Modeling the Effects of Rainfall Intensity on the Heteroscedastic Traffic Speed Dispersion on Urban Roads

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Abstract

The heteroscedasticity refers to a collection of random variables with sub-population that have different dispersions from others. The variable dispersion could be quantified by measures of statistical dispersion such as standard deviation or coefficient of standard deviation. This study aims to model the effects of rainfall intensity on the heteroscedastic traffic speed dispersion on urban roads. The traffic and rainfall intensity data were collected by a selected video traffic detector and its nearest rainfall station in Hong Kong, respectively. The coefficient of variation of speed (CVS) was employed to measure the vehicular traffic speed dispersion. The analysis shows that the empirical values of CVS typically range from 0.05 to 0.2 at different traffic densities and rainfall intensities, and the exponential function provides a good fit to traffic speed data under both dry and rain conditions. A generalized function of CVS with the effects of rainfall intensity is proposed, calibrated and validated with different sets of empirical data. The calibration and validation results show that the proposed generalized function of CVS fits well with the empirical data. The empirical findings and the generalized function of CVS proposed in this study may benefit for assessing and modeling the level-of-service performance of urban roads in Pacific Rim cities similar to Hong Kong with relatively high annual rainfall intensity.

Keywords: Traffic speed dispersion; Traffic speed variance; Coefficient of variation of speed; Heteroscedasticity; Rainfall Intensity.

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Introduction

Speed is a fundamental traffic performance measure of the roadway system, and speed dispersion is a key variable in traffic safety analysis. Speed dispersion, which is typically measured by speed variance or coefficient of variation of speed (CVS), has been found to be one of the major factors affecting traffic safety, in particular vehicle accidents. For example, recent study by Quddus (2013) shows that a 1% increase in speed variation is associated with a 0.3% increase in accident rates observed in a major road network in London.

Rain has significant detrimental effects on road traffic (Lam et al. 2008; Shao et al. 2008; Billot et al. 2009, 2010; Sumalee et al. 2011a), especially on traffic speed in Hong Kong (Tam et al. 2007; Lam et al. 2013) which has the highest average annual rainfall among some major Pacific Rim cities (Lam et al. 2013). Recently, Lam et al. (2013) investigated the relationships between traffic speed, flow, and density under various rainfall conditions on urban roads in Hong Kong. It has been found that rainfall intensity has significant impacts on urban road key traffic stream parameters, e.g. free-flow speed, speed at capacity and roadway capacity. Moreover, a generalized speed-flow-density function with rainfall intensity effects was proposed, and can possibly be used for assessing the performance of urban roads and modeling drivers' route choice behavior with different rainfall intensity.

The objectives of this study are to extend previous work of Lam et al. (2013) by investigating the heteroscedastic traffic speed dispersion on urban roads and constructing a generalized speed dispersion function with rainfall intensity effects. The heteroscedasticity, which is the absence of homoscedasticity, refers to a collection of random variables with sub-population that have different dispersions from others. The variable dispersion could be quantified by measures of statistical dispersion such as standard deviation or coefficient of standard deviation. In the particular field of traffic engineering, the heteroscedastic traffic speed dispersion means that values of the statistical measure of traffic speed dispersion are varied at different traffic densities. Fig. 1 shows an illustrative example of the heteroscedastic traffic speed dispersion under different weather conditions in Hong Kong.

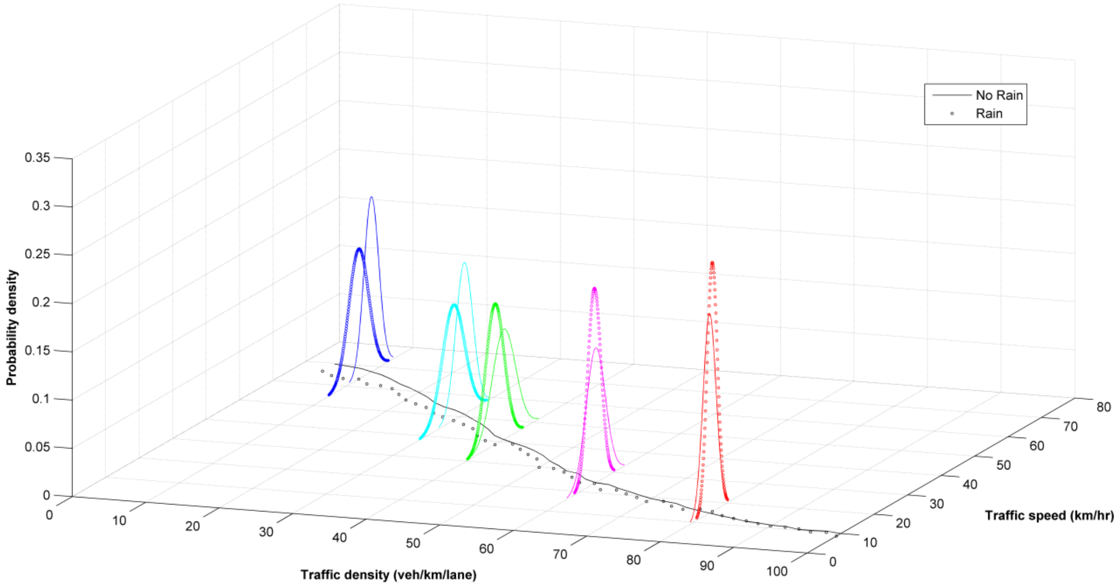

Fig. 1 Illustrative the heteroscedastic traffic speed dispersion in Hong Kong

Two features distinguish this study from the previous relevant studies. First, numerous studies (May 1990; Treiber and Helbing 1999; Wang et al. 2007; Chung and Recker 2010, 2014; Li et al. 2012a; Wang et al. 2013a, 2013b) investigated the empirical characteristics of speed dispersion on different types of roadways in various cities. However, those studies were analyzed either based on dry/no rain condition or without considering the effects of adverse weather conditions. In view of the detrimental effects of adverse weather on road traffic characteristics (Lam et al. 2008; Shao et al. 2008; Billot et al. 2009, 2010; Sumalee et al. 2011a; Tam et al. 2007; Lam et al. 2013), this study attempts to investigate the effects of rainfall intensity on the heteroscedastic traffic speed dispersion on urban roads in Hong Kong. The empirical findings of this study may benefit for empirical analysis and mathematical modeling (Ngoduy 2009; Sumalee et al. 2011b; Chen et al. 2013) in other Pacific Rim cities such as Bangkok similar to Hong Kong with relatively high annual rainfall intensity.

Second, although previous studies investigated the empirical characteristics of speed dispersion, however, there are only a few studies (Wang et al. 2007; Chung and Recker 2010, 2014; Wang et al. 2013a, 2013b) which have recently started to examine the relationship between traffic speed dispersion and traffic density under dry and/or no rain condition. This paper aims to propose a generalized function of vehicular traffic speed dispersion with the relationship of traffic density under different rainfall intensities. The generalized function proposed in this paper can be considered as an extension of the existing studies (Wang et al. 2007; Chung and Recker 2010, 2014; Wang et al. 2013a, 2013b). In addition, the generalized function proposed in this paper may also benefit for improving the urban traffic control and management, e.g. the analysis of road traffic safety under rainy condition (e.g. Li et al. 2012b) and the impacts of adverse weather on travel time reliability (Sumalee et al. 2011a).

The rest of this article is divided into five sections. The second section reviews the previous related studies on the relationship between traffic speed dispersion and accident rates, the empirical characteristics of traffic speed dispersion, and the impacts of adverse weather on traffic characteristics. The third section describes the different sets of data used for calibration and validation of the generalized function proposed in this paper. The fourth section presents the empirical characteristics of traffic speed dispersion at different traffic densities and rainfall intensities. The fifth section provides the calibration and validation results of the generalized traffic speed dispersion function with taking account the effects of rainfall intensity. The final section provides conclusion together with suggestions for further study.

Literature Review

Although numerous studies investigate the relationship between traffic speed and accidents, see Aarts and Schagen (2006) for a review, however, there are only a few studies that examined the relationship between traffic speed dispersion/variation and accidents in the past few decades. Lave (1985) firstly pointed out that “variance kills, not speed”. He used speed variance as the statistical measure of speed dispersion. It was found that accident rates were associated with traffic speed variance, not speed. Garber and Gadiraju (1989) and Taylor et al. (2000) also confirmed the above findings in Lave (1985) by using data in U.S. and U.K. respectively. Garber and Gadiraju (1989) showed that a strong relationship between speed variance and traffic accidents exists on interstate highways. While Taylor et al. (2000) collected aggregated speed and accident data in UK and found that the relationship exists on different road types, e.g. congested roads in town, inner city roads, sub-urban roads, and outer suburban roads. Recently, the study by Quddus (2013) shows that a 1% increase in speed variation is associated with a 0.3% increase in accident rates observed in a major road network in London.

Besides the relationship between traffic speed dispersion/variation and accidents, several studies also investigated the empirical characteristics of the speed dispersion. The standard deviation of speed (SDS) and coefficient of variation of speed (CVS) were widely used as the indicators of speed dispersion. May (1990) indicated that CVS might range from approximately zero to something on the order of the reciprocal of the mean speed, and normally range from 8% to 17% in the empirical studies. Del Castillo and Benitez (1995) used CVS as an indicator to distinguish stationary and unstable periods of traffic flows. Shankar and Mannering (1998) investigated the relationship between lane mean speeds and speed deviations based on a structural model. The model indicated that in-lane mean speeds were positively affect in-lane speed deviations. Treiber and Helbing (1999) modeled traffic speed dispersion by assuming that CVS is positively correlated with traffic density. The results showed that the values of CVS were about 10% under the free-flow condition. Park and Ritchie (2004) investigated the relationship between freeway speed variance, lane changing and vehicle heterogeneity. The statistical analysis indicated that lane changing behavior has significant impact on section speed variability. In addition, they also pointed out that long vehicles also have considerable influence on speed variance. Wang et al. (2007) presented characteristics of speed dispersion in urban freeway traffic in Nanjing, China. The results showed that traffic density could be modeled as an exponential function of CVS. In addition, the values of CVS varied from 7% to 32%. Recently studies (Chung and Recker 2010, 2014) confirmed the exponential function of CVS with traffic density in Wang et al. (2007) and also showed the possible application of speed dispersion in measuring freeway level of service and air emissions evaluation. Recently, Li et al. (2012a) and Wang et al. (2013a, 2013b) investigated the stochastic modeling of the fundamental diagram by correlating the traffic speed variance with traffic density and free flow speed.

Previous studies investigated the relationship between traffic speed dispersion and accidents, and the characteristics of traffic speed dispersion. However, those studies were all conducted in a general case without specifying the effects of weather conditions. In fact, previous studies showed that adverse weather conditions in the form of rain have significant impact on traffic characteristics (Lam et al. 2008; Shao et al. 2008; Billot et al. 2009, 2010; Sumalee et al. 2011a). The traffic characteristics include free-flow speed, road capacity, speed at capacity, etc. With rainfall intensity increases, the key traffic stream parameters such as free-flow speed, speed at capacity, and road capacity (or maximum flow) decrease. The specific traffic characteristics under different rainfall intensities are critical factors affecting traffic operations and safety. Thus, integrating the impact of rain into traffic modeling and management is essential and important, especially in the urbanized area with high annual rainfall intensity, e.g. Hong Kong (Tam et al. 2007; Lam et al. 2013).

Data

The traffic data used in this study was collected from Gloucester Road located in the north of Hong Kong Island. Gloucester Road is a major section of a primary distributor, which belongs to one road type of urban roads in Hong Kong with high capacity junctions, segregated pedestrian facilities and limited frontage access (Transport Department 2011). The road section, as shown in Fig. 2, is a five-lane (one-way) primary distributor with a speed limit of 70 km/h. The number of annual average daily vehicles using it was 172, 560 and 168, 560 (two-way) for years 2009 and 2010, respectively (Transport Department 2010, 2011).

There are five traffic lanes and a bus stop in the eastbound direction in the selected road section. In this study, the study area of the selected road section was confined to the second and the third traffic lanes (see Fig. 2), which are towards the most congested tunnel, Cross Harbor Tunnel, in Hong Kong. The traffic data under different congestion conditions and rainfall intensities can be easily obtained at these two lanes for investigation and modeling of the impact of rainfall intensity on the heteroscedastic traffic speed dispersion.

Two types of data, traffic data (e.g., space mean speed and vehicular traffic flow), and rainfall intensity data, were collected and matched into a common database for empirical analysis of rainfall effects.

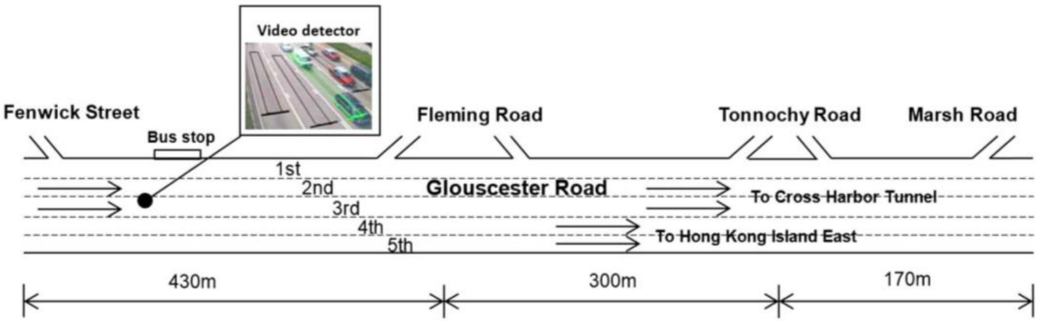

Fig. 2 Schematic diagram of the selected urban road section (Lam et al. 2013)

Traffic data

Traffic data were collected by the Hong Kong Journey Time indication System (JTIS). JTIS is operated by the Hong Kong Transport Department, and provides the average journey time estimates on major routes in Hong Kong with an updating interval at every two minutes (Tam and Lam 2011). Autoscope video traffic detectors were installed at the major roads in Hong Kong to collect the real-time traffic data, such as time mean speeds, space mean speeds, and traffic counts for estimation of journey times at selected routes crossing the harbor.

As shown in Fig. 2, a video traffic detector is installed at the selected road section. In this study, space mean speeds and traffic flows on the two selected traffic lanes were obtained from the selected traffic detector. As the traffic density data are not available from the JTIS traffic detector dataset, the traffic density is equal to traffic flow divided by space mean speed.

The traffic data adopted for this study is at 2 minute time interval in the period of July 2009 to December 2010 in the selected urban road section in Hong Kong. In order to avoid possible data noise, the traffic data was aggregated at 10 minute time interval in this study.

Weather data

This study use hourly rainfall precipitation to measure the rainfall intensity. Hourly rainfall intensity data were collected from the Happy Valley rainfall station, which is the nearest rainfall station to the selected video traffic detector. The distance between the Happy Valley rainfall station and the selected traffic detector is 1.7 km. The hourly rainfall precipitation data collected at the Happy Valley station were used to represent the weather conditions of the selected urban road section. The period in which rainfall intensity data was collected is same as that in which traffic data was extracted from the JTIS.

The differences between the aggregation intervals of traffic data and rainfall intensity data were noted. In this study we assume that the rainfall intensity is uniformly distributed in each hour time interval.

Filtering of invalid data

The collected traffic data and rainfall precipitation data were filtered to ensure the data validity. On one hand, we synchronized traffic data and weather data, and eliminated traffic data corresponding to the missing rainfall precipitation data. Conversely, the rainfall precipitation data corresponding to invalid traffic data was also eliminated from the database. On the other hand, both traffic data and rainfall precipitation data collected during traffic accidents were removed to avoid possible inaccurate traffic speed-flow-density relationship due to the bottlenecks caused by traffic accidents (Huang et al. 2008). In this study, we assumed traffic accidents can be removed within one hour after the occurrence of traffic accidents, and removed the corresponding traffic data and rainfall precipitation data at that time. In addition, we also eliminated the outlier observations due to malfunction of the traffic detector.

Empirical Observations of the Heteroscedastic Traffic Speed Dispersion with Rainfall Intensity Effects

The Statistical Measure

The coefficient of variation of speed (CVS) is employed in this study to statistically measure traffic speed dispersion. CVS is defined as the standard deviation divided by the sample mean speed (May 1990). In other words, it could be considered as a normalized standard deviation. This feature makes it widely accepted to measure traffic speed dispersion in previous studies (May 1990; Del Castillo and Benitez 1995; Treiber and Helbing 1999; Wang et al. 2007; Chung and Recker 2010, 2014).

This study uses CVS as a statistical indicator to measure and compare traffic speed dispersion at different traffic densities and rainfall intensities. In order to calculate the values of CVS at different traffic densities and rainfall intensities, we divide the collected traffic data into m subgroups. For each subgroup i at traffic density k_i and rainfall intensity r, the sample mean speed, $\bar{v}_i(k_i, r)$, sample speed variance, ${s_i}^2(k_i, r)$, and sample standard deviation of speed $s_i(k_i, r)$ can be calculated as follows:

$$\bar{v}_i(k_i, r) = \frac{\sum_{m=1}^{N_i} v_m(k_i, r)}{N_i} \tag{1}$$

$${s_i}^2(k_i, r) = \frac{\sum_{m=1}^{N_i} (v_m(k_i, r) - \bar{v}(k_i, r))^2}{N_i - 1} \tag{2}$$

$$s_i(k_i, r) = \sqrt{{s_i}^2(k_i, r)} \tag{3}$$

where i = subgroup i at traffic density k_i and rainfall intensity r
m = The speed observation m in subgroup i whereas $1 \leq i \leq N_i$

N_i = total number of speed observations in subgroup i
$\bar{v}_i(k_i, r)$ = sample mean speed of subgroup i
$v_m(k_i, r)$ = speed of the observation m in subgroup i
${s_i}^2(k_i, r)$ = sample speed variance of subgroup i
$s_i(k_i, r)$ = sample standard deviation of speed of subgroup i

The value of $\bar{v}_i(k_i, r)$ can be calculated either by the equation (1), or by a generalized speed-density function with rainfall intensity effects proposed by Lam et al. (2013), see the equation below.

$$\bar{v}_i(k_i, r) = \exp[-0.044 \cdot r^{0.296} + 4.260 - \theta(\frac{k_i}{k_j})^{\delta}] \tag{4}$$

where i = subgroup i at traffic density k_i and rainfall intensity r
k_j = jam density
N_i = total number of speed observations in subgroup i
θ, δ = parameter to be calibrated
$\bar{v}_i(k_i, r)$ = sample mean speed of subgroup i

Then, the value of sample coefficient of variation of speed, $CVS_i(k_i, r)$, at traffic density k_i and rainfall intensity r can be calculated in the equation below.

$$CVS_i(k_i, r) = \frac{s_i(k_i, r)}{\bar{v}_i(k_i, r)} \tag{5}$$

where i = subgroup i at traffic density k_i and rainfall intensity r
$CVS_i(k_i, r)$ = sample coefficient of variation of speed of subgroup i
$\bar{v}_i(k_i, r)$ = sample mean speed of subgroup i
$s_i(k_i, r)$ = sample standard deviation of subgroup i

The value of $CVS_i(k_i, r)$ can be interpreted as the percentage of the standard deviation of the speed sample to the sample mean speed at traffic density k_i and rainfall intensity r. As indicated by May (1990), the range of CVS might range from approximately zero to something on the order of the reciprocal of the square root of the mean speed.

It should be noted that, in order to use CVS as the statistical indicator to measure traffic speed dispersions at different traffic densities and rainfall intensities, the issue of sample size should be addressed. The equation (5) requires the accurate approximation of sample mean speed $\bar{v}_i(k_i, r)$ and sample standard deviation of speed $s_i(k_i, r)$, respectively. In general, the approximation accuracy increases with the sample size N_i enlarged. As a practical matter, if $N_i > 30$, the standard deviation of the sample is numerically substituted for the standard deviation of the population (May 1990).

The Empirical Observations

In order to calculate the values of $CVS_i(k_i, r)$ at traffic density k_i and rainfall intensity r by equation (1) - (5), we firstly divide the collected traffic data into four categories based on rainfall intensity: dry ($rainfall\ intensity = 0\ mm/hr$), light rain ($rainfall\ intensity \in (0,0.5]\ mm/hr$), medium rain ($rainfall\ intensity \in (0.5,2.5] mm/hr$), and heavy rain ($rainfall\ intensity > 2.5\ mm/hr$). Then, for each category of rainfall intensity, the collected traffic speed data is grouped with density aggregation level 5 veh/km/lane to satisfy the sample size requirement (May 1990). Due to the limitation of samples, the values of $CVS_i(k_i, r)$ at different categories of rainfall intensity are only calculated till the density $k_{20} = 100$ veh/km/lane. The empirical traffic speed data and the heteroscedastic traffic speed dispersion measured by $CVS_i(k_i, r)$ at different traffic densities in different categories of rainfall intensity can be seen in Fig. 3.

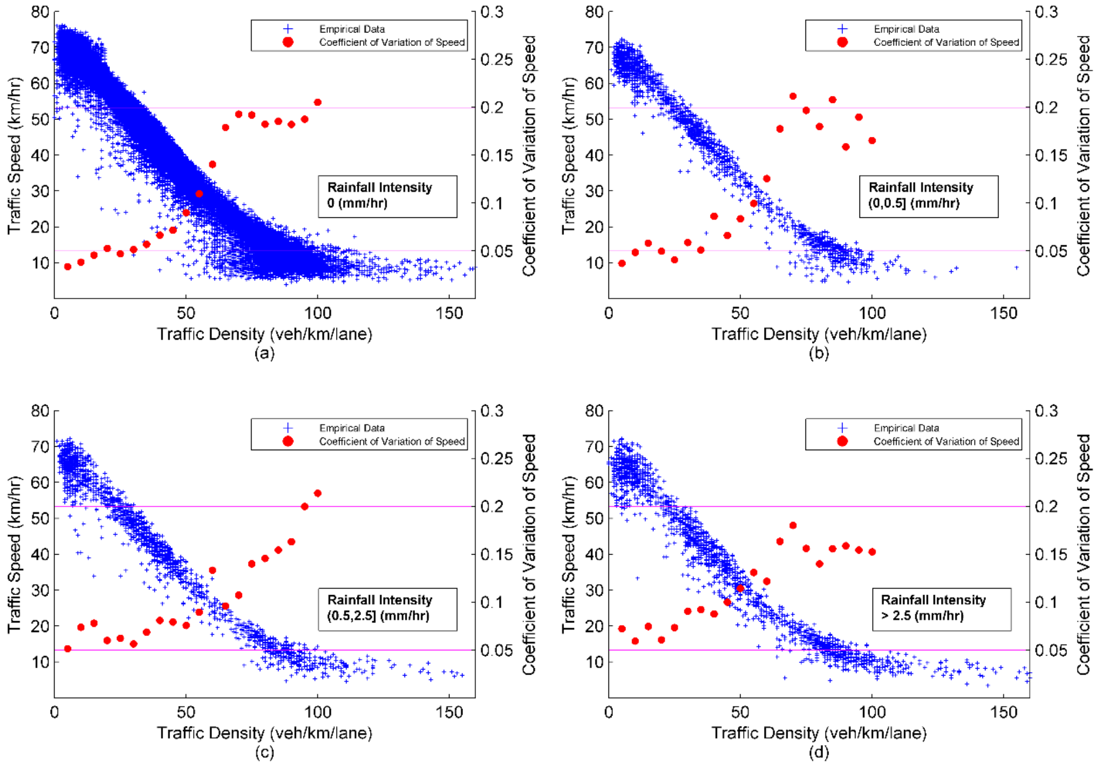

Fig. 3 The empirical data and CVS at different traffic densities and rainfall intensities

Fig. 3 shows that the values of CVS typically ranges from 0.05 to 0.2 in different categories of rainfall intensity. Such range is reasonable by comparing the empirical observations of the CVS values in previous studies (May 1990; Wang et al. 2007; Chung and Recker 2010). May (1990) indicated that CVS normally ranged from 0.08 to 0.17 in the empirical studies. Wang et al. (2007) observed traffic density distributed from 0.07 to 0.32. While Chung and Recker (2010) found that CVS ranged from 0.08 to as large as 0.5 on the 10-lane Interstate 80 in northern California.

Fig. 3 also presents that the exponential relationship between CVS and traffic density, which observed in previous studies (Wang et al. 2007; Chung and Recker 2010, 2014) under dry/no-rain condition, also exists under rain condition. The values of CVS grows exponentially with the increase of traffic density in each category of rainfall intensity, and a sharp increase of the value of CVS happens when traffic density approaching the critical density (e.g. 70-80 veh/km/lane) where roadway capacity usually obtained. In other words, the density area of maximum speed variance is also the density area of maximum throughput (Benjaafar et al. 1997). This is easy to understand, when traffic density increases, the potential vehicle conflicts such as acceleration, deceleration or lane-changing also increase (Park and Ritchie 2004). The increased conflicts make traffic stream stop-and-go and result in speed dispersion increase.

In addition to the general exponential relationship, there are several interesting findings by comparing the values of CVS among different categories of rainfall intensity. On one hand, it can be observed that no significant difference exists between the patterns in Fig. 3 (a) and (b). This is easy to understand because driving behavior might not be changed between light rain condition and dry condition. However, with the rainfall intensity increase, the values of CVS in non-congested status (e.g. traffic density $k \leq 50$ veh/km/lane) under medium and heavy rainfall conditions are significantly higher than those values under dry/light rain conditions. Fig. 3 (c) and (d) show that the values of CVS whereas traffic density $k \leq 50$ veh/km/lane are all higher than 0.05, while the corresponding values in Fig. 3 (a) and (b) are equal or even lower than 0.05. On the other hand, with the rainfall intensity increase, the values of CVS in congested status (e.g. traffic density $k > 70$ veh/km/lane) under medium/heavy rainfall conditions are slightly lower than those values under dry/light rain conditions. For example, it is obvious that, under heavy rainfall condition in Fig. 3 (d), the average value of CVS whereas traffic density $k > 70$ veh/km/lane is only 0.15, while the corresponding value under dry condition in Fig. 3 (a) is 0.18.

The above findings might be caused by the heterogeneity and consistency of driving behavior under different rainfall conditions. As explained by Treiber and Helbing (1999), under dry/no-rain condition, the speed dispersion for free traffic at low densities is caused mainly by variations of the individual desired speed. The main reason for the considerable speed dispersion for congested traffic at high densities is the variation of the time headway. The mechanism explained by Treiber and Helbing (1999) can also be used to explain the above findings under different rainfall conditions. The difference is that, in non-congested status under rainfall conditions, the variations of the individual desired speed might be increased due to drivers' response behavior to the rainfall conditions. While in congested status under rainfall conditions, drivers might be more cautious and keep larger time headways due to the visibility issue. This may somehow reduce the variation of the time headway. In addition, other factors may also influence speed dispersion under rainfall conditions. For example, the lane-changing behavior. Due to the poor visibility and wet roadway pavement conditions, drivers might prefer to follow the vehicle ahead instead of lane changing, which may reduce potential vehicle conflicts and speed variations than dry conditions.

Modeling the Heteroscedastic Traffic Speed Dispersion with Rainfall Intensity Effects

This section aims to model the empirical observations in previous section. It is desirable to find appropriate a generalized function to represent the heteroscedastic traffic speed dispersion with rainfall intensity effects. In this section, we first calibrate a conditional exponential function which is widely used in previous studies (May 1990; Wang et al. 2007; Chung and Recker 2010, 2014). A generalized exponential function is proposed, calibrated and validated with collected traffic speed data.

A Conditional Exponential Function

The empirical observations in previous section shows that the value of CVS grows exponentially as traffic density increase with rainfall intensity effects. Such observations confirm previous findings (Wang et al. 2007; Chung and Recker 2010, 2014) which show that exponential function is a good fit to the relationship between CVS and traffic density. Thus, a conditional exponential function of CVS with rainfall intensity effects is proposed in this section. The equation can be seen below.

$$CVS_r(k) = \alpha_r \cdot e^{\beta_r \cdot k} \tag{6}$$

where $CVS_r(k)$ = coefficient of variation of speed with rainfall intensity r and traffic intensity k

α_r, β_r = parameters need to be calibrated

Fig. 4 shows the calibrated conditional CVS functions with different rainfall intensities. The calibration results prove that the conditional expoential function is also a good to fit to the collected traffic data under different categories of rainfall intensity. All the coefficients of determination (R^2) are higher than 0.8, implying that over 80% of the calibrated traffic speed values fit well with the observed traffic speed data. It should be noted that those outliers are values of CVS happen around critical density or capacity. As explained in previous section, when traffic flows approaching the roadway capacity, the increased vehicle conflicts make the traffic as a stop-and-go process thus increase the speed dispersion.

Moreover, Fig. 4 also illustrate the empirical observations in previous section about the values of CVS in different categories of rainfall intensity. With the rainfll intensity increases, the values of CVS in non-congested status under medium and heavy rainfall conditions are significantly higher than those values under dry/light rain conditions (see the red arrow in Fig. 4); while the values of CVS in congested status under medium/heavy rainfall conditions are lower than those values under dry/light rain conditions (see the blue arrow in Fig. 4).

Finally, it can also be observed that there is a relationship between the coefficients of the conditional CVS equation and the rainfall intensity. With the rainfall intensity increase, the slopes of the exponential function decrease while the initial value of CVS increase. In the exponential function, β_r controls the slope while α_r refers to the initial value of CVS. It can be seen in Fig. 4 that, with the rainfall intensity increase, the coefficient of α_r increases while the coefficient of β_r decreases. This relationship helps us to formulate a generalized CVS function with the rainfall intensity effects in the next section.

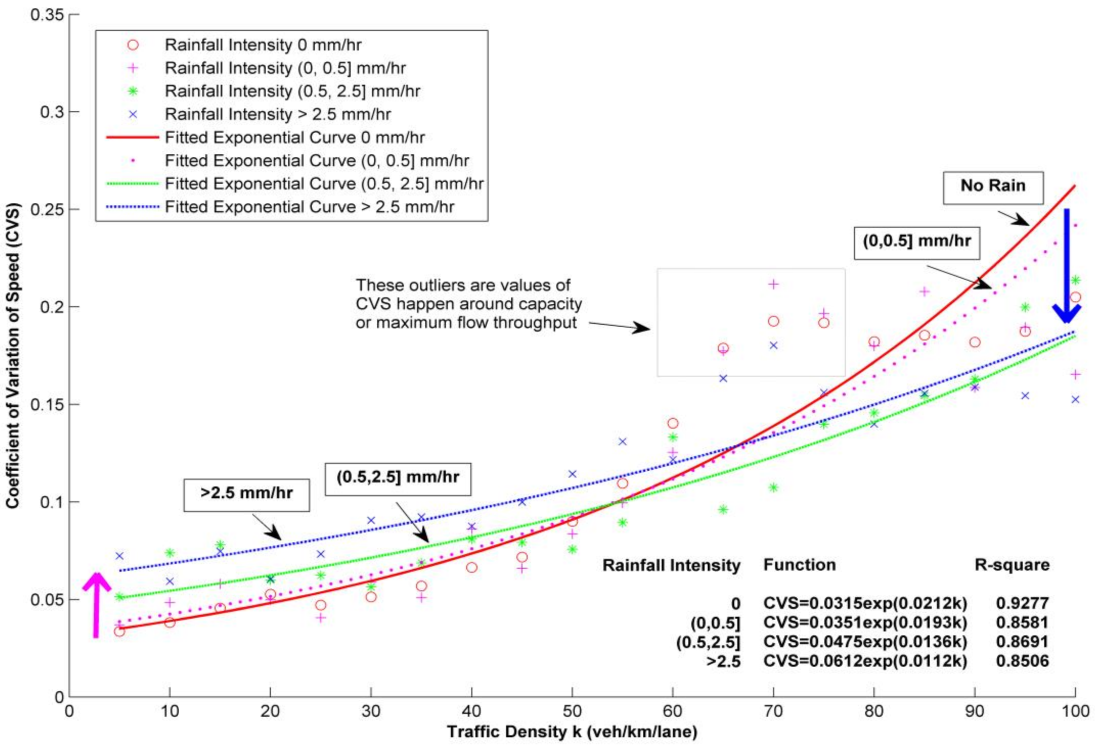

Fig. 4 The calibrated conditional functions of CVS with different rainfall intensities

A Generalized Exponential Function

In this section, a generalized exponential function of $CVS(r,k)$ with rainfall intensity effects is proposed based on the empirical observations in the previous section. The equation can be seen below.

$$CVS(r,k) = (\alpha \cdot r + \alpha_0) \cdot e^{(\beta \cdot r + \beta_0) \cdot k} \tag{7}$$

where $CVS(r,k)$ = coefficient of variation of speed with rainfall intensity r and traffic intensity k

$\alpha,\ \beta$ = parameters need to be calibrated

$\alpha_0,\ \beta_0$ = parameters in dry condition with rainfall intensity $r = 0$

$\alpha_0 = 0.0315;\ \beta_0 = 0.0212$ (values can be found in Fig. 4)

Equation (7) was calibrated with the valid observed traffic density, rainfall intensity and the calculated values of CVS from July 2009 to December 2010. By the nonlinear regression method, the values of parameter α and β was found to be 0.005433 and -0.002112, respectively. The coefficient of determination (R^2) is 0.766, representing a good fit for the observed traffic data.

Fig. 5 illustrates the calibrated relationship between traffic density and CVS for different rainfall intensities. The calibrated surface fit well with the observed empirical features in previous sections. It can be seen that at the same traffic density in non-congested condition, the values of CVS increase with rainfall intensity increases;

while the values of CVS decrease with rainfall intensity increase at the same traffic density in congested condition.

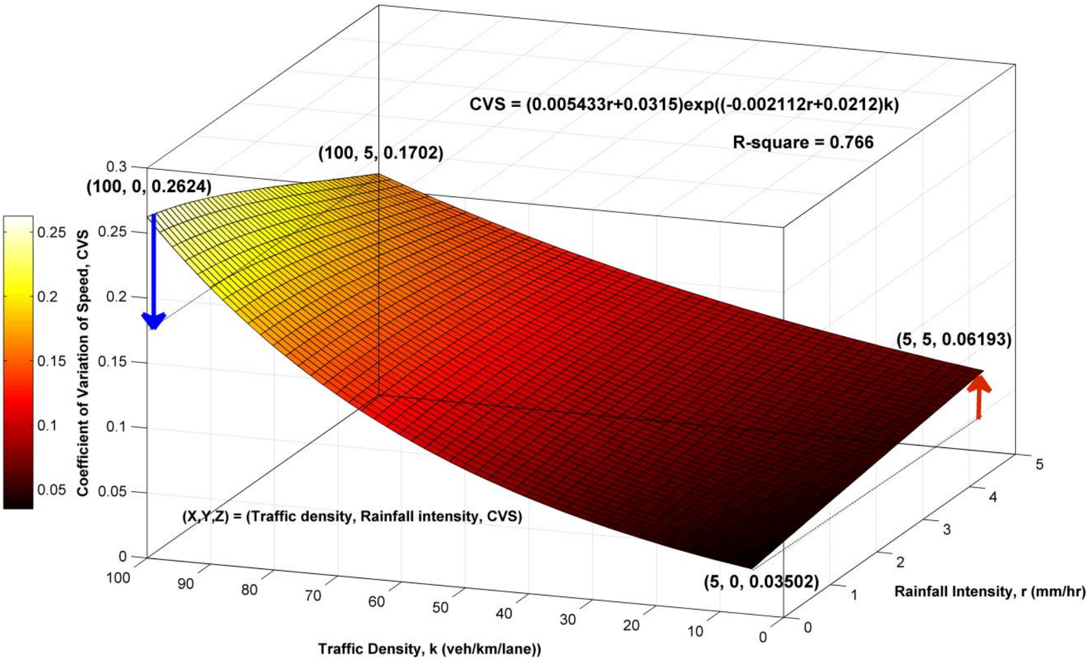

Fig. 5 Calibrated the generalized function of CVS with different rainfall intensities

Validation of the Proposed Function

To validate the calibrated generalized exponential function of CVS with rainfall intensity effects, an independent set of traffic and rainfall intensity data was collected at the same selected road section in the year 2011. The collected datasets were preprocessed using the same method described in the third section of this paper, for filtering the invalid data before validation.

Fig. 6 presents the traffic flows estimated by the generalized exponential function of CVS with rainfall intensity effects against the observed values of CVS under respective dry and rainy conditions. Both the coefficient of determination (R^2) are higher than 0.7, implying that over 70% of the estimated traffic flows fit well with the observed traffic flows. The mean absolute error (MAE) and the mean absolute percentage error (MAPE) which reflects the predictability accuracy of the calibrated generalized speed-flow function are also indicated in Fig. 6. It should be noted that those outliers are values of CVS happen around critical density or capacity.

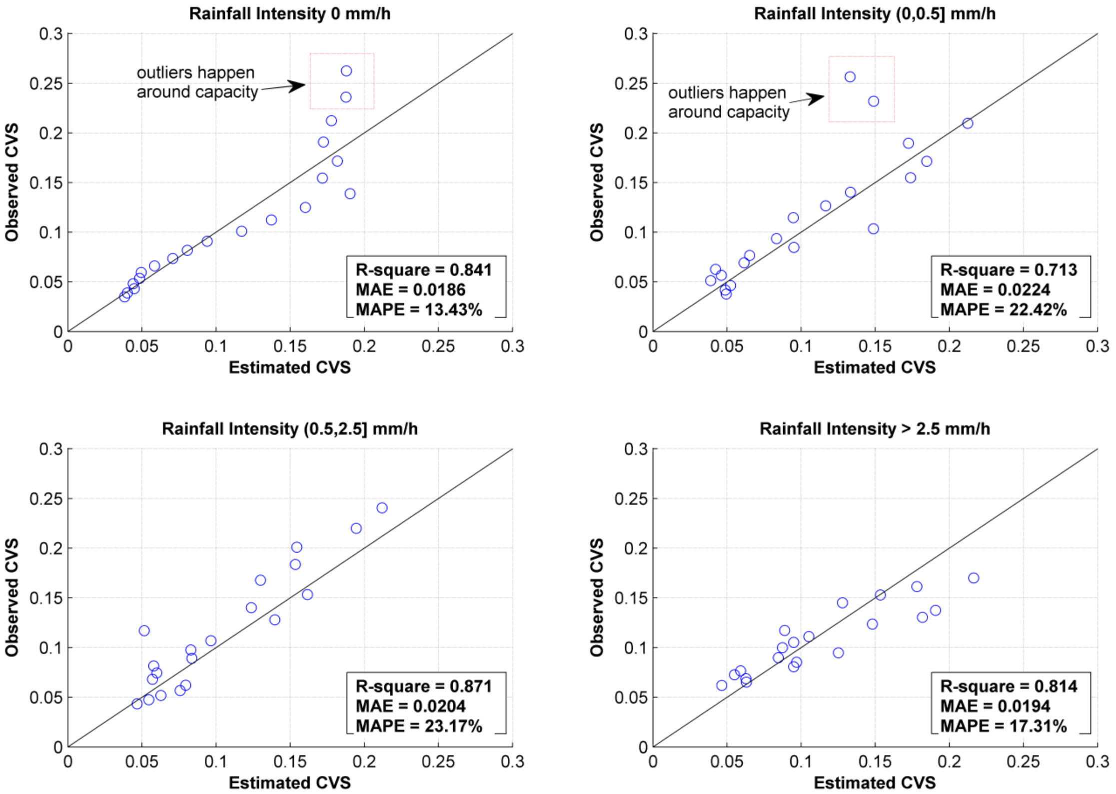

Fig. 6 Observed traffic speed versus estimated traffic speed by the calibrated generalized function of CVS with rainfall intensity effects

Conclusions

In this study, the impact of rainfall intensity on the heteroscedastic traffic speed dispersion was analyzed. The traffic and rainfall intensity data collected by a selected video detector and its nearest rainfall station in Hong Kong was used for the analyses. The coefficient of variation of speed (CVS) was used to measure traffic dispersions at different traffic densities and rainfall intensities. The empirical observations of the impacts of rainfall intensity on the heteroscedastic traffic speed dispersion were presented, and a generalized exponential function was proposed to model the observed characteristics of the heteroscedastic traffic speed dispersion with the effects of rainfall intensity. The major conclusions of this study are as follows:

- The empirical values of CVS typically range from 0.05 to 0.2 at different traffic densities and rainfall intensities.
- When rains, the values of CVS at low traffic densities are higher than those values under dry condition; while the values of CVS at high traffic densities are lower than those values under dry condition.
- The difference of values of CVS under different rainfall intensities might be caused by the heterogeneity and consistency of driving behavior under different rainfall conditions.
- The expoential function is proved to be a good fit to traffic data not only under dry condition in previous studies (Wang et al. 2007; Chung and Recker 2010, 2014), but also under different rainfall intensities.
- A generalized exponential function of CVS with rainfall intensity effects is

proposed, calibrated and validated. The calibration and validation shows that the proposed generalized function of CVS fits well with empirical data.

The observed patterns of the heteroscedastic traffic speed dispersions in Hong Kong and the proposed generalized exponential function with the effects of rainfall intensity may benefit traffic operations and management in metropolitan areas similar to Hong Kong. However, the findings of this study cannot be generalized because they are based on the analysis of a single set of data of traffic speed from a specific video detector in a particular area. For other areas, a sensitivity analysis based on the calibrated models is recommended. Moreover, to have a reliable generalized exponential function with the effects of rainfall intensity, more empirical data from different locations are required. These data should be used to better calibrate and compare current state-of-the-practice and state-of-the-art traffic models.

Moreover, besides traffic modeling, the behavior analysis is a much more fundamental issue for better understanding of the drivers' behavior when rains. In this study, the authors offer several tentative explanations for the difference of the empirical patterns of the heteroscedastic traffic speed dispersions. However, such hypotheses need additional rigorous tests supplemented with individual information from surveys of different drivers. The current traffic data do not contain such information. Possible future work would be to conduct driving behavior surveys in Hong Kong.

Finally, further study can be carried out to explore potential applications of the proposed generalized exponential function of traffic speed dispersion with considering the effects of rainfall intensity. One possible application is to analyze the relationship between traffic speed dispersion, adverse weather and accident rates (e.g. Li et al. 2012b). The other potential application is to quantify the level of service of urban roads in relation to the rainfall intensity. This may be beneficial to an improved understanding of traffic flow characteristics on urban roads and new applications of intelligent transportation systems such as real-time traffic control and management (e.g. Billot et al. 2010; Chung and Recker 2014).

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