Modelling volume changes of sand under thermal loads: a preliminary attempt

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Abstract

Within the critical state framework, some thermo-mechanical models have been developed to simulate thermal volume changes of saturated soils. Most of the existing models are verified through experimental results of silt and clay, and they cannot well capture the behaviour of sand under thermal loads. In this study, an elastoplastic model is newly proposed to predict thermally-induced volume changes of saturated sand. The key component of the new model is the introduction of a state dependent threshold temperature, T_R, which can be determined experimentally. This T_R is found to increase with an increase in the state parameter, which is defined as the difference between the current void ratio and the critical state void ratio. Depending on the state parameter, heating would induce overall contraction at temperatures below T_R, but only cause overall expansion when the temperature is higher than T_R. When the maximum temperature experienced by soil specimen, T_y, is lower than T_R, heating would induce not only elastic but also plastic strains. The plastic modulus becomes larger when the difference between T_y and T_R is smaller. When T_y is larger than T_R, sand response becomes essentially elastic. The new model is applied to simulate the behaviour of Toyoura sand under heating at different stresses and densities. It is revealed that the measured and computed results are fairly consistent. The new model is able to well capture the state-dependent volume changes of sand under thermal loads.

Keywords

Constitutive relations; temperature effects; sands; deformation

Introduction

Some elastoplastic models have been reported in the literature (Hueckel & Baldi, 1990; Cui et al., 2000; Laloui & Cekerevac, 2008; Abuel-Naga et al., 2009; Zhou & Ng, 2015) to describe thermo-mechanical soil behaviour, which is crucial to the performance of energy structures (Vega & McCartney, 2014). Most of the existing models assume that heating-induced soil volume changes are strongly affected by the over-consolidation ratio (OCR). For normally consolidated and highly over-consolidated soils, plastic contraction and elastic expansion would be predicted, respectively. For slightly over-consolidated soil, the specimen is assumed to firstly expand and then contract. The above typical predictions are generally consistent with the behaviour of fine-grained soils (clay and silt), at least at a qualitative level. As far as the authors are aware, however, the existing models have not been verified through experimental results of sand.

To investigate thermally induced volume changes of sand, Kosar (1983) carried out a series of temperature-controlled oedometer tests on an oil sand at a medium dense state. From his data, one can deduce that there is a threshold temperature T_R, which is estimated to be about 35°C, below which the sand specimen contracted. In contrast, the specimen expanded when the temperature was above the T_R Similar observation can also be found from the data reported by Agar (1984). In these two studies, the effects of soil state (i.e., void ratio and stress state) were not considered. Recently, Ng et al. (2016) reported a series of temperature-controlled triaxial tests on Toyoura sand at different soil states. When the soil state was almost on the critical state line in the e – p' plane, where e and p' are void ratio and mean effective stress respectively, sand specimen contracted in the temperature range from 23 to 35°C and it expanded at temperatures between 35 and 50°C. A T_R can be identified from this test. On the contrary, for a denser specimen far below the critical state line, only expansion was observed when the temperature changed from 23 to 50°C. This implies that the T_R is state-dependent. It is evident that the observed experimental behaviour of sand is different from the typical predictions by using existing thermo-mechanical models.

In the present study, a thermo-mechanical model is developed for saturated sand at isotropic stress condition. To capture the elastoplastic volume changes induced by thermal loads, some new formulations such as state-dependent T_R are introduced and defined. Moreover, details of model formulations, calibration of soil parameters and model verification are presented.

Mathematical formulations

State parameter

The current model is developed for saturated sand in the p'-T-e plane, where p', T and e are mean effective stress, temperature and void ratio, respectively. The state parameter ψ, which was proposed by Been & Jefferies (1985), is adopted to describe the state of soil:

$$\psi = e - e_c \tag{1}$$

where e_c is the critical state void ratio corresponding to the current mean effective stress. Parameter ψ quantifies how far the soil state is from the critical sate line in the e-p' plane. For soil states on the 'dry' and 'wet' sides of the critical state line, the values of ψ are negative and positive, respectively. So far, ψ has been widely used for in the development of constitutive models (Xiao et al., 2013; Zhou & Ng, 2016).

State-dependent threshold temperature

As stated in the Introduction, previous studies consistently reveal that there is a T_R, below which a sand specimen contracts during the first heating (Kosar, 1983; Agar, 1984; Ng et al., 2016). On the contrary, the specimen expands when the temperature is higher than T_R. Moreover, Ng et al. (2016) found that T_R increases with an increasing stress but with a decreasing density. Based on these observations, the concept of state-dependent T_R is proposed and incorporated in the current model as follows:

$$T_R = T_0(1+\psi) \tag{2}$$

where T_0 is a reference temperature, which is equal to the value of T_R for soil state on the critical state line in the $e-p'$ plane (i.e., $\psi = 0$), as shown in Figure 1.

On the other hand, it is assumed that the yield temperature T_y equals to the maximum temperature in the stress history. T_y serves as the boundary of elastic range during heating. When the current temperature is lower than T_y, thermally induced soil response is assumed to be elastic. When the current temperature reaches T_y, sand response would be elastoplastic at temperatures below T_R. On the contrary, only elastic expansion occurs when the current temperature and T_y exceed T_R. According to Figure 1 and equation (2), sand specimen with a higher state parameter exhibits elastoplastic behaviour at a larger temperature increment (see Figure 1). This is consistent with the experimental results of Ng et al. (2016). Furthermore, the accumulation rate of plastic thermal strain is assumed to decrease when the difference between T_y and T_R decreases. To quantify the difference between T_y and T_R, a new variable R

is defined as follows:

$$R = 1 - T_y / T_R \tag{3}$$

According to equation (3), the value of R is close to 1 when T_y is far below T_R. During the first heating process, T_y continuously increases. On the contrary, as revealed by existing experimental results (Agar, 1984; Ng et al., 2016), the change in soil void ratio is very small (less than 0.01) during heating and cooling. It is therefore reasonable to expect that thermally induced evolution of T_R is almost negligible (see equation (2)). Consequently, the value of R would continuously decrease during heating. It should be pointed out that when the current model is used to predict soil behaviour under thermal loads, T_R and ψ are not assumed constant. T_R depends on ψ (see equation (2)), which would change with the accumulation of thermal volumetric strain.

Elastoplasticity

The current model is developed for sand specimen at the isotropic stress condition. Only volumetric strain $d\varepsilon_v$ (zero deviator strain) is induced by heating. The elastic and plastic components of $d\varepsilon_v$ are determined through a decoupled approach:

$$d\varepsilon_v = d\varepsilon_v^e + d\varepsilon_v^p \tag{4}$$

where $d\varepsilon_v^e$ and $d\varepsilon_v^p$ are elastic and plastic increments of volumetric strains, respectively. They are calculated using the following two equations:

$$d\varepsilon_v^e = -\alpha_s \, dT \tag{5}$$

$$d\varepsilon_v^p = \frac{1}{K_T^p} dT \tag{6}$$

where α_s is the volumetric thermal expansion coefficient of soil skeleton; K_T^p is the plastic modulus for thermal loading. Based on experimental results of Ng et al. (2016), α_s can be assumed to be constant for simplicity. Parameter K_T^p is strongly affected by soil state and the following equation is proposed for it:

$$\frac{1}{K_T^p} = \alpha_s \, m \langle R \rangle^2 \left(p' / p_a \right)^{0.5} \tag{7}$$

where m is a parameter controlling the accumulation rate of plastic thermal strain; p_a is the atmospheric pressure (103 kPa). $\langle R \rangle = R$ if $R > 0$ and $\langle R \rangle = 0$ if $R \leq 0$. The McCauley bracket $\langle \; \rangle$ is used to avoid the prediction of non-zero plastic thermal strain when $T_y > T_R$, as previous experimental results suggest that sand specimen does not show plastic contraction if the temperature exceeds T_R (Agar, 1984; Ng et al., 2016). The dependency of plastic moduli on $p'^{-0.5}$ is consistent with many existing elastoplastic models for sand (for example, Dafalias & Manzari (2004)).

It should be noted that thermal volume changes are not only dependent on state parameter. They are also affected by some other factors such as loading rate and stress path (Towhata et al., 1993; Burghignoli et al., 2000; Coccia & McCartney, 2016a; Coccia & McCartney, 2016b).

Calibration of soil parameters

Three parameters are defined in the proposed model: T_0, m and α_s. According to equation (2), parameter T_0 can be calibrated through one heating and cooling test with determinable T_R and ψ as follows:

$$T_0 = \frac{T_R}{1+\psi} \tag{8}$$

Based on equation (5), the value of parameter α_s is expressed as

$$\alpha_s = -\frac{d\varepsilon_v^e}{dT} \tag{9}$$

It is assumed in the current model that soil response at temperatures above T_R is elastic. Hence, the value of α_s can be calculated from the relationship between thermal strain and soil temperature in this temperature range where $T_y > T_R$. Alternatively, by assuming that soil response during the cooling phase is elastic, the value of α_s can be calibrated from the results of a cooling test.

Equations (4) through (6) suggest that the incremental thermal strain of soil is equal to zero when $\alpha = 1/K_T^p$. From the soil state with $d\varepsilon_v/dT = 0$, parameter m can be calibrated using the following equation:

$$m = \frac{1}{\langle R \rangle^2 \left(p'/p_a\right)^{0.5}} \tag{10}$$

Based on equations (8) through (10), all of the three model parameters can be calibrated from experimental results. These equations are applied to calibrate the model parameters for Toyoura sand later.

Verification of the proposed model

It should be noted that there are limited studies of thermally induced volume changes of sand (Kosar, 1983; Agar, 1984; Ng et al., 2016). Among these studies, as far as the authors are aware, only Ng et al. (2016) reported sufficient information for calibrating advanced new constitutive model for sands. One of the four tests carried out by Ng et al. (2016), was used to calibrate the newly proposed model, whereas the other three remaining tests were used to verify the new model independently.

Figure 2 shows stress paths of the four heating tests reported by Ng et al. (2016). In each test, soil specimen with a given density was prepared using the air pluviation method (Vaid & Negussey, 1988). Three relative densities were considered with D_r values of 20%, 70% and 90%. Each specimen was saturated and then compressed to the predefined stress (p' = 50 and 200 kPa) under the isotropic stress condition. At a controlled density and stress, the ψ of each specimen is calculated using equation (1) with $e_c = 0.934 - 0.019\,(p'/101)^{0.7}$, which was measured and proposed by Li & Wang (1998). The values of ψ fall in the range of -0.01 (for the soil state lies almost on the critical state line) and -0.29 (for the soil state locates on the 'dry' side of the critical state) (see Table 1).

After consolidation, each specimen was heated in slow steps under the drained condition (23→30→ 40→ 50°C). The temperature of water circulated surrounding soil specimen was increased instantaneously at each thermal stage. For each temperature increment, it took about 4 hours for the soil temperature to reach an equilibrium state. Soil temperature was maintained at the target value for 6 hours, which is sufficient for all specimens to reach the equilibrium state. Thermally induced volume changes of each specimen were measured using a GDS controller. More details of the test program and procedures were reported by Ng et al. (2016).

From the results of one heating test (D20S200 in Table 1: ψ = -0.01; D_r = 20%; p' = 200 kPa), the three model parameters are determined using equations (8) through. It can be seen from Figure 3 that starting from about 45°, the specimen D20S200 expands at a rate of 1×10^{-5} per °C, suggesting that T_0 = 45°C, m = 30 and $\alpha_s = 1\times10^{-5}$ per °C. Moreover, the accumulation rate of thermal strain is zero at about 38°C. This observation suggests that at 38°C, the incremental plastic contraction is equal to the incremental elastic expansion. Based on the soil state at this condition (i.e., $d\varepsilon_v/dT = 0$), the value of m can be calculated using equation (10): m = 30. These parameters are then applied to predict thermally induced volume changes of Toyoura sand at three other states (D70S200, D90S200 and D20S50 in Table 1).

Figure 3 compares measured and computed thermal strains of Toyoura sand at a confining stress of 200 kPa but with three different initial densities (D_r = 20%, 70% and 90%). It is clear that at each density, the measured and computed results are fairly consistent. The proposed model is able to well capture the state-dependent volume changes of sand under heating. When soil state is almost on the critical state (D_r = 20%; ψ = -0.01), the value of T_R calculated using equation (2) is about 45°C. The computed soil response is therefore elastoplastic in the temperature range of 23 to 45°C. Moreover, the values of R and $1/K_T^p$ decrease (see equation (3) and (7)) when the temperature increases from 23 to 45°C. Consequently, the computed accumulation rate of plastic thermal strain becomes smaller as soil temperature increases. When temperature exceeds 45°C, R and $1/K_T^p$ are zero, with elastic expansion computed by the proposed model. This type of behaviour (first contraction and then expansion during heating) may not be predicted by existing thermo-mechanical models (Hueckel & Baldi, 1990; Cui et al., 2000; Laloui & Cekerevac, 2008; Abuel-Naga et al., 2009; Zhou & Ng, 2015), as discussed in the Introduction. When the state parameter of soil decreases from -0.01 (D_r = 20%) to -0.19 (D_r = 70%), the value of T_R decreases from 45 to 36°C. The values of R, $1/K_T^p$ and thermal strains are therefore much smaller, consistent with the measured results. When soil state is far below the critical state line (D_r = 90%; ψ = -0.29) in the e - p' plane, the calculated T_R (32°C) is close to the initial value of T_y (23°C). Hence, R and $1/K_T^p$ are very small, and soil specimen shows continuous overall expansion during the heating process.

Figure 4 compares measured and computed thermal strains of Toyoura sand with the same initial density (D_r = 20%) but at different confining pressures (p' = 50 and 200 kPa). When the confining pressure increases from 50 to 200 kPa, the measured thermal contraction of sand shows a significant increase. At both stress states, the accumulation of thermal strain under heating is well captured by the proposed model. The capability of the proposed model for simulating stress effects is mainly attributed to its two key features. Firstly, the value of ψ is larger at 200 kPa than that at 50 kPa (see Table 1). With a higher ψ, T_R is higher (see equation (2)), suggesting a wider elastoplastic range under heating. Secondly, stress effects on $1/K_T^p$ are considered through the term $(p'/p_a)^{0.5}$ in equation (7). As the confining pressure increases, the value of $1/K_T^p$ becomes smaller. With a wider elastoplastic range and a smaller $1/K_T^p$, the plastic thermal contraction is therefore larger.

Summary and conclusions

A new elastoplastic model is developed to predict the volume change behaviour of sand under thermal loads. The proposed model incorporates the concept of threshold temperature T_R, which is observed to increase with an increase in state parameter. When the maximum temperature experienced by a specimen T_y is lower than the T_R, both elastic expansion and plastic contraction would be induced by heating. The plastic modulus is higher when the difference between T_y and T_R is smaller. When T_y exceeds T_R, soil responses are assumed to be elastic expansion.

The new model is calibrated by one test and applied to simulate the behaviour of three other tests on Toyoura sand during heating at different states (a function of both stress and density). It is evident that the measured and computed thermal strains are found to be consistent at different states generally.

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Notation

K_T^p	plastic modulus
R	Variable defined as 1 - T_y / T_R
T	soil temperature
T_R	threshold temperature
Ty	yield temperature
T_0	a reference temperature
$d\varepsilon_v$, $d\varepsilon_v^e$, $d\varepsilon_v^p$	incremental volumetric strain: total, elastic and plastic
e	current void ratio
e_c	void ratio at the critical state
m	soil parameter controlling the magnitude of plastic thermal strain
p'	mean effective stress
p_a	atmospheric pressure (101 kPa)
q	deviator stress
α_s	thermal expansion coefficient of soil skeleton
υ	specific volume
ψ	state parameter of soil

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Table 1. Details of experimental program

Specimen Identity	Relative density D_r	Void ratio e	Mean effective stress p': kPa	State parameter ψ	Stress path (see Figure 2)
D20S200	20%	0.89	200	-0.012	$C \rightarrow F \rightarrow F_1$
D70S200	70%	0.71	200	-0.192	$B \rightarrow E \rightarrow E_1$
D90S200	90%	0.62	200	-0.282	$A \rightarrow D \rightarrow D_1$
D20S50	20%	0.90	50	-0.021	$C \rightarrow G \rightarrow G_1$

Note: The values of D_r, e and ψ prior to thermal cycle are shown in this table.

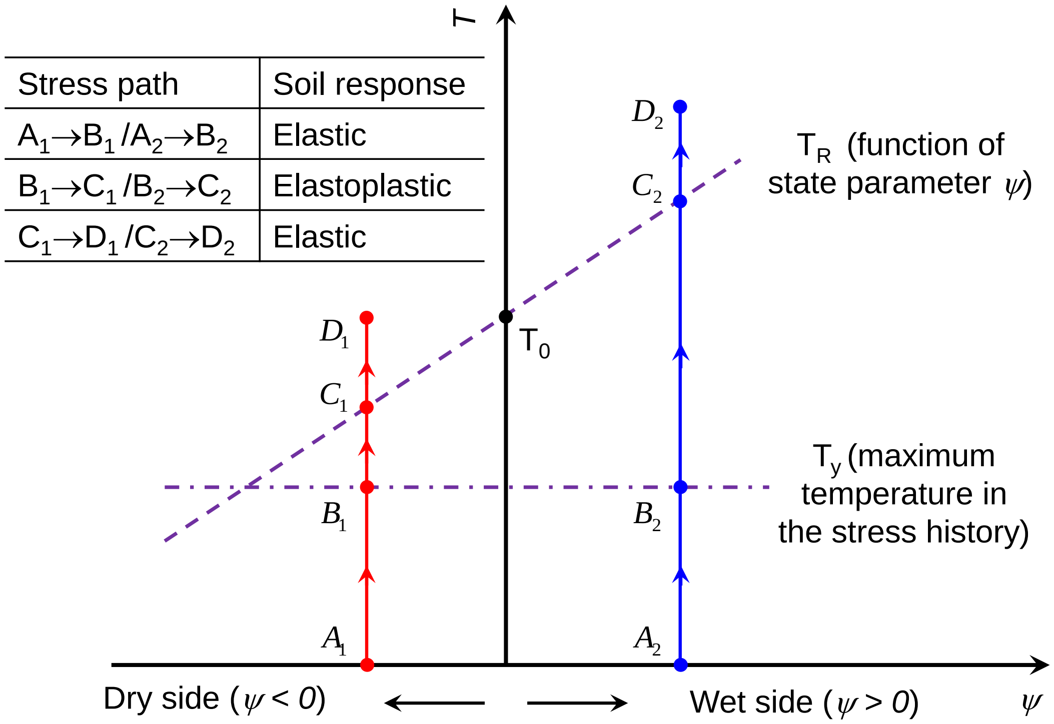

Figure 1. Threshold temperature T_R and yield temperature T_y in the $T - \psi$ plane

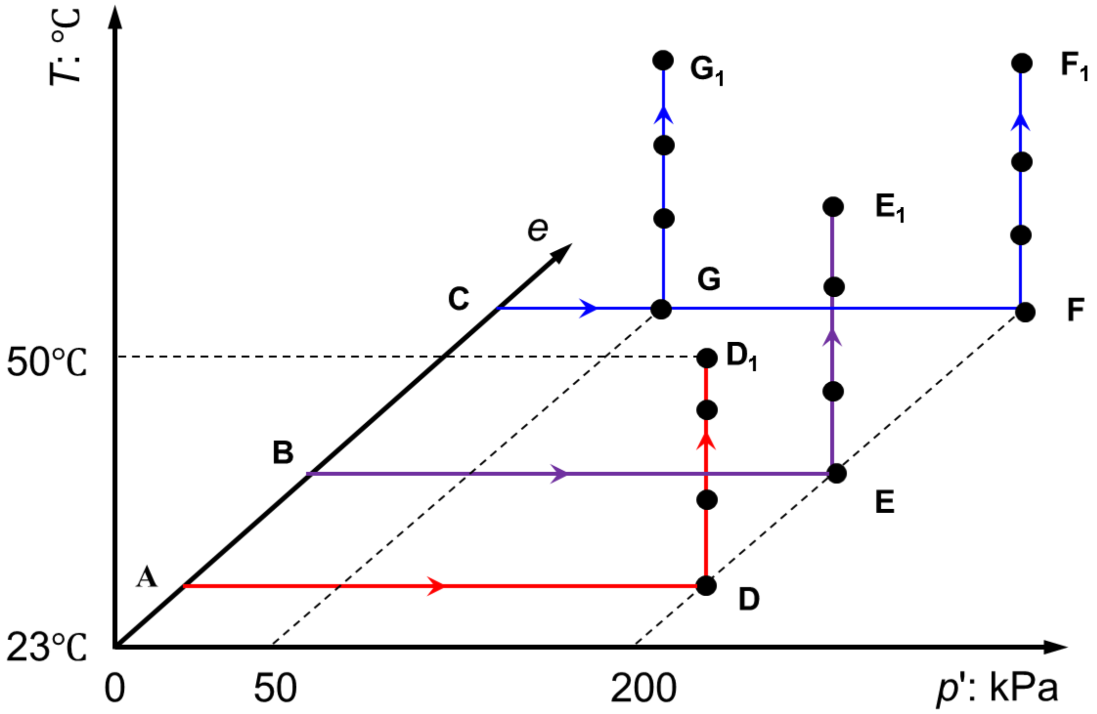

Figure 2. Stress paths of heating tests on saturated Toyoura sand

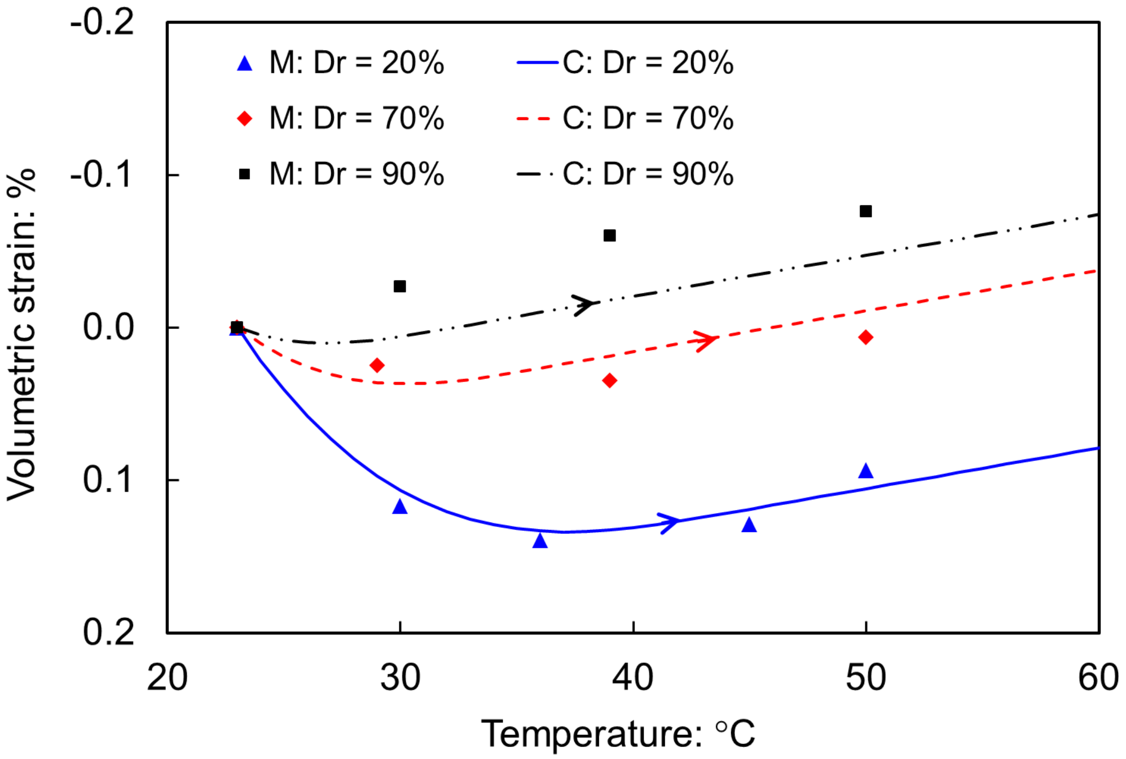

Figure 3. Comparisons between measured (M) and computed (C) thermal strains at different initial densities and at a confining stress of 200 kPa

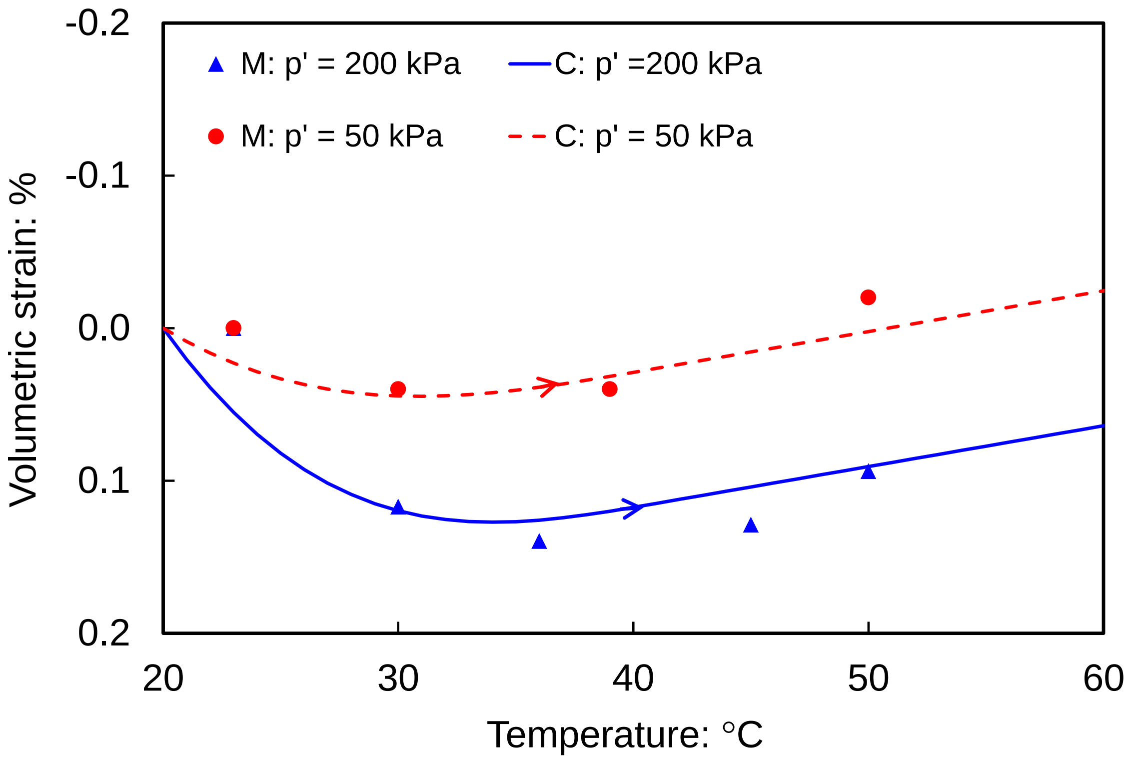

Figure 4. Comparisons between measured (M) and computed (C) thermal strains at different confining stresses with an initial D_r of 20%