Modelling intra-household interactions in household's activity-travel scheduling behaviour

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Activity-travel scheduling models can be used to investigate individuals' activity and travel decisions such as activity types, activity start time, activity duration, and departure time. Considerable evidence suggests that intra-household behavioural interactions exist in household activity-travel scheduling behaviour, which means an individual's decisions are affected by other household members' behaviour. However, most existing analytical activity-travel scheduling studies focus on one-individual level, and assume that each household member makes activity-travel decisions independently. As a result, the estimation of activity participation may be biased. In this study, a household activity-travel scheduling model is proposed to investigate the interactions between two household members. Markov Decision Process (MDP) is employed to provide a framework of dynamic discrete choice process that allows the household's decisions to have complex interdependence over time. The impact of intra-household interactions on individual's activity-travel scheduling behaviour is explicitly explored, and the variation in intra-household interactions across activity types is thoroughly examined by the proposed household MDP model.

Keywords: intra-household interaction; Markov Decision Process; activity-travel scheduling

1. Introduction

To model within-day dynamics in activity-travel scheduling, Markov Decision Process (MDP) is an expressive framework for formulating complicated inter-temporal choices in activity-travel scheduling (Puterman 1994). The advantage of MDP is that it does not need to consider each activity-travel schedule individually. There is thus no need to enumerate all feasible schedules. Another merit of the MDP formulation is that it takes into account the expected utility that can be obtained in the near future. The individual makes optimal decision at present with the knowledge that he will also make optimal decisions in the future.

In the literature, to model activity-travel choices in congested transport networks, the super-network representation is adopted by many studies and some network equilibrium models were proposed (Liao et al. 2010; Ramadurai and Ukkusuri 2010; Liao et al. 2013; Fu and Lam 2014; Fu, Lam, and Meng 2014; Liu et al. 2015). To some extent, these activity-based super-network equilibrium models can be considered as special cases of the MDP model for activity-travel scheduling. Each node in a super-network can be represented by a state in the MDP model. Each link in the super-network is an activity or travel decision that connects one state to another. Each route in the super-network is an ordered set of states and decisions, which constitutes an activity-travel schedule. In the MDP model, the feasible schedules are defined by local rules for each state and decision. Directly imposing rules on routes makes the super-network models computationally intractable. The major advantage of the MDP model is that it is much easier to define local rules for feasible states and decisions than to define rules for feasible routes in a super-network.

Based on the framework of the MDP proposed by Rust (1994), some structural dynamic discrete choice models have been proposed (Aguirregabiria and Mira 2010). Jonsson and Karlström (2005) and Xiong (2013) applied MDP in activity scheduling behaviour. Charypar and Nagel (2005) applied MDP in daily activity plan. Cirillo and Xu (2011) presented an extensive review of MDP models in transportation research. However, the above models do not consider interdependencies of household members in activity scheduling.

It is well recognised that the interaction between household members would influence individuals' activity choices particularly in a congested transportation network. Some types of activities can be allocated to a particular household member. Household members also jointly participate in activities to obtain higher overall utility for the entire household. However, in the literature, most activity-based models assumed that each individual's choice is independent of that of other individuals (Ben-Akiva and Bierlaire 1999). This assumption is, however not satisfied in the context of household activity-travel scheduling. The interdependencies between household members indeed influence the activity participation of each household member, so the intra-household interactions should be modelled in activity-travel scheduling models.

The complex nature of inter-personal dependencies results in many studies using the simulation technique. For example, Miller and Roorda (2003) and Roorda et al. (2008) presented comprehensive simulation models for household activity-travel scheduling. Arentze and Timmermans (2009) developed a need-based model of activity generation for a multi-day planning period taking account of household members' interactions. Dubernet and Axhausen (2013) included joint travels in a multi-agent micro-simulation. Apart from simulation models, a number of econometric models have also been proposed with the aim of exploring the intra-household behavioural interactions in relation to activity-travel choice behaviour, using structural equation modelling or the random utility approach. For example, the study of out-of-home activities and travel durations by Golob and McNally (1997), a time allocation model for two-individual households that accounts for joint activity participation by Gliebe and Koppelman (2002), and the work of Zhang et al. (2009) in which different household utility functions are introduced to represent household members' joint decision making interactions.

Compared to the development of activity-based simulation models and econometric models, fewer studies have been devoted to developing mathematical analytical models to consider intra-household interactions. The motivation of this paper is to investigate household's decision-making process with consideration of intra-household interactions. An analytical model for scheduling household's activity-travel behaviour is presented in this paper. To model the dynamics in activity scheduling and provide a framework of dynamic discrete choice process, we formulate the series of decisions made by the household as a MDP. The proposed MDP model allows the household's decisions to have complex interdependence over time.

Even though the household consists of different individuals, the household is assumed to act as a single decision-making unit. Hence, the household activity choice fits into the modelling framework of discrete choice. A household utility function can be used to capture the household's total utility with consideration of intra-household interactions (Zhang et al. 2002; Zhang et al. 2009).

The structure of this paper is as follows. Assumptions are firstly given in Section 2. Section 3 presents a household MDP model which captures the intra-household interactions. Section 4 describes the solution algorithm for the proposed model. Numerical examples illustrating the proposed model and algorithm are provided in Section 5. Conclusions are drawn in Section 6, together with suggestions for further research.

2. Assumptions

In order to facilitate essential ideas without loss of generality, the following assumptions are made in this paper.

A1: The individuals in a household make activity-travel decisions jointly (Zhang et al. 2005). Each individual voluntarily takes actions to implement the decision.

A2: Each individual has different preference over activity-travel schedules and the preference is represented by an individual utility function (Axhausen and Gärling 1992; Ettema et al. 2007).

A3: Household members are honest in revealing their preferences. The individual preferences are therefore public information within the household.

A4: With knowledge of individual preferences, the joint decision-making process seeks to maximize the total utility of the household (Lam and Yin 2001; Lam and Huang 2002; Zhang et al. 2005; Huang and Lam 2005; Li et al. 2010).

Assumption A1 ensures that the joint decisions are effectively implemented by household members. With regard to Assumption A2, the individual utility functions can be used to derive the household utility function for activity-travel schedules. Assumptions A3 and A4 eliminate the possible strategic behaviour adopted by the individuals to gain advantages within the household. Without these assumptions, the household activity-travel scheduling process becomes a more general problem that game theory is needed to account for the individuals' strategic behaviour.

3. Model formulation

3.1. *Markov Decision Process*

Travellers make activity-travel choices repeatedly over time. The choices depend on the contextual situations, such as time of the day and the traveller's location. The contextual situations can be represented by the state in the MDP model. A rational traveller attempts to anticipate the future situations and gain more utility for the whole day. This behaviour is captured by the objective of the MDP model. With appropriate definition of state, state transition and objective function, the activity-travel scheduling behaviour is grounded in a rigorous mathematical framework and viewed from a broader perspective. Details of the decision variables in MDP model for activity-travel scheduling can be found in Xiong et al. (2011).

Each household member participates in activities in parallel. The combination of optimal decisions of all household members is not always optimal in terms of the welfare of the entire household. Typically, making an activity-travel decision for one household member constrains the decisions available for the other. For example, if a household member drives the only car of the household, the other member cannot choose private car as the travel mode.

A household with two members, indexed by $i \in \{1, 2\}$, is considered in this paper. The model formulation can be easily generalized to a household with three or more members. Let A_i be the set of daily activities for individual i in the household. Each individual can undertake a subset of the activities in A_i. The activities in A_i are categorized into two types based on the flexibility of participation, i.e. compulsory and non-compulsory activities, which will be discussed in Section 3.3.

The activity-travel scheduling of household member i is formulated as an individual MDP model, denoted by M_i. S_i, D_i, p_i and R_i are the state set, decision set, transition probability function and utility function of individual MDP model M_i. The two household members share the same discount ratio for future utility, denoted by β.

In this study, the activity-travel scheduling behaviour of the entire household is defined as a household MDP model. The state set of the household MDP is a proper subset of the cross product of the household members' state sets, i.e., $S = \left\{ (s_1, s_2) \mid t_{s_1} = t_{s_2}, s_1 \in S_1, s_2 \in S_2 \right\}$. t_{s_1} and t_{s_2} indicate the time of day. It can be seen that for any household state, the times of household members' states are the same.

Time is discretised and a 24-hour day is evenly divided into T time episodes, denoted by $\{1, \ldots, T\}$. The state s_i includes five variables that describe the individual's contextual situations, i.e. time of the day t_{s_i}, the current location of the individual w_{s_i}, the set of uncompleted activities A_{s_i}, the on-going activity a_{s_i} and its remaining time e_{s_i}. The state of household member i is thus a 5-tuple $s_i = \left(t_{s_i}, w_{s_i}, A_{s_i}, a_{s_i}, e_{s_i} \right)$. If the state of a household member is (*9AM, Office, Work,* {*Shopping, Home*}, *Work, 9 hours*), it indicates that the household member works in the office at 9AM and will keep working for 9 hour. Two additional activities, *Shopping* and *Home* (i.e. in-home activity), need to be undertaken in the remainder of the day.

At each decision epoch, a decision d_i is selected from a decision set D_i. There are two types of decisions, activity decision and travel decision. If d is an activity decision, d is an ordered pair (a,h), where a denotes the activity to be participated in and h denotes the chosen activity duration. Since the subsequent activity is a component of the decision, the order of activity participation is determined by the decision. If d is a travel decision, d is an ordered pair (z,m), where z is the destination of the trip and m denotes the travel mode.

The decision set of the household MDP model D, is a proper subset of the cross product of the individual household members' decision sets, i.e., $D \subseteq D_1 \times D_2$. Each household member's decision set is defined as follows.

A household member's activity decision consists of the choice of activity type and duration:

$$D_{activity}(s_i) = \left\{ (a,h) \mid a \in A_{s_i}, [t_a, t_a + h] \subset [\underline{t}_a, \bar{t}_a] \right\} \tag{1}$$

$[\underline{t}_a, \bar{t}_a]$ is a given time window indicating that the activity should be conducted within time $\underline{t}_a$ and time $\bar{t}_a$. Suppose that the individual decision of a household member is (*Shopping, 1 hour*). The indication is that the household member will do shopping for 1 hour.

Let $B(z)$ denote the set of available activities at location $z \in W$. A household member's travel decision consists of the choice of trip destination and travel mode:

$$D_{travel}(s_i) = \left\{ (z,m) \mid A_{s_i} \cap B(z) \neq \varnothing, z \in W \setminus \{ w_{s_i} \}, m \in M(w_{s_i}, z) \right\} \tag{2}$$

$M(w_s, \text{z})$ is the available travel modes from w_s to z.

The union of the individual's travel decisions and activity decisions gives all the activity-travel decisions that the individual can make:

$$D_{new}(s_i) = D_{activity}(s_i) \cup D_{travel}(s_i) \tag{3}$$

Each household member can participate in activities in parallel. The household is at a decision epoch whenever a household member has completed an activity. The other member, however, may have not completed his/her on-going activity. Formally, if the current state $s = (s_1, s_2)$ is a decision epoch, each household member either takes a decision from $D_{new}(s_i)$ or continues the on-going activity. A special decision set is defined to account for the on-going activity of individual household member $D_{pre}(s_i) = \left\{ (a_{s_i}, e_{s_i}) \mid e_{s_i} > 0 \right\}$. Then the set of feasible decisions for household member i is expressed as:

$$D(s_i) = \begin{cases} D_{new}(s_i) & s_i \in \mathrm{I}_i \\ D_{pre}(s_i) & s_i \notin \mathrm{I}_i \end{cases} \tag{4}$$

where $\mathrm{I}_i = \left\{ s_i \mid e_{s_i} = 0, s_i \in S_i \right\}$ is the set of decision epochs for individual i.

The set of feasible decisions for the household is the cross product of that of the two household members:

$$D(s) = D(s_1) \times D(s_2) \tag{5}$$

To allow the possibility of simply waiting for a household member to complete an on-going activity, the individual's decision set $D(s_i)$ is augmented with a *wait* decision. The *wait* decision has a variable duration equal to the time until the next decision epoch. Travel decision is treated as a special activity *travel* with travel time as the activity duration. Let $Y(s,d)=\left\{\left(a_{d_i},h_{d_i}\right) | d_i \in D(s_i), \forall i=1,2\right\}$ denote the set of the on-going activities and their remaining times. The next decision epoch is the earliest time after which any on-going activity is completed (denoted as τ_d^s):

$$\tau_d^s = \min_{(a,h)\in Y(s,d)} h \tag{6}$$

Once the decision is made, the household receives an immediate utility, and the household's participation of activity in subsequent state is determined by the decision. When a household decision $d=(d_1,d_2)$ is made in state $s=(s_1,s_2)$, the subsequent state of household member i is updated as follows:

$$s_i' = \begin{cases} \left(t_{s_i}+\tau_d^s, w_{s_i}, A_{s_i} \setminus \left\{a_{s_i}\right\}, a_{d_i}, e_{s_i}-\tau_d^s\right) & d_i \in D_{activity}(s_i) \\ \left(t_{s_i}+\tau_d^s, z_{d_i}, A_{s_i}, travel, e_{s_i}-\tau_d^s\right) & d_i \in D_{travel}(s_i) \\ \left(t_{s_i}+\tau_d^s, w_{s_i}, A_{s_i}, a_{s_i}, e_{s_i}-\tau_d^s\right) & d_i \in D_{pre}(s_i) \end{cases} \tag{7}$$

The subsequent state is a random variable due to stochastic travel time. To capture the travel time uncertainty, the subsequent state is specified by a transition probability $p(s'|s,d)$ rather than a deterministic transition. The state transition probability for household member i (denoted by $p_i(s_i'|s_i,d_i)$) is equal to the probability that travelling from w_{s_i} to z_{d_i} by travel mode m_{d_i}. The transition probability function of the household's state is defined as:

$$p(s'|s,d) = p_1(s_1'|s_1,d_1)\cdot p_2(s_2'|s_2,d_2) \tag{8}$$

The total discounted household utility of making household decision d at state s is expressed by:

$$r(s,d) = \sum_{k=1}^{\tau_d^s} \beta^{k-1} r(s,d,k) \tag{9}$$

where $r(s,d,k)$ is the immediate utility that the household obtains at time k, and $\beta \in [0,1]$ is the discount factor for future utility and is constant over time. Different values of β indicate a variety of behaviour patterns. If $\beta=0$, the household is only concerned with immediate utility. If $\beta=1$, the household places the same values on the immediate utility and the future utility of activities within the same day. If $\beta>0$, the current decision depends on the future utility. This dependency reveals forward-looking behaviour.

According to assumption A4, at each decision epoch, the household makes a decision to maximize the weighted sum of the immediate utility and the expected future utility. The expected maximum utility is calculated by solving the recursive equation:

$$\overline{V}(s) = \mathrm{E}\left[\max_{d\in D(s)}\left\{r(s,d)+\varepsilon(d)+\beta^{\tau_d^s}\cdot\overline{V}(s')\right\}\right] \tag{10}$$

where $r(s,d)$ is the deterministic component of utility and $\varepsilon(d)$ is a zero-mean random variable due to unobserved characteristics. The dimension of $\varepsilon(d)$ is determined by the number of alternatives in $D(s)$. The detailed calculation of $\varepsilon(d)$ is illustrated in Xiong et al. (2011).

The household decisions over the whole day constitute the household's daily activity-travel schedule.

3.2. *Household utility function*

A household utility function is adopted to represent the household joint preference with consideration of intra-household interactions. The immediate utility that the household obtains at time k is decomposed as

$$r(s,d,k)=\sigma_1 \cdot r_1(s_1,d_1,k)+\sigma_2 \cdot r_2(s_2,d_2,k)+r_J(s,d,k) \qquad (11)$$

It can be seen from equation (11) that the household activity utility is the summation of weighted household members' utility and an interaction effect. σ_i $(i=1,2)$ denotes the weight parameter representing the relative influence of household member i. r_i $(i=1,2)$ is the individual immediate utility that household member i can obtain when conducting the activity independently:

$$r_i(s_i,d_i,k)=\begin{cases}\mu\left(a_{d_i},t_{s_i}+k\right) & \text{if } d_i \in D_{activity}(s_i)\\ \alpha\left(m_{d_i}\right) & \text{if } d_i \in D_{travel}(s_i)\end{cases} \qquad (12)$$

where $\mu(\cdot)$ and $\alpha(\cdot)$ denote the activity and travel utility respectively.

r_J indicates the interaction effect of household members:

$$r_J(s,d,k)=\rho \cdot r_1(s_1,d_1,k)\cdot r_2(s_2,d_2,k) \qquad (13)$$

where ρ measures the level of interaction between household members' activities. A detailed interpretation of the household utility function and alternative formulations representing different decision making strategies can be found in Zhang et al. (2002, 2009).

The interaction coefficient ρ takes non-zero values if the two household members jointly conduct activity a at location w, i.e., $a_{s_1}=a_{s_2}=a$ and $w_{s_1}=w_{s_2}=w$ for any state $s \in S$. Otherwise, (13) is equal to zero and the household utility (11) is reduced to the weighted sum of individuals' utilities.

To model differences in intra-household interactions across activities, distinct interaction coefficient ρ can be specified for each activity. Activities that require companionship and collaboration among household members have a positive intra-household interaction coefficient. Some routine activities that only need to be undertaken by any one of the individuals are specified with a negative ρ. In summary, there exists positive interaction between household members if $\rho > 0$, negative interaction if $\rho < 0$, and no interaction if $\rho = 0$. Figure 1 depicts the household utility with different interaction coefficients. It can be seen from Figure 1 that the household can obtain more utility from joint activity participation if the household members have positive interaction.

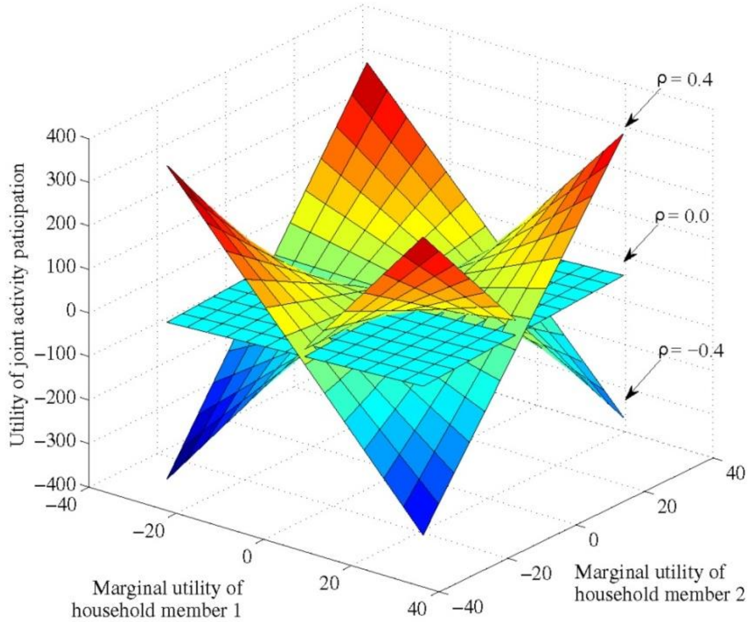

Figure 1. Utility of household's joint activity under different interaction coefficients.

3.3. *Compulsory and non-compulsory activities*

The daily activities can be categorized into two types based on the flexibility of participation, i.e. compulsory and non-compulsory activities. All the compulsory activities are allocated to a specific household member and should be completed within the planning horizon. The non-compulsory activities are optional. Imposing these constraints on activity choice demonstrates the flexibility of the MDP framework.

To model the compulsory and non-compulsory activities, the original household state is augmented with an additional component G_s, denoting the set of non-compulsory activities. Activities in G_s can be undertaken by any household member or be skipped. The original set of daily activities A_{s_i} is redefined to include compulsory activities that must be completed by individual i.

The sets of non-compulsory activities in the subsequent state s' are updated as follows:

$$G_{s'} = \begin{cases} G_s & d_i \notin D_{activity}(s_i), \forall i \\ G_s \setminus \{a_{s_1}, a_{s_2}\} & \text{otherwise} \end{cases} \tag{14}$$

The other components of the subsequent state are updated according to state transition equation (7).

At each decision epoch, individual i can select a compulsory activity or a non-compulsory one. Thus, the individual's activity decision set is expressed as:

$$D_{activity}(s_i) = \left\{(a,h) \mid a \in A_{s_i} \cup G_s, [t_a, t_a + h] \subset [\underline{t}_a, \bar{t}_a]\right\} \tag{15}$$

To ensure that any individual completes the compulsory activities in A_{s_i}, for any absorbing state $s \in S_*$, the set of uncompleted compulsory activities should be empty, $A_{s_1} = A_{s_2} = \varnothing, \forall (s_1, s_2) \in S_*$.

4. Solution algorithms

Given the optimal solutions of the individual's MDP models, one heuristic solution of household MDP model is directly combining the optimal solutions of individual's MDP models. However, due to the intra-household interactions and constraints on household decisions, this method may be suboptimal and even results in infeasible household decisions.

The following solution algorithm narrows down the household's decision space via dynamic merging the solutions of the individual MDP models (Singh and Cohn 1998). The dynamic merging algorithm finds the optimal solution to the household MDP model by directly performing value iteration on the household state and decision set. The pseudo code of the proposed algorithm is presented in Figure 2.

For solving a household MDP model $M = (M_1, M_2)$, the individual MDPs M_1 and M_2 should be first solved using the algorithm presented in Xiong et al. (2011). Then the optimal values of M_1 and M_2, $V(s_i), \forall s_i \in S_i, i = 1,2$ are used to construct the initial lower and upper bounds (denoted as $V_0^L(s)$ and $V_0^U(s)$ respectively) in the dynamic merging algorithm. The details of the dynamic merging algorithm can be found in Singh and Cohn (1998).

for each state $s \in S$ **do**
    set $(s_1, s_2) \leftarrow s$
    set $V_0(s) \leftarrow 0$
    set $V_0^L(s) \leftarrow \max\{V(s_1), V(s_2)\}$ and $V_0^U(s) \leftarrow V(s_1) + V(s_2)$
set $k \leftarrow 0$
repeat
    set $k \leftarrow k+1$
    for each state $s \in S$ **do**
        update the lower and upper bounds:

$$V_k^L(s) \leftarrow \max_{d \in D_{k-1}(s)} \left\{ r(s,d) + \sum_{s' \in S} p(s'|s,d) \cdot V_{k-1}^L(s'|s,d) \right\}$$

$$V_k^U(s) \leftarrow \max_{d \in D_{k-1}(s)} \left\{ r(s,d) + \sum_{s' \in S} p(s'|s,d) \cdot V_{k-1}^U(s'|s,d) \right\}$$

        update the value of the household state:

$$V_k(s) \leftarrow \max_{d \in D_{k-1}(s)} \left\{ r(s,d) + \sum_{s' \in S} p(s'|s,d) \cdot V_{k-1}(s'|s,d) \right\}$$

        update the set of competitive decisions:

$$D_k(s) \leftarrow \left\{ d \in D_{k-1}(s) \mid Q_k^U(s,d) \geq V_t^L(s) \right\}$$

where $Q_k^U(s,d) = r(s,d) + \sum_{s' \in S} p(s'|s,d) \cdot V_{k-1}^U(s'|s,d)$

until $|D_k(s)| = 1$ for all $s \in S$ **or** $|V_k(s) - V_{k-1}(s)| < \epsilon$

for each state $s \in S$ **do**

set $\pi(s) \leftarrow \arg\max_{d \in D_k(s)} \sum_{s' \in S} F(s'|s,d) \left[r(s,d) + \varepsilon(d) + V_k(s'|s,d) \right]$

return V_k and π

Figure 2. Dynamic merging algorithm for household MDP model.

The efficiency of dynamic merging is gained by constructing lower and upper bounds on the optimal values of the household states. The bounds are initially constructed based on the optimal solution of individual MDP models and then incrementally updated using value iteration. If the upper bound of household decision d is less than the lower bound of another household decision, the decision d is strictly dominated and can be excluded from the household decision set. The algorithm terminates when there is only one household decision remaining in set $D_k(s)$ for each household state s, or when the expected utility changes by a small amount in an iteration.

5. Numerical examples

Figure 3 shows a 4-node transport network. 10,000 behaviourally homogeneous households are considered and each household is composed of two adults: Individual 1 and Individual 2. Node H represents the residential location. Node W1 and W2 represent the workplaces of Individual 1 and 2 respectively. For simplicity, travel time is assumed deterministic and the congestion effect is captured by a BPR function,

$$\tau_l(f_l(t)) = t_l^0 \times \left(1 + 0.15 \left(\frac{f_l(t)}{5000} \right)^4 \right) \tag{16}$$

where $f_l(t)$ is the flow on link l at time t.

The equivalent disutility of travelling for one hour is $\alpha = 60$. The discount ratio of the future utility is set to $\beta = 0.99$. The entire day (24 hours) is divided into 288 intervals each with five minutes.

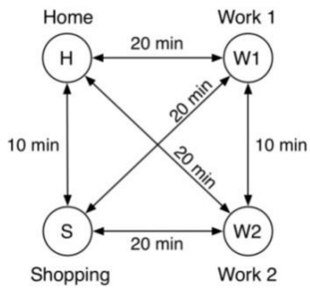

Figure 3. The test network.

The utility of pursuing an activity varies over the course of the day. The optimal starting time and the duration of activity depend on the temporal profile of activity utility. The bell-shaped marginal utility function proposed in (Ettema and Timmermans 2003) is adopted in this example:

$$g(a,t)=\frac{\gamma_a\lambda_a U_a^{\max}}{\exp\left[\gamma_a\left(t-\xi_a\right)\right]\cdot\left\{1+\exp\left[-\gamma_a\left(t-\xi_a\right)\right]\right\}^{\lambda_a+1}} \tag{17}$$

where t is the time of day, $U_a^{\max}$ is the maximum accumulated utility of activity a, and γ_a, λ_a, ξ_a are activity-specific parameters. The parameters can be estimated on the basis of survey data. This function captures not only activity characteristics but also activity participation time. Many related studies have adopted this type of function for modelling the marginal utility of activity (Zhang et al. 2005; Li et al. 2010).

Three types of activities are considered in the example: *Home*, *Work*, and *Shopping*. The parameters of utility function for each activity are presented in Table 1. Figure 4 depicts the temporal profiles of the individual's marginal activity utility.

Table 1. Parameters in utility function for each activity.

Activity	Parameters of utility function			
	$U_a^{\max}$	γ_a	λ_a	ξ_a (min)
Home	1000	0.006	1.0	0
Work	800	0.010	1.0	720
Shopping by Individual 1	**180**	0.032	1.0	1110
Shopping by Individual 2	**60**	0.032	1.0	1110

The two household members have distinct preferences for shopping activity, as shown by the bold values in Table 2. It can be found that individual 1 is more willing to shop than Individual 2.

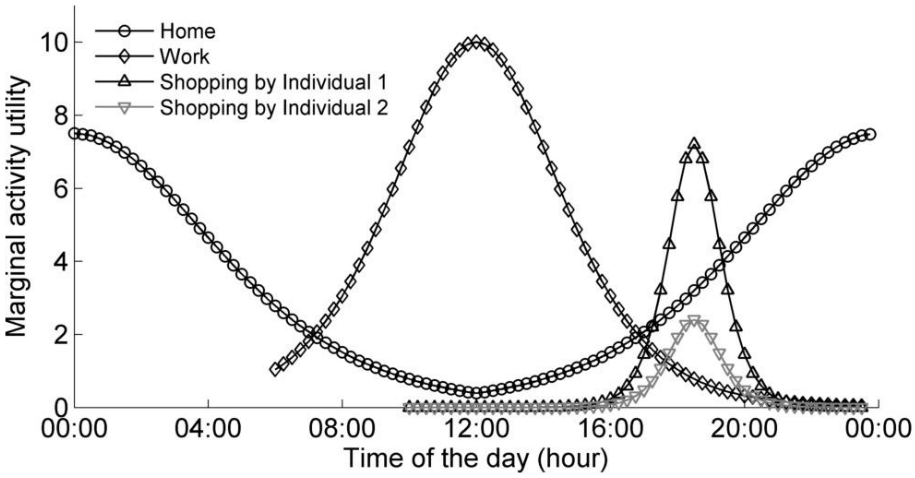

Figure 4. Temporal profiles of individual's marginal activity utility

The welfare of the individuals in a household is treated equally important. The weight parameter σ_i, representing the relative influence of household member i is thus assigned the same value: $\sigma_1 = \sigma_2 = 1.0$. The interaction coefficients of *Home* and *Work* activities are zero. The interaction coefficient of *Shopping* is denoted by ρ. The solution algorithm is implemented in A Mathematical Programming Language (AMPL) and used to solve the examples in this section. The model is examined with different intra-household interactions. The key findings are discussed as follows.

First, the interaction coefficient of *Shopping* is set to 0. Each individual makes activity and travel decisions independently. The activity-travel pattern of each individual reflects the underlying individual preference. Figure 5 illustrates the activity participation of Individuals 1 and 2 over time of the day. Since Individual 1 gains a higher level of utility from shopping, individual 1 goes shopping after work. Individual 2 prefers to return home directly after work.

The patterns of activity participation for Individuals 1 and 2 depicted in Figure 5 are used as the base scenario for further analysis. The results of the positive and negative intra-household interactions are discussed and compared with the base scenario in the following discussions.

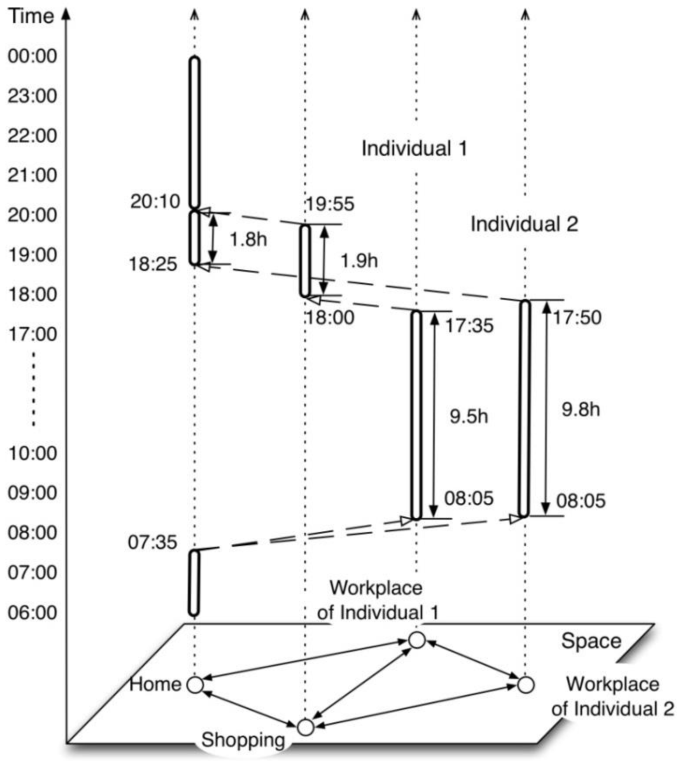

Figure 5. Activity participation of a two-person household (ρ = 0).

If a household considers shopping as a non-compulsory activity with positive intra-household interaction, the two household members will prefer to participate in the activity together to interact more with each other. Thus, the interaction coefficient should take a positive value, $\rho = 0.2$ for example. The overall household's utility should be higher than the sum of individuals' utilities from independent activity participation. The extra utility received by the household is measured by (13).

Figure 6 shows the extra utility for different shopping times and durations. For a given duration of shopping, an optimal time gives the maximum utility. For shopping before 18:00, the utility increases rapidly with the duration of shopping. However, after 18:00 the gain of utility for spending an extra unit of time on shopping approaches zero. This tendency is illustrated by the contour lines parallel with y-axis between 18:00 and 19:00. This observation demonstrates that joint activity has an optimal timing and duration.

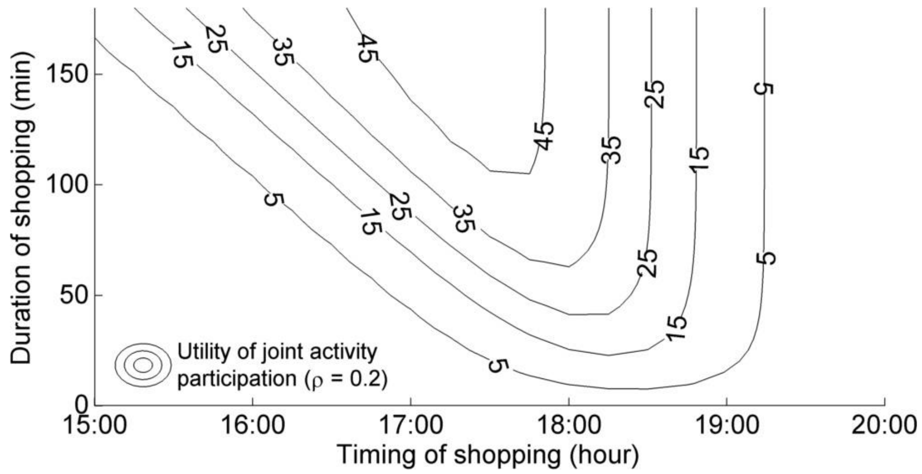

Figure 6. The utility of joint activity participation.

Since joint participation provides a higher overall household utility than independent participation, the activity-travel pattern of Individual 2 changes. Figure 7 shows that Individual 2 joins the shopping activity with Individual 1. The activity-travel pattern of Individual 1 does not show significant variation with ρ increases.

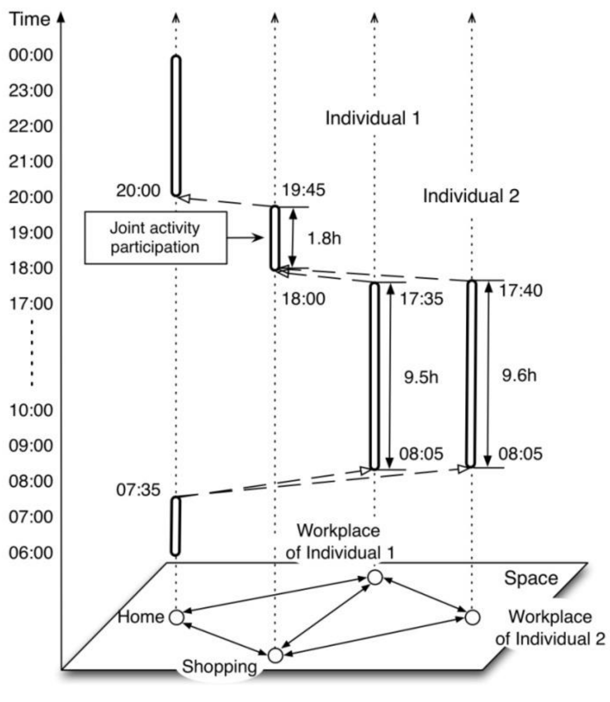

Figure 7. Activity participation of a two-person household (ρ = 0.2).

Table 2 presents the allocation of time to activities and travel for a large range of interaction coefficients. The duration of shopping for Individual 2 increases rapidly when ρ is increased from 0.0 to 0.2. However, this trend slows down when ρ approaches 0.5. Individual 2 spends less time on in-home activity to compensate the increased time in shopping. The working duration of Individual 2 is always maintained at 8 hours to 9 hours.

Table 2. Durations of activities and travel for different values of ρ (Individual 2).

ρ	Home	Work	Shopping	Travel	Total
0.0	13.8	9.4	0.0	0.8	24.0
0.2	11.9	9.2	1.8	1.1	24.0
0.5	11.7	8.9	2.3	1.1	24.0
1.0	11.6	8.4	2.9	1.1	24.0

If the household considers shopping as a non-compulsory activity with negative intra-household interaction, only one household member will take action to complete the shopping activity and the entire household benefits from that action. If a household member has done the shopping task, the benefit of another shopping trip is negligible and the cost of the trip is significant, particularly in a congested transportation network.

The action of any household member is thus substitutable within the household. The interaction coefficient takes a negative value in this case. Figure 8 depicts the utility of independent activity participation with homogenous individual preferences over shopping.

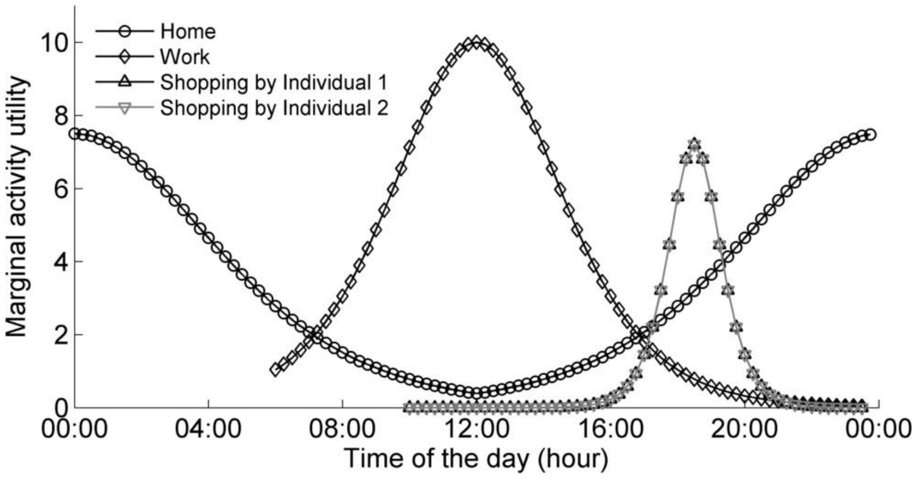

Figure 8. Temporal profiles of individual's marginal activity utility with homogenous preferences.

The simulated time-space path in Figure 9 shows the possible activity-travel patterns. The two individuals share the same temporal profile of marginal utility for shopping. The household can assign the shopping task to either individual and get the same overall utility. The probability of conducting shopping activity for any individual is 50%.

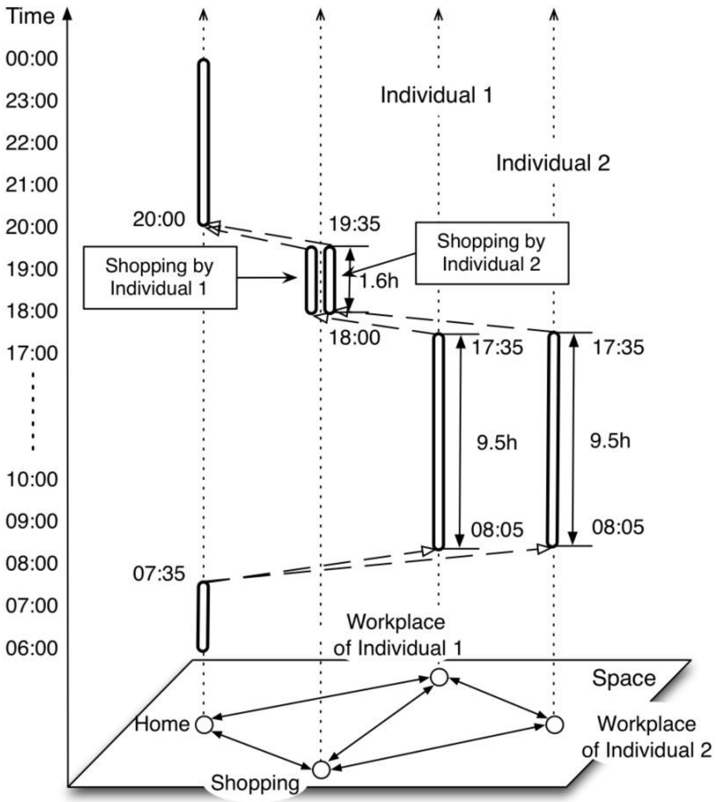

Figure 9. Activity participation over time of the day (ρ = -0.2).

6. Conclusions

A household activity-travel scheduling model with consideration of intra-household interactions has been presented in this paper. Markov Decision Process (MDP) was employed to model household's activity-travel scheduling behaviour as MDP can provide a modelling framework that allows the household's decisions to have complex interdependency over time. In the proposed MDP model, the household's choice reveals a dependency on the time of day and the activities that have been conducted before the current choice. The household also takes into account the future utility that they can obtain. The focus of this paper is capturing the household's complicated activity-travel decisions over time with explicit consideration of intra-household interaction.

The proposed model allows decomposing the household's utility into two components, i.e. the utility of engaging in an activity independently and the utility derived from the joint activity participation with other household members. An efficient solution algorithm was developed for solving the household MDP model. The numerical results showed that the proposed model can be used to investigate household' activity and travel behavior with consideration of intra-household interactions. If the intra-household interaction for an activity is not considered, joint activity participation would be underestimated or overestimated. If the intra-household interaction for an activity is positive, household members would prefer to participate in the activity jointly to obtain higher household utility. If the intra-household interaction for an activity is negative, this activity would be assigned to one household member.

In the proposed MDP model, the set of activity programs is predefined. Location choices are not incorporated in the proposed model. How to overcome these limitations by using other approaches should be explored in further research. Model calibration and validation should be conducted with empirical data (Chow and Recker 2012). Statistical methods such as maximum likelihood method can be employed to calibrate the parameters of the MDP model (Fu, Lam, and Xiong 2015). Time-series data are required. The dataset should include household members' activity choices and geographic locations over time. Another potential extension of the household MDP model is to consider day-to-day dynamics in activity-travel scheduling. The effect of certain activities can persist for multiple days and thus the activities participated in one particular day can influence the later activity-travel schedules (Arentze and Timmermans 2009; Cirillo and Axhausen 2010; Chow and Nurumbetova 2015). It should be noted that activity-travel schedules on weekdays and weekends differ significantly. Compulsory activities, such as work and school are regular occurrences on weekdays, while some non-compulsory activities, such as physical exercise, are usually performed at the weekend.

In this study, all activities are categorized into two types: compulsory and non-compulsory. It would be of interest to further categorize activities into even smaller groups based on their socio-economic characteristics. In line with the contentions of Bradley and Vovsha (2005), the variation in intra-household interactions across activity types can then be examined at a finer level of detail. This study captures the intra-household interactions of a two-person household using a simple multiplicative formulation. Alternative formulations representing different decision making strategies should be considered in further studies, such as the winner taking it all or maximizing the utility of any household member. Additionally, different types of households should be considered in further research, such as two full-time workers with children, non-worker or part-time worker, and two retired persons (Vovsha et al. 2004). The various characteristics of different household members affect the household's activity-travel patterns. The number of feasible household states increases exponentially with the number of household members. The computational burden is the major difficulty for modelling households with three or more individuals. However, approximate dynamic programming with interpolation can be employed to alleviate the computational burden as indicated by Keane and Wolpin (1994).

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