

Truck routing and platooning problem considering time-varying traffic conditions

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ABSTRACT

Truck platooning enabled by connected and automated vehicle (CAV) has shown great potential to reduce fuel consumption and save road space. However, the realization of these favorable benefits requires precise synchronization of the routes and schedules for the truck fleet subject to constantly changing traffic conditions caused by various factors, e.g., rush hour. Therefore, this study investigates the truck routing and platooning problem in consideration of the time-varying travel speeds. The problem is to seek time-compatible routes and schedules for the truck fleet while minimizing the fuel consumption via the formation of platoons over the entire trip without violating the service time window. We develop a mixed-integer linear programming (MILP) model for the proposed problem, which can be solved by state-of-the-art solvers like Gurobi. Numerical experiments are conducted to verify the efficacy of the proposed model and demonstrate the necessity to consider time-varying traffic conditions in truck routing and platooning.

INTRODUCTION

Truck platooning, enabled by CAV, allows a string of virtually connected trucks following each other in small proximity (see Figure 1 for an example), which could deliver various benefits including enhanced traffic safety, increased roadway capacity, and notably, reduced fuel consumption and related emissions via lowered air drag friction over the trucks. However, such favorable platoons cannot be established without precise synchronization of trucks' routes and schedules. In fact, the travel time of trucks required to traverse a road segment may change from one period to another of a day caused by various factors such as rush hour, imposing a significant burden to route and schedule the truck fleet to platoon. Particularly for the freight transport services operating in crowded large cities or on major highways, considering the time-varying traffic

conditions in the decision-making of routing and scheduling for the truck fleet to reap the fuel-saving benefits of platooning is one of the most pressing challenges faced by the relevant system operators.



Figure 1. An illustration of a three-truck platoon (Larson et al., 2016).

Literature review. Attracted by the considerable benefits, a growing number of studies have been devoted to truck platooning from various aspects, especially its technical issues, such as the automation driving technology and control policies, the design and safe operation of platoons by specifying vehicles' speeds and intervehicle gaps, as well as the evaluation of platoons' performance through experimental studies (Abou Harfouch et al., 2017; Alam et al., 2015a; Alam et al., 2015b; Bonnet and Fritz, 2000; Goli and Eskandarian, 2020; Lee et al., 2021; Liu et al., 2019; Shladover, 2019). Though the research scopes and methodologies of these studies may differ, they endeavor to achieve the same objective that forming and maintaining safe, stable and profitable platoons as much as possible from the technical perspective. In the real-life application, however, the establishment and the performance of truck platooning not only is influenced by the technical capabilities but as well depends on the routing and scheduling of platooning in the transportation network. Unfortunately, the research reviewed by Bhoopalam et al. (2018) revealed that the high-level decision-making in truck platooning to maximize the benefits of platooning has yet received underestimated attention. Among these limited studies, Larson et al. (2013) firstly introduced local controllers placed at road intersections to maximize platooning opportunities by adjusting the speeds of trucks approaching the intersections. Later, some studies further considered the adjustments to the routes that trucks take to facilitate platooning (Larsson et al., 2015; Liang et al., 2013; Liang et al., 2014). For example, Larsson et al. (2015) were the first to formulate the platooning problem as a mixed-integer programming model, which aims to route the truck fleet to their respective destinations using the least fuel over the entire trip and proved its NP-hardness. However, they merely modelled and solved a special case of platooning problem where deadlines of delivery tasks are discarded. After that, the model in Larsson et al. (2015) were used as a benchmark model to be extended from several aspects (Boysen et al., 2018; Larson et al., 2016;

Luo et al., 2018; Nourmohammadzadeh et al., 2016; Sokolov et al., 2017; Stehbeck, 2019; Sun and Yin, 2019; Van De Hoef et al., 2017). For example, Larson et al. (2016) extended the model by incorporating implied constraints deriving from the inherent properties of the platooning problem to find the optimal solutions without resorting to heuristics. Boysen et al. (2018) examined the impact of some practical factors on the fuel-saving performance of platooning using a basic case where all trucks share the same itinerary.

Most aforementioned investigations into the truck platooning problem have been undertaken using the assumption that the travel speeds are constant. However, this assumption may be far from reality, particularly for urban areas or major highways, where the travel speeds may typically vary with the time of day due to the rush hours. Therefore, the resulting plans made without considering the time-varying traffic conditions may lack reliability and applicability in real-life scenarios. Despite wide recognition of its importance, the time-varying travel times have received little attention in the decision-making of routing and platooning so far.

Objectives and contributions. To bridge the above research gaps, this paper investigates the truck routing and platooning problem considering time-varying traffic conditions (referred to as TRPTC problem hereafter). We assume that trucks are allowed to detour and wait at some intersections for platooning opportunities to conserve fuel. The following trucks in a platoon will experience fuel reduction, whereas the leading truck as well as trucks traveling alone experience no fuel savings. We also assume that the planning horizon is partitioned into a set of periods and the travel speed on each arc varies from one period to another with predictable patterns. The objective of this study is to minimize the total fuel costs for the truck fleet over the entire trip by determining the time-compatible routes, schedules, and platooning plans under the time-varying travel speeds without violating the service time windows. To achieve this objective, we formulate a novel mixed-integer linear programming (MILP) model. Numerical experiments are conducted to demonstrate the efficacy of the proposed model and validate the necessity to consider time-varying traffic conditions in truck routing and platooning.

The rest of the paper is organized as follows: assumptions, notations, and problem description are elaborated in Section 2, followed by the formulation of a MILP model for the TRPTC problem in Section 3. Numerical experiments are conducted to demonstrate the efficacy of the proposed model in Section 4. Lastly, Section 5 concludes the paper and presents possible future research.

ASSUMPTIONS, NOTATIONS, AND PROBLEM STATEMENT

We define the TRPTC problem over a highway network represented by a directed and connected graph $G=(\mathbf{V},\mathbf{E})$, where \mathbf{V} is the set of nodes in the graph and $\mathbf{E}\subset \mathbf{V}\times \mathbf{V}$ is the set of edges. Each edge $(i,j)\in \mathbf{E}$ is associated with a non-negative length w_{ij} . Let \mathbf{N} be the set of involved trucks to be routed. Each truck $n\in \mathbf{N}$ is assigned a transport mission, consisting of an origin node o_n , a

destination node d_n , and a service time window, i.e., the earliest departure time σ_n and latest arrival time τ_n . The planning horizon is partitioned into the same set of periods $\mathbf{K} = \{1, 2, \dots, k, \dots, |\mathbf{K}|\}$ for each edge in this study. For a period $k \in \mathbf{K}$, b_k and e_k represent the beginning and ending time, respectively. We denote the travel speed on edge (i, j) during period k as v_{ijk} . The travel time of truck n on edge (i, j) in period k is denoted by δ_{ij}^{nk} and the total required travel time of this arc depends on the actual periods that they are traveled. Besides, the incurred fuel cost per kilometer for a truck to traverse edge (i, j) is denoted by c_{ij} . The following trucks in platoons can save a fraction η of the normal fuel used by trucks traveling alone. To exactly formulate the proposed problem, we will elaborate on the truck platooning and the time-varying travel of edges in the following subsections.

Truck platooning. We consider that trucks can save fuel when they form a platoon, i.e., when they start traversing a shared edge in their paths at the same time. Trucks are allowed to take detours or wait at nodes for some time for platooning opportunities as long as the service time window are respected. For a truck $n \in \mathbf{N}$, its waiting time at a node $i \in \mathbf{V}$ is denoted by u_{in} . To determine the travel routes and time schedules for the truck fleet to platoon, we introduce a binary route variable $x_{ijn}, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N}$ and a continuous time variable $t_{ijn}, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N}$ to indicate whether truck n traverses edge (i, j) during its trip and the time when truck n starts traversing edge (i, j) , respectively. To specify the platooning plans, we need define a binary platoon variable p_{ij}^{nm} to indicate whether truck n travels adjacently behind truck m over edge (i, j) . To form a platoon, several trucks should start traversing the same edge (i, j) from node i at the same time, i.e.,

$$-M(1 - P_{ij}^{mm}) \leq t_{ijn} - t_{ijm} \leq M(1 - P_{ij}^{mm}), \forall (i, j) \in \mathbf{E}, n, m \in \mathbf{N}, n \geq m \quad (1)$$

$$2P_{ij}^{mm} \leq x_{ijn} + x_{ijm}, \forall (i, j) \in \mathbf{E}, n, m \in \mathbf{N}, n \geq m \quad (2)$$

Eq. (1) enforces that if trucks n and m platoon on edge (i, j) , they must start traversing edge (i, j) at the same time. Eq. (2) is the flow requirement for trucks n and m if they platoon on edge (i, j) . In addition, to specify the relative position relationship between trucks n and m if they are platooned, we should have the following constraints:

$$p_{ij}^{nm} + p_{ij}^{mn} \leq 1, \quad \forall (i, j) \in \mathbf{E}, n \neq m \in \mathbf{N} \quad (3)$$

$$\sum_{m \in \mathbf{N}} P_{ij}^{nm} \leq 1, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N} \quad (4)$$

$$\sum_{n \in \mathbf{N}} P_{ij}^{nm} \leq 1, \forall (i, j) \in \mathbf{E}, m \in \mathbf{N} \quad (5)$$

Eq. (3) guarantees that truck n travels either behind or ahead of truck m . Eq. (4) and Eq. (5) stipulate that a truck follow at most one vehicle and is followed by at most one truck on an edge, respectively.

Time-varying travel of edges. Under time-varying traffic conditions, the required travel time for truck n to traverse an edge (i, j) depends on when this truck departs from node i and the periods it actually experiences on (i, j) . To determine the schedules for the truck fleet, in addition to the binary decision variable $y_{ij}^{nk}, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N}, k \in \mathbf{K}$, indicating whether truck n will traverse edge (i, j) in time interval k , we need introduce a continuous variable d_{ij}^{nk} to represent the distance traveled on edge (i, j) in period k of truck n . If edge (i, j) is traversed in the trip of truck n , $\sum_{k \in \mathbf{K}} d_{ij}^{nk} = w_{ij}$ must be satisfied. Besides, we should have Eq. (6) to ensure that d_{ij}^{nk} cannot exceed the length of edge (i, j) :

$$d_{ij}^{nk} \leq w_{ij} \cdot y_{ij}^{nk}, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N}, k \in \mathbf{K} \quad (6)$$

The following Eq. (7) expresses the travel time of truck n on edge (i, j) in period k :

$$\delta_{ij}^{nk} = d_{ij}^{nk} / v_{ijk}, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N}, k \in \mathbf{K} \quad (7)$$

Then, the total travel time for truck n to traverse edge (i, j) is $\sum_{k \in \mathbf{K}} \delta_{ij}^{nk}$. Note that all the notations used throughout this study are provided in Appendix for readability.

OPTIMIZATION MODEL BUILDING

With the above notations and preliminary analysis, the proposed TRPTC problem in this study can be formulated by the following optimization model:

[TRPTC]

$$\min_{\mathbf{x}, \mathbf{y}, \mathbf{p}} \sum_{n \in \mathbf{N}} \sum_{(i, j) \in \mathbf{E}} c_{ij} w_{ij} (x_{ijn} - \eta \sum_{m \in \mathbf{N}} p_{ij}^{nm}) \quad (8)$$

subject to Constraints (1)-(7), and

$$\sum_{\{j|(i,j) \in \mathbf{E}\}} x_{ijn} - \sum_{\{j|(j,i) \in \mathbf{E}\}} x_{jin} = \begin{cases} 1 & \text{if } i = o_n \\ -1 & \text{if } i = d_n \\ 0 & \text{otherwise} \end{cases}, \forall i \in \mathbf{V}, n \in \mathbf{N} \quad (9)$$

$$x_{ijn} \geq y_{ij}^{nk}, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N}, k \in \mathbf{K} \quad (10)$$

$$x_{ijn} \leq \sum_{k \in \mathbf{K}} y_{ij}^{nk}, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N} \quad (11)$$

$$\sum_{k \in \mathbf{K}} d_{ij}^{nk} = w_{ij} \cdot x_{ijn}, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N} \quad (12)$$

$$y_{ij}^{nk_1} \leq M(2 - y_{hi}^{nk_2} - x_{hin}), \forall (h, i), (i, j) \in \mathbf{E}, n \in \mathbf{N}, k_1, k_2 \in \mathbf{K}, k_1 < k_2 \quad (13)$$

$$\sum_{(i,j) \in \mathbf{E}} \delta_{ij}^{nk} \leq e_k - b_k, \forall n \in \mathbf{N}, k \in \mathbf{K} \quad (14)$$

$$t_{ijn} \leq e_k - \delta_{ij}^{nk} + M \cdot (1 - y_{ij}^{nk}), \forall (i, j) \in \mathbf{E}, n \in \mathbf{N}, k \in \mathbf{K} \quad (15)$$

$$t_{ijn} \geq b_k - M \cdot (1 - y_{ij}^{nk}), \forall (i, j) \in \mathbf{E}, n \in \mathbf{N}, k \in \mathbf{K} \quad (16)$$

$$t_{ijn} - t_{hin} \geq \sum_{k \in \mathbf{K}} \delta_{hi}^{nk} + u_{in} - M \cdot (2 - x_{hin} - x_{ijn}), \forall (i, j), (h, i) \in \mathbf{E}, n \in \mathbf{N}, i \neq o_n \neq d_n \quad (17)$$

$$t_{ijn} - t_{hin} \leq \sum_{k \in \mathbf{K}} \delta_{hi}^{nk} + u_{in} + M \cdot (2 - x_{hin} - x_{ijn}), \forall (i, j), (h, i) \in \mathbf{E}, n \in \mathbf{N}, i \neq o_n \neq d_n \quad (18)$$

$$t_{o_n in} \geq \sigma_n + u_{o_n n} - M \cdot (1 - x_{o_n in}), \forall (o_n, i) \in \mathbf{E}, n \in \mathbf{N} \quad (19)$$

$$\tau_n \geq t_{id_n n} + \sum_{k \in \mathbf{K}} \delta_{id_n}^{nk} + u_{d_n n} - M \cdot (1 - x_{id_n n}), \forall (i, d_n) \in \mathbf{E}, n \in \mathbf{N} \quad (20)$$

$$t_{ijn} \leq M \cdot x_{ijn}, \forall (i, j) \in \mathbf{E}, n \in \mathbf{N} \quad (21)$$

$$u_{in} \leq M \cdot \sum_{j \in \mathbf{N}} (x_{ijn} + x_{jin}), \forall i \in \mathbf{V}, n \in \mathbf{N} \quad (22)$$

$$\begin{cases} x_{ijn} \in \{0, 1\}, y_{ij}^{nk} \in \{0, 1\}, p_{ij}^{nm} \in \{0, 1\} \\ d_{ij}^{nk} \geq 0, \delta_{ij}^{nk} \geq 0, t_{ijn} \geq 0, u_{in} \geq 0 \end{cases}, \forall (i, j) \in \mathbf{E}, n, m \in \mathbf{N}, k \in \mathbf{K} \quad (23)$$

The objective function shown by Eq. (8) is to minimize the total fuel cost incurred on all edges traversed by the whole fleet. Constraints (1)-(7) are elaborated in Section 2. Constraint (9) ensures flow conservation for each truck. Constraints (10)-(11) jointly bound the relationship between x_{ijn} and y_{ij}^{nk} . Constraint (12) ensures that the edge (i, j) in a trip must be fully covered by truck n if n traverses it. Constraint (13) guarantees that a truck will never traverse a pair of adjacent edges without respecting the time constraint. Constraint (14) stipulates that the total travel time of a truck in a period k is shorter than the length of it, i.e., $e_k - b_k$. Constraint (15) ensures that if edge (i, j) is traversed by truck n in period k , the departure time of this truck should be earlier than the ending time of this period minus the travel time of edge (i, j) in that period. Similarly, constraint

(16) guarantees that if an edge (i, j) is traversed by truck n in period k , the starting time of traversing this edge of this truck should be larger than the beginning time of this period. Constraints (17) and (18) ensure that the entering time for consecutive nodes in a truck's route should increase with the edge traversal time plus waiting time. Constraint (19) calculates the time when a truck enters its first edge. Constraint (20) forces each truck to be in its destination by a given deadline. Constraint (21) ensures that if there is no flow on an edge, the enter time must be zero. Similarly, constraint (22) ensures that if there is no flow through a node, the waiting time must be zero. Constraint (23) defines the domains of the decision variables.

NUMERICAL EXPERIMENTS

This section presents the numerical experiments to evaluate the performance of our proposed model. We solve the model by directly using Gurobi 9.0 and the maximum CPU runtime is set to be 3600 seconds. Note that the truck routing and platooning problem we investigated in this study is computationally challenging due to its NP-hardness. Therefore, we only test small-sized instances in the numerical experiments.

Test instances. Three types of randomly generated undirected graphs with different numbers of nodes, i.e., $|\mathbf{V}| = \{10, 30, 50\}$, are used as the representations of highway networks. The length of each edge (i, j) , i.e., w_{ij} , measured by *km*, is chosen as a uniformly random integer from the set $\{30, 31, \dots, 60\}$. Figure 2 is an example for the randomly generated highway networks.

As for the truck fleet, we consider two different fleet scenarios, i.e., $|\mathbf{N}| = \{5, 10\}$. The pair of origin/destination nodes, i.e., o_n and d_n , for each truck is randomly chosen from the nodes of the networks. Besides, the origin time σ_n for each truck is drawn uniformly from $[0, 720]$, while the latest arrival times are calculated as:

$$\tau_n = \sigma_n + (1 + \varphi) \cdot \zeta_n, \forall n \in \mathbf{N} \quad (24)$$

where ζ_n is the minimum time required for truck n to travel from its origin to its destination. φ is a parameter to control the tightness of the time window for each truck and we set $\varphi = 1$ in this paper. Kindly note that for a given highway network and a fleet size, five instances with randomly generated parameters will be created. As such, a total of 30 random instances will be used to assess the performance of the proposed model.

Regarding the time-varying traffic conditions, we partition the 1440-minute planning horizon into six time periods, i.e., $[0, 480]$, $[480, 600]$, $[600, 960]$, $[960, 1140]$, $[1140, 1320]$ and $[1320, 1440]$, to represent the pre-morning peak, morning peak, off-peak, evening peak, after-evening peak and off-peak of a day, respectively. Each period is associated with a travel speed range (km/h), i.e., $[80, 100]$, $[30, 60]$, $[70, 90]$, $[30, 60]$, $[60, 80]$, and $[80, 100]$, to factor the time-varying traffic conditions of a day. For simplicity, we assume that the traffic congestion pattern during the day is the same on each edge. The travel speed of a truck on each edge in each period

is randomly generated from the corresponding speed range. Besides, the fuel consumption cost c_{ij} per kilometer is set to be 1 and the fuel reduction rate η for the following vehicles in a platoon is set to 10% in this study.

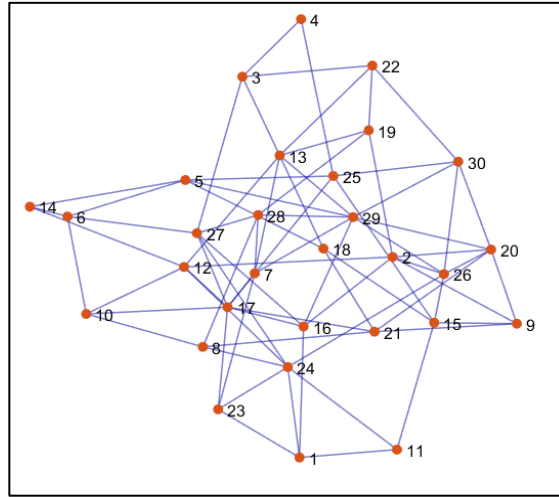


Figure 2. An example of a 30-node highway network.

Assessment of the model. Table 1 tabulates the average results of the objective function values and CPU runtimes of the 30 instances obtained by Gurobi. According to Table 1, we observe that even a small increase in the fleet size would lead to a sharp increase in computational time. For example, the scenario with $|\mathbf{N}|=5$ can be optimally solved within a few seconds on average, whereas when the fleet size merely increases to 10, averagely requires 40 minutes are required, a remarkably longer computational time. Another main observation is that the network size also significantly affects CPU runtimes. Specifically, for the scenario with $|\mathbf{V}|=10$ and $|\mathbf{N}|=10$, it merely takes dozens of seconds to obtain the optimal solutions. However, Gurobi cannot even find a feasible solution for these 10 trucks within 1 hr when the network size increases to 50. The exponentially increased CPU runtimes for Gurobi fully demonstrate the complex structure of our proposed model.

Table 1. Numerical results.

Problem instances		Computational results	
Network size	Fleet size	Optima 1	CPU time 1 (s)
10	5	196.99	1
	10	379.45	87
30	5	256.58	3
	10	533.27	2534
50	5	359.42	11

	10	-	-
Average	-	345.14	527

Results comparison to the model with fixed traffic conditions. To numerically justify the necessity of considering time-varying traffic conditions in the trucks' routing and platooning, we further obtain the solution results for the model without taking the time-varying traffic conditions into account. That is, for each edge, the travel speed is the same all day for the truck fleet. We fix the travel speed of trucks to be 80 km/h in this subsection. The average results are reported in Table 2 for comparison. The variations of the average objective function values under time-varying and fixed traffic conditions are further visualized in Figure 3. According to Table 2 and Figure 3, we observe that the consideration of time-varying traffic conditions brings about a moderate increase (averagely 1.74%) in the total fuel costs. This may be explained by the fact that the routes and time schedules without the consideration of varied traffic congestion may be infeasible for the trucks to platoon and thus the fuel savings are always overestimated. This finding indicates that the time-dependent traffic conditions does influence the platooning opportunities in real-life scenarios. These findings demonstrate the necessity of considering time-varying traffic conditions in the truck routing and platooning problem. In addition, as indicated by ΔCPU in Table 2, we see that the computational time obtained by the model without taking the time-varying traffic conditions into account is averagely 26.54% shorter than that achieved by the model considering time-varying traffic conditions.

Table 2. Comparison results of different models.

Problem instance		Computational results		Comparison	
Network size	Fleet size	Optima 2	CPU time 2 (s)	ΔObj	ΔCPU
10	5	195.43	1	0.79%	0.00%
	10	374.05	62	1.44%	-28.74%
30	5	251.23	2	2.13%	-33.33%
	10	522.37	1896	2.09%	-25.18%
50	5	341.76	6	2.24%	-45.45%
	10	-	-	-	-
Average	-	336.96	393	1.73%	-26.54%

$$\Delta\text{Obj} = (\text{Optima1} - \text{Optima 2}) / \text{Optima 2}$$

$$\Delta\text{CPU} = (\text{CPU time2} - \text{CPU time1}) / \text{CPU time1}$$

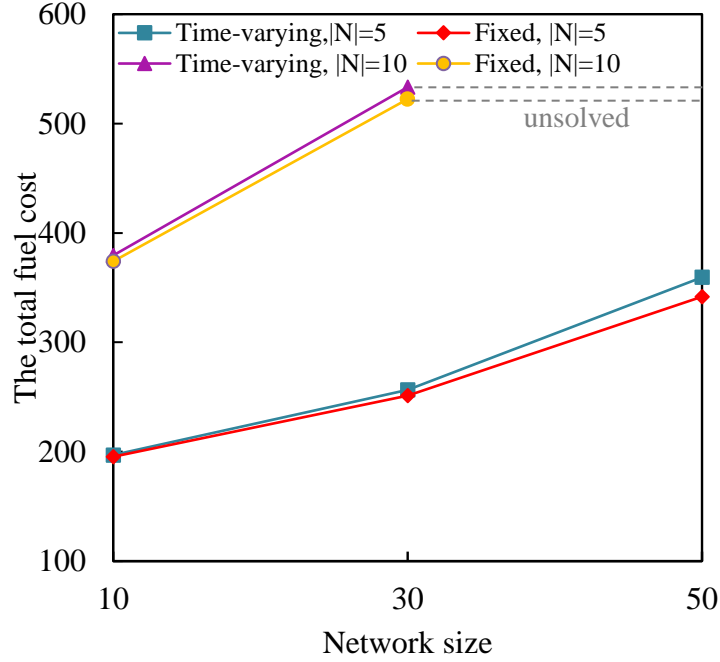


Figure 3. Variations of the total fuel costs under different models with the increase of network size.

CONCLUSIONS

In this study, we investigate the truck routing and platooning problem considering time-varying traffic conditions. The problem was formulated as a MILP model and was solved directly by Gurobi. Totally 30 randomly generated instances were used in the numerical experiments to validate the efficacy of the proposed model and demonstrate the necessity of considering time-varying traffic conditions in the truck routing and platooning. Nevertheless, this study only addressed small-scaled instances of the proposed problem. To cope with larger-sized instances, more compact mathematical models and tailored-designed efficient algorithms are highly expected in future related research.

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APPENDIX

Symbol	Description
<i>Indices and sets:</i>	
$\mathbf{G} = (\mathbf{V}, \mathbf{E})$	Graph with node set \mathbf{V} and edge set \mathbf{E}
\mathbf{V}	Set of network nodes
\mathbf{E}	Set of network edges
\mathbf{N}	Set of trucks
\mathbf{K}	Set of periods of the planning horizon
(i, j)	Index for edge
i, j	Indices for nodes
n, m	Indices for trucks
k	Index for time period
l	Number of time intervals, $l = \text{card}(\mathbf{K})$
<i>Known parameters:</i>	
o_n	Origin node for truck $n \in \mathbf{N}$
d_n	Destination node for truck $n \in \mathbf{N}$
w_{ij}	The time required to traverse the edge $(i, j) \in \mathbf{E}$
σ_n	Earliest departure time for truck $n \in \mathbf{N}$
τ_n	Latest arrival time for truck $n \in \mathbf{N}$
v_{ijk}	Travel speed on edge (i, j) in time interval $k \in \mathbf{K}$
$[b_k, e_k]$	Beginning and ending time of time interval $k \in \mathbf{K}$
e_l	The ending time of the last time interval
ζ_n	Minimum time required for a truck to travel from its origin to its destination
c_{ij}	Unit fuel cost of traversing edge $(i, j) \in \mathbf{E}$
η	Fuel reduction rate for the trailing trucks in a platoon
φ	A parameter to control the tightness of the time window for each truck
M	A sufficiently large enough positive number
<i>Decision variables:</i>	
x_{ijn}	Binary variable indicating whether truck $n \in \mathbf{N}$ will traverse edge (i, j)
y_{ij}^{nk}	Binary variable indicating whether truck $n \in \mathbf{N}$ will traverse edge (i, j) in time interval $k \in \mathbf{K}$
p_{ij}^{nm}	Binary variable indicating whether truck $n \in \mathbf{N}$ will follow truck $m \in \mathbf{N}$ over edge (i, j)
d_{ij}^{nk}	Continuous variable indicating the traveled distance of truck $n \in \mathbf{N}$ on edge (i, j) in period $k \in \mathbf{K}$
δ_{ij}^{nk}	Continuous variable indicating the traveled time of truck $n \in \mathbf{N}$ on edge (i, j) in period $k \in \mathbf{K}$
t_{ijn}	Continuous variable indicating the time when truck $n \in \mathbf{N}$ enters edge

u_{in}	(i, j) Continuous variable indicating the waiting time of truck $n \in \mathbf{N}$ at node $i \in \mathbf{V}$
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