Advances in Engineering Software, Vol. 38, No. 3, 2007, pp. 145-149

Reliability and performance-based design by artificial neural network

K.W. Chau
Department of Civil & Structural Engineering, Hong Kong Polytechnic University, Hunghom, Kowloon, Hong Kong
Email: cekwchau@polyu.edu.hk

Abstract
Whilst conventional approach in structural design is based on reliability-calibrated factored design formula, performance-based design customizes a solution to the specific circumstance. In this work, an artificial neural network approach is employed to determine implicit limit state functions for reliability evaluations in performance-based design and to optimally evaluate a set of design variables under specified performance criteria and corresponding desired reliability levels in design. Case examples are shown for reliability design. Through the establishment of the response and reliability databases, for specified target reliabilities, structural response computations are integrated with the evaluation of design parameters and design can be accomplished. By employing this methodology, with the same performance requirements, pertinent design parameters can be altered in order to evaluate feasible design alternatives, to explore the usage of various structural materials and to define required material quality control.

Keywords: Design parameters, Neural network, Performance-based design, Structural reliability

Introduction

Recent catastrophic structural failures due to earthquakes and cyclones have uncovered the limitations of contemporary design practices and revealed the requirements for novel concepts and methodologies for structural performance evaluation and design. One of the key issues in evaluation and design process is a proper consideration and handling of the large uncertainty in both the loadings and the complex structural behaviour, in particular for those within the non-linear range (Wen, 2001).

With the above background, it is strongly believed that the engineering profession entails new paradigms in order to furnish the best potential service to society compatible to the advent of the computer era. In fact, the previous transition from Allowable Stress Design to Partial Factors Method (PFM) has contributed partly to this development. Although PFM can be regarded semi-probabilistic with respect to load and resistance factors, it is still deterministic from the designer's point of view. The recent advancement and development of computer technology provides a good chance of the transition from the PFM to a structural reliability assessment concept based on a fully probabilistic approach. Besides, nowadays the emergence and wide popularity of powerful desktop computer with high operating speed renders the development of efficient customized design within reach.

In previous research on performance-based design, the probability of failure is usually evaluated by importance sampling simulation around a design point acquired through an approximate analysis using a first order response surface and FORM (Melchers, 1999). The evaluation of the performance function is usually performed through a local interpolation procedure. It may be difficult to develop a global response surface over the entire variable domain in many problems, especially in non-linear structural dynamics. For instance, it may not be possible to represent peaks and valleys representing resonance at different frequencies with sufficient accuracy. As a developing and promising technology, the capability of an artificial neural network to cope with uncertainty in complex situations, has been seized upon for wide ranging applications in recent years (Rumelhart, Hinton & Williams, 1986; Garrett, 1994; Chau & Cheng, 2002; Chau, 2004a, 2004b). With the recent advances in artificial intelligence (AI) technology, it is possible to develop an approximating limit state function or response surface, which can fulfil the above purpose, with a neural network approach.

This work delineates an approach to reliability estimation, which integrates a response surface methodology and artificial neural network. The major contribution of this study is mainly on the application of artificial neural network on the inverse performance-based design to determine design parameters that satisfy reliability constraints, which has not been attempted previously by other researchers.

Performance-based design

Performance-based design represents the consideration of different performance criteria and the evaluation of design parameters with a view to fulfilling them, with comparable target reliabilities over the entire service life of a structure. Thus it is able to customize an engineering solution to the specific circumstance at hand. Requirements are defined through performance criteria and the accomplishment of target reliability levels. The corresponding design parameters of the structure fulfilling these criteria are assessed through the actual exceedence probability in all the required limit states over its service life. This approach is capable of taking full advantage of the latest advancements in reliability theory as well as sophisticated computational tools in structural analysis (Rackwitz, 2001; Esteva et al., 2002; Cheng et al., 2006; Rackwitz, 2006).

Performance or limit state function

A key to success of the performance-based design is the efficiency in determining the attained reliability for a specific combination of design parameters. In the evaluation of reliability or failure probability of a structural system, the failure criterion for the system can be represented as a limit state function with respect to pertinent random design variables. A limit state function is in general expressed as follows:

$$g(x)\begin{cases} >0: & criterion \quad for \quad safety \\ \leq 0: & criterion \quad for \quad failure \end{cases} \tag{1}$$

where G(x) and x $=(x_1,x_2,\ldots,x_m)^T$ represents a performance or limit state function and a vector of intervening random variables involved in the system, respectively. g(x) is in fact a structural response corresponding to x, which is often difficult to be represented explicitly. For structures subject to dynamic loading or with high complexity, a complicated structural analysis or a modelling of the system behaviour should be executed to evaluate g(x). The determination of structural failure probability entails repeated retrievals for the limit state function g(x), which may be very time-consuming since it possibly involves the computation of nonlinear structural responses. Owing to this, appropriate prediction techniques are entailed to evaluate the correlation between the structural responses and random design parameters for a specific limit state.

The usual steps for prediction of the limit state function is as follows. The actual structural responses g(x) are at first computed in some points within the interested region. An approximating function g'(x), also called a response surface, is then employed to mimic the real limit state function g(x). In previous works, an interpolation method (Schuëller, Bucher, Bourgund & Ouypornprasert, 1989) or a regression analysis (Foschi, Li & Zhang, 2002) have been used in fitting such a response surface. Since a real limit state function g(x) may take any arbitrary shape, it is in fact difficult to find an ideal function g'(x) which can fit g(x) perfectly throughout the entire domain.

For a small region around a specified design point, a first order form of Taylor series expansion may be able to represent the limit state function locally. Yet, more than one critical failure region may exist for a complex limit state function. Thus, if g(x) is complicated and a whole-area fitting is entailed, a simple first or second order polynomial may not be adequate.

Methodology

In this work, two back-propagation neural networks are employed. It firstly delineates an approach to reliability estimation, which integrates a response surface methodology and artificial neural network. The central step of the methodology is to represent the performance or limit state function, through discrete data corresponding to a set of variable vectors, previously acquired from compiled deterministic and algorithmic analyses. As such, the compilation and gleaning of structural response data is undertaken independently of the reliability computation. Whilst the former step may be computationally intensive, the latter step only entails a pattern recognition process under AI technology with less computational effort. This renders the possibility of a fast evaluation of reliability and efficient implementation of performance-based design. A virtually global surface is matched to structural response data for various combinations of the design variables. Whilst the implementation of performance-based approaches is based on an explicit specification of criteria, employing a structural response analysis for the computation of the performance requirements and reliability analysis to determine the attained exceedence probability in each limit state, in solving practical design problems, an inverse performance-based approach is more convenient.

Under this approach, the performance criteria and corresponding target reliability levels are given and a set of design parameters to optimally satisfy the criteria with the desired reliabilities are to be determined. The design variables can also be evaluated directly by the second neural network with pattern recognition between the attained reliabilities and the target specifications for all limit states. This renders the development of useful and pragmatic performance-based design software which is of fundamental significance for its general acceptance in real engineering problems.

Neural network for evaluation of response surface

In this work, an artificial neural network approach is employed to determine performance functions for reliability evaluations in performance-based design. A commercially available software package, MathLab, is employed to facilitate the analysis (Gilat, 2005). Back-propagation has generally been the most popular method used to train nonlinear, multilayered neural networks to perform function approximation and pattern classification. Here, a three-layer back-propagation neural network is used. It has been shown that ANNs with one hidden layer can approximate any function, given that sufficient degrees of freedom are provided (Bebis and Georgiopoulos, 1994).

The output consists of a single neuron representing the response surface g(x). The input represents the random variables x_i in the input layer. Moody and Yarvin (1992) have compared the performance of several transfer functions and concluded that the sigmoidal transfer functions performed better than other functions, particularly when the data were noisy and contained mildly non-linear relationships. The S-shaped sigmoid curve is used as a transfer function on each neuron to represent the input-output relation in the hidden layer and output layer whilst a linear function is employed for the input layer.

The back-propagation learning rule is used to adjust the weights and biases of the network in order to minimize the mean squared error between the real response surface and that predicted from the neural network model. In the learning process, some actual training data are applied and the error between the output of the network and every response is computed. The squared error E is written as a function of the weighting coefficients. A back-propagation error algorithm is used to minimize the error by continually changing the values of the network weights and biases in the direction of steepest descent with respect to error (Rumelhart, Widrow, & Lehr, 1994). The minimization of the squared error E proceeds until E converges to within a preset tolerance for all test points.

Neural network for inverse performance-based design

For specified target reliabilities, structural response computations can be integrated with the evaluation of design parameters. These response computations are performed first through a separate computational tool in order to constitute a response database, which is then used in the reliability evaluation. If the performance criteria and the corresponding target reliability levels are given, a set of design parameters which optimally meet the requirements with the desired reliabilities can be evaluated. This entails an optimization to minimize the sum of square of differences between the target reliabilities and attained reliabilities for the performance criteria as follows:

$$E = \sum_{i=1}^{n} \{\beta_i^T - \beta_i(d)\}^2 \tag{2}$$

where E is the objective function to be minimized, β_i^T is the target reliability, $\beta_i(d)$ is the attained reliability for a set of design parameters d, and n is the number of performance criteria.

The inverse performance-based design problem is in general quite time-consuming since repetitive computations of attained reliabilities are usually required for a specific combination of design parameters. In order to overcome this difficulty, a reliability database can be efficiently established for a variety of design parameters during the optimization through a neural network approach.

A similar three-layer back-propagation neural network as that for evaluation of response surface is employed. However, the number of units involved in the input layer, the hidden layer and the output layer differ. The output consists of several neurons representing the design parameters whilst the input represents the target reliability levels for all the performance criteria together with all the remaining random variables. With the proper establishment of the response and reliability databases, this inverse performance-based approach permits the development of very efficient customized design.

Case examples

Example 1 – Inverse performance-based design

A mass concrete gravity pier of width b constructed under water depth h is subject to a horizontal berthing load F, as shown in Figure 1. Two limit states are considered in this case, namely, overturning of the pier and sliding of the pier on the seabed. The design parameters are the mean and standard deviation or coefficient of variation (C.O.V.) of the weight W of the pier, which is assumed to be normally distributed. Two performance criteria are considered in this case, representing sliding stability and overturning stability, as follows:

$$g_1 = W \tan\theta - F(x)\varphi \quad (3)$$

$$g_2 = W - \frac{4hF(x)\varphi}{b} \quad (4)$$

where x is the vector of random variables associated with berthing loading, θ is the friction angle at the seabed, φ is a random variable representing the model error in computation of F.

Given these two criteria for performance, two reliability levels are imposed: $\beta_1 = 1.5$ for g_1 and $\beta_2 = 2.0$ for g_2. As shown in Table 1, in this case, the number of units in the input layer and the output layer are seven and two, respectively. The variables are assumed normal and random with a given coefficient of variation. The reliability indices are determined by assuming a standard normal distribution function. Since the estimation of failure probability entails repeated call for the limit state functions, the computation is very expensive. Hence, a response database for the structural analysis is constructed using the commercially available algorithmic package ABAQUS (Hibbitt et al, 1998), including 1,000 combinations of seven random variables.

In the beginning, the network has a learning process, where some actual data from the response database are provided, and the output of the network and every response is calculated. The learning rate and momentum term are set to be 0.1 and 0.9, respectively. The tolerance of the square error is 10^{-4}. Moreover, the reliability indices are computed for 1,000 combinations of design parameters, which constitute a reliability database. For inverse performance-based design, the error between the output of the network and the target reliability index is minimized. Table 2 shows the results of the optimization. The attained reliabilities at the optimum parameters are $\beta_1 = 1.49$ for g_1 and $\beta_2 = 2.01$ for g_2.

Example 2 – Case containing correlated variables

This neural network is employed to simulate the following three limit state functions g_1, g_2 and g_3 involving four random variables, x_1, x_2, x_3 and x_4:

$$g_1 = x_1^2 - 4x_2 - 2x_3x_4 \quad (5)$$

$$g_2 = 2x_1x_4 - x_2x_3 \quad (6)$$

$$g_3 = x_1x_2x_4 - 2x_3 \quad (7)$$

As shown in Table 3, amongst the eight random variables, i.e., mean values and coefficients of variation for each of them, three of them are chosen as the design parameters. Variables x_1 and x_3 are assumed to have a correlation coefficient of 0.8. Table 4 shows the results of inverse performance-based design for this example. The target reliability indices are validated by the benchmarking forward reliability computation FORM. The processing time is 2,008s for a Intel Pentium (800MHz) processor, which is comparatively much faster than those for the benchmarking FORM or Monte Carlo techniques. When comparison is made with other inverse reliability algorithms such as Der Kiureghian et al. (1994), this method has advantages in the imposition of fewer restrictions. It should be noted that the number of design parameters can be a multiple number instead of a single parameter. Moreover, this algorithm can deal with problems involving correlated variables.

Conclusions

In this work, an artificial neural network approach is employed to determine implicit performance functions for reliability evaluations in performance-based design and to optimally evaluate design variables under specified performance criteria and pertinent target reliability levels in inverse performance-based design. It is proven that, with the learning process, a neural network can be applied to make a good fitness on a whole area approximation in simulation of actual limit state functions. Through the inverse performance-based design approach, while the performance requirements are maintained the same, pertinent design parameters can be altered in order to evaluate feasible design alternatives, explore the usage of various structural materials and define required material quality control. Although the traditional codified design approach may still be prevalent at present and remain a useful format of satisfying minimum requirements, the performance-based design approach has the capability to assist in evaluating decisions or innovations. It is believed that the efficiency in evaluating these design parameters is of principal significance for the ultimate general adoption of performance-based design in solving day-to-day practical structural design problems.

References

Bebis, G., & Georgiopoulos, M. (1994). Feed-forward neural networks: Why network size is so important. *IEEE Potentials*, October/November, 27-31.

Chau, K.W. (2004a). River stage forecasting with particle swarm optimization," *Lecture Notes in Computer Science*, 3029, 1166-1173.

Chau, K.W. (2004b). Rainfall-runoff correlation with particle swarm optimization algorithm. *Lecture Notes in Computer Science*, 3174, 970-975.

Chau, K.W., & Cheng, C.T. (2002). Real-time prediction of water stage with artificial neural network approach. *Lecture Notes in Artificial Intelligence*, 2557, 715-715.

Cheng, G., Xu, L. & Jiang, L. (2006). A sequential approximate programming strategy for reliability-based structural optimization**.** *Computers & Structures*, 84(21), 1353-1367.

Der Kiureghian, A., Zhang, Y., & Li, C.C. (1994). Inverse reliability problem. *Journal of Engineering Mechanics, ASCE*, 120(5), 1154-1159.

Esteva, L., Díaz-López, O., García-Pérez, J., Sierra, G. & Ismael, E. (2002). Life-cycle optimization in the establishment of performance-acceptance parameters for seismic design. *Structural Safety*, 24(2-4), 187-204.

Foschi, R.O., Li, H., & Zhang, J. (2002). Reliability and performance-based design: a computational approach and applications. *Structural Safety*, 24(2-4), 205-218.

Garrett, Jr. J.H. (1994). Where and why artificial neural networks are applicable in civil engineering. *Journal of Computing in Civil Engineering*, 8(2), 129-130.

Gilat, Amos. (2005). *MATLAB: an introduction with applications*. Hoboken, N.J.: Wiley.

Hibbitt, T., Karlsson & Sorensen, Inc. (1998). *ABAQUS User's Manual*, Version 5.8. Pawtucket: Hibbitt, Karlsson & Sorensen, Inc.

Melchers, R.E. (1999). *Structural reliability analysis and prediction*. Chichester: Wiley.

Moody, J., & Yarvin, N. (1992). Networks with learned unit response functions. In: Moody, J.E., Hanson, S.J., Lippmann, R.P. (Eds.), *Advances in Neural Information Processing Systems 4*. Morgan Kaufmann, San Mateo, CA.

Rackwitz, R. (2001). Reliability analysis – a review and some perspectives. *Structural Safety*, 23(4), 365-395.

Rackwitz, R. (2006). The effect of discounting, different mortality reduction schemes and predictive cohort life tables on risk acceptability criteria. *Reliability Engineering & System Safety*, 91(4), 469-484.

Rumelhart, D.E., Widrow, B., & Lehr, M.A. (1994). The basic ideas in neural networks. *Communications of the ACM*, 37(3), 87-92.

Rumelhart, D.E., Hinton, G.E., & Williams, R.J. (1986). Learning internal representation by error propagation. In *Parallel Distributed Processing: Foundations*, Vol. 1, Cambridge: MIT Press.

Schuëller, G.I., Bucher, C.G., Bourgund, U., & Ouypornprasert, W. (1989). On efficient computational schemes to calculate structural failure probabilities. *Probabilistic Engineering Mechanics*, 4(1), 10-18.

Yu, X.H., & Chen, G.A. (1997). Efficient backpropagation learning using optimal learning rate and momentum. *Neural Networks*, 10(3), 517-527.

Wen, Y.K. (2001). Reliability and performance-based design. *Structural Safety*, 23(4), 407-428.

Table 1. Input and output units for example 1

Input units	Output units
Target reliability β_1^T for sliding stability g_1	Mean value of W
Target reliability β_2^T for overturning stability g_2	C.O.V. of W
Mean of width b	--
Mean of water depth h	--
Mean of horizontal berthing load F	--
C.O.V. of horizontal berthing load F	--
Mean of friction angle at seabed θ	--

Table 2. Results of inverse performance-based design for example 1

Limit state	Target reliability β_i^T	Attained reliability $\beta_i(d)$	Design parameters
Sliding stability	1.5	1.49	mean W = 7015 MN
Overturning stability	2.0	2.01	C.O.V. = 0.31

Table 3. Input and output units for example 2

Input units	Output units
Mean value of x_1 = 6.0	Mean value of x_2
Mean value of x_4 = 1.0	Mean value of x_3
C.O.V. of x_2 = 0.2	C.O.V. of x_1
C.O.V. of x_3 = 0.1	--
C.O.V. of x_4 = 0.1	
Target reliability β_1^T for g_1 = 3.0	--
Target reliability β_2^T for g_2 = 3.5	--
Target reliability β_3^T for g_3 = 4.0	--

Table 4. Results of inverse performance-based design for example 2

Limit state	Target reliability β_i^T	Attained reliability $\beta_i(d)$	Design parameters	
			Neural network	FORM
g_1	3.0	2.98	mean x_2 = 3.308	mean x_2 = 3.289
g_2	3.5	3.51	mean x_3 = 1.993	mean x_3 = 1.984
g_3	4.0	4.02	C.O.V. of x_1 = 0.831	C.O.V. of x_1 = 0.833

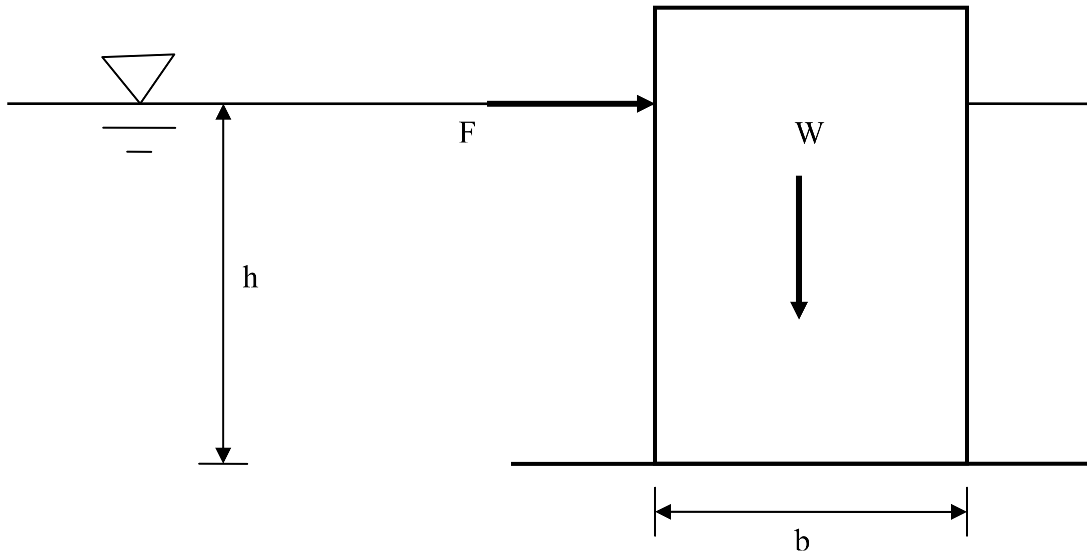

Figure 1. Platform under berthing loading