asset to the pre-determined acceptable level of performance, S_1.

The LCC module calculates the total discounted incurred costs to the community based on the state and the mode of the asset each year. In this process, a step-by-step procedure is followed. This procedure determines the costs associated with maintaining the asset and, consequently, its lifecycle costs. To account for the uncertain nature of the events that the asset experiences and the uncertainty associated with the costs of retrofitting or repairing assets, a Monte Carlo sampling is conducted. Note that, in Monte Carlo sampling, all the events within the investment horizon of T occur at discrete time intervals. To further clarify this procedure, the algorithm of this analysis is provided in Table A1 in Appendix A. The LCC model comprises of sub-models to incorporate various costs related to an asset. Five major types of costs are assumed considered to be effective in the life cycle of the asset. These costs, without loss of generality, are simplified to be employed, simulated, and analyzed.

Cost sub-models

The costs associated with operating an asset are classified into the following categories: opportunity costs, user costs, maintenance costs, retrofit costs, losses, and recovery costs. In the context of this study, opportunity costs equal to the benefits that the money used for retrofit could have earned by being invested elsewhere. To estimate the opportunity costs, in this manuscript, it is assumed that instead of investing in seismic retrofits, managers can invest in alternatives, the most highly valued of which periodically earn benefits expressed as a percentage of the invested capital and denoted by r_a. Therefore, the opportunity costs over the investment horizon can be expressed using:

$$C_m = C_R \times \left[\left(\frac{1 + r_a}{1 + r}\right)^n - 1\right] \tag{10}$$

where C_m is the opportunity cost over the investment horizon, C_R is the retrofit cost, n is the remaining years in the investment horizon, r is the discount rate, and r_a is the benefits that could have been acquired by investing in the most highly valued alternative and expressed as a percentage of invested capital per period.

User costs are incurred by society when the functionality of an asset is disrupted (e.g., when the loss of functionality of the transportation system increases the travel time of the commuters). In this study, losses incurred by the community are classified as the social losses (e.g., casualties, reduction of life-quality, and mental injuries) as well as the direct and indirect economic losses (e.g., loss of capital and business interruptions) (Bradley et al. 2008; Sudaryo et al. 2012).

In the proposed framework, cost sub-models are used to calculate the expected whole life cost of an asset given various policies (i.e., retrofit plans) and under various uncertainties such as the occurrence time of an earthquake and the market volatilities. Consistent with the approach prescribed by HAZUS – MH2.1 (FEMA-NIBS: Federal Emergency Management Agency (FEMA) by the National Institute of Building Sciences 2003), user, maintenance, retrofit, and recovery costs and the losses sustained by the community, are expressed as a function of replacement costs and state of the asset understudy.

Cost aggregation model

Monte Carlo sampling is used to simulate various events (e.g., deterioration, retrofit actions, hazards, and recovery actions) that can occur throughout a given investment horizon and calculate the associated costs incurred by the community. Due to the high number of uncertain variables that affect the costs carried by the community over the life-cycle of assets, the number of simulations (i.e., the sample size of the uncertain variables) should be carefully calculated since it affects confidence in the results. Each sample represents a realization of the probable events that can occur throughout the lifecycle of the asset under study. The asset model returns the replacement costs of the asset (i.e., all costs for building a duplicate of the asset) and its states throughout a life cycle. Total costs at each step depend on the mode of the asset. Four modes are defined to describe the condition of the asset at each step: damaged, undergoing recovery, undergoing retrofit, and operational. An asset can experience each mode multiple times during its life. Fig. 3 demonstrates various modes of an asset during its life cycle.

For each sample, Eq. (11) is used to calculate the whole life cost of the asset. The whole life cost of the asset is the sum of the discounted relevant costs over its life as determined by the mode of the asset.

$$C^j = \sum_{i}^{T} [C_u(t) + C_{rt}(t) + C_{rc}(t) + C_m(t) + C_l(t)]/(1+r)^i \quad (11)$$

where C^j is the WLC of the asset in the j^{th} simulation, r is the discount rate, C_u is the user costs, C_{rt} is the retrofit costs, C_{rc} is the recovery costs, C_m is the maintenance costs, and C_l characterizes the losses.

Investment valuation module

In this research, the flexibility to implement or delay retrofit actions is characterized as a real option. The flexibility can enhance the value of an investment. Fig. 4 illustrates the benefit of incorporating options in retrofit investment. It depicts the histogram of the whole life cost of an asset over the remainder of its life. The current state of the asset is S_2 and the owner can only choose between retrofitting the asset now or never.

In this scenario, not only does retrofit at the current time reduce the expected WLC of the asset from \$7.11M to \$6.34M, but it also decreases the standard deviation of the costs, which is a measure of the investment risk. Nevertheless, the immediate implementation of the retrofit action means forfeiting all the possible benefits that the financial resources used for retrofit can generate if invested in other ventures. Real options analysis provides the investors with the opportunity to explore the value that can be potentially gained by delaying the investment. Yao and Jaafari (2003) studied the applicability of binomial or trinomial lattice and dynamic programming to real options analysis. In complex projects with higher degrees of uncertainty, decision-makers should assess all the possible scenarios. To this end, scholars (e.g., Cheah and Liu 2006; Yao and Jaafari 2003) have used Monte Carlo sampling and decision trees in ROA.

In this study, the multinomial lattice model (Fig. 5) is used in the process of analyzing the value of a retrofit option and identifying the best time to exercise it given the possible changes in the state of the asset and the probable costs associated with its operation, maintenance, retrofits, and post-hazard restoration.

At each step and for each possible state of the asset, the lifecycle costs associated with the immediate retrofit of the asset, which is referred to as scenario (Ψ_1), is compared against two probable outcomes of deferring the retrofit: 1) a hazard occurs during the current period after which, depending on the intensity of the hazard and the extent of the damage to its components, it may enter the recovery or operational (scenario (Ψ_2)); 2) no hazard occurs during the current period, which allows asset managers to take action in the future (scenario (Ψ_3)). The decision tree shown in Fig. 6 characterizes these scenarios. The cost associated with a decision at each node of the tree is driven by Eq. (12)

$$EC^t(D_s) = Min\left\{ \begin{array}{l} EC^t(\Psi_1), P_e \times EC^t(\Psi_2) + (1 - P_e) \times [E^tC\ (\Psi_3) \\ + \sum_{j=0}^{n-i+1} P(S_i, S_{i+j}) \times EC^{t+1}(D_{S_{i+j}})] \end{array} \right\} \tag{12}$$

where $EC^t(\Psi_k)$ is the expected cost of scenario k, $k = 1,2,3$, at t, P_e is the probability of occurrence of hazards in the next year from t, n is the number of states, S_i is the state at the intended node, $P(S_i, S_{i+j})$ is the transition probability from state S_i to S_{i+j}, $EC(D_{S_{i+j}})$ is the expected cost of the decision at state S_{i+j} at $t + 1$. For each given year over the investment horizon, the exercise boundary identifies the state beyond which the value of the immediate implementation of the retrofit intervention exceeds the value of its deferment and triggers the implementation of the retrofit intervention. Therefore, whenever the asset state passes the threshold determined by the exercise boundary, the immediate implementation of the retrofit becomes preferred to its deferment since it will be associated with a lower overall life-cycle cost.

Illustrative example

The proposed framework is applied to the Chamran bridge (Table 2) in the INSURER community. INSURER is a virtual community that serves as a testbed for the algorithms developed at the Center for Infrastructure Sustainability and Resilience Research (INSURER) at the Sharif University of Technology. The building stock comprises 17418 buildings of various occupancies such as single and multifamily residential, retail, finance, healthcare, educational, light and heavy manufacturing, and public services. Infrastructure systems that serve this community are transportation, power, water, gas, and communications networks. The transportation infrastructure of the community consists of 20 km of urban roads and nine bridges. Chamran bridge, which imitates a real bridge, is used as an example to showcase the application of the proposed framework. Table 2 presents the characteristics of this bridge.

The INSURER community is exposed to moderate to severe seismic hazards created by a fault with seismic characteristics similar to those of the Kahrizak fault in Tehran, Iran. Table 3 summarizes the state-dependent parameters used in this framework.

The number of iterations in the Monte Carlo simulation affects confidence in the results and should be carefully estimated. Based on the central limit theorem, the average of a large number of estimations (e.g., the calculated LCC in each scenario in this study) after n iterations follow a normal distribution characterized as follows:

$$N\sim(\mu_{\bar{x}}, \frac{\sigma}{\sqrt{n}}) \tag{13}$$

In this case, the confidence interval of estimation of the mean is $\pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ (Law and Kelton 2000). To calculate the minimum required number of iterations, R, 100 rounds of sampling is conducted to find the initial estimates of average, μ_0, and standard deviation, σ_0. One percent of the initial estimation of μ is considered to be the maximum allowable confidence interval, ϵ. Therefore, minimum number of iterations is calculated as follows

$$R \geq \left(\frac{Z_{\frac{\alpha}{2}} \times \sigma_0}{0.01 * \mu_0}\right) \tag{14}$$

The proposed framework is applied to evaluate and compare the three policies. In addition to retrofitting now or never, three alternative policies are assessed. According to policy **A**, the asset is retrofitted when it reaches a certain service level. Based on policy **B**, the asset is retrofitted at a pre-set time. According to policy **C**, the asset undergoes retrofit actions when its state reaches the threshold determined by the exercise boundary characterized by the real options analysis. The policies mentioned above are used in several rounds of simulation to identify the optimal policy. The following section presents the simulation results.

Results

Fig. 7 depicts the WLC of the asset as a function of the year at which it is retrofitted, assuming that policy B, which requires a retrofit at a pre-set date regardless of the asset state, is followed. As shown by Fig. 7, retrofitting the bridge six years from the time of this analysis (i.e., 2020) yields the minimum WLC (i.e., $4.51 Million) in the remaining life cycle.

Next, using the proposed real options framework, an exercise boundary is derived. Fig. 8 shows the state of the asset beyond which the value of implementing the retrofit action exceeds the value of deferring its implementation to the following period.

Fig. 9 demonstrates the likelihood distribution of the event that the retrofit action is implemented in any year over the investment horizon.

Fig. 10 demonstrates the histogram of the costs associated with various strategies and policies. The results are also summarized in Table 4. As demonstrated, using the exercise boundary as a guide for making the retrofit decisions yields the lowest WLC.

Fig. 10 shows that abandoning the retrofit yields the highest WLC in comparison to other policies. This can be attributed to the losses incurred by the community due to the damage the seismically vulnerable asset can sustain in the aftermath of a future earthquake. If the asset is retrofitted when it reaches the pre-determined minimum acceptable state (**Policy A**), the WLC will be reduced by 16.4% compared to the immediate implementation of the retrofit (i.e., in the year 2020). The histogram of **Policy B**, which involves plans the retrofit actions based on the optimization of the LCC, is similar to that of **Policy C**. Nevertheless, the mean WLC of the latter is 8.8% lower than that of the former. It is noteworthy that the relatively high standard deviation of the WLC of implementing each of the policies mentioned above is due to the significantly high costs associated with a rare event (i.e., the earthquake).

Fig. 11 demonstrated the WLC of the asset over the study period under various policies with varying degradation and occurrence rates. Fig. 11(a) demonstrates that a higher degradation rate leads to a higher WLC since the state of an asset is a determinant of its ownership and operation costs. Similarly, Fig. 11(b) shows that higher frequencies of the earthquake occurrence increase the WLC except for the case in which the asset manager abandons seismic retrofits altogether. The expected costs of Policy C, real options-based retrofit, is of the highest importance to this discussion. In all cases, the optimal timing of retrofit actions as determined by the real options analysis yields the lowest expected WLC. The reduction of costs by Policy C compared to Policy B was up to 14% in some cases.

Conclusions

Due to budget limitations and the underlying uncertainties about various aspects of the state and performance of infrastructure systems, asset managers planning to invest in seismic retrofits must use proper investment decision-making methods to ensure that funds are appropriately allocated, and vulnerability reduction goals are realized. These investment valuation methods should be capable of generating proper retrofit plans for seismically vulnerable infrastructure systems. This study was motivated by the need to address a gap in the existing body of knowledge stemming from the need for a valuation method that enables investors to determine whether they should delay an improvement and when it becomes financially sound to adopt a seismic retrofit action. The theoretically well-founded and practically useful framework presented in this study is tailored to investment decision making for seismic retrofits. The proposed investment valuation model uses real options analysis to determine the optimal time to invest in retrofit interventions while considering the degradation rate of the assets, frequency, magnitude, and consequences of hazards, as well as the fluctuations in the recovery and retrofit costs over time. It determines the optimal time to invest in retrofit actions by conducting a trade-off between the total costs of immediate implementation and its deferment. It provides an exercise boundary beyond which the retrofit of the asset is beneficial.

Chamran bridge in INSURER virtual community was chosen to showcase the capabilities of the proposed framework. The results of this analysis showed that implementing retrofit plans based on outcomes of ROA (**Policy C**) reduces the WLC of the asset in comparison to fixed strategies (**Policy A**) and optimized retrofit plans (**Policy B)**. Employing **Policy C** instead of **Policy B** (conventional retrofit plans by conducting optimization) can reduce the WLC of the asset by 8.8% in the remaining operational years. The results of this study showed that the benefits of ROA are not limited to reducing overall costs. As shown by Fig. 9, there is a 23% likelihood that using the exercise boundary as the basis for deciding about the retrofit implementation leads to no action over the lifecycle of the asset. In comparison with rigid plans, ROA can reduce the misappropriation of available resources. A sensitivity analysis conducted by perturbing the degradation rate of the asset and the occurrence rate of the earthquake showed that regardless of the extent of variations of the parameters mentioned above, implementation of retrofit interactions based on the plans devised by real options analysis yields the lowest WLC.

The proposed framework can be enhanced in a variety of ways. Future studies can focus on developing models that appropriately characterize the degradation process of various assets.

There is a need for appropriate models that determine the post-earthquake functionality of the asset based on the extent of the damage that it may sustain in the face of the probable future earthquake. The proposed framework can be extended to include models that properly quantify the economic, social, environmental, and socioeconomic costs of hazards. It can also be extended to determine the optimal timing of vulnerability reduction interventions for multiple assets in a network (e.g., multiple bridges in an urban transportation network) and interdependent assets (e.g., components of the electricity network that support the water distribution network). Lastly, the proposed framework can be applied to determine the optimal timing of vulnerability reduction interventions of other assets such as buildings and components of lifelines (e.g., water, electricity, and gas networks).

Appendix A

Table A1. Pseudo-code for life cycle cost analysis in the proposed framework

Input: Asset's characteristics	
Input: Environmental and hazards' characteristics	
$Z = 0$	// Z: Holder for simulations' results (total costs)
for $j \in \{1, 2, \ldots, N\}$ **do**:	// N: Number of simulations
$Y = \{\}$	// Y: Holder for each simulation's result
RT = A generated retrofit scenario	
H = A generated hazard sample	
for t $\in \{0, 1, \ldots, T\}$ **do**:	// T: Investment horizon
if t **in** H:	
find S', M as a response to hazard	// S': State after hazard, M: Mode
RC = Generate recovery scenario	
if t **in** RC:	
find S', M as a result of recovery	// S': State after recovery
if t **in** RT:	
find S', M as a result of retrofit	// S': State after retrofit
else:	// The asset undergoes degradation
find S', M as a result of degradation	// S': State after degradation
$Y[t] = \{S', M\}$	// Storing each year's results
$S = S'$	// Assigning S' to state of next year
end for	
V = Estimate replacement costs in years	
C^j = calculate costs given Y and V	// Calculate all costs given simulations results and replacement costs estimation
$Z = (Z \times (j-1) + C^j)/j$	// Updating the average of total costs

```
end for
return Z
```

Data Availability Statement

All data that support the findings of this study are available from the corresponding author upon reasonable request.

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