Numerical Algebra, Control and Optimization

doi:10.3934/naco.2022019

ON THE SIMULTANEOUS DESIGN OF BROADBAND BEAMFORMER FILTERS AND CONFIGURATION

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(Communicated by Jiongmin Yong)

Abstract. For signal enhancement, beamforming remains to be an essential technique for many applications. In the design process, the microphone locations are prescribed and the signal from a target location is being enhanced. While the filter coefficients can be readily optimized, it is found that the signal enhancement capability depends significantly on the array configuration. Therefore, it is advantageous to consider both filters and microphone positions as design variables. In this paper, this problem is addressed. We formulate the beamformer design problem as a non-linear least square problem and propose Gauss-Newton algorithm to update both filters and configuration simultaneously during iterations. We illustrate by several designs to demonstrate the effectiveness of the proposed method.

1. **Introduction.** Beamforming [25, 9] is a signal processing technique to filter noise spatially so that the received signals from a desired location can be enhanced. It has been studied extensively due to their wide applications in many areas such as biomedicine, speech recognition, acoustics and wireless communications in which especially multiuser MIMO systems [12, 13]. In the literature, various techniques [1, 7] have been developed to design beamformer coefficients under certain predefined design conditions. One traditional method is to collect calibration signals and optimize on the least-squares error and signal-to-noise ratio. If the array configuration is fixed, the steering vectors of the desired signals can be estimated together with their direction-of-arrivals [22]. This leads to another way of exploring the signal variance matrix to suppress interference, such as the Capon beamformers and the linearly constrained minimum variance (LCMV) beamformers [16, 14, 15]. The noise signal can be blocked as well with a procedure similar to echo cancellation, which is the generalized sidelobe canceler (GSC) beamforming technique [20, 6].

2020 *Mathematics Subject Classification.* Primary: 90C30, 65K05; Secondary: 68U99.

Key words and phrases. Beamformer optimization, array configuration optimization.

This paper is supported by RGC Grant PolyU. 152200/14E and 152245/18E, and PolyU grant G-UAHF. The first author is also supported by National Natural Science Foundation of China 12171168, Natural Science Foundation of Guangdong Province 2021A1515010368 and 2020A1515010489, the Foundation of Department of Education of Guangdong Province 2020ZDZX3004.

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The near-field-far-field reciprocity relationship can be explored for designing near-field beamformers [8]. More elaborated acoustic signal propagation models have also be employed. The beamformer design problem can be formulated as an semi-infinite optimization problem similar to the design of multidimensional digital filters; various optimization methods, such as linear programming techniques [19, 23], quadratic programming techniques [21, 11] and second-order cone programming [2] and semi-definite programming [4] have been applied.

It is known that the array response changes significantly when the microphone positions change (see, for example, [24]). For example, the performance of the designed beamformers can be enhanced significantly if the configuration of the beamforming array is optimized [5, 17] which ends up in an end-fire array. On the other hand, for hanging microphones in three-dimensional design, the spherical array provided better performance [18]. The design of spherical array has been further studied in [10]. Clearly the positioning of microphones plays an important role in the beamforming design problem. However, unlike optimizing the filter coefficients, in which the problem is convex, the objective function is highly nonlinear when the position is considered in the optimization process. As a result, existing methods for filter design are not applicable. In the literature, the current approaches in optimizing microphone positions are employing a two-level cycles with the outer cycle being the positioning optimization while the inner cycle design the beamformer for a fixed configuration. Indeed, the design process can be carried out in a way that the filter coefficients and the microphone positions are optimized simultaneously in a single cycle. In this way, the method can converge rapidly to obtain the design. In view of this, in this paper, we formulate the beamformer design problem as a non-linear least square problem to optimize on the filters and microphone positions simultaneously. The gradient information is derived analytically with respect to the positions and a Gauss-Newton algorithm is proposed to solve the problem. Unlike Newton's method, the Gauss-Newton algorithm can only be used to minimize a sum of squared functions with the advantage that second derivatives, which can be challenging to compute, are not required. We demonstrate that the resulting method can converge very fast comparing with other methods. By adding microphone positions to be the design variables, we can obtain much better frequency responses.

The rest of the paper is organized as follows. In Section 2, we formulate the beamforming design problem together with microphone location problem. In Section 3, The algorithm for beamforming design problem is presented. For illustration, several examples are solved in Section 4.

2. **Formulation.** Denote the position vector of the source signal by $\boldsymbol{s}$ and the position vector of the i-th microphone by $\boldsymbol{r}_i$. Let $\boldsymbol{\gamma} = (\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_N)$, the region of all possible locations is given by $\Gamma^N = \Gamma \times \cdots \times \Gamma$. Figure 1 depicts the structure of a near-field beamformer. The sound signal is received by the microphone array and processed by the FIR filters. Using a sound propagation model, the transfer function from the source to the i-th microphone is given by

$$H_i(\boldsymbol{s}, f, \boldsymbol{\gamma}) = \frac{1}{\|\boldsymbol{s} - \boldsymbol{\gamma}_i\|} e^{-j2\pi f \|\boldsymbol{s} - \boldsymbol{\gamma}_i\|/c}. \tag{1}$$

The array response is therefore given by

$$\boldsymbol{H}(\boldsymbol{s}, f, \boldsymbol{\gamma}) = (H_1(\boldsymbol{s}, f, \boldsymbol{\gamma}), \ldots, H_N(\boldsymbol{s}, f, \boldsymbol{\gamma}))^\intercal. \tag{2}$$

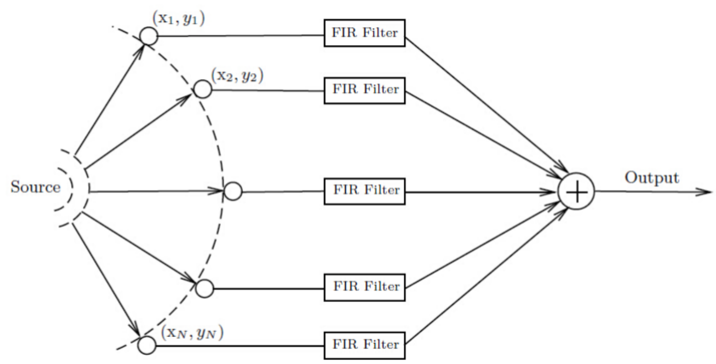

FIGURE 1. The structure of a near-field beamformer

Denote the filter response by

$$\boldsymbol{d}_0(f) = \left(1, e^{-j2\pi f/f_s}, \ldots, e^{-j2\pi f(L-1)/f_s}\right)^{\mathsf{T}}, \tag{3}$$

where f_s is the sampling rate. Let the filter coefficients be

$$\boldsymbol{w} = (\boldsymbol{w}_1^{\mathsf{T}}, \ldots, \boldsymbol{w}_N^{\mathsf{T}})^{\mathsf{T}}, \tag{4}$$

where each $\boldsymbol{w}_i$ has a length of L, the beamformer response is given by

$$G(\boldsymbol{s}, f, \boldsymbol{\gamma}) = \boldsymbol{w}^{\mathsf{T}} \boldsymbol{d}(\boldsymbol{s}, f, \boldsymbol{\gamma})$$

with

$$\boldsymbol{d}(\boldsymbol{s}, f, \boldsymbol{\gamma}) = \boldsymbol{H}(\boldsymbol{s}, f, \boldsymbol{\gamma}) \otimes \boldsymbol{d}_0(f),$$

where $\otimes$ is the Kronecker product.

Let $G_d(\boldsymbol{s}, f, \boldsymbol{\gamma})$ be a desired response imposed on the beamformer, and consider a region $\Omega = \cup_{i=1}^m \Omega_i$ in the space-frequency domain where each Ω_i is a convex set and $\Omega_i \cap \Omega_j = \emptyset$ for $i \neq j$. Here, we choose the desired response function as

$$G_d(\boldsymbol{s}, f) = \begin{cases} e^{-j2\pi f\left(\frac{\|\boldsymbol{s}-\boldsymbol{r}_c\|}{c} + \frac{L-1}{2}T\right)}, & \text{if } (\boldsymbol{r}, f) \text{ is in passband region} \\ 0, & \text{if } (\boldsymbol{r}, f) \text{ is in stopband region} \end{cases}$$

where $\boldsymbol{r}_c = \frac{1}{N}\sum_{i=1}^N \boldsymbol{r}_i$ is the coordinate for the center element. Following [23], we expand the complex functions as

$$\boldsymbol{d}(\boldsymbol{s}, f, \boldsymbol{\gamma}) = \boldsymbol{d}_1(\boldsymbol{s}, f, \boldsymbol{\gamma}) + j\boldsymbol{d}_2(\boldsymbol{s}, f, \boldsymbol{\gamma}),$$

$$G_d(\boldsymbol{s}, f, \boldsymbol{\gamma}) = G_{d_1}(\boldsymbol{s}, f, \boldsymbol{\gamma}) + jG_{d_2}(\boldsymbol{s}, f, \boldsymbol{\gamma}),$$

and denote

$$u(\boldsymbol{s}, f, \boldsymbol{\gamma}) = \boldsymbol{w}^{\mathsf{T}} \boldsymbol{d}_1(\boldsymbol{s}, f, \boldsymbol{\gamma}) - G_{d_1}(\boldsymbol{s}, f, \boldsymbol{\gamma}),$$

$$v(\boldsymbol{s}, f, \boldsymbol{\gamma}) = \boldsymbol{w}^{\mathsf{T}} \boldsymbol{d}_2(\boldsymbol{s}, f, \boldsymbol{\gamma}) - G_{d_2}(\boldsymbol{s}, f, \boldsymbol{\gamma}).$$

Let $\Omega_d = \{(\boldsymbol{s}^1, f^1), \ldots, (\boldsymbol{s}^p, f^p)\} \subset \Omega$ is a set of dense grid points. The design problem can be formulated as

$$\min_{\boldsymbol{w} \in \mathbb{R}^{NL}, \boldsymbol{\gamma} \in \Gamma^N} \sum_{(\boldsymbol{s}^i, f^i) \in \Omega} u^2(\boldsymbol{s}^i, f^i, \boldsymbol{\gamma}) + v^2(\boldsymbol{s}^i, f^i, \boldsymbol{\gamma}). \tag{5}$$

After rearranging the terms, the beamforming design problem can be summarized in the following.

Problem. *Find a coefficient vector* $\boldsymbol{w} \in \mathbb{R}^{NL}$ *and a position vector* $\boldsymbol{\gamma} \in \Gamma^N$ *such that the cost function*

$$\|\boldsymbol{D}(\boldsymbol{s}, f, \boldsymbol{\gamma})\boldsymbol{w} - \boldsymbol{G}_d(\boldsymbol{s}, f, \boldsymbol{\gamma})\|_2^2$$

is minimized, where

$$\boldsymbol{D}(\boldsymbol{s}, f, \boldsymbol{\gamma}) = (\boldsymbol{d}_1(\boldsymbol{s}^1, f^1, \boldsymbol{\gamma}), \cdots, \boldsymbol{d}_1(\boldsymbol{s}^p, f^p, \boldsymbol{\gamma}), \boldsymbol{d}_2(\boldsymbol{s}^1, f^1, \boldsymbol{\gamma}), \cdots, \boldsymbol{d}_2(\boldsymbol{s}^p, f^p, \boldsymbol{\gamma}))^T,$$

$$\boldsymbol{G}_d(\boldsymbol{s}, f, \boldsymbol{\gamma}) = (G_{d_1}(\boldsymbol{s}^1, f^1, \boldsymbol{\gamma}), \cdots, G_{d_1}(\boldsymbol{s}^p, f^p, \boldsymbol{\gamma}), G_{d_2}(\boldsymbol{s}^1, f^1, \boldsymbol{\gamma}), \cdots, G_{d_2}(\boldsymbol{s}^p, f^p, \boldsymbol{\gamma}))^T.$$

Due to γ (the unknown positions) being the decision variables, this is a nonlinear least-squares optimization problem. In order to solve the problem iteratively, the first order gradient information is needed. Define

$$Ł(\boldsymbol{z}) = \boldsymbol{D}(\boldsymbol{s}, f, \gamma)\boldsymbol{w} - \boldsymbol{G}_d(\boldsymbol{s}, f, \gamma),$$

where $\boldsymbol{z} = (\boldsymbol{w}^T, x_1, \cdots, x_N, y_1, \cdots, y_N)^T$. The first order Taylor expansion of $L(\boldsymbol{z})$ about the kth iterate $\boldsymbol{z}^{(k)}$ at each iteration where k is

$$Ł_i(\boldsymbol{z}) = Ł_i(\boldsymbol{z}^{(k)}) + \sum_j \frac{\partial L_i(\boldsymbol{z}^{(k)})}{\partial \boldsymbol{z}_j}(\boldsymbol{z}_j - \boldsymbol{z}_j^{(k)}).$$

In the following section, the gradients are established.

3. Algorithm.

In this section, we describe the use of the Gauss-Newton algorithm, which is a modification of Newton's method, for finding a minimum of the beamformer design problem. Unlike Newton's method, the Gauss–Newton algorithm can only be used to minimize a sum of squared functions with the advantage that second derivatives, which can be challenging to compute, are not required.

Lemma 3.1. *For* $Ł(\boldsymbol{z}) = \boldsymbol{D}(\boldsymbol{s}, f, \gamma)\boldsymbol{w} - \boldsymbol{G}_d(\boldsymbol{s}, f, \gamma)$, *let* $\boldsymbol{J}_{ij} = \frac{\partial Ł_i(\boldsymbol{z}^{(k)})}{\partial \boldsymbol{z}_j}$. *Then we have*

$$\boldsymbol{J} = \begin{pmatrix} \boldsymbol{D} & \boldsymbol{K} \end{pmatrix}$$

where $\boldsymbol{D}_{ij}(\boldsymbol{s}, f, \boldsymbol{\gamma}^{(k)}) = \frac{\partial Ł_i}{\partial \boldsymbol{w}_j}(\boldsymbol{z}^{(k)})$, *and*

$$\boldsymbol{K} = \begin{pmatrix} \frac{\partial L_{1R}}{\partial x_1} & \cdots & \frac{\partial L_{1R}}{\partial x_N} & \frac{\partial L_{1R}}{\partial y_1} & \cdots & \frac{\partial L_{1R}}{\partial y_N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial L_{pR}}{\partial x_1} & \cdots & \frac{\partial L_{pR}}{\partial x_N} & \frac{\partial L_{pR}}{\partial y_1} & \cdots & \frac{\partial L_{pR}}{\partial y_N} \\ \frac{\partial L_{1I}}{\partial x_1} & \cdots & \frac{\partial L_{1I}}{\partial x_N} & \frac{\partial L_{1I}}{\partial y_1} & \cdots & \frac{\partial L_{1I}}{\partial y_N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial L_{pI}}{\partial x_1} & \cdots & \frac{\partial L_{pI}}{\partial x_N} & \frac{\partial L_{pI}}{\partial y_1} & \cdots & \frac{\partial L_{pI}}{\partial y_N} \end{pmatrix}.$$

Proof. Let $\boldsymbol{J}_{ij} = \frac{\partial Ł_i(\boldsymbol{z}^{(k)})}{\partial \boldsymbol{z}_j}$ and the vector of increments $\triangle \boldsymbol{z}_j = \boldsymbol{z}_j - \boldsymbol{z}_j^{(k)}$, then we have

$$Ł_i(\boldsymbol{z}) = Ł_i(\boldsymbol{z}^{(k)}) + \sum_j \boldsymbol{J}_{ij} \triangle \boldsymbol{z}_j.$$

To minimize the cost function $\|Ł(\boldsymbol{z})\|^2$, the gradient is set to zero to obtain

$$2\sum_i (Ł_i(\boldsymbol{z}^{(k)}) + \sum_s \boldsymbol{J}_{is} \triangle \boldsymbol{z}_s)\boldsymbol{J}_{ij} = 0.$$

That is

$$\sum_i \sum_s \boldsymbol{J}_{ij} \boldsymbol{J}_{is} \triangle \boldsymbol{z}_s = -\sum_i \boldsymbol{J}_{ij} Ł_i(\boldsymbol{z}^{(k)}).$$

Then we have

$$\triangle \boldsymbol{z} = -(\boldsymbol{J}^T\boldsymbol{J})^{-1}\boldsymbol{J}^T\text{Ł}(\boldsymbol{z}^{(k)}),$$
$$\boldsymbol{z}^{(k+1)} = \boldsymbol{z}^{(k)} + \triangle \boldsymbol{z} = \boldsymbol{z}^{(k)} - (\boldsymbol{J}^T\boldsymbol{J})^{-1}\boldsymbol{J}^T\text{Ł}(\boldsymbol{z}^{(k)}).$$

Define

$$\beta_1^{ij} = \cos a_j^i \cdot \cos b^i - \sin a_j^i \cdot \sin b^i, \quad \beta_2^{ij} = -\sin a_j^i \cdot \cos b^i - \cos a_j^i \cdot \sin b^i,$$
$$\beta_3^{ij} = \sin a_j^i \cdot \cos b^i \cdot \frac{2\pi f^i}{c},, \quad \beta_4^{ij} = \cos a_j^i \cdot \sin b^i \cdot \frac{2\pi f^i}{c},$$
$$\beta_5^{ij} = \cos a_j^i \cdot \cos b^i \cdot \frac{2\pi f^i}{c},, \quad \beta_6^{ij} = \sin a_j^i \cdot \sin b^i \cdot \frac{2\pi f^i}{c},$$
$$\beta_7^{i} = \frac{1}{N}\sin 2\pi f^i(\frac{\|\boldsymbol{s}^i - \boldsymbol{r}_c\|}{c} + \frac{L-1}{2}T) \cdot \frac{2\pi f^i}{c} \cdot \frac{s_x^i - r_{cx}}{\|\boldsymbol{s}^i - \boldsymbol{r}_c\|},$$
$$\beta_8^{i} = \frac{1}{N}\sin 2\pi f^i(\frac{\|\boldsymbol{s}^i - \boldsymbol{r}_c\|}{c} + \frac{L-1}{2}T) \cdot \frac{2\pi f^i}{c} \cdot \frac{-r_{cy}}{\|\boldsymbol{s}^i - \boldsymbol{r}_c\|}.$$
$$\beta_9^{i} = \frac{1}{N}\cos 2\pi f^i(\frac{\|\boldsymbol{s}^i - \boldsymbol{r}_c\|}{c} + \frac{L-1}{2}T) \cdot \frac{2\pi f^i}{c} \cdot \frac{s_x^i - r_{cx}}{\|\boldsymbol{s}^i - \boldsymbol{r}_c\|},$$
$$\beta_{10}^{i} = \frac{1}{N}\cos 2\pi f^i(\frac{\|\boldsymbol{s}^i - \boldsymbol{r}_c\|}{c} + \frac{L-1}{2}T) \cdot \frac{2\pi f^i}{c} \cdot \frac{-r_{cy}}{\|\boldsymbol{s}^i - \boldsymbol{r}_c\|},$$

where $a_j^i = \frac{2\pi f^i \|\boldsymbol{s}^i - \boldsymbol{r}_j\|}{c}$, $b^i = \frac{2\pi f^i (q-1)}{fs}$. If $(\boldsymbol{r}, f)$ in the stopband region, we can derive

$$\frac{\partial \text{Ł}_{iR}}{\partial x_j} = \sum_{q=1}^{L} w_{jq}\left(\beta_1^{ij}\frac{s_x^i - r_{jx}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|^3} + \frac{\beta_3^{ij}}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} \cdot \frac{s_x^i - x_j}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} + \frac{\beta_4^{ij}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|} \cdot \frac{s_x^i - x_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|}\right)$$

$$\frac{\partial \text{Ł}_{iR}}{\partial y_j} = \sum_{q=1}^{L} w_{jq}\left(\beta_1^{ij}\frac{-y_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|^3} + \frac{\beta_3^{ij}}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} \cdot \frac{-y_j}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} + \frac{\beta_4^{ij}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|} \cdot \frac{-y_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|}\right)$$

$$\frac{\partial \text{Ł}_{iI}}{\partial x_j} = \sum_{q=1}^{L} w_{jq}\left(\beta_2^{ij}\frac{s_x^i - r_{jx}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|^3} + \frac{\beta_5^{ij}}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} \cdot \frac{s_x^i - x_j}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} + \frac{\beta_6^{ij}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|} \cdot \frac{s_x^i - x_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|}\right)$$

$$\frac{\partial \text{Ł}_{iI}}{\partial y_j} = \sum_{q=1}^{L} w_{jq}\left(\beta_2^{ij}\frac{-y_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|^3} + \frac{\beta_5^{ij}}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} \cdot \frac{-y_j}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} + \frac{\beta_6^{ij}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|} \cdot \frac{-y_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|}\right)$$

If $(\boldsymbol{r}, f)$ in the passband region, since $x_c = \frac{1}{N}\sum_{j=1}^{N} x_j$ and $y_c = \frac{1}{N}\sum_{j=1}^{N} y_j$, we have

$$\frac{\partial \text{Ł}_{iR}}{\partial x_j} = \sum_{q=1}^{L} w_{jq}\left(\beta_1^{ij}\frac{s_x^i - r_{jx}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|^3} + \frac{\beta_3^{ij}}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} \cdot \frac{s_x^i - x_j}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} + \frac{\beta_4^{ij}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|} \cdot \frac{s_x^i - x_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|}\right) - \beta_7^i$$

$$\frac{\partial \text{Ł}_{iR}}{\partial y_j} = \sum_{q=1}^{L} w_{jq}\left(\beta_1^{ij}\frac{-y_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|^3} + \frac{\beta_3^{ij}}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} \cdot \frac{-y_j}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} + \frac{\beta_4^{ij}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|} \cdot \frac{-y_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|}\right) - \beta_8^i.$$

$$\frac{\partial \text{Ł}_{iI}}{\partial x_j} = \sum_{q=1}^{L} w_{jq}\left(\beta_2^{ij}\frac{s_x^i - r_{jx}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|^3} + \frac{\beta_5^{ij}}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} \cdot \frac{s_x^i - x_j}{\|\boldsymbol{s} - \boldsymbol{r}_j\|} + \frac{\beta_6^{ij}}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|} \cdot \frac{s_x^i - x_j}{\|\boldsymbol{s}^i - \boldsymbol{r}_j\|}\right) - \beta_9^i.$$

$$\frac{\partial Ł_{iI}}{\partial y_j} = \sum_{q=1}^{L} w_{jq}\left(\beta_2^{ij}\frac{-y_j}{\|\boldsymbol{s}^i-\boldsymbol{r}_j\|^3} + \frac{\beta_5^{ij}}{\|\boldsymbol{s}-\boldsymbol{r}_j\|}\cdot\frac{-y_j}{\|\boldsymbol{s}-\boldsymbol{r}_j\|} + \frac{\beta_6^{ij}}{\|\boldsymbol{s}^i-\boldsymbol{r}_j\|}\cdot\frac{-y_j}{\|\boldsymbol{s}^i-\boldsymbol{r}_j\|}\right) - \beta_{10}^i.$$

□

Using Lemma 3.1, we can summarize the final algorithm as follows:

Algorithm 3.1. The Gauss-Newton algorithm for beamformer design:

Step 1: Choose a $\boldsymbol{z}^{(0)}$. Set $k = 0$.

Step 2: Compute

$$Ł(\boldsymbol{z}^{(k)}) = \boldsymbol{D}(\boldsymbol{s}, f, \boldsymbol{\gamma}^{(k)})\boldsymbol{w}^{(k)} - \boldsymbol{G}_d(\boldsymbol{s}, f, \boldsymbol{\gamma}^{(k)}),$$

and

$$\boldsymbol{J} = \begin{pmatrix}\boldsymbol{D} & \boldsymbol{K}\end{pmatrix}.$$

Step 3 :

Let $z^{(k+1)} = \boldsymbol{z}^{(k)} - (\boldsymbol{J}^T\boldsymbol{J})^{-1}\boldsymbol{J}^T Ł(\boldsymbol{z}^{(k)})$, if $\|Ł(\boldsymbol{z}^{(k+1)} - Ł(\boldsymbol{z}^{(k)}\| < \epsilon$, then stop. Otherwise we set $k = k+1$ and return to step 2.

4. **Numerical Examples.** In this section we provide examples to demonstrate the performance of the algorithm which is implemented in MATLAB. We choose the desired response function as

$$G_d(\boldsymbol{r}, f) = \begin{cases} e^{-j2\pi f\left(\frac{\|\boldsymbol{r}-\boldsymbol{r}_c\|}{c} + \frac{L-1}{2}T\right)}, & \text{if } (\boldsymbol{r}, f) \text{ is in passband region,} \\ 0, & \text{if } (\boldsymbol{r}, f) \text{ is in stopband region,} \end{cases}$$

where $\boldsymbol{r}_c$ is the reference central microphone location.

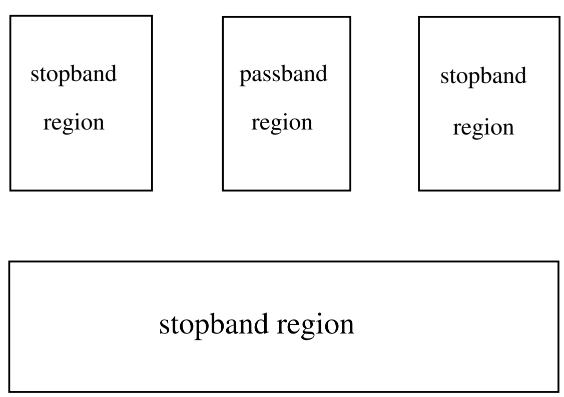

FIGURE 2.

In the first example, we consider that the initial microphone array is an equispaced linear array with five elements that can be seen in Figure 3, where the diamond point denotes the speaker position and the circle points denote the microphone array positions. Here we consider each filter has 7 taps. The passband region is defined as

$$\{(x, f) : -0.4m \le x \le 0.4m,\ 0.5kHz \le f \le 1.5kHz\}$$

while the stopband region is the union of several parts as

$$\{(x, f) : 1.5m \leq |x| \leq 2.5m,\ 0.5kHz \leq f \leq 1.5kHz\},$$
$$\{(x, f) : -2.5m \leq |x| \leq 2.5m,\ 2.5kHz \leq f \leq 4kHz\}.$$

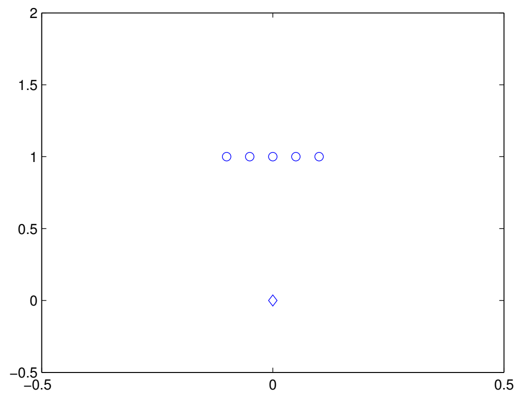

FIGURE 3. Initial array configuration (Ex1)

After using the proposed Gauss-Newton method, the final array configuration can be found in Figure 4. In this example, Table 1 shows the advantages of using Gauss-Newton method instead of the package in Matlab for solving non-linear least squares problems. The convergence history of the algorithm is shown in Figure 5. We can find that the running time of using Gauss-Newton method is significantly less than using the package in Matlab. The amplitude of the actual response $G(\boldsymbol{r}, f)$ is shown in Figure 6 with -16.9637(dB) stopband ripple. Actually, if we do not consider microphone positions which means we just use the microphone configuration in Figure 3, the amplitude of the actual response $G(\boldsymbol{r}, f)$ is shown in Figure 7 with -11.3925(dB) stopband ripple.

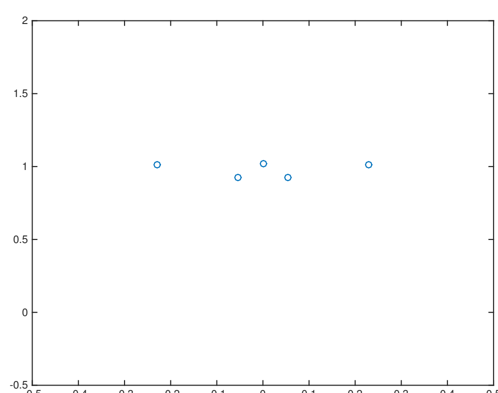

FIGURE 4. Final array configuration (Ex1)

	Gauss-Newton	Matlab Nonlinear LS
running time	8.7995(s)	1073.2(s)

TABLE 1. Comparison of the running times (Ex1)

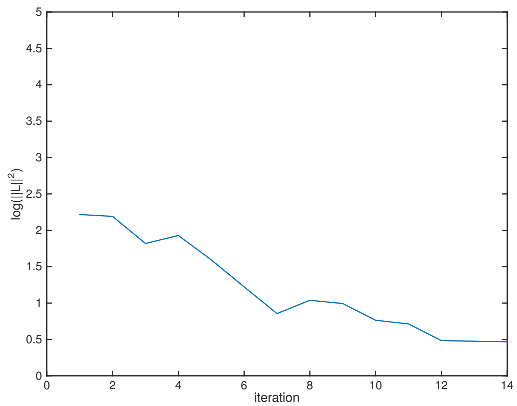

FIGURE 5. Convergence history of the algorithm

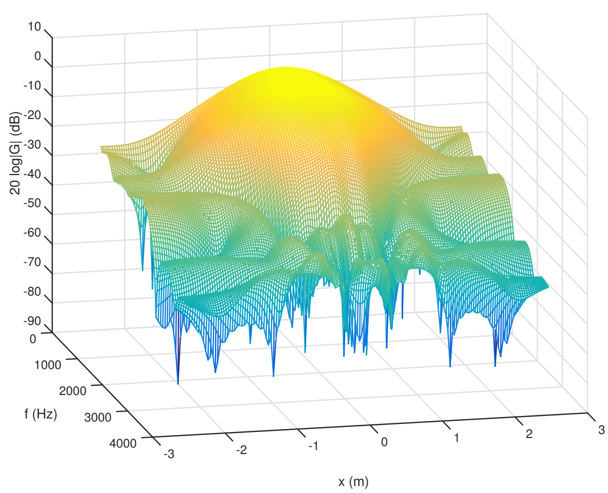

FIGURE 6. Amplitude of $G(\boldsymbol{r}, f)$ where $N = 5$, $L = 7$ (Ex1) with considering microphone positions.

In the second example, we extend the length of the filters significantly so that each filter has 26 taps. The initial configuration is showed in Figure 8 for this example. The passband region is defined as

$$\{(x, f) : -0.4m \leq x \leq 0.4m,\ 0.3kHz \leq f \leq 2.5kHz\}$$

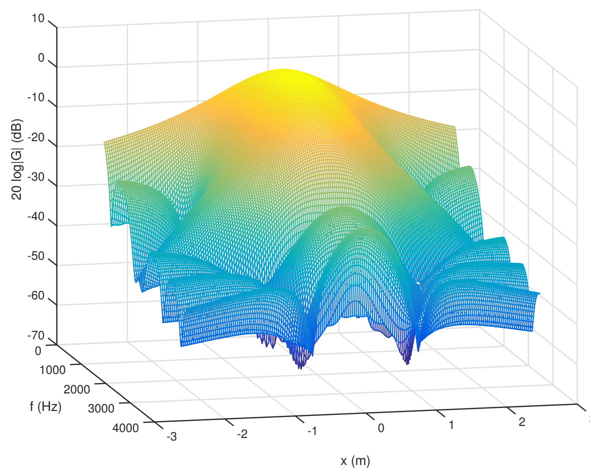

FIGURE 7. Amplitude of $G(\boldsymbol{r}, f)$ where $N = 5$, $L = 7$ (Ex1) without considering microphone positions.

while the stopband region is the union of several parts as

$$\{(x, f) : 1.8m \leq |x| \leq 2.5m,\ 0.3kHz \leq f \leq 2.5kHz\},$$

$$\{(x, f) : -2.5m \leq |x| \leq 2.5m,\ 3kHz \leq f \leq 4kHz\}.$$

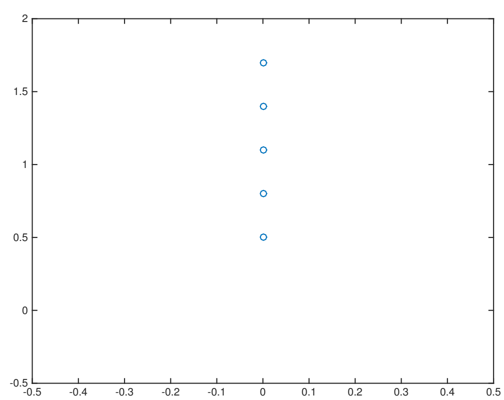

FIGURE 8. Initial array configuration (Ex2)

After using the proposed Gauss-Newton method, the final array configuration can be found in Figure 9. The amplitude of the actual response $G(\boldsymbol{r}, f)$ is shown in Figure 10 with -29.7699(dB) stopband ripple. Actually, if we do not consider microphone positions which means we just use the microphone configuration in Figure 8, the amplitude of the actual response $G(\boldsymbol{r}, f)$ is shown in Figure 11 with -12.1544(dB) stopband ripple.

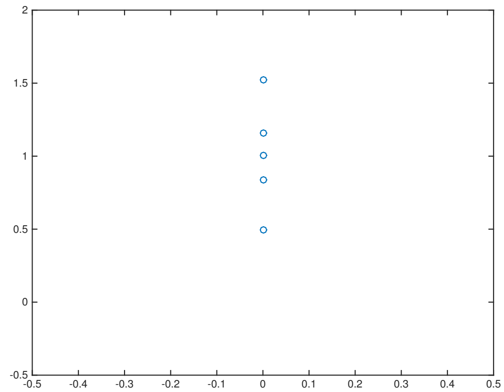

FIGURE 9. Final array configuration (Ex2)

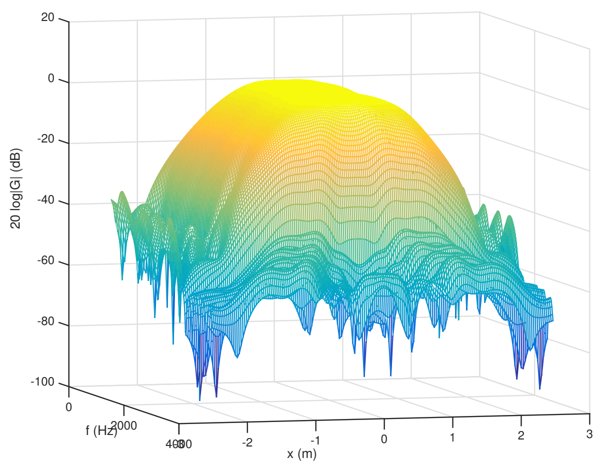

FIGURE 10. Amplitude of $G(\boldsymbol{r}, f)$ where $N = 5$, $L = 26$ (Ex2) with considering microphone positions.

5. **Concluding remarks.** In this paper, we have formulated beamformer design problem with simultaneous design of filter coefficients and array configuration. The problem is formulated as a nonlinear least-squares optimization problem. First order gradient is derived analytically and a Gauss-Newton method is applied for solving the problem. Numerical examples have demonstrated that the designs have improved significantly when the configuration is added as a decision variable, and the computational time is much faster than existing solver. As a future extension, it would be of interest to investigate global optimization techniques for solving the problem and the accelerated gradient method [13, 3].

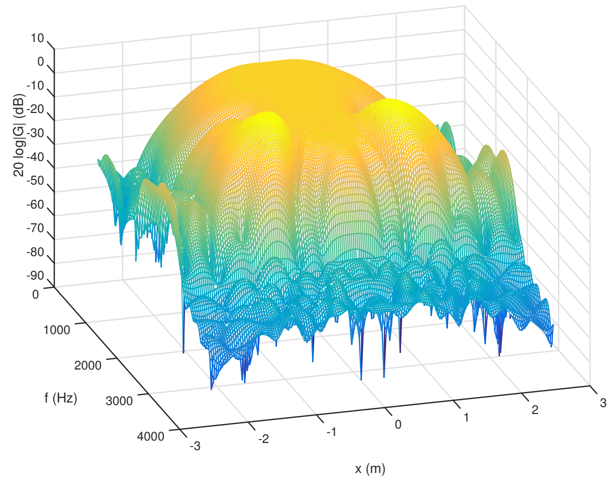

FIGURE 11. Amplitude of $G(\boldsymbol{r}, f)$ where $N = 5$, $L = 26$ (Ex2) without considering microphone positions.

REFERENCES

[1] M. Brandstein and D. Ward, *Microphone Arrays: Signal Processing Techniques and Applications*, Springer Verlag, Berlin, 2001.

[2] S. C. Chan and H. Chen, Uniform concentric circular arrays with frequency-invariant characteristics-theory, design, adaptive beamforming and DOA estimation, *IEEE Trans. Signal Processing*, **55** (2007), 165–177.

[3] H. H. Dam, A. Cantoni and B. Li, A fast low complexity method for optimal zero-forcing beamformer MU-MIMO system, *IEEE Signal Processing Letters*, **22** (2015), 1443–1447.

[4] Z. G. Feng, K. F. C. Yiu and S. Nordholm, A two-stage method for the design of near-field broadband beamformer, *IEEE Trans. on Signal Processing*, **59** (2011), 3647–3656.

[5] Z. G. Feng, K. F. C. Yiu and S. Nordholm, Placement design of microphone arrays in near-field broadband beamformers, *IEEE Trans. on Signal processing*, **60** (2012), 1195–1204.

[6] S. Gannot, D. Burshtein and E. Weinstein, Signal enhancement using beamforming and nonstationarity with applications to speech, *IIEEE Trans. Signal Processing*, **49** (2001), 1614–1626.

[7] W. Kellermann, *Beamforming for Speech and Audio Signals*, in *Handbook of Signal Processing in Acoustics* (eds. D. Havelock, S. Kuwano, and M. Vorlander), Springer, New York, 2008.

[8] R. Kennedy, D. Ward and T. Abhayapala, Nearfield beamforming using radial reciprocity, *IEEE Trans. Signal Processing*, **47** (1999), 33–40.

[9] H. Krim and M. Viberg, Two decades of array signal processing research: The parametric approach, *IEEE Signal Processing Magazine*, **13** (1996), 67–94.

[10] C. C. Lai, S. Nordholm and Y. H. Leung, Design of steerable spherical broadband beamformers with flexible sensor configurations, *IEEE Transactions on Audio, Speech and Language Processing*, **21** (2013), 427–438.

[11] B. Lau, Y. H. Leung, K. L. Teo and V. Sreeram, *Minimax filters for microphone arrays*, *IEEE Transactions on Circuits and Systems II*, **46** (1999), 1522–1525.

[12] B. Li, C. Wu, H. H. Dam, A. Cantoni and K. L. Teo, A parallel low complexity zero-forcing beamformer design for multiuser MIMO systems via a regularized dual decomposition method, *IEEE Transactions on Signal Processing*, **63** (2015), 4179–4190.

[13] B. Li, H. H. Dam, A. Cantoni and K. L. Teo, A first-order optimal zero-forcing beamformer design for multiuser MIMO systems via a regularized dual accelerated gradient method, *IEEE Communications Letters*, **19** (2015), 195–198.

[14] B. Li, Y. Rong, J. Sun and K. L. Teo, A distributionally robust minimum variance beamformer design, *IEEE Signal Processing Letters*, **25** (2018), 105–109.

[15] B. Li, Y. Rong, J. Sun and K. L. Teo, A distributionally robust linear receiver design for multi-access space-time block coded MIMO systems, *IEEE Transactions on Wireless Communications*, **16** (2017), 464–474.

[16] J. Li and P. Stoica, *Robust Adaptive Beamforming*, John Wiley, Inc. Hoboken, New Jersey, 2006.

[17] Z. B. Li and K. F. C. Yiu, Beamformer configuration design in reverberant environments, *Engineering Applications of Artificial Intelligence*, **47** (2016), 81–87.

[18] Z. B. Li, K. F. C. Yiu and Z. G. Feng, A hybrid descent method with genetic algorithm for microphone array placement design, *Applied Soft Computing*, **13** (2013), 1486–1490.

[19] Y. C. Lim and Y. Lian, The optimum design of one and two-dimensional FIR filters using the frequency response masking technique, *IEEE Trans. Cir. and Sys. II*, **40** (1993), 88–95.

[20] S. Nordholm and Y. H. Leung, Performance limits of the broadband generalized sidelobe cancelling structure in an isotropic noise field, *J. of the Acoustical Soc. of America*, **107** (2000), 1057–1060.

[21] S. Nordholm, V. Rehbock, K. L. Teo and S. Nordebo, *Chebyshev approximation for the design of broadband beamformers in the near field*, *IEEE Transactions on Circuits and Systems II*, **45** (1998), 141–143.

[22] J. Ryan and R. Goubran, Array optimization applied in the near field of a microphone array, *IEEE Transactions on Speech Audio Processing*, **8** (2000), 173–178.

[23] K. F. C. Yiu, X. Q. Yang, S. Nordholm and K. L. Teo, Near-field broadband beamformer design via multidimensional semi-infinite linear programming techniques, *IEEE Transactions on Speech and Audio Processing*, **11** (2003), 725–732.

[24] K. F. C. Yiu, M. J. Gao and Z. G. Feng, Design of near-field broadband beamformer using semi-definite programming, *Int. J. Innov. Comput. Inform. Con.*, **8** (2012), 3755–3768.

[25] B. Van Veen and K. Buckley, Beamforming: a versatile approach to spatial filtering, *IEEE ASSP Magazine*, **5** (1988), 4–24.

Received May 2022; Final revision May 2022; early access July 2022.

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