The Implications of Utilizing Market Information and Adopting Agricultural Advice for Farmers in Developing Economies

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To alleviate poverty in developing countries, governments and non-governmental organizations (NGOs) disseminate two types of information: (1) agricultural advice to enable farmers to improve their operations (cost reduction, quality improvement, and process yield increase); and (2) market information about future price/demand to enable farmers to make better production planning decisions. This information is usually disseminated free of charge. While farmers can use the market information to improve their production plans without incurring any (significant) cost, adopting agricultural advice to improve operations requires upfront investment, for example, equipment, fertilizers, pesticides, and higher quality seeds. In this paper, we examine whether farmers should use market information to improve their production plans (or adopt agricultural advice to improve their operations) when they engage in Cournot competition under both uncertain market demand and uncertain process yield.

Our analysis indicates that both farmers will use the market information to improve their profits in equilibrium. Hence, relative to the base case in which market information is not available, the provision of market information can improve the farmers' total welfare (i.e., total profit for both farmers). Moreover, when the underlying process yield is highly uncertain or when the products are highly heterogeneous, the provision of market information is welfare-maximizing in the sense that the maximum total welfare of farmers is attained when both farmers utilize market information in equilibrium. Furthermore, in equilibrium, whether a farmer adopts the agricultural advice depends on the size of the requisite upfront investment. More importantly, we show that agricultural advice is not always welfare improving unless the upfront investment is sufficiently low. This result implies that to improve farmers' welfare, governments should consider offering farmer subsidies.

Keywords: emerging markets; social responsibility; operational improvements; competitive production strategies

1 Introduction

Agriculture plays an important role in emerging economies. For example, the agricultural sector accounts for 50% and 74% of the total workforce in India and Kenya, respectively. However, the farmers in these regions remain poor because they lack opportunities to improve their operations so that they can produce higher yield, more available and better quality crops at lower cost. The lack of market information and agricultural advice often results in market inefficiency, poor yields, and huge crop wastage, all of which damage farmers' earnings and livelihoods. Without agricultural advisory information, farmers may not be able to make proper planning decisions in areas such as pest control and soil depletion, which often lead to low and uncertain yields. Moreover, without information about future market price trends, farmers cannot make effective production quantity decisions, which in turn can affect their realized profit. These factors further aggravate the difficult situation faced by those at the bottom of the pyramid, as this large population of farmers has few income sources (Jensen (2007)).

Recognizing these challenges faced by farmers, many governments have developed agricultural extension services by providing the following two types of information: (1) agricultural advice that can help farmers to improve their operations (i.e., *how to grow?*) by reducing operating costs, improving quality, and increasing process yield;[1] and (2) market (price/demand) forecast information that can help farmers to make better long-term production planning decisions (i.e., *how much to grow?*).[2] For example, the Indian government provides both types of information on its website (www.india.gov.in/topics/agriculture) for free.

Due to a lack of Internet access, many governments and NGOs disseminate agricultural advice and market information through different channels including radio, television, and call centers. For example, in Kenya and Mali, an NGO launched a weekly hour-long radio program called Mali Shambani that discusses farming techniques and market price trends, etc. This free radio program also offers an interactive call-in component for farmers to ask agricultural questions via phone or SMS messaging. In the same vein, the India Ministry of Agriculture launched the Kisan Call Centers in 2004 to deliver extension services to farmers over the phone. This free service enables Indian farmers to use their phones to seek advice and to gain access to information posted on the Internet. In another example, in India the NGO Digital Green (digitalgreen.org) distributes farming advisory information through online videos and DVDs delivered to farmers free of charge.

It is certainly a big step forward for more farmers to gain access to agricultural advice and market information. However, even if the information is free of charge, farmers need

[1] Agricultural advisory information includes: (a) tools and equipment for seedbed preparation, sowing, planting and harvesting; (b) high quality seeds and the safe use of approved pesticides and fertilizers; (c) pest management and locust control; and (d) soil and water conservation.

[2] Some governments also provide information about the current market prices in different markets to help farmers to make better short-term selling decisions (i.e., when/where to sell?). For the sake of tractability, we do not model this type of information in this paper.

to decide whether to use such market information when making their production plans, especially when they compete under uncertain market demand and uncertain process yield. Moreover, because the adoption of agricultural advice often requires upfront investment (tools, seeds, fertilizers, etc.), farmers need to decide whether it is cost-effective to do so.

These observations motivate us to examine a situation in which two risk-neutral farmers engage in Cournot competition under uncertain market demand and uncertain process yield. (Note that our model can be extended to multiple farmers by considering the "proportion" of the farmers who adopt the advice (or utilize the market information).) We provide three justifications for selecting Cournot competition as our modeling choice. First, Carter and MacLaren (1994) argue that Cournot competition is a good approximation of the real economic decision-making for managing perishable products, especially when the production quantity cannot be changed quickly in advance of sales (e.g., fruits, vegetables) or for managing products with lengthy production processes (e.g., tree crops). Second, Deodhar and Sheldon (1996), Dong et al. (2006), and others provide empirical evidence to support the existence of Cournot competition in various agricultural product markets, such as malting barley and banana. Third, if one interprets a "farmer" in our model as a marketing board (or a marketing cooperative) that represents a group of farmers in countries such as India and South Africa, then our model captures the quantity competition between two marketing boards. In this context, each marketing board sets the aggregate production quantity to control the market price. To do so, each marketing board may impose a quota on each farmer's production quantity (Nieuwoudt 1987).[3]

To alleviate poverty, we consider the case in which the government offers either agricultural advice or market information but not both.[4] The analysis for the case in which the government offers only market information is simpler because no upfront investment is involved. However, for the case in which the government offers only agricultural advice, we need to model the upfront investment associated with the adoption of the agricultural advice *endogenously*.

Our model is intended to examine the following research questions:

1. Should all farmers use market information to plan their production in equilibrium?

2. Does market information improve farmers' welfare?

3. Should all farmers adopt agricultural advice to improve their operations in equilibrium when upfront investment is involved?

4. Does agricultural advice improve farmers' welfare?

To analyze the implications of providing market information and/or agricultural advice, we first establish a unified approach that combines market information and agricultural advice

[3]We thank the Department Editor for suggesting we interpret our model in the context of quantity competition between two cooperatives.

[4]The analysis associated with the case in which the government offers both agricultural and market information is intractable because it involves 16 subgames.

without considering the upfront investment. This unified approach enables us to investigate the interactions among various types of operational improvement induced by the two types of information. Specifically, we find that the effect of yield improvement is complementary to quality improvement and market demand forecast accuracy. In addition, when the process yield is highly uncertain, yield improvement and cost reduction are always complementary. However, when the uncertainty of the yield is relatively low, yield improvement and cost reduction are complementary if and only if the unit cost is large. Finally, we show that information accuracy improvement has no effect on quality improvement and cost reduction. By examining the equilibrium outcomes associated with the unified approach, we are able to separately investigate whether farmers should utilize market information and whether farmers should adopt agricultural advice. Our equilibrium analysis enables us to obtain the following results. First, we show that the provision of market information always improves the farmers' total welfare (i.e., the sum of the profits of all farmers) and that farmers should use market information to improve their production planning in equilibrium. However, as all farmers use market information to plan their production, we find that market information can maximize farmers' welfare when the underlying process yield is highly uncertain or when the products are highly heterogeneous. Second, in equilibrium, whether a farmer should adopt the agricultural advice is dependent on the size of the requisite upfront investment. More importantly, we show that agricultural advice is not always welfare improving, unless the upfront investment is sufficiently low. This result implies that to improve farmers' welfare, governments should consider offering farmer subsidies.

As such, our paper makes two major contributions to the existing literature on socially responsible operations. First, we examine the value of market information when farmers engage in quantity competition under both uncertain market demand and uncertain process yield. Second, by endogenizing the investment decision, we examine the value of agricultural advice.

The rest of this paper is organized as follows. In Section 2, we review the relevant literature. Section 3 presents a unified framework that enables us to analyze two separate settings (market information and agricultural advice) by analyzing a single model. We also analyze the underlying subgames and the meta-game in this section. In Section 4, we consider the case in which the government offers only market information and analyze the behavior of each farmer in equilibrium. Section 5 deals with the case in which the government offers only agricultural advice and each farmer needs to pay an upfront investment if he chooses to adopt the advice. Concluding remarks are provided in Section 6 and all proofs are relegated to the Appendix.

2 Literature review

There is limited modeling literature on making socially responsible operations because this topic is an emerging research area in operations management. Accordingly, most of the relevant articles are recent. For example, Chen et al. (2013a) examine the ITC e-Choupal network and discuss how it substantially changes the information and material flows. They show that the implicit agreement between the contracted farmers and the ITC behaves like a formal contract and it is in the ITC's best interest to provide all of the farmers with its services. Chen et al. (2013b) further study the peer-to-peer information sharing in Avaaj Otalo. They articulate why and when farmers have incentives to share demand and price information with other competing farmers, and why those who provide answers to others may be criticized and under-appreciated. Specifically, they show that the responses of the knowledgeable farmers are always less informative than those of the experts. (See Sodhi and Tang (2012) for a comprehensive survey.) Chen and Tang (2013) examine the value of public and private information for the economic development of agricultural business. They show that farmers are more responsive to the public/private signal when the public/private signal is more accurate. Therefore, when the public signal becomes more accurate, the effect of the private signal on the farmers' welfare decreases. However, all of the aforementioned studies assume that farmers utilize the information they receive. In contrast, we provide the option for each farmer to decide whether to utilize the market information. Because upfront investment is involved, we allow each farmer to decide whether to adopt certain agricultural advice.

In addition to examining the adoption of agricultural advice, we examine the utilization of market information under competition. Therefore, our paper is also related to the literature on information sharing in oligopolies. This line of research mainly examines whether firms have incentives to share their private information with competitors. The private information may concern either uncertain common value such as demand intercepts or uncertain private value such as costs. For example, Gal-Or (1985 and 1986) examine whether competing firms should share common demand intercept or production cost information with each other. Her results suggest that the incentive for information sharing crucially depends on the content of information (demand versus cost) and the nature of competition (quantity versus price). Raith (1996) provides a comprehensive survey of this research stream. Vives (1988) studies a large market in which firms engage in Cournot (quantity) competition and have access to private signals about the uncertain market demand. He shows that information is aggregated inefficiently and there is welfare loss even if the market is asymptotically competitive. The same informational setting is adopted by Li (2002). Using a two-tier supply chain relationship, he examines whether a downstream retailer has an incentive to share information with the upstream supplier, because the supplier may pass this information to other competing retailers. Armantier and Richard (2003) empirically show that sharing cost information in the airline industry can benefit the airlines without hurting the consumers. Zhu (2004) explores a B2B exchange that provides an online platform for information transmission. He shows that whether a firm should join the B2B exchange depends on the cost heterogeneity, product differentiation, and the degree of uncertainty. Hueth and Marcoul (2006) consider the information sharing among agricultural intermediaries. They show that even if information sharing can increase the profit of each firm, firms will conceal information in equilibrium. Jansen (2010) studies the information sharing in R&D competition. He shows that the incentive to disclose information depends on whether the winner firm of an R&D race is capable of appropriating the full revenue of its innovation. In this paper, we explore an entirely different context. First, we consider both the uncertain common value (e.g., market potential) and the uncertain private value (e.g., the process yield). Second, although some of our results can shed light on the farmers' incentive for information sharing, we do not explicitly examine this issue. Instead, we consider the case in which the government offers market information and agricultural advice to farmers. More importantly, we focus on the issue of whether each farmer should utilize market information and whether each farmer should adopt agricultural advice when both market demand and process yield are uncertain.

3 The model

Consider two farmers who produce and sell the same crop in a common market.[5] Each farmer i, $i = 1, 2$, incurs a production cost cq_i, where c is the unit production cost and q_i is the production quantity (a decision variable). In the base case, when farmer i processes q_i units, farmer i's actual output is $z_i q_i$, where z_i is the uncertain process yield such that $E(z_i) = \mu_y$, $\mu_y \leq 1$, and $Var(z_i) = \sigma_y^2$. We assume that z_1 and z_2 are independent random variables. For notational convenience, we define S^2 and C_y as the second moment and coefficient of variation (CV) of z_i, respectively, so that $S^2 = E(z_i^2) = \sigma_y^2 + \mu_y^2$ and $C_y = \sigma_y/\mu_y$.

To capture the quantity competition under uncertain market demand, we assume that the uncertain market price

$$P = M - (z_1 q_1 + z_2 q_2),$$

where $M > 0$ corresponds to the uncertain market potential.[6] We also assume that M is independent of z_i, $i = 1, 2$, and normally distributed with a mean of μ_m and a variance of σ_m^2, that is, $M \sim N(\mu_m, \sigma_m^2)$. Consider the case in which neither product is available in the market (i.e., $q_1 = q_2 = 0$) so that the market price equals M. In this case, if we let one farmer i produce an infinitesimal amount (and the other farmer produces nothing), then the gross margin of farmer i is $E(Mz_i - c) = \mu_m \mu_y - c$. For notational convenience, hereafter we denote $g = \mu_m \mu_y - c$, which represents the expected gross margin when only one farmer exists in the market (i.e., without considering quantity competition).

We now model the implications of the aforementioned information that is intended to help farmers make better production planning decisions and improve their operations.

Market information. The government offers information I that would enable farmers to improve the accuracy of their forecast of the market potential M. To facilitate our analysis, we assume that (M, I) are bivariate normally distributed so that

$$(M, I) \sim N(\mu_m, \mu_I, \sigma_m^2, \sigma_I^2, \rho),$$

where ρ is the correlation coefficient between the market potential and market information and $\rho \in (-1, 1)$. We also assume that both M and I are independent of z_i, $i = 1, 2$. Each

[5] Our analysis can be extended to the case in which there are $n > 2$ farmers.

[6] For ease of exposition, we set the price elasticity to 1. However, our model can be extended to the case of $P = M - b(z_1 q_1 + z_2 q_2)$, where $b > 0$.

farmer can use information I to "update" her forecast on M. By considering the conditional expectation and conditional variance, we get

$$E(M|I) = \mu_m + \rho\frac{\sigma_m}{\sigma_I}(I - \mu_I) \ \text{ and } Var(M|I) = \sigma_m^2(1-\rho^2).$$

By noting that $Var(M|I) \leq Var(M)$, we can conclude that each farmer can use information I to obtain a more accurate forecast of the market potential.

Agricultural Advice. If a farmer adopts this agricultural advice by making an upfront investment K, she can enjoy three benefits that are described as follows.

1. **Cost reduction.** Each farmer reduces her unit production cost from c to βc, where $\beta \leq 1$.

2. **Quality improvement.** Each farmer improves her product quality so that the average market potential is increased from μ_m to $\alpha\mu_m$, where $\alpha \geq 1$. Therefore, the improved market potential, denoted by αM, follows the normal distribution with mean $\alpha\mu_m$ and variance σ_m^2, that is, $\alpha M \sim N(\alpha\mu_m, \sigma_m^2)$.

3. **Process yield improvement.** The process yield of farmer i, $i = 1, 2$ is increased from z_i to $z_i^{'}$, where $E(z_i^{'}) = \gamma\mu_y \geq E(z_i)$ $(\gamma \geq 1)$ and $Var(z_i^{'}) = \sigma_y^2$.[7] To ensure that the improved yield is bounded by 1, $\gamma\mu_y \leq 1$ is required. Similar to S^2 and g associated with the regular yield z_i as defined earlier, we let $S^{'2} = \sigma_y^2 + \gamma^2\mu_y^2$ and $g^{'} = \alpha\gamma\mu_m\mu_y - \beta c$.[8]

Table 1 summarizes the notations used in this paper. According to our model description, market information and agricultural advice affect farmers in two different ways. First, when utilizing market information I, each farmer i can use the updated market uncertainty $(M|I)$ to determine her production quantity q_i. Second, when adopting agricultural advice, each farmer incurs an upfront investment K. However, when deciding on her production quantity q_i, farmer i enjoys three benefits associated with quality improvement via α, cost reduction via β, and process yield improvement via γ. Although market information and agricultural advice affect farmers in different ways, we now present a unified approach so that we can use one generic model to analyze their implications. Specifically, we introduce our unified approach and analyze the equilibrium outcomes of our generic model in the next section. We then use the equilibrium outcomes of our generic model to separately examine the implications of the provision of market information and agricultural advice in the subsequent sections.

[7] For ease of exposition, we assume that the quality and yield improvements affect only the mean value of z_i but not the variance. However, the structure of the results remains the same when we relax this assumption.

[8] Because $\alpha \geq 1$, $\gamma \geq 1$ and $\beta \leq 1$, we have $S^{'} \geq S$ and $g^{'} \geq g$. Also, to ensure that both $E(M|I)$ and the equilibrium production quantity in our analysis are non-negative, we assume that after adopting the advice, the "improved" gross margin $g^{'}$ is large enough so that $g^{'} + 2\rho\frac{\sigma_m}{\sigma_I}(I - \mu_I) > 0$.

Table 1: List of Notations

Notation	Definition
M	market potential (a random variable)
(μ_m, σ_m^2)	mean and variance of market potential
I	market information (a random variable)
(μ_I, σ_I^2)	mean and variance of market information
$\rho = Corr(M, I)$	correlation coefficient between market information and market potential
c	regular unit production cost
K	upfront investment cost for adopting agricultural advice
α	quality improvement parameter, $\alpha \geq 1$
β	cost reduction parameter, $0 < \beta \leq 1$
γ	yield level improvement parameter, $\gamma \geq 1$
q_i	production quantity of farmer i , $i = 1, 2$ (a decision variable)
z_i	regular process yield of farmer i, $i = 1, 2$ (a random variable)
$(\mu_y, \sigma_y^2, S^2, C_y)$	mean, variance, second moment and coefficient of variation (CV) of regular process yield z_i, $i = 1, 2$, where $S^2 = \sigma_y^2 + \mu_y^2$, $C_y = \sigma_y/\mu_y$
$z_i^{'}$	improved process yield of farmer i, $i = 1, 2$
$S^{'2} = \sigma_y^2 + \gamma^2\mu_y^2$	second moment of $z_i^{'}$, $i = 1, 2$
$g = \mu_m\mu_y - c$	gross margin "before" adopting agricultural advice (without considering quantity competition)
$g^{'} = \alpha\gamma\mu_m\mu_y - \beta c$	gross margin "after" adopting agricultural advice (without considering quantity competition)

4 A Unified Approach

In this section, we introduce a unified approach that combines market information and agricultural advice. Recall that the adoption of agricultural advice involves the upfront investment cost K while the adoption of market information does not. If we suppress the upfront investment K that is associated with the adoption of agricultural advice, we can characterize the meta-game between the two farmers as a two-person game in which each player has to decide whether to utilize market information (respectively, whether to adopt agricultural advice). For tractability, we shall assume that under our unified approach, each farmer either utilizes both market information and agricultural advice (denoted by Y), or utilizes nothing at all (denoted as N).[9] Consequently, there are four pairs of strategies: $(N, N), (Y, N), (N, Y)$, and (Y, Y). For each of these four pairs of strategies, there is a corresponding subgame in which both farmers engage in Cournot (quantity) competition. For each subgame, we need to determine the production quantity and the expected payoff of each farmer in equilibrium. We use superscript to denote the equilibrium outcome of each

[9] If both market information and agricultural advice are available, each farmer has four options to choose from; i.e., whether to utilize market information or not and whether to adopt agricultural advice or not. Under this setting, these two farmers engage in $4 \times 4 = 16$ corresponding subgames. The analysis of these 16 subgames and the comparisons among equilibrium outcomes of these 16 subgames would become very tedious.

subgame. For example, for subgame (Y,N), let q_i^{YN} be the production quantity and π_i^{YN} be the expected payoff of each farmer i in equilibrium, $i = 1,2$.

Our unified approach for analyzing the implications of market information and agricultural advice can be described as follows. First, we solve all four subgames by determining the expected payoff of each farmer assuming that both market information and agricultural advice are available. Second, to analyze the implications of market information, we first determine the expected payoff of each farmer for the case in which only market information is available by setting $\alpha = \beta = \gamma = 1$. To determine whether each farmer "utilizes" market information in equilibrium, we solve the 2x2 meta-game by using the expected payoffs determined in those four subgames, as shown in Table 2.

Table 2: Farmers' Expected Payoffs

Farmer 1 \ Farmer 2	N (do not utilize/adopt)	Y (utilize /adopt)
N (do no utilize/adopt)	π_1^{NN}, π_2^{NN}	π_1^{NY}, π_2^{NY}
Y (utilize/adopt)	π_1^{YN}, π_2^{YN}	π_1^{YY}, π_2^{YY}

Next, to analyze the implications of agricultural advice, we first determine the expected payoff of each farmer for the case in which only agricultural advice is available by setting $\rho = 0$. To determine whether each farmer "adopts" agricultural advice in equilibrium, we solve the meta-game by using the expected payoffs by accounting for the upfront investment K as determined in the four subgames, as shown in Table 2.

4.1 Analysis of Subgames

Using the above mentioned unified approach, we now proceed to analyze the four subgames that correspond to those 4 pairs of strategies: $(N,N), (Y,N), (N,Y)$, and (Y,Y). Due to symmetry, the analysis associated with subgame (Y,N) is identical to that of subgame (N,Y). Therefore, it suffices to analyze subgames $(N,N), (Y,Y)$, and (Y,N). After determining the equilibrium outcomes of all four subgames, we can then solve the meta-game as depicted in Table 2.

Under our unified approach, subgame (Y,Y) is the most complex because it involves both the utilization of market information and the adoption of agricultural advice. For any revealed market information I, the expected profit of farmer i needs to take into account all three benefits associated with the adoption of agricultural advice; namely, the effective market price becomes $P' = \alpha M - (z_i' q_i + z_{3-i}' q_{3-i})$, the effective output becomes $z_i' q_i$, and the effective production cost becomes $\beta c q_i$. Therefore, the expected profit of farmer i under strategy (Y,Y) can be expressed as

$$\begin{aligned} \pi_i(q_i|I) &= E\{(\alpha M - z_i' q_i - z_j' q_j) z_i' q_i - \beta c q_i | I\} \\ &= g' q_i + \gamma \rho \mu_y \frac{\sigma_m}{\sigma_I}(I - \mu_I) q_i - S'^2 q_i^2 - \gamma^2 \mu_y^2 q_i q_j | I, \text{ for } i,j = 1,2, j \neq i. \end{aligned} \quad (1)$$

Because the other strategies are special cases of strategy (Y, Y), we can use the above expression to determine the farmer's profit under the other strategies. Given the expected profit as stated in (1), we are now ready to solve all of the subgames.

4.1.1 Subgame (N, N)

The case in which neither farmer utilizes market information (nor adopts agriculture advice) under strategy (N, N) corresponds to the case that $\alpha = \beta = \gamma = 1$ and $\rho = 0$ so that $g' = g$ and $S^{'2} = S^2$. By utilizing (1), the expected profit of farmer i can be written as

$$\pi_i(q_i) \quad = \quad gq_i - S^2 q_i^2 - \mu_y^2 q_i q_j, \quad i = 1, 2, \ j \neq i.$$

By noting that $\pi_i(q_i)$ is concave, we obtain farmer i's best response function as follows:

$$q_i(q_j) = \frac{g - \mu_y^2 q_j}{2S^2}, \quad i = 1, 2, \ j \neq i.$$

Solving the best response functions of both farmers, we obtain the following proposition.

Proposition 1. *The equilibrium outcomes associated with strategy* $(N,\ N)$ *satisfy*

$$q_i^{NN} \quad = \quad \frac{g}{2S^2 + \mu_y^2}, \quad i = 1, 2, \tag{2}$$

$$\pi_i^{NN} \quad = \quad \frac{S^2 g^2}{(2S^2 + \mu_y^2)^2}, \quad i = 1, 2, \tag{3}$$

where $g = \mu_m \mu_y - c$ *and* $S^2 = \sigma_y^2 + \mu_y^2$. *Both the expected profit* π_i^{NN} *and production quantity* q_i^{NN} *are increasing in* μ_m *and decreasing in* σ_y. *Moreover,* π_i^{NN} *and the expected output* $\mu_y q_i^{NN}$ *are both increasing in* μ_y.

Proposition 1 implies that without utilizing market information (or without adopting agricultural advice), a higher market potential μ_m or a lower yield uncertainty (via higher μ_y or lower σ_y) can enable both farmers to produce more and earn more. However, the production quantity q_i^{NN} is not necessarily increasing in μ_y. On the one hand, a higher expected yield μ_y can certainly enable each farmer to generate more output with the same input. On the other hand, to avoid over supply that drives down the market price, a higher expected yield μ_y can also cause each farmer to produce less in equilibrium. Although the production quantity q_i^{NN} is not monotone in μ_y for farmer i, the expected output $\mu_y q_i^{NN}$ is increasing in μ_y. Therefore, a higher expected yield will always lead to a larger expected output even when a farmer produces less.[10]

[10] We thank the senior editor for pointing out the monotonicity of the expected output with respect to the expected yield.

4.1.2 Subgame (Y, Y)

When both farmers utilize market information (or adopt agriculture advice) under strategy (Y, Y), the expected profit of each farmer for any given information I is given in (1). In this case, it is easy to check that the best response function of farmer i is

$$q_i(q_j)|I = \frac{g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2}} - \frac{\gamma^2\mu_y^2 q_j}{2S^{'2}}, \quad i = 1, 2, \ j \neq i.$$

Recall that $g^{'} = \alpha\gamma\mu_m\mu_y - \beta c$ and $S^{'2} = \sigma_y^2 + \gamma^2\mu_y^2$. By considering the best response functions of both farmers, we have the following result.

Lemma 1. *For any given market information I, the ex post equilibrium outcomes associated with strategy (Y, Y) satisfy*

$$\begin{aligned} q_i^{YY}|I &= \frac{g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)}{\sigma_I(2S^{'2} + \gamma^2\mu_y^2)}, \ \textit{for } i = 1, 2, \\ \pi_i^{YY}|I &= \frac{S^{'2}[g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)]^2}{\sigma_I^2(2S^{'2} + \gamma^2\mu_y^2)^2}, \ \textit{for } i = 1, 2. \end{aligned}$$

Based on Lemma 1, we can obtain the ex ante equilibrium outcomes as follows.

Proposition 2. *The ex ante equilibrium outcomes associated with strategy (Y, Y) satisfy*

$$q_i^{YY} = \frac{g^{'}}{2S^{'2} + \gamma^2\mu_y^2}, \ \textit{for } i = 1, 2, \tag{4}$$

$$\pi_i^{YY} = \frac{S^{'2}(g^{'2} + \gamma^2\rho^2\mu_y^2\sigma_m^2)}{(2S^{'2} + \gamma^2\mu_y^2)^2}, \ \textit{for } i = 1, 2. \tag{5}$$

Both q_i^{YY} and π_i^{YY} are increasing in α and decreasing in β. Furthermore, π_i^{YY} is increasing in ρ^2, γ and σ_m.

Proposition 2 reveals that when farmers utilize market information (and adopt agricultural advice), they can earn more when the market information becomes more informative (i.e., as ρ^2 increases) or when the agricultural advice becomes more beneficial (i.e., as α and γ increase, or as β decreases). This result is intuitive.

Recall that $E(M|I) = \mu_m + \rho\frac{\sigma_m}{\sigma_I}(I - \mu_I)$ and $Var(M|I) = \sigma_m^2(1-\rho^2)$. Then, we can use $E_I E(M|I) = \mu_m$ and (4) to show that the ex ante production quantity is independent of market information via ρ. Next, observe that market information I enables each farmer to reduce the variance of market potential M from $Var(M)$ to $Var(M|I)$, where $Var(M|I) = \sigma_m^2(1-\rho^2) = Var(M) - \rho^2\sigma_m^2$. By noting that the term $\rho^2\sigma_m^2$ represents the reduction of variance when a farmer utilizes market information, it is intuitive to see that each farmer's expected profit increases in relation to the amount of variance reduction $\rho^2\sigma_m^2$. Finally, similar to Lemma 1, when the expected yield μ_y increases, each farmer may reduce the production quantity q_i^{YY} in equilibrium to avoid over supply that drives down market price.

4.1.3 Subgame (Y, N)

Under strategy (Y, N), farmer 1 is the only farmer who utilizes market information (and adopts agricultural advice). Thus, farmer 1's market potential becomes αM while farmer 2' market potential remains M. By noting that farmer 2's process yield is z_2, we can derive the expected profit of farmer 1 by replacing $z_2^{'}$ with z_2 in (1), getting

$$\begin{aligned}\pi_1(q_1) &= E\{(\alpha M - (z_1^{'}q_1 + z_2q_2))z_1^{'}q_1 - \beta c q_1|I\} \\ &= g^{'}q_1 + \gamma\rho\mu_y\frac{\sigma_m}{\sigma_I}(I-\mu_I)q_1 - S^{'2}q_1^2 - \gamma\mu_y^2 q_1 q_2|I. \qquad (6)\end{aligned}$$

However, although farmer 2 does not utilize market information or adopt agricultural advice, she knows that farmer 1 observes $I = \mu_I$ (in expectation). Therefore, farmer 2's expected profit is

$$\begin{aligned}\pi_2(q_2) &= E\{(M - (z_1^{'}q_1|I=\mu_I) - z_2q_2)z_2q_2 - cq_2\} \\ &= gq_2 - S^2q_2^2 - (\gamma\mu_y^2 q_1|I=\mu_I)q_2. \qquad (7)\end{aligned}$$

By deriving the first order conditions of (6) and (7), we can obtain the following best response functions:

$$q_1(q_2)|I = \frac{g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2}} - \frac{\gamma\mu_y^2 q_2}{2S^{'2}}. \qquad (8)$$

$$q_2(q_1) = \frac{g - \gamma\mu_y^2(q_1|I=\mu_I)}{2S^2}. \qquad (9)$$

Based on the above best response functions, we can derive the ex post equilibrium outcomes whose expressions hinge upon the value of C_y, the coefficient of variation of the process yield.

Lemma 2. *For any given market information I, the ex post equilibrium outcomes associated with strategy $(Y,\ N)$ satisfy*

$$q_1^{YN}|I = \begin{cases}\frac{2S^2g^{'}-\gamma\mu_y^2 g}{4S^2S^{'2}-\gamma^2\mu_y^4} + \frac{\gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2}}, & if\ C_y^2 > \frac{\gamma g^{'}}{2g} - \gamma^2, \\ \frac{g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2}}, & otherwise,\end{cases} \qquad (10)$$

$$q_2^{YN} = \begin{cases}\frac{2S^{'2}g-\gamma\mu_y^2 g^{'}}{4S^2S^{'2}-\gamma^2\mu_y^4}, & if\ C_y^2 > \frac{\gamma g^{'}}{2g} - \gamma^2, \\ 0, & otherwise,\end{cases} \qquad (11)$$

$$\pi_1^{YN}|I = \begin{cases}S^{'2}\left[\frac{2S^2g^{'}-\gamma\mu_y^2 g}{4S^2S^{'2}-\gamma^2\mu_y^4} + \frac{\gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2}}\right]^2, & if\ C_y^2 > \frac{\gamma g^{'}}{2g} - \gamma^2, \\ \frac{[g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)]^2}{4\sigma_I^2 S^{'2}}, & otherwise,\end{cases} \qquad (12)$$

$$\pi_2^{YN} = \begin{cases}\frac{S^2(2S^{'2}g-\gamma\mu_y^2 g^{'})^2}{(4S^2S^{'2}-\gamma^2\mu_y^4)^2}, & if\ C_y^2 > \frac{\gamma g^{'}}{2g} - \gamma^2, \\ 0, & otherwise.\end{cases} \qquad (13)$$

When farmer 1 is the only farmer who benefits from utilizing market information (via ρ) and from adopting agricultural advice (via α, β, and γ), Lemma 2 reveals that farmer 1 can afford to force farmer 2 to exit the market when the process yield uncertainty is sufficiently low; i.e., when $C_y^2 \leq \frac{\gamma g^{'}}{2g} - \gamma^2$.

By using Lemma 2, we get the following proposition.

Proposition 3. *The ex ante equilibrium outcomes of farmer 1 associated with strategy* (Y, N) *satisfy*

$$q_1^{YN} = \begin{cases} \frac{2S^2 g^{'} - \gamma\mu_y^2 g}{4S^2 S^{'2} - \gamma^2\mu_y^4}, & if\ C_y^2 > \frac{\gamma g^{'}}{2g} - \gamma^2, \\ \frac{g^{'}}{2S^{'2}}, & otherwise, \end{cases} \tag{14}$$

$$\pi_1^{YN} = \begin{cases} \frac{S^{'2}(2S^2 g^{'} - \gamma\mu_y^2 g)^2}{(4S^2 S^{'2} - \gamma^2\mu_y^4)^2} + \frac{\gamma^2\rho^2\mu_y^2\sigma_m^2}{4S^{'2}}, & if\ C_y^2 > \frac{\gamma g^{'}}{2g} - \gamma^2, \\ \frac{g^{'2} + \gamma^2\rho^2\mu_y^2\sigma_m^2}{4S^{'2}}, & otherwise. \end{cases} \tag{15}$$

The ex ante equilibrium outcomes of farmer 2 associated with strategy (Y, N) *are given in Lemma 2, as stated in* (11) *and* (13).

Proposition 3 has the same interpretation as Lemma 2: in equilibrium, farmer 1 can afford to force farmer 2 to exit the market when the process yield uncertainty is sufficiently low.

Below we compare the equilibrium outcomes of farmer 1 (who utilizes and adopts) and that of farmer 2 (who does not utilize or adopt).

Corollary 1. *When only farmer 1 chooses to invest, then* $q_1^{YN} \geq q_2^{YN}$ *if* $\gamma = 1$.

Corollary 1 reveals that farmer 1 will produce more than farmer 2 when the adoption of agricultural advice will not increase the process yield ($\gamma = 1$). However, it is not always true that farmer 1 will produce more. This is the case especially when farmer 1 is concerned about over supply that will drive down the market price. When $\gamma > 1$, farmer 1 can process less input than farmer 2 and yet generate a higher output than farmer 2. Therefore, farmer 1 may produce less.

Finally, by symmetry, we can use Proposition 3 to obtain the equilibrium outcomes associated with strategy (N, Y), where $q_1^{NY} = q_2^{YN}$, $q_2^{NY} = q_1^{YN}$, $\pi_1^{NY} = \pi_2^{YN}$, and $\pi_2^{NY} = \pi_1^{YN}$.

In summary, we have determined each farmer's expected profit in equilibrium associated with each subgame. By using these expected profits, we can specify the payoffs associated with the 2x2 meta-game depicted in Table 2. We now proceed to solve this meta-game to examine the conditions under which a farmer will utilize market information (or adopt agricultural advice).

4.2 Meta-game Analysis

By examining each farmer's expected profit in equilibrium associated with subgames (N, N), (Y, Y), (Y, N), and (N, Y) in Propositions 1, 2, and 3, we can establish the following lemma via direct comparison.

Lemma 3. *The farmer's expected profits associated with subgames (N, N), (Y, Y), (Y, N), and (N, Y) possess the following properties.*

1. $\pi_1^{YN}(=\pi_2^{NY}) \geq \pi_1^{YY}(=\pi_2^{YY}) \geq \pi_1^{NN}(=\pi_2^{NN}) \geq \pi_1^{NY}(=\pi_2^{YN})$.
2. $\pi_1^{YN} - \pi_1^{NN}(=\pi_2^{NY} - \pi_2^{NN}) \geq \pi_1^{YY} - \pi_1^{NY}(=\pi_2^{YY} - \pi_2^{YN})$.

By noting from Lemma 3 that farmer 1's expected profit satisfies $\pi_1^{YN} \geq \pi_1^{NN}$ and $\pi_1^{YY} \geq \pi_1^{NY}$ (and similarly for farmer 2), we can conclude that a farmer can always increase her expected profit by utilizing market information (or adopting agricultural advice) regardless of the strategy selected by the other farmer. Also, by noting that $\pi_1^{YN} - \pi_1^{NN} \geq \pi_1^{YY} - \pi_1^{NY}$ and $\pi_2^{NY} - \pi_2^{NN} \geq \pi_2^{YY} - \pi_2^{YN}$, we can conclude that by utilizing market information (or adopting agricultural advice), the increase in the expected profit of a farmer is higher when the other farmer chooses not to utilize market information (or not to adopt agricultural advice).

Lemma 3 reveals that without considering the upfront investment K associated with the adoption of agricultural advice, a farmer can always benefit from utilizing market information (or adopting agricultural advice). This observation enables us to compare the expected payoffs shown in Table 2 by using the inequalities established in Lemma 3. By doing so, we can solve the meta-game as follows.

Corollary 2. *Without considering the upfront investment K associated with the adoption of agricultural advice, strategy (Y, Y) is the unique equilibrium; i.e., both farmers utilize market information (or adopt agricultural advice) in equilibrium.*

Knowing that market information (or agricultural advice) is beneficial to each farmer when the farmers engage in Cournot competition under both demand and process yield uncertainty, Corollary 2 is a natural consequence, that is, both farmers will utilize market information (or adopt agricultural advice) in equilibrium.

4.3 Welfare Improvement

When both farmers utilize market information (or adopt agricultural advice) in equilibrium, the government (or NGO) can measure the farmers' welfare improvement due to the provision of market information (or agricultural advice) against the base case under strategy (N, N). In this case, we define the farmers' welfare improvement according to the term $(\pi_1^{YY} - \pi_1^{NN}) + (\pi_2^{YY} - \pi_2^{NN})$. Due to symmetry (i.e., $\pi_1^{YY} = \pi_2^{YY}$ and $\pi_1^{NN} = \pi_2^{NN}$), it is sufficient for us to focus our analysis on $(\pi_1^{YY} - \pi_1^{NN})$. Comparing the expressions for the farmers' expected profits stated in Propositions 1 and 2, we get the following corollary.

Corollary 3. *Without considering the upfront investment K associated with the adoption of agricultural advice, market information (or agricultural advice) is welfare improving:* $(\pi_i^{YY} - \pi_i^{NN}) > 0$ *for* $i = 1, 2$. *Also, the welfare improvement* $(\pi_i^{YY} - \pi_i^{NN})$ *is decreasing in* β, *and increasing in* α, γ, *and* ρ^2.

Corollary 3 reveals that both farmers benefit from utilizing market information (or adopting agricultural advice) in equilibrium. Hence, market information is welfare improving. Without considering the upfront investment K associated with the adoption of agricultural advice, agricultural advice is also welfare improving. (We shall examine the effect of the upfront investment K on the farmers' welfare in Section 5.) It is intuitive to see that the welfare improvement $(\pi_i^{YY} - \pi_i^{NN})$ increases as the benefits associated with market information (via ρ^2) and agricultural advice (via α, γ, and β) increase.

To further investigate the interaction effects among quality improvement, cost reduction, process yield improvement, and forecast accuracy improvement, we establish the following corollary.

Corollary 4. *Without considering the upfront investment K associated with the adoption of agricultural advice, the welfare improvement* $(\pi_i^{YY} - \pi_i^{NN})$, $i = 1, 2$, *possesses the following properties.*

1. *The welfare improvement* $(\pi_i^{YY} - \pi_i^{NN})$ *is supermodular in* (α, γ) , (γ, σ_m) *and* (γ, ρ^2); *i.e.,*
$$\frac{\partial^2(\pi_i^{YY} - \pi_i^{NN})}{\partial\alpha\partial\gamma} > 0, \ \frac{\partial^2(\pi_i^{YY} - \pi_i^{NN})}{\partial\gamma\partial\sigma_m} > 0 \ \textit{and} \ \frac{\partial^2(\pi_i^{YY} - \pi_i^{NN})}{\partial\gamma\partial\rho^2} > 0.$$

2. *There exist threshold points* $\bar{C}_y$ *and* $\bar{c}$ *such that when* $C_y/\gamma > \bar{C}_y$, *the welfare improvement* $(\pi_i^{YY} - \pi_i^{NN})$ *is always submodular in* (β, γ). *But when* $C_y/\gamma \leq \bar{C}_y$, *the welfare improvement is submodular in* (β, γ) *if and only if* $c > \bar{c}$.

3. *The welfare improvement* $(\pi_i^{YY} - \pi_i^{NN})$ *is modular in* (α, ρ^2) *and* (β, ρ^2); *i.e.,*
$$\frac{\partial^2(\pi_i^{YY} - \pi_i^{NN})}{\partial\alpha\partial\rho^2} = 0 \ \textit{and} \ \frac{\partial^2(\pi_i^{YY} - \pi_i^{NN})}{\partial\beta\partial\rho^2} = 0.$$

The first statement of Corollary 4 has the following implications. First, quality improvement (via α) and process yield improvement (via γ) are complementary, that is, they generate a "compounding effect" on the farmer's welfare. In this case, the process yield improvement results in a larger expected output, which intensifies the market competition, while the quality improvement leads to a larger market potential, thereby softening the market competition.

Second, process yield improvement (via γ) is more beneficial in terms of welfare improvement when market uncertainty is higher (i.e., when σ_m is larger) or when the market information is more accurate (i.e., when ρ^2 is larger). By noting that the term $\rho^2\sigma_m^2$ represents the reduction of variance when a farmer utilizes market information, this result implies that when the use of market information is more effective in improving the forecast accuracy (via $\rho^2\sigma_m^2$), the farmers have more incentives to improve the process yield.

Next, we examine the second statement of Corollary 4. Note that $C_y/\gamma \equiv \frac{\sigma_y}{\gamma\mu_y}$ represents the coefficient of variation of the "improved process yield" and that the unit cost reduces as β decreases. The second statement of Corollary 4 shows that cost reduction (via β) and process yield improvement (via γ) are complementary when: (1) the improved process yield is highly uncertain, or (2) the improved process yield is relatively stable but the unit cost c is sufficiently large. These results can be explained as follows. First, according to Proposition 2, the effect of yield improvement (via γ) on the production quantity q_i^{YY} is ambiguous. Positively, the yield improvement can enable each farmer to obtain the same output by reducing her production quantity. This indirect cost reduction causes both farmers to produce more. Negatively, the yield improvement may also intensify the market competition and drive down the market price, causing both farmers to produce less in equilibrium. The cost reduction (via β) is complementary to the positive effect but is substituted by the negative effect of yield improvement. Furthermore, from (4), we can easily show that $\partial^2 q_i^{YY}/\partial\gamma\partial c > 0$, which implies that when the unit cost is larger, q_i^{YY} is more likely to increase in γ. This also indicates that when the unit cost is larger, the positive effect tends to be stronger than the negative effect. Next, observing from the best response of both farmers under strategy (Y, Y), if one farmer produces one extra unit, the other's best response is to reduce her production quantity by $\gamma^2\mu_y^2/2S'^2$. This quantity change can measure the product substitution between the two farmers. We refer to this as a substitution factor. By noting that $\gamma^2\mu_y^2/2S'^2 = 1/2(C_y^2/\gamma^2 + 1)$ is decreasing in C_y/γ (the coefficient of variation of the improved process yield), we know that the more uncertain the process yield, the less fierce the market competition. Thus, when the process yield is highly uncertain (i.e., $C_y/\gamma > \bar{C}_y$) such that the market competition is relatively mild, the positive effect of yield improvement always dominates its negative effect. Therefore, the cost reduction and yield improvement are complementary. However, when the process yield is relatively stable, cost reduction and yield improvement are complementary if and only if the unit cost is large enough such that the positive effect of yield improvement can dominate its negative effect.

Finally, the third statement of Corollary 4 shows that the improvement in forecast accuracy has no effect on quality improvement or cost reduction. Therefore, the government prefers the combination of process yield improvement and demand forecast improvement over the combination of demand forecast improvement and cost reduction or quality improvement.

5 The Implications of Market Information and Agricultural Advice

Based on the unified approach and the corresponding analysis presented in Section 4, we now examine the implications of market information and agricultural advice separately.

5.1 Utilization of Market Information

In this section, we consider the case in which the government only provides market information I that is intended to help farmers improve their production planning. Without the benefits associated with agricultural advice, we have $\alpha = \beta = \gamma = 1$. In this case, $S' = S$, $g' = g$ and $C_y^2 > 0 > \frac{\gamma g'}{2g} - \gamma^2$. Based on Propositions 1, 2, 3, and Lemma 2, both farmers' expected profit under each subgame can be simplified as:

$$
\begin{aligned}
\pi_1^{NN} &= \pi_2^{NN} = \frac{S^2 g^2}{(2S^2+\mu_y^2)^2}, \\
\pi_1^{YY} &= \pi_2^{YY} = \frac{S^2(g^2+\rho^2\mu_y^2\sigma_m^2)}{(2S^2+\mu_y^2)^2}, \\
\pi_1^{YN} &= \pi_2^{NY} = \frac{S^2 g^2}{(2S^2+\mu_y^2)^2} + \frac{\rho^2\mu_y^2\sigma_m^2}{4S^2}, \qquad (16) \\
\pi_1^{NY} &= \pi_2^{YN} = \frac{S^2 g^2}{(2S^2+\mu_y^2)^2}. \qquad (17)
\end{aligned}
$$

By direct comparison, we establish the following result.

Corollary 5. *When the government provides market information I only,*

1. *The farmer's expected profits satisfy $\pi_1^{YN}(=\pi_2^{NY}) > \pi_1^{YY}(=\pi_2^{YY}) > \pi_1^{NN} = \pi_1^{NY}(=\pi_2^{NN} = \pi_2^{YN})$. Furthermore, $\pi_1^{YN} - \pi_1^{NN}(=\pi_2^{NY} - \pi_2^{NN}) > \pi_1^{YY} - \pi_1^{NY}(=\pi_2^{YY} - \pi_2^{YN})$.*
2. *Both farmers use the market information in equilibrium to make better production planning decisions.*

Because "market information only" is a special case of the unified model, the above corollary is a sharper statement of Lemma 3 and Corollary 2. Next, the inequality $\pi_1^{YN}(=\pi_2^{NY}) > \pi_1^{YY}(=\pi_2^{YY})$ in Corollary 5 implies that the farmer with the market information has no incentives to share the market information with his competitors, which is consistent with the results obtained by Gal-Or (1985, 1986). As stated in Gal-Or (1985, 1986), firms gain from sharing private value (e.g., unit production cost) but lose from sharing common value (e.g., common demand). However, Gal-Or (1985, 1986) addresses the above issue from the viewpoint of profit maximization. In our context, the market potential is a common value to both farmers and we are interested in examining whether the government has incentives to distribute the market information in terms of welfare maximization. That is, is the market information welfare maximizing in the sense that the maximum total welfare of the farmers is attained when both farmers utilize the market information in equilibrium? Recall from Corollary 3 that market information is welfare improving: $(\pi_i^{YY} - \pi_i^{NN}) > 0$ for $i = 1, 2$. However, Corollary 5 reveals that strategy (Y, Y) is the equilibrium but $\pi_1^{YN} > \pi_1^{YY}$ and $\pi_2^{YN} < \pi_2^{YY}$. Therefore, it remains unclear whether $(\pi_1^{YY} + \pi_2^{YY})$ dominates the farmers' total welfare associated with all other strategies. We now examine this question.

We first observe from Corollary 3 that $(\pi_1^{YY}+\pi_2^{YY}) > (\pi_1^{NN}+\pi_2^{NN})$. Due to the symmetry between strategies (Y, N) and (N, Y), we can conclude that market information is welfare maximizing if $(\pi_1^{YY}+\pi_2^{YY})$ is greater than $(\pi_1^{YN}+\pi_2^{YN})$. Given that $g = \mu_m \mu_y - c$ and $S^2 = \sigma_y^2 + \mu_y^2$, we can establish a simple condition under which market information is welfare maximizing: the maximum total welfare of the farmers is attained when both farmers use the market information in equilibrium under strategy (Y, Y).

Proposition 4. *When the government provides market information I only, the provision of this market information is welfare maximizing if and only if the coefficient of variation of the process yield $\frac{\sigma_y}{\mu_y} \equiv C_y > \sqrt{\frac{\sqrt{2}-1}{2}}$.*

Proposition 4 reveals that the market information is welfare maximizing when the regular process yield is highly uncertain. Therefore, if the uncertainty of the process yield is relatively high, even when the farmer with the market information prefers concealing this information, the government has incentives to distribute market information to both farmers to maximize the farmers' welfare. However, when the uncertainty of the process yield is relatively low, neither the government nor the farmer with the market information wants to reveal this information. A close look of the best responses under the four subgames shows that when the government only provides market information, the substitution factor between the two farmers' products is $\mu_y^2/2S^2$, which can be rewritten as $1/2(C_y^2+1)$. This implies that when the regular process yield is highly uncertain, the quantity competition is somewhat mild. In view of this, Proposition 4 actually reveals that the provision of market information is welfare maximizing if and only if the market competition is relatively mild.

Proposition 4 specifies the condition under which the government's provision of market information is welfare maximizing. However, one may wonder whether this result will hold when the products are heterogeneous (measured in terms of substitutability level). To examine this issue, we consider the following inverse demand function to capture product heterogeneity. Specifically, the price of farmer i's product P_i satisfies:

$$P_i = M - (z_i q_i + t z_j q_j),\ i, j = 1, 2, i \neq j,$$

where t measures the level of substitutability between products. Without loss of generality, we assume that $t \in [0,\ 1]$ so that a low (high) value of t corresponds to the case in which the products are less (more) substitutable. (Note that the (homogeneous) products are perfect substitutes when $t = 1$.)

Corollary 6. *Suppose the farmers' products are heterogenous so that the market price becomes $P_i = M - (z_i q_i + t z_j q_j)$. Then:*

1. *The provision of market information is always welfare improving.*
2. *When $t < 2(\sqrt{2}-1)$, the provision of market information is always welfare maximizing.*
3. *When $t \geq 2(\sqrt{2}-1)$, the provision of market information is welfare maximizing if and only if $C_y > \sqrt{\frac{(\sqrt{2}+1)t-2}{2}}$.*

Analogous to Proposition 4 associated with the homogenous product case, the first statement of Corollary 6 reveals that when products are heterogeneous, the provision of market information is still welfare improving. In regard to welfare maximization, the second and third statements of Corollary 6 generalize the result stated in Proposition 4 that corresponds to the case when $t = 1$. When t decreases, the products become less substitutable, competition becomes less fierce, and the strategies chosen by the two farmers are less correlated. Therefore, when the level of substitutability is sufficiently low (i.e., when $t < 2(\sqrt{2} - 1)$), one farmer's strategy has little effect on the profit of the other farmer. Consequently, the provision of market information can be welfare-maximizing. Nevertheless, when the product substitutability is relatively high (i.e., $t \geq 2(\sqrt{2} - 1)$), competition becomes more fierce, and the farmers' strategies are more correlated. In this case, we obtain a similar result to that stated in Proposition 4: market information can maximize the total welfare of the farmers if and only if the process yield is highly uncertain, in which case the market competition is relatively soft.

In summary, when the government provides market information I only, we find that both farmers use the market information under strategy (Y, Y) in equilibrium. Also, we show that the market information is certainly welfare improving. However, the market information is welfare maximizing in the sense that the maximum total welfare of farmers is attained under strategy (Y, Y) if and only if the process yield is highly uncertain. In addition, we generalize our model by considering product heterogeneity. We show that if the level of product substitutability is low, the provision of market information is always welfare maximizing. However, when the level of product substitutability is high, the provision of market information is welfare maximizing if and only if the process yield is highly uncertain.

5.2 Adoption of Agricultural Advice

In this section, we examine the case in which the government only offers agricultural advice that is intended to help farmers to improve quality, reduce cost, and improve process yield. Without the benefits associated with market information I, we have $\rho = 0$. Without considering the upfront investment K associated with the adoption of agricultural advice, we can use Propositions 1, 2, 3, and Lemma 2 to show that both farmers' expected profit under each strategy can be simplified as:

$$
\begin{aligned}
\pi_1^{NN} &= \pi_2^{NN} = \frac{S^2 g^2}{(2S^2+\mu_y^2)^2},\\
\pi_1^{YY} &= \pi_2^{YY} = \frac{S'^2 g'^2}{(2S'^2+\gamma^2\mu_y^2)^2},\\
\pi_1^{YN} &= \pi_2^{NY} = \begin{cases} \frac{S'^2(2S^2g'-\gamma\mu_y^2 g)^2}{(4S^2S'^2-\gamma^2\mu_y^4)^2}, & if\ C_y^2 > \frac{\gamma g'}{2g}-\gamma^2,\\ \frac{g'^2}{4S'^2}, & otherwise.\end{cases}\\
\pi_1^{NY} &= \pi_2^{YN} = \begin{cases} \frac{S^2(2S'^2 g-\gamma\mu_y^2 g')^2}{(4S^2S'^2-\gamma^2\mu_y^4)^2}, & if\ C_y^2 > \frac{\gamma g'}{2g}-\gamma^2,\\ 0, & otherwise.\end{cases}
\end{aligned}
$$

By direct comparison and applying Lemma 3, we obtain the following proposition.

Proposition 5. *When the government offers agricultural advice only,*

1. *Without considering the upfront investment K, the farmers' expected profits satisfy $\pi_1^{YN}(=\pi_2^{NY}) > \pi_1^{YY}(=\pi_2^{YY}) > \pi_1^{NN}(=\pi_2^{NN}) > \pi_1^{NY}(=\pi_2^{YN})$. Furthermore, $\pi_1^{YN}-\pi_1^{NN}(=\pi_2^{NY}-\pi_2^{NN}) > \pi_1^{YY}-\pi_1^{NY}(=\pi_2^{YY}-\pi_2^{YN})$.*
2. *By incorporating the upfront investment K, the equilibrium strategy of the meta-game as depicted in Table 2 can be characterized as follows.*[11]

$$
Equilibrium = \begin{cases} (Y,Y), & if\ K < \pi_1^{YY}-\pi_1^{NY},\\ (Y,N)\ and\ (N,Y), & if\ \pi_1^{YY}-\pi_1^{NY} \le K \le \pi_1^{YN}-\pi_1^{NN},\\ (N,N), & if\ K > \pi_1^{YN}-\pi_1^{NN}.\end{cases}
$$

Proposition 5 shows that the upfront investment K has a direct effect on whether a farmer adopts agricultural advice in equilibrium. Specifically, both farmers adopt agricultural advice when the upfront investment K is low (i.e., when $K < \pi_1^{YY}-\pi_1^{NY}$), and no farmer adopts agricultural advice when the upfront investment K is high (i.e., when $K > \pi_1^{YN}-\pi_1^{NN}$). When K is in a moderate range (i.e., when $\pi_1^{YY}-\pi_1^{NY} \le K \le \pi_1^{YN}-\pi_1^{NN}$), one farmer will adopt agricultural advice by making the upfront investment K.[12]

Without considering the upfront investment cost K, Corollaries 3 and 4 reveal that agricultural advice is welfare improving when both farmers adopt agricultural advice in equilibrium under strategy (Y,Y). However, when incorporating the upfront investment K,

[11] Due to symmetry, it suffices to state only the conditions that are based on the expected profits of farmer 1.

[12] Here, the game becomes a coordination game and has two pure Nash equilibria and one mixed Nash equilibrium (Fudenberg and Tirole 1991). In the mixed Nash equilibrium, farmer i, $i=1,2$, chooses to invest with probability $\frac{\pi_1^{YN}-\pi_1^{NN}-K}{\pi_1^{NY}-\pi_1^{YY}+\pi_1^{YN}-\pi_1^{NN}}$.

Proposition 5 reveals that the equilibrium strategy hinges upon K. To examine whether agricultural advice is welfare improving when accounting for the upfront investment K, we consider the following scenarios.

First, consider the case in which K is high (i.e., when $K > \pi_1^{YN} - \pi_1^{NN}$), agricultural advice does not improve the farmers' welfare because no farmer will adopt the advice.

Second, when K is moderate (i.e., when $\pi_1^{YY} - \pi_1^{NY} < K < \pi_1^{YN} - \pi_1^{NN}$), we have two pure equilibria (Y, N) and (N, Y). Due to symmetry, it suffices to examine whether agricultural advice is welfare improving when only farmer 1 adopts the advice in equilibrium under strategy (Y, N). In other words, will $(\pi_1^{YN} - K) + \pi_2^{YN} > \pi_1^{NN} + \pi_2^{NN}$ when K satisfies $\pi_1^{YY} - \pi_1^{NY} < K < \pi_1^{YN} - \pi_1^{NN}$? Note that $\pi_2^{YN} = \pi_1^{NY}$ and $\pi_2^{NN} = \pi_1^{NN}$. The condition $(\pi_1^{YN} - K) + \pi_2^{YN} > \pi_1^{NN} + \pi_2^{NN}$ can be simplified as $\pi_1^{YN} + \pi_1^{NY} - 2\pi_1^{NN} > K$. Combining this simplified condition and the range within which K lies (i.e., $\pi_1^{YY} - \pi_1^{NY} < K < \pi_1^{YN} - \pi_1^{NN}$) along with $\pi_1^{NN} > \pi_1^{NY} (= \pi_2^{YN})$ (see the first statement of Proposition 5), we can conclude that agricultural advice is welfare improving under strategy (Y, N) if and only if the value of K falls within the range $\pi_1^{YY} - \pi_1^{NY} < K < \pi_1^{YN} + \pi_2^{YN} - 2\pi_1^{NN}$. Note that this range exists only when $\pi_1^{YY} - \pi_1^{NY} < \pi_1^{YN} + \pi_2^{YN} - 2\pi_1^{NN}$, which may not hold in general.[13] Consequently, we can conclude that agricultural advice may not be welfare improving under strategy (Y, N) (and strategy (N, Y)).

Third, consider the case in which K is low (i.e., when $K < \pi_1^{YY} - \pi_1^{NY}$). In this case, Proposition 5 reveals that both farmers adopt agricultural advice under strategy (Y, Y). Hence, agricultural advice is welfare improving if and only if $(\pi_1^{NN} + \pi_2^{NN}) < (\pi_1^{YY} - K) + (\pi_2^{YY} - K)$. By symmetry, this condition can be simplified as $K < \pi_1^{YY} - \pi_1^{NN}$. Also, observe from the first statement of Proposition 5 that $\pi_1^{YY} - \pi_1^{NN} < \pi_1^{YY} - \pi_1^{NY}$. Combining the simplified condition with this observation, we can conclude that agricultural advice is welfare improving if and only if the upfront investment is sufficiently low (i.e., when $K < \pi_1^{YY} - \pi_1^{NN}$).

Based on the implications of the aforementioned three scenarios, we establish the following statement.

Corollary 7. *Depending on the upfront investment K, agricultural advice is not necessarily welfare improving. However, to ensure that adopting agricultural advice (Y, Y) improves farmers' total welfare, the government should consider offering subsidies so that the "effective" upfront investment to be borne by each farmer is below the threshold $(\pi_1^{YY} - \pi_1^{NN})$.*

Corollary 7 reveals that offering agricultural advice alone is not sufficient to ensure that the total welfare of farmers will be improved unless either the upfront investment K is sufficiently low (i.e., $K < \pi_1^{YY} - \pi_1^{NN}$) or the government offers subsidies so that the "effective"

[13] To elaborate, given that $\pi_1^{NY} = \pi_2^{YN}$, the condition $\pi_1^{YY} - \pi_1^{NY} < \pi_1^{YN} + \pi_2^{YN} - 2\pi_1^{NN}$ can be simplified as $2(\pi_1^{NN} - \pi_1^{NY}) < \pi_1^{YN} - \pi_1^{YY}$, where the right hand side represents the net gain of farmer 1 by being the only one adopting the agricultural advice instead of both farmers utilizing the agricultural advice, and the left hand side represents the net gain of the farmers when they choose not to adopt the agricultural advice. Therefore, when the benefits associated with the agricultural advice are small (via small γ, α , ρ^2, or large β), this condition may not hold. For example, it can be checked that $\pi_1^{YN} - \pi_1^{YY} - 2(\pi_1^{NN} - \pi_1^{NY})$ is negative when $\mu_m = 20$, $c = 3$, $\alpha = 1.3$, $\beta = 0.7$, $\rho = 0.4$, $\sigma_y = 0.7$, $\sigma_m = 0.2$, $\mu_y = 0.7$, and $\gamma < 1.05$.

upfront investment to be borne by each farmer is below the threshold $\pi_1^{YY} - \pi_1^{NN}$. This result may help justify the farmer subsidies offered in developing countries.[14]

6 Conclusion

In this paper, we presented a unified framework for analyzing the implications when the government offers market information that can help farmers to make better (long-term) production planning decisions (or agricultural advice that can help farmers to improve product quality, reduce production cost, and enhance process yield). By considering the case in which farmers engage in Cournot competition under both demand and process yield uncertainty, we showed that without considering the upfront investment, both farmers would utilize market information (or adopt agricultural advice) in equilibrium. We also showed the complementary effects associated with different benefits (quality improvement, cost reduction, and process yield increase).

We then used the results of our general model to analyze the case in which the government only offers market information. We found that both farmers would utilize the market information in equilibrium and that the market information is welfare improving. Moreover, the market information is welfare maximizing when the process yield is highly uncertain.

For the case in which the government only offers agricultural advice, we showed that each farmer will adopt agricultural advice in equilibrium only when the upfront investment is below a certain threshold. We also found that agricultural advice is not necessarily welfare improving. To ensure that agricultural advice can improve the total welfare of farmers, we showed that farmer subsidies are essential especially when upfront investment is high.

This paper is an initial attempt to examine the implications of market information on agricultural advice. However, our model can be extended to the case in which the government offers both market information and agricultural advice, although the analysis becomes intractable because it involves the comparison of 16 different expected profits. Nevertheless, our approach can enable us to analyze this case numerically. Furthermore, our results can shed some light on the interdependencies between market information and agricultural advice. For example, Corollary 4 states that more accurate market information can enhance the effect of yield improvement but has no influence on the effects of quality improvement and cost reduction. Therefore, the government prefers to subsidize the farmers to improve their process yield rather than improve quality and reduce cost when the market information is accurate.

Besides providing market information about future market prices that can help farmers to make long term production planning decisions, various governments are now offering

[14]For example, to encourage farmers to purchase various types of agricultural equipment to help reduce their production costs, the Department of Agriculture and Cooperation of India offers subsidies in the form of 50% of the equipment cost. See farmech.gov.in/FarmerGuide/BI/11.htm for details. In another example, to improve quality and process yield, small Kenyan farmers can purchase fertilizers from the government owned National Cereals and Produce Board at subsidized prices (i.e., 32% below market price). See http://partnews.brownbag.me/2013/04/15/kenyan-subsidized-fertilizer-explained/ for details.

current market price information that can help farmers to make short-term selling decisions (when and where to sell). It is of interest to analytically examine the implications of utilizing the current market price information for farmers in emerging markets especially because the relevant empirical findings have been mixed. For example, Mittal et al. (2010) find that by utilizing current market price information, farmers enjoy higher incomes. However, Fafchamps and Minten (2011) find no evidence supporting this claim under a different experimental setting.

In this paper, we focus on the case in which each farmer either adopts all of the agricultural advice by making a full investment or adopts none of the advice. However, different types of advice may require different amounts of investment.[15] Therefore, in the event that each farmer can choose to adopt a particular subset of advice by making the requisite investment, the analysis quickly becomes tedious because the number of subsets of advice grows exponentially. For this reason, we defer this issue as a topic of future research.

In this paper, we examined the benefits of farm subsidies in the context of developing countries, where farmers tend to be poor and have little access to financial services and formal training. This context is drastically different from that of developed countries where farmers are relatively wealthy, powerful, and well-trained (Smith 2012). In this case, the farmers can obtain financial support through different financing channels (see http://smallfarm.about.com/od/otherresources/a/farmgrants.htm), and have some control over the market price. For example, in 2013, farmers in Australia strategically held back on wheat sales to maintain a high market price (Thukral and Packham 2014). Currently, there is an on-going debate over whether the government should offer subsidies to farmers in developed countries (Edwards 2009).[16] Because the contexts are very different, there is a need to develop a different model to investigate the value of farm subsidies in developed economies, and we leave this question for future research.

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[15] We thank the referee for pointing out this scenario.

[16] In the United States, the government offers millions of dollars in subsidies to domestic farmers and agribusinesses to supplement their income and help them manage their production and maintenance costs.

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Appendix: Proofs

Proof of Proposition 1. As the best response functions of both farmers have the same structure, the equilibrium is symmetric, that is, $q_1^{NN} = q_2^{NN}$. Therefore,

$$q_i^{NN} = \frac{g - \mu_y^2 q_i^{NN}}{2S^2},$$

which yields

$$q_i^{NN} = \frac{g}{2S^2 + \mu_y^2}.$$

Consequently,

$$\pi_i^{NN} = q_i^{NN}\left[g - (S^2 + \mu_y^2) q_i^{NN}\right] = \frac{S^2 g^2}{(2S^2 + \mu_y^2)^2}.$$

Recall that $S^2 = \sigma_y^2 + \mu_y^2$ and $g = \mu_m \mu_y - c$, it is easy to see that π_i^{NN} and q_i^{NN} are increasing in μ_m and decreasing in σ_y. Because $\frac{dS^2}{d\mu_y} = 2\mu_y$ and $\frac{dg}{d\mu_y} = \mu_m$, we have

$$\begin{aligned}
\frac{d\pi_i^{NN}}{d\mu_y} &= \frac{(\frac{dS^2}{d\mu_y} g^2 + S^2 \frac{dg^2}{d\mu_y})(2S^2 + \mu_y^2) - 2S^2 g^2 (2\frac{dS^2}{d\mu_y} + 2\mu_y)}{(2S^2 + \mu_y^2)^3} \\
&= \frac{2\sigma_y^2 g(\mu_m \sigma_y^2 + \mu_y c) + 2S^2 g \mu_m \sigma_y^2 + 4S^2 g \mu_y c + 2\mu_y S^2 g c}{(2S^2 + \mu_y^2)^3} \\
&> 0.
\end{aligned}$$

$$
\begin{aligned}
\frac{d(\mu_y q_i^{NN})}{d\mu_y} &= q_i^{NN} + \mu_y \frac{dq_i^{NN}}{d\mu_y} \\
&= \frac{g}{2S^2+\mu_y^2} + \frac{\mu_y \left(\frac{dg}{d\mu_y}(2S^2+\mu_y^2) - g(\frac{dS^2}{d\mu_y} + 2\mu_y) \right)}{(2S^2+\mu_y^2)^2} \\
&= \frac{g}{2S^2+\mu_y^2} + \frac{\mu_y(\mu_m(2S^2+\mu_y^2) - 4g\mu_y)}{(2S^2+\mu_y^2)^2} \\
&= \frac{g(2S^2+\mu_y^2) + \mu_y\mu_m(2S^2+\mu_y^2) - 4g\mu_y^2}{(2S^2+\mu_y^2)^2} \\
&= \frac{2g\sigma_y^2 + 2\mu_y\mu_m S^2 + \mu_m\mu_y^3 - g\mu_y^2}{(2S^2+\mu_y^2)^2} \\
&= \frac{2g\sigma_y^2 + 2\mu_y\mu_m S^2 + c\mu_y^2}{(2S^2+\mu_y^2)^2} > 0.
\end{aligned}
$$

□

Proof of Lemma 1. Observing the best response functions $q_i(q_j)$, we know that in equilibrium $q_1^{YY}|I = q_2^{YY}|I$. Therefore,

$$q_i^{YY}|I = \frac{g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2}} - \frac{\gamma^2\mu_y^2 q_i^{YY}|I}{2S^{'2}},$$

which yields

$$q_i^{YY}|I = \frac{g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2} + \sigma_I\gamma^2\mu_y^2}.$$

Substituting the above equation into $\pi_i(q)|I$, we obtain

$$
\begin{aligned}
\pi_i^{YY}|I &= (q_i^{YY}|I)\left[g^{'} + \gamma\rho\mu_y\frac{\sigma_m}{\sigma_I}(I-\mu_I) - (S^{'2} + \gamma^2\mu_y^2)(q_i^{YY}|I)\right] \\
&= \frac{g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2} + \sigma_I\gamma^2\mu_y^2}\left[g^{'} + \gamma\rho\mu_y\frac{\sigma_m}{\sigma_I}(I-\mu_I) - \frac{(S^{'2}+\gamma^2\mu_y^2)(g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I))}{2\sigma_I S^{'2} + \sigma_I\gamma^2\mu_y^2}\right] \\
&= \frac{S^{'2}[g^{'}\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)]^2}{\sigma_I^2(2S^{'2}+\gamma^2\mu_y^2)^2}.
\end{aligned}
$$

□

Proof of Proposition 2. From Lemma 1, it is easy to obtain the ex ante equilibria (4) and (5). Note that only $g^{'}$ is dependent of α and β, and $g^{'}$ is increasing in α and decreasing in β. Therefore, both q_i^{YY} and π_i^{YY} are increasing in α and decreasing in β. Recall that

$S'^2 = \sigma_y^2 + (\gamma\mu_y)^2$ and $g' = \alpha\gamma\mu_m\mu_y - \beta c$. Thus, $\frac{dS'^2}{d\gamma} = 2\gamma\mu_y^2$ and $\frac{dg'}{d\gamma} = \alpha\mu_m\mu_y$. Therefore,

$$\begin{aligned}
\frac{d\pi_i^{YY}}{d\gamma} &= \frac{[\frac{dS'^2}{d\gamma}(g'^2 + \gamma^2\mu_y^2\rho^2\sigma_m^2) + 2S'^2(\alpha\mu_m\mu_y g' + \gamma\rho^2\mu_y^2\sigma_m^2)](2S'^2 + \gamma^2\mu_y^2)}{(2S'^2 + \gamma^2\mu_m^2)^3} \\
&\quad - \frac{-4S'^2(g'^2 + \gamma^2\mu_y^2\rho^2\sigma_m^2)(\frac{dS'^2}{d\gamma} + \gamma\mu_y^2)}{(2S'^2 + \gamma^2\mu_m^2)^3} \\
&= \frac{(2\gamma^2\mu_y^2\sigma_y^2 + 4\sigma_y^4)\gamma\rho^2\mu_y^2\sigma_m^2 + 2g'(\alpha\gamma^2\mu_m\mu_y^3\sigma_y^2 + 2\alpha\mu_m\mu_y\sigma_y^4 + 3\gamma\mu_y^2\beta c S'^2 + \gamma\mu_y^2\sigma_y^2\beta c)}{(2S'^2 + \gamma^2\mu_y^2)^3} \qquad (18) \\
&> 0.
\end{aligned}$$

Finally, it is easy to verify that π_i^{YY} is increasing in ρ^2 and σ_m. □

Proof of Lemma 2. From (8), given q_2,

$$(q_1|I = \mu_I) = \frac{g'}{2S'^2} - \frac{\gamma\mu_y^2 q_2}{2S'^2}.$$

Substituting it into (9),

$$q_2 = \frac{g}{2S^2} - \frac{\gamma\mu_y^2(g' - \gamma\mu_y^2 q_2)}{4S^2 S'^2}.$$

As the production quantity must be nonnegative, solving the above equation we obtain

$$q_2^{YN} = \begin{cases} \frac{2S'^2 g - \gamma\mu_y^2 g'}{4S^2S'^2 - \gamma^2\mu_y^4}, & if\ 2S'^2 g - \gamma\mu_y^2 g' > 0, \\ 0, & otherwise. \end{cases}$$

Because $2S'^2 g - \gamma\mu_y^2 g' = \mu_y^2[2(\gamma^2 + C_y^2)g - \gamma g']$, $2S'^2 g - \gamma\mu_y^2 g' > 0$ is equivalent to $C_y^2 > \frac{\gamma g'}{2g} - \gamma^2$. Plugging q_2^{YN} into (8), we obtain (10). Then

$$(q_1^{YN}|I = \mu_I) = \begin{cases} \frac{2S^2 g' - \gamma\mu_y^2 g}{4S^2S'^2 - \gamma^2\mu_y^4}, & if\ C_y^2 > \frac{\gamma g'}{2g} - \gamma^2, \\ \frac{g'}{2S'^2}, & otherwise. \end{cases}$$

When $C_y^2 > \frac{\gamma g'}{2g} - \gamma^2$, from (6) and (7), we can get

$$\begin{aligned}
\pi_2^{YN} &= q_2^{YN}\left[g - (S^2 q_2^{YN} + \gamma\mu_y^2(q_1^{YN}|I=\mu_I))\right] \\
&= \frac{2S^{'2}g - \gamma\mu_y^2 g'}{4S^2S^{'2} - \gamma^2\mu_y^4}\left[g - \frac{2S^2S^{'2}g - \gamma\mu_y^2 S^2 g'}{4S^2S^{'2} - \gamma^2\mu_y^4} - \frac{2\gamma\mu_y^2 S^2 g' - \gamma^2\mu_y^4 g}{4S^2S^{'2} - \gamma^2\mu_y^4}\right] \\
&= \frac{S^2(2S^{'2}g - \gamma\mu_y^2 g')^2}{(4S^2S^{'2} - \gamma^2\mu_y^4)^2}. \\
\pi_1^{YN}|I &= (q_1^{YN}|I)[g' + \gamma\rho\mu_y\frac{\sigma_m}{\sigma_I}(I-\mu_I) - ((q_1^{YN}|I)S^{'2} + \gamma\mu_y^2 q_2^{YN})] \\
&= \left[\frac{2S^2g' - \gamma\mu_y^2 g}{4S^2S^{'2} - \gamma^2\mu_y^4} + \frac{\gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2}}\right]\left[g' + \gamma\rho\mu_y\frac{\sigma_m}{\sigma_I}(I-\mu_I) - \left(\frac{2S^2S^{'2}g' - \gamma\mu_y^2 S^{'2}g}{4S^2S^{'2} - \gamma^2\mu_y^4}\right.\right. \\
&\quad \left.\left. + \frac{\gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I} + \frac{2S^{'2}\gamma\mu_y^2 g - \gamma^2\mu_y^4 g'}{4S^2S^{'2} - \gamma^2\mu_y^4}\right)\right] \\
&= S^{'2}\left[\frac{2S^2g' - \gamma\mu_y^2 g}{4S^2S^{'2} - \gamma^2\mu_y^4} + \frac{\gamma\rho\mu_y\sigma_m(I-\mu_I)}{2\sigma_I S^{'2}}\right]^2.
\end{aligned}$$

When $C_y^2 \le \frac{\gamma g'}{2g} - \gamma^2$, it can be easily shown that $\pi_2^{YN} = 0$ and

$$\pi_1^{YN}|I = (q_1^{YN}|I)\left[g' + \gamma\rho\mu_y\frac{\sigma_m}{\sigma_I}(I-\mu_I) - (q_1^{YN}|I)S^{'2}\right] = \frac{[g'\sigma_I + \gamma\rho\mu_y\sigma_m(I-\mu_I)]^2}{4\sigma_I^2 S^{'2}}.$$

$\square$

Proof of Corollary 1. Suppose $\gamma = 1$. Then, $S^2 = S^{'2}$. When $C_y^2 > \frac{\gamma g'}{2g} - \gamma^2$, according to Lemma 2 and Proposition 3,

$$q_1^{YN} - q_2^{YN} = \frac{2S^2g' - \gamma\mu_y^2 g - 2S^{'2}g + \gamma\mu_y^2 g'}{4S^2S^{'2} - \gamma^2\mu_y^4} = \frac{(2S^2 + \gamma\mu_y^2)(g' - g)}{4S^2S^{'2} - \gamma^2\mu_y^4} \ge 0.$$

When $C_y^2 \le \frac{\gamma g'}{2g} - \gamma^2$, $q_2^{YN} = 0$. Thus, when $\gamma = 1$, $q_1^{YN} \ge q_2^{YN}$. $\square$

Proof of Lemma 3. We first consider the scenario $C_y^2 > \frac{\gamma g'}{2g} - \gamma^2$. Let

$$L_1 = \frac{S'g'}{(2S^{'2} + \gamma^2\mu_y^2)}, \text{ and } L_2 = \frac{S'(2S^2g' - \gamma\mu_y^2 g)}{(4S^2S^{'2} - \gamma^2\mu_y^4)}.$$

Then, from (5) and (15), we have

$$\pi_1^{YY} = L_1^2 + \frac{S^{'2}\gamma^2\rho^2\mu_y^2\sigma_m^2}{(2S^{'2} + \gamma^2\mu_y^2)^2}, \tag{19}$$

$$\pi_1^{YN} = L_2^2 + \frac{\gamma^2\rho^2\mu_y^2\sigma_m^2}{4S^{'2}}. \tag{20}$$

Note that $\pi_1^{NY} = \pi_2^{YN}$. From (13), we have

$$\begin{aligned}
\sqrt{\pi_1^{NN}} - \sqrt{\pi_1^{NY}} &= \frac{Sg}{(2S^2+\mu_y^2)} - \frac{S(2S^{'2}g - \gamma\mu_y^2 g^{'})}{(4S^2S^{'2} - \gamma^2\mu_y^4)} \\
&= \frac{[2\mu_y^2(S^2\gamma g^{'} - S^{'2}g) + \gamma\mu_y^4(g^{'} - \gamma g)]S}{(2S^2+\mu_y^2)(4S^2S^{'2} - \gamma^2\mu_y^4)}, \qquad (21) \\
L_2 - L_1 &= \frac{S^{'}(2S^2g^{'} - \gamma\mu_y^2 g)}{(4S^2S^{'2} - \gamma^2\mu_y^4)} - \frac{S^{'}g^{'}}{(2S^{'2} + \gamma^2\mu_y^2)} \\
&= \frac{[2\gamma\mu_y^2(S^2\gamma g^{'} - S^{'2}g) + \gamma^2\mu_y^4(g^{'} - \gamma g)]S^{'}}{(4S^2S^{'2} - \gamma^2\mu_y^4)(2S^{'2} + \gamma^2\mu_y^2)}, \qquad (22)
\end{aligned}$$

As $\alpha \geq 1$, $\beta \leq 1$, and $\gamma \geq 1$,

$$\begin{aligned}
g^{'} - \gamma g &= \gamma\mu_m\mu_y(\alpha - 1) - (\beta - \gamma)c \geq 0, \\
S^2\gamma g^{'} - S^{'2}g &= \gamma(\sigma_y^2 + \mu_y^2)(\alpha\gamma\mu_m\mu_y - \beta c) - (\sigma_y^2 + \gamma^2\mu_y^2)(\mu_m\mu_y - c) \\
&= \sigma_y^2[\gamma(\alpha\gamma\mu_m\mu_y - \beta c) - (\mu_m\mu_y - c)] + (\alpha - 1)\gamma^2\mu_m\mu_y^3 + (\gamma - \beta)\gamma\mu_y^2 c \\
&\geq 0.
\end{aligned}$$

Thus, $\pi_1^{NN} \geq \pi_1^{NY}$ and $L_2 \geq L_1$. Because

$$\frac{\gamma^2\rho^2\mu_y^2\sigma_m^2}{4S^{'2}} - \frac{S^{'2}\gamma^2\rho^2\mu_y^2\sigma_m^2}{(2S^{'2} + \gamma^2\mu_y^2)^2} = S^{'2}\gamma^2\rho^2\mu_y^2\sigma_m^2\left(\frac{1}{2S^{'2}} - \frac{1}{2S^{'2} + \gamma^2\mu_y^2}\right)\left(\frac{1}{2S^{'2}} + \frac{1}{2S^{'2} + \gamma^2\mu_y^2}\right) \geq 0,$$

from (19) and (20), we have $\pi_1^{YN} - \pi_1^{YY} \geq L_2^2 - L_1^2 \geq 0$. Note that when $\alpha = \gamma = \beta = 1$ and $\rho = 0$, $\pi_i^{YY} = \pi_i^{NN}$. Then, based on Proposition 2, we can derive that $\pi_1^{YN} \geq \pi_1^{YY} \geq \pi_1^{NN} \geq \pi_1^{NY}$. Naturally, due to symmetry, $\pi_2^{NY} \geq \pi_2^{YY} \geq \pi_2^{NN} \geq \pi_2^{YN}$.

Below, we prove that $\pi_1^{YN} - \pi_1^{NN} \geq \pi_1^{YY} - \pi_1^{NY}$. From (21) and (22), we get

$$\begin{aligned}
\frac{\pi_1^{YN} - \pi_1^{YY}}{\pi_1^{NN} - \pi_1^{NY}} &\geq \frac{L_2^2 - L_1^2}{\pi_1^{NN} - \pi_1^{NY}} \\
&= \frac{L_2 - L_1}{\sqrt{\pi_1^{NN}} - \sqrt{\pi_1^{NY}}}\frac{L_2 + L_1}{\sqrt{\pi_1^{NN}} + \sqrt{\pi_1^{NY}}} \\
&= \frac{\gamma S^{'}(2S^2 + \mu_y^2)}{S(2S^{'2} + \gamma^2\mu_y^2)}\frac{L_2 + L_1}{\sqrt{\pi_1^{NN}} + \sqrt{\pi_1^{NY}}}.
\end{aligned}$$

We can show that

$$\begin{aligned}
\sqrt{\pi_1^{NN}}+\sqrt{\pi_1^{NY}} &= \frac{Sg}{(2S^2+\mu_y^2)}+\frac{S(2S^{'2}g-\gamma\mu_y^2 g^{'})}{(4S^2S^{'2}-\gamma^2\mu_y^4)} \\
&= \frac{[8S^2S^{'2}g-2\mu_y^2(S^2\gamma g^{'}-S^{'2}g)-\gamma\mu_y^4(g^{'}+\gamma g)]S}{(2S^2+\mu_y^2)(4S^2S^{'2}-\gamma^2\mu_y^4)} \\
L_2+L_1 &= \frac{S^{'}(2S^2g^{'}+\gamma\mu_y^2 g)}{(4S^2S^{'2}-\gamma^2\mu_y^4)}+\frac{S^{'}g^{'}}{(2S^{'2}+\gamma^2\mu_y^2)} \\
&= \frac{[8S^2S^{'2}g^{'}+2\gamma\mu_y^2(S^2\gamma g^{'}-S^{'2}g)-\gamma^2\mu_y^4(g^{'}+\gamma g)]S^{'}}{(4S^2S^{'2}-\gamma^2\mu_y^4)(2S^{'2}+\gamma^2\mu_y^2)}.
\end{aligned}$$

Because $g^{'}\geq g\gamma$ and $S^2\gamma g^{'}-S^{'2}g\geq 0$,

$$\frac{8S^2S^{'2}g^{'}}{\gamma}+2\mu_y^2(S^2\gamma g^{'}-S^{'2}g)-\gamma\mu_y^4(g^{'}+\gamma g)\geq 8S^2S^{'2}g-2\mu_y^2(S^2\gamma g^{'}-S^{'2}g)-\gamma\mu_y^4(g^{'}+\gamma g),$$

which yields

$$\frac{L_2+L_1}{\sqrt{\pi_1^{NN}}+\sqrt{\pi_1^{NY}}}\geq\frac{\gamma S^{'}(2S^2+\mu_y^2)}{S(2S^{'2}+\gamma^2\mu_y^2)}.$$

Moreover, because

$$\begin{aligned}
\gamma^2S^{'2}(2S^2+\mu_y^2)^2-S^2(2S^{'2}+\gamma^2\mu_y^2)^2 &= (4S^2S^{'2}-\gamma^2\mu_y^4)(\gamma^2S^2-S^{'2}) \\
&= (4\sigma_y^4+4\gamma^2\mu_y^2\sigma_y^2+4\mu_y^2\sigma_y^2+3\gamma^2\mu_y^4)(\gamma^2-1)\sigma_y^2 \\
&> 0,
\end{aligned}$$

$\frac{\gamma^2S^{'2}(2S^2+\mu_y^2)^2}{S^2(2S^{'2}+\gamma^2\mu_y^2)^2}\geq 1$. Thus, $\pi_1^{YN}-\pi_1^{YY}\geq\pi_1^{NN}-\pi_1^{NY}$. Equivalently, $\pi_1^{YN}-\pi_1^{NN}\geq\pi_1^{YY}-\pi_1^{NY}$. Again, by symmetry, $\pi_2^{NY}-\pi_2^{NN}\geq\pi_2^{YY}-\pi_2^{YN}$.

Now we consider the scenario $C_y^2\leq\frac{\gamma g^{'}}{2g}-\gamma^2$. As $C_y^2\geq 0$, $g^{'}\geq 2\gamma g$. Thus,

$$\begin{aligned}
\pi_1^{YN}-\pi_1^{YY} &= \frac{g^{'2}+\gamma^2\rho^2\mu_y^2\sigma_m^2}{4S^{'2}}-\frac{S^{'2}(g^{'2}+\gamma^2\rho^2\mu_y^2\sigma_m^2)}{(2S^{'2}+\gamma^2\mu_y^2)^2} \\
&= \frac{(g^{'2}+\gamma^2\rho^2\mu_y^2\sigma_m^2)\gamma^2\mu_y^2(4S^{'2}+\gamma^2\mu_y^2)}{4S^{'2}(2S^{'2}+\gamma^2\mu_y^2)^2}>0.
\end{aligned}$$

When $C_y^2\leq\frac{\gamma g^{'}}{2g}-\gamma^2$, $\pi_1^{NY}=\pi_2^{YN}=0$. Therefore, $\pi_1^{YN}\geq\pi_1^{YY}\geq\pi_1^{NN}\geq\pi_1^{NY}$. By symmetry, $\pi_2^{NY}\geq\pi_2^{YY}\geq\pi_2^{NN}\geq\pi_2^{YN}$. Next, we can show that

$$\begin{aligned}
\pi_1^{YN}-\pi_1^{YY}-(\pi_1^{NN}-\pi_1^{NY}) &= \pi_1^{YN}-\pi_1^{YY}-\pi_1^{NN} \\
&\geq \frac{g^{'2}\gamma^2\mu_y^2(4S^{'2}+\gamma^2\mu_y^2)}{4S^{'2}(2S^{'2}+\gamma^2\mu_y^2)^2}-\frac{S^2g^2}{(2S^2+\mu_y^2)^2} \\
&= \frac{g^{'2}\gamma^2\mu_y^2(4S^{'2}+\gamma^2\mu_y^2)(2S^2+\mu_y^2)^2-4S^{'2}S^2g^2(2S^{'2}+\gamma^2\mu_y^2)^2}{4S^{'2}(2S^{'2}+\gamma^2\mu_y^2)^2(2S^2+\mu_y^2)^2}.
\end{aligned}$$

Recall that $C_y^2 \leq \frac{\gamma g^{'}}{2g} - \gamma^2$ is equivalent to $2S^{'2}g - \gamma\mu_y^2 g^{'} \leq 0$, thus,

$$\begin{aligned}
\pi_1^{YN} - \pi_1^{YY} - (\pi_1^{NN} - \pi_1^{NY}) &\geq \frac{2S^{'2}gg^{'}\gamma(4S^{'2}+\gamma^2\mu_y^2)(2S^2+\mu_y^2)^2 - 4S^{'2}S^2g^2(2S^{'2}+\gamma^2\mu_y^2)^2}{4S^{'2}(2S^{'2}+\gamma^2\mu_y^2)^2(2S^2+\mu_y^2)^2} \\
&= \frac{2S^{'2}g[g^{'}\gamma(4S^{'2}+\gamma^2\mu_y^2)(2S^2+\mu_y^2)^2 - 2S^2g(2S^{'2}+\gamma^2\mu_y^2)^2]}{4S^{'2}(2S^{'2}+\gamma^2\mu_y^2)^2(2S^2+\mu_y^2)^2}.
\end{aligned}$$

Note that

$$\begin{aligned}
g^{'}\gamma(4S^{'2}+\gamma^2\mu_y^2)(2S^2+\mu_y^2)^2 &\geq 16g^{'}\gamma S^{'2}(S^2)^2 + 4\gamma^3 g^{'}\mu_y^2(S^2)^2 + 4\gamma^3\mu_y^4 S^2 g^{'}, \\
2S^2g(2S^{'2}+\gamma^2\mu_y^2)^2 &= 8(S^{'2})^2S^2g + 8S^{'2}S^2g\gamma^2\mu_y^2 + 2\gamma^4\mu_y^4S^2g.
\end{aligned}$$

Because $S^2\gamma g^{'} - S^{'2}g \geq 0$, $2S^{'2}g - \gamma\mu_y^2 g^{'} \leq 0$ and $g^{'} \geq 2\gamma g$,

$$\begin{aligned}
g^{'}\gamma(4S^{'2}+\gamma^2\mu_y^2)(2S^2+\mu_y^2)^2 - 2S^2g(2S^{'2}+\gamma^2\mu_y^2)^2 &\geq 8S^{'2}S^2(2g^{'}\gamma S^2 - S^{'2}g) + 4\gamma^2\mu_y^2S^2(\gamma\mu_y^2g^{'} - 2S^{'2}g) \\
&\quad +2\gamma^3\mu_y^2S^2(2g^{'}S^2 - \gamma\mu_y^2 g) \\
&\geq 0.
\end{aligned}$$

Therefore, $\pi_1^{YN} - \pi_1^{YY} \geq \pi_1^{NN} - \pi_1^{NY}$, which is equivalent to $\pi_1^{YN} - \pi_1^{NN} \geq \pi_1^{YY} - \pi_1^{NY}$. By symmetry, $\pi_2^{NY} - \pi_2^{NN} \geq \pi_2^{YY} - \pi_2^{YN}$. □

Proof of Corollary 3. By noting that $\rho \neq 0$ and π_i^{NN} is independent of α, β, γ, and ρ, Corollary 3 can be easily derived from Proposition 2 and Lemma 3. □

Proof of Corollary 4. Note that π_1^{NN} is independent of those parameters. We only need to focus on π_1^{YY}. From (5) and (18), we have

$$\begin{aligned}
\frac{\partial^2\pi_1^{YY}}{\partial\gamma\partial\rho^2} &= \frac{\gamma\mu_y^2\sigma_m^2(2\gamma^2\sigma_y^2\mu_y^2+4\sigma_y^4)}{(2S^{'2}+\gamma^2\mu_y^2)^3} > 0, \quad \frac{\partial^2\pi_1^{YY}}{\partial\gamma\partial\sigma_m} = \frac{2\gamma\sigma_m\rho^2\mu_y^2(2\gamma^2\sigma_y^2\mu_y^2+4\sigma_y^4)}{(2S^{'2}+\gamma^2\mu_y^2)^3} > 0, \\
\frac{\partial^2\pi_1^{YY}}{\partial\gamma\partial\alpha} &= \frac{2g^{'}(\gamma^2\mu_m\mu_y^3\sigma_y^2+2\mu_m\mu_y\sigma_y^4) + 2\mu_m\mu_y\gamma(\alpha\gamma^2\mu_m\mu_y^3\sigma_y^2+2\alpha\mu_m\mu_y\sigma_y^4+3\gamma\mu_y^2\beta cS^{'2}+\gamma\mu_y^2\sigma_y^2\beta c)}{(2S^{'2}+\gamma^2\mu_y^2)^3} \\
&> 0. \\
\frac{\partial^2\pi_1^{YY}}{\partial\alpha\partial\rho^2} &= \frac{\partial^2\pi_1^{YY}}{\partial\beta\partial\rho^2} = 0.
\end{aligned}$$

Moreover,

$$\begin{aligned}
\frac{\partial^2 \pi_1^{YY}}{\partial\beta\partial\gamma} &= \frac{-2c(\alpha\gamma^2\mu_m\mu_y^3\sigma_y^2 + 2\alpha\mu_m\mu_y\sigma_y^4 + 3\gamma\mu_y^2\beta c S^{'2} + \gamma\mu_y^2\sigma_y^2\beta c) + 6g^{'}\gamma\mu_y^2 c S^{'2} + 2g^{'}\gamma\mu_y^2\sigma_y^2 c}{(2S^{'2} + \gamma^2\mu_y^2)^3} \\
&= \frac{-2c\alpha\gamma^2\mu_m\mu_y^3\sigma_y^2 - 4c\alpha\mu_m\mu_y\sigma_y^4 + 2c(g^{'} - \beta c)(3\gamma\mu_y^2 S^{'2} + \gamma\mu_y^2\sigma_y^2)}{(2S^{'2} + \gamma^2\mu_y^2)^3} \\
&= \frac{2c[-\alpha\gamma^2\mu_m\mu_y^3\sigma_y^2 - 2\alpha\mu_m\mu_y\sigma_y^4 + (\alpha\gamma\mu_m\mu_y - 2\beta c)(3\gamma\mu_y^2 S^{'2} + \gamma\mu_y^2\sigma_y^2)]}{(2S^{'2} + \gamma^2\mu_y^2)^3} \\
&= \frac{2c[-2\alpha\mu_m\mu_y\sigma_y^4 + 3\alpha\gamma^2\mu_m\mu_y^3 S^{'2} - 2\beta c(3\gamma\mu_y^2 S^{'2} + \gamma\mu_y^2\sigma_y^2)]}{(2S^{'2} + \gamma^2\mu_y^2)^3} \\
&= \frac{2c[\alpha\mu_m\mu_y(3\gamma^2\mu_y^2 S^{'2} - 2\sigma_y^4) - 2\beta c(3\gamma\mu_y^2 S^{'2} + \gamma\mu_y^2\sigma_y^2)]}{(2S^{'2} + \gamma^2\mu_y^2)^3} \\
&= \frac{2c[\alpha\mu_m\mu_y^5(3\gamma^4 + 3\gamma^2 C_y^2 - 2C_y^4) - 2\beta c(3\gamma\mu_y^2 S^{'2} + \gamma\mu_y^2\sigma_y^2)]}{(2S^{'2} + \gamma^2\mu_y^2)^3} \\
&= \frac{2c[-2\alpha\mu_m\mu_y^5\gamma^4(\frac{C_y^2}{\gamma^2} + \frac{\sqrt{33}-3}{4})(\frac{C_y^2}{\gamma^2} - \frac{\sqrt{33}+3}{4}) - 2\beta c(3\gamma\mu_y^2 S^{'2} + \gamma\mu_y^2\sigma_y^2)]}{(2S^{'2} + \gamma^2\mu_y^2)^3}.
\end{aligned}$$

Let $\bar{C}_y = (\sqrt{\sqrt{33}+3})/2$. Obviously, when $C_y/r > \bar{C}_y$, the above equation is definitely negative. Otherwise, because the term inside the bracket in the numerator is decreasing in c, there exists a $\bar{c}$ such that $\frac{\partial^2 \pi_1^{YY}}{\partial\beta\partial\gamma} < 0$ if and only if $c > \bar{c}$. □

Proof of Corollary 5. Because $\alpha = \beta = \gamma = 1$, $g = g^{'}$, $S^2 = S^{'2}$, and $\gamma g^{'} - 2\gamma^2 g \leq 0$. Based on (13), we then have

$$\pi_1^{NY} = \frac{S^2(2S^{'2}g - \gamma\mu_y^2 g^{'})^2}{(4S^2 S^{'2} - \gamma^2\mu_y^4)^2} = \frac{S^2 g^2}{(2S^2 + \mu_y^2)^2} = \pi_1^{NN}.$$

By symmetry, we can obtain that $\pi_2^{NN} = \pi_2^{YN}$. Furthermore, because $\rho \neq 0$, it follows easily from Lemma 3 that $\pi_i^{YY} > \pi_i^{NN}$, $\pi_1^{YN} - \pi_1^{NN} > \pi_1^{YY} - \pi_1^{NY}$, and $\pi_2^{NY} - \pi_2^{NN} > \pi_2^{YY} - \pi_2^{YN}$. Finally, due to $\pi_1^{YY} > \pi_1^{NY}$ and $\pi_1^{YN} > \pi_1^{NN}$, the best response of farmer 1 is to invest. By symmetry, farmer 2's best response is the same as farmer 1. Thus, (Y, Y) is the unique equilibrium. □

Proof of Proposition 4. From (16) and (17), we have

$$\pi_1^{YN} = \frac{S^2 g^2}{(2S^2 + \mu_y^2)^2} + \frac{\rho^2\mu_y^2\sigma_m^2}{4S^2}, \text{ and } \pi_2^{YN} = \frac{S^2 g^2}{(2S^2 + \mu_y^2)^2}.$$

Thus,

$$\begin{aligned}
\pi_1^{YY} + \pi_2^{YY} - (\pi_1^{YN} + \pi_2^{YN}) &= \frac{2S^2(g^2 + \rho^2\mu_y^2\sigma_m^2)}{(2S^2 + \mu_y^2)^2} - \frac{2S^2 g^2}{(2S^2 + \mu_y^2)^2} - \frac{\rho^2\mu_y^2\sigma_m^2}{4S^2} \\
&= \frac{\rho^2\mu_y^2\sigma_m^2(2\sqrt{2}S^2 - 2S^2 - \mu_y^2)(2\sqrt{2}S^2 + 2S^2 + \mu_y^2)}{4S^2(2S^2 + \mu_y^2)^2} \\
&= \frac{\rho^2\sigma_m^2\mu_y^4[2(\sqrt{2}-1)C_y^2 + 2\sqrt{2} - 3](2\sqrt{2}S^2 + 2S^2 + \mu_y^2)}{4S^2(2S^2 + \mu_y^2)^2}.
\end{aligned}$$

Therefore, $\pi_1^{YY} + \pi_2^{YY} > \pi_1^{YN} + \pi_2^{YN}$ if and only if $C_y > \sqrt{\frac{\sqrt{2}-1}{2}}$. $\square$

Proof of Corollary 6. By noting that $\alpha = \beta = \gamma = 1$ and $P_i = M - (z_i q_i + t z_j q_j),\ i, j = 1, 2, i \neq j$, we can obtain the farmers' expected profits associated with the four subgames, which are summarized in following table.

Table 3: Summary of the results under heterogenous products

	Expected profit
(N, N)	$\pi_i(q_i) = gq_i - S^2q_i^2 - t\mu_y^2 q_i q_j,\ i, j = 1, 2, i \neq j,$
(Y, Y)	$\pi_i(q_i \mid I) = gq_i + \rho\mu_y\frac{\sigma_m}{\sigma_I}(I - \mu_I)q_i - S^2q_i^2 - t\mu_y^2 q_i q_j \mid I$
(Y, N)	$\pi_1(q_1) = gq_1 + \rho\mu_y\frac{\sigma_m}{\sigma_I}(I - \mu_I)q_1 - S^2q_1^2 - t\mu_y^2 q_1 q_2 \mid I$
	$\pi_2(q_2) = gq_2 - S^2q_2^2 - t(\mu_y^2 q_1 \mid I = \mu_I)q_2$
(N, Y)	$\pi_1(q_1) = gq_1 - S^2q_1^2 - tq_1(\mu_y^2 q_2 \mid I = \mu_I)$
	$\pi_2(q_2) = gq_2 + \rho\mu_y\frac{\sigma_m}{\sigma_I}(I - \mu_I)q_2 - S^2q_2^2 - t\mu_y^2 q_1 q_2 \mid I$

By considering the first order condition, we can obtain the equilibrium outcomes, which are summarized in Table 4.

Table 4: Summary of the results under heterogenous products

	Best response	Equilibrium outcomes (ex post)
(N, N)	$q_i(q_j) = \frac{g - t\mu_y^2 q_j}{2S^2}$	$q_i^{NN} = \frac{g}{2S^2 + t\mu_y^2},\ \pi_i^{NN} = \frac{S^2g^2}{(2S^2 + t\mu_y^2)^2}$
(Y, Y)	$q_i(q_j) \mid I = \frac{g\sigma_I + \rho\mu_y\sigma_m(I - \mu_I)}{2\sigma_I S^2} - \frac{t\mu_y^2 q_2}{2S^2}$	$q_i^{YY} = \frac{g}{2S^2 + t\mu_y^2},\ \pi_i^{YY} = \frac{S^2(g^2 + \rho^2\mu_y^2\sigma_m^2)}{(2S^2 + t\mu_y^2)^2}$
(Y, N)	$q_1(q_2) \mid I = \frac{g\sigma_I + \rho\mu_y\sigma_m(I - \mu_I)}{2\sigma_I S^2} - \frac{t\mu_y^2 q_2}{2S^2}$	$q_1^{YN} = \frac{g}{2S^2 + t\mu_y^2},\ \pi_1^{YN} = \frac{S^2g^2}{(2S^2 + t\mu_y^2)^2} + \frac{\rho^2\mu_y^2\sigma_m^2}{4S^4}$
	$q_2(q_1) = \frac{g - t\mu_y^2(q_1 \mid I = \mu_I)}{2S^2}$	$q_2^{YN} = \frac{g}{2S^2 + t\mu_y^2},\ \pi_2^{YN} = \frac{S^2g^2}{(2S^2 + t\mu_y^2)^2}$
(N, Y)	$q_1(q_2) = \frac{g - t\mu_y^2(q_2 \mid I = \mu_I)}{2S^2}$	$q_1^{NY} = \frac{g}{2S^2 + t\mu_y^2},\ \pi_1^{NY} = \frac{S^2g^2}{(2S^2 + t\mu_y^2)^2}$
	$q_2(q_1) \mid I = \frac{g\sigma_I + \rho\mu_y\sigma_m(I - \mu_I)}{2\sigma_I S^2} - \frac{t\mu_y^2 q_1}{2S^2}$	$q_2^{NY} = \frac{g}{2S^2 + t\mu_y^2},\ \pi_2^{NY} = \frac{S^2g^2}{(2S^2 + t\mu_y^2)^2} + \frac{\rho^2\mu_y^2\sigma_m^2}{4S^4}$

It is easy to see that $\pi_i^{YY} > \pi_i^{NN}$ and $\pi_1^{YY} + \pi_2^{YY} > \pi_1^{NN} + \pi_2^{NN}$. Furthermore, due to symmetry, to check whether the provision of market information is welfare maximizing, we only need to compare the total welfare of farmers under strategies (Y, Y) and (Y, N). By direct comparison,

$$\begin{aligned}
\pi_1^{YY} + \pi_2^{YY} - (\pi_1^{YN} + \pi_2^{YN}) &= \frac{2\rho^2\mu_y^2\sigma_m^2}{(2S^2 + t\mu_y^2)^2} - \frac{\rho^2\mu_y^2\sigma_m^2}{4S^4} \\
&= \frac{\rho^2\mu_y^2\sigma_m^2[8S^4 - (2S^2 + t\mu_y^2)^2]}{4S^4(2S^2 + t\mu_y^2)^2} \\
&= \frac{\rho^2\mu_y^2\sigma_m^2[2\sqrt{2}S^2 - (2S^2 + t\mu_y^2)][2\sqrt{2}S^2 + (2S^2 + t\mu_y^2)]}{4S^4(2S^2 + t\mu_y^2)^2} \\
&= \frac{\rho^2\mu_y^4\sigma_m^2[2(\sqrt{2} - 1)C_y^2 + 2(\sqrt{2} - 1) - t][2\sqrt{2}S^2 + (2S^2 + t\mu_y^2)]}{4S^4(2S^2 + t\mu_y^2)^2}.
\end{aligned}$$

Thus, when $t < 2(\sqrt{2} - 1)$, the above equation is always positive. When $t \geq 2(\sqrt{2} - 1)$, $\pi_1^{YY} + \pi_2^{YY} > \pi_1^{NY} + \pi_2^{NY}$ if and only if $C_y > \sqrt{\frac{t - 2(\sqrt{2}-1)}{2(\sqrt{2}-1)}}$, which can be simplified as $C_y > \sqrt{\frac{(\sqrt{2}+1)t - 2}{2}}$. □

Proof of Proposition 5. The first statement follows easily from the proof of Lemma 3. For the second statement, first consider the case that $K \leq \pi_1^{YY} - \pi_1^{NY}$. According to Lemma 3, when $K \leq \pi_1^{YY} - \pi_1^{NY}$, $\pi_1^{YY} - K > \pi_1^{NY}$, and $\pi_1^{YN} - K > \pi_1^{NN}$, which implies that the best response of farmer 1 is to adopt and invest. By symmetry, farmer 2's best response is also to adopt and invest. Therefore, (Y, Y) is the unique equilibrium. The other two cases can be derived similarly and we omit the details here. □