Simultaneous optimization of fuel surcharges and transit service runs in a multi-modal transport network: a time-dependent activity-based approach

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Abstract

This paper addresses the simultaneous optimization problem of fuel surcharges and transit service runs for energy sustainability of multi-modal transport network using a time-dependent activity-based approach. To model commuters' choices of trip chain, travel mode, departure time, route, and activity timing and duration over the times of a day, a time-dependent activity and multi-modal travel choice equilibrium problem is first addressed and formulated as an equivalent variational inequality problem. A new model for optimizing the fuel surcharges and transit vehicle runs is proposed to maximize the total social net benefit of the multi-modal transport system. The proposed model explicitly considers the interaction between the fuel surcharges and transit service runs and the commuters' activity-travel scheduling behavior. A heuristic solution algorithm is then developed to solve the proposed model. Finally, an illustrative example is given to show the application of the proposed model with various sensitivity tests. Insightful findings are presented with particularly the effects of the fuel surcharges and transit service improvement on the performance of the multi-modal transport system in terms of the modal split, fuel consumption, and total social net benefit.

Keywords: Multi-modal transport network; time-dependent activity-based approach; fuel surcharges; number of transit vehicle runs; variational inequality.

1. Introduction

Energy and environmental issues have recently attracted considerable public attention from around the world. There is a wide consensus that transportation is a major consumer of energy, mostly in the form of fossil fuels, and a major contributor to climate change due to vehicular pollution emissions (Nagurney, 2000; Ge et al., 2014). According to the latest report by the US Department of Energy (USEIA, 2010), the transportation sector consumes approximately 28% of all end-use energy in the United States. The environmental act of European Union (EUROPA, 2008) also indicated that in Europe, about 25% of total energy-related carbon dioxide emissions, 60% of carbon monoxide emissions, and more than half of total nitrogen oxide emissions come from transportation. The continuing growth in the number of vehicles is exacerbating the energy and environmental problems.

Fuel surcharge (or petrol tax) policy, which imposes an additional charge on the consumption of fuels, has been recently proposed as an effective instrument in some Asian countries or areas, such as China, to combat the increasingly serious energy and environmental problems in congested road networks. For instance, the empirical study that was conducted in Hong Kong by Hung (2006) showed that the introduction of the fuel tax policy promoted car owners to switch to clean fuels. Sterner (2007) found that the fuel surcharges can significantly restrain the growth in fuel demand and the associated carbon emissions. The empirical study of Gallo (2011) indicated that the implementation of the fuel surcharge policy can reduce vehicular greenhouse gas emissions through pushing car users towards more fuel-efficient vehicles. Kim et al. (2011) further revealed that the gasoline taxes reduced the carbon dioxide emissions in Korea.

These previous related studies mainly focused on the effects of the fuel surcharge policy on the vehicular pollution emissions and the energy demand. Little attention has been paid to modeling its effects on commuters' activity and travel choices, particularly on their shift between modes in a multi-modal transport network. In reality, the fuel surcharge policy, as an economic measure to manage travel demand, may change individuals' scheduled trip plans and activity participations, including their choices of trip chain, travel mode, departure time, route, and timing and duration of activities (Arentze et al., 2013; Habib, 2013; Etemadnia et al., 2014). Therefore, this raises some intriguing and important issues: what are the effects of the fuel surcharge policy on the individuals' activity and travel schedules? How to determine the fuel surcharge rate for environmentally sustainable transport system?

On the other hand, the introduction of the fuel surcharges may induce some of commuters to shift to public transit modes due to the increased auto travel cost. As a result, the passenger demand for transit modes increases. To gain the support of the public to the fuel surcharge policy, the local authority should improve the level of transit services by making use of the revenue generated from the fuel surcharges so as to increase the number of transit vehicle runs to meet the increasing passenger demand. Recently, Li et al. (2010) showed that the transit service schedules can significantly affect passengers' activity and travel choices. An adequate number of transit vehicle runs does not only provide passengers with timely linkages to reach their desired activity destinations, but also enhances the usage efficiency of transit vehicles. Therefore, there is a need to reveal the interaction among the fuel surcharges, transit vehicle runs, and the activity and travel choices of commuters in multi-modal transport networks, particularly in transit-oriented cities in Asia, such as Hong Kong, Beijing and Wuhan in China.

To answer these intriguing problems, a time-dependent activity-based approach can be adopted to model the responses of commuters to the fuel surcharge policy and transit vehicle scheduling in a multi-modal transport network over the times of a day. In the literature, the activity-based modeling approaches can be classified into two main categories: microscopic simulation approach and macroscopic network modeling approach. The microscopic simulation approach is useful for practical applications, but they are inadequate for revealing general properties of a problem because their results depend very much on the values of input parameters and the network topology structures in the simulation scenarios. Example studies include Kitamura et al. (2000), Henson et al. (2009), Hatzopoulou et al. (2011), and Pendyala et al. (2012). The network modeling approach can analytically address the effects of relevant policies and make a general conclusion regarding a policy directly from the model's properties. Sample studies include Lam and Yin (2001), Lam and Huang (2003), Huang and Lam (2005), Li et al. (2010), Fu and Lam (2014), and Fu et al. (2014). In this paper, the network modeling approach is adopted.

Two objectives of this paper are: (1) to develop a time-dependent multi-modal network equilibrium model that simultaneously considers the commuters' choices of trip chain, travel mode, departure time, route, and activity timing and duration as well as the effects of the fuel surcharges and transit vehicle scheduling, and (2) to optimize the fuel surcharges and the number of transit vehicle runs so as to improve the performance of the multi-modal transport system in terms of total social net benefit of the system. The major contributions made in this study are as follows. Firstly, a time-dependent activity and multi-modal transport network equilibrium problem is presented and formulated as an equivalent variational inequality problem. It explicitly incorporates the effects of the fuel surcharges and transit service scheduling and the commuters' choices of trip chain, travel mode, departure time, route, and activity timing and duration over the times of a day. Secondly, on the basis of the time-dependent network equilibrium, a model for optimizing the fuel surcharges and the number of transit vehicle runs is proposed to maximize the total social net benefit of the multi-modal transport system. A heuristic solution algorithm is developed to solve the proposed model. Thirdly, the effects of the fuel surcharge policy on individual's activity time allocation and transport system performance, in terms of modal split between auto and transit modes, total fuel consumption, and total social net benefit, are numerically examined for environmentally sustainable transport planning purpose.

The remainder of this paper is organized as follows. The next section describes basic components of the proposed model, including model assumptions, some basic concepts, activity utility, and travel disutility by mode. Section 3 presents the time-dependent activity and multi-modal transport network equilibrium problem. It follows with the model for optimizing the fuel surcharges and the number of transit vehicle runs. In Section 4, a heuristic solution algorithm is presented to solve the proposed model. Section 5 provides a numerical example to illustrate the essential merits of the proposed model. Finally, conclusion is given in Section 6 together with recommendations for further study.

2. Basic considerations

2.1. Assumptions

To facilitate the presentation of the essential ideas of this paper, the following assumptions are made.

A1 The whole study horizon [0, *T*] is discretized into equally spaced intervals, sequentially numbered $t \in T = \{0,1,\cdots,\bar{T}\}$, and Δ is the length of an interval so that $\bar{T}\Delta = T$. The study horizon in this paper is from 00:00 to 24:00 and is discretized into 1440 intervals each with one minute.

A2 All individuals are assumed to be homogenous in terms of their values of time. They can complete their trips by two alternative modes: auto and public transit modes. The transfer or exchange between modes, such as park-and-ride, is not considered in this paper, but will be explored in a future study.

A3 The set of feasible trip chains is assumed to be pre-specified. This assumption has also been adopted by related studies, such as that of Maruyama and Sumalee (2007), and Li et al. (2010, 2014), but can be relaxed by incorporating a dynamic activity/trip chain generator into the model (Arentze et al., 2011; Habib and Miller, 2009).

A4 The daily activity-travel schedules of trip-makers involve the following decisions: which trip chain to choose, when to depart from origin (e.g., home), which mode to take to reach an activity location, and how long to spend participating in each activity. Trip-makers base their decisions about activity and travel schedules on a tradeoff between the utility or benefits derived from activity participation at different locations and the disutility incurred by journey between activity locations. Here, it is assumed that all individuals in the system are utility-maximizing decision makers, that is, they schedule their activity-travel patterns to maximize their own net utility during a day, thus leading to a time-dependent version of Wardrop's user equilibrium.

A5 The objective for optimizing the fuel surcharges and the number of transit vehicle runs is to maximize the total social net benefit of the multi-modal transport system. The total social net benefit is defined as the difference of total user net utility in the system minus total transit operating cost.

A6 Transit vehicles in the network are assumed to fully follow a run schedule (Tong and Wong, 1999; Tong et al., 2001; Poon et al., 2004; Li et al., 2010), and stochastic disturbances in the in-vehicle travel time due to supply or demand uncertainty are not considered, although they will be explored in a future study.

A7 Fuel consumption of auto or transit vehicle travel is assumed to be a linear function of the distance traveled. This assumption has also been adopted by previous related studies, such as Liu et al. (2009) and Li et al. (2012).

2.2. Some useful concepts

Consider a multi-modal transportation network *G* = (*N*, *A*, *L*), where *N* is the set of nodes, *A* is the set of links, and *L* is the set of transit lines. Let *R* be the set of all trip origin nodes, and *r* be an *origin* of trip, $r \in R \subseteq N$. Each trip is associated with a trip chain. A *trip chain* consists of an ordered set of *sojourn* nodes (i.e. activity nodes). Individuals pursue their activities in these sojourn nodes. Let *I* denote the set of all activity nodes (or locations), and *i* denote a specific activity location in a trip chain, $i \in I \subseteq N$. Let *w* denote a trip chain that traverses *I* sojourn nodes, i.e. $w = \{1, 2, ..., i, ..., I\}$. Let W_r be the set of all chains starting at origin *r*. A trip chain that begins and ends at the same node is called a *tour*. A *path* in the network is a sequence of nodes and links. Let $P_{i,i+1}$ be the set of all paths between activity nodes *i* and $i+1$ (i.e. both are regarded as a pair of origin and destination nodes), and P_w be the set of all paths in trip chain *w*. P_w is then the combination of all possible paths between each pair of consecutive activity nodes in trip chain *w*, i.e. $P_w = \left\{P_{1,2} \times P_{2,3} \times \cdots \times P_{i,i+1} \times \cdots \times P_{I-1,I}\right\}$, where "$\times$" denotes the Cartesian product.

As an illustration, Figure 1 shows a typical home-based trip chain (tour), "home – work – shop – home", and its paths. It can be seen that there are two paths between home and work (i.e., p1 and p2) and between work and shop (i.e., p3 and p4), and one path between shop and home (i.e., p5). As a result, there are totally four paths for this chain, i.e., p1 – p3 – p5; p1 – p4 – p5; p2 – p3 – p5; and p2 – p4 – p5.

For presentation purpose, the multi-modal network *G* that consists of auto and transit modes is partitioned into two sub-networks, i.e. auto sub-network $G^{\text{au}} = (N^{\text{au}}, A^{\text{au}})$ and transit sub-network $G^{\text{tr}} = (N^{\text{tr}}, A^{\text{tr}}, L)$, where N^{au} and A^{au} are the sets of nodes and links in the auto sub-network, and N^{tr} and A^{tr} are the sets of nodes and links in the transit sub-network, respectively. In this paper, unless specified otherwise, the superscripts "au" and "tr" imply the auto and transit modes, respectively.

In the schedule-based (or run-based) transit sub-network, a transit vehicle *run* is a trip made by a transit vehicle from the beginning to the end of a transit line. A *transit line* is a group of vehicles that run back and forth between two transit stations, and can be described by the itinerary and the vehicle runs on that itinerary. The *transit link* is defined as each pair of adjacent transit nodes (i.e. a *line segment*) along a transit line. When there are several common lines directly connecting two nodes, each of them forms a distinct transit link. A *transit path* is described by a series of transit links.

From **A4**, the daily activity-travel schedules of individuals in a multi-modal network involve the following decisions: which trip chain to choose, when to depart from origin, which mode to take to reach an activity location, and how long to perform each activity. Individuals base their decisions about activity and travel schedules on a tradeoff between the utility derived from activity participation at different locations and the disutility incurred by travel between activity locations. The utility that is gained from an activity depends on the start time of that activity and the duration that it lasts. The activity *start time*, *end time*, and activity *duration* satisfy the following relationship (see Figure 2):

$$\tau_i^E = \tau_i^S + \tau_i, \quad \forall i \in I \text{, and} \tag{1}$$

$$\tau_{i+1}^S = \tau_i^E + T_{i,i+1}, \quad \forall i \in I , \tag{2}$$

where τ_i^S is the start time of activity *i*, which is also the arrival time at activity location *i*, τ_i is the duration for performing activity *i*, and τ_i^E is the end time of activity *i* (i.e. the departure time from activity location *i*). $T_{i,i+1}$ is the travel (or journey) time from activity location *i* to activity location $i+1$. In a multi-modal scenario, $T_{i,i+1}$ is related to the travel mode used, which is defined later. For ease of presentation, we define an *activity schedule pattern* as all possible combinations of activity start time, activity duration, and activity end time, denoted as $\boldsymbol{\tau} = \left\{ \tau_i^S, \tau_i, \tau_i^E, \ \forall i \in I \right\}$, and define Θ as the set of all activity schedule patterns in the network.

2.3. Utility of path in a trip chain

Let $U_{pwr}^{m}(t,\boldsymbol{\tau})$ be the (net) utility of path *p* in chain *w* starting at origin *r* with travel mode *m* (auto or transit mode), departure time *t*, and activity schedule pattern $\boldsymbol{\tau}$. It is the difference between the total utility derived from all instances of activity participation along the chain minus the total disutility of travel along the chain, and is expressed as

$$U_{pwr}^{m}(t,\boldsymbol{\tau})=\sum_{i\in I}U_i\left(\tau_i^S,\tau_i\right)\delta_{iw}-\sum_{i\in I}C_{h,i,i+1}^{m}\left(\tau_i^E\right)\delta_{pwr}^{h,i,i+1},\quad \forall p\in P_w, w\in W_r, r\in R, t\in T, \boldsymbol{\tau}\in\Theta, m, \tag{3}$$

where $U_i\left(\tau_i^S,\tau_i\right)$ is the utility achieved by a commuter performing activity *i* at time τ_i^S for duration τ_i. $C_{h,i,i+1}^{m}\left(\tau_i^E\right)$ is the disutility of traveling along path *h* from activity location *i* at interval τ_i^E (i.e. activity *i*'s end time $\tau_i^E=\tau_i^S+\tau_i$) to activity location $i+1$ by mode *m*. δ_{iw} equals 1 if activity location *i* is on chain *w*, and 0 otherwise. $\delta_{pwr}^{h,i,i+1}$ equals 1 if path *h* between activity nodes *i* and $i+1$ is a part of path *p* in chain *w* starting at origin *r*, and 0 otherwise. The utility of activity and the disutility of travel by mode are, respectively, defined as below.

2.4. Utility of activity

Individuals gain utility or benefits from participation in an activity that is dependent on the activity start time and duration (Joh et al., 2002; Ettema and Timmermans, 2003). Let $MU_i(t)$ denote the marginal utility of activity *i* that is the utility gained from participation in one time unit of activity *i* at time *t*. Thus, the utility achieved by a commuter performing activity *i* with start time *t* and duration τ_i can be defined as

$$U_i\left(t,\tau_i\right)=\int_t^{t+\tau_i}MU_i(x)dx,\quad \forall\tau_i, i\in I, t\in T, \tag{4}$$

where $MU_i(x)$ can be measured by a bell-shaped marginal utility function recently proposed by Joh et al. (2002) and generalized by Li et al. (2010) as follows.

$$MU_i(t)=U_i^0+\frac{\rho_i\lambda_i U_i^{\max}}{\exp\left(\rho_i\left(t-\xi_i\right)\right)\left(1+\exp\left(-\rho_i\left(t-\xi_i\right)\right)\right)^{\lambda_i+1}},\quad \forall i\in I, t\in T, \tag{5}$$

where U_i^0 is the baseline utility level of activity *i*. $U_i^{\max}$ is the maximum utility of activity

i, and ρ_i, λ_i and ξ_i are activity-specific parameters. The parameter ξ_i determines the time of day at which the marginal utility reaches its maximum value (i.e., the inflection point), ρ_i determines the slope or steepness of the curve, and λ_i determines the relative position of the inflection point. These parameters can be estimated based on survey data.

2.5. Disutility of travel by auto mode

We now define the travel disutility, $C_{h,i,i+1}^{\text{au}}(t)$, of commuters who depart from activity location i at interval t to activity location $i+1$ along path h by auto mode. It consists of the travel time cost on the road and the operating cost (fuel cost) of the vehicles, i.e.,

$$C_{h,i,i+1}^{\text{au}}(t) = \alpha_1 T_{h,i,i+1}^{\text{au}}(t) + \Phi_{h,i,i+1}^{\text{au}}, \quad \forall h \in P_{i,i+1}^{\text{au}}, i \in I, t \in T, \tag{6}$$

where α_1 is the commuter's value of travel time, which is used to convert travel time into equivalent monetary units. $P_{i,i+1}^{\text{au}}$ is the set of all paths between activity nodes i and $i+1$ in the auto sub-network. $T_{h,i,i+1}^{\text{au}}(t)$ is the travel time of commuters departing from activity location i at interval t to activity location $i+1$ along path h by auto. $\Phi_{h,i,i+1}^{\text{au}}$ is the fuel cost of path h by auto from activity location i to activity location $i+1$, which is measured in terms of monetary units. Following **A7**, $\Phi_{h,i,i+1}^{\text{au}}$ is a linear function of the distance traveled, and thus not related to the departure time interval.

Travel time $T_{h,i,i+1}^{\text{au}}(t)$ in Equation (6) can be calculated by

$$T_{h,i,i+1}^{\text{au}}(t) = \sum_{a \in A^{\text{au}}} \sum_{k \geq t} T_a^{\text{au}}(k) \delta_{at}^{h,i,i+1}(k), \quad \forall h \in P_{i,i+1}^{\text{au}}, i \in I, t \in T, \tag{7}$$

where $T_a^{\text{au}}(k)$ is the travel time of auto on link a during interval k. $\delta_{at}^{h,i,i+1}(k)$ equals 1 if the commuters departing from activity location i at interval t to activity location $i+1$ via path h arrive link a at interval k, and 0 otherwise.

The link travel time experienced by commuters who enter link a during interval k can be expressed as a function of all inflows entering that link by interval k (Chen and Hsueh, 1998; Lam et al., 2006), i.e.,

$$T_a^{\mathrm{au}}(k) = f\left(v_a^{\mathrm{au}}(1), v_a^{\mathrm{au}}(2), \cdots, v_a^{\mathrm{au}}(k)\right), \quad \forall a \in A^{\mathrm{au}}, k \in T , \tag{8}$$

where $v_a^{\mathrm{au}}(k)$ is the inflow of auto that enters link *a* during interval *k*, which is given by

$$v_a^{\mathrm{au}}(k) = \sum_{i \in I} \sum_{h \in P_{i,i+1}^{\mathrm{au}}} \sum_{t \in T} v_{ah,i,i+1,t}^{\mathrm{au}}(k), \quad \forall a \in A^{\mathrm{au}}, k \in T , \tag{9}$$

where $v_{ah,i,i+1,t}^{\mathrm{au}}(k)$ is the inflow to link *a* during interval *k* that departs from location *i* at interval *t* to location $i+1$ along path *h*. The link inflow, $v_{ah,i,i+1,t}^{\mathrm{au}}(k)$, can be determined in terms of path inflows as follows:

$$v_{ah,i,i+1,t}^{\mathrm{au}}(k) = f_{h,i,i+1}^{\mathrm{au}}(t)\delta_{at}^{h,i,i+1}(k), \quad \forall a \in A^{\mathrm{au}}, h \in P_{i,i+1}^{\mathrm{au}}, i \in I, k \in T, t \in T , \tag{10}$$

where $f_{h,i,i+1}^{\mathrm{au}}(t)$ is the path inflow who departs from location *i* at interval *t* to location $i+1$ via path *h* by auto, which can be given by

$$f_{h,i,i+1}^{\mathrm{au}}(t) = \sum_{r \in R} \sum_{w \in W_r} \sum_{p \in P_w^{\mathrm{au}}} \sum_{k \in T} q_{pwr}^{\mathrm{au}}(k)\delta_{h,i,i+1,k}^{pwr}(t), \quad \forall h \in P_{i,i+1}^{\mathrm{au}}, i \in I, t \in T , \tag{11}$$

where $q_{pwr}^{\mathrm{au}}(k)$ is the auto travel demand departing from origin *r* at interval *k* and using path *p* in chain *w*. $\delta_{h,i,i+1,k}^{pwr}(t)$ equals 1 if the commuters departing from origin *r* along path *p* in chain *w* at interval *k* enter path *h* between location *i* and location $i+1$ at interval *t*, and 0 otherwise. $q_{pwr}^{\mathrm{au}}(k)$ can be represented as

$$q_{pwr}^{\mathrm{au}}(k) = \sum_{\boldsymbol{\tau} \in \Theta(w)} q_{pwr}^{\mathrm{au}}(k, \boldsymbol{\tau}), \quad \forall p \in P_w^{\mathrm{au}}, w \in W_r, r \in R, k \in T , \tag{12}$$

where $q_{pwr}^{\mathrm{au}}(k, \boldsymbol{\tau})$ is the auto travel demand departing at interval *k* from origin *r* along path *p* in chain *w* with activity schedule pattern $\boldsymbol{\tau}$.

The fuel cost of auto, $\Phi_{h,i,i+1}^{\mathrm{au}}$, from activity location *i* to activity location $i+1$ along path *h* is a linear function of the distance traveled, i.e.

$$\Phi_{h,i,i+1}^{\mathrm{au}} = \left(\eta_0 + \eta_1\right)\phi^{\mathrm{au}} D_{h,i,i+1}^{\mathrm{au}}, \quad \forall h \in P_{i,i+1}^{\mathrm{au}}, i \in I , \tag{13}$$

where η_0 is the baseline price level per unit of fuel consumption, η_1 is the fuel surcharges, and ϕ^{au} is the fuel consumption of auto per kilometer. $D_{h,i,i+1}^{\mathrm{au}}$ is the length of path *h* between activity locations *i* and $i+1$ in the auto sub-network, which is expressed as

$$D_{h,i,i+1}^{\mathrm{au}} = \sum_{a\in h} D_a^{\mathrm{au}} \delta_a^{h,i,i+1}, \quad \forall h \in P_{i,i+1}^{\mathrm{au}}, i \in I \,, \tag{14}$$

where $\delta_a^{h,i,i+1}$ equals 1 if link a on path h between location i and location $i+1$, and 0 otherwise.

2.6. Disutility of travel by transit mode

The disutility of travel by transit mode consists of the following components: in-vehicle travel time, waiting time, in-vehicle crowding discomfort cost, access/egress time (i.e. walking time), and fare (Tong and Wong, 1999; Poon et al., 2004; Sumalee et al., 2009; Li et al., 2009). The travel disutility, $C_{h,i,i+1}^{\mathrm{tr}}(t)$, of commuters departing at interval t from activity location i to activity location $i+1$ along transit path h can be defined as

$$C_{h,i,i+1}^{\mathrm{tr}}(t) = \beta_1 T_{h,i,i+1}^{\mathrm{tr}} + \beta_2 G_{h,i,i+1}^{\mathrm{tr}}(t) + \beta_3 W_i^{\mathrm{tr}}(t) + \beta_4 \left(\Lambda_i^{\mathrm{tr}} + \Lambda_{i+1}^{\mathrm{tr}}\right) + \theta_{h,i,i+1}^{\mathrm{tr}}, \quad \forall h \in P_{i,i+1}^{\mathrm{tr}}, i \in I, t \in T \,, \tag{15}$$

where $T_{h,i,i+1}^{\mathrm{tr}}$ is the in-vehicle travel time on transit path h between activity location i and activity location $i+1$. $G_{h,i,i+1}^{\mathrm{tr}}(t)$ is the in-vehicle crowding discomfort cost for commuters departing at interval t from activity location i to activity location $i+1$ along transit path h. $W_i^{\mathrm{tr}}(t)$ is the average passenger waiting time at location i. Λ_i^{tr} and $\Lambda_{i+1}^{\mathrm{tr}}$ are the access time to transit station at activity location i, and the egress time from transit station at activity location $i+1$, respectively. $\theta_{h,i,i+1}^{\mathrm{tr}}$ is the transit fare on transit path h between activity location i and activity location $i+1$. The parameters β_1, β_2, β_3 and β_4 are, respectively, the value of in-vehicle travel time, value of crowding discomfort, value of waiting time, and the value of walking time, which are all measured in monetary value per unit time.

The in-vehicle travel time, $T_{h,i,i+1}^{\mathrm{tr}}$, on transit path h from activity location i to activity location $i+1$ can be calculated by

$$T_{h,i,i+1}^{\mathrm{tr}} = \sum_{a\in A^{\mathrm{tr}}} T_a^{\mathrm{tr}} \delta_a^{h,i,i+1}, \quad \forall h \in P_{i,i+1}^{\mathrm{tr}}, i \in I \,, \tag{16}$$

where T_a^{tr} is the in-vehicle travel time of transit vehicle on link a. According to **A6**, transit vehicles always operate on schedule. T_a^{tr} is thus equal to the difference between the departure time of a transit vehicle run from the downstream node of transit link a and the time that the vehicle arrives at the upstream node of that link. Let a^- and a^+ be the upstream and downstream nodes of transit link *a*, respectively. T_a^{tr} can then be expressed as

$$T_a^{\text{tr}} = t_{\text{dep},a^+}^{\text{tr}}(j) - t_{\text{arr},a^-}^{\text{tr}}(j), \quad \forall a \in A^{\text{tr}}, j \in J , \tag{17}$$

where *j* is the *j*th transit vehicle run, and *J* is the set of all transit vehicle runs. $t_{\text{arr},a^-}^{\text{tr}}(j)$ and $t_{\text{dep},a^+}^{\text{tr}}(j)$ represent the arrival time of the *j*th vehicle run at the upstream node a^- of transit link *a* and the departure time from the downstream node a^+ of link *a*, respectively.

The in-vehicle crowding discomfort cost $G_{h,i,i+1}^{\text{tr}}(t)$ can be calculated by

$$G_{h,i,i+1}^{\text{tr}}(t) = \sum_{a \in A^{\text{tr}}} \sum_{k \geq t} g_a^{\text{tr}}(k) \delta_{at}^{h,i,i+1}(k), \quad \forall h \in P_{i,i+1}^{\text{tr}}, i \in I, t \in T , \tag{18}$$

where $g_a^{\text{tr}}(k)$ is the in-vehicle crowding discomfort cost on transit link *a* during interval *k*, which can be estimated by (see, e.g. Lo et al., 2003; Li et al., 2009)

$$g_a^{\text{tr}}(k) = T_a^{\text{tr}} \left(b_0 + b_1 \left(\frac{v_a^{\text{tr}}(k)}{C_a^{\text{tr}}} \right)^{n_1} \right), \quad \forall a \in A^{\text{tr}}, k \in T , \tag{19}$$

where T_a^{tr} can be given by Equation (17), and C_a^{tr} is the capacity of link *a*. b_0 is the baseline discomfort level of transit services, and b_1 and n_1 are the positive calibrated parameters of the in-vehicle discomfort function. $v_a^{\text{tr}}(k)$ is the passenger flow on link *a* during interval *k*, which is given by

$$v_a^{\text{tr}}(k) = \sum_{h \in P_{i,i+1}^{tr}} \sum_{t \in T} \sum_{i \in I} v_{ah,i,i+1,t}^{\text{tr}}(k), \quad \forall a \in A^{\text{tr}}, k \in T , \tag{20}$$

where $v_{ah,i,i+1,t}^{\text{tr}}(k)$ is the passenger flow on link *a* during interval *k* that departs from location *i* at interval *t* to location $i+1$ along transit path *h*. It can be given by

$$v_{ah,i,i+1,t}^{\text{tr}}(k) = f_{h,i,i+1}^{\text{tr}}(t) \delta_{aht}^{i,i+1}(k), \quad \forall a \in A^{\text{tr}}, h \in P_{i,i+1}^{\text{tr}}, i \in I, k \in T, t \in T , \tag{21}$$

where $f_{h,i,i+1}^{\text{tr}}(t)$ is the passenger flow departing from location *i* at interval *t* to location $i+1$ via path *h* by transit mode, which can be expressed as

$$f_{h,i,i+1}^{\text{tr}}(t) = \sum_{r \in R} \sum_{w \in W_r} \sum_{p \in P_w^{\text{tr}}} \sum_{k \in T} q_{pwr}^{\text{tr}}(k) \delta_{h,i,i+1,k}^{pwr}(t), \quad \forall h \in P_{i,i+1}^{\text{tr}}, i \in I, t \in T , \tag{22}$$

where $q_{pwr}^{\mathrm{tr}}(k)$ is the passenger demand departing from origin r at interval k along path p in chain w by transit mode. It can be represented as

$$q_{pwr}^{\mathrm{tr}}(k)=\sum_{\boldsymbol{\tau}\in\Theta} q_{pwr}^{\mathrm{tr}}(k,\boldsymbol{\tau}),\quad \forall p\in P_w^{\mathrm{tr}}, w\in W_r, r\in R, k\in T, \tag{23}$$

where $q_{pwr}^{\mathrm{tr}}(k,\boldsymbol{\tau})$ is the transit passenger demand departing at interval t from origin r along path p in chain w with activity schedule pattern $\boldsymbol{\tau}$.

Average waiting time of passengers, $W_i^{\mathrm{tr}}(t)$, at location i can be determined by the difference between the arrival time of passengers at the transit station that is located in location i and the time that the first coming transit vehicle run arrives at the upstream node a^{-} of transit link a originating at activity node i, i.e.

$$W_i^{\mathrm{tr}}(t)=t+\Lambda_i^{\mathrm{tr}}-t_{\mathrm{arr},a^{-}}^{\mathrm{tr}}(\hat{j}),\quad \forall i\in I, t\in T, \tag{24}$$

where $t+\Lambda_i^{\mathrm{tr}}$ represents the arrival time at transit station of commuters who end activity i at interval t, and $\hat{j}$ is the first arriving transit vehicle run, which is determined by

$$\hat{j}=\min\left\{ j \,\middle|\, t+\Lambda_i^{\mathrm{tr}}-t_{\mathrm{arr},a^{-}}^{\mathrm{tr}}(j)\geq 0, \forall j\in J\right\}. \tag{25}$$

Walking time Λ_i^{tr} for access to or egress from the transit station at activity node i can be calculated by

$$\Lambda_i^{\mathrm{tr}}=\frac{D_i^{\mathrm{tr}}}{V_0},\quad \forall i\in I, \tag{26}$$

where D_i^{tr} is the average walking distance for access to or egress from the transit station at activity location i. V_0 is the average walking speed of passengers.

3. Model formulation

3.1. Time-dependent activity and multi-modal transport network equilibrium

According to **A4**, each individual in the multi-modal transportation system schedules his/her activity-travel pattern to maximize his/her own net utility during a day. This leads to a time-dependent Wardrop's user equilibrium of joint choices of trip chain, travel mode, departure time, route, and activity timing and duration.

Definition 1. At equilibrium, for each trip origin, the net utility of all of the used combinations of trip chain, travel mode, departure time, route, and activity timing and duration are equal and maximal, and the net utility of any unused combination of trip chain, travel mode, departure time, route, and activity timing and duration is smaller than or equal to the maximum.

Definition 1 means that for a given origin *r*, no individual would be better off by unilaterally changing his/her trip chain, travel mode, departure time, route, or activity schedule pattern. The activity-travel scheduling equilibrium can mathematically be expressed in a complementary form as

$$\left(U_{pwr}^{m*}(t,\boldsymbol{\tau})-U_{r}^{\max}\right)q_{pwr}^{m*}(t,\boldsymbol{\tau})=0,\quad \forall p\in P_{w}, w\in W_{r}, t\in T, \boldsymbol{\tau}\in\Theta, m\in\{\text{au},\text{tr}\}, \tag{27}$$

$$U_{pwr}^{m*}(t,\boldsymbol{\tau})\leq U_{r}^{\max},\quad \forall p\in P_{w}, w\in W_{r}, t\in T, \boldsymbol{\tau}\in\Theta, m\in\{\text{au},\text{tr}\}, \tag{28}$$

$$q_{pwr}^{m*}(t,\boldsymbol{\tau})\geq 0,\quad \forall p\in P_{w}, w\in W_{r}, t\in T, \boldsymbol{\tau}\in\Theta, m\in\{\text{au},\text{tr}\}, \tag{29}$$

where the superscript "*" represents the user equilibrium state, and $U_{r}^{\max}$ is the maximal net utility received by individuals at origin *r* from their activity and travel schedules during a day.

The following proposition shows that the multi-modal activity-travel scheduling equilibrium conditions (27)-(29) is equivalent to a variational inequality (VI) problem.

Proposition 1. A time-dependent travel demand pattern $\left\{q_{pwr}^{m*}(t,\boldsymbol{\tau})\right\}$ in the context of daily activity-travel schedules reaches an equilibrium state if and only if it solves the following VI problem:

$$\sum_{r\in R}\sum_{w\in W_{r}}\sum_{m\in\{\text{au},\text{tr}\}}\sum_{p\in P_{w}^{m}}\sum_{\boldsymbol{\tau}\in\Theta}\sum_{t\in T}U_{pwr}^{m*}(t,\boldsymbol{\tau})\left(q_{pwr}^{m}(t,\boldsymbol{\tau})-q_{pwr}^{m*}(t,\boldsymbol{\tau})\right)\leq 0,\quad \forall q_{pwr}^{m}(t,\boldsymbol{\tau})\in\Omega, \tag{30}$$

where the variables with the asterisk in the above VI represent the optimal solutions. Ω is the feasible set of travel demand variables $q_{pwr}^{m*}(t,\boldsymbol{\tau})$, which satisfies

$$\Omega = \left\{ \sum_{w \in W_r} \sum_{m \in \{\text{au},\text{tr}\}} \sum_{p \in P_w^m} \sum_{\boldsymbol{\tau} \in \Theta} \sum_{t \in T} q_{pwr}^m(t, \boldsymbol{\tau}) = Q_r, \quad \forall r \in R \right\}, \tag{31}$$

where Q_r is the total travel demand at origin *r*.

It can easily be shown that the VI problem (30) indeed reproduces the multi-modal activity and travel choice equilibrium conditions (27)-(29). For the detailed proof of this proposition, we refer the reader to Friesz et al. (1993), Wie et al. (1995), Chen (1998), and Lam et al. (2006). It should be pointed out that VI (30) is path-based, hence path enumeration is required in the solution algorithm. Note that the path travel disutility functions defined in the previous sections are non-linear and non-convex (Huang and Lam, 2002). VI (30) is thus non-convex, implying that multiple local solutions may exist.

3.2. Optimization model for fuel surcharges and transit service runs

As previously stated, the authority aims to maximize the total social net benefit of the multi-modal transportation system by determining the optimal fuel surcharges and transit vehicle runs. The social net benefit, denoted as *Z*, is the difference of the total user net utility in the system minus the total transit operating cost, represented as

$$Z = \sum_{r \in R} \sum_{w \in W_r} \sum_{p \in P_w^{\text{au}}} \sum_{\boldsymbol{\tau} \in \Theta(w)} \sum_{t \in T} U_{pwr}^{\text{au}}(t, \boldsymbol{\tau}) q_{pwr}^{\text{au}}(t, \boldsymbol{\tau}) + \sum_{r \in R} \sum_{w \in W_r} \sum_{p \in P_w^{\text{tr}}} \sum_{\boldsymbol{\tau} \in \Theta(w)} \sum_{t \in T} U_{pwr}^{\text{tr}}(t, \boldsymbol{\tau}) q_{pwr}^{\text{tr}}(t, \boldsymbol{\tau})$$
$$+ \sum_{i \in I} \sum_{h \in P_{i,i+1}^{\text{au}}} \sum_{t \in T} \eta_1 \phi^{\text{au}} D_{h,i,i+1}^{\text{au}} f_{h,i,i+1}^{\text{au}}(t) + \sum_{i \in I} \sum_{h \in P_{i,i+1}^{\text{tr}}} \sum_{t \in T} \theta_{h,i,i+1}^{\text{tr}} f_{h,i,i+1}^{\text{tr}}(t)$$
$$- \sum_{a \in A^{\text{tr}}} \left(\gamma_a^0 + \gamma_a^1 J_a \right) - \sum_{a \in A^{\text{tr}}} \eta_0 \phi^{\text{tr}} D_a^{\text{tr}} J_a \,, \tag{32}$$

where J_a is the number of transit vehicle runs on transit link *a*. γ_a^0 is the fixed transit operating cost (e.g. capital cost) on transit link *a*, and γ_a^1 is the variable transit operating cost (not including fuel cost) per transit vehicle run (e.g. vehicle maintenance costs and crew wages). ϕ^{tr} is the fuel consumption of transit vehicle per kilometer, and D_a^{tr} is the length of transit link *a*. The first two terms on the right-hand side of Equation (32) are the total net utility of auto users and transit passengers in the system, respectively. The third and fourth terms are the total fuel surcharges paid by auto users and the total transit fare revenue, respectively. The purpose for adding these two terms is to offset/cancel the fuel surcharges and transit fares that are included in the first two terms, i.e., to exclude the fuel surcharges and transit fares in the objective function. This is because the payment of both the fuel surcharges and fares implies only a transfer of money from users to the authority within the system and not a deadweight loss. The last two terms are the total transit operating cost, which is assumed to be a linear function of the number of transit vehicle runs. Specifically, the fifth term is the transit operating cost except the fuel cost, and the last term is the total fuel cost of transit vehicles.

In view of the above, the social net benefit maximization model for the fuel surcharge η_1 and transit service runs $\mathbf{J} = \{J_a, a \in A^{\text{tr}}\}$ can be formulated as

$$\max_{\eta_1, \mathbf{J}} \ Z\left(\eta_1, \mathbf{J}\right), \tag{33}$$

where $\left\{q_{pwr}^m(t, \boldsymbol{\tau})\right\}$ and $\left\{f_{h,i,i+1}^m(t)\right\}$ in the objective function Z are determined by solving

$$\sum_{r \in R} \sum_{w \in W_r} \sum_{m \in \{\text{au,tr}\}} \sum_{p \in P_w^m} \sum_{\boldsymbol{\tau} \in \Theta} \sum_{t \in T} U_{pwr}^{m*}(t, \boldsymbol{\tau}) \left(q_{pwr}^m(t, \boldsymbol{\tau}) - q_{pwr}^{m*}(t, \boldsymbol{\tau})\right) \le 0, \quad \forall q_{pwr}^m(t, \boldsymbol{\tau}) \in \Omega. \tag{34}$$

The optimization model (33)-(34) is actually a bi-level mathematical programming problem with a VI equilibrium constraint. It is intrinsically non-convex and hence it might be difficult to solve for a global optimum. In the next section, a heuristic solution algorithm is developed to solve this model.

4. Solution algorithm

Thus far, many heuristic solution algorithms for solving bi-level programming problems can be found (see, e.g. Luo et al., 1996). Here, the Hooke-Jeeves approach, a multidimensional search procedure, is adapted for solving the model (33)-(34). It does not require explicit knowledge of the derivative information of the objective function (33) with regard to the decision variables $\mathbf{y} = \left(\eta_1, \mathbf{J}\right)$ (in the following, we denote $|\mathbf{y}|$ as the dimension of vector **y**). The basic idea underlying the approach is to perform two types of searches in turn, i.e. an exploratory search and a pattern search. The step-by-step procedure is as follows:

Step 0. Initialization.

0.1. Choose an initial solution $\mathbf{y}^{(0)}$. Let $\mathbf{y}=\mathbf{y}^{(0)}$, solve the multi-modal activity and travel choice equilibrium problem (34), then calculate the social net benefit objective function $Z(\mathbf{y})$ by Equation (32).

0.2. Choose an initial step size $\sigma>0$ and the acceleration factor ω. Set $\zeta=1$.

0.3. Set the indices $\psi=1$ and s = 0.

Step 1. Exploratory search.

1.1. If $s>|\mathbf{y}|$ (the dimension of decision variables), go to Step 2.

1.2. Let $\hat{\mathbf{y}}=\mathbf{y}+\zeta\sigma e_j$ (e_j is a vector with 1 in the *j*th position and 0 elsewhere), solve the multi-modal activity and travel choice equilibrium problem (34) with $\hat{\mathbf{y}}$ and compute $Z(\hat{\mathbf{y}})$.

1.3. If $Z(\hat{\mathbf{y}})>Z(\mathbf{y})$, let $\mathbf{y}=\hat{\mathbf{y}}$, $Z(\mathbf{y})=Z(\hat{\mathbf{y}})$, $\psi=\psi+1$, $\zeta=1$ and go to Step 1.1. Otherwise, go to Step 1.4.

1.4. If $\zeta=1$, let $\zeta=-1$ and go to Step 1.2. Otherwise (i.e. $\zeta=-1$), let $\psi=\psi+1$, $\zeta=1$, and go to Step 1.1.

Step 2. Pattern search.

2.1. If $Z(\hat{\mathbf{y}})>Z(\mathbf{y}^{(s)})$, let $\mathbf{y}^{(s+1)}=\hat{\mathbf{y}}$, $Z(\mathbf{y}^{(s+1)})=Z(\hat{\mathbf{y}})$, $\mathbf{y}=\mathbf{y}^{(s)}+\omega(\mathbf{y}^{(s+1)}-\mathbf{y}^{(s)})$, $\psi=1$, $s=s+1$ and go to Step 1. Otherwise, go to Step 2.2.

2.2. If σ is "sufficiently" small, stop and output the optimal solution $\mathbf{y}^{(s)}$. Otherwise, let $\sigma=0.5\sigma$, $\psi=1$, $\mathbf{y}=\mathbf{y}^{(s)}$ and go to Step 1.

The Hooke-Jeeves approach above can converge to an optimal solution of the optimization problem (33)-(34) (see, e.g. Bazaraa et al., 2006, p.370). In Steps 0.1 and 1.2, the multi-modal activity and travel choice equilibrium problem (34) can be solved by the following solution approach.

Step 0. *Initialization*. Choose initial values for travel demand pattern $\mathbf{q}^{(0)}=\left\{q_{pwr}^{m(0)}(t,\boldsymbol{\tau})\right\}$. Set iteration counter $\kappa=0$.

Step 1. Calculate the utility of each path in each chain for each of auto and transit modes according to Equations (3)-(26) that are defined in Sections 2.3-2.6.

Step 2. Find the maximum-utility combination of trip chain, travel mode, departure time, route, and activity schedule pattern for each trip origin.

Step 3. Assign the total demand originating from each origin to the maximum-utility combination by performing all-or-nothing loading, and then yield the auxiliary travel demand pattern $\hat{q}^{(\kappa)} = \left\{ \hat{q}_{pwr}^{m(\kappa)}(t, \boldsymbol{\tau}) \right\}$.

Step 4. *Updating*. Update the travel demand pattern by

$$q_{pwr}^{m(\kappa+1)}(t, \boldsymbol{\tau}) = q_{pwr}^{m(\kappa)}(t, \boldsymbol{\tau}) + \left(\hat{q}_{pwr}^{m(\kappa)}(t, \boldsymbol{\tau}) - q_{pwr}^{m(\kappa)}(t, \boldsymbol{\tau}) \right) \Big/ (\kappa + 1) .$$

Step 5. *Convergence check*. If the relative gap $GAP = \| \mathbf{q}^{(\kappa+1)} - \mathbf{q}^{(\kappa)} \| / \| \mathbf{q}^{(\kappa)} \|$ is smaller than a pre-specified tolerance, then stop. Otherwise, set $\kappa = \kappa + 1$ and go to Step 1.

In Step 0, the initial travel demand pattern can be set to zero, which signifies an empty network. The CPU time that is required for the proposed algorithm is mainly dominated by Step 2, because searching of the longest paths for the auto and transit sub-networks is required in this step.

5. Illustrative example

5.1. Problem setting

The example network, which involves two alternative modes (i.e. auto and bus) and four types of activities, is given in Figure 3. The activities concerned include staying at home, eating (a before-work activity), working, and shopping (an after-work activity). The input parameters for the marginal utility functions of these four activities are shown in Table 1. The associated marginal utility curves are plotted in Figure 4. There are four home-based trip chains (or tours), i.e. {home – work – home}, {home – work – shopping – home}, {home – eating – work – home}, and {home – eating – work – shopping – home}.

The auto travel time $T_a^{\text{au}}(t)$ on link *a* can be computed by the following Bureau of Public Roads (BPR) function:

$$T_a^{\text{au}}(t) = T_a^0 \left(1 + 0.15 \left(\frac{v_a^{\text{au}}(t)}{C_a^{\text{au}}} \right)^{4.0} \right), \quad \forall a \in A^{\text{au}}, t \in T , \tag{35}$$

where T_a^0 is the free-flow travel time of auto on link *a*, and C_a^{au} is the capacity of link *a*.

The input data for the link travel time functions of auto and bus are shown in Table 2. It is assumed that the total number of commuters originating from home location is 2000 persons, and the walking time for access to or egress from each activity location is 3 minutes. The baseline price level per unit of fuel consumption η_0 is CN$ 8.0 per liter ("CN$" stands for the Chinese currency and US$1.0 approximates CN$6.20 on Jan 1, 2015). The commuters' value of time is CN$15 per hour, and the values of waiting and walking times are CN$30 per hour. Other model parameters are: $\gamma_a^0 = 50$ (CN$), $\gamma_a^1 = 40$ (CN$/run), $b_0 = 0.0$, $b_1 = 0.15$, $n_1 = 4.0$, $\phi^{tr} = 0.3$ (liter/veh-km), $\phi^{au} = 0.092$ (liter/veh-km). The proposed solution algorithm was coded in programming language C and run on a personal computer with an Intel Pentium 1.4-GHz CPU and 1 GB of RAM. In the numerical test, about 920 seconds of CPU time is required.

5.2. Discussion of results

Table 3 shows the optimal solutions before and after the implementation of fuel surcharges in terms of the modal split, fuel consumption, activity time allocation, and total social net benefit. It can be seen that the resultant optimal fuel surcharge is CN$0.92 per liter. The introduction of the fuel surcharges leads to an increase in the optimal bus service runs by 8 runs per hour for bus line 1 (from 12 to 20) and by 15 runs per hour for bus line 2 (from 15 to 30). This is because after the introduction of the fuel surcharges, some travel demand (a total of 216 commuters) switches from the auto mode to the bus mode due to an increased auto travel cost, leading to an increase in the bus modal split by 10.8% (from 30.9% to 41.7%). Specifically, the bus travel demands for chains "H-E-W-S-H", "H-W-S-H", "H-E-W-H", and "H-W-H" increase by 34, 67, 46 and 69 passengers, respectively. The auto travel demands for chains "H-E-W-S-H", "H-W-S-H", and "H-E-W-H" decrease by 159, 44, and 32, respectively. As a result, the total number of commuters using the longest chain ''H-E-W-S-H'' decreases by 125 (from 747 to 622), whereas those using other short chains (i.e. "H-W-S-H", "H-E-W-H", and "H-W-H") increase by 23, 14 and 88, respectively. These results show that the introduction of fuel surcharges would have an important effect on the modal split of commuters and their activity schedules. It should be pointed out that the decrease in the number of activities/trips of commuters may induce some secondary effects, such as an increase in the number of their activities/trips during weekends or in the number of their household members' activities/trips.

Such secondary effects can be investigated in a household-level activity scheduling framework (Bhat and Misra, 1999; Ho and Mulley, 2013), which is left for further study.

Table 3 also shows the effects of implementing the fuel surcharge scheme on the fuel consumption of the multi-modal transportation system. It can be seen that introducing the fuel surcharges can cause a significant decrease in the auto fuel consumption by 544 liters per day (from 3473 to 2929), but a slight increase in the bus fuel consumption by 114 liters per day (from 294 to 408). As a result, the total fuel consumption of the system decreases by 430 liters per day (from 3767 to 3337). This is attributable to the decrease in the auto travel demand and the increase in the bus runs, as stated above. The last row of Table 3 also shows that the change of the activity-travel schedules due to the fuel surcharge schemes can induce an increase in the total social net benefit by CN$ 0.02×10^5 per day. This means that the authority can improve the performance of the transportation system in terms of the total social net benefit and total fuel consumption by the joint implementation of fuel surcharges and transit service improvement (e.g. increasing the bus service runs).

In addition, it can be seen in Table 3 that the implementation of the fuel surcharge schemes can affect the time uses of individuals and their time allocations to various activities. In particular, the effects on the average duration of staying at home and average shopping duration are remarkable. Specifically, the average home-stay duration increases by 0.31 hour, and the average shopping duration decreases by 0.21 hour. However, the changes in the average journey time, average eating duration, and average work duration are trivial. These observations are because the fuel surcharge schemes drive part of travel demand to shift from the fast mode (e.g. auto) to the slow mode (i.e. bus), leading to an increase in the average journey time of commuters. In addition, commuters shift from the longer activity chain (e.g. H-E-W-S-H) to the shorter activity chain (e.g. H-W-H), implying a decrease in the average shopping duration (by 0.21 hour) and average eating duration (by 0.06 hour).

Figure 5 plots the profiles of the total fuel consumption of the transportation system during the morning peaks (6:30 to 9:30) before and after the implementation of the fuel surcharges. It can be noted in Figure 5a that there are two fuel-consumption peaks (i.e., around 6:45 and 8:15). This is because there are two associated departure peaks during the morning peaks: one is for eating purpose, and the other is for working purpose (see Figure 4). In addition, for each of the departure peaks the fuel consumption rate under the fuel surcharge scenario is lower than that without the fuel surcharge. As a result, the fuel consumption accumulation curve for the fuel surcharge scenario is below that for the no-fuel-surcharge scenario, as shown in Figure 5b. Again, this is due to the shift of travel demand from auto mode to bus mode after introducing the fuel surcharges. Figure 5 shows that the proposed model provides a potential avenue to forecast the time-varying fuel consumption in a multi-modal transport network over the times of a day.

6. Conclusion and further studies

This paper adopted a time-dependent activity-based modeling approach to investigate the simultaneous optimization problem of the fuel surcharges and transit service runs in a multi-modal transport system. A bi-level programming model was proposed to capture the interaction between the fuel surcharges and transit service runs, and the commuters' activity-travel choice behavior. In this paper, the activity and travel choice equilibrium problem was presented as an equivalent variational inequality formulation for modeling commuters' choices on trip chain, travel mode, departure time, route, and activity timing and duration over the times of a day. A heuristic solution algorithm was developed to solve the proposed bi-level programming model for determining the optimal fuel surcharges and transit vehicle runs in the multi-modal transport system.

The numerical results on a simple network have shown that the introduction of the fuel surcharges would change the modal split of travel demand and commuters' activity timing and duration. A joint implementation of the fuel surcharge and transit service improvement can lead to a sustainable modal split of the multi-modal transport system and enhance the performance of the system in terms of total social net benefit and total fuel consumption of the multi-modal transport system. The proposed model provides a useful tool for estimating the time-varying profile of the total fuel consumption in a multi-modal transport network over the times of a day, and can be used to model competitive multi-modal transport services and to evaluate various energy-related and/or environmentally sustainable transport policies at the strategic planning level.

Although the numerical results on a small network illustrate the essential merits and properties of the proposed model, case studies on realistic large-scale networks are required for further validation of the proposed model. Further research work may also focus on the following directions.

1. The activity utility functions play an important role in the proposed activity-based multi-modal transport model in this paper. In order to make use of the proposed model for practical applications in reality, there is a need to calibrate empirically the parameters of the activity utility functions.
2. It was assumed in this paper that the transit vehicles can be operated on schedule. In practice, the uncertainties in network supply and/or demand can result in adjustment of the transit vehicle timetables (i.e. transit vehicles cannot fully follow the timetables). It is thus necessary to incorporate the effects of the network uncertainty in the activity-based multi-modal transport model (Rasouli and Timmermans, 2012; Fu and Lam, 2014).
3. This paper mainly focused on individual’s activity and travel choice behavior. However, it is important to extend the proposed model to consider the households’ activity and travel choice behavior and the interaction between household’s members (Yoon and Goulias, 2010; Bhat et al., 2013), which is left for a future study.
4. Recently, the study by Arentze and Timmermans (2012) has addressed the importance of incorporating psychological factors in travel demand models. The proposed model can be further extended to consider the effects of the psychological/mental factors on joint decision-making behavior and energy consumption.
5. Studies have shown that mode-specific accessibility (Lei et al., 2012) and land use pattern (Shiftan, 2008; de Abreu E Silva et al., 2012; Harding et al., 2012) can significantly influence the activity and travel choice behavior of commuters. It is thus meaningful to investigate the joint impacts of transport mode’s accessibility and land use on the activity and travel scheduling behavior of commuters.

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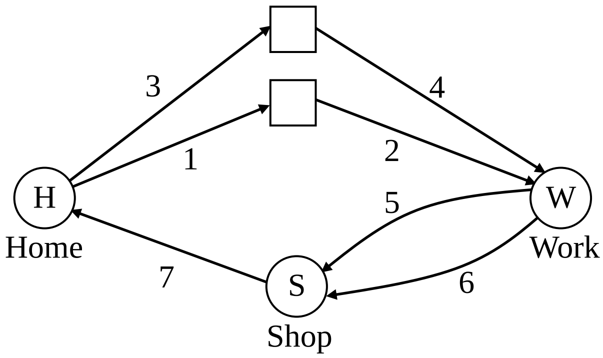

Paths between activity locations:
Home to work: p1 = 1 – 2; p2 = 3 – 4
Work to shop: p3 = 5; p4 = 6
Shop to home: p5 = 7

Paths for the trip chain:
p1 – p3 – p5
p1 – p4 – p5
p2 – p3 – p5
p2 – p4 – p5

Figure 1. A home-based trip chain and its paths.

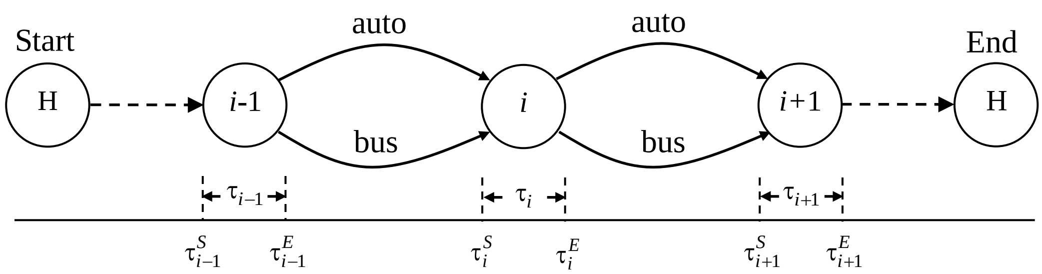

Figure 2. Activity schedule pattern for a trip chain.

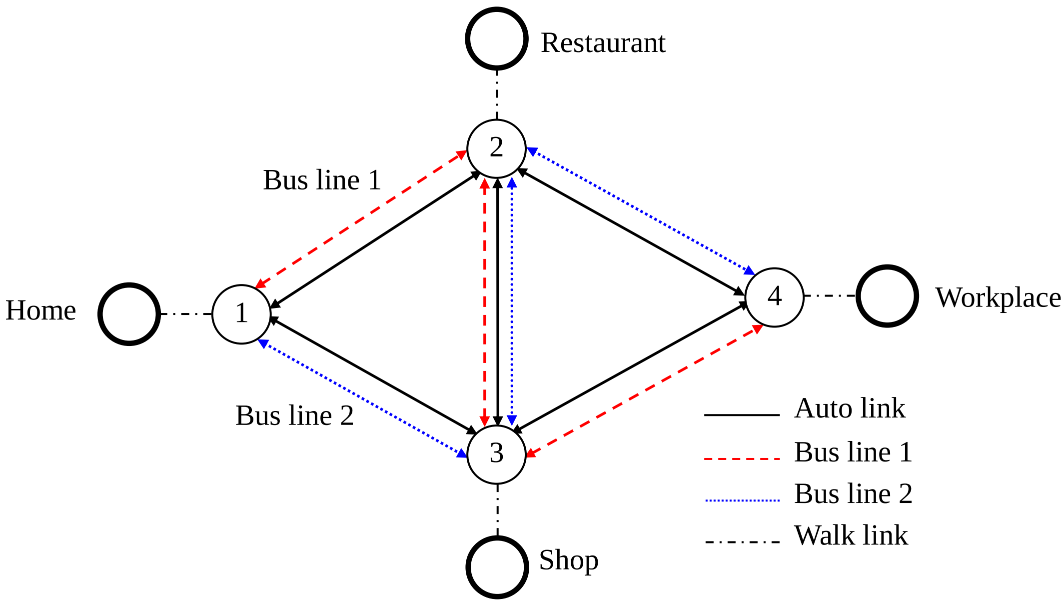

Figure 3. Example network.

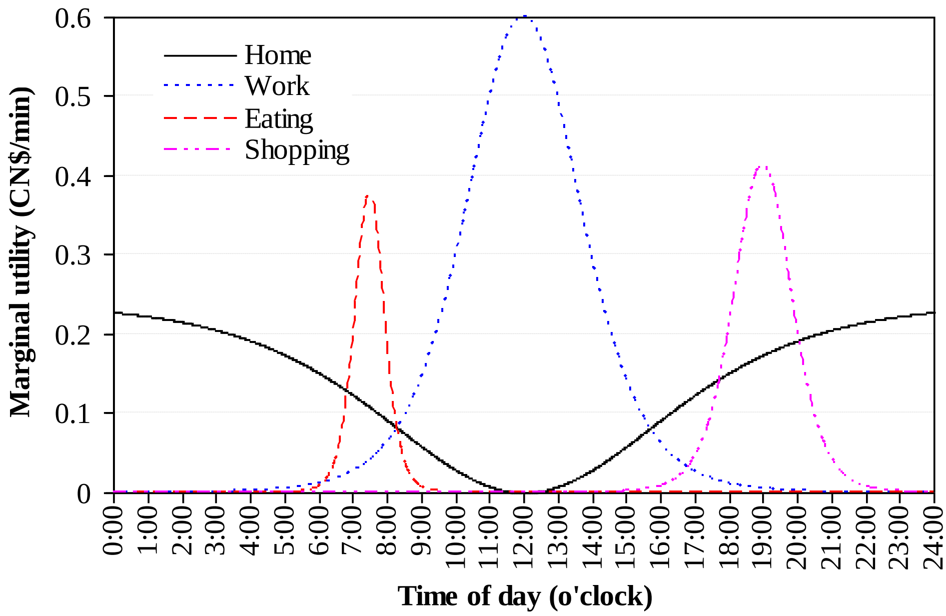

Figure 4. The marginal utility curves for four activities.

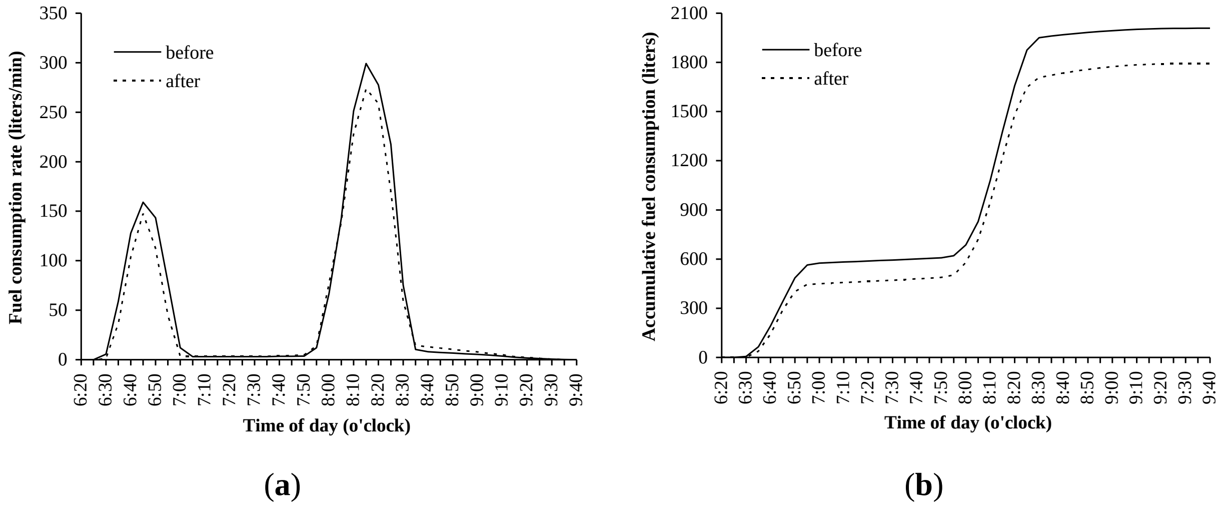

Figure 5. Profiles of fuel consumption during the morning peaks before and after the implementation of fuel surcharges: (a) fuel consumption rate; (b) accumulative fuel consumption.

Table 1 Input parameters for the marginal utility functions of four activities

Parameter	Home	Work	Eating	Shopping
ρ	-0.006	0.015	0.060	0.030
λ	1.0	1.0	1.0	1.0
ξ	720	720	450	1140
$U^{\max}$	160	160	25	55
U^{0}	0.24	0.00	0.00	0.00

Table 2 Input data for the travel time functions of auto and bus modes

Auto				bus		
Link No.	Length of link (km)	Free-flow time (min)	Capacity of link (veh/h)	Link No.	Length of link (km)	Travel time (min)
(1, 2)	7	10	3000	(1, 2)	7	20
(1, 3)	7	10	3000	(1, 3)	7	20
(2, 4)	7	10	3000	(2, 4)	7	20
(3, 4)	7	10	3000	(3, 4)	7	20
(2, 3)	7	10	3000	(2, 3)	7	20

Table 3 Comparison of optimal solutions before and after implementing fuel surcharges

	Before	After	After − Before
Optimal fuel surcharge (CN$/liter)	0	0.92	0.92
Optimal number of runs for bus line 1 (runs/h)	12	20	8
Optimal number of runs for bus line 2 (runs/h)	15	30	15
Modal split of travel demand			
Auto	1382 (69.1%)	1166 (58.3%)	-216 (-10.8%)
Bus	618 (30.9%)	834 (41.7%)	216 (+10.8%)
Auto travel demand for different chains			
H-E-W-S-H	522	363	-159
H-W-S-H	334	290	-44
H-E-W-H	314	282	-32
H-W-H	212	231	19
Bus travel demand for different chains			
H-E-W-S-H	225	259	34
H-W-S-H	116	183	67
H-E-W-H	184	230	46
H-W-H	93	162	69
Total travel demand for different chains			
H-E-W-S-H	747	622	-125
H-W-S-H	450	473	23
H-E-W-H	498	512	14
H-W-H	305	393	88
Total fuel consumption by mode (liters/day)			
Auto	3473	2929	-544
Bus	294	408	114
Total fuel consumption (liters/day)	3767	3337	-430
Average duration for different activities (h)			
Home	12.49	12.80	0.31
Eating	0.54	0.48	-0.06
Work	7.67	7.58	-0.09
Shopping	2.30	2.09	-0.21
Average travel (or journey) time	1.00	1.05	0.05
Total social net benefit (CN$/day)	5.67×10^5	5.69×10^5	0.02×10^5

Note: H = Home, E = Eating, W = Work, and S = Shopping.