



28 **Keywords:** site investigation; trace elements; geogenic arsenic; spatial variability; restricted  
29 maximum likelihood; soil remediation.

30

## 31 **1. Introduction**

32 Soil contamination by metals/metalloids poses an increasing threat to human health and  
33 environmental quality across the globe (Tóth et al., 2016; Xia et al., 2017; Plessl et al., 2017).  
34 Among the potentially toxic elements, arsenic (As) has received considerable attention over  
35 the last few decades due to its high toxicity and environmental risk (Bundschuh et al., 2013;  
36 Ehlert et al., 2016; Sandhi et al., 2017). Geochemical properties of As are complex because of  
37 its various chemical species and amphoteric nature (Yang et al., 2017; Wu et al., 2017),  
38 which makes it challenging for proper assessment and remediation (Rahman et al., 2017).  
39 Soil can naturally possess high concentrations of As due to weathering of the parent materials,  
40 volcanic eruptions, and forest fires (Beiyan et al., 2017; Li et al., 2017). In recent years,  
41 industrialization and urbanization have also transferred As from used products into the  
42 environment, resulting in many industrial contaminated sites (Tsang et al., 2014; Gallego et  
43 al., 2016; Wcisło et al., 2016). Consequently, many studies have focused on urban  
44 contaminated soils with As from anthropogenic sources, such as agricultural activities and  
45 industrial and mining processes (Rieuwerts et al., 2014; Cao et al., 2016; Yoon et al., 2016;  
46 González-Fernández et al., 2018). Although geogenic As-containing soil/sediment is a  
47 common problem worldwide (Fendorf et al., 2010; Yang et al., 2014), including Hong Kong  
48 (Li et al., 2017; Cui et al., 2018), there has been limited discussion on the characterization of  
49 spatial variations of geogenic As and the corresponding implications on  
50 management/remediation of As-containing sites.

51 To develop cost-effective management and risk mitigation recommendations, the  
52 concentration distribution and variability features (or spatial correlation patterns) of the trace

53 elements should be established foremost through identifying, mapping, and monitoring  
54 processes (Bednářová et al., 2016; Pan et al., 2017). It is because the remediation processes,  
55 especially for extraction-based approaches, are often expensive and highly dependent on the  
56 types of contaminants and estimated amounts of contaminated soils that need to be treated  
57 (Bolan et al., 2014; Tsang and Yip, 2014; Wan et al., 2016). In practice, however, it is  
58 difficult to accurately predict the concentrations of metals/metalloids due to their complex  
59 spatial distribution patterns, including the occasional occurrence of ‘hotspots’ with high  
60 levels of anthropogenic contamination or geogenic formation. Legislations in various  
61 countries (e.g., China, the United States, the Netherlands, Australia, and New Zealand)  
62 advocate the use of probabilistic sampling schemes (e.g., square grid, simple random,  
63 stratified random) for contaminated site assessments (Waterhouse, 1980; Bell et al., 1983;  
64 Gilbert, 1987; Horta et al., 2015), which is similarly applied in Hong Kong (HK EPD, 2011).  
65 However, these methods focused on detecting high concentration regions of anthropogenic  
66 contaminants and quantifying the extent of such hotspots. This is considered appropriate  
67 when there is prior knowledge about the contaminants involved, their transport mechanisms,  
68 and the human activities causing the contamination.

69 For trace elements of geogenic nature, the determination of sampling strategies and  
70 characterization of their spatial distributions may require different techniques because their  
71 existence are not caused by anthropogenic activities (Li et al., 2015; Cui et al., 2018). To this  
72 end, various methods such as Geostatistics, multivariate methods, and Geographic  
73 Information System (GIS) mapping have been applied to identify and reveal the distributions  
74 of these trace elements (Lark, 2000; Lark & Cullis, 2004; Santra et al., 2012; Antunes &  
75 Albuquerque, 2013; Hao et al., 2016; Chakraborty et al., 2017; Boente et al., 2017).  
76 Geostatistics has been developed for application in various disciplines, and is represented by  
77 techniques including various types of kriging (ordinary/disjunctive/indicator kriging),

78 global/local polynomial interpolation (G/LPI), inverse distance weighting (IDW), nearest  
79 neighbour interpolation (NNI), radial basis functions (RBF), sequential Gaussian simulation  
80 (SGS), etc. Each of them involves different statistical assumptions. Despite the growing  
81 literature of these methods, there are major limitations associated with their application for  
82 site investigation in an urban setting. For instance, many development sites in densely-  
83 populated cities may span across hundreds of metres to tens of kilometres, within which a  
84 large number (i.e., hundreds to thousands) of samples may be needed to provide adequate  
85 precision for meaningful geostatistical analyses to aid the site development plans. Meanwhile,  
86 the concentrations of trace elements may display three-dimensional spatial variations across  
87 the subsurface soil domain, which should be properly accounted for in such analyses. Many  
88 previous studies (e.g., [Santra et al., 2012](#); [Chakraborty et al., 2012](#); [Chakraborty et al., 2017](#);  
89 [Zhang & Yang, 2017](#)) discussed the accuracy of various geostatistical approaches through  
90 cross validation measures and indicators such as root-mean-square-error (RMSE) and mean  
91 percentage error (MPE). However, the uncertainty associated with As distribution across the  
92 site is rarely discussed in detail. While quantification of uncertainty is essential from the  
93 project management perspective, such estimates are often difficult to verify.

94 This study extends the integrated framework for spatial variability analyses from our  
95 recent studies ([Liu et al., 2017](#); [Liu & Leung, 2017](#)), incorporating the restricted maximum  
96 likelihood (REML) method with a three-dimensional, anisotropic autocorrelation structure,  
97 tailored for analysing the concentrations of trace elements in soils. Effectiveness of the  
98 approach is illustrated by the implementation on a major development site in Hong Kong,  
99 where borehole sampling of As is performed in multiple stages. The current study articulates  
100 the spatial extent of the geogenic As, and proposes a rational approach to quantify the  
101 associated uncertainty, hence improving the effectiveness of geoenvironmental sampling  
102 strategy for site assessment and remediation.

103

## 104 **2. Methodology**

### 105 **2.1. Soil sampling and analysis**

106 A new development site located in the New Territories in Hong Kong is discussed in this  
107 study to illustrate the three-dimensional variations of As concentrations, and how the  
108 variability can be characterized by the proposed framework in this study. Two stages of  
109 geoenvironmental investigation were performed within this development area, with their key  
110 information summarized in Table 1. In order to identify potential contamination at the site,  
111 the Stage 1 investigation was performed (Fig. 1a) at an early stage of the project, which  
112 included drilling of 35 boreholes, with 388 soil samples retrieved from different depths for  
113 the testing of As concentrations (HK CEDD, 2015). The locations of boreholes had been  
114 strategically selected for broad coverage across the development area (approximately 1,600 m  
115 × 2,700 m on plan), considering both site accessibility and the locations of future structures.  
116 Within the development area, there was a smaller site (around 100 m × 200 m) of particular  
117 concern in the project. The site was where one of the first structures (Building A shown in  
118 Fig. 1) would be constructed, and the As concentrations at this location had to be assessed to  
119 formulate appropriate mitigation measures. However, during the Stage 1 investigation, no  
120 boreholes had been drilled within this site due to accessibility issues at that stage. Predictions  
121 were therefore made by the proposed approach (Section 2.2), utilizing all 388 sample values  
122 obtained from the Stage 1, and their corresponding spatial information. Shortly before  
123 construction of Building A, the Stage 2 investigation was conducted with 12 additional  
124 boreholes and 205 samples across the site (Fig. 1b). These additional samples are treated in  
125 this study as verification data for independent assessments of the accuracy of the proposed  
126 approach.

127 During the geoenvironmental investigation, the average depth of boreholes is  
128 approximately 20 m (with minimum of 2 m and maximum of 52 m), where soil samples were  
129 retrieved every 0.5 – 2 m along the depth of borehole. Soil samples obtained at varying depth  
130 of the boreholes were sealed to be air-tight at the site, frozen with ice packs in freezing boxes  
131 before and during transportation, and stored at 4°C in the dark in cold chamber. Selected  
132 samples were freeze-dried within two weeks upon sample delivery and stored at -20°C in a  
133 refrigerator before spectroscopic analysis (Beiyuan et al., 2017; Li et al., 2017). The As  
134 concentrations in the digested samples were determined by using an Inductively Coupled  
135 Plasma-Atomic Emission Spectrometry (ICP-AES, Perkin Elmer Optima 3300DV), with the  
136 limit of detection of 1 mg/kg. In addition, 20% random replicates and spiked samples were  
137 included for quality control of sample analysis, and NIST Reference Soil 2710A (containing  
138 1540 mg/kg As) were used for quality assurance. The As recovery rate was 91-93% after  
139 digestion and ICP-OES analysis, thus suggesting good recovery and reliability/reproducibility.

140 During the Stage 1 investigation, the laboratory tests revealed variations of As  
141 concentrations ranging from 1 mg/kg to 1,220 mg/kg dry soil in the 388 samples. The  
142 enormous variations in concentrations, with the occasional detection of high levels of As,  
143 warranted careful assessments of contamination levels during the planning stages of the  
144 development. In particular, it was important to make predictions of As levels at the Building  
145 A site in order to formulate the mitigation measures before the construction.

146

## 147 **2.2. Geostatistical characterization method**

148 Based on the locations of the 388 Stage 1 samples and their corresponding As concentrations,  
149 the spatial variability features can be established for better prediction at the Building A site  
150 using geostatistical approach. In many previous attempts of geostatistical characterization of  
151 trace elements (e.g. Zupan et al. 2000; Burgos et al. 2006; Yang et al., 2009, etc.), a

152 semivariogram ( $\gamma$ ) is developed for the spatial variable,  $\mathbf{z}$ , that represents concentrations of  
153 the trace elements at different locations  $\mathbf{x}$ :

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [z(x_i) - z(x_i + h)]^2 \quad (1)$$

154 where  $z(x_i)$  represents the value of  $\mathbf{z}$  at location  $x_i$ ;  $N(h)$  is the number of pairs of samples  
155 that are separated at a distance  $h$  (in any direction). In many of these studies, however, the  
156 spatial trend was not explicitly discussed or considered in the analyses. However, the  
157 accuracy of the semivariogram analyses can be affected if the ‘deterministic’ trend is not  
158 properly defined based on the site data. In addition, depending on the site history and  
159 geochemical properties of the specific trace element, its spatial variations may display  
160 features of three-dimensional anisotropy: the concentrations may be relatively uniform in one  
161 direction, but show more variations in other directions in the subsurface domain. These  
162 features cannot be rigorously considered by Eq. (1).

163 This study extends the integrated approach from our recent work (Liu et al., 2017; Liu &  
164 Leung, 2017) for three-dimensional spatial variability analyses of As concentrations in soils.  
165 In these previous studies, the restricted maximum likelihood (REML) is incorporated with  
166 rigorous statistical tests to ensure that the assumptions on normality and stationarity are  
167 satisfied, while the optimal polynomial order for the trend can be determined, and with  
168 potential outliers in the dataset identified in an integrated framework of spatial data analysis.  
169 Liu & Leung (2017) also showed that patterns of spatial variations observed in various  
170 directions of natural soil/rock properties can be interpreted together with geological settings  
171 of the site. In the current study, the framework is implemented with considerations of three-  
172 dimensional spatial correlation structure, where the estimates of As concentration profiles can  
173 be sequentially updated when additional data become available from the geoenvironmental  
174 investigation. The proposed approach will be compared with traditional geostatistical

175 methods in characterizing the spatial variations of As-containing soils. While the analyses  
 176 enhance our understanding on the distributions of geogenic As in soils, this study further  
 177 elaborates on their significance to site-specific uncertainty quantification and management  
 178 strategies. In the current work, the spatial distributions of As, referred to as  $\mathbf{z}$ , consists of a  
 179 large-scale ‘deterministic’ trend across the site, and residual effects that represent the local  
 180 deviations from the trend:

$$\mathbf{z} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (2)$$

181 where  $\mathbf{X}$  is a matrix containing the spatial coordinates of sample points in three dimensions.  $\boldsymbol{\beta}$   
 182 is the vector of trend coefficients, and  $\mathbf{X}\boldsymbol{\beta}$  combine to produce the trend.  $\boldsymbol{\varepsilon}$  represent the  
 183 residuals, or deviations from the trend, which is often found to be spatially correlated: the  
 184 values of  $\varepsilon_i$  and  $\varepsilon_j$  are often similar if the locations  $i$  and  $j$  are close to each other, while  
 185 greater variations are observed for larger separation distances. To simulate such effects, the  
 186 Gaussian (squared exponential) autocorrelation function is adopted:

$$R(h) = \exp \left[ - \left( \frac{h_x^2 + h_y^2}{\theta_1^2} \right) - \frac{h_z^2}{\theta_2^2} \right] \quad (3)$$

187 where  $h_x$ ,  $h_y$  and  $h_z$  are the separation distances in  $x$ ,  $y$  and  $z$  directions, respectively.  $\theta_1$   
 188 and  $\theta_2$  are range parameters in the lateral and vertical directions, also known as  
 189 autocovariance distances. The significance of Eq. (3) lies on its flexibility in modelling  
 190 anisotropic spatial correlations, as different values of  $\theta_1$  and  $\theta_2$  may arise as a result of the  
 191 geogenic origin of the trace element in nature. This has not been considered by previous  
 192 REML analyses (e.g. [Lark & Cullis, 2004](#)), and the importance of such features will be  
 193 illustrated by the case study presented in later sections. Considering all sampling points  
 194 within the site,  $R(h)$  for different  $i$  and  $j$  locations combine to form the matrix  $\mathbf{R}$ , which  
 195 represents the smooth scale spatial variations in As concentrations. Apart from such spatial  
 196 variations, there is also contribution from the ‘white noise’ (nugget) effect in the overall



197 variance of concentrations. This white noise may be due to handling/measurement errors for  
 198 the specimens, or other random natural variations that do not correlate with separation  
 199 distances. The variance contributions from the smooth scale variations and white noise  
 200 effects are represented by  $\sigma_e^2$  and  $\sigma_n^2$ , respectively. The variance of  $\boldsymbol{\varepsilon}$ , denoted as  $\mathbf{V}$  herein,  
 201 can then be expressed as:

$$\mathbf{V} = \sigma_e^2 \mathbf{R} + \sigma_n^2 \mathbf{I} = (\sigma_e^2 + \sigma_n^2)[s\mathbf{R} + (1 - s)\mathbf{I}] \quad \text{where } 0 \leq s = \frac{\sigma_e^2}{(\sigma_e^2 + \sigma_n^2)} \leq 1 \quad (4)$$

202 and  $\mathbf{I}$  is the identity matrix. Therefore, the objectives of the analyses are to obtain the trend  
 203 coefficients,  $\boldsymbol{\beta}$ , and parameters that characterize three-dimensional spatial variability of  $\boldsymbol{\varepsilon}$  (i.e.,  
 204  $s, \theta_1, \theta_2$ ), based on the measured data  $\mathbf{z}$ . The REML method is adopted for this purpose, and  
 205 the approach mainly involves finding the set of parameters  $\boldsymbol{\Theta} = \{s, \theta_1, \theta_2\}$  that maximizes the  
 206 following log-likelihood function:

$$L(\boldsymbol{\Theta}|\mathbf{y}) = -\frac{n-p}{2} \log(2\pi) - \frac{1}{2} \log|\mathbf{V}| - \frac{1}{2} \log|\mathbf{W}| - \frac{1}{2} \mathbf{y}^T \mathbf{V}^{-1} \mathbf{Q} \mathbf{y} \quad (5)$$

207 where  $\mathbf{W} = \mathbf{X}^T \mathbf{V}^{-1} \mathbf{X}$ ;  $\mathbf{Q} = \mathbf{I} - \mathbf{X} \mathbf{W}^{-1} \mathbf{X}^T \mathbf{V}^{-1}$ ;  $\mathbf{y} = (\mathbf{I} - \mathbf{X}(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T) \mathbf{z}$ , and represents the  
 208 filtered dataset with the trend components filtered out. The maximization can be treated as an  
 209 optimization problem, i.e. to obtain the set of  $\boldsymbol{\Theta}$  parameters that best matches the observed  
 210 data  $\mathbf{z}$ . This can be achieved by various optimization techniques; the current study adopts the  
 211 differential evolution algorithm, and details of this heuristic algorithm can be found in [Storn](#)  
 212 [& Price \(1997\)](#). With  $\boldsymbol{\Theta}$  determined, the autocorrelation structure of  $\boldsymbol{\varepsilon}$  is well defined. The  
 213 trend coefficients  $\boldsymbol{\beta}$  can then be estimated (i.e.  $\hat{\boldsymbol{\beta}}$ ) using general least squares method, and  
 214 predictions at unsampled locations,  $\hat{\mathbf{z}}$ , and the corresponding uncertainty in prediction  
 215 (prediction variance  $\sigma_z^2$ ), can be estimated based on the best linear unbiased prediction  
 216 (BLUP) technique ([Atkinson et al., 2008](#); [Santra et al., 2012](#)):

$$\hat{\mathbf{z}} = \mathbf{X}_0 \hat{\boldsymbol{\beta}} + \mathbf{K}^T \mathbf{V}^{-1} \hat{\boldsymbol{\varepsilon}} \quad (6a)$$

$$\sigma_z^2 = \text{diag}[\mathbf{K}_0 - \mathbf{K}^T \mathbf{V}^{-1} \mathbf{K} + \mathbf{M}^T (\mathbf{X}^T \mathbf{V}^{-1} \mathbf{X})^{-1} \mathbf{M}] \quad (6b)$$

217 where  $\mathbf{X}_0$  is the matrix containing spatial coordinates of unsampled (prediction) locations;  $\mathbf{K}$   
 218 is the covariance matrix between observations and predictions, i.e.  $\mathbf{K} = \text{cov}[\mathbf{z}(\mathbf{x}), \mathbf{z}(\mathbf{x}_0)]$ ,  
 219  $\mathbf{K}_0 = \text{cov}[\mathbf{z}(\mathbf{x}_0), \mathbf{z}(\mathbf{x}_0)^T]$  and  $\mathbf{M} = \mathbf{X}_0^T - \mathbf{X}^T \mathbf{V}^{-1} \mathbf{K}$ . In general, the level of uncertainty is low  
 220 near existing samples, and increases with separation distance away from these sample points.  
 221 In other words, the uncertainty at unsampled areas varies in three dimensions across the site  
 222 domain, and is dependent on both spatial locations of all existing samples, the trend of the  
 223 variations,  $\boldsymbol{\beta}$ , and autocorrelation features represented by  $\boldsymbol{\Theta}$ .

224

### 225 **3. Results and Discussion**

#### 226 **3.1. Spatial variability analysis based on Stage 1 samples**

227 The As concentrations obtained from the 388 samples of Stage 1 investigation are first  
 228 analyzed using the proposed REML approach. The sample values are log-transformed, as  
 229 positive-value variables in geochemistry often follow the lognormal distribution. Figure 2  
 230 shows the statistical distributions of the As data and the log-transformed values, which  
 231 further validates the assumption of lognormal distribution. Regarding the deterministic trend  
 232 function, a quadratic structure for lateral directions and linear trend for vertical direction are  
 233 adopted to represent the large-scale variations across the site ( $\mathbf{X}\boldsymbol{\beta}$ ). Using the observed data  $\mathbf{z}$   
 234 (vector with 388 components), the  $\boldsymbol{\Theta}$  parameters that maximizes the log-likelihood function  
 235 (Eq. 5) are found to be  $s = 0.57$ ,  $\theta_1 = 130.9$  m and  $\theta_2 = 4.2$  m. These are the parameters  
 236 that characterize the spatial variability of As concentrations, and the large difference between  
 237  $\theta_1$  and  $\theta_2$  illustrates the strong anisotropy effects regarding the influence range the  
 238 correlation: the variations are more abrupt along the vertical direction (small  $\theta_2$ ), compared  
 239 to a more gradual transition along the lateral directions. Figure 3a shows the corresponding  
 240 autocorrelation functions ( $R$ ), decoupled in the vertical and lateral directions. As a

241 comparison with more conventional geostatistical approaches, a separate analysis is  
 242 conducted using the method of moments (MoM), where the autocovariance is defined by:

$$C(h) = \frac{1}{N(h)} \sum_{i=1}^{N(h)} [z(x_i) - t(x_i)][z(x_i + h) - t(x_i + h)] \quad (7)$$

243 with  $\mathbf{t}$  representing the trend component vector (i.e.  $\mathbf{t} = \mathbf{X}\boldsymbol{\beta}$ ). The similarity between Eqs. (1)  
 244 and (7) should be noted, as the MoM approach is closely related to the determination of  
 245 semivariogram in geostatistics (Reilly & Gelman, 2007; Oliver & Webster, 2014). Together  
 246 with the assumption of isotropic spatial feature ( $\theta_1 = \theta_2$ ), there is no need to separate  $h_x$ ,  $h_y$   
 247 and  $h_z$ , and the corresponding MoM approach is similar to the geostatistical approach  
 248 adopted by Zupan et al. (2000), Burgos et al. (2006) and Yang et al. (2009). The spatial  
 249 correlation structure obtained by this MoM analysis is shown in Fig. 3b for comparison. The  
 250 accuracies of the two approaches will be compared and discussed in more detail in the next  
 251 section.

252 The proposed REML approach allows the prediction of As concentrations at unsampled  
 253 locations ( $\hat{\mathbf{z}}$ ), and quantification of the associated uncertainty through the prediction variance  
 254 ( $\sigma_z^2$ ), across the three-dimensional subsurface domain of the entire development site. For  
 255 example, the ground surface at the Building A site is at +15 mPD (mPD: metres above Hong  
 256 Kong Principal Datum, which is 1.230 m below Mean Sea Level), and Fig. 4 shows the  
 257 predictions of As concentrations and the prediction variance at three different depths  
 258 (+10 mPD, 0 mPD, -10 mPD).

259 Based on the raw data of measured As concentrations from Stage 1 investigation, the  
 260 sample variance is 1.99 (in log-space, since the concentrations are assumed to follow  
 261 lognormal distribution). Figure 4b shows that the value of prediction variance across the  
 262 Building A site is relatively uniform, with a magnitude around 1.6, which illustrates two  
 263 interesting aspects of the spatial correlation analyses. Firstly, the difference between raw data

264 variance ( $\approx 1.99$ ) and prediction variance of residuals ( $\approx 1.6$ ) arises from the removal of the  
265 deterministic trend ( $\mathbf{X}\boldsymbol{\beta}$ ), or the de-trending process, in the proposed approach. Meanwhile,  
266 the almost uniform prediction variance across Building A site means that the uncertainty is  
267 not heavily influenced (reduced) by any ‘nearby’ existing samples. In fact, the closest  
268 borehole from Stage 1 investigation is more than 100 m away from the site, while the  
269 horizontal range of influence ( $\theta_1$ ) for each sample is only around 131 m according to the  
270 analyses. In other words, considering the spatial features of As in this region, the Stage 1  
271 boreholes and samples are not located close enough to the Building A site to aid the  
272 assessment of contamination levels there. These spatial correlation features also carry  
273 important implications to the predictive capabilities of various approaches, as will be  
274 illustrated in the next section.

275

### 276 **3.2. Predictions and validations with Stage 2 investigation data**

277 While a number of previous studies described the geostatistical characterization of trace  
278 elements such as As, there is limited discussion on the accuracy of the approaches,  
279 particularly within the context of predictive capabilities verified through subsequent geo-  
280 environmental investigation. In this study, the two aspects of predictions, namely the best  
281 estimates ( $\hat{\mathbf{z}}$ ) and uncertainty quantification ( $\sigma_z^2$ ) by Eqs. (6a) and (6b), are compared with the  
282 additional sample data retrieved from Stage 2 investigation at the Building A site. The  
283 advantages of the proposed approach over ‘conventional’ techniques of MoM are also  
284 illustrated through the verification process.

285 Comparisons between predicted and measured As concentrations can be made relatively  
286 easily for the Stage 2 sample data. On the contrary, it is more difficult to verify the  
287 ‘uncertainty’ estimates, since the prediction variance varies across the site, but there is only  
288 one measured value, and therefore one prediction error, at any location in the subsurface

289 domain. An indirect approach to verify the  $\sigma_z^2$  estimates is to normalize the prediction error  
290 ( $\tilde{\varepsilon}_i = \hat{z}_i - z_i$ ) for each location  $i$  by the value of  $\sigma_{z,i}$  evaluated at that location. The  
291 normalized errors associated with all Stage 2 samples can then be compared with the standard  
292 normal distribution. This comparison is based on the proposition that with a proper  
293 characterization of uncertainty, the prediction errors across the domain should follow a  
294 normal distribution, with the variance denoted by the corresponding  $\sigma_{z,i}^2$  at the location.

295 Figure 5 shows the histograms of normalized prediction errors (in log-space) using both  
296 the proposed REML approach (Eqs. 2-6), and the conventional MoM approach (Eq. 7). In  
297 Fig. 5a, the predictions are made solely based on the Stage 1 data (388 samples), and both  
298 approaches generally underestimated the As concentrations at the Building A site, with mean  
299 error smaller than zero. As mentioned earlier, considering the horizontal spatial range ( $\theta_1$ ) of  
300 131 m, and that the closest borehole is more than 100 m away, it is not surprising that both  
301 approaches cannot accurately predict the As concentrations at Building A, as the ‘knowledge’  
302 of Stage 1 samples do not reach far enough to provide useful information at that site.

303 In the next modelling scenario, data from two of the twelve Stage 2 boreholes is  
304 incorporated into the geostatistical analyses, and predictions are then made at the locations of  
305 the remaining ten boreholes. Since these boreholes are within the Building A site with plan  
306 dimensions in the same order as  $\theta_1$ , incorporating such additional information is expected to  
307 enhance the accuracy of the approach. In each analysis, both the  $\hat{z}$  and  $\sigma_z^2$  estimates are  
308 updated using sample information from the two additional boreholes. Also, for a  
309 comprehensive comparison, all possible combinations of locations of the two ‘known’  
310 boreholes are simulated, and the corresponding normalized prediction errors at the ten  
311 ‘unknown’ borehole locations are summarized in the histograms in Fig. 5b. With the  
312 knowledge from two additional boreholes, predictions by the proposed REML approach are  
313 improved, both in terms of the best estimates (mean error closer to zero) and with error

314 distribution closely resembling the standard normal distribution. In contrast, results by the  
315 MoM are not significantly improved, mainly due to the fact that the three-dimensional nature  
316 of spatial variability cannot be properly reflected (Fig. 3) and assimilated into the prediction  
317 model. The similar exercise is then repeated for two other scenarios: (1) incorporating sample  
318 data from four boreholes and making predictions at the remaining eight borehole locations  
319 (Fig. 5c); and (2) incorporating sample data from six boreholes and making predictions at the  
320 remaining six locations (Fig. 5d). In both cases, the improvement by the REML approach is  
321 more substantial than the conventional MoM approach: the mean prediction error is close to  
322 zero using the REML approach, and the distributions of normalized error roughly follow the  
323 standard normal distribution. Applying this concept to realistic project situations, it would be  
324 desirable to continuously refine the predictions at unsampled locations, based on new sample  
325 data as the investigation programme progresses.

326 As a further comparison between the proposed REML approach and conventional MoM  
327 analyses, Fig. 6 shows the results of one simulated scenario incorporating data from two  
328 boreholes of Stage 2 investigation, to predict the As concentrations at the other ten borehole  
329 locations. In this example, 41% of the measured values fall within  $\pm 30\%$  of the REML  
330 predictions, which is a significantly higher percentage than using the conventional method.  
331 Similarly, Fig. 7 shows the simulated scenario with knowledge of six boreholes and  
332 prediction of the other six, in which case almost 60% of the measured values fall within  
333  $\pm 30\%$  of the REML predictions. In fact, the substantial variability in the natural occurrence of  
334 As makes it unlikely for any modelling approach to precisely pinpoint their concentrations at  
335 all locations, and the occasional existence of extreme values does comply with classical  
336 statistical theories. Nonetheless, the improved predictive power of the proposed approach,  
337 and the capabilities to rationally quantify and represent the uncertainty (Fig. 5), mean that it

338 can become a useful tool in assessing the risks associated with the occurrence of As, or  
339 potentially other contaminants or trace elements, at a development site.

340 Figures 8 and 9 show the contours of predicted As concentrations, and the associated  
341 prediction variance (in log-space) after drilling and sampling at the six and twelve Stage 2  
342 boreholes at the Building A site. It should be noted that the colour scales in Figs. 3, 8 and 9  
343 are consistent with each other, which facilitate comparison between the estimates as more  
344 information becomes available. Also, combining the best estimates with prediction variance  
345 (after back-transformation to original space), the range of predictions with different  
346 confidence levels (e.g., mean  $\pm$  standard deviation) can be established, which provide an  
347 alternative means to demonstrate the risk levels associated with As occurrence across the site.

348

### 349 **3.3. Effectiveness of sampling in reducing uncertainty of As concentrations**

350 The proposed approach also provides information regarding the effectiveness of additional  
351 samples in reducing uncertainty. From a risk management perspective, this would be a useful  
352 indicator on the required number of samples to adequately characterize As distributions  
353 across the site. Following previous discussions, the uncertainty indicator, i.e. the prediction  
354 variance, varies spatially in three dimensions across the entire subsurface domain at all  
355 unsampled locations, and can only be shown in a series of contours. In this section, a simpler  
356 definition is adopted, and denoted as the prediction variance reduction factor (PVRF). PVRF  
357 is calculated by first obtaining the average prediction variance across the site (e.g.,  
358 Building A), and then evaluating the change of this average variance upon drilling and  
359 sampling at a new borehole location. This quantifies the significance of each borehole (and  
360 its samples) in reducing the overall uncertainty. To quantify the effectiveness of a particular  
361 borehole,  $PVRF_i$  is defined by:

$$\text{PVRF}_i(\%) = \frac{PV_{i-1} - PV_i}{PV_{i-1}} \times 100\% \quad (8)$$

362 where  $PV_i$  is the average prediction variance across the site after drilling borehole  $i$ . Therefore,  
 363 PVRF may be interpreted as the percentage reduction in overall uncertainty associated with  
 364 each successive borehole. A low level of  $\text{PVRF}_i$  refers to a low impact from borehole  $i$  in  
 365 further reducing uncertainty in As variations. By evaluating the value of PVRF associated  
 366 with each borehole, its effectiveness on the overall uncertainty can be quantified. It is then  
 367 possible to make informed decisions regarding the appropriate number of boreholes and  
 368 samples, to balance the need to characterize site uncertainty and the time and resources  
 369 associated with drilling, sampling and testing at each additional borehole location.

370 Figure 10 shows the PVRF values obtained for the twelve boreholes at the Building A  
 371 site. The data point represents the average value obtained by simulation of 1000 realizations,  
 372 with the error bars illustrating the standard deviations. These realizations are necessary since  
 373 the calculation of PVRF is a sequential process, and depends on the order of  
 374 drilling/sampling at the twelve locations. For example, sampling at the first borehole will  
 375 reduce the overall uncertainty by about 16% on average, while the subsequent boreholes will  
 376 become less effective as more and more information becomes available within Building A  
 377 site. In particular, after drilling and sampling at the sixth borehole location, the uncertainty  
 378 reduction by each subsequent borehole will be generally less than 5%. Figures 5d and 7 also  
 379 show that with the sample data from six boreholes, the As distributions and uncertainty can  
 380 be predicted with reasonable accuracy using the proposed approach. Such information can  
 381 become useful when site-specific geoenvironmental sampling strategies are devised.

382

#### 383 **4. Conclusions**

384 This study outlines the geostatistical approach using REML, which has been extended for  
 385 consideration of three-dimensional spatial autocovariance structure. The proposed approach



386 is applied to investigate the spatial distributions of geogenic As concentrations at a  
387 development site in Hong Kong, which involved 388 samples and 205 samples retrieved  
388 during two separate stages of geoenvironmental investigation. Data from the multi-stage  
389 investigation are utilized in the verification of the proposed approach, both regarding the best  
390 estimates of As levels and the associated uncertainty at previously unsampled areas. The  
391 proposed approach is shown to produce more accurate predictions than conventional  
392 geostatistical approaches, even with a relatively small dataset in a large development area.  
393 The importance of site-specific characterization of spatial variability is also highlighted, as  
394 the accuracy of predictions depend heavily on correlation parameters, geometry of the site  
395 and locations of existing sample information. The proposed approach also addresses another  
396 key consideration in development sites, which is the quantification of uncertainty at  
397 unsampled locations. The estimated prediction variance can be condensed into a simple  
398 indicator defined as PVRF in this study, which quantifies the effectiveness of  
399 geoenvironmental sampling in reducing the uncertainty levels of As concentrations across the  
400 site. This helps to supplement the conventional approach that rely on qualitative expert  
401 opinions, as it provides quantitative indication to support the decision-making process  
402 associated with the necessity and strategies of future sampling, considering the tolerable risk  
403 levels and financial setup of the project.

404

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**Table**

Table 1 Site area and sampling information for Stages 1 and 2 investigations

	Stage 1 (Fig. 1a)	Stage 2 (Fig. 1b)
Target area	Entire development area (1,600 m × 2,700 m)	Building A site (100 m × 200 m)
No. of boreholes	35	12
No. of samples	388	205
Average spacing of boreholes	870 m	57 m
Vertical sampling interval	0.5 – 2 m	0.5 – 2 m



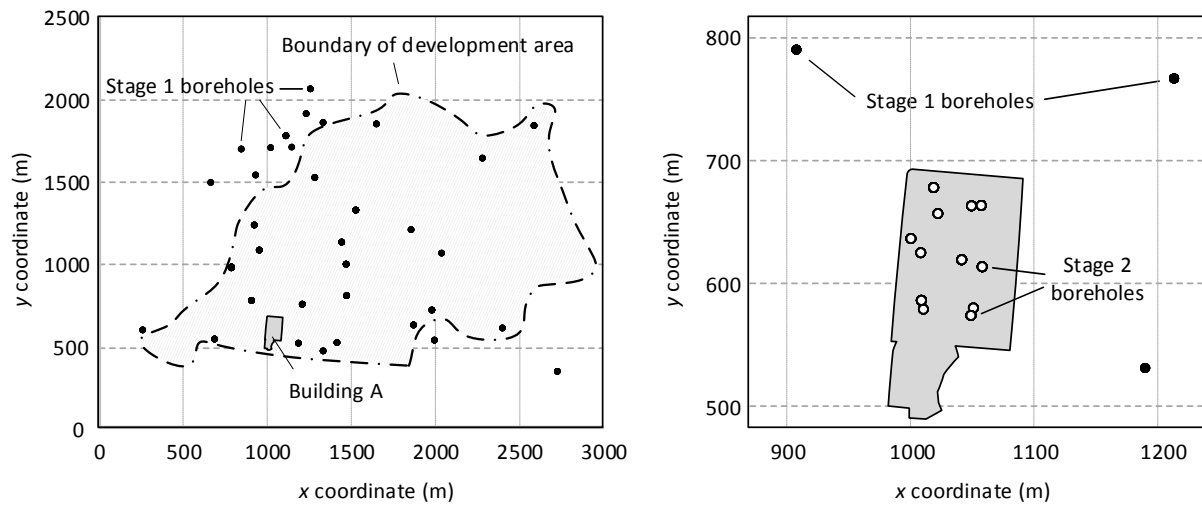


Figure 1 Site area and sampling locations for Stages 1 and 2 investigations

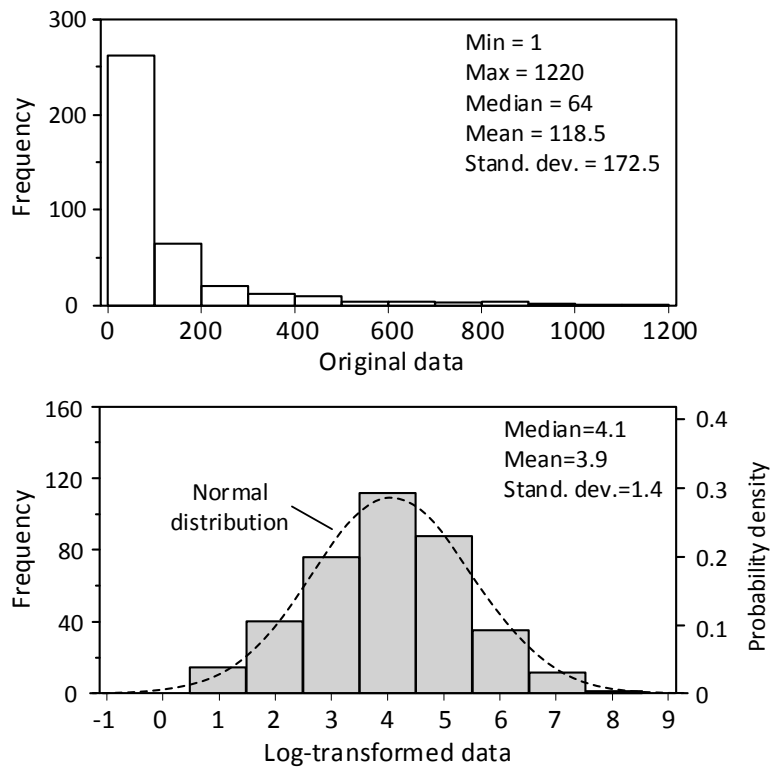


Figure 2 Statistical distribution of As concentrations from Stage 1 investigation

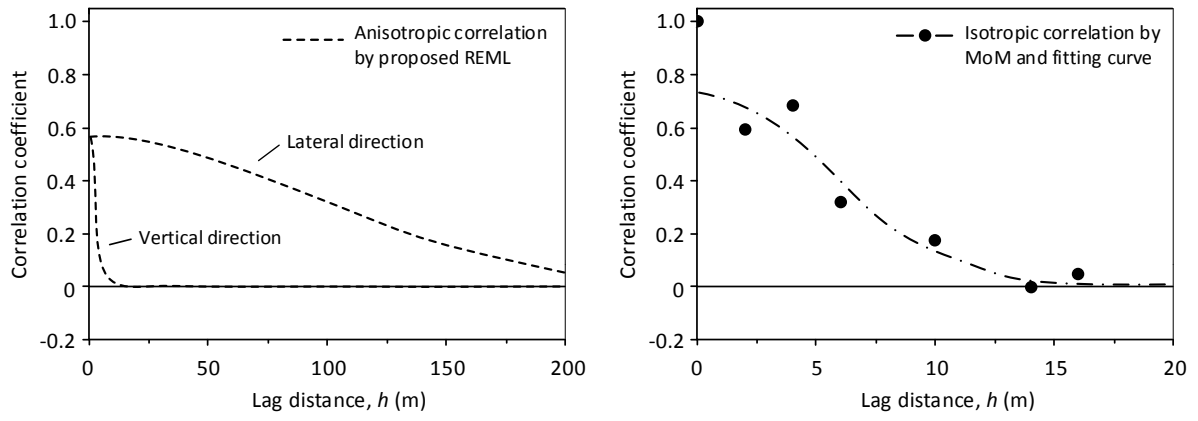


Figure 3 Autocorrelation structure estimated by proposed approach and conventional MoM

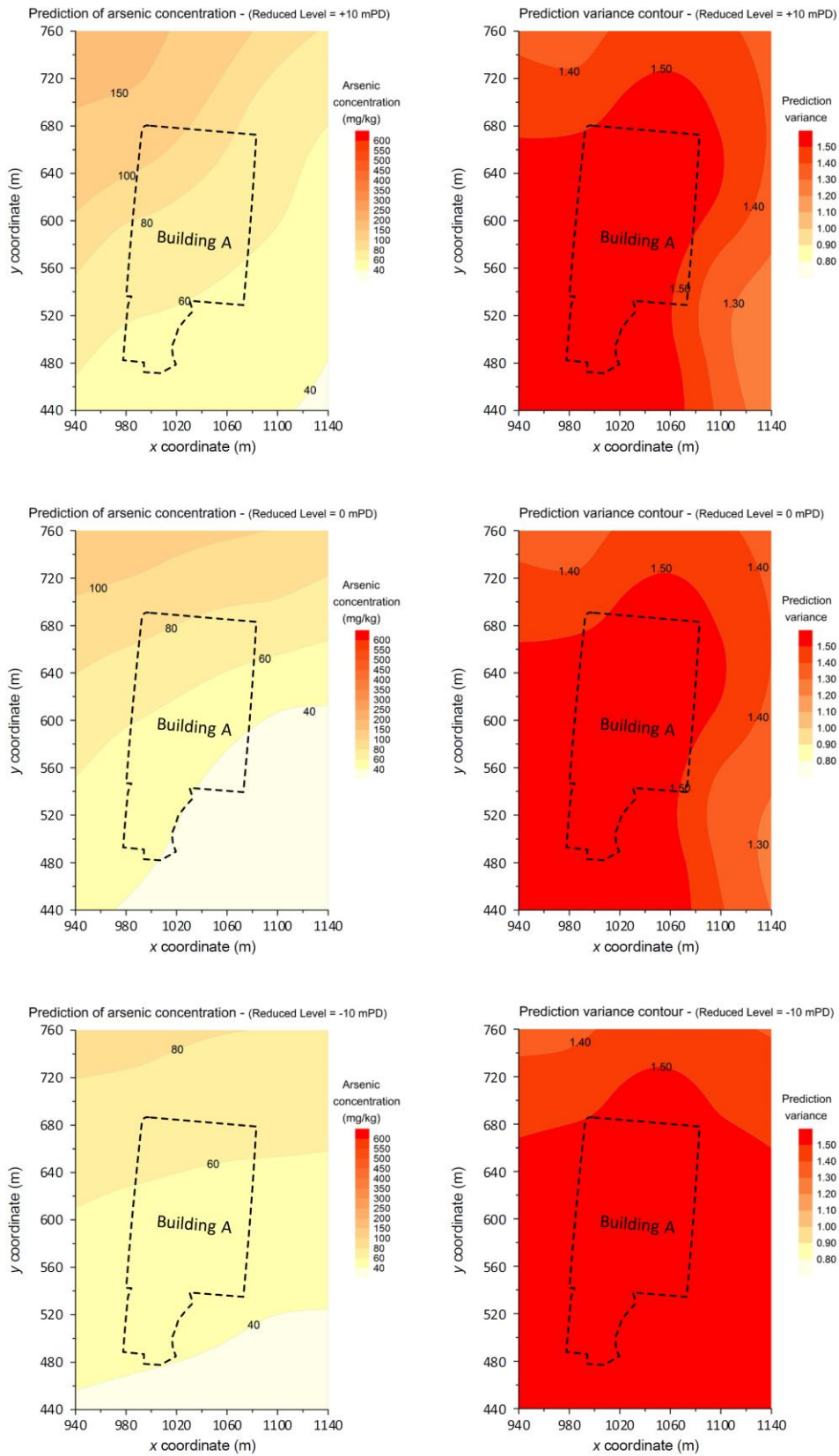


Figure 4 Predicted As concentrations (left) and prediction variance (right) using Stage 1 samples

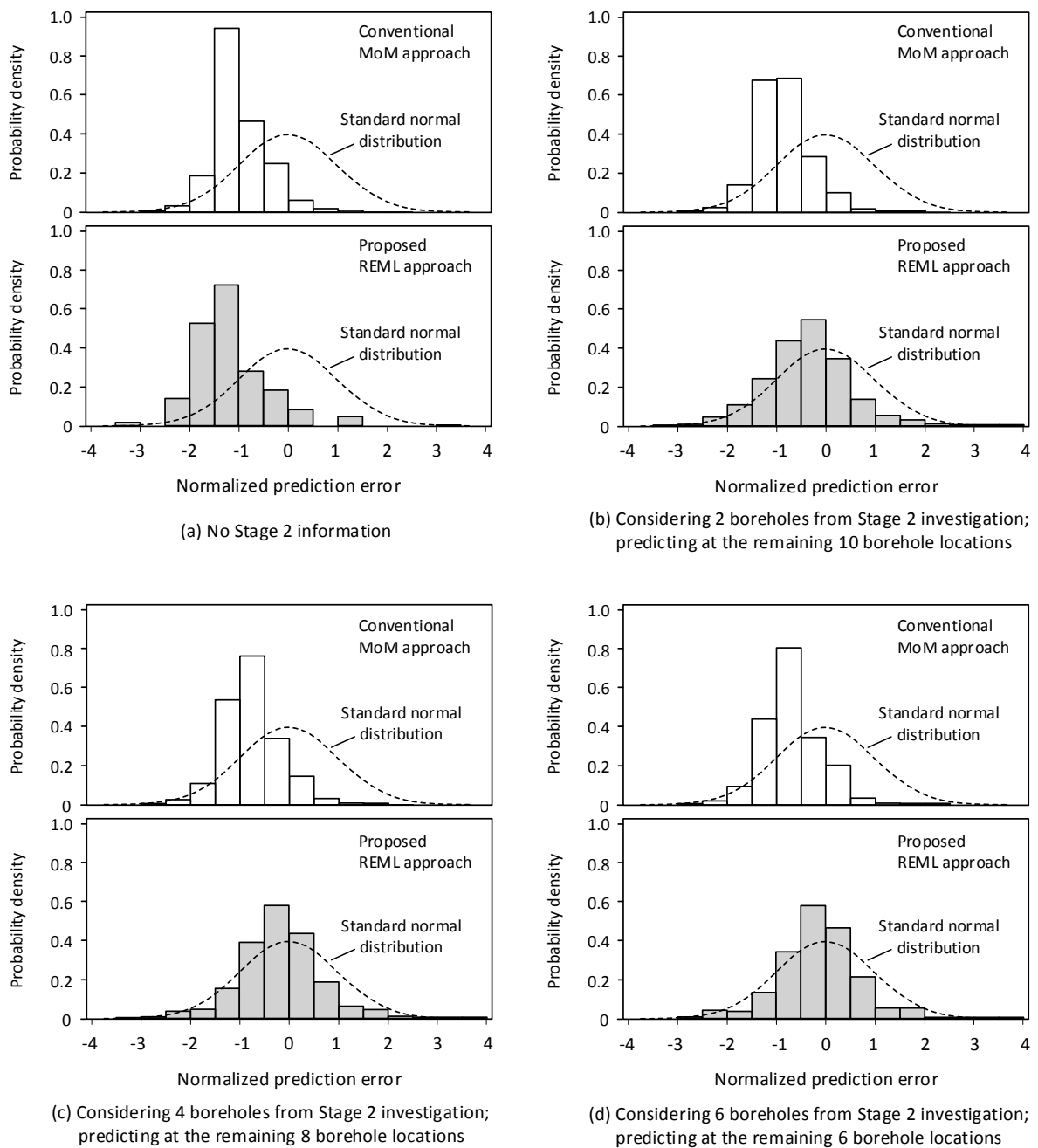


Figure 5 Histograms of normalized prediction errors: (a) with no stage 2 sample data; (b) incorporating two Stage 2 borehole data; (c) incorporating four Stage 2 borehole data; (d) incorporating six Stage 2 borehole data

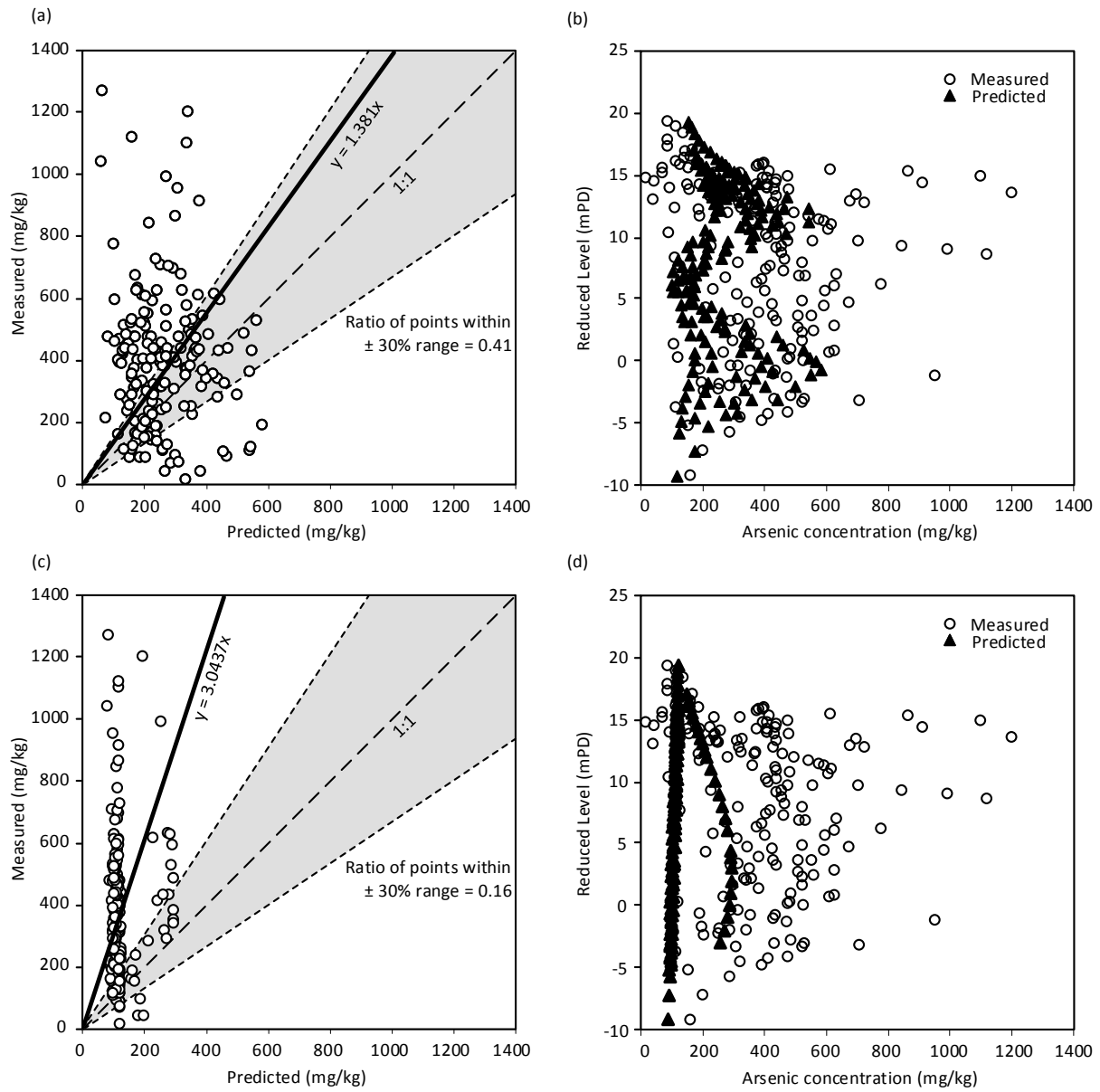


Figure 6 Comparisons between measured As concentration with (a, b) REML and (c, d) MoM predictions incorporating two Stage 2 borehole data

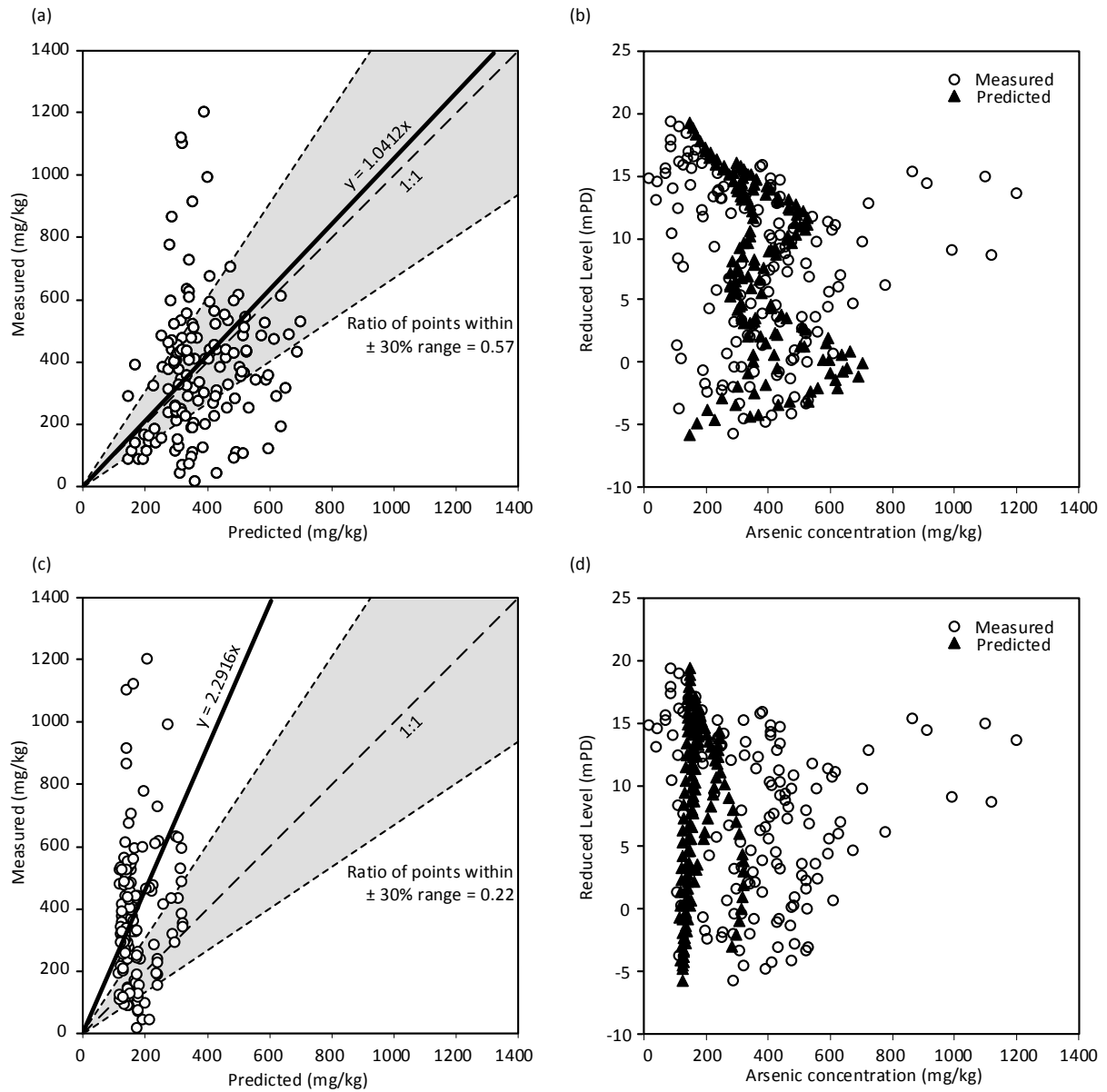


Figure 7 Comparisons between measured As concentration with (a, b) REML and (c, d) MoM predictions, incorporating six Stage 2 borehole data

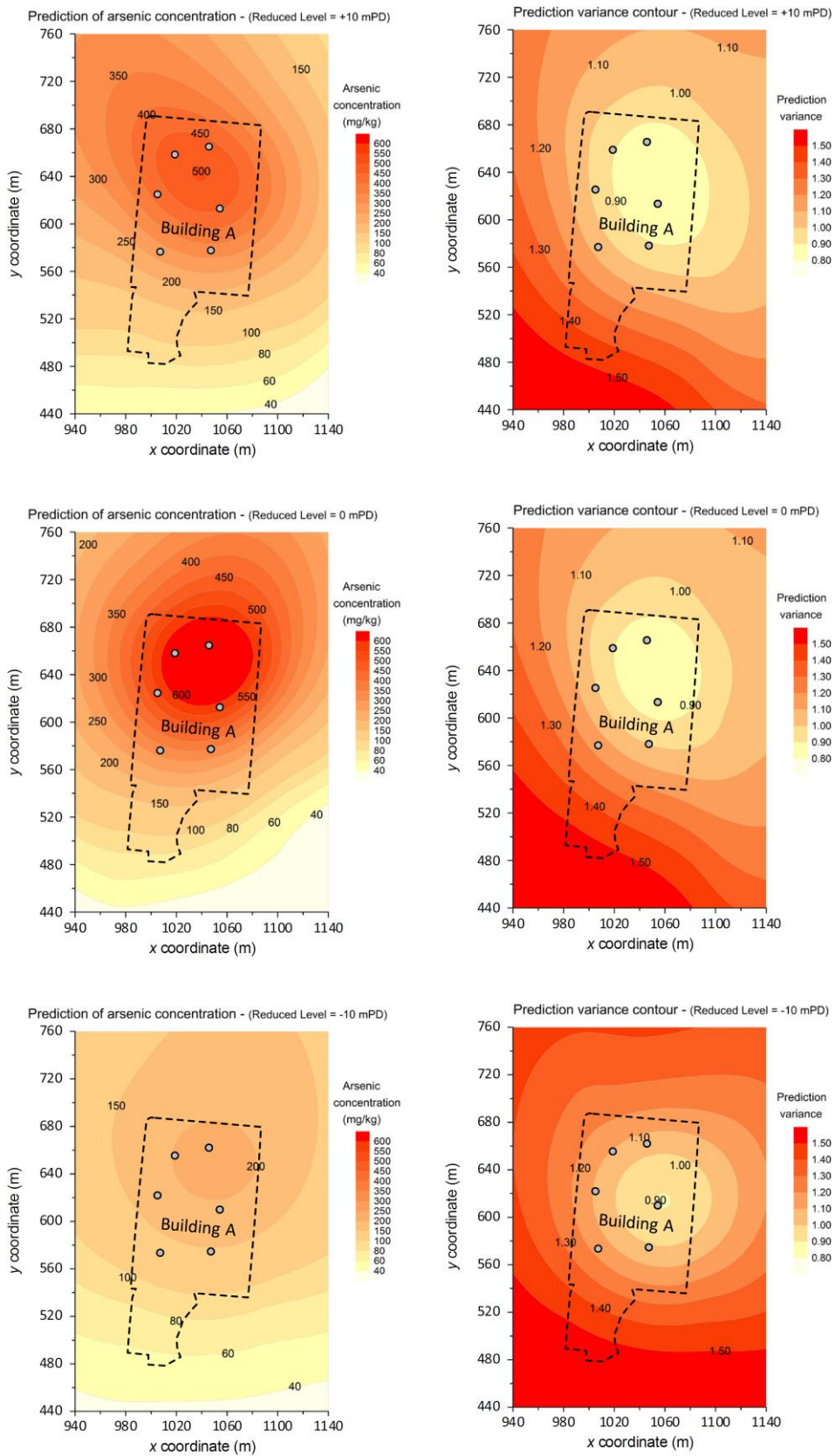


Figure 8 Predicted As concentrations (left) and prediction variance (right) after drilling and sampling at six borehole locations (grey circles) in Stage 2



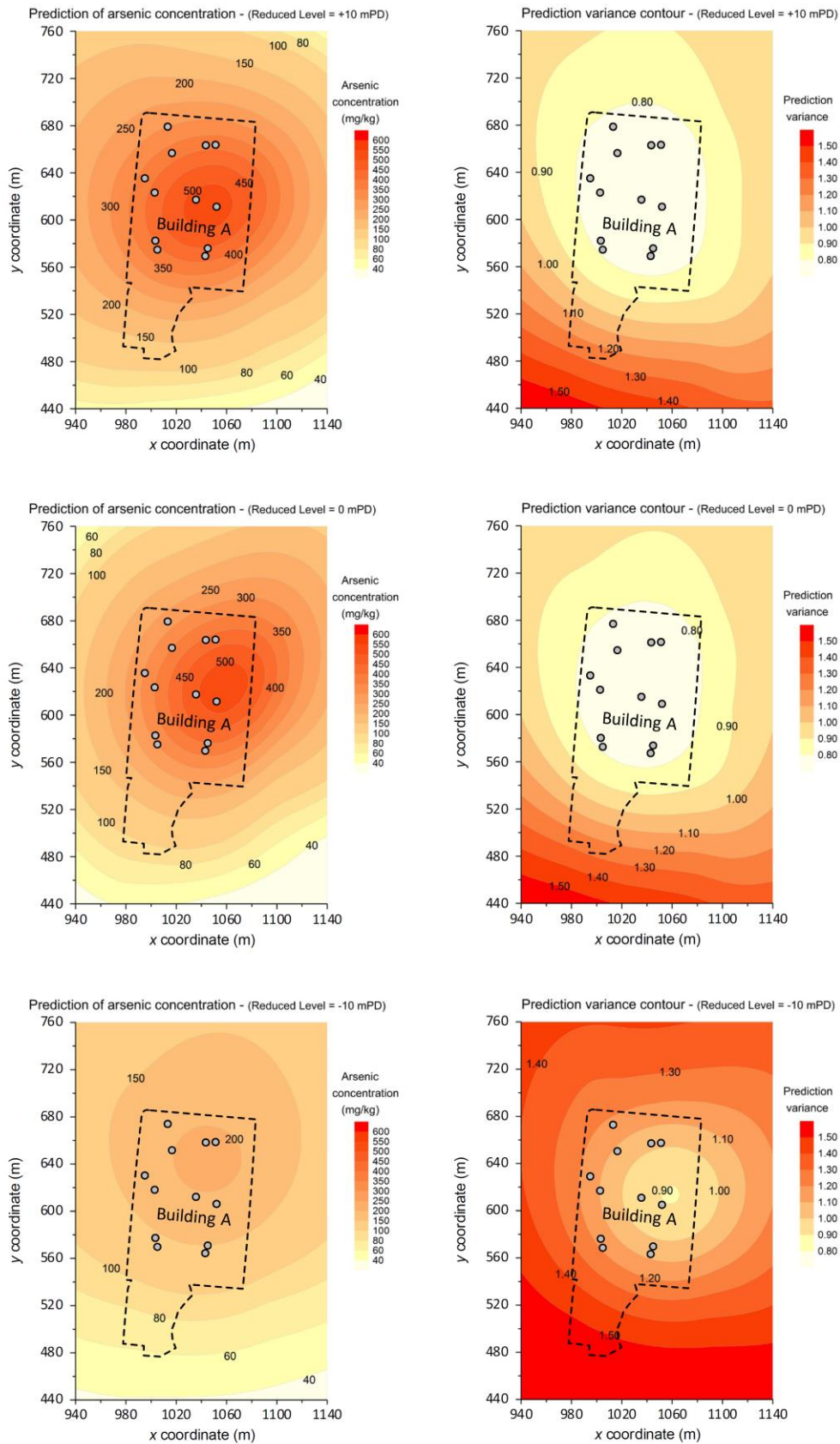


Figure 9 Predicted As concentrations (left) and prediction variance (right) after drilling and sampling at twelve borehole locations (grey circles) in Stage 2

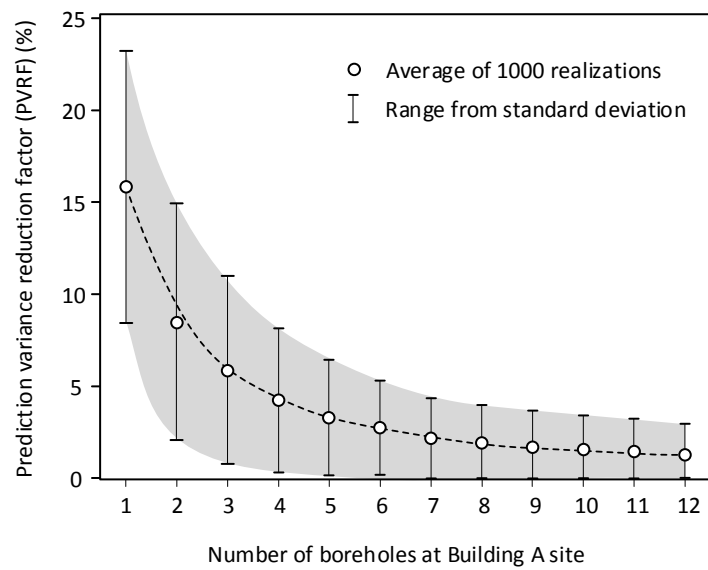


Figure 10 Changes of PVRF with sampling at successive borehole locations