

Lightning Surge Propagation on a Grounded Vertical Conductor

Yuxuan Ding, Ya Ping Du, and Mingli Chen

Abstract—This letter addresses lightning surge propagation on a vertical conductor connected to the earth via a ground electrode. As the vertical conductor does not support the TEM mode of wave propagation, surge propagation is characterized with time- and position-variant surge impedance and attenuation coefficient of current. The reflection of a lightning surge at the ground then is addressed with the approach similar to the travelling wave theory. Expressions of the reflected surge excited by the current with a ramp or an arbitrary waveform are derived. Note that the reflected surge consists of two components with different attenuation rates. The traditional reflection coefficient is not applicable to the grounded vertical conductor. The results are numerically validated with the finite-difference time domain and partial element equivalent circuit methods.

Index Terms—Ground, lightning, surge, vertical conductor.

I. INTRODUCTION

VERTICAL structures on the ground, such as towers and buildings, are subject to lightning strikes. Severe lightning electromagnetic environments may cause malfunction or even damage to vulnerable equipment in the vicinity. Therefore, it is of significant important to evaluate the lightning surges along the structures to protect the equipment on the towers or in the buildings.

Traditionally, a vertical tower is represented by surge impedance in the lightning surge analysis, where a number of experimental and theoretical studies have been carried out [1]–[5]. In [6] and [7], the tower was modeled as a distributed parameter line or a transmission line with constant impedance. In these studies, it was assumed that the surge current would not attenuate during its propagation on the tower. However, it was reported in [8] and [9] that a vertical line above the ground would not support the TEM mode of surge propagation. The surge current is attenuated during its propagation even on a lossless line, although the surge voltage remained unchanged. Several impedances at a line discontinuity were numerically investigated using the partial element equivalent circuit (PEEC) method, and applied to evaluate surge propagation at the discontinuity [10] and [11]. However, generalized mathematical models for such impedances were not available. Du and Ding

[9] introduced more general characteristic parameters for a vertical line, i.e., the time- and position-variant surge impedance and the attenuation coefficient of current. The surge propagation along the vertical line could be able to be determined explicitly with characteristic impedance and attenuation coefficient, similar to the travelling wave theory in [12].

This letter presents an extended discussion on the surge propagation on a grounded vertical conductor resulting from the ground reflection, with the characteristic parameters of a vertical line [9]. The conductor is connected to the earth via a ground electrode, and is subject to a lightning stroke at its top end. With the surge impedance and attenuation coefficient of current, the propagation mechanism on the grounded conductor can be revealed. More importantly, explicit expressions of the surges resulting from the ground reflection are derived under the source current with a ramp or an arbitrary waveform. Finally, numerical validation with both finite-difference and time-domain (FDTD) method [13] and PEEC method [14] is presented.

II. REFLECTION UNDER A RAMP-WAVEFORM CURRENT SOURCE

Fig. 1(a) is a typical configuration of the grounded vertical conductor subject to a lightning stroke. The lightning stroke is represented with current source I_s connected on the top end of the conductor ($z = 0$). The current source can generate the lightning return stroke current propagating upwards in the lightning channel, and the lightning discharge current propagating downwards to the ground on the vertical conductor. Notably, the vertical conductor is connected to a ground electrode represented with ground resistance R . A reflected surge is generated when the downward current reaches the ground surface ($z = z_0$). It was reported in [14] that the material parameters of the soil, such as conductivity and permittivity, have little influence on the surge propagation on a vertical line. Therefore, the analysis is performed on the line connected to a perfect ground via ground resistance R , as shown in Fig. 1(a).

A. Scenario I: Ground Resistance $R = 0$

By using the method of image, the perfect ground is substituted with the images placed under the ground surface. As seen in Fig. 1(b), the conductor is extended to the source image at $z = 2z_0$. The source image is connected to the lead wire image on the other end. Since this configuration is symmetrical with respect to the ground surface, upper part of the line configuration is selected for discussion, as shown in Fig. 1(c). The surge propagation over such a line is able to be described with the theory of the vertical transmission line proposed in [9].

The work was supported the Research Grants Council of the HKSAR under Project No. 152044/14 and Project No. 152038/15. (Corresponding author: Ya Ping Du.)

The Authors are with the Department of Building Services Engineering, The Hong Kong Polytechnic University Hung Hom, Kowloon, Hong Kong (e-mail: yx.ding@connect.polyu.hk; ya-ping.du@polyu.edu.hk; mingli.chen@polyu.edu.hk).

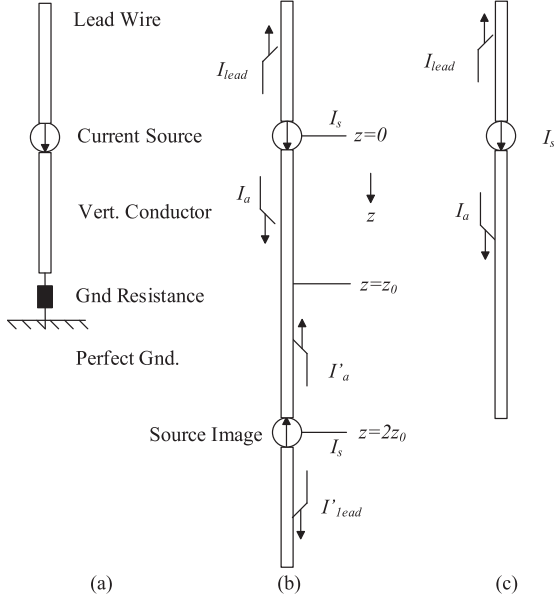


Fig. 1. Configurations of a vertical grounded conductor subject to lightning (a) original config.; (b) config. with the image; (c) config. with the downward source current.

It is assumed that the conductor is perfectly conductive. Under a ramp current source I_s applied at $z = 0$ and $t = 0$, surge current $I_a(z, t)$ on the conductor with radius r can be expressed with attenuation coefficient $p_a(z, t)$ [9], i.e.,

$$I_a(z, t) = I_a\left(0, t - \frac{z}{c}\right) \cdot p_a(z, t) \quad t \geq z/c \quad (1)$$

where $I_a(0, t - z/c)$ is the retarded current without the attenuation, and equaling to $I_s(t - z/c)$. c is the speed of an electromagnetic field wave in the free space. With surge impedance $Z_a(z, t)$ [9], electric potential $\phi_a(z, t)$ is given as

$$\phi_a(z, t) = I_s\left(t - \frac{z}{c}\right) \cdot p_a(z, t) \cdot Z_a(z, t). \quad (2)$$

Both $p_a(z, t)$ and $Z_a(z, t)$ are the characteristic parameters of a vertical transmission line. They can be determined numerically using an efficient iterative procedure given in Appendix. These parameters are varying with wire radius, but not affected by the slope of a ramp waveform.

It should be noted that the resultant surge on the conductor is the sum of surges I_a and I'_a , which are symmetrical with respect to the ground surface. These components have the same expressions, but arise from the current source at $z = 0$ and its image at $z = 2z_0$, respectively. As a result, the total surge current above the ground ($0 \leq z \leq z_0$) is expressed as

$$\begin{aligned} I(z, t) &= I_a(z, t) + I'_a(z, t) \\ &= I_s\left(t - \frac{z}{c}\right) p_a(z, t) + I_s\left(t - \frac{2z_0 - z}{c}\right) p_a(2z_0 - z, t). \end{aligned} \quad (3)$$

B. Scenario II: Ground Resistance $R \neq 0$

Fig. 2(a) shows the simplified configuration of a grounded conductor terminated with a ground resistance. Resistance R is inserted in series between the conductor and its image. In this scenario, additional surge current I_Δ and its image I'_Δ will be generated when a lightning current reaches the ground.

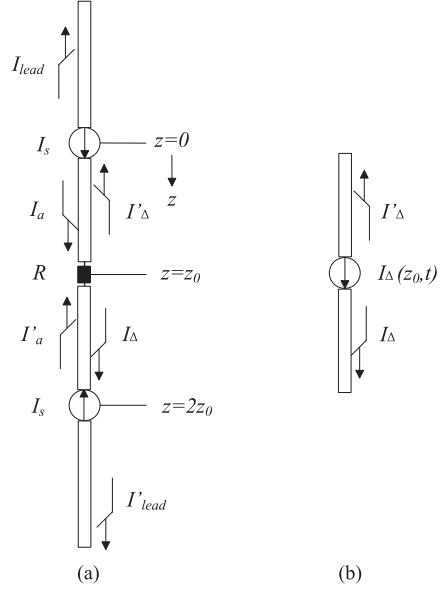


Fig. 2. Configuration of a vertical line with resistance subject to lightning (a) config. with the image; (b) config. for additional reflected currents.

The propagation starts from the ground surface in the opposite directions. The total surge current in (3) is revised as

$$I(z, t) = I_a(z, t) + I_a(2z_0 - z, t) + I_\Delta(z, t). \quad (4)$$

As seen in Fig. 2(a), both I_Δ and I'_Δ are symmetrical with respect to the ground surface. This is equivalent to the configuration where a virtual current source $I_\Delta(z_0, t)$ is placed at $z = z_0$, as illustrated in Fig. 2(b). Based on the theory of the vertical transmission line [9], the upward surge current can be expressed as

$$I_\Delta(z, t) = I_\Delta\left(z_0, t - \frac{z_0 - z}{c}\right) \cdot p_a\left(z_0 - z, t - \frac{z_0}{c}\right). \quad (5)$$

Attenuation coefficient in (5), resulting from the virtual source $I_\Delta(z_0, t)$ applied at $z = z_0$ and $t = z_0/c$, have the same expression as that used in (1). Similar to (2), electric potential on the bottom end of the line is given as

$$\phi_\Delta(z_0, t) = -I_\Delta(z_0, t) \cdot Z_a\left(0, t - \frac{z_0}{c}\right). \quad (6)$$

In (6), the negative sign arises from the upward propagation of I_Δ . Similarly, surge impedance in (6) yielding from the virtual source has the same expression of that in (2).

The current in (4) yields a resistive voltage on the ground resistance as well

$$V_R = (I_a(z_0, t) + I'_a(z_0, t) + I_\Delta(z_0, t)) \times R. \quad (7)$$

The symmetric property leads to the fact that $I_a(z_0, t) = I'_a(z_0, t)$. As the electric potential of the earth is equal to zero, i.e., $V_R + \phi_\Delta(z_0, t) = 0$, additional current component $I_\Delta(z_0, t)$ can be obtained as

$$\begin{aligned} I_\Delta(z_0, t) &= \frac{-2R}{Z_s(t - t_0) + R} I_a(z_0, t) \\ &= \frac{-2R}{Z_s(t - t_0) + R} I_s(t - t_0) \cdot p_a(z_0, t). \end{aligned} \quad (8)$$

where $Z_s(t) = Z_a(0, t)$ and $t_0 = z_0/c$.

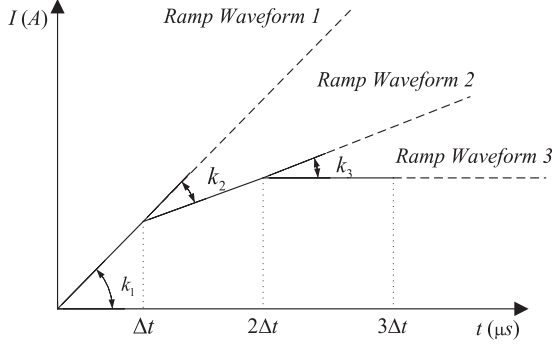


Fig. 3. Decomposition of an arbitrary waveform into ramp waveforms.

Conventionally, the reflection coefficient β of the surge current can be defined as

$$\beta(z_0, t) = \frac{Z_s(t - t_0) - R}{Z_s(t - t_0) + R} \quad (9)$$

where β is similar to the traditional reflection coefficient of a TEM line. However, (9) is not able to be directly applied to the vertical line, since two reflected components shown in (4) attenuate at different rates during their upward propagation.

With (5) and (8), total surge current in (4) can be expressed explicitly for $t \geq t_0$,

$$I(z, t) = I_s(t_1)p_a(z, t) + I_s(t_3)p_a(2z_0 - z, t) - I_s(t_3)p_a(z_0 - z, t) \frac{2R}{Z_s(t_3) + R} p_a(z_0 - z, t - t_0) \quad (10)$$

where $t_1 = t - z/c$, $t_2 = t - (z_0 - z)/c$, and $t_3 = t_2 - z_0/c$.

Noted that the electric potential does not attenuate during its propagation [9] on a single conductor, i.e., $\phi_a(z, t) = \phi_a(0, t - z/c)$, the following identities can be obtained:

$$I_a(z, t)Z_a(z, t) = I_s(t - z/c)Z_s(t - z/c) \\ p_a(z, t)Z_a(z, t) = Z_s(t - z/c). \quad (11)$$

The total electric potential on the conductor can be expressed as

$$\phi(z, t) = I_a(z, t)Z_a(z, t) - I_a(2z_0 - z, t)Z_a(2z_0 - z, t) - I_a(z, t)Z_a(z_0 - z, t - t_0) \\ = I_s(t_1)Z_s(t_1) - I_s(t_3)Z_s(t_3) \cdot \left(1 - \frac{2R \cdot p_a(z_0, t_2)}{Z_s(t_3) + R}\right). \quad (12)$$

III. REFLECTION WITH AN ARBITRARY-WAVEFORM CURRENT

As shown in Fig. 3, an arbitrary waveform can be expressed approximately by a series of ramp waveforms with different time delays, as follows:

$$I(t) = \sum_{j=0}^N k_j \cdot r(t - j\Delta t) \quad (13)$$

where $r(t - j\Delta t)$ is a unit ramp function with the slope of one applied at $t = j\Delta t$. Δt is the time step used in the evaluation

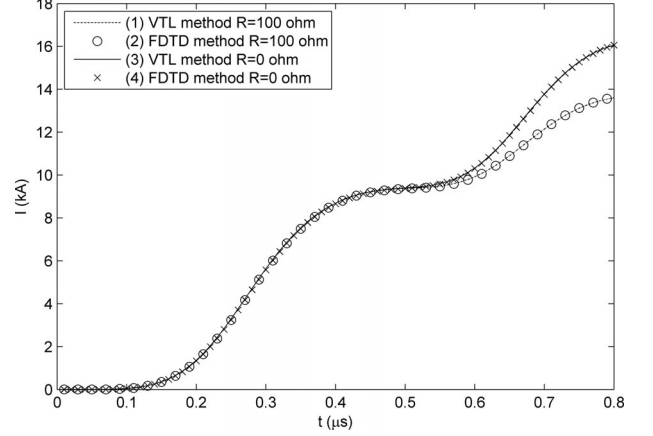


Fig. 4. Surge currents calculated with the FDTD method and with (15).

of surges. In particular, the slope k_j can be expressed as

$$k_j = \begin{cases} \frac{I(\Delta t)}{\Delta t} & j = 0 \\ \frac{I[(j+1)\Delta t] - 2I[j\Delta t] + I[(j-1)\Delta t]}{\Delta t} & j \neq 0 \end{cases} \quad (14)$$

The surge impedance and attenuation coefficient do not vary with the slope of a ramp source current [9]. Both surge current and electric potential on the line arising from an unit ramp current applied at $j\Delta t$ can be directly obtained through (10) and (12). The surge current and electric potential under the arbitrary-waveform current source can be given by

$$I_{arbi}(z, t) = \sum_{j=1}^N k_j I(z, t - j\Delta t) \\ \phi_{arbi}(z, t) = \sum_{j=1}^N k_j \phi(z, t - j\Delta t). \quad (15)$$

This expression indicates that both potential and current excited by an arbitrary-waveform current source can be calculated with the surge responses of a ramp current source, or characteristic parameters $p_a(z, t)$, and $Z_a(z, t)$ of a vertical line.

IV. SIMULATION RESULTS AND COMPARISON

The electric potential and surge current on the grounded conductor have been yielded with (15), assuming that the characteristic parameters $p_a(z, t)$ and $Z_a(z, t)$ of a vertical line have been given. In this section, simulation results of the surges on the line shown in Fig. 1(a) will be presented. In the simulation, the ground is located at $z_0 = 120$ m. The conductor radius is equal to 20 mm. The source current has the 10 kA magnitude and the 0.25/100 μ s waveform.

To validate the proposed formulas, the comparison with the FDTD [11] or the PEEC method [12] has been performed. The FDTD method is generally applicable to the surge current analysis exclusively. Figs. 4 and 5 show the surge current and electric potential at the middle point of the line ($z = 60$ m) calculated by the FDTD method, the PEEC method, and the proposed method (VTL) under an impulse current source. Two different scenarios of ground resistance have been simulated, i.e., a) $R = 0 \Omega$ and b) $R = 100 \Omega$. It can be observed that the surge current and electric potential match the results from the FDTD and PEEC methods very well. The average error of the surge current and electric potential is generally less than 1%.

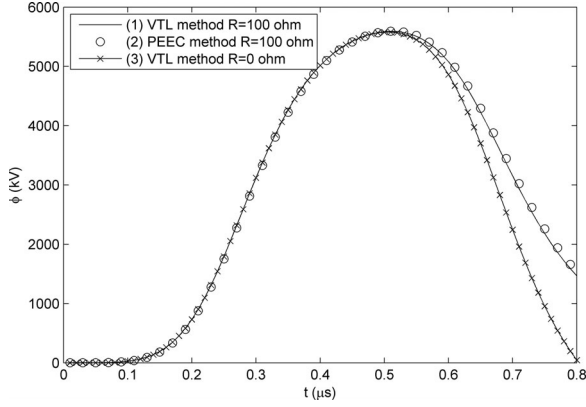


Fig. 5. Electric potentials calculated with the PEEC method and with (15).

V. CONCLUSION

This letter has addressed the lightning surge propagation on a vertical conductor grounded via a ground electrode. The expressions of the surges on a grounded conductor have been derived under the source current with a ramp waveform, as well as an arbitrary waveform. Similar to the traditional travelling theory, lightning surges on the conductor can be simply determined by ground resistance and characteristic parameters of a single line, i.e., the attenuation coefficient and the surge impedance. The reflected surge consists of two components attenuating at different rates during the upward propagation. The traditional reflection coefficient is not applicable to the reflected surge on the vertical conductor. The proposed formulas have been validated with the FDTD and PEEC methods, and good agreements have been observed.

APPENDIX

Let $z_i = i\Delta z$, $t_j = j\Delta t$ (position index i and time index $j = 0, \dots, N$), and $\Delta z = c\Delta t$ in the configuration shown in Fig. 1(c). Discrete current $I_{i,j} = I_a(z_i, t_j)$ on the line then is expressed as

$$I_{i,j} = p_{i,j} \cdot I_{0,j-i}, \quad (\text{A1})$$

where $p_{i,j} = p_a(z_i, t_j)$ and $I_{0,j-i} = I_s(t_j - z_i/c)$ at the origin. Source current I_s has a ramp waveform. Subsequently, magnetic potential $A_{i,j} = A_z(z_i, t_j)$ is expressed as

$$\begin{aligned} A_{i,j} &= \frac{\mu_0}{4\pi} \int_{\frac{z_i - ct_j}{2}}^{\frac{z_i + ct_j}{2}} \frac{I_a(l', t - |z_i - l'|/c)}{\sqrt{(z_i - l')^2 + r^2}} dl' \\ &= \frac{\mu_0}{4\pi} \left[\sum_{k=0}^{(j-i)/2} I_{0,j-i-2k} \times p_{k,j-i-k} \times f_{1,i}(k) \right. \\ &\quad + \sum_{k=0}^i I_{0,j-i} \times p_{k,j-i+k} \times f_{2,i}(k) \\ &\quad \left. + \sum_{k=i+1}^{(j+i)/2} I_{0,j+i-2k} \times p_{k,j+i-k} \times f_{2,i}(k) \right] \quad (\text{A2}) \end{aligned}$$

where

$$f_{1,i}(k) = \ln \frac{k + 1.5 + i + \sqrt{(k + 1.5 + i)^2 + (r/\Delta z)^2}}{k + 0.5 + i + \sqrt{(k + 0.5 + i)^2 + (r/\Delta z)^2}},$$

$$f_{2,i}(k) = \begin{cases} \ln \frac{|k-i|+0.5+\sqrt{(|k-i|+0.5)^2+(r/\Delta z)^2}}{|k-i|-0.5+\sqrt{(|k-i|-0.5)^2+(r/\Delta z)^2}}, & i \neq k \\ 2 \ln \frac{1}{r/\Delta z}, & i = k \end{cases}.$$

With initial values $p_{i,j}^{(0)} = 1 - j/N$, calculate $A_{i,j}$ with (A2), and update $p_{i,j}^{(m+1)}$ at the $(m+1)$ th iteration with

$$\Delta p_{i,j}^{(m+1)} = \Delta p_{i,j}^{(m)} \times e^{\gamma \times \frac{A_{i,j}^{(m)} - A_{i+1,j+1}^{(m)}}{A_{i,j}^{(m)}}} \quad (\text{A3})$$

where $\Delta p_{i,j}^{(m)} = p_{i+1,j+1}^{(m)} - p_{i,j}^{(m)}$ at the m th iteration, $p_{0,j}^{(m)} = 1$ at the source point, and γ is an damping coefficient. Note that the magnetic potential does not attenuate during its propagation [9], i.e., $A_{i,j} = A_{i+1,j+1}$. Once $A_{i,j}^{(m)} - A_{i+1,j+1}^{(m)}$ is nearly zero, the iterative process is completed.

As $A_{i,j} = 0$ for $i \geq j$, both electric potential and surge impedance on the perfect conductor are expressed as

$$\begin{aligned} \phi_{i,j} &= c \cdot \sum_{k=i}^j (A_{k,j} - A_{k,j-1}) = c \cdot A_{i,j} \\ Z_{i,j} &= \phi_{i,j} / I_{i,j}. \end{aligned} \quad (\text{A4})$$

The surge current, electric potential, and surge impedance can then be calculated with (A1), (A2), and (A4), respectively.

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