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A stochastic model updating strategy-based improved response surface model and advanced Monte Carlo simulation

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ABSTRACT

To improve the accuracy and efficiency of computation model for complex structures, the stochastic model updating (SMU) strategy was proposed by combining the improved response surface model (IRSM) and the advanced Monte Carlo (MC) method based on experimental static test, prior information and uncertainties. Firstly, the IRSM and its mathematical model were developed with the emphasis on moving least-square method, and the advanced MC simulation method is studied based on Latin hypercube sampling method as well. And then the SMU procedure was presented with experimental static test for complex structure. The SMUs of simply-supported beam and aeroengine stator system (casings) were implemented to validate the proposed IRSM and advanced MC simulation method. The results show that (1) the SMU strategy hold high computational precision and efficiency for the SMUs of complex structural system; (2) the IRSM is demonstrated to be an effective model due to its SMU time is far less than that of traditional response surface method, which is promising to improve the computational speed and accuracy of SMU; (3) the advanced MC method observably decrease the samples from finite element simulations and the elapsed time of SMU. The efforts of this paper provide a promising SMU strategy for complex structure and enrich the theory of model updating.

1. Introduction

In engineering simulation analysis, it is always difficult for the established model to accurately simulate real engineering problem. The precision improvement of simulation model has resulted in the development of model updating technology [1]. Finite element model updating (FEMU) is one important model updating technique to reduce the error between the finite element (FE) model and the corresponding real-structure in the light of test data. As an inverse optimal problem, structural FEMU method gets a rapidly development recently. Friswell [2,3], Peter et al. [4], studied on the FEMU in structural dynamics and adjusted structural parameters using a minimum variance estimator; Zapico-Valle et al., advanced a new FEMU in structural dynamics [5]; Ahmadian et al., developed the modeling and updating methods for large surface-to- surface joints in the awe-mace structure [6]; Modak, focused on the model updating using uncorrelated modes [7]; Jin et al., proposed a new multi-objective approach for FEMU [8]. Besides, genetic algorithm is also applied to FEMU and dynamic FEMU [9,10]. From the above efforts, the existing updating methods are deterministic model updating which regards the influencing parameters as the specific values.

In fact, uncertainties are ubiquitous and inevitable in many aspects of geometric sizes, manufacture, assembling, joint stiffness design, material property, and so forth [11–13], which promote the emergence of uncertain analysis method with respect to uncertain parameters. Roy et al., given an overview of a comprehensive framework with respect to uncertainties [14]; Park et al., discussed the quantification of model uncertainty using Bayesian approach [15]; Fonseca et al., completed uncertainty identification by the maximum likelihood method [16]; Schuëller et al., studied on the uncertainty analysis of a large-scale satellite FE model [17]. To improve the precision of FEMU, the relative theories and method were also developed for stochastic model updating (SMU) with the consideration of random and uncertain factors [18]. For instance, Mares et al., investigated the theory and application of SMU [19,20]; Husain et al., adopted the perturbation method to study SMU [21]; Bao et al., presented a Monte Carlo (MC) simulation-based inverse propagation method for SMU [22]; Beck et al., focused on Bayesian updating of structural models and reliability using Markov Chain MC simulation [23]; Rui et al., proposed an efficient statistically equivalent reduced method for stochastic model updating [24].

However, for complex structure with large-scale uncertain parameters, the FEMU is unacceptable due to low computational efficiency for an excess of FE simulations and superabundant loop computation. It is urgent to seek a new model updating method to improve computational efficiency. One viable alternative to FE model is response surface (RS) method, which needs less FE calculations and holds rapid simulation of RS function, and has been employed to deterministic model updating without considering uncertain parameters by Ren [25,26], Chakraborty [27] and Fang [28]. Recently, RS method was applied to the SMU of uncertain parameters to select important parameters as updating variables by probabilistic analysis [29]. For example, Fang et al., proposed a SMU method for parameter variability quantification based on RS method and MC simulation [30] and also investigated the parameter variability estimation using stochastic RS model updating [31]; Romero et al., constructed a RS model for uncertainty propagation based on progressive-lattice-sampling experimental design [32]; besides, artificial neural network and Kriging model were investigated for RS model updating [33,34].

Currently RS method-based least-square method (LSM) is frequently-used and extensively studied, however, many shortages exist yet for the SMU of complex structure: (1) prior information is not utilized for the existing RS methods which limit the application of RS methods, so that RS model-based LSM is unable to perfectly approximate the real-structure model [27]; (2) traditional MC simulation methods used widely in SMU-based RS method has no memory capability on random sampling for input variables. Therefore, it is possible that the accuracy of SMU for complex structure is unaccepted. For the first issue, based on structural mechanics equations and dimensional analysis principle, the improved RS model (IRSM) is proposed based on moving least-square method (MLSM) by takes a full consideration of the relationship between mathematical and physical principles, the laws of statistics and the prior (posterior) information, which attempts to make the RS function closer to the mechanism model of real-structure and then improve the precision of SMU. Additionally, MLSM remedies the insufficiency of LSM [27], which is towardly to establish the high-precision RS model of complex structure. For the second one, an advanced MC simulation with Latin hypercube sampling method (LHSM) is presented to extract the samples of random parameters (uncertainties) which requires less samplings for fitting RS function and potentially improve computing speed owe to avoid repeated sampling for memory ability [35].

Dynamic measurements have been proved to be valid in providing valuable reference data for FEMU [3,17]. However, it is difficult to separate modeling errors from stiffness-related variables and mass-related variables. Static data have only related with stiffness rather than mass parameters so that the precision of static data is ensured [25]. If more response information is provided, the updated model built-based static data holds high-precision and high-reliability because the static data are easily achieved and affected by noise level [27]. Therefore, the structural static responses are important and necessary for a successful and reliable SMU.

The objective of this paper is to attempt to explore a SMU strategy based IRSM-based MLSM and advanced MC simulation-based LHSM by using structural static responses. The proposed methods were applied to the SMUs of simply sup- ported beam and aeroengine stator system (casings) based on the measured static responses. By the SMUs, the IRSM and advanced MC simulation are demonstrated to be effective and reasonable in improving the precision and efficiency of the SMU of complex structure.

In what follows, Section 2 investigates the basic theory of SMU including IRSM-based MLSM and advanced MC simulation. In Section 3 the SMU procedure is given. Section 4 focuses on the SMU and validation of simply supported beam based on the proposed method. The SMU and validation of aeroengine stator system are implemented in Section 5. Themain conclusions are given in Section 6.

2. Basics theory for stochastic modeling updating

2.1 Improved response surface method, IRSM

In this subsection, the IRSM is developed based on MLSM and prior information. MLSM is used to search for the efficient coefficients of RS model. Prior information is applied to establish the reliable RS model.

An advanced method for regression is MLSM, which introduces a weighted LSM that has various weights with respect to the position of approximation. Therefore, the coefficients of a RS model are functions of the location and hereby should be calculated for each location. This procedure is interpreted as a local approximation [36]. The basic principles of LSM and MLSM are shown in Fig. 1. In Fig. 1, the dotted curve is obtained from classical LSM. Therefore, for the scattered data, only one best approximation curve can be obtained based on traditional LSM. Conversely, with MLSM one approximation function exists there at one calculation point, i.e., there is a subsequent function at each different calculation point. Therefore, for MLSM the coefficients of the RS model are not constants but are variables of the calculation positions. This locally weighted approximation can be performed with respect to effective data near the calculation location, and the data can be weighted according to the distance from the calculation location. It is obvious that the weighted coefficients of the RS model acquired from MLSM hold higher precision than these from LSM, so that the structured RS model-based MLSM better approximate to real model or function and has higher approximation precision. Therefore, higher approximation precision is the strength of advantage of using this method. Generally, the subsequent functions are so-called as the weighting function. In the next moment, the numerical derivation of the coefficients of the RS model with MLSM is discussed for uncertain system.

There are many investigations on SMUs with the emphasis on the correlation (covariance matrix) of input parameters by using perturbation techniques [37–39]. Especially the study presented by Govers and Link [39] demonstrated that the correlation of input parameters slightly influence the results of SMU by adjusting covariance matrix from uncertain experimental modal data. In this paper, this study of SMU is presented by ignoring the correlation of input parameters.

For uncertain system, suppose there are n-response values y_i with respect to the changes of x_{ij}, which denote the ith observation of the jth dependent variable x_j. Assume that the error term $\boldsymbol{\varepsilon}$ (the vector of the errors of response) in the model has $E(\varepsilon)=0$, $Var(\varepsilon)=\sigma^2$ and that the $\{\boldsymbol{\varepsilon} i\}_{i=1,2,\ldots,n}$ are the uncorrelated random errors of n response values (observation) from experiments.

As an alternative mathematical model to FE models for complex structure, RS method is used to solve a complicated and low-efficient design problem. Compared to traditional sensitivity based FEMU method [2–10], the RS method holds the strengths of easy implementation and high computational efficiency. With RS model, the following matrix form can express the relationship between the responses and the variables:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon} \tag{1}$$

where $\boldsymbol{y}$ is the $n\times1$ vector of the observations where n is the number of experiments; $\boldsymbol{X}$ denotes the $n\times p$ matrix of the level of the independent variables in which $p=k+1$ is the number of parameter including the number of coefficients k (degree of freedom of regression) and one constant term for RS model; ε is the $n\times1$ vector of the errors of responses; β is the $p\times1$ vector of the regression coefficients.

For uncertain system in Eq. (1), the least-squares function $L(x)$ is defined by the sum of weighted errors, i.e.,

$$L(\boldsymbol{x}) = \sum_{i=1}^{n} \omega_i \epsilon_i^2 = \boldsymbol{\epsilon}^T \boldsymbol{W}(\boldsymbol{x})\boldsymbol{\epsilon} = \left(\boldsymbol{y} - \boldsymbol{X\beta}\right)^T \boldsymbol{W}(\boldsymbol{x})\left(\boldsymbol{y} - \boldsymbol{X\beta}\right) \tag{2}$$

in where ωi is the weighting of the ith response value; $\boldsymbol{x}$ is the vector of approximation location; $\boldsymbol{W}(\boldsymbol{x})$ signifies the weighting matrix for functions at the location $\boldsymbol{x}$. Here, note that $\boldsymbol{W}(\boldsymbol{x})$ is the diagonal weighting matrix of the function of the location in the MLSM and can be constructed by

$$\boldsymbol{W}(\boldsymbol{x}) = \begin{bmatrix} \omega(\boldsymbol{x}-\boldsymbol{x}_1) & 0 & \cdots & 0 \\ 0 & \omega(\boldsymbol{x}-\boldsymbol{x}_2) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \omega(\boldsymbol{x}-\boldsymbol{x}_n) \end{bmatrix}$$
$$= diag\left(\omega(\boldsymbol{x}-\boldsymbol{x}_1), \omega(\boldsymbol{x}-\boldsymbol{x}_2), \cdots, \omega(\boldsymbol{x}-\boldsymbol{x}_n)\right) \tag{3}$$

where x_I ($I=1, 2, \ldots, n$) denotes the vector of Ith sampling (or experiment) points.

There are several kinds of weighting functions such as constant, linear, quadratic, high-order polynomials, and exponential functions, which are defined in Eq. (4).

$$\omega(\boldsymbol{x}-\boldsymbol{x}_I)=\omega(d)$$
$$=\begin{cases} \text{if } d/R_I \geq 1, & \begin{cases} \text{constant} & 1 \\ \text{linear} & 1-d/R_I \\ \text{quadratic} & 1-(d/R_I)^2 \\ \text{4th–poly} & 1-6(d/R_I)^2+8(d/R_I)^3-3(d/R_I)^4 \\ \text{exponential} & \exp(1-d/R_I) \end{cases} \\ \text{if } d/R_I \geq 1, & 0 \end{cases} \tag{4}$$

here d is the distance between $\boldsymbol{x}$ and $\boldsymbol{x}_I$; $\omega(d)$ is the weighting function at the distance d; R_I indicates the size of an approximation region. For instance, a fourth-order polynomial weighting function is expressed by a bell-shaped figure. The weighting function has 1 (maximum value) at 0 normalized distance and 0 (minimum value) outside of 1 normalized distance, namely, $\omega(0)=1$, $\omega(d/R_I>1)=0$. Also the function decreases smoothly from 1 to 0.

To minimize $L(x)$, the least-squares estimators must satisfy

$$\left.\frac{\partial L(\boldsymbol{x})}{\partial \beta}\right|_{\boldsymbol{b}} = -2\boldsymbol{X}^T\boldsymbol{W}(\boldsymbol{x})y+2\boldsymbol{X}^T\boldsymbol{W}(\boldsymbol{x})\boldsymbol{X}\boldsymbol{b}=0 \tag{5}$$

where $\boldsymbol{b}$ is the least-squares estimator of $\boldsymbol{\beta}$ (or coefficient vector of RS model). Thus coefficients $\boldsymbol{b}(\boldsymbol{x})$ of the RS model can be obtained by the matrix operation:

$$\boldsymbol{b}(\boldsymbol{x})=\left[\boldsymbol{X}^T\boldsymbol{W}(\boldsymbol{x})\boldsymbol{X}\right]^{-1}\boldsymbol{X}^T\boldsymbol{W}(\boldsymbol{x})\boldsymbol{y} \tag{6}$$

In this step, it is important to note that the coefficient $\boldsymbol{b}(\boldsymbol{x})$ is a function of the location or position $\boldsymbol{x}$. Note that a procedure to calculate $\boldsymbol{b}(\boldsymbol{x})$ is a local approximation, and “moving” processes perform a global approximation through the whole design domain.

Most of existing RS functions like quadratic polynomials does not fully utilize prior information. Besides, for complex structures, the classical mechanics and complexity influence the analysis of structure issue. On account of these facts, the authentic and reliable updated results can be achieved only if the modifying process of complex structural model considers the relationship between mathematical and physical principle, the randomness of impact factors and the prior information. Along with the heuristic thought, an IRSM with MLSM is proposed based on the prior information with respect to the characteristics of static response. For instance, ① structural stiffness distribution with mechanical properties of materials (elasticity modulus, Poisson ratio), boundary condition and geometrical shape; ② mass distribution with respect to material density, geometrical shape and lumped mass distribution; ③ loading characteristic defined by sizes, position and variation. The aforementioned prior knowledges can provide an insight reference for selecting updated parameters, establishing RS model and accuracy testing. Hence the significant factors and the perfect RS function are likely to be obtained. The RS function-based dimensional analysis reveals three favorable strengths of specific physical meaning, easy fitting and optimization and wide application.

When $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{P}$ denote the vectors of mass, stiffness and load, respectively, generalized structural static responses $\boldsymbol{y}$(matrix) is

$$\boldsymbol{y}=\frac{\boldsymbol{P}}{\boldsymbol{K}}=f\left(\boldsymbol{P},\boldsymbol{K}\right) \tag{7}$$

As displayed from the above analysis, the prior information and dimensional analysis principle can pledge the efficiency and reliability of the RS function of real-structure. In the construction of RS function, central composite design (CCD) method is effective in the formation of polynomials [28]. Thus, the CCD method is adopted for stochastic RS model updating in this paper.

2.2 Advanced Monte Carlo simulation

MC method is widely applied to probabilistic statistical analysis [30]. In the process of SMU, MC method is used to describe the uncertainty propagation between input variables and output response [17]. Presently, direct sampling method (DSM) is widely used in MC simulation. Unfortunately, the DSM loses the capability of memory for the random sampling of variables, seriously influencing the efficiency of extracting samples and fitting RS function. Especially with the requirement of even-increasing precision for more samples, the computational precision becomes unacceptable. An illustration of a sample set with DSM is shown in Fig. 2, which has 15 samples for two random variables x_1 and x_2 with the standard uniform distribution. In Fig. 2, two sample points is too close each other. However, as an advanced MC simulation technique, LHSM can isometrically divide random sample points as shown in Fig. 3 with the same sample number and distributions. Besides, the Latin hypercube has good memory capability in sampling process. Relative to DSM, LHSM also needs fewer simulations loops, only 20% to 40% of the computational load of DSM [35]. Thus, the LHSM combining the IRSM are adopted to discuss SMU in this paper.

3. Stochastic model updating procedure

With respect to the advanced MC simulation-based LHSM and the MLSM, the SMU procedure based on IRSM for complex structures is illustrated in Fig. 4, which includes six steps: FE analysis, design of experiment, establishment of RS model, advanced MC simulation, uncertainty condition and propagation, and test-simulation correlation.

Firstly, the initial FE model is established in light of objective structure and updated parameters;

And then the CCD method is employed to obtain the sample points of significant parameters, and then these sample points are applied to fit the improved RS model by prior information, dimensional analysis and MLSM.

Thirdly, the statistical properties are obtained with the fitted RS model and advanced MC simulation.

The next step performs inverse optimization. The objective functions are formed containing the relative errors of means and standard deviations between analytical and measured responses, respectively, i.e.,

$$\begin{cases} f\left(\overline{x}_{mean}\right) = \sum_{i=1}^{n}\left(\frac{x_i^{IRSM} - x_i^{\exp}}{x_i^{\exp}}\right)^2 \\ f\left(\sigma_{Std}\right) = \sum_{i=1}^{n}\left(\frac{\sigma_i^{IRSM} - \sigma_i^{\exp}}{\sigma_i^{\exp}}\right)^2 \end{cases} \quad (8)$$

where x_i^{IRSM} and σ_i^{IRSM} denote the ith mean and standard deviation predicted by IRSM, x_i^{exp} and σ_i^{exp} the ith mean and standard deviation estimated by experimental measurements.

Therefore, the mean and covariance of updated input parameters are estimated synchronously through uncertainty inverse propagation, and the output responses of simulation converge towards the stochastic test data.

Finally, the analysis of simulation and test correlation is implemented to evaluate the precision of model.

An important aspect for the efficiency of the fit provided by the MLS approximation is the selection of the weights $\varphi\,(x-x_i)$ for the distance measure in Eq. (3). These weights define the relative importance of each component of the position x for selecting $\boldsymbol{W}(\boldsymbol{x})$ and establishing the moving character of the interpolation over the support points. The weights should (1) establish a normalization for the different components of $\boldsymbol{x}$ but more importantly (2) provide higher weights for components that have larger influence on the stochastic performance. The first task may be easily established by a weight inversely proportional to the variance σ_i^2 component according to the variance of the support points $\{x_i;\ i = 1, 2, \ldots, n\}$; the scaled components have then unity variance. This is equivalent to a normalization of vector $\boldsymbol{x}$.

For the second task, the components that have greater importance on the stochastic sampling process should be prioritized. This may be established by comparing the marginal target distributions to the marginal proposal distribution. Taflanidis et al. [40], gives the rule of weight selection by the relative information entropy based on information about the sensitivity of the sampling process with respect to each of the model parameters. In this paper, the approach is applied to determine the MLS weights and improve the response surface approximation accuracy. In the process of fitting function weight based on this rule, several kinds of weighting functions such as constant, linear, quadratic, high-order polynomials, and exponential functions in Eq. (4) are regarded as the models of MLSM weight function. Through the comparison of these models for each calculation point, an efficient model with the smallest error is as the general form of weight function. And then based on the approach, the specific weight function is quantified for each calculation point.

4. Numerical example

4.1 Simply-supported beam structure

The SMU of a simply-supported beam is executed to verify the effectiveness of the proposed method. The simply supported beam is shown in Fig. 5. Wherein, length l=0.3 m, elastic modulus E=206 GPa, density ρ=7.800 kg/m^3 and Poisson ratio μ=0.3 were selected as input parameters. A static force (P=2500 N) enforces the middle (point C) of the beam, which causes the maximum vertical displacement at point C. Owing to the fact that numerical measurement (numerical solution) is more effective to validate the developed method, the maximum vertical displacements $y_{\max}$ is taken as the response of SMU.

4.2 Improved RS modeling of the beam

Subject to a vertical downward force P in Fig. 5, in the view of theoretical mechanics the maximum vertical displacement $y_{\max}$ at the point C (x=l/2) is obtained

$$y_{\max} = y|_{x=\frac{l}{2}} = -\frac{Pl^3}{4Ebh^3} \tag{9}$$

The width b∈[4 mm, 20 mm] and height h∈[30 mm, 50 mm] were selected as the updated parameters and the maximum displacement $y_{\max}$ was regarded as the response. The CCD method is used to get the sample points of these parameters. With quadratic polynomials, the establishment of IRSM is as follows:

Firstly, with MLSM, the quadratic polynomial RS function with and without cross item are regressed as

$$f = 584.58973 - 7.36897x_1 - 16.89972x_2 + 0.07257x_1x_2 + 0.04773x_1^2 + 0.14092x_2^2 \tag{10}$$

$$f = 453.48323 - 7.18763x_1 - 10.99983x_2 + 0.09146x_1^2 + 0.10102x_2^2 \tag{11}$$

where x_1 and x_2 are b and h, respectively.

For the simply-supported beam, the geometric parameters of width b and height h only effect the inertia moment I which is the only parameter effecting the stiffness distribution of the beam when the elasticity modulus E is determined. In light of dimensional analysis principle, the IRSM of the simply-supported beam is assumed as

$$f = \beta_0 b^{\beta_1} h^{\beta_2} \tag{12}$$

To further simplify the IRSM, the Napierian logarithm of Eq. (12) was denoted by $\widehat{f}$

$$\widehat{f} = \ln(f) = \ln(\beta_0 b^{\beta_1} h^{\beta_2}) = \ln(\beta_0) + \beta_1 \ln(b) + \beta_2 \ln(h) \tag{13}$$

Suppose

$$\begin{cases} \widehat{\beta_0} = \ln(\beta_0) \\ x_1 = \ln(b) \\ x_2 = \ln(h) \end{cases} \tag{14}$$

subsequently, the IRSM is rewritten as

$$\widehat{f} = \widehat{\beta_0} + \beta_1 x_1 + \beta_2 x_2 \tag{15}$$

By the MLSM with the aid of CCD method, the IRSM is fitted as

$$\widehat{f} = 18.22138 - 1.0x_1 - 3.0x_2 \tag{16}$$

Due to Eqs. (14) and (12) is written as

$$f = e^{\widehat{f}} = e^{\widehat{\beta_0} + \beta_1 x_1 + \beta_2 x_2} = e^{\widehat{\beta_0}} e^{\beta_1 x_1} e^{\beta_2 x_2} = e^{18.22138} b^{-1} h^{-3} = 8.193 \times 10^7 \frac{1}{bh^3} \tag{17}$$

To check the accuracy of traditional RS methods with cross item (Eq. (10), RSM1) and without cross item(Eq. (11), RSM2) as well as IRSM (Eq. (16), based on the same design variables and sample points, the computed results of three RS methods are listed in Table 1 referencing to the numerical solutions from Eq. (9). To validate the generalization and robustness of IRSM, 5 groups of samples obtained from Eq. (9) were used to evaluate the output responses of three RS methods, as shown in Table 2. The error comparison of traditional RS methods and IRSM is revealed in Fig. 6. Besides, the RS nephograms of numerical simulation, RSM1, RSM2 and IRSM are illustrated in Fig. 7, in which displacement response denotes the maximum response of the beam.

4.3 *Stochastic model updating of uncertain parameters*

The geometric parameters b and h were assumed to follow the Gaussian distribution as shown in Table 3. 15 groups of stochastic testing samples and 5000 simulating samples for b and h were extracted and the mean and standard deviation of response were gained in Table 3. Based on the advanced MC simulation and MLSM, the SMU-based IRSM was implemented instead of the FEM to converge towards the stochastic numerical values with inverse uncertainty propagation. The updated results are also summarized in Table 3. The simulation histories and histogram of initial and updated response y_{max} are illustrated in Fig. 8. To evaluate prediction precision, the probability density curves of the numerical testing, initial and updated displacements are shown in Fig. 9.

4.4 Discussions

From Table 1, the maximum displacement response y_{max} based on the traditional RS methods are close to numerical solutions where the maximum and minimum errors are 2.75% and 0.16% for RSM1 and 8.55% and 0.09% for RSM2, re- spectively. However, the considerable improvement has been achieved by using IRSM because all the errors are less than 0.01%, which highly proves the accuracy and reliability of IRSM. As shown in Table 2 and Fig. 6, all the predicting errors of IRSM are still less than 0.01% when the parameters b and h are out of the bounds of design variables, which also fully proves the robustness and prediction capability of IRSM. However, the traditional RS methods (RSM1 and RSM2) are not able to accurately predict the response. As shown in Fig. 7, the IRSM is very similar to numerical solutions while the traditional RS methods hold large difference.

As shown in Table 3 and Fig. 8, the updated input parameters are closer to the numerical test values as well as the output response y_{max} better agree with the test values, comparing with the initial RS model. As revealed in Fig. 9, the probability density curve of the initial displacement significantly differs from test value. However, the probability density curve of updated displacement is quite similar to the test response. Therefore, the updated RS model is better coinciding with the test results than the initial model.

To sum up, the above conclusions full verifies the precision and validity of IRSM. In addition, it is shown that the accuracy of the fitted RS model is greatly influenced by the characteristics of structure, which requires considering the relationship of mathematical and physical principle, the statistical law and the prior information.

5. Stochastic model updating of aeroengine stator system

5.1 FE model and static testing of aeroengine casings

The SMU of aeroengine stator system is to be finished to further validate the IRSM and advanced MC simulation. The stator system of an advanced turbofan engines was selected as the object of SUM. The stator system consists of many components with different function and materials as shown in Fig. 10, including intermediate casing, high pressure com- pressor casing, extension casing, combustion casing, turbine casing, bypass outside casing, afterburner diffuser and after- burner cylinder, and so forth. These casings are assembled together with bolted joints to transfer force, moment and energy from turbine to compressor. It is difficult to simulate the real mechanical characteristics of these bolted joints that are a major error source. To reduce the error, it is necessary to establish an accurate FE model of aeroengine stator system and update this model by adopting SMU technology and uncertainties. In the FE model of casing in Fig. 11, the spring elements are employed to represent the bolted joints, and 8 independent variables k_1–k_8 are chosen as updating parameters which are the connection stiffness among different casings. Based on static testing, the 8 updating parameters, following the Gaussian distribution, were achieved where their initial mean and standard deviation (Std.) are given in Table 4. The static stiffness of 4 important sections (B, C, D and E) on aeroengine stator system were measured to estimate the accuracy of the initial FE model where the means and standard deviations of the 4 output responses are obtained in Table 5 by repetitive measurements on a set of nominally identical structures.

5.2 Improved RS modeling and stochastic model updating

5.2.1 Improved RS model of output responses

8 Independent variables (k_1–k_8) and 4 important measured values on different sections are regarded as input parameters and output responses. Based on the CCD and advanced MC method, 50 samples were extracted to build the RS functions with the MLSM for the responses of different sections, by considering the randomness of 8 parameters.

5.2.2 Uncertainty quantification analysis

Through the SMU of aeroengine stator system with the hybrid optimization and advanced MC method, the updated results for the initial and updated parameters are obtained for 8 stiffness and 4 displacements as shown in Tables 4 and 5, respectively. The corresponding histograms of the initial and updated output responses are shown in Fig. 12,

As seen in Tables 4 and 5, the updated mean and standard deviation of 4 static responses (B, C, D, and E) converge towards the stochastic test values. The errors of responses largely reduce where the maximum errors of means and standard deviations diminish to 3.47% and 14.9% from 30.68% and 129.9%, respectively. As demonstrated from Fig. 12, the discrepancy between the initial and updated output responses further lessen and the responses obey the Gaussian distribution.

5.3 Verification of the proposed IRSM and advanced MC simulation

To validate the effectiveness of the proposed IRSM and advanced MC simulation, the FE model, traditional RS method and IRSM are adopted to execute the SMUs of 4-point displacements on aeroengine stator system with static testing sam-ples by using the computer with 16 code CPUs (2.66 GHz) and RAM 32 GB. The results of analytical comparison with the test data and initial model are located in Fig. 13 and the computing time of the point response B with three ap-proaches is shown in Table 6. It is noted that the FEM of aeroengine stator sys-tem includes 179,717 elements and 311,421 nodes and each static calculation of FE model costs about 265 s. The number of required samples denotes the num-ber of samples the fitting RS models need. Fitting time is the time-consumption of establishing RS models. Simulated time indicates the elapsed time of 5000 simulation on the established RS model. Total time is the sum of fitting time and simulated time.

5.4 Discussions

As shown from Fig. 13, the probability density curves of the 4 initial output responses are significantly different from the test responses. However, the probability density curves of the 4 updated output responses quite approximate to the test responses, which full validate that the proposed IRSM and advanced MC simulation hold high accuracy in the SMU of complex structure. The reason is that the MLSM improves the precision of established RS model, and the application of prior information further makes the constructed RS model closer to the real structural characters as well.

As known in Table 6, (1) the establishment of IRSM only needs 31 FE simulations, far less than (84 samples) traditional RS method. The conclusion supports the effectiveness of Latin hypercube sampling method (LHSM) in building RS model;

(2) due to fewer fitting samples, the fitting time of IRSM only costs 8.215×10^3s while the traditional RS method needs 2.226×10^4s, which fully reveals the fast regression capability of MLSM in shaping RS model; (3) the simulated time of RS model for IRSM is much less than that of traditional RS method, which demonstrate the high efficiency of the IRSM in large-scale simulations of complex structure and it is also verified by the total computing time; (4) it is also illustrated that the SMU-based RS method holds higher simulating speed and efficiency than FE model.

From the SMUs of simply-supported beam and aeroengine static system, the SMU strategy of combining IRSM-based MLSM and advanced MC simulation with LHSM holds high computational efficiency and precision in the SMU of complex structure.

6. Conclusions

The objective of this effort is to explore a stochastic model updating (SMU) strategy with fusing the improved response surface model (IRSM) and advanced Monte Carlo (MC) simulation for complex structure based on static test data and uncertainties. Firstly, the IRSM was proposed based on moving least-square method (MLSM) and prior information. And then the advanced MC simulation method was presented with Latin hypercube sampling method (LHSM). The SMUs of simply-supported beam and aeroengine stator system were implemented to validate the proposed methods. Some conclusions are summarized as follows:

(1) The explored SMU strategy holds high computational efficiency and accuracy in the SMU of complex structure, which provides a promising way to establish accurate computational model.

(2) The proposed IRSM is high precision and cost-effective in the establishment and simulation of improved RS model due to the memory ability of MLSM, and the adoption of prior information and uncertainties as well.

(3) The advanced MC simulation method is likely to extract effective samples, reduce the number of required samples and simulated time.

(4) The reliable model of aeroengine stator system is achieved, which is towards to improve the simulation and computation research of SMU.

(5) The SMU procedure is provided for the model updating of other structure and components.

The study of this paper enriches and develops the model updating theory and method. The application proposed SMU technology needs to further be validated in other fields.

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Figures

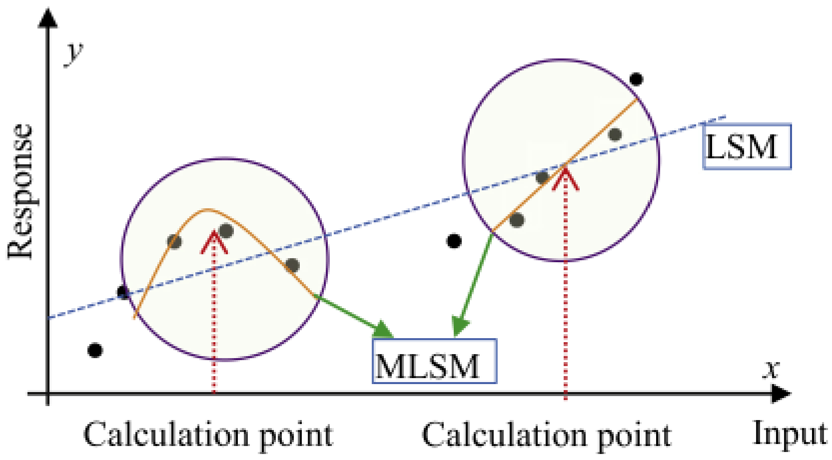


Fig.1. Basic principles of LSM and MLSM.

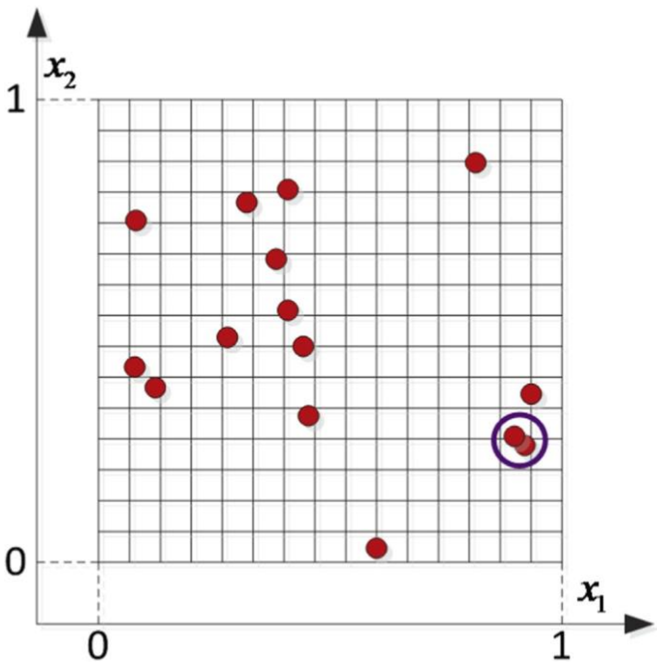


Fig. 2. Sample set generated with DSM.

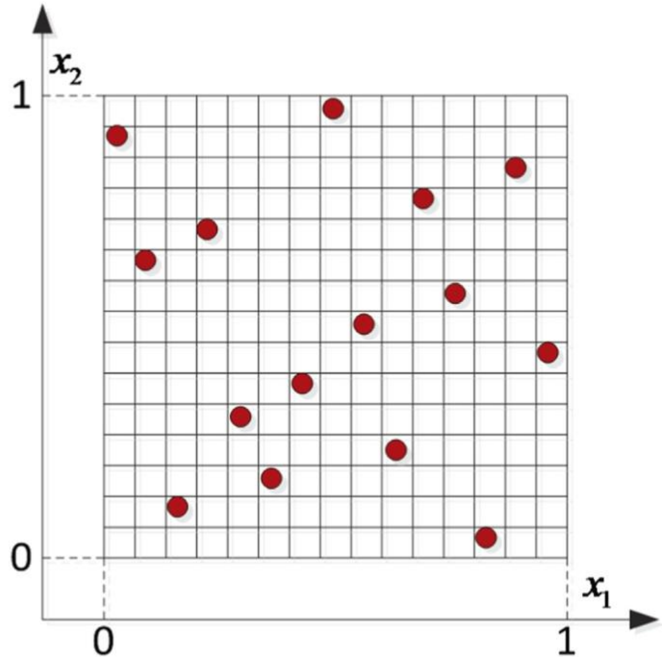


Fig. 3. Sample set generated with LHSM.

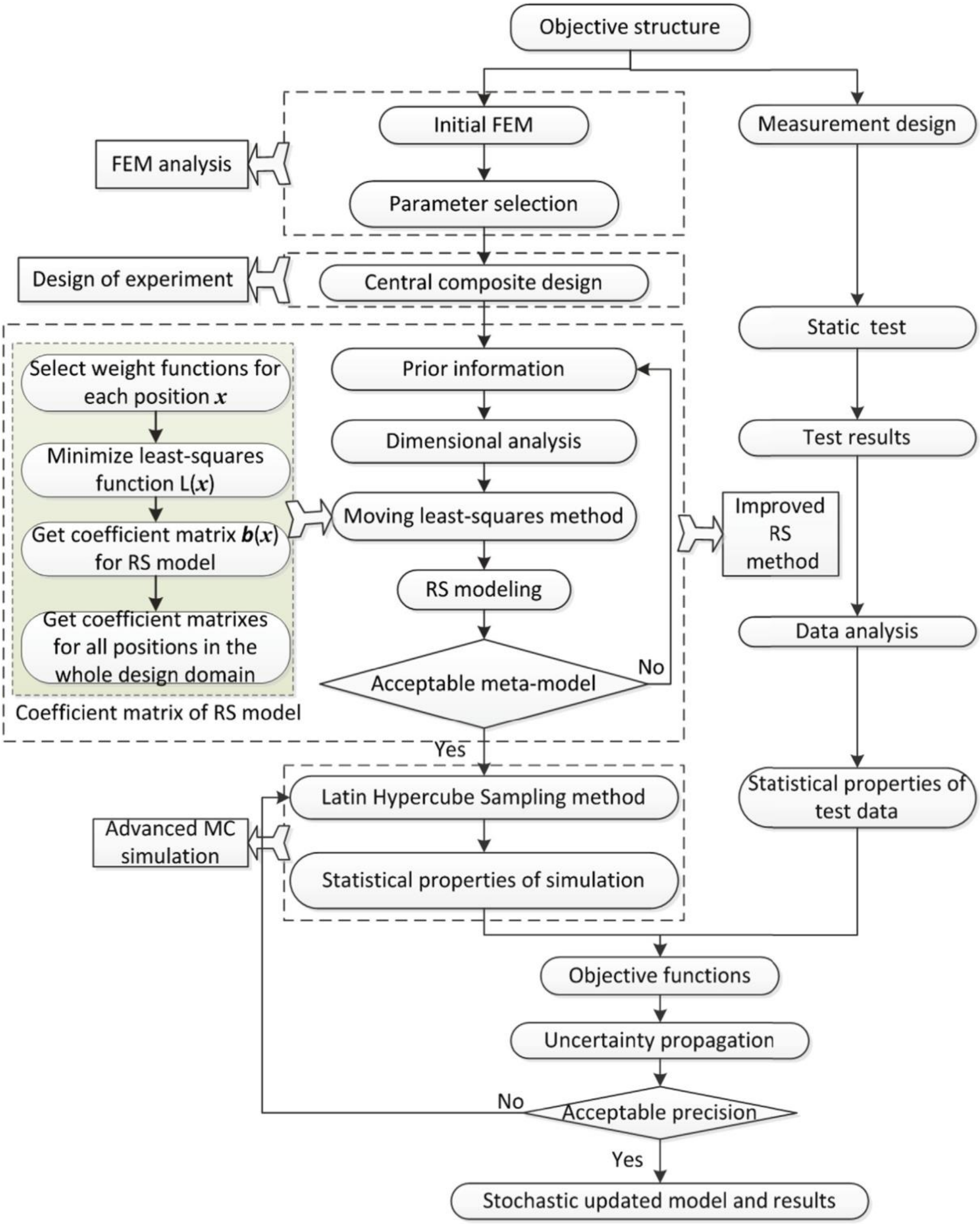


Fig. 4. SMU procedure of IRSM with CCD method, advanced MC simulation and MLSM.

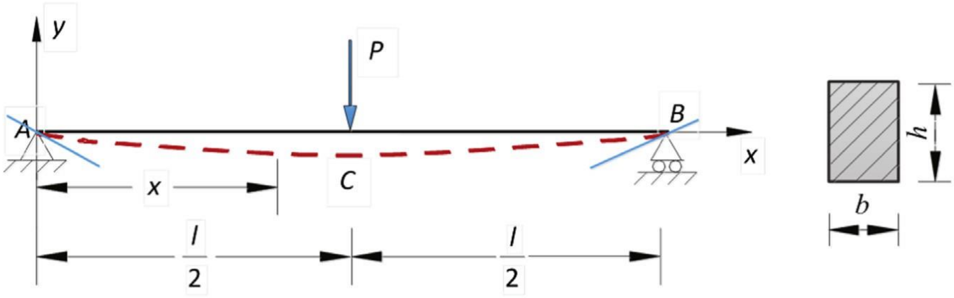


Fig. 5. Numerical simply-supported beam.

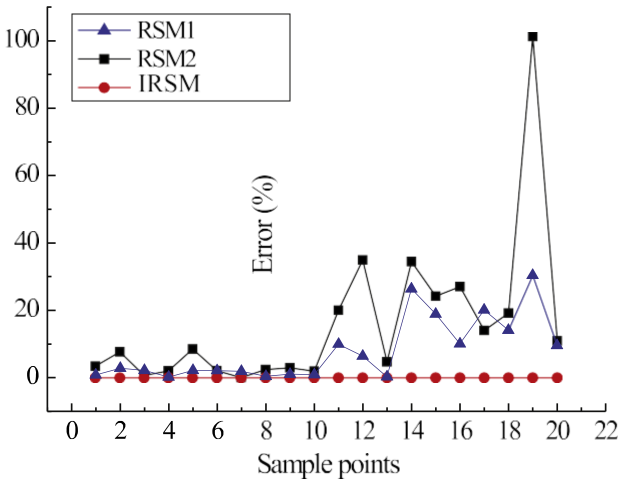


Fig. 6. Error comparison for traditional RS methods and IRSM.

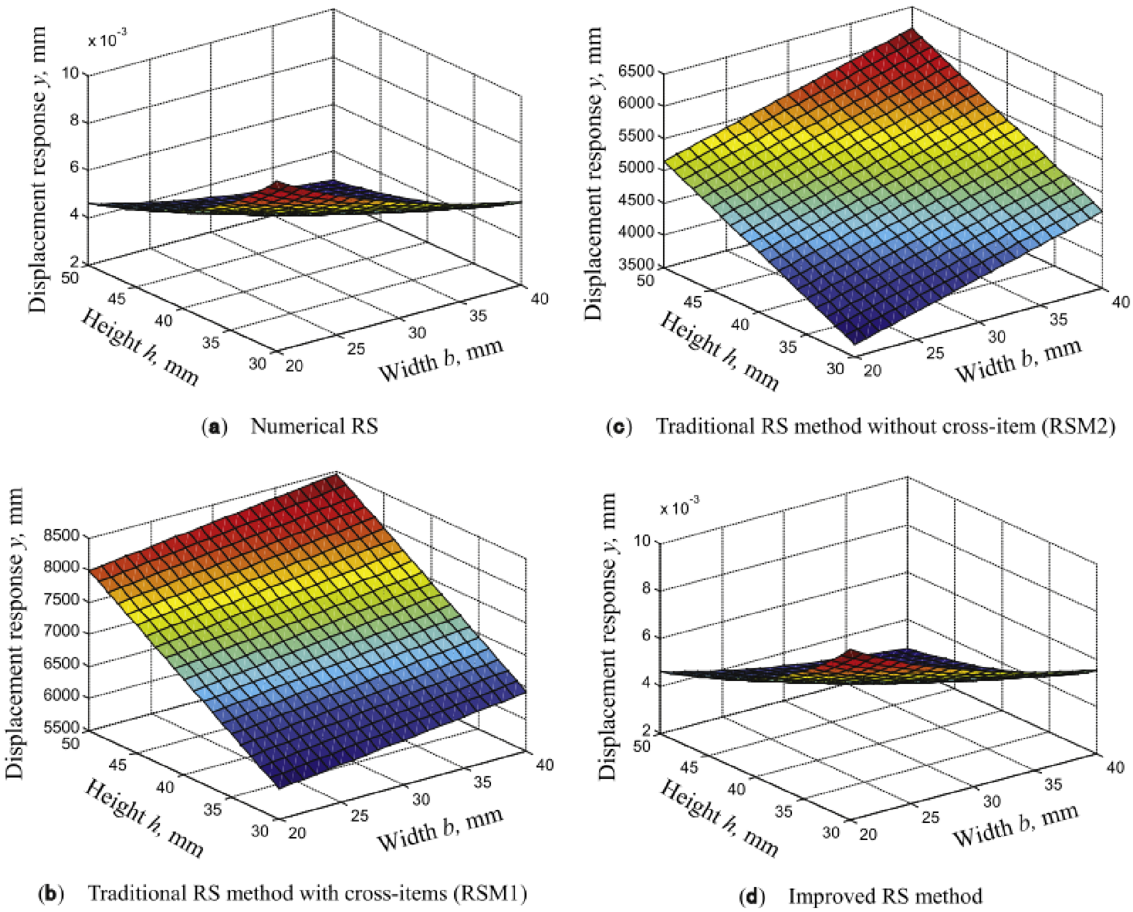


Fig.7. RS nephograms of different RS methods.

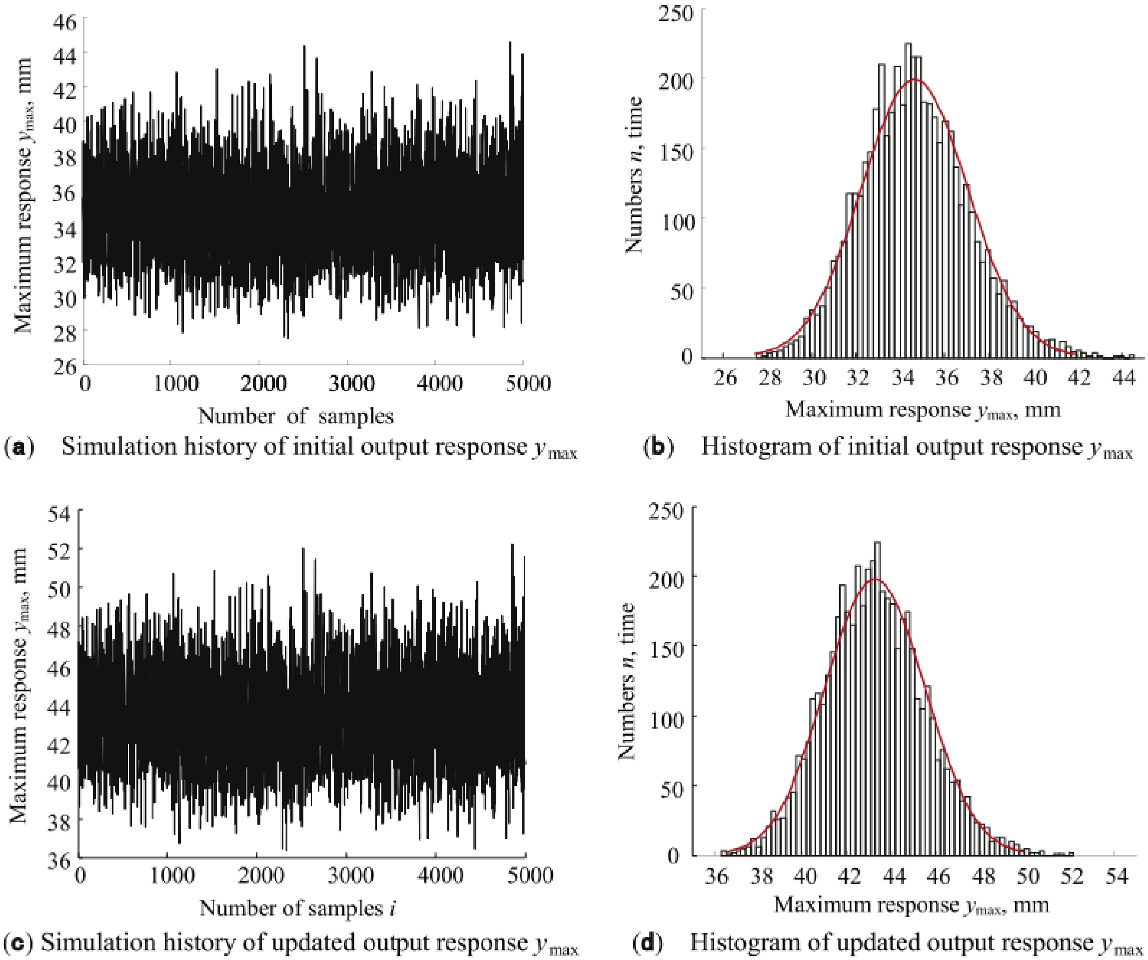


Fig.8. Simulation histories and histogram of initial and updated output responses.

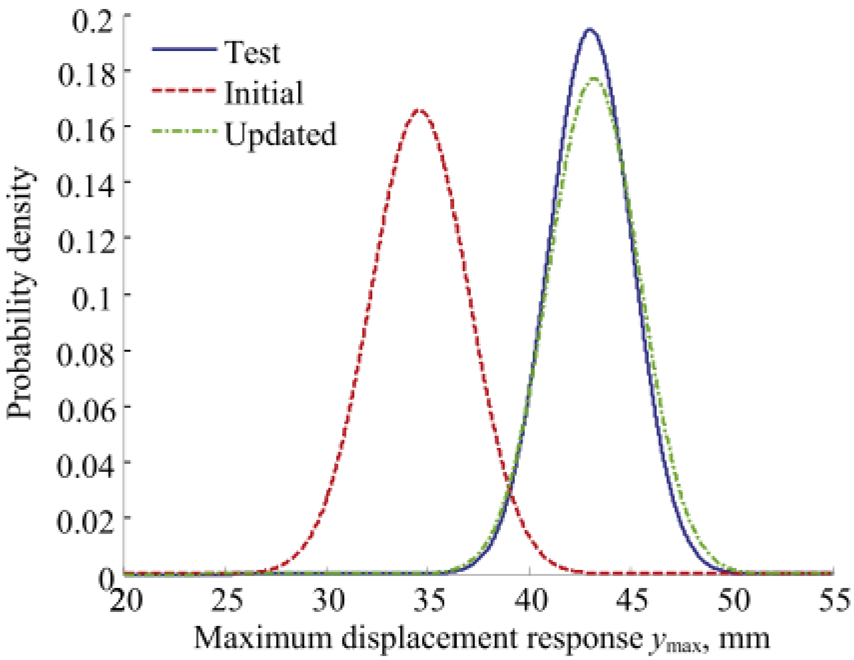


Fig. 9. Numerical, initial and updated probability density curve of output responses.

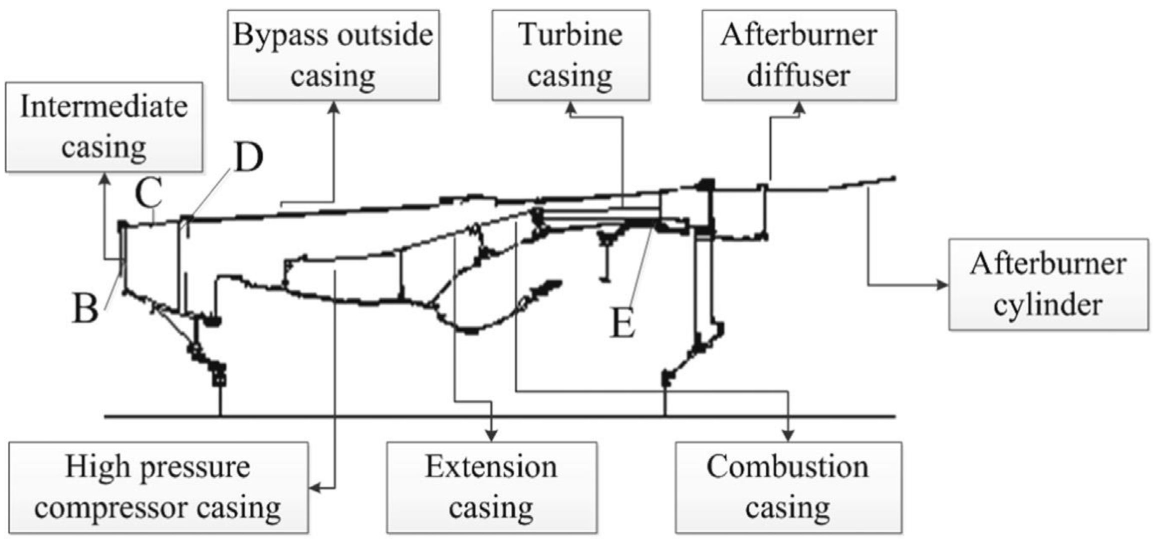


Fig. 10. Structure of an aeroengine stator system.

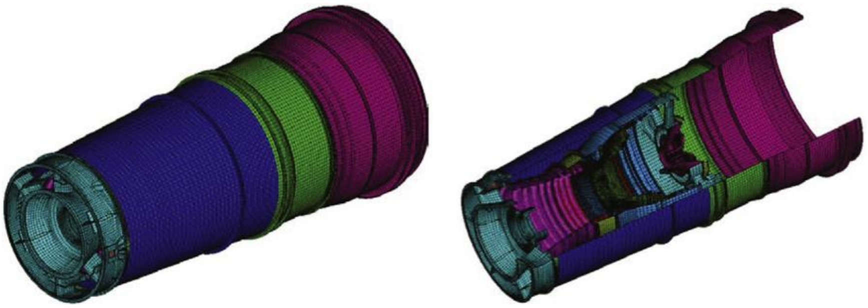


Fig. 11. FE model of aeroengine stator system.

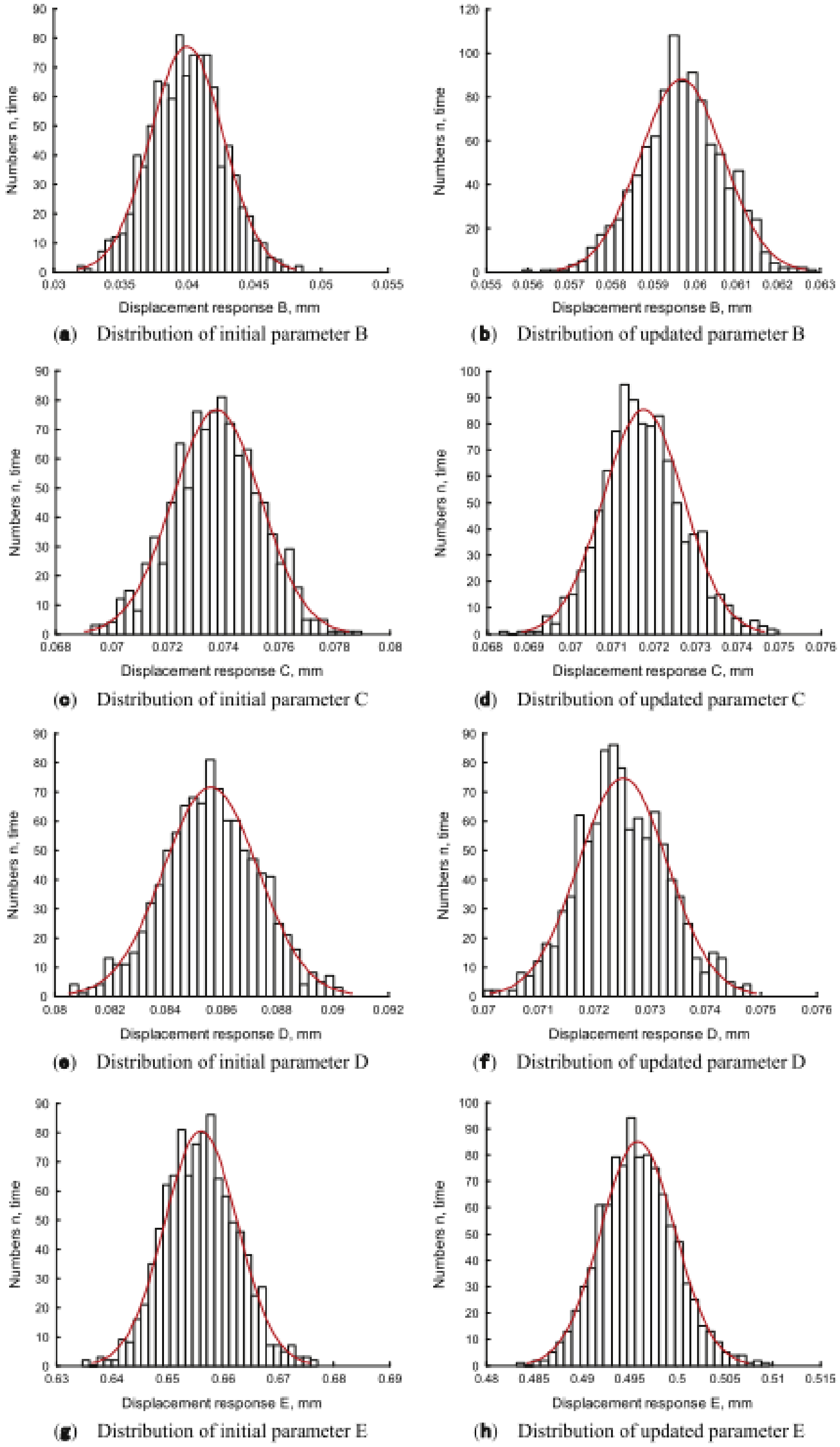
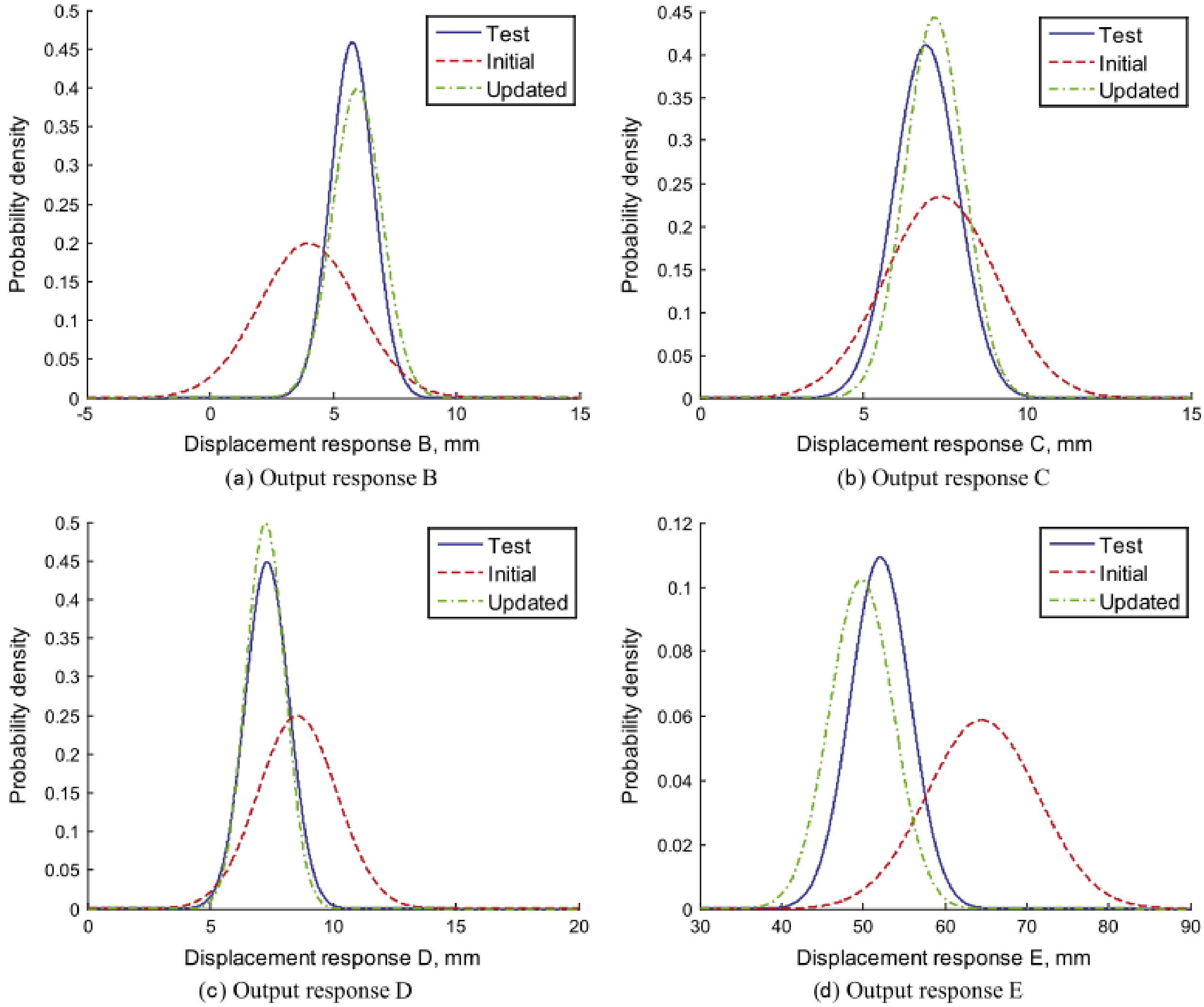


Fig. 12. Stochastic distribution of initial and updated output static responses.



(a) Output response B

(b) Output response C

(c) Output response D

(d) Output response E

Fig. 13. Test, initial and updated probability density curve of output responses.

Tables

Number	Parameters		Numerical solution	Traditional RSM1		Traditional RSM2		IRSM	
	b, mm	h, mm		Response	Error, %	Response	Error, %	Response	Error, %
1	23	37	70.314 3	69.739 5	−0.82	67.848 3	−3.51	70.325	0.01
2	40	48	18.517 9	19.026 5	2.75	17.092 8	−7.70	18.520 8	0.01
3	30	45	29.965 2	29.322 7	−2.14	29.739 2	−0.75	29.969 8	0.01
4	40	32	62.498 1	62.599 6	0.16	63.809 6	2.10	62.507 6	0.01
5	25	43	41.212 7	42.082 6	2.11	44.738 3	8.55	41.219	0.01
6	39	41	30.476 2	29.834 7	−2.10	31.127 5	2.14	30.480 8	0.01
7	30	47	26.300 4	25.806 8	−1.88	26.323 6	0.09	26.304 4	0.01
8	28	43	36.797 1	36.926 3	0.35	37.724	2.52	36.802 7	0.01
9	31	39	44.547 3	45.007 6	1.03	43.227 9	−2.96	44.554 1	0.01
10	30	40	42.665 4	43.044 8	0.89	41.813 2	−2.00	42.671 9	0.01

Table 1. Comparison of numerical solutions with the maximum displacement responses of traditional RS methods and IRSM.

Number	Parameters		Numerical solution	Traditional RSM1		Traditional RSM2		IRSM	
	b, mm	h, mm		Response	Error, %	Response	Error, %	Response	Error, %
1	19	29	176.778 3	130.217 7	−26.34	115.897 1	−34.44	176.805 3	0.01
2	18	32	138.884 6	112.724 2	−18.84	105.182 8	−24.27	138.905 8	0.01
3	26	51	23.751 6	26.137	10.04	30.170 8	27.03	23.755 2	0.01
4	30	53	18.341 2	22.023 1	20.07	20.924 8	−14.09	18.344	0.01
5	19	33	119.972 3	103.082 3	−14.08	96.945 9	−19.19	119.990 7	0.01

Table 2. Comparison of prediction ability of numerical solutions and three RS methods.

Types	Variables	Test values, mm		Initial values, mm		Updated values, mm	
		Mean	Std.	Mean (error, %)	Std. (error, %)	Mean(error, %)	Std. (error, %)
Input	b	30.034	0.3404	32 (6.50)	0.48 (41.01)	30.864 (2.76)	0.3701 (8.70)
	h	39.876	0.4785	42 (5.30)	0.756 (57.99)	39.469 (−1.02)	0.5237 (9.40)
Output	y_{max}	43.0655	2.0508	34.6410 (−19.56)	2.4098 (17.50)	43.2276 (0.38)	2.2535 (9.90)

Table 3. Distributions of input parameters and output response for test, initial values and updated value.

Input variables, $\times 10^3$ N/mm		K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
Initial	Mean	20	30	30	40	40	20	20	30
	Std.	3	4.5	4.5	6	6	3	3	4.5
Updated	Mean	17.22	37.88	40.06	34.72	32.55	28.94	34.72	43.4
	Std.	1.154	1.174	1.963	0.729	2.539	0.984	2.118	1.345

Table 4. Initial and updated means and standard deviations of 8 input stiffness parameters.

Output, 10^{-2} mm	Test		Initial				Updated			
	Mean	Std.	Mean	Error, %	Std.	Error, %	Mean, %	Error, %	Std.	Error, %
B	5.77	0.87	4.00	−30.68	2.0	129.9	5.97	3.47	1.0	14.9
C	6.91	0.97	7.36	6.51	1.7	75.3	7.17	3.76	0.9	−7.2
D	7.32	0.89	8.55	16.80	1.6	79.8	7.25	−0.96	0.8	−10.1
E	52.1	3.65	64.56	23.92	6.8	86.3	49.79	−4.43	3.9	6.8

Table 5. Test, initial and updated means and standard deviations of 4 output displacement parameters.

Methods	Fitting RS models		Simulation of RS models	Total time, s
	Number of required samples	Fitting time, s	Simulated time, s	
FE model	–	–	–	1.325×10^6
Traditional RS method	84	2.226×10^4	324	$1.258\ 4 \times 10^4$
Improved RS method	31	8.215×10^3	138	8.353×10^3

Table 6. Computing time of three methods for SMU of point B response on aeroengine stator system.