

# **Effects of boundary conditions on cyclic thermal strains of clay and sand**

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**Abstract:** The thermal oedometer has been widely adopted to determine soil deformation under heating and cooling. Very often an oedometer ring made of steel is used to laterally confine a soil specimen. To investigate and quantify the effects of boundary conditions (i.e., thermal expansion and contraction) of an oedometer ring on soil thermal strains, in this study steel and invar rings were adopted for comparisons. The former has a linear thermal expansion coefficient (TEC) of  $1.5 \times 10^{-5} \text{ m}/(\text{m } ^\circ\text{C})$ , which is within the typical range of  $5 \times 10^{-6}$  to  $3.5 \times 10^{-5} \text{ m}/(\text{m } ^\circ\text{C})$  for soil skeletons. The linear TEC of the latter is  $5.0 \times 10^{-7} \text{ m}/(\text{m } ^\circ\text{C})$ , which is substantially smaller than those of soils. Axial strains of saturated normally consolidated clay and loose sand subjected to thermal cycles were measured in these two types of oedometer rings. The accumulated plastic strains of clay and sand measured in the steel ring were 32% and 11% larger than those measured in the invar ring, respectively. As expected, the expansion and contraction of the steel ring itself resulted in additional soil deformation during thermal cycles. It was also found that a popular equation, which assumes that the soil volume remains constant during the expansion and contraction of the oedometer ring, overestimates the influence of boundary conditions on the incremental axial strain of soil by more than threefold.

**Key words:** oedometer test; strain; temperature; repeated loading

## Introduction

The thermal oedometer has been widely used to study soil volume change behaviour under thermal loading (Romero et al., 2003; Abuel-Naga et al., 2007; François and Laloui, 2010; Vega and McCartney, 2015; Di Donna and Laloui, 2015). In most of the previous tests, an oedometer ring made of steel was used to laterally confine a soil specimen. The linear thermal expansion coefficient (TEC) of steel ( $1.5 \times 10^{-5} \text{ m}/(\text{m } ^\circ\text{C})$ ) is comparable to that of the soil skeleton ranging from  $5 \times 10^{-6}$  to  $3.5 \times 10^{-5} \text{ m}/(\text{m } ^\circ\text{C})$  (Campanella and Mitchell, 1968; Vega and McCartney, 2015; Ng et al., 2016). But during heating and cooling, the steel ring experiences expansion and contraction, imposing non-negligible lateral strain on a soil specimen. To estimate the influence of lateral strain on the measured axial strain of a soil specimen, Romero (1999) and Abuel-Naga et al. (2007) proposed the following equation:

$$\Delta \varepsilon_a = 2\alpha_{or}\Delta T \quad (1)$$

where  $\Delta \varepsilon_a$  is the additional axial strain of a soil specimen induced by the expansion and contraction of the oedometer ring.  $\alpha_{or}$  and  $\Delta T$  are the linear TEC of the oedometer ring and the change in temperature, respectively. This equation assumes that the volume of a soil specimen remains constant despite the expansion and contraction of the oedometer ring. Given this assumption, the equation may not be able to accurately capture the behaviour of soil which shows significant volume changes under thermal and mechanical loads. Up to now, however, the Equation 1 has not been experimentally verified.

In this study, a series of tests were carried out to measure the thermal axial strains of normally consolidated clay and loose sand under cyclic heating and cooling. To consider the effects of boundary conditions on the measured axial strains of soil specimens, two types of oedometer rings (steel and invar rings) were used. The linear TECs of steel and invar are  $1.5 \times 10^{-5}$  and  $5.0 \times 10^{-7}$  m/(m °C), respectively. The TEC of steel is close to that of the soil skeleton, while the TEC of invar is 30 times smaller and hence its expansion and contraction can be ignored. Given that the Young's modulus of the invar and steel are about 145 GPa and 180 GPa, respectively, the high Young's modulus of the two metals can maintain zero lateral conditions under axial mechanical loading (ASTM D2435/D2435M-11). Thus, these two rings were used to investigate the cyclic thermal behaviour of saturated clay and sand. In addition, Equation 1 was used to evaluate the experimental results.

### **Test apparatus**

Figure 1 shows the schematic diagram of a thermal oedometer, which is modified from a conventional oedometer by adding a heating and cooling system. The system consists of a heating and cooling unit, a water bath, a pump, a spiral tube, a water tank and thermocouples. During testing, the water temperature in the bath is controlled by the heating and cooling unit. Water with a controlled initial temperature is circulated through the spiral tube which runs around the soil specimen. The soil specimen is heated or cooled through heat exchange with the circulating water. To reduce energy loss and to minimize soil temperature fluctuation, the oedometer is insulated with

foam boards. At thermal equilibrium state, the fluctuation of soil temperature in the oedometer is less than 0.1°C.

### **Test material and specimen preparation**

Through isotropic tests, Ng et al. (2016) found that the thermal volume change behaviour of the coarse-grained soil is qualitatively different from those of fine-grained soils. Oedometer and isotropic tests show that the heating-induced volume changes of fine-grained soils are strongly affected by the overconsolidation ratio (Hueckel and Baldi, 1990; Delage et al., 2000; Cekerevac and Laloui, 2004). For normally consolidated and highly overconsolidated soils, plastic contraction and elastic expansion are respectively predicted. Lightly overconsolidated soils are assumed to expand first before contracting. On the contrary, the coarse-grained soil such as loose and medium dense sand contract first before expanding beyond a certain temperature in isotropic tests (Ng et al., 2016). Clearly the thermal volume change behaviour of the coarse-grained soil is qualitatively different than those of fine-grained soils. Hence, both coarse-grained and fine-grained soils were tested in the current study to investigate the influence of boundary conditions on soil strain. A clay of low plasticity (ASTM D2487-11) and Toyoura sand were adopted. The physical properties of the clay are summarized in Table 1. The properties of Toyoura sand were reported by Verdugo and Ishihara (1996).

The clay specimens were prepared using the static compaction method. The compaction water content and initial dry density were 9.2% and 1.18 g/cm<sup>3</sup>,

respectively. The clay was first oven-dried and then passed through a 2 mm British Standard (BS) sieve. The prepared soil was evenly spread on a plastic plate and de-aired water was sprayed on it to increase its water content to 9.2%, which is identical to the in-situ water content. The wetted soil was mixed thoroughly with a blender. Soil particles clumped together during this process. To prepare homogeneous specimens, the mixture was passed through the 2 mm BS sieve again and kept in a sealed plastic bag for moisture equalisation. The Toyoura sand specimens with an initial relative density of 21% were prepared using the air pluviation method.

### **Test program and procedures**

Two series of cyclic heating and cooling tests were carried out. In the first series, two identical clay specimens (CS and CI) were tested along the same thermo-mechanical paths using oedometer rings made of steel and invar, respectively. The measured soil strains were compared to reveal the influence of boundary conditions on the thermal axial strain of saturated clay. Similarly, the second series of tests (SS and SI) were designed to study the effects of boundary conditions on the thermal axial strain of saturated sand. The test program is summarized in Table 2.

After specimen preparation, de-aired water was added to the cell to submerge and saturate the specimen for 24 hours. To achieve a higher degree of saturation, a vacuum of -80 kPa was applied and maintained in the oedometer cell to remove air bubbles within soil specimens. These saturation procedures were adopted for both clay and sand specimens. Moreover, one-dimensional soil deformation during the

saturation stage was measured using a LVDT. For the tested clay and sand specimens, the deformation was only 0.03% and 0.05% (loose sand), respectively. After 24 hours, the specimens were compressed to an effective vertical stress of 50 kPa. The consolidation process was considered to be completed when the incremental axial strain was less than 0.025% within 24 hours (Romero et al., 2003). According to critical state soil mechanics (Wood, 1990; Been and Jefferies, 1985), soil behaviour depends not only on the void ratio but also on the stress level. In this study, the void ratio and effective axial stress of the sand specimens before applying thermal loading were 0.885 (relative density of 21%) and 50 kPa, respectively. The soil state thus fell on the dry side of the critical state line, as measured by Li and Wang (1998). For the clay specimens, the soil state fell on the wet side of the critical state line. The value of the state parameter (Been and Jefferies, 1985) for each specimen is calculated and summarized in Table 2.

After mechanical consolidation, the specimens were subjected to cyclic thermal loads under drained conditions at temperatures ranging from 15 to 70°C. The rates of thermal loading and unloading suggested by Di Donna and Laloui (2015) were used in the current study: 2 and 5°C/hour during heating and cooling, respectively. The test was terminated when the incremental plastic strain of a thermal cycle was less than 0.025% (close to the accuracy of the LVDT measurements,  $\pm 0.02\%$ ). Following this criterion, five and three thermal cycles were applied to the clay and sand specimens, respectively.

## Interpretation of experimental results

### *Effects of boundary conditions on thermal strain of clay*

Figure 2(a) shows the thermal axial strain of clay measured in the oedometer ring made of steel. The measured soil settlement during mechanical loading is also shown in Figure 2(a). The effective vertical stress was increased from zero to 50 kPa for consolidation. After 48 hours, the rate of axial strain accumulation was  $7.2 \times 10^{-4}\%$ /hour. The consolidation was considered to be completed following the criterion (i.e.,  $1.0 \times 10^{-3} \%$ /hour) proposed by Romero et al. (2003). The axial strain at equilibrium state was 16.7%. During the first heating process, an axial strain of 0.43% was induced. Only a minuscule amount of axial strain (less than 0.05%) was recovered during the subsequent cooling process. As the number of thermal cycles increased, the irreversible axial strain accumulated but at a decreasing rate. After four thermal cycles, the soil response almost reached a stable state, at which the incremental irreversible strain was negligible. A similar trend of plastic strain accumulation, induced by cyclic thermal loads, was reported by previous researchers who conducted isotropic (Campanella and Mitchell, 1968) and oedometer tests (Donna and Laloui, 2015; Vega and McCartney, 2015).

Figure 2(b) shows the thermal axial strain of clay tested in the oedometer ring made of invar. The soil settlement during mechanical loading is also shown in Figure 2(b). After 48 hours, the rate of axial strain accumulation induced by mechanical loading was  $4.1 \times 10^{-4}\%$ /hour, which satisfies the termination criterion of consolidation



(i.e.,  $1.0 \times 10^{-3}\%$ /hour). In addition, an axial strain of 16.9% was induced by the mechanical loading. The measured soil behaviour, such as the accumulation of plastic strain with an increasing number of thermal cycles, follows a similar trend to that in Figure 2(a). At a given number of thermal cycles, however, the irreversible axial strain measured in the steel ring was consistently larger than that determined in the invar ring. For example, with five thermal cycles, the accumulated plastic axial strain in the steel ring (i.e., 0.66%) was approximately 32% larger than that in the invar ring (i.e., 0.50%). The reason is that the TEC of steel is 30 times larger than that of invar. During heating, the steel ring significantly expands, while the expansion of the invar ring is negligible. The vertical surfaces characterising the exterior and interior boundaries of the steel ring (considering a vertical cross section passing through the diameter of the ring) move horizontally towards the exterior boundary surface. Consequently, the soil specimen may be subjected to a lower constraint from the steel ring, inducing additional settlement of the soil specimen. A gap could potentially form between a soil specimen and the oedometer ring. This occurrence may further increase the thermal contraction of the specimen when it is put in the steel ring as compared to the invar ring.

Using a thermal oedometer equipped with a steel ring, Vega and McCartney (2015) investigated the volume change behaviour of a saturated recompacted silt under cyclic thermal loads. They found that there was an obvious delay of temperature variation between the steel ring and soil. Due to this delay, the steel ring

expanded or contracted before the soil specimen did, affecting the transient response of the soil specimen. In the current study, similar transient effects were observed in the tests using the steel ring, but not in the ones involving the invar ring.

Figure 3 shows the difference between axial strains measured in the steel ring ( $\Delta\epsilon_{a,steel}$ ) and the invar ring ( $\Delta\epsilon_{a,invar}$ ). Results calculated from Equation 1 are also included in the figure for comparison. During the heating process,  $\Delta\epsilon_{a,steel}$  was consistently larger than  $\Delta\epsilon_{a,invar}$  as expected. Their difference narrowed significantly as the number of thermal cycles increased. During the cooling process, the difference between  $\Delta\epsilon_{a,steel}$  and  $\Delta\epsilon_{a,invar}$  was almost constant. More importantly, Equation 1 clearly overestimates the difference between  $\Delta\epsilon_{a,steel}$  and  $\Delta\epsilon_{a,invar}$  by more than threefold for both processes. Furthermore, during the heating process, the difference between calculated and measured results increased with an increase in the number of thermal cycles (see Figure 2). This is likely because Equation 1 is deduced based on the constant volume assumption. However, the heating-induced expansion of the steel ring would alter the lateral soil stress and hence induce soil volume changes. Equation 1 overestimates  $\Delta\epsilon_{a,steel}-\Delta\epsilon_{a,invar}$  when the soil specimen expands and underestimates it when the soil specimen contracts. According to Chiu and Ng (2003), the volumetric strains of saturated isotropic soil depend on the changes in mean effective stress ( $p'$ ) and the stress ratio ( $q/p'$ ):

$$\Delta\epsilon_v = f(\Delta p') + g[\Delta(q/p')] \quad (2)$$

where  $\Delta\epsilon_v$  is the total volumetric strain;  $f(p')$  and  $g(q/p')$  are the functions of

$p'$  and  $q/p'$ , respectively. During the first heating process, the release of lateral constraint causes a reduction in  $p'$  and an increase in  $q/p'$ , considering that the coefficient of lateral stress at rest ( $K_0$ ) is generally less than 1 for normally consolidated soil. The reduction in  $p'$  results in elastic volumetric expansion, while the increase in  $q/p'$  induces plastic volumetric contraction of the normally consolidated soil. The differences between the calculated and measured results are relatively small, because the elastic expansion and plastic contraction offset each other. With an increase in the number of thermal cycles, the over-consolidation ratio of the soil specimen increases due to the accumulation of plastic volumetric contraction. The plastic volumetric contraction induced by an increase in  $q/p'$  becomes smaller, and the elastic volume change would dominate soil behaviour. Consequently, the differences between the calculated and measured results increase.

During the cooling process, the differences between measured and calculated results are independent of the number of thermal cycles. This is likely because a limited amount of plastic contraction is induced by shearing during this process. The constant difference between calculated and measured results is mainly due to the elastic contraction of the soil specimen induced by the increase in lateral constraint.

#### *Effects of boundary conditions on thermal strain of sand*

Figure 4(a) shows the thermal axial strain of sand tested in the oedometer ring made of steel. During the first heating process, the sand specimen showed continuous settlement and an axial strain of 0.10% was induced. This observation is different

from that reported by Ng et al. (2016) who conducted isotropic tests on Toyoura sand with similar state parameters ( $\Psi = -0.021$  and  $-0.040$  in the previous and current studies, respectively). They found that at constant isotropic stress, the sand contracted at temperatures from 20 to 35°C but dilated at temperatures from 35 to 55°C. The discrepancy between the results of the two studies suggests that the measurements with the steel ring do not reflect the actual thermal strain of sand, due to the change in boundary conditions. During the heating process, the steel ring would show an obvious expansion, while the expansion of the invar ring is negligible. The different thermal responses of the steel and invar rings would induce different levels lateral constraint on soil specimens. The expansion of the steel ring would induce additional settlement of soil specimens. On the other hand, the plastic axial strain of sand accumulated at a decreasing rate with an increasing number of thermal cycles, similar to the response of clay (see Figure 2).

Figure 4(b) shows the thermal axial strains of sand tested in the oedometer ring made of invar. Different from the results of tests using the steel ring (see Figure 4(a)), the sand specimen in the invar ring contracted at temperatures from 23 to 50°C and then dilated at temperatures from 50 to 70°C during the first heating process. It can be seen from Figure 4(b) that sand specimen in the invar ring contracts during cooling. The trend is similar to that obtained from isotropic tests, as discussed above. Moreover, a comparison between Figures 4(a) and 4(b) reveals that the irreversible axial strain induced by three thermal cycles was approximately 11% larger in the steel

ring (i.e., 0.10%) than in the invar ring (i.e., 0.09%). The difference is attributed to the thermal expansion and contraction of the steel ring.

Furthermore, it can be seen from Figure 4 that during the cooling process, sand specimens in the steel and invar rings experienced heave and settlement, respectively. The observation of cooling-induced heave seems to contradict existing theoretical and isotropic test results, which generally suggest that soil contracts during cooling (Cui et al., 2000; François and Laloui, 2008; Zhou and Ng, 2015; Campanella and Mitchell, 1968; Coccia and McCartney, 2016). This discrepancy can be explained by the fact that the actual soil thermal contraction is offset by the non-negligible contraction of the steel ring.

Figure 5 shows the measured and calculated differences between axial strains of sand specimens measured in tests using the steel ring ( $\Delta\epsilon_{a,steel}$ ) and the invar ring ( $\Delta\epsilon_{a,invar}$ ). Different from the results in Figure 3, during not only the heating process but also the cooling process, the difference between  $\Delta\epsilon_{a,steel}$  and  $\Delta\epsilon_{a,invar}$  was almost constant. This is likely because the sand was much stiffer than the clay at the range of stresses and temperatures considered. Shearing-induced plastic contraction is expected to be negligible. Therefore, a constant difference (dependent on the elastic expansion or contraction of soil specimens) between the measured and calculated results was observed during both heating and cooling. Furthermore, Equation 1 overestimates the difference between  $\Delta\epsilon_{a,steel}$  and  $\Delta\epsilon_{a,invar}$  by more than threefold for both processes.

## **Conclusions**

Measurements obtained from oedometer tests using invar and steel rings show that the settlement of clay and sand accumulates with the number of thermal cycles, but at a decreasing rate until it reaches a stable state. The accumulated plastic axial strains of normally consolidated clay and loose sand measured in the steel ring are approximately 32% and 11% larger than those recorded in the invar ring, respectively. The larger soil settlement measured in the steel ring is due to the fact that the linear thermal expansion coefficient of steel is 30 times larger than that of invar. Thus, vertical strains measured in a steel ring should be treated with great caution due to the marked role of the oedometer-soil interaction.

An existing well-known equation which considers the influence of boundary conditions on soil axial strains was experimentally evaluated. The equation assumes that the soil volume remains constant during the thermal expansion and contraction of the oedometer ring. Our results show that the equation can overestimate the influence of boundary conditions on axial strains by over threefold.

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## **List of Figures**

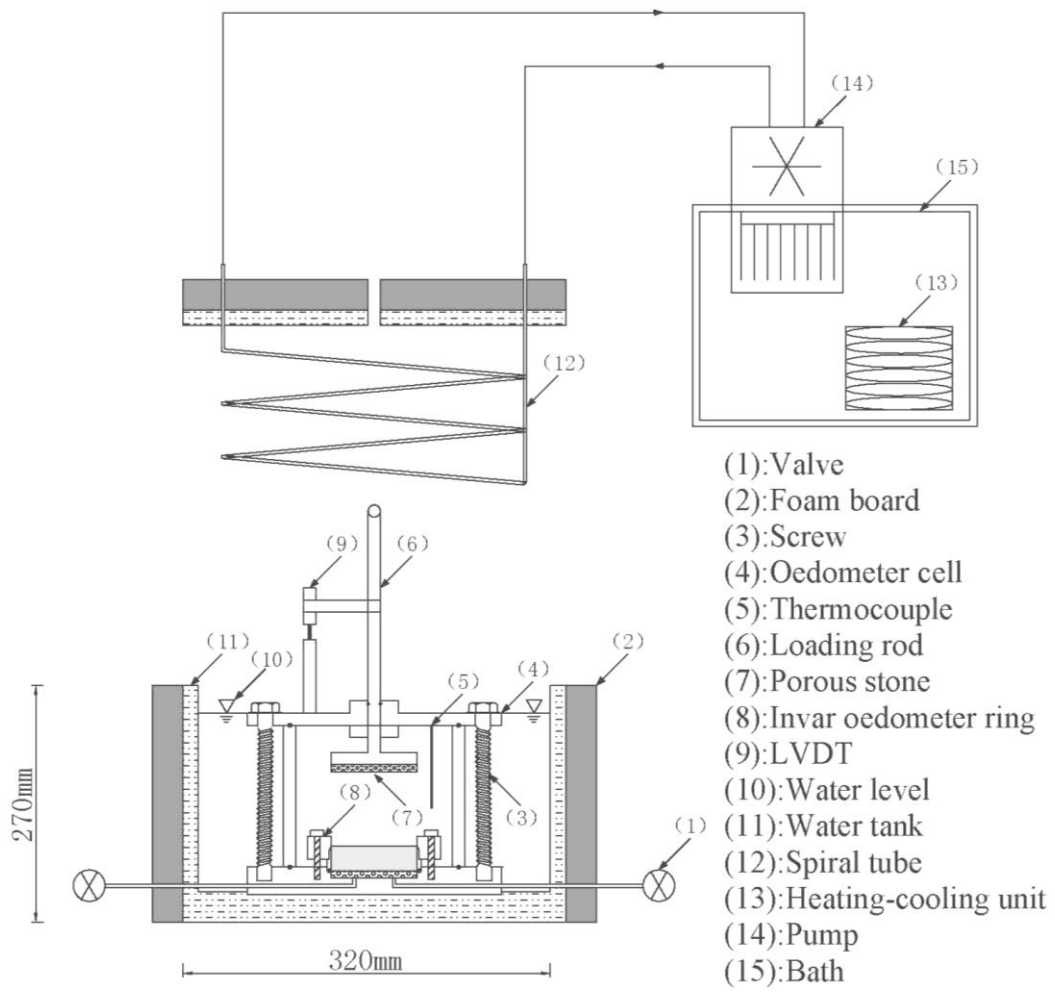
Figure 1. Schematic diagram of the thermal oedometer

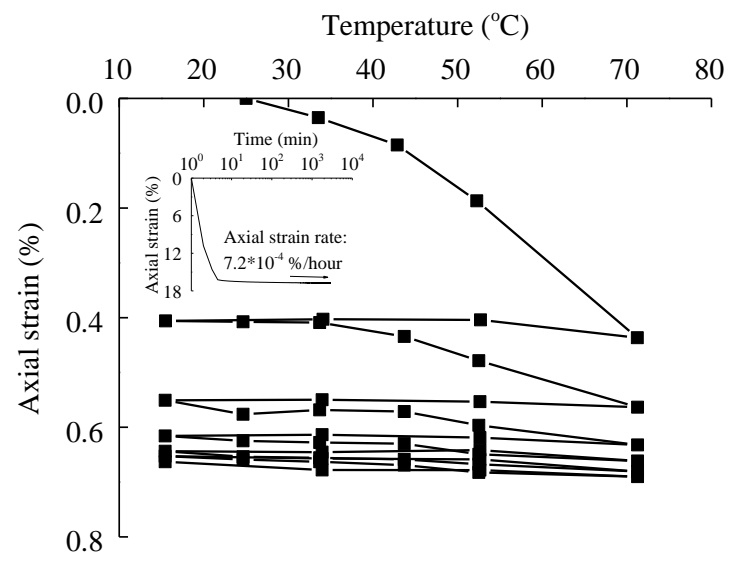
Figure 2. Thermal axial strains of NC clay measured in oedometer rings made of (a) steel; (b) invar

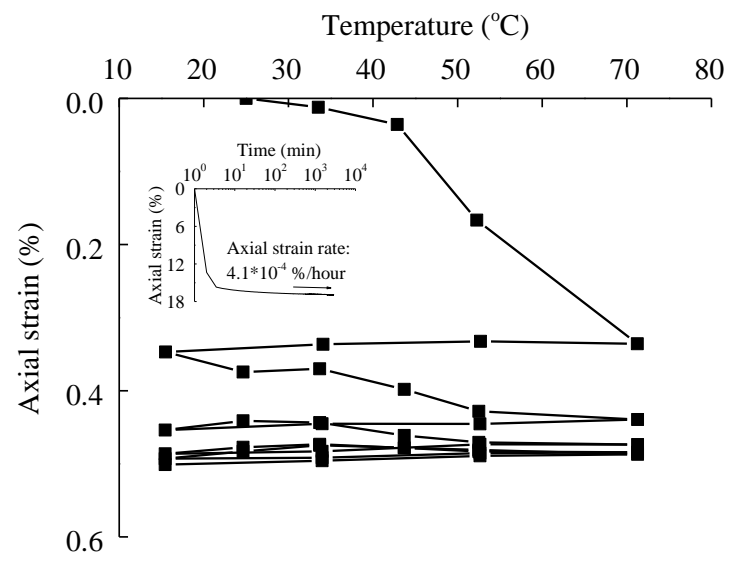
Figure 3. Comparison of measured and calculated differences in thermal axial strains of clay in steel and invar rings

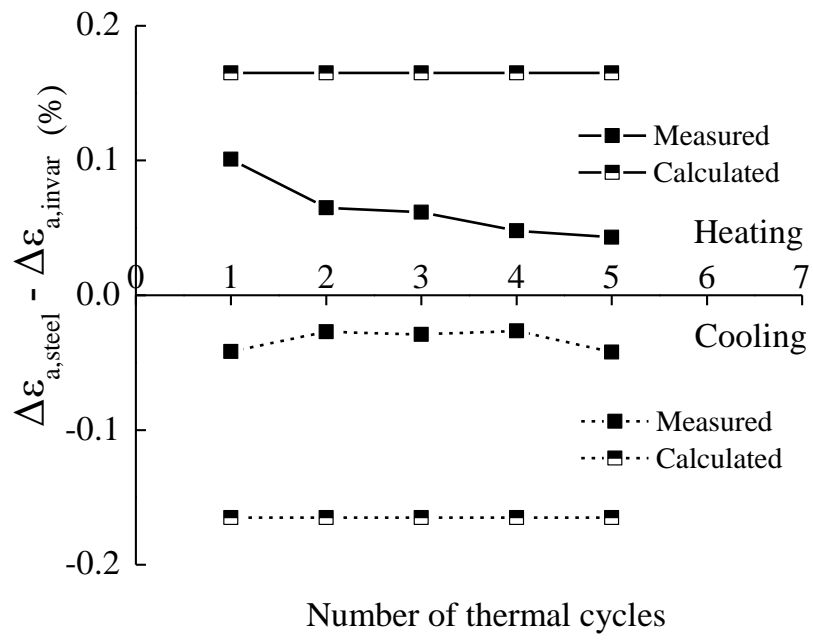
Figure 4. Thermal axial strains of Toyoura sand measured in oedometer rings made of (a) steel; (b) invar

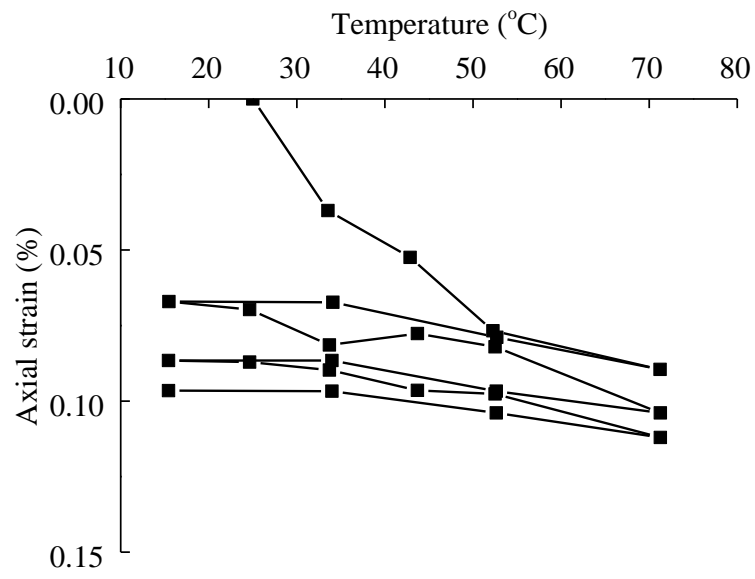
Figure 5. Comparison of measured and calculated differences in thermal axial strains of Toyoura sand in steel and invar rings

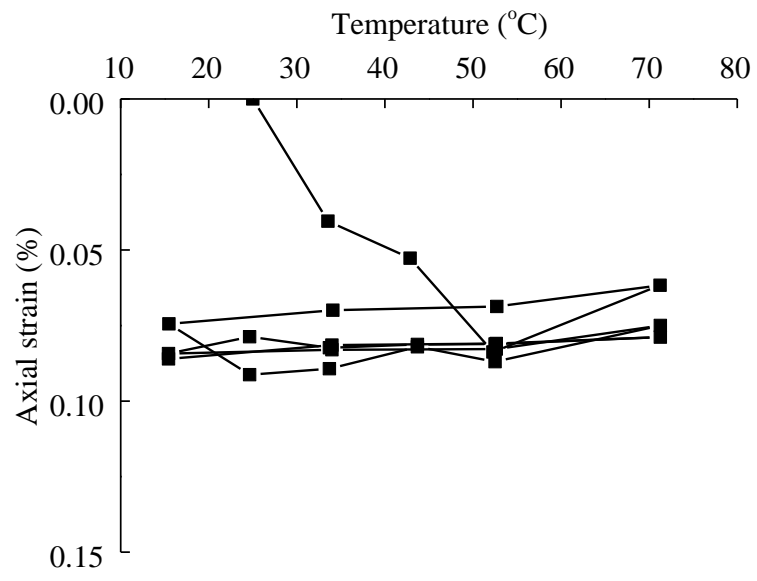




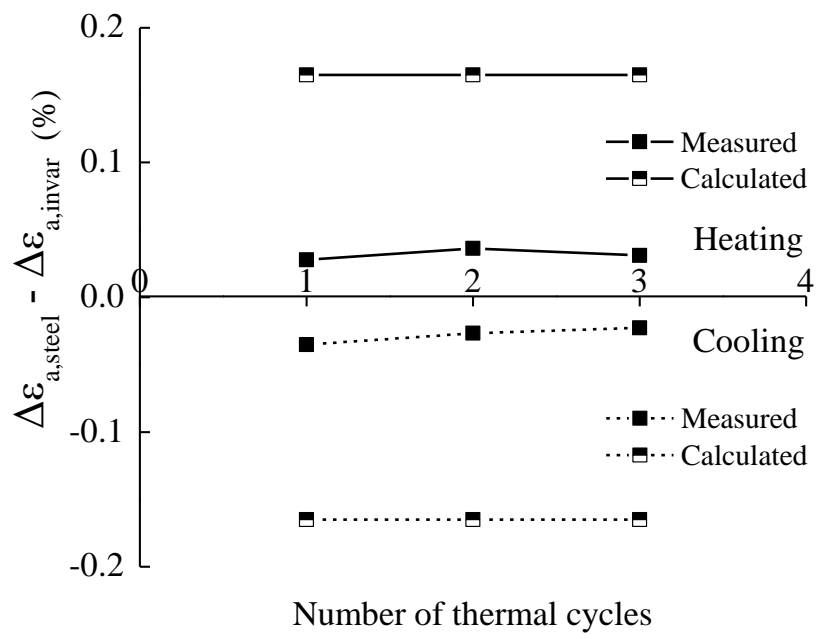












## **List of Tables**

Table 1. Physical properties of the clay tested

Table 2. Test program and void ratio of soil specimens at each state

Table 1 Physical properties of the clay tested

Parameter	Value
Specific gravity	2.69
Liquid limit (%)	39
Optimum water content (%)	18.1
Maximum density (g/cm <sup>3</sup> )	1.68
Plasticity limit (%)	19
Plasticity index (%)	17
Clay particle fraction (<0.002 mm, %)	28
Silt particle fraction (0.002-0.063 mm, %)	72

Table 2 Test program and void ratio of soil specimens at each state

Series	Test ID	Soil type	Material of oedometer ring	$e_0$ (initial state)	$e_1$ (after mechanical consolidation)	$\Psi$ (after mechanical consolidation)	$e_f$ (after thermal cycles)
I	CS	Clay	Steel	1.276	0.894	0.061	0.882
	CI		Invar	1.271	0.887	0.054	0.878
II	SS	Sand	Steel	0.887	0.885	-0.040	0.883
	SI		Invar	0.889	0.886	-0.037	0.884

Note:

1. Thermal expansion coefficient of (1) invar:  $5.0 \times 10^{-7}$  m/(m °C); (2) steel:  $1.5 \times 10^{-5}$  m/(m °C); and (3) soil skeleton:  $5 \times 10^{-6}$ - $3.5 \times 10^{-5}$  m/(m °C) (Campanella and Mitchell, 1968; Vega and McCartney, 2015; Ng et al., 2016)
2. The measured critical state line of clay is  $e = 1.170 - 0.096 \ln(p')$ , where  $e$  and  $p'$  are the void ratio and mean effective stress, respectively. For Toyoura sand, the critical state line follows  $e = 0.934 - 0.019(p'/101)^{0.7}$  (Li and Wang, 1998)