Tradable location tax credit scheme for balancing traffic congestion and environmental externalities

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Abstract

This paper proposes a novel urban system equilibrium model for design of a tradable location tax credit scheme to balance traffic congestion and environmental externalities. In the proposed model, the interaction between location tax credit scheme and households' residential location choices is explicitly considered. It is assumed that the authority initially allocates the credits to all households in a uniform way, and the households pay a certain amount of credits for housing consumptions, relying on the traffic congestion and environmental externalities that they cause. The credits are traded among households through a free market and the tax revenue due to credit trading is considered as a part of household's income to be redistributed through household's income budget constraint. For a given credit scheme, households' residential location choices and housing market structure in terms of housing prices and space can be endogenously determined by the urban system equilibrium and the price per unit of credit is governed by the credit market equilibrium. A social welfare maximization model is presented to determine the total amount of credits issued. The results show that implementation of the tradable location tax credit scheme can rationalize the urban residential density and promote the efficiency of the urban system in terms of social welfare.

Keywords: Traffic congestion and environmental externalities; tradable location tax credit scheme; redistribution of tax revenue; households' residential location choices.

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1. Introduction

1.1. Background and motivation

The past decade has witnessed the rapid expansion in the size of some Chinese cities, such as Beijing, Shanghai and Hong Kong, due to rapid urbanization and economic growth. For instance, the urban area of Beijing city has currently reached 1,386 square kilometers, which is three times larger than that of 400 square kilometers of that city in the 1990s. The 2012 report given by the Chinese Academy of Science (CAS, 2012) showed that the average journey time of commuters from their home locations to their places of work in Beijing has increased from about 36 minutes in 2009 to about 52 minutes in 2011. The expansion in the city’s size has led to a more decentralized urban structure, thus causing a longer average journey distance, a higher average travel cost, and a heavier traffic congestion externality.

Recently, the average height of buildings in many Chinese cities also dramatically increases due to shortage of the land resources available for housing development. This leads to an excessive population agglomeration and thus an excessively rapid growth in urban residential density, particularly at the locations close to the central business district (CBD). For example, during the 2000-2010 decade, the average household residential density of Shanghai city has dramatically increased from 19,181 persons per square kilometer to 23,000 persons per square kilometer. The population agglomeration can reduce the city’s size and thus save households’ average transportation expenses per year. However, it has also been shown that an increase in residential density would incur some negative effects, such as increase of neighborhood noise disturbance, decay of neighborhood sanitation services, shortage of communal facilities and higher neighborhood crime rate, and thus causes a decrease in the environmental quality for residents, and also a decrease in household’s residential satisfaction degree or utility level. These negative effects on the surrounding environmental quality are referred to as environmental externality in this paper. The environmental externality of a location is related to the residential density of that location. For instance, the residential density in Hong Kong is almost 34,000 persons per square kilometer and the average housing floor area per person is less than 13.9 square meters, implying a low-level environmental quality and a serious environmental externality in Hong Kong.

Apparently, the city’s size and the household residential density are dependent on the trade-off between traffic congestion externality and environmental externality. The traffic congestion externality favors a compact residential distribution, whereas the environmental externality favors a decentralized residential distribution. Specifically, a high-density and compact city usually has a good accessibility to the CBD, and thus a low average commuting cost for the households in the urban area. A high residential density, however, can cause serious environmental externality and thus low environmental quality in the urban areas. In contrast, a low-density and decentralized city may satisfy households' preferences for environmental quality, but also incur a high average commuting cost and high traffic congestion externality due to a long-distance journey. This raises some intriguing and important problems: how does urban residential distribution affect traffic congestion and environmental externalities and what residential distribution is preferred in terms of social welfare maximization? The answers to these problems have significant implications for sustainable urban development, particularly for large cities in the developing countries of Asia, such as Beijing, Shanghai, and Hong Kong.

Tradable credit scheme, as a promising substitution for congestion pricing scheme, has recently been proposed as a useful tool for regulation of externalities (e.g., Yang and Wang, 2011; Wang and Yang, 2012; Wang et al., 2012). In comparison with the congestion pricing scheme, the tradable credit scheme has two significant merits. First, a financial transfer from the public to the government is not involved in this scheme, which enhances the public acceptability of the scheme. Second, an initial uniform allocation of the credits among urban residents can promote the fairness of the scheme and thus its political acceptability. Along this line, a tradable location tax credit scheme is proposed in this paper to balance the trade-off between the traffic congestion and environmental externalities.

1.2. Literature review

To address the tradable location tax credit scheme issue, advanced analytical models should be developed to investigate the effects of this scheme on households' residential location choices and housing market. In this regard, the classical monocentric urban models (e.g., see Alonso, 1964; Muth, 1969) have been recognized as a benchmark representation of urban spatial structure due to its ability to describe the land resource allocation of a city in a simple and tractable way. In the classical urban models, it was usually assumed that the travel costs of commuters were a function of their travel distances (i.e., traffic congestion effects are ignored). The congestion-free assumption may cause a significant bias in the prediction capability of these models, and thus restricts their applications in practice. For example, congestion pricing issues cannot be addressed under the congestion-free assumption.

In the literature, there are a number of variations of the classical urban models with considering the traffic congestion effects (e.g., see Solow, 1972; Anas and Xu, 1999; Li et al., 2013; Li and Guo, 2015). Congestion pricing has been proposed as a feasible solution to the growing traffic congestion problem because of its potential to internalize traffic congestion externality (e.g., see Lindsey, 2009; Li et al., 2012c; Guzman et al., 2014). For example, the cordon toll pricing issues have been widely explored through using a continuum modeling approach (e.g., see Mun et al., 2005; Ho et al., 2005, 2013; Verhoef, 2005; Li et al., 2012a, 2014; De Lara et al., 2013; Li and Guo, 2015). More recently, tradable credit schemes have been advocated as a promising substitution for congestion pricing schemes for the regulation of congestion externality (e.g., see Yang and Wang, 2011; Zhang et al., 2011; Wu et al., 2011, 2012; Wang and Yang, 2012; Wang et al., 2012, 2014; He et al., 2013; Nie and Yin, 2013; Shirmohammadi et al., 2013; Tian et al., 2013; Xiao et al., 2013; Ye and Yang, 2013; Bao et al., 2014; Liu et al., 2014; Nie, 2015; Zhu et al., 2015). In the tradable credit schemes, travel demand is regulated through the allocation of tradable credits or permits and the price of credit trading. In a typical setting, a government agency identifies eligible households and distributes credits among them. Auto users are then required to pay a certain number of credits to access transportation facilities. They can purchase or sell credits in the market created by the government agency, which, however, does not intervene in credit transactions. The role of the agency is to determine the initial allocation of credits and subsequent credit charges for use of transportation facilities.

These above-mentioned studies mainly focused on the management of traffic congestion externality caused by vehicular usage. The effects of environmental externality, which is related to urban residential density, on the environmental quality and thus on households' residential location choices have not been investigated in these previous related studies. However, some studies have showed that the environmental externality due to a high residential density has a significant impact on the households' residential location choices and thus on the urban spatial structure. For instance, Wabe (1971), Mirrlees (1972), Schuler (1974) and Kanemoto (1980) found that dense inhabitation environment can breed various adversities, such as noise, fire, air pollution (bad air quality), and antisocial behavior, and thus has a negative effect on human health, wellbeing, and education (e.g., see Nagar and Paulus, 1997; Gomez-Jacinto and Hombrados-Mendieta, 2002; Solari and Mare, 2012). Richardson (1977), Tauchen (1981) and Fujita (1989) further addressed the importance of incorporating the environmental externality in the urban spatial structure model. This means that considering the traffic congestion externality only may underestimate the externality level that each household actually generates, thus leading to a distorted urban system. It is, therefore, very important to take into account the effects of both traffic congestion and environmental externalities on the urban spatial structure in the urban models so as to correct the market distortion.

1.3. Problem statement and contributions

This paper proposes a tradable location tax credit scheme to balance the traffic congestion and environmental externalities in a linear monocentric city. In this tradable credit scheme, the local authority initially allocates a certain amount of location tax credits to all households in the city in a uniform way. The households pay a certain amount of credits for their housing consumptions, which depends on the traffic congestion and environmental externalities that they cause. The credits can then be traded among households through a free market. The problem addressed in this paper is: how to determine the total amount of credits initially issued so as to maximize the social welfare of the urban system? To solve this problem, a new urban system equilibrium model is developed, which considers both the traffic congestion externality and the environmental externality. With use of the urban system equilibrium model, a social welfare maximization model is proposed to find the optimal total amount of credits and the corresponding price per unit of credit to create an efficient and environment-friendly urban system.

The major contributions made in this paper are as follows. First, a new urban system equilibrium model is presented, in which the effects of location-dependent environmental amenities and traffic congestion on households' residential location choices are considered. Second, a tradable location tax credit scheme is presented to rationalize the household residential location distribution across the city so as to balance the traffic congestion and environmental externalities, together with a comparison with the first-best tax pricing scheme. An analytical model for optimizing the total amount of credits distributed is proposed to maximize the urban system's total social welfare. Third, the redistribution issue of the tax revenue generated by the location tax credit trading scheme is explicitly considered through the household's income budget constraint, which was hardly concerned in previous travel credit trading studies. One exception is the study of Wu et al. (2012), which incorporates the effects of tax revenue on travelers' trip generation, mode and route choices.

The remainder of this paper is organized as follows. In the next section, the urban system equilibrium problem without tax is first formulated, in which the variation of environmental quality across the urban system is taken into account. It follows with presentation of the first-best tax pricing scheme in Section 3 as a benchmark for design of the tradable location tax credit scheme. Section 4 proposes a tradable location tax credit model to maximize the total social welfare, which is a promising substitution of the first-best tax pricing scheme. In Section 5, a numerical example is provided to illustrate the applications of the proposed model. Finally, conclusions and recommendations for further studies are given in Section 6.

2. Urban system equilibrium without tax

2.1. Assumptions

To facilitate the presentation of essential ideas without loss of generality, the following basic assumptions are made in this paper.

A1 The city concerned is assumed to be linear, closed and monocentric, implying that the total number of households in the city is exogenously given and fixed, and all job opportunities cluster in a highly compact city center, i.e., CBD (Central Business District). It is also assumed that the land is owned by a public landowner (e.g., the government) and the total differential rent from land (i.e., the total net rent from land after deducting the opportunity cost of the land in the city) is redistributed to the households in the city. The value of land at/beyond the city boundary is equal to its opportunity cost. These assumptions have been widely adopted in the field of urban economics, such as Alonso (1964), Muth (1969), Fujita (1989), Verhoef (2005) and Li et al. (2012a,b, 2013).

A2 Following Fujita (1989), Richardson (1977) and Tauchen (1981), the environmental quality is assumed to be a strictly decreasing function of the household residential density. This assumption is not unreasonable because (i) environmental quality can be reflected by the surrounding noise level, sanitation service level, safety level and communal facility level, which are closely related to the household residential density; and (ii) this assumption reflects households' preferences for a low-density region due to cleaner, quieter and safer surroundings compared to a high-density region. The negative effects on the surrounding environmental quality, such as increase of neighborhood noise disturbance, decay of neighborhood sanitation services, and shortage of communal facilities noise, are referred to as environmental externality in this paper.

A3 There are three types of stakeholders in the urban system: the authority, property developers and households. The authority is assumed to determine the total amount of credits issued so as to maximize the total social welfare of the urban system. The property developers are assumed to determine their capital investment intensities to maximize their own net profit. All households are assumed to be homogeneous in terms of income level and aim to maximize their own utility within the capital budget constraint. The income of each household is assumed to be fixed and given exogenously. The household utility is assumed to follow a Cobb-Douglas function regarding housing spaces, non-housing goods consumption and environmental quality (e.g., see Beckmann, 1969; Solow, 1972; Richardson, 1977; Tauchen, 1981; Anas, 1982; Fujita, 1989).

A4 In the tradable location tax credit scheme, the authority initially allocates a certain amount of location tax credits to all households in the city in a uniform way (e.g., see Yang and Wang, 2011; Wang and Yang, 2012). The credit charging scheme is assumed to be location-dependent. Specifically, the amount of credits that each household should pay depends on the total externality generated by that household, including the traffic congestion and environmental externalities. The price of unit credit can be determined by the credit trading market.

A5 This study mainly focuses on commuting journeys of workers between their homes and their workplaces, which are compulsory (or obligatory) trips. Thus, a household's number of trips is not affected by various factors, such as the household's residential location and income level. This means that the demand for travel to the CBD per day is given and fixed. We assume that the average daily number of commuters (or workers) per household is represented as η. For example, $\eta = 1$ indicates that each household makes an average of one trip to the CBD per day. The assumption that there is one commuter per household has also been adopted in previous related studies (e.g., Anas and Xu, 1999; Li et al., 2012b, 2013; Li and Guo, 2015).

2.2. Travel cost

In a linear monocentric city as assumed in **A1**, each location is characterized by the accessibility to the CBD (i.e., the distance from the CBD) and the environmental quality which is decreasing with the residential density. We denote x as the distance of a location from the CBD and $n(x)$ as the household residential density (i.e., the number of households per unit of land area) at location x. Let $q(x)$ be the hourly density of travel demand (i.e., number of commuters per unit of land area) at location x, and ξ be the peak-hour factor (i.e., the ratio of peak-hour flow to the average daily flow), which is used to convert the travel demand from a daily to an hourly basis. According to **A5**, η is the average daily number of trips to the CBD per household. $q(x)$ can thus be defined as

$$q(x) = \xi\eta n(x) = \delta n(x), \tag{1}$$

where $\delta\,(=\xi\eta)$ is the average number of peak-hour trips to the CBD per household. The hourly travel demand $Q(x)$ at location x can thus be given as

$$Q(x) = \int_x^{x_f} q(w)dw = \delta\int_x^{x_f} n(w)dw, \tag{2}$$

where x_f is the distance from the urban boundary to the CBD (or the length of the corridor).

Let $t\left(Q(x)\right)$ represent the travel time per unit of distance around location x, where $Q(x)$ is the traffic volume at location x as defined above. Suppose that $t\left(Q(x)\right)$ is a strictly increasing function of traffic volume $Q(x)$ at location x and can be estimated using the following Bureau of Public Roads (BPR) function

$$t\left(Q(x)\right) = t_0\left[1.0 + a_1\left(\frac{Q(x)}{K}\right)^{a_2}\right], \tag{3}$$

where t_0 is the free-flow travel time per unit of distance, K is the capacity of the corridor and a_1 and a_2 are positive parameters.

The travel time from location x to the CBD, $T(x)$, can then be expressed as

$$T(x) = \int_0^x t\left(Q(w)\right)dw\,. \tag{4}$$

Denote $\Lambda(x)$ as the monetary travel cost from location x to the CBD. Following Wang et al. (2004), Liu et al. (2009) and Li et al. (2012a), $\Lambda(x)$ can be assumed to be a linear function of the distance traveled, i.e.,

$$\Lambda(x) = \Lambda_0 + \Lambda_1 x\,, \tag{5}$$

where Λ_0 and Λ_1 are, respectively, the fixed cost (e.g., the parking charge in the CBD per work trip) and variable cost (e.g., fuel cost per unit of distance) of travel.

Let $\Psi(x)$ be the one-way travel cost from location x to the CBD, which consists of travel time cost and monetary travel cost. $\Psi(x)$ can then be expressed as

$$\Psi(x) = \sigma T(x) + \Lambda(x)\,, \tag{6}$$

where $T(x)$ and $\Lambda(x)$ are, respectively, given by Eqs. (4) and (5), and σ is the value of travel time, which is used to convert the travel time into equivalent monetary units.

In the urban system, each commuter makes one two-way commuting journey between his/her place of residence and a workplace located in the CBD. Given the one-way average travel cost $\Psi(x)$, the average annual round trip cost $\Phi(x)$ between location x and the CBD can be calculated by

$$\Phi(x) = 2\omega\Psi(x)\,, \tag{7}$$

where the number "2" denotes a two-way journey between location x and the CBD, and ω is the average annual number of trips to the CBD per household, which can be estimated using survey data.

2.3. Households' residential location choices

In the basic model of Solow (1972) and Anas (1982), the residential environments are assumed to be the same across the city, and thus it fails to investigate the influence of environmental amenities on the households' residential location choices. In this study, we introduce the locational variation in environmental amenities and denote the environmental quality at location x as $E\left(x\right)$. Following Richardson (1977) and Fujita (1989), $E(x)$ is given by the reciprocal of the residential density at location *x*, namely

$$E(x) = 1/n(x). \tag{8}$$

Eq. (8) reflects households' preferences for a low-density region according to **A2**.

According to **A3**, each household is assumed to choose a residential location to maximize its own utility subject to a capital budget constraint. Similar to the classical urban models (e.g., see Beckmann, 1969; Solow, 1972; Fujita, 1989), this paper adopts a Cobb-Douglas form of utility function, specified as

$$U(x) = u\left(z, g, E\right) = z(x)^{\alpha} g(x)^{\beta} E(x)^{\gamma}\ , \alpha > 0, \beta > 0, \alpha + \beta = 1, \tag{9}$$

where $U(x)$ is the annual utility of households at location x. α, β and γ are positive elasticity parameters. $z(x)$ is the annual composite non-housing goods consumed per household at location x, for which the price is normalized to 1. $E(x)$, which is given by Eq. (8), is the environmental quality at location *x*, and $g(x)$ is the annual consumption of housing per household at location x, which is measured in square kilometers of floor space. Eq. (9) shows that the household's utility increases with the increase in the consumptions of the housing and non-housing goods or the improvement of environmental quality.

Let *N* be the total number of households in the city concerned and *R* be the total differential rent from land. The annual income of each household is then its non-land income *Y* plus a returned differential rent R/N according to **A1**. The household utility maximization problem can thus be represented as

$$\max_{z,g}\ U\left(x\right) = z(x)^{\alpha} g(x)^{\beta} E\left(x\right)^{\gamma}, \tag{10}$$

$$\text{s.t.} \quad Y + \frac{R}{N} - \Phi(x) - z(x) - p(x)g(x) = 0, \tag{11}$$

where $p(x)$ is the average annual rental price per unit of housing floor area at location *x*, and $\Phi(x)$, which is determined by Eq. (7), denotes the average annual round-trip cost for households living at location x. Following **A1**, the public landownership assumption implies

$$R = \int_0^{x_f} \left(r(x) - r_a\right) dx, \tag{12}$$

where *R* represents the total differential rent, x_f denotes the distance of city's fringe from the CBD, r_a is a constant opportunity cost of the land , and $r(x)$ is the rent or value per unit of land area at location *x*, which is dependent on property developers' housing production behavior discussed later.

For presentation purpose, we define the net income of a household living at location x, $I(x)$, as the annual income of a household deducted by the total travel cost, i.e.,

$$I(x) = Y + \frac{R}{N} - \Phi(x) . \tag{13}$$

Next, we define the bid rent, $\psi(I(x), u, E(x))$ as the maximal rent that a household can pay for residing at location x with environmental quality $E(x)$ while enjoying a fixed utility level u, which can be mathematically expressed as

$$\psi(I(x), u, E(x)) = \max_{z,g} \left\{ \frac{I(x) - z}{g} \middle| U(z, g, E) = u \right\} = \max_{g} \left\{ \frac{I(x) - Z(g, u, E)}{g} \right\}, \tag{14}$$

where $Z(g, u, E)$ is the solution to $U(z, g, E) = u$ for z, which is a function of annual housing consumption g, annual utility level u, and the environmental quality E. Accordingly, $g(\cdot)$ and $p(\cdot)$ can be obtained as a function of common utility level u and environmental quality $E(\cdot)$, as follows:

$$g(x, u, E) = \left(\alpha^{\alpha} u^{-1}\right)^{-1/\beta} I(x)^{-\alpha/\beta} E\left(x\right)^{-\gamma/\beta}, \text{ and} \tag{15}$$

$$p(x, u, E) = \psi(I(x), u, E) = \beta\left(\alpha^{\alpha} u^{-1}\right)^{1/\beta} I(x)^{1/\beta} E\left(x\right)^{\gamma/\beta} . \tag{16}$$

It can be seen from Eq. (16) that an improvement in the environmental quality or accessibility to the CBD (i.e., a decrease in transportation cost to the CBD) can contribute to an increase in the housing price. Substitute Eqs. (15) and (16) into Eq. (11), one obtains

$$z(x) = \alpha I(x) . \tag{17}$$

2.4. Property developers' housing production behavior

Following Song and Zenou (2006), the property developers are assumed to behave in a Cobb-Douglas form of housing production function, as follows:

$$D = \theta C^{1/2} L^{1/2} , \tag{18}$$

where θ is a positive parameter, and D is the housing output. L and C are, respectively, the land and capital inputs. Thus, the housing production function per unit of land can be written as

$$h\left(S(x)\right)=\frac{D}{L}=\theta\sqrt{S(x)}\,, \tag{19}$$

where $S \equiv C/L$ represents the capital input per unit of land, and thus $h\left(S(x)\right)$ is the housing output per unit of land.

Let *k* be the price of capital (i.e., the interest rate). The net profit per unit of land area, $\pi(x)$, at location *x* can then be given by

$$\pi(x)=p(x)h\left(S(x)\right)-\left(r(x)+kS(x)\right), \tag{20}$$

where the price of unit housing $p(\cdot)$ is given by Eq. (16) and $r(x)$ is the rent or value per unit of land area at location *x*. The first term on the right-hand side of Eq. (20) is the total revenue from housing leases. The final two terms are the land rent cost and the capital cost, respectively.

Each property developer in the housing market aims to maximize its own net profit by determining the optimal capital investment intensity, expressed as

$$\max_{S}\ \pi(x)=p(x)\theta\sqrt{S}-\left(r(x)+kS\right). \tag{21}$$

The first-order optimality condition of the maximization problem (21) yields

$$\frac{\partial\pi}{\partial S}=\frac{1}{2}\theta p(x)S^{-1/2}-k=0\,. \tag{22}$$

Substituting $p(\cdot)$ in Eq. (16) into Eq. (22) produces the capital investment intensity as a function of utility level *u* and environmental quality $E(x)$, namely

$$S(x,u,E)=\left((2k)^{-1}\beta\theta\left(\alpha^{\alpha}u^{-1}\right)^{1/\beta}I(x)^{1/\beta}E(x)^{\gamma/\beta}\right)^{2}. \tag{23}$$

Combining Eqs. (19) and (23), we have

$$h\left(S(x,u,E)\right)=(2k)^{-1}\beta\theta^{2}\left(\alpha^{\alpha}u^{-1}\right)^{1/\beta}I(x)^{1/\beta}E(x)^{\gamma/\beta}. \tag{24}$$

The household residential density $n(\cdot)$ at location *x* can thus be calculated by

$$n(x)=\frac{h\left(S(x,u,E)\right)}{g(x,u,E)}=(2k)^{-1}\beta\theta^{2}\left(\alpha^{\alpha}u^{-1}\right)^{2/\beta}I(x)^{(1+\alpha)/\beta}E(x)^{2\gamma/\beta}, \tag{25}$$

which defines the equilibrium residential density across the city.

Substituting Eq. (8) into Eq. (25), we can obtain

$$n(x)=\left((2k)^{-\beta}\left(\beta\theta^{2}\right)^{\beta}\left(\alpha^{\alpha}u^{-1}\right)^{2}I(x)^{(1+\alpha)}\right)^{1/(\beta+2\gamma)}. \tag{26}$$

Thus, we can calculate the environmental quality at any location x, as below.

$$E(x)=\left((2k)^{\beta}\left(\beta\theta^{2}\right)^{-\beta}\left(\alpha^{\alpha}u^{-1}\right)^{-2}I(x)^{-(1+\alpha)}\right)^{1/(\beta+2\gamma)}. \tag{27}$$

Substituting Eq. (27) into Eqs. (15) and (16) respectively, we can obtain the equilibrium housing price $p(\cdot)$ and housing space $g(\cdot)$, as follows.

$$p(x)=\left((2k)^{\gamma}\beta^{(\beta+\gamma)}\theta^{-2\gamma}\alpha^{\alpha}u^{-1}I(x)^{(1+\gamma)}\right)^{1/(\beta+2\gamma)}, \text{ and} \tag{28}$$

$$g(x)=\left((2k)^{-\gamma}\left(\beta\theta^{2}\right)^{\gamma}\left(\alpha^{\alpha}u^{-1}\right)^{-1}I(x)^{(\gamma-\alpha)}\right)^{1/(\beta+2\gamma)}. \tag{29}$$

Similarly, the equilibrium capital input per unit of land $S(\cdot)$ and the housing output per unit of land $h\left(S(\cdot)\right)$ can be given by

$$S(x)=\left((2k)^{-(\beta+\gamma)}\beta^{(\beta+\gamma)}\theta^{\beta}\alpha^{\alpha}u^{-1}I(x)^{(1+\gamma)}\right)^{2/(\beta+2\gamma)}, \text{ and} \tag{30}$$

$$h\left(S(x)\right)=\left((2k)^{-(\beta+\gamma)}\left(\beta\theta^{2}\right)^{(\beta+\gamma)}\alpha^{\alpha}u^{-1}I(x)^{(1+\gamma)}\right)^{1/(\beta+2\gamma)}. \tag{31}$$

Note that under perfect competition, the property developers earn zero profit, Eq. (20) thus equals 0, yielding

$$r(x)=\left(\left(2^{-1}\beta\right)^{2(\beta+\gamma)}\left(k^{-1}\theta^{2}\right)^{\beta}\left(\alpha^{\alpha}u^{-1}\right)^{2}I(x)^{2(1+\gamma)}\right)^{1/(\beta+2\gamma)}. \tag{32}$$

2.5. Housing market equilibrium

At equilibrium, all households should be within the urban area, expressed as

$$\int_0^{x_f} n(x)dx = N\,, \tag{33}$$

where *N* is the total number of households in the city concerned.

Substituting Eq. (26) into Eq. (33) and solving it for *u*, we then have

$$u = \left(\frac{1}{N} \int_0^{x_f} \left((2k)^{-\beta} \left(\beta\theta^2\right)^{\beta} \alpha^{2\alpha} I(x)^{(1+\alpha)} \right)^{1/(\beta+2\gamma)} dx \right)^{(\beta+2\gamma)/2} . \tag{34}$$

In addition, the equilibrium rent per unit of land area at the city's fringe x_f equals the agricultural rent or opportunity cost of the land, i.e.,

$$r\left(x_f\right) = r_a\,, \tag{35}$$

where r_a is the constant opportunity cost of the land.

Substituting Eq. (32) into Eq. (35) yields

$$\left(\left(2^{-1}\beta\right)^{2(\beta+\gamma)} \left(k^{-1}\theta^2\right)^{\beta} \left(\alpha^{\alpha} u^{-1}\right)^2 I(x_f)^{2(1+\gamma)} \right)^{1/(\beta+2\gamma)} = r_a\,. \tag{36}$$

Remark 1. The combination of Eqs. (34), (36), and (26)-(32) determines the housing market equilibrium. Specifically, given the value of net income $I(x)$, one can determine the values of utility level u and city boundary x_f by solving the system of Eqs. (34) and (36). The equilibrium rent per unit of housing space $p(x)$, the equilibrium amount of housing space $g(x)$, and the equilibrium household residential density $n(x)$ can thus be determined by Eqs. (28), (29), and (26), respectively.

3. First-best tax pricing scheme

For completeness purpose, in this section the first-best tax pricing scheme is presented as a benchmark for design of the tradable location tax credit scheme. As previously stated, the local government aims to maximize the total social welfare (SW) of the urban system by determining the optimal tax pricing scheme. Note that the total differential land rents are reallocated to the households in the city through a lump-sum transfer, which are thus not included in the social welfare. The social welfare can thus be defined as the total utility of all households in the urban system. Denote $l(x)$ as the location tax levied on each household at location *x*, and $\Gamma = \{l(x), x \in (0, x_f)\}$ as the associated tax pricing scheme across the city. The social welfare maximization problem can then be mathematically expressed as

$$\max_{\Gamma} SW^{so} = N \cdot u . \tag{37}$$

where the superscript "SO" represents the "social optimum" case with the first-best tax pricing scheme. The common utility level u can be determined by solving the urban system equilibrium with the tax pricing scheme Γ.

It is well-known that the optimal Pigouvian congestion charging scheme requires that all externalities including the traffic congestion and environmental externalities should be internalized to correct the distorted market. This means that the household at a location should pay a location tax which equals the externalities imposed by the household at that location on all other households in the urban system. We denote $l_t(x)$ and $l_e(x)$ as the traffic congestion externality and the environmental externality generated by a household at location *x*, respectively. Then, the location tax levied on each household $l(x)$ can be given by

$$l(x) = l_t(x) + l_e(x) . \tag{38}$$

The government is responsible for the collection of location tax. It is assumed that the tax revenue is uniformly redistributed to each household in the urban system, i.e. each household obtains the average tax revenue, denoted by *G*, as follows:

$$G = \frac{1}{N}\int_0^{x_f} l(x)n(x)dx . \tag{39}$$

Denote $I^{so}(x)$ as the net income of each household at location *x* after introducing the first-best tax pricing scheme, which can be given by

$$I^{so}(x) = Y + \frac{R}{N} + G - \Phi(x) - l(x) , \tag{40}$$

where the total differential rent R is determined by Eq. (12), and $l(x)$ is given by Eq. (38).

Note that when a commuter passes through location x, he/she will cause an additional travel cost $t'\left(Q(x)\right)$ to each existing commuter at location x. This means that the total traffic congestion externality caused by a commuter traveling from location x to the CBD is $\int_0^x t'\left(Q(w)\right)Q(w)dw$. Accordingly, the average annual traffic congestion externality, $l_t\left(x\right)$, caused by a commuter living at location x can be calculated by

$$l_t(x) = 2\omega\sigma\int_0^x t'\left(Q(w)\right)Q(w)dw = 2\omega\sigma a_1 a_2 t_0 K^{-a_2}\int_0^x \left(Q(w)\right)^{a_2} dw, \tag{41}$$

where σ is used to convert the traffic congestion externality into equivalent monetary units.

Similarly, when a household migrates to location x from other location, this household will cause an extra cost of $\frac{\partial Z}{\partial E}\frac{\partial E}{\partial n}$ to the existing households living at location x, which is referred to as environmental externality (see Fujita, 1989). Thus, the total environmental externality at location x, $l_e(x)$, caused by an additional household can be given by

$$l_e(x) = n(x)\frac{\partial Z(g,u,E)}{\partial E}\frac{\partial E}{\partial n} = \gamma I^{so}(x)\ . \tag{42}$$

Eq. (42) implies that the environmental externality equals 0 for $\gamma = 0$.

Combining Eqs. (38), (40), and (42), we can obtain

$$I^{so}(x) = \frac{1}{1+\gamma}\left(Y + \frac{R}{N} + G - \Phi(x) - l_t(x)\right). \tag{43}$$

Remark 2. According to Eqs. (41) and (42), as the distance from the CBD increases, the traffic congestion externality generated by each household at location x, $l_t(x)$, increases, while the environmental externality generated by each household at location x, $l_e(x)$, decreases. An increase in γ (the elasticity parameter of environmental quality in the household utility function) results in an increase in the environmental externality. The reason behind this is that a larger γ actually implies a more important degree of the environmental quality in the household utility.

Proposition 1. *With the first-best tax pricing scheme given by Eqs. (38), (41) and (42), the traffic congestion and environmental externalities in the urban system are fully internalized, thus leading to the SO urban system solution* $\left\{u^*, x_f^*, n^*(x), g^*(x), p^*(x), r^*(x), S^*(x)\right\}$ *for problem (37), as follows:*

$$g^*(x) = \left((2k)^{-\gamma} \left(\beta\theta^2\right)^{\gamma} \left(\alpha^{\alpha}/u^*\right)^{-1} I^{so}(x)^{(\gamma-\alpha)} \right)^{1/(\beta+2\gamma)}, \tag{44}$$

$$S^*(x) = \left((2k)^{-(\beta+\gamma)}\beta^{(\beta+\gamma)}\theta^{\beta}\, \alpha^{\alpha}/u^*\, I^{so}(x)^{(1+\gamma)}\right)^{2/(\beta+2\gamma)}, \tag{45}$$

$$p^*(x) = \left((2k)^{\gamma}\beta^{(\beta+\gamma)}\theta^{-2\gamma} \left(\alpha^{\alpha}/u^*\right) I^{so}(x)^{(1+\gamma)}\right)^{1/(\beta+2\gamma)}, \tag{46}$$

$$r^*(x) = \left(\left(2^{-1}\beta\right)^{2(\beta+\gamma)} \left(k^{-1}\theta^2\right)^{\beta} \left(\alpha^{\alpha}/u^*\right)^2 I^{so}(x)^{2(1+\gamma)} \right)^{1/(\beta+2\gamma)}, \tag{47}$$

$$n^*(x) = \left((2k)^{-\beta} \left(\beta\theta^2\right)^{\beta} \left(\alpha^{\alpha}/u^*\right)^2 I^{so}(x)^{(1+\alpha)} \right)^{1/(\beta+2\gamma)} \quad \text{, and} \tag{48}$$

$$\int_0^{x_f^*} n^*(x) = N, \tag{49}$$

where the net income $I^{so}(x)$ is determined by Eq.(43). At the city boundary (i.e., $x = x_f^*$), the land rent $r^*(x_f^*)$ is equal to $r_a - l_e(x_f^*)n^*(x_f^*)$.

The proof of Proposition 1 is provided in Appendix A. As for the first-best tax pricing scheme, we have the following properties.

Proposition 2. *With the first-best tax pricing scheme, the environmental quality $E^*(x)$ is an increasing function of the distance x from the CBD. The residential density $n^*(x)$, housing price $p^*(x)$, land rent $r^*(x)$, and capital input per unit of land $S^*(x)$ are decreasing functions of the distance x from the CBD. However, the monotonicity of housing space function $g^*(x)$ depends on the value of ($\alpha - \gamma$).*

Proposition 3. *With the first-best tax pricing scheme, if $l_t(x_f^*) > l_e(x_f^*)$ holds for the city boundary x_f^*, then there must exist some critical location $\bar{x}$ such that the environmental externality is equal to the traffic congestion externality at location $\bar{x}$, and the traffic congestion externality is higher (or lower) than the environmental externality outside (or inside) the critical location $\bar{x}$. That is, there exists one location $\bar{x} \in [0, x_f^*]$ such that the following conditions are satisfied:*

$$l_t(x)\begin{cases} < l_e(x), & \text{if } x < \bar{x}, \\ = l_e(x), & \text{if } x = \bar{x}, \\ > l_e(x), & \text{if } x > \bar{x}. \end{cases} \tag{50}$$

The proofs of Propositions 2 and 3 are given in Appendices B and C, respectively. They will be further illustrated in the numerical example later.

4. Credit-based charging scheme

Recently, tradable credit scheme has been advocated as a promising substitution for the congestion pricing scheme from the viewpoint of the regulation of congestion externality (e.g., Yang and Wang, 2011; Wang and Yang, 2012). Tradable credit scheme has the benefit of not involving financial transfer from the public to the government, which makes such scheme more acceptable for the public compared to the congestion pricing scheme. In this section, we discuss the tradable credit scheme.

4.1. Urban system equilibrium with tradable location tax credit scheme

Given a credit scheme, households' residential location choices and housing market structure in terms of housing prices and space can be endogenously determined by the urban system equilibrium, and the price per unit of credit can be given by the credit market equilibrium. This credit scheme is characterized by the total amount of credits and the credit charging scheme. For regulation of externalities and improvement of market efficiency, the credit charging scheme at any location is set according to the level of traffic congestion and environmental externalities that households actually generate at that location. The total amount of credits is determined by the government.

Let $\tau(x)$ represent the amount of credits that each household at location x should pay, and τ_{tot} represent the total annual amount of credits initially distributed. Following **A4**, a uniform credit distribution scheme is adopted for fairness at the initial stage. Thus, the annual amount of credits initially allocated to each household (denoted by τ_0) in the urban system can be obtained by

$$\tau_0 = \tau_{tot}/N, \tag{51}$$

where N represents the total number of households in the urban system. Note that the credit constraint requires that the total amount of credits consumed can't exceed the total amount supplied, i.e.,

$$\int_0^{x_f} \tau(x)n(x)dx \le \tau_{tot}\,, \tag{52}$$

where the left-hand side of Eq. (52) denotes the total amount of credits that households have to pay, and the right-hand side denotes the total amount of credits supplied.

Let ε represent the price of unit credit in the credit trading market. After implementing the location tax credit scheme, the budget constraint of each household residing at location x in Eq. (11) can thus be rewritten as

$$Y + \frac{R}{N} + \varepsilon\left(\tau_0 - \tau(x)\right) - \Phi(x) - z(x) - p(x)g(x) = 0\,, \tag{53}$$

where $\varepsilon\left(\tau_0 - \tau(x)\right)$ denotes the annual household's net revenue generated from credit trading, which may be positive or negative. $\varepsilon\left(\tau_0 - \tau(x)\right) > 0$ implies that a household at location x has redundant credits after deducting the credits paid for its externalities, and thus gain a positive net revenue from the credit sale; Otherwise, the household has to buy some extra credits in the credit trading market and thus gains a negative net revenue from the credit sale.

Remark 3. It can be seen from Eq. (53) that the households' net revenue from credit trading is explicitly incorporated in their budget constraints, which can affect their residential location choices and thus the housing market structure. Accordingly, the location-dependent credit charging scheme can be served as a financial tool to adjust the households' residential location choices and housing market according to the externality level of each location. Specifically, in households' budget constraint, $\varepsilon\tau_0$ stands for the redistributed tax revenue due to the credit trading. The effects of the tax revenue redistribution on taxers' consumption behavior have been seldom considered in the previous credit-based scheme studies, such as Yang and Wang (2011), Wang and Yang (2012), and Wang et al. (2012).

The household utility maximization problem (10)-(11) with tradable credit scheme can thus be reformulated as

$$\max_{z,g} \quad U\left(x\right) = z(x)^{\alpha} g(x)^{\beta} E\left(x\right)^{\gamma}\,, \tag{54}$$

$$\text{s.t.} \quad Y + \frac{R}{N} + \varepsilon\left(\tau_0 - \tau(x)\right) - \Phi(x) - z(x) - p(x)g(x) = 0. \tag{55}$$

The property developers' housing production behavior is similar to that in the basic urban system equilibrium (see Section 2.4), and is thus omitted here. For ease of presentation, we denote $I^c(x)$ as the net income of households at location *x* after introducing the credit scheme, which can be expressed as

$$I^c(x) = Y + \frac{R}{N} + \varepsilon(\tau_0 - \tau(x)) - \Phi(x), \tag{56}$$

where $\tau(x)$ denotes the amount of credits that each household at location *x* should pay. According to **A4**, $\tau(x)$ can be given by

$$\tau(x) = l_t(x) + l_e(x), \tag{57}$$

where $l_t\left(x\right)$ and $l_e(x)$, respectively, represent the average annual traffic congestion and environmental externalities caused by each household at location *x*, given by

$$l_t(x) = 2\omega\sigma a_1 a_2 t_0 K^{-a_2} \int_0^x \left(Q(w)\right)^{a_2} dw, \text{ and} \tag{58}$$

$$l_e(x) = n(x) \frac{\partial Z(g,u,E)}{\partial E} \frac{\partial E}{\partial n} = \gamma I^c(x). \tag{59}$$

In addition, the credit market equilibrium conditions can be represented by

$$\int_0^{x_f} \tau(x) n(x) dx = \tau_{tot}, \text{ if } \varepsilon > 0, \text{ and} \tag{60}$$

$$\int_0^{x_f} \tau(x) n(x) dx \le \tau_{tot}, \text{ if } \varepsilon = 0. \tag{61}$$

Eqs. (60) and (61) are the credit market clearing conditions. The former guarantees that the equilibrium price of credit is positive only when all issued credits are consumed, and the latter implies that the equilibrium price of credit is 0 when the supply of credits exceeds the demand. These conditions are consistent with those in Yang and Wang (2011).

The equilibrium solutions of the urban system with the credit scheme can be obtained by solving Eqs. (26)-(33) in which the net income of each household $I(x)$ is replaced by $I^c(x)$. The equilibrium price of unit credit can be determined by the credit market equilibrium conditions (60) and (61).

4.2. A tradable location tax credit scheme for social optimum

The previous section presents the urban system equilibrium with tradable location tax credit scheme, in which the credit charging scheme is set endogenously according to the externality level of each location and the total amount of credits is given exogenously. The authority's responsibility is to determine the optimal total amount of credits issued, τ_{tot}, so as to maximize the social welfare of the urban system. As stated before, the social welfare is defined as the total utility of all households within the urban system. Denote SW^c as the total social welfare of the urban system after implementing the tradable location tax credit scheme. Then, the social welfare maximization problem can be formulated as

$$\max_{\tau_{tot}} \ SW^c = N \cdot u , \tag{62}$$

where the superscript "c" represents the "tradable location tax credit scheme" case. The common utility level u can be determined by solving the urban system equilibrium with tradable location tax credit scheme proposed in the Section 4.1. On the one hand, if τ_{tot} is so large that it makes credit supply exceed the demand, then implementation of the credit scheme has no effects on externality management and thus on social welfare improvement. On the other hand, if τ_{tot} is quite small, deficiency of credits will bid up the equilibrium price of one credit, and thus make externality overpriced and further decrease the social welfare. The following proposition shows the property of the SO credit scheme that maximizes the social welfare.

Proposition 4. *Denote* $\left\{ l(x), 0 \le x \le x_f^* \right\}$ *as the first-best tax pricing scheme and* $n^*(x)$ *as the resultant residential density across the city. Then, the optimal total amount of credits for problem (62) can be given by* $\tau_{tot} = \int_0^{x_f^*} l(x) n^*(x) dx$*, and the resultant equilibrium leads to the maximal social welfare with the first-best tax pricing scheme(i.e.,* SW^{so}*). Furthermore, the equilibrium price of one credit equals 1 (i.e.,* $\varepsilon = 1$*).*

Proof. Let $\left\{ n^*(x), g^*(x), p^*(x), r^*(x), S^*(x), x_f^*, u^* \right\}$ represent the SO solution given by Eqs. (44)-(49). Apparently, it satisfies all the urban system equilibrium conditions presented in Section 4.1. As for the credit market equilibrium condition, the total amount of credits consumed can be determined by $\int_0^{x_f^*} \tau(x) n^*(x) dx$, where $\tau(x)$ is given by

$$\tau(x) = l_t(x) + l_e(x) = l(x)\,. \tag{63}$$

The total amount of credits consumed within the urban system can thus be given by

$$\int_0^{x_f^*} \tau(x) n^*(x) dx = \int_0^{x_f^*} l(x) n^*(x) dx\,. \tag{64}$$

Combing Eq. (64) with the assumption $\tau_{tot} = \int_0^{x_f^*} l(x) n^*(x) dx$, we have

$$\int_0^{x_f^*} \tau(x) n^*(x) dx = \tau_{tot}\,. \tag{65}$$

Eq. (65) implies that the total amount of credits consumed is equivalent to the total amount of credits initially issued and thus the credit market equilibrium condition (60) holds. It can be seen that $\left\{ n^*(x), g^*(x), p^*(x), r^*(x), S^*(x), x_f^*, u^* \right\}$ satisfies both the urban system equilibrium and the credit market equilibrium, and is thus the urban system equilibrium solution with tradable location tax credit scheme. Specifically, this equilibrium solution leads to the maximal social welfare SW^{so} and thus achieves the social optimum. As such, the corresponding total amount of credits $\tau_{tot} = \int_0^{x_f^*} l(x) n^*(x) dx$ is the optimal solution of problem (62), and such credit scheme is referred to as the SO credit scheme.

As stated in Section 4.1, the equilibrium residential density $n^*(x)$ can be given by

$$n^*(x) = \left((2k)^{-\beta} \left(\beta\theta^2 \right)^{\beta} \left(\alpha^{\alpha} / u^* \right)^2 I^c(x)^{(1+\alpha)} \right)^{1/(\beta+2\gamma)}\,. \tag{66}$$

On the other hand, $n^*(x)$ indicates the residential density under the first-best tax pricing scheme, and can be given by Eq.(48). Combining Eqs. (66) and (48), we have $I^c(x) = I^{so}(x)$ for any location x in the city, i.e.,

$$Y + \frac{R}{N} + \varepsilon(\tau_0 - \tau(x)) - \Phi(x) = Y + \frac{R}{N} + G - \Phi(x) - l(x),\ \ \forall x \in [0, x_f]\,. \tag{67}$$

For any location x with $\tau(x) \neq \tau_0$, the equilibrium price of unit credit can be determined by

$$\varepsilon = \frac{G - l(x)}{\tau_0 - \tau(x)} = \frac{\frac{1}{N}\int_0^{x_f^*} l(x)n^*(x)dx - l(x)}{\frac{\tau_{tot}}{N} - \tau(x)} = \frac{\frac{1}{N}\int_0^{x_f^*} l(x)n^*(x)dx - l(x)}{\frac{1}{N}\int_0^{x_f^*} l(x)n^*(x)dx - l(x)} = 1. \qquad (68)$$

In view of the above, the total amount of credits $\tau_{tot} = \int_0^{x_f^*} l(x)n^*(x)dx$ is the optimal solution of problem (62) and the equilibrium price of one credit equals 1. This completes the proof.

5. Numerical study

In this section, we apply an example to illustrate the properties of the proposed model and the contributions of this study. The numerical example is intended to ascertain the effects of the elasticity parameter of environmental quality on the social welfare and the city size. It is also used to investigate the optimal total amount of credits issued, the households' net revenue from credit trading and the effects of the SO credit scheme on the urban system performance. Table 1 shows the baseline values (i.e., base case) of the model's input parameters. In the following analysis, unless specifically stated otherwise, the input data are the same with those of the base case.

5.1. Sensitivity analysis to the elasticity parameter of environmental quality

We first conduct a sensitivity analysis about the elasticity parameter γ of environmental quality, which indicates the importance of environmental factor in the household utility. Specifically, we compare its impacts on the urban system performance before and after implementing the SO credit scheme, which can be determined by solving the social welfare maximization model (62). Figs. 1 (a) and (b), respectively, show the changes of the total annual social welfare and the city size with the parameter γ. In Fig. 1, the solid lines represent the no credit scheme scenario and the dotted lines represent the SO credit scheme scenario. It can be seen that for both scenarios, as γ increases, the total annual social welfare decreases, whereas the city size increases. This is because an increase in γ implies an increased importance of the environmental quality in the household utility function (see Eq. (9) or (54)). The city thus becomes more decentralized (i.e., urban sprawl) in order to improve the urban environmental quality. As a result, the social welfare of the urban system decreases.

It can be noted in Fig.1 (a) that for all values of γ, the implementation of the SO credit scheme leads to a higher social welfare and a higher household utility level compared to the no credit scheme scenario. The credit scheme can thus benefit to all the households in the urban system in terms of their utility level. Fig.1 (b) shows that for a small value of γ, introducing the SO credit scheme leads to urban concentration (see the left-hand side of point A). However, for a large value of γ, the SO credit scheme induces urban sprawl (see the right-hand side of point A). The city size remains unchanged at point A when γ equals 0.03. These findings differ from the result presented in previous related studies, i.e., congestion pricing always reduces city size (e.g., see Verhoef, 2005; De Lara et al., 2013; Li et al., 2014). This is not surprising because the previous related studies considered only the traffic congestion externality, but ignored the effects of the environmental externality. The ignorance of the environmental externality will underestimate the externality level that households actually generate, thus leading to a distorted urban system.

Table 2 further compares the performances of the urban system with and without the SO credit scheme for different values of γ. It can be seen that for γ = 0.00, 0.03, 0.10 and 0.25, the implementation of the SO credit scheme, respectively, results in an increase in the annual social welfare by 1.16%, 1.22%, 1.30% and 1.49%, an increase in annual total net income by 5.36%, 5.44%, 5.04% and 2.13%, and a decrease in annual total externality by 36.57%, 30.60%, 22.17% and 12.69%, compared to no credit scheme scenario. When γ equals 0.03, after implementation of the credit scheme, the city length remains unchanged (i.e., 25.58 km), but the average annual housing price increases by 8.29% and the average housing space per household decreases by 2.62%.

5.2. SO tradable location tax credit scheme

Fig. 2 shows the changes of the annual social welfare and the corresponding equilibrium price per unit of credit with the total amount of credits issued for a fixed value of γ (e.g., $\gamma = 0.25$). It can be seen that as the total amount of credits increases, the annual social welfare of the urban system first increases and then decreases, whereas the price per unit of credit always decreases. The annual social welfare achieves the maximum at point B^* with the total amount of credits of 15.69×10^9, which equals the total annual externality under social optimum as shown in Table 2. The resultant total maximum annual social welfare is 2.87×10^6 utility units per year and the price per unit of credit is $1.0 per credit. This illustrates the result of Proposition 4. In addition, it can also be observed that when the total amount of credits exceeds 18.0×10^9 (i.e., point B_0), the equilibrium price of the credit becomes zero, thus the credit scheme fails to improve the performance of the urban system.

5.3. Trade-off between traffic congestion and environmental externalities

Fig. 3 plots the traffic congestion and environmental externalities generated by each household along the linear corridor after implementing the SO credit scheme, i.e., the total amount of credits is 15.69×10^9. It can be seen that as the distance from the CBD increases, the environmental externality caused by one household always decreases, but the traffic congestion externality always increases. This is because, as the distance from the CBD increases, the residential density decreases, which implies an increase in the environmental quality and thus a decrease in the environmental externality. However, the average journey distance increases, implying an increase in the traffic congestion externality. As a result, the traffic congestion and environmental externality curves intersect at point $\bar{x}$ with a distance of 2.07 kilometers from the CBD and a (traffic congestion or environmental) externality level of $14,000 per year. When $x < \bar{x}$ (or $x > \bar{x}$), the environmental externality is higher (or lower) than the traffic congestion externality. This further illustrates the result of Proposition 3.

Fig. 3 also shows the total externality (i.e., the sum of traffic congestion and environmental externalities) first increases and then decreases, and reaches the highest level of $35,300 per year at location $x^* = 10.73$ kilometers from the CBD. This means that the households residing at the middle of the transportation corridor generate highest externality than those at the suburb and close to the CBD.

5.4. Household's net revenue from credit scheme

Fig. 4 indicates the net revenue of each household along the corridor from credit trading. The net revenue, represented by $\varepsilon\left(\tau_0 - \tau(x)\right)$, is the market price of unit credit multiplied by the net credits (i.e., the difference between the amount of credits initially distributed and that paid). Fig. 4 shows that the households residing at the middle of the corridor (i.e., between locations C_1 and C_2) have a negative net revenue due to their high total externality (see Fig. 3), whereas the households living at the suburb and nearby the CBD have a positive net revenue due to their low total externality (see Fig. 3). Specifically, the households living at the CBD can obtain the maximum revenue of \$13,380 per year from selling the redundant credits, whereas the households living in the middle of the corridor (i.e., point D) have to buy extra credits for their externalities of up to \$3,890 per year. In addition, the households residing at locations C_1 and C_2 just run out of their distributed credits (i.e., the break-even).

5.5. Effects of SO credit scheme on urban system performance

We now investigate the effects of the SO credit scheme on the urban system performance. In Fig. 5, the solid and dotted curves represent the results under the no credit scheme scenario and the SO credit scheme scenario, respectively. Fig. 5(a) shows the household residential distributions with and without the SO credit scheme. It can be observed that the two curves intersect at points E_1 and E_2 with a distance of 3.48 km and 28.45 km from the CBD, respectively. The corresponding household residential densities are, respectively, 29,800 and 5,000 households per square kilometer. This means that the SO credit scheme has no effect on the household residential densities of locations E_1 and E_2. The household residential density increases for $x < E_1$ (i.e., the left-hand side of point E_1) and $x > E_2$ (i.e., the right-hand side of point E_2), but decreases for $E_1 < x < E_2$ (i.e., the middle section between points E_1 and E_2). This is because the implementation of the SO credit scheme causes a positive profit (or a negative profit) from credit trading for residents living at the suburb and nearby the CBD area (or the middle of the corridor) as shown in Fig. 4, thus encouraging households to move to the low-credit areas with low externalities (i.e., the suburb and the CBD area). As a result, the environmental quality in the middle area of the city (i.e., between locations F_1 and F_2) gets an improvement, but the environmental quality at the suburb and nearby the CBD area get a slight deterioration, as shown in Fig. 5(b). In the meantime, the housing space per household in the middle area of the city (i.e., the area between J_1 and J_2) increases, whereas those in the suburb and nearby the CBD area (i.e., the right-hand side of J_2 and the left-hand side of J_1) decrease, as shown in Fig. 5(c). In addition, the housing prices per unit of floor space nearby the CBD area and in the suburb (i.e., the left-hand side of K_1 and the right-hand side of K_2) increase, but that in the middle area of the city (i.e., the area between K_1 and K_2) decreases, as shown in Fig. 5(d).

6. Conclusions and further studies

This paper proposed a tradable location tax credit scheme, which is considered as a promising substitution for the first-best tax pricing scheme, for balancing traffic congestion and environmental externalities in a linear monocentric city. It was assumed that each household in the urban system pays a certain amount of credits for internalizing the externalities it causes due to use of congested roads and crowding neighborhoods. The authority aims to determine the total amount of credits to maximize the social welfare of the urban system. For a given credit scheme, the households' residential location choices and housing market structure in terms of housing prices and space can be endogenously determined by the urban system equilibrium, and the price per unit of credit can be calculated by the credit market equilibrium. On the basis of the urban system equilibrium, a social welfare maximization model was then proposed for finding the SO credit scheme.

A numerical example was provided for illustrating the properties of the proposed model. Some important findings and new insights have been obtained. First, when considering both traffic congestion and environmental externalities, introduction of the SO credit scheme may cause urban expansion or agglomeration. This finding differs from the result in previous studies that congestion pricing always reduces city size (e.g., see Verhoef, 2005; De Lara et al., 2013; Li et al., 2014). Second, there is a tradeoff between the traffic congestion externality and the environmental externality. The highest level of the total externality occurs at the middle area of the transportation corridor. Third, the households residing in the suburb and the CBD area can gain a positive profit from the SO credit scheme, whereas the households residing in the middle area of the city suffer a loss from this charging scheme. In addition, the SO credit scheme can increase the total social welfare of the urban system through internalizing the externalities generated by households. The proposed model can serve as a useful tool for long-term strategic planning of sustainable urban economic development, and for evaluation of various transport and housing policies.

Although the proposed model in this paper provides a new avenue for the urban system analysis, further extensions should be made in the following directions. First, a monocentric urban structure was assumed in this paper. However, modern cities are generally composed of multiple business and commercial centers. This assumption can thus be relaxed to consider polycentric urban structures in a further study. Second, the households in the urban system are assumed to be homogenous in terms of their income level and utility function. However, studies showed that the income level has a significant impact on the households' residential location choices. Therefore, there is a need to extend the proposed model to take into account multiple household classes with different income levels. Third, a uniform initial distribution scheme for the location tax credits was adopted in this paper. This assumption can be relaxed to consider other distribution schemes, such as location-based or income-based distribution schemes. The effects of initial distribution schemes on the urban system can thus be explored in a further study.

Acknowledgements

The authors would like to thank three anonymous referees for their helpful comments and suggestions on an earlier draft of the paper. The work described in this paper was jointly supported by grants from the National Natural Science Foundation of China (71501150, 71525003, 71222107, 71471013), the Fundamental Research Funds for the Central Universities (WUT: 2015VI014), the Research Grants Council of the Hong Kong Special Administrative Region, China (152074/14E), Research Institute for Sustainable Urban Development (RISUD) of The Hong Kong Polytechnic University (Project No. H-ZG3X), and the National Research Foundation of Korea grant funded by the Korea government (MSIP) (NRF-2010-0028693).

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Appendix A: Proof of Proposition 1

The social welfare maximization problem (37) is difficult to be solved directly. Fortunately, according to Fujita (1989), we can resort to its compensated problem, which minimizes the total cost subject to a pre-specified utility for each household. Let u be the pre-specified utility, and TC be the total cost, which is the sum of the transportation cost, non-housing goods cost, land opportunity cost and the capital cost for housing production. Thus, the total cost minimization problem (cf. Fujita, 1989) can be described as

$$\min_{x_f, n(x), S(x), g(x)} TC = \int_0^{x_f} \left(\left(\Phi(x) + Z\left(g(x), u, E\left(n(x)\right)\right) \right) n(x) + kS(x) + r_a \right) dx \,, \tag{A.1}$$

$$\text{s.t.} \qquad n(x) = \frac{\theta\sqrt{S(x)}}{g(x)} \,, \tag{A.2}$$

$$Q(x) = \delta \int_x^{x_f} n(w) dw \,, \tag{A.3}$$

$$\int_0^{x_f} n(x) dx = N \,. \tag{A.4}$$

In Eq. (A.1), $Z\left(g(x), u, E\left(n(x)\right)\right)$ is the solution to $U\left(z(x), g(x), E\left(n(x)\right)\right) = u$ for $z(x)$, $E\left(n(x)\right)$ is the environmental quality determined by Eq. (8), and $\Phi(x)$ is the annual transportation cost of each household at location x, which is a function of the travel demand $Q(x)$. Eq. (A.2) represents the residential density constraint, Eq. (A.3) represents the hourly travel demand constraint, and Eq. (A.4) represents the population constraint. Note that Eqs. (A.3) and (A.4) can be further expressed as

$$dQ(x)/dx = -\delta n(x) \,, \text{ where } Q(0) = \delta N \,, \text{ and } Q(x_f) = 0 \,. \tag{A.5}$$

The total income of all households minus the total cost in Eq. (A.1) produces the social surplus, specified as $\Omega = YN - TC$. Note that the total income is a constant, thus the total cost minimization problem is equivalent to the social surplus maximization problem, which can be described as

$$\max_{x_f, n(x), S(x), g(x)} \Omega = \int_0^{x_f} \left(\frac{Y - \Phi(x) - Z\left(g(x), u, E\left(n(x)\right)\right)}{g(x)} \theta\sqrt{S(x)} - kS(x) - r_a \right) dx \,, \tag{A.6}$$

s.t. Eqs. (A.2) and (A.5).

In Eq. (A.6), the annual transportation cost of each household incorporates both time and monetary costs, i.e., $\Phi(x) = 2\omega\sigma T(x) + 2\omega\Lambda(x)$, in which $T(Q(x))$ is given by Eq. (4). Note that $T(0) = 0$. Therefore, according to integration by parts and $dT(x)/dx = t(Q(x))$, we have

$$\int_0^{x_f} T(x)n(x)dx = -\frac{1}{\delta}\int_0^{x_f}\left(T(x)\frac{dQ(x)}{dx}\right)dx = \frac{1}{\delta}\int_0^{x_f} t(Q(x))Q(x)dx, \tag{A.7}$$

From Eq. (A.7), the social surplus maximization problem (A.6) can be reformulated as

$$\max_{\substack{x_f, n(x),\\ S(x), g(x)}} \Omega = \int_0^{x_f}\left(\frac{Y - 2\omega\Lambda(x) - Z(g(x), u, E(n(x)))}{g(x)}\theta\sqrt{S(x)} - \frac{2\omega\sigma}{\delta}t(Q(x))Q(x) - kS(x) - r_a\right)dx, \tag{A.8}$$

s.t. Eqs. (A.2) and (A.5),

which is an optimal control problem with $Q(x)$ as the state variable. The associated Hamiltonian function can thus be constructed as

$$H = \frac{Y - 2\omega\Lambda(x) - Z(g(x), u, E(n(x)))}{g(x)}\theta\sqrt{S(x)} - \frac{2\omega\sigma}{\delta}t(Q(x))Q(x) - kS(x) - r_a - \delta\lambda(x)n(x). \tag{A.9}$$

In Eq. (A.9), the multiplier $\lambda(x)$ has the economic implication of the shadow price of travel demand (i.e., $Q(x)$). The associated Lagrangian function can thus be formulated as

$$\begin{aligned}\Theta &= H + l_e(x)\left(n(x) - \frac{\theta\sqrt{S(x)}}{g(x)}\right)\\ &= \frac{Y - \delta\lambda(x) - 2\omega\Lambda(x) - l_e(x) - Z(g(x), u, E(n(x)))}{g(x)}\theta\sqrt{S(x)} - \frac{2\omega\sigma}{\delta}t(Q(x))Q(x) - kS(x) + l_e(x)n(x) - r_a,\end{aligned} \tag{A.10}$$

where the Lagrangian multiplier $l_e(x)$ has the economic implication of the environmental externality per household at location *x*, which represents the extra cost to the existing households caused by an additional household at location x.

For presentation purpose, we define $I(x) = Y - \delta\lambda(x) - 2\omega\Lambda(x) - l_e(x)$. Then, according to the maximum principle (see Long and Vousden, 1977), the following conditions must be satisfied.

$$\begin{cases} \dfrac{\partial\Theta}{\partial S} = \dfrac{I(x) - Z\left(g(x), u, E\left(n(x)\right)\right)}{2g(x)\sqrt{S(x)}}\theta - k = 0, \\ \dfrac{\partial\Theta}{\partial g} = \dfrac{1/\alpha\, u^{1/\alpha} g(x)^{-\beta/\alpha} E\left(n(x)\right)^{-\gamma/\alpha} - I(x)}{\left(g(x)\right)^2} = 0, \\ \dfrac{\partial\Theta}{\partial n} = -\dfrac{\partial Z(g,u,E)}{\partial E}\dfrac{\partial E}{\partial n} n(x) + l_e(x) = 0, \end{cases} \tag{A.11}$$

$$\begin{cases} \dfrac{d\lambda(x)}{dx} = -\dfrac{\partial\Theta}{\partial Q} = \dfrac{2\omega\sigma}{\delta}\left(t'\left(Q(x)\right)Q(x) + t\left(Q(x)\right)\right), \\ \dfrac{\partial\Theta}{\partial l_e} = n(x) - \dfrac{\theta\sqrt{S(x)}}{g(x)} = 0, \end{cases} \tag{A.12}$$

$$\frac{dQ(x)}{dx} = \frac{\partial\Theta}{\partial\lambda} = -\delta n(x)\text{, and} \tag{A.13}$$

$$\Theta_{x=x_f} = \frac{I(x_f) - Z\left(g(x_f), u, E\left(n(x)\right)\right)}{g(x_f)}\theta\sqrt{S(x_f)} - kS(x_f) + l_e(x_f)n(x_f) - r_a = 0\,, \tag{A.14}$$

where Eq. (A.11) represents the extreme-value conditions, Eq. (A.12) represents the adjoint equations, Eq. (A.13) represents the ordinary differential equation, and Eq. (A.14) represents the terminal condition, which states the zero-profit condition at the urban fringe.

Note that Eq. (A.12) implies

$$\lambda(x) = \frac{2\omega\sigma}{\delta}\int_0^x t'\left(Q(w)\right)Q(w)dw + \frac{2\omega\sigma}{\delta}\int_0^x t\left(Q(w)\right)dw + A = \frac{1}{\delta}\left(l_t(x) + 2\omega\sigma T(x) + B\right), \tag{A.15}$$

where $l_t(x) = 2\omega\sigma\int_0^x t'\left(Q(w)\right)Q(w)dw$ implies the tax for the traffic congestion externality, $B = \delta A$ is a constant, which implies the population tax (or subsidy if negative) for achieving a pre-specified utility. Then, substituting Eq. (A.15) into $I(x)$ yields

$$I(x) = Y - B - \Phi(x) - l_t(x) - l_e(x)\,. \tag{A.16}$$

Reorganizing Eqs. (A.11)-(A.13) yields the optimal solution, as follows:

$$S^*(x) = \left((2k)^{-(\beta+\gamma)}\beta^{(\beta+\gamma)}\theta^{\beta}\alpha^{\alpha}u^{-1}I(x)^{(1+\gamma)}\right)^{2/(\beta+2\gamma)}, \tag{A.17}$$

$$g^*(x) = \left((2k)^{-\gamma}\left(\beta\theta^2\right)^{\gamma}\left(\alpha^{\alpha}u^{-1}\right)^{-1} I(x)^{(\gamma-\alpha)}\right)^{1/(\beta+2\gamma)}, \tag{A.18}$$

$$n^*(x) = \left((2k)^{-\beta}\left(\beta\theta^2\right)^{\beta}\left(\alpha^{\alpha}u^{-1}\right)^{2} I(x)^{(1+\alpha)}\right)^{1/(\beta+2\gamma)}, \tag{A.19}$$

$$l_e(x) = \gamma I(x)\,, \tag{A.20}$$

$$l_t(x) = 2\omega\sigma a_1 a_2 t_0 K^{-a_2} \int_0^x \left(Q(w)\right)^{a_2} dw \,. \tag{A.21}$$

where the superscript "$*$" represents the optimal solution for the social surplus maximization problem, and $\{l_t(x), l_e(x)\}$ is the optimal tax pricing scheme. The resultant housing price can thus be determined by

$$p^*(x) = \frac{I(x) - Z\left(g(x), u, E\left(n(x)\right)\right)}{g(x)} = \left((2k)^\gamma \beta^{(\beta+\gamma)} \theta^{-2\gamma} \left(\alpha^\alpha u^{-1}\right) I(x)^{(1+\gamma)}\right)^{1/(\beta+2\gamma)} . \tag{A.22}$$

Note that under perfect competition, the property developers earn zero profit, thus the land price can be derived as follows:

$$r^*(x) = \left(\left(2^{-1}\beta\right)^{2(\beta+\gamma)} \left(k^{-1}\theta^2\right)^\beta \left(\alpha^\alpha u^{-1}\right)^2 I(x)^{2(1+\gamma)}\right)^{1/(\beta+2\gamma)} . \tag{A.23}$$

The terminal condition (A.14) can thus be expressed in a more succinct way, as follows:

$$r^*(x_f^*) + l_e(x_f^*) n^*(x_f^*) - r_a = 0 \,. \tag{A.24}$$

In addition, the initial condition (A.5) must be satisfied, thus we have

$$\int_0^{x_f} n^*(x) dx = N \,. \tag{A.25}$$

Note that Eq. (A.25) can determine the unique value of the population tax or subsidy B given a target utility u, and inversely determine the unique value of u given the population tax B.

Denote V as the resultant maximal social surplus for achieving a pre-specified utility u, as follows:

$$V = \Omega\left(x_f^*, n^*(x), S^*(x), g^*(x)\right). \tag{A.26}$$

According to the Envelop Theorem, the derivative of V with regard to the parameter u can be derived as

$$\frac{dV}{du} = \int_0^{x_f} -\frac{\partial Z\left(g(x), u, E\left(n(x)\right)\right)}{\partial u} n(x) dx = -1/\alpha \int_0^{x_f} u^{\beta/\alpha} g(x)^{-\beta/\alpha} E\left(n(x)\right)^{-\gamma/\alpha} dx < 0 \,, \tag{A.27}$$

which means obtaining a higher target utility (i.e., u) requires higher cost and thus lower social surplus. Since the social surplus cannot be negative, the maximal target utility (denoted by u^*) can thus be obtained when the maximal social surplus achieves zero, as follows:

$$V = \int_0^{x_f^*} \left(r^*(x) - r_a\right) dx + \int_0^{x_f^*} \left(l_t(x) + l_e(x)\right) n^*(x) dx + B \cdot N = 0 \,. \tag{A.28}$$

Accordingly, the value of B (i.e., population tax or subsidy) can be obtained, as follows:

$$B=-\frac{\int_0^{x_f^*}\left(r^*(x)-r_a\right)dx+\int_0^{x_f^*}\left(l_t(x)+l_e(x)\right)n^*(x)dx}{N}=-\left(\frac{R}{N}+G\right), \tag{A.29}$$

where $R=\int_0^{x_f}\left(r(x)-r_a\right)dx$ implies the total differential rent from land lease, and $G=\frac{1}{N}\int_0^{x_f}\left(l_t(x)+l_e(x)\right)n(x)dx$ implies the average tax per household.

Eq. (A.29) means that the maximized utility can be obtained (referred to as social optimum), when both the land revenue and tax revenue are reallocated to all the households as a subsidy. Substituting Eqs. (A.20) and (A.29) into Eq. (A.16) yields the net income of a household under social optimum (denoted by $I^{so}(x)$), as follows:

$$I^{so}(x)=\frac{1}{1+\gamma}\left(Y+\frac{R}{N}+G-\Phi(x)-l_t(x)\right). \tag{A.30}$$

Substituting $I^{so}(x)$ for $I(x)$ in Eqs. (A.17)-(A.19), and (A.22)-(A.23) yields the value of $\left\{S^*(x),g^*(x),n^*(x),p^*(x),r^*(x)\right\}$ under social optimum. Also, the values of city length x_f^* and the maximal utility u^* can be determined by solving the system of Eqs. (A.24) and (A.25). This completes the proof of Proposition 1.

Appendix B: Proof of Proposition 2

Taking derivative of Eq. (43) with regard to *x*, we obtain

$$\frac{dI^{so}(x)}{dx}=-\frac{1}{1+\gamma}\left(2\omega\sigma t\left(Q(x)\right)+2\omega\Lambda_1+2\varpi\sigma a_1a_2t_0K^{-a_2}\left(Q(x)\right)^{a_2}\right)<0\,,\ x\in[0,x_f^*]. \tag{B.1}$$

This implies that the net income of each household decreases with the distance *x* from the CBD. Taking derivative of Eq. (48) with regard to location *x*, we have

$$\frac{dn^*(x)}{dx}=(1+\alpha)/(\beta+2\gamma)\left((2k)^{-\beta}\left(\beta\theta^2\right)^{\beta}\left(\alpha^{\alpha}/u^*\right)^2 I^{so}(x)^{2(\alpha-\gamma)}\right)^{1/(\beta+2\gamma)}\frac{dI^{so}(x)}{dx}<0\,. \tag{B.2}$$

This implies that as the distance *x* from the CBD increases, the household residential density $n^*(x)$ decreases. Thus, based on the definition of the environmental quality given by $E^*(x)=1/n^*(x)$, taking derivation yields

$$\frac{dE^*(x)}{dx}=\frac{d\left(1/n^*(x)\right)}{dx}=-\frac{1}{n^*(x)^2}\frac{dn^*(x)}{dx}>0. \tag{B.3}$$

It means that as the distance x from the CBD increases, the environmental quality $E^*(x)$ is improved. Therefore, the households living in the suburb enjoy a better environmental quality than those close to the CBD. Taking derivative of Eqs. (44) and (45) gives

$$\frac{dg^*(x)}{dx}=\frac{\gamma-\alpha}{\beta+2\gamma}\left((2k)^{-\gamma}\left(\beta\theta^2\right)^{\gamma}\left(\alpha^{\alpha}/u^*\right)^{-1}I^{so}(x)^{-(\gamma+1)}\right)^{1/(\beta+2\gamma)}\frac{dI^{so}(x)}{dx}, \text{ and} \tag{B.4}$$

$$\frac{dS^*(x)}{dx}=\frac{2(1+\gamma)}{\beta+2\gamma}\left((2k)^{-(\beta+\gamma)}\beta^{(\beta+\gamma)}\theta^{\beta}\,\alpha^{\alpha}/u^*\,I^{so}(x)^{(1+\alpha)/2}\right)^{2/(\beta+2\gamma)}\frac{dI^{so}(x)}{dx}<0. \tag{B.5}$$

Eq. (B.4) means that the housing space of each household, $g^*(x)$, increases with the increase in the distance x from the CBD for $\gamma<\alpha$, but decreases for $\gamma>\alpha$. Eq. (B.5) implies that the property developers' capital input density per unit land, $S^*(x)$, decreases as the distance x from the CBD increases.

Similarly, taking derivatives of Eqs. (46) and (47) with regard to x, one can obtain

$$\frac{dp^*(x)}{dx}=\frac{1+\gamma}{\beta+2\gamma}\left((2k)^{\gamma}\beta^{(\beta+\gamma)}\theta^{-2\gamma}\left(\alpha^{\alpha}/u^*\right)I^{so}(x)^{(\alpha-\gamma)}\right)^{1/(\beta+2\gamma)}\frac{dI^{so}(x)}{dx}<0, \tag{B.6}$$

$$\frac{dr^*(x)}{dx}=\frac{2(1+\gamma)}{\beta+2\gamma}\left(\left(2^{-1}\beta\right)^{2(\beta+\gamma)}\left(k^{-1}\theta^2\right)^{\beta}\left(\alpha^{\alpha}/u^*\right)^2 I^{so}(x)^{(1+\alpha)}\right)^{1/(\beta+2\gamma)}\frac{dI^{so}(x)}{dx}<0. \tag{B.7}$$

Consequently, the housing price $p^*(x)$ and the land rent $r^*(x)$ decrease with the distance from the CBD. This completes the proof of Proposition 2.

Appendix C: Proof of Proposition 3

We define $F(x)$ as the difference between the traffic congestion externality and the environmental externality at location x, i.e., $F(x)=l_t(x)-l_e(x)$. Taking the first-order derivative of $F(x)$ with regard to x, we have

$$\frac{dF(x)}{dx}=2\varpi\sigma a_1 a_2 t_0 K^{-a_2}\left(Q(x)\right)^{a_2}-\gamma\frac{dI^{so}(x)}{dx}. \tag{C.1}$$

From Proposition 2, $\dfrac{dI^{so}(x)}{dx} < 0$, we thus have $F'(x) > 0$, implying that $F(x)$ is an increasing function with regard to the distance x from the CBD. In addition, at the CBD (i.e., $x = 0$), $l_t(0) = 0$ and $l_e(0) = \gamma I^{so}(0) > 0$ hold. We thus obtain $F(0) < 0$ at location $x = 0$. On the other hand, at the city boundary (i.e., $x = x_f^*$), $F\left(x_f^*\right) = l_t\left(x_f^*\right) - l_e\left(x_f^*\right) > 0$ holds. By the intermediate value theorem (Smith and Minton, 2012), there must exist an $\bar{x} \in (0, x_f^*)$ such that $F(\bar{x}) = 0$ or $l_t\left(\bar{x}\right) = l_e\left(\bar{x}\right)$ holds. This completes the proof of this proposition.

Table 1 Input parameters for the numerical illustration.

Symbol	Definition	Baseline value
σ	Value of travel time (\$/h)	20
t_0	Free-flow travel time per unit of distance (h/ km)	0.02
K	Capacity of the corridor (veh/h)	18,000
a_1, a_2	Parameters in travel time function	0.15 and 4.0
Λ_0	Fixed component of monetary travel cost (\$)	10
Λ_1	Variable component of monetary travel cost (\$/veh-km)	1.0
η	Average daily number of trips to the CBD per household	1.0
ξ	Peak-hour factor	10%
ω	Average annual number of trips to the CBD per household	365
α, β, γ	Parameters in households' utility function	0.7, 0.3, 0.25
r_a	Opportunity cost of land in the city (\$/km^2/year)	10,000,000
Y	Annual household income (\$/year)	60,000
N	Total number of households in the city	500,000
k	Price of capital (i.e., the interest rate)	5%
θ	Parameter in housing production function	2×10^{-5}

Table 2 Comparison of urban system performances with and without the SO credit scheme for different values of γ.

Performance index	$\gamma=0$		$\gamma=0.03$		$\gamma=0.1$		$\gamma=0.25$	
	No credit scheme	SO credit scheme	No credit scheme	SO credit scheme	No credit scheme	SO credit scheme	No credit scheme	SO credit scheme
Urban length (km)	**23.92**	**22.74**	**25.58**	**25.58**	**28.29**	**30.9**	**31.28**	**38.28**
Average residential density (households/km^2)	20903	21988	19547	19547	17674	16181	15985	13062
Average annual housing price (\$/m^2/year)	320.8	353.3	302.8	327.9	275.1	287.9	244.3	241.3
Average annual land price (\$/m^2/year)	156.8	173.8	142.2	150.0	120.9	116.2	98.94	82.57
Average housing space per household (m^2/household)	46.77	44.74	48.06	46.8	49.74	49.9	50.68	52.39
Total annual externality (billion \$)	**6.70**	**4.25**	**8.40**	**5.83**	**11.91**	**9.27**	**17.97**	**15.69**
Total annual environmental externality (billion \$)	0	0	0.73	0.77	2.28	2.39	5.16	5.27
Total annual traffic congestion externality (billion \$)	6.70	4.25	7.67	5.06	9.63	6.88	12.81	10.42
Annual total net income[1] (billion \$)	**25.00**	**26.34**	**24.25**	**25.57**	**22.80**	**23.95**	**20.63**	**21.07**
Average household utility level (utility units)	**74.45**	**75.31**	**54.14**	**54.79**	**26.07**	**26.41**	**5.65**	**5.74**
Annual social welfare (10^6 utility units)	**37.23**	**37.65**	**27.07**	**27.40**	**13.04**	**13.21**	**2.827**	**2.869**

[1] Annual total net income = $\int_0^{x_f} I(x)n(x)dx$.

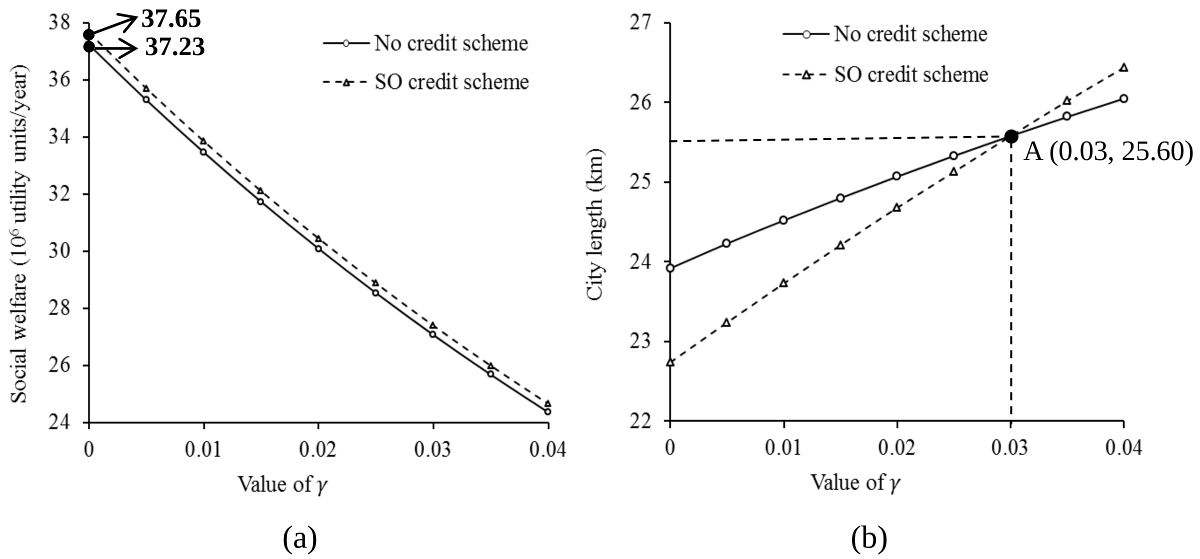

Fig. 1. Sensitivity analysis with respect to γ under no credit scheme and the SO credit scheme scenarios: (a) change of annual total social welfare with γ; (b) change of city length with γ.

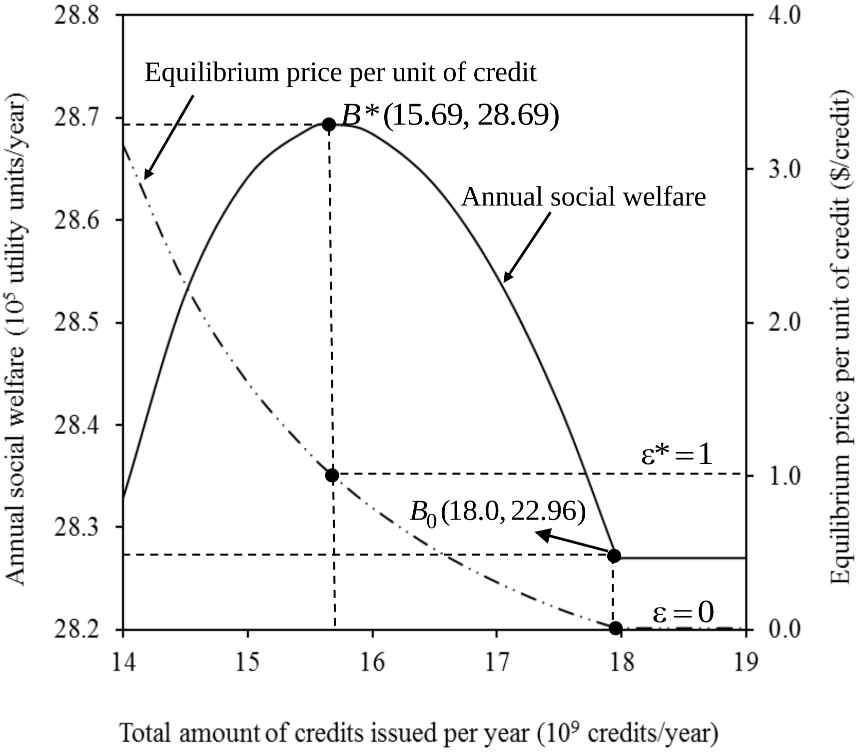

Fig. 2. Annual social welfare and price per unit of credit against total amount of credits issued.

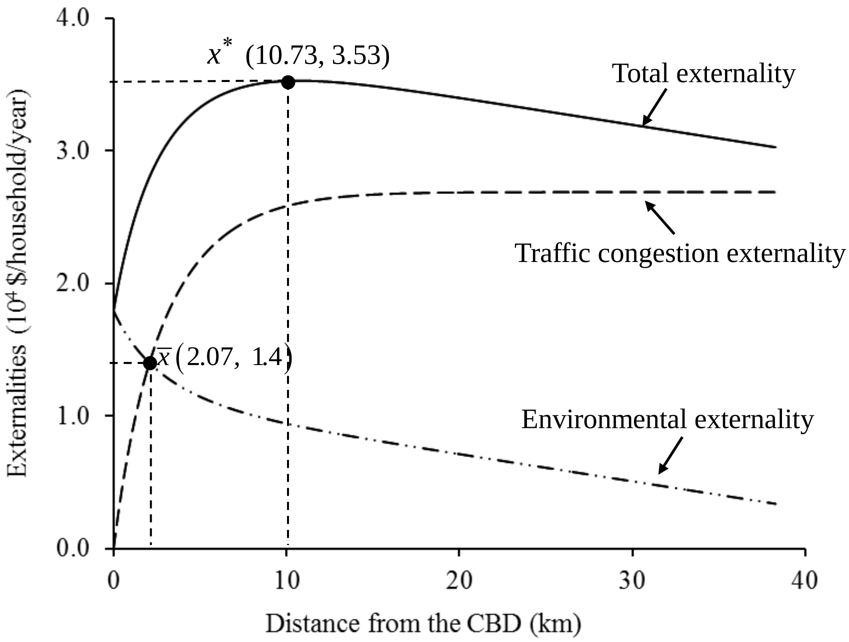

Fig. 3. Annual externalities generated by each household along the linear corridor.

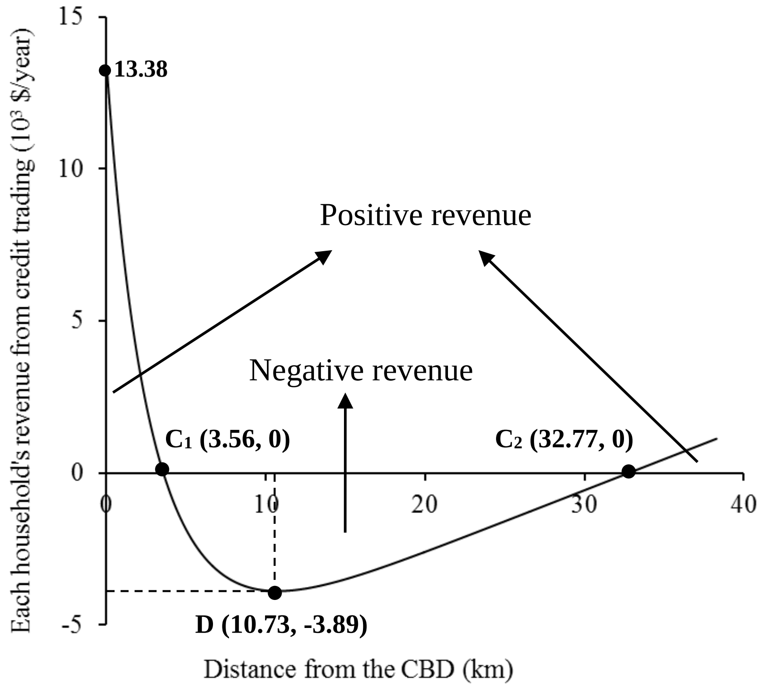

Fig. 4. Average net revenue of each household from credit trading at different locations.

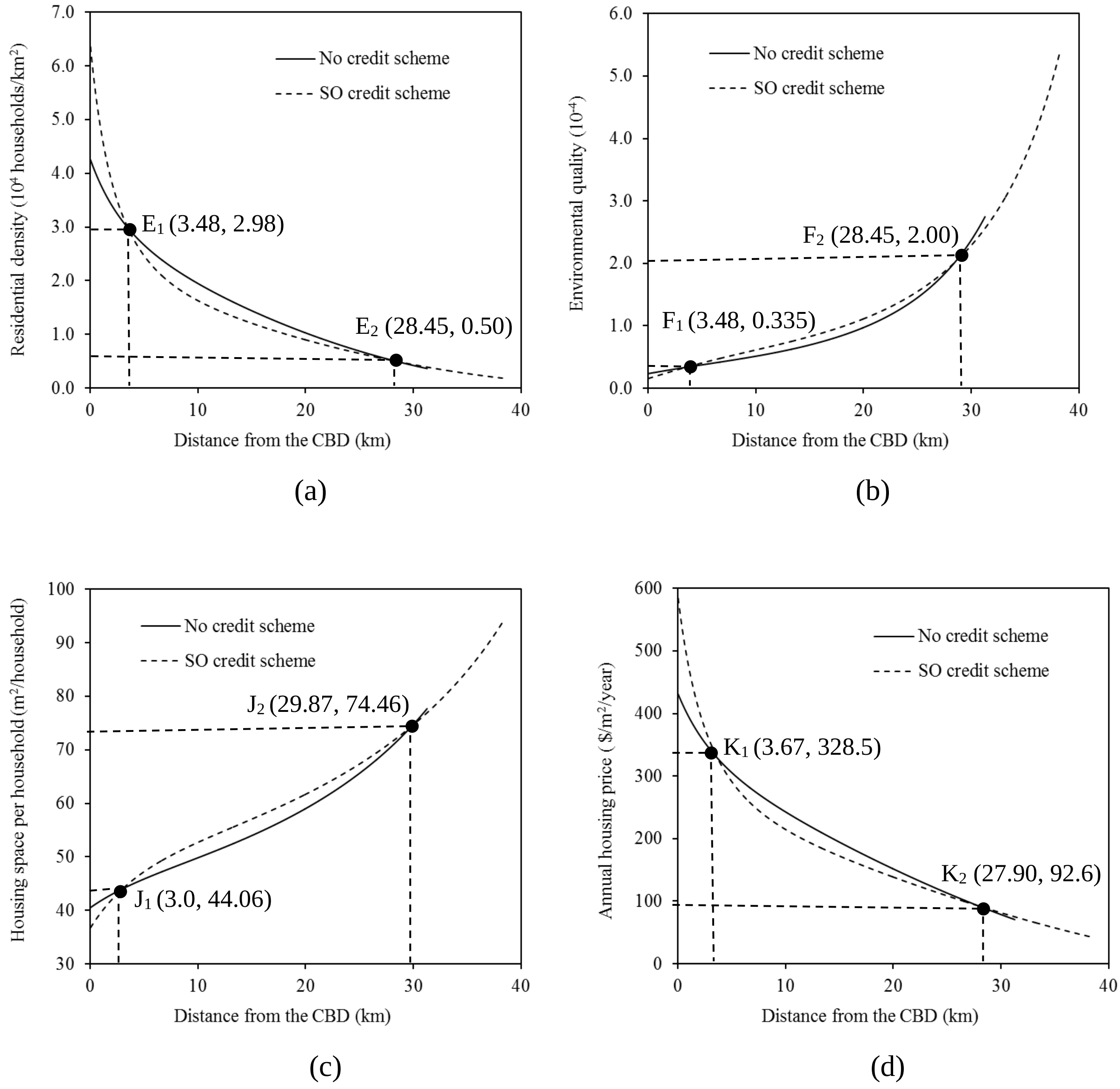

Fig. 5. Comparison of urban system performances before and after implementing the SO credit scheme: (a) residential density; (b) environmental quality; (c) housing space per household; (d) housing price.