

# Development and Performance of New Simplified Method for Soft Soil with Creep Under Multi-Staged Loading

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## ABSTRACT

In this paper, a new simplified method (de-coupled method) is presented for calculating consolidation settlements of a soft soil layer with creep under multi-staged loadings. All equations of this new simplified method are derived for a general multi-staged loading case. Different stress-states of the soil layer are considered. At the same time, a fully coupled finite element computer program with an Elastic Visco-Plastic (EVP) model and Hypothesis A method are used to calculate the consolidation settlements of the same soil layer with the same conditions. Then, the settlement results using this new simplified method are presented and compared with results from the finite element (FE) simulations and the Hypothesis A method. Based on the comparison, it is demonstrated that the new simplified method is much better than the Hypothesis A method. In addition, this new simplified method is used in the *Berthierville* test site subjected to three-staged loading. The comparison of calculated results and measured data demonstrates that the new simplified method is easy to use by simple spread-sheet calculation and has a good accuracy for the situations analyzed in this study.

**Keywords:** consolidation settlement, creep, simplified method, Hypothesis B, multi-staged loading, ramp loading

## 1. INTRODUCTION

In general, the elastic visco-plastic (EVP) constitutive model is the most suitable one to describe the general stress-strain-time relationship of clayey soils (Bjerrum, 1967; Yin and Graham, 1989, 1994, 1999; Yin *et al.*, 2010; Feng *et al.*, 2017). Thus, numerous researchers have attempted to determine the consolidation settlement of soft soil ground by using the EVP model (Zhu and Yin, 2000; Zhu *et al.*, 2001; Yin and Hicher, 2008; Yin and Karstunen, 2008; Karstunen and Yin, 2010; Yin *et al.*, 2011). However, the numerical methods, such as finite difference method or the finite element (FE) method, are necessary for solving the highly nonlinear differential equations in the coupled consolidation and creep when the settlement problem is concerned (Zhu *et al.*, 2001; Yin and Graham, 1996; Vermeer and Neher, 1999; Nash and Ryde, 2001; Nash and Brown, 2013). In the coupled analysis of EVP constitutive model with the proper numerical methods, it is regarded that the creep and consolidation occur simultaneously, which is consistent with the opinions of Hypothesis B (Ladd, *et al.*, 1977; Degago, *et al.*, 2011). In practice, the creep settlement (also termed as “secondary consolidation”) is calculated when the consolidation stage is completed, which follows the assumption of Hypothesis A (Ladd, *et al.*, 1977; Mesri and Vardhanabhuti, 2006). Degago *et al.* (2011) summarized previous data in laboratory tests and field monitoring, and access whether the creep occurs in the consolidation. They found that most data in the literature supports the Hypothesis B. To build a bridge between the research and the practice, a new simplified method was proposed based on Hypothesis B and validated for the consolidation settlement calculation of soft soil layers following the EVP constitutive relationship and “equivalent time” concept (Feng and Yin, 2017, 2018; Yin and Feng, 2017). In this new simplified method, the average degree of consolidation is calculated by the empirical equations of Terzaghi’s theory for one-layered soil subjected to the instant loading and Zhu and Yin method (1999, 2005) for double-layered soil under time-dependent loading. It has

72 been demonstrated that this new simplified method is an approximate approach by hand  
73 calculation of the consolidation settlement of clayey soils exhibiting creep, which is very  
74 valuable in the reclamation design. Since this new simplified method was just proposed  
75 recently, it was validated for one soil layer, double soil layers, and one soil layer with vertical  
76 drain under an instant/ramp loading. Recently, Feng and Yin (2019) took into account the  
77 nonlinear compressibility in the new simplified method for the thick soil layer subjected to a  
78 ramp loading. It is found that previous works mainly focus on an instant loading or a ramp  
79 loading, while the loading condition is very different in practice. There is an obligation to  
80 develop this new simplified method in calculating the ground settlement of soft soil layers  
81 under various loading conditions for its further application.

82  
83 In projects, such as the reclamations, the loadings are gradually applied with time (Feng et al.,  
84 2019; Yao et al., 2019). For many cases, the first staged loading would be kept a certain  
85 period, afterwards, next staged loading would be applied and maintained, termed as  
86 multi-staged loading. Terzaghi (1943) presented the practical methods of the graphical  
87 construction to consider the construction time factor. Olson (1977) derived a mathematical  
88 solution for one-dimensional (1-D) consolidation of homogeneous soils subjected to a simple  
89 ramp loading. Zhu and Yin (1999, 2005) obtained the analytical solutions for double-layered  
90 soil under the single time-dependent loading and considered the depth-dependent vertical  
91 total stress specially. Afterwards, researchers such as Conte and Troncone (2009), Walker and  
92 Indraratna (2009), Lu *et al.* (2011), Lei *et al.* (2015, 2016), Ni *et al.* (2019) focused on the  
93 multi-staged ramp loading and made some meaningful achievements. Therefore, the  
94 multi-staged loading is necessary to be considered when the total consolidation settlement is  
95 concerned in field projects, especially for the soft soils.

In this paper, the new simplified method is developed to calculate the consolidation settlement for soft soils with creep under a general multi-staged loading. The equations are derived for both normally consolidated state (NCS) and over-consolidated state (OCS) using the “equivalent time” concept. The typical soft soil layer (4m) from *Berthierville* test site with different stress-strain states (over-consolidation ratio,  $OCR=1, 1.5$ , and  $2$ ) are calculated and verified. Then, different construction periods (30 days and 365 days), different loading stages (two-, three-, and four-staged loadings) are calculated and compared with the FE simulations. And its accuracy is also examined based on the results of finite element modelling (FEM). Afterwards, this new simplified method is utilized in the *Berthierville* test site and compared with the measured data in the site to demonstrate its feasibility.

## 2. THE NEW SIMPLIFIED METHOD FOR CONSOLIDATION SETTLEMENT OF SOIL LAYER SUBJECTED TO MULTI-STAGED LOADING

Figure 1 shows the schematic diagram of multi-staged loading including the construction period ( $t_{c1}, t_{c2}, t_{c3} \dots$ ) and the consolidation duration of each loading ( $t_1, t_2, t_3 \dots$ ). The definition of these parameters will be used in the following derivations and expressions.

In the new simplified calculation method, the total settlement,  $S_{totalB}$ , is the summation of consolidation settlement,  $S_{consolidation}$ , and creep settlement,  $S_{creep}$ . A general equation of this new simplified method for 1-D consolidation settlement for clayey soils can be expressed by:

$$S_{totalB} = \sum_{j=1}^m (S_{consolidation-j} + S_{creep-j}) \quad (1)$$

In this equation,  $j$  is the stage number in the multi-staged loading. This new simplified method is developed from the idea of the EVP model based on the “equivalent time” concept, which is proposed by Bjerrum (1967), Yin and Graham (1989, 1994). The term of “B” in the total

settlement is utilized in this new simplified method because the creep is considered in consolidation stage following the Hypothesis B. The details of Hypothesis A and Hypothesis B can be referred in Yin and Feng (2017), Feng and Yin (2017).

## **2.1 For Stage 1 in Multi-staged Loading**

### **2.1.1 The Consolidation Settlement of Soil Layer**

The consolidation settlement is related to the parameter values calculated from the nonlinear stress-strain relationship and the average degree of consolidation of the soil layer. The coefficient of volume compressibility,  $m_v$ , is usually adopted to describe the volume change per unit volume with respect to the increase of effective stress, and it is a basis to determine the coefficient of consolidation,  $c_v = \frac{k_z}{\gamma_w m_v}$  (Craig, 2004). Due to the variation of the initial vertical effective stress with depth, the soil layer is divided into the  $i$ -sublayer with a suitable thickness (e.g. 0.5 m). For each sublayer, it is necessary to determine the stress state by comparing the final effective stress and the pre-consolidation pressure: it is an OCS for  $\sigma'_{z0,i} + \Delta\sigma'_{z1,i} < \sigma'_{zp,i}$ ; and it is a NCS for  $\sigma'_{z0,i} + \Delta\sigma'_{z1,i} \geq \sigma'_{zp,i}$ .  $\sigma'_{z0,i}$ ,  $\Delta\sigma'_{z1,i}$ , and  $\sigma'_{zp,i}$  are initial effective vertical stress, vertical effective stress increment in stage 1 (as shown in Figure 1), pre-consolidation pressure for each sublayer, respectively.

In Figure 2, the initial stress-strain state is at point 0, the consolidation settlement is related to soil compression on the instant time line (plotted in Figure 2a) or reference time line (plotted in Figure 2b). The total consolidation settlement is the sum of that of each sublayer, and sublayer settlement is calculated based on the stress-strain state with respect to pre-consolidation pressure at the center of each sublayer in the new simplified Hypothesis B method:

$$S_{f1,i} = \varepsilon_{f1,i} H_i = \frac{C_e}{V} \log \left( \frac{\sigma'_{z0,i} + \Delta \sigma'_{z1,i}}{\sigma'_{z0,i}} \right) H_i \quad \text{for OCS} \quad (2)$$

$$S_{f1,i} = \varepsilon_{f1,i} H_i = \left\{ \frac{C_e}{V} \log \left( \frac{\sigma'_{zp,i}}{\sigma'_{z0,i}} \right) + \frac{C_c}{V} \log \left( \frac{\sigma'_{z0,i} + \Delta \sigma'_{z1,i}}{\sigma'_{zp,i}} \right) \right\} H_i \quad \text{for NCS} \quad (3)$$

$$m_{v1} = \frac{1}{n} \sum_{i=1}^n \frac{\varepsilon_{f1,i}}{\Delta \sigma'_{z1,i}} \quad (4)$$

$$S_{f1} = \sum_{i=1}^n S_{f1,i} \quad (5)$$

$$S_{consolidation-1} = \begin{cases} \frac{\Delta \sigma'_{z,t}}{\Delta \sigma'_{z1}} U_{z1}(t) S_{f1} & \text{for } 0 \leq t \leq t_{c1} \\ U_{z1}(t) S_{f1} & \text{for } t_{c1} < t \end{cases} \quad (6)$$

where  $H_i$  is the thickness of sublayer,  $m_{v1}$  is the coefficient of volume compressibility of sublayer under the stage 1 loading,  $\varepsilon_{f1}$  is the final strain of sublayer under the effective stress of stage 1 loading,  $V$  is the specific volume,  $V = 1 + e_o$ ,  $C_e$  is the rebounding index, which is the slope of unloading-reloading line in the linear portion of the  $e - \log(\sigma'_z)$  plot, and  $C_c$  is the slope of normal consolidation line (NCL), termed as compression index. The values of  $C_e$  and  $C_c$  are easily obtained from the standard oedometer test with a duration of 24 hours (1 day),  $S_{consolidation-1}$  is the consolidation settlement under stage 1 loading,  $U_{z1}$  is the average degree of consolidation with the correction for the construction period.  $\Delta \sigma'_{z,t}$  is the vertical stress increment at time of  $t$  (within the construction period). In this study, the empirical method suggested by Terzaghi (1943) is adopted to consider the construction period, expressed as follows:

$$U_z(t) = \begin{cases} U'_z(t/2) & \text{when } t < t_{c1} \\ U'_z(t - t_{c1}/2) & \text{when } t \geq t_{c1} \end{cases} \quad (7)$$

where  $U'_z$  is the average degree of consolidation based on the instant loading, similar to Yin

and Feng (2017).

### 2.1.2 The Calculation of Creep Settlement

The creep settlement is calculated by using the following equation:

$$S_{creep-1} = \alpha S_{creep,f1} + (1 - \alpha) S_{creep,d1} \quad (8)$$

where  $S_{creep,f1}$  is the creep settlement with respect to final effective stress under stage 1 loading ignoring the coupling of the excess pore water pressure and creep,  $S_{creep,d1}$  is the delayed creep settlement under stage 1 loading due to the coupling of the excess pore water pressure. “Delayed” means that the  $S_{creep,d1}$  will occur when  $U_z = 98\%$  in the field.  $\alpha$  is a parameter for calculating the creep settlement, whose value is within 0~1. In this study, we take  $\alpha = U_z$  as a simplification. As shown in Figure 2, the creep compression is vividly expressed by the equivalent time lines, which is directly related to creep parameters,  $C_{ae}/V$  and  $t_o$ . In this study, the default value of  $t_o$  is 1 day. The creep strain rate on the equivalent time line is independent of the stress path, which combines the *OCS* and *NCS*, expressed as:

$$S_{creep,f1} = \sum_{i=1}^n S_{creep,f1,i} = \sum_{i=1}^n \varepsilon_{creep,f1,i} H_i \quad (9)$$

$$\varepsilon_{creep,f1,i} = \frac{C_{ae}}{V} \log \left( \frac{t_o + t_{e,i}}{t_o + \Delta t_{e1,i}} \right) \quad \text{for } OCS \quad (10)$$

$$\varepsilon_{creep,f1,i} = \frac{C_{ae}}{V} \log \left( \frac{t_o + t_e}{t_o} \right) \quad \text{for } NCS \quad (11)$$

where  $\Delta t_{e,i}$  and  $t_{e,i}$  are determined when  $t$  is larger than  $t_{c1}/2$  from the following equations:

$$\Delta t_{e1,i} = t_o \times 10^{\left( \frac{(\varepsilon_{f1,i} - \varepsilon_{zp,i}) V}{C_{ae}} \right)} \left( \frac{\sigma'_{z0,i} + \Delta \sigma'_{z1,i}}{\sigma'_{zp,i}} \right)^{\frac{C_c}{C_{ae}}} - t_o \quad (12)$$

$$t_{e,i} = t - \frac{t_{c1}}{2} - t_o + \Delta t_{e1,i} \quad \text{for } OCS \quad (13)$$



$$t_e = t - \frac{t_{cl}}{2} - t_o \quad \text{for NCS} \quad (14)$$

The delayed creep settlement,  $S_{creep,d1}$ , is related to the  $S_{creep,f1}$  in all cases above but delayed by the time of  $t_{EOP,field}$ :

$$S_{creep,d1} = \sum_{i=1}^n S_{creep,d1,i} = \sum_{i=1}^n \varepsilon_{creep,d1,i} H_i \quad (15)$$

For OCS, the delayed creep settlement occurs after the time of  $t_{EOP,field}$ , there is a relationship between the delayed creep strain and total creep strain with the help of “equivalent time” concept:

$$\begin{aligned} \varepsilon_{creep,d1,i} &= \frac{C_{ae}}{V} \log \left( \frac{t_o + t_{e,i}}{t_o + \Delta t_{e1,i}} \right) - \frac{C_{ae}}{V} \log \left( \frac{t_o + \Delta t_{e1,i} + t_{EOP,field} - t_o}{t_o + \Delta t_{e1,i}} \right) \\ &= \frac{C_{ae}}{V} \log \left( \frac{t_o + t_{e1,i}}{t_o + \Delta t_{e1,i} + t_{EOP,field} - t_o} \right) \\ &= \frac{C_{ae}}{V} \log \left( \frac{t_o + t_{e1,i}}{\Delta t_{e1,i} + t_{EOP,field}} \right) \end{aligned} \quad (16)$$

Similarly, the delayed creep settlement at NCS can be calculated as follows:

$$\begin{aligned} \varepsilon_{creep,d1,i} &= \frac{C_{ae}}{V} \log \left( \frac{t_o + t_e}{t_o} \right) - \frac{C_{ae}}{V} \log \left( \frac{t_o + t_{EOP,field} - t_o}{t_o} \right) \\ &= \frac{C_{ae}}{V} \log \left( \frac{t_o + t_e}{t_{EOP,field}} \right) \end{aligned} \quad (17)$$

At the NCS, it can be observed that the calculation of delayed creep settlement is similar to the “secondary consolidation” settlement in traditional Hypothesis A method. Comparatively, this delayed creep settlement can also be used in the OCS with the same creep parameter and equivalent time.

## 2.2 For Stage 2 in the Multi-staged Loading

### 2.2.1 The Consolidation Settlement of Soil Layer

The pre-consolidation pressure at the center of each sublayer is influenced by the creep strain in stage 1 loading and it should be carefully determined. For OCS, as plotted in Figure 2(a), the pre-consolidation pressure of each sublayer in stage 2 is derived as:

$$\begin{aligned}
\varepsilon_{zp2,i} &= \varepsilon_{z1,t1,i} + \frac{C_e}{V} \log \left( \frac{\sigma'_{zp2,i}}{\sigma'_{z0,i} + \Delta\sigma'_{z1,i}} \right) = \varepsilon_{zp,i} + \frac{C_c}{V} \log \left( \frac{\sigma'_{zp2,i}}{\sigma'_{zp,i}} \right) \\
\varepsilon_{z1,t1,i} - \varepsilon_{zp,i} &= \frac{C_c}{V} \log \left( \frac{\sigma'_{zp2,i}}{\sigma'_{zp,i}} \right) - \frac{C_e}{V} \log \left( \frac{\sigma'_{zp2,i}}{\sigma'_{z0,i} + \Delta\sigma'_{z1,i}} \right) \\
\varepsilon_{z1,t1,i} - \varepsilon_{zp,i} &= \left\{ \frac{C_c}{V} - \frac{C_e}{V} \right\} \log(\sigma'_{zp2,i}) - \frac{C_c}{V} \log(\sigma'_{zp,i}) + \frac{C_e}{V} \log(\sigma'_{z0,i} + \Delta\sigma'_{z1,i}) \\
\log(\sigma'_{zp2,i}) &= \left\{ \varepsilon_{z1,t1,i} - \varepsilon_{zp,i} \right\} \frac{V}{(C_c - C_e)} + \frac{C_c}{(C_c - C_e)} \log(\sigma'_{zp,i}) - \frac{C_e}{(C_c - C_e)} \log(\sigma'_{z0,i} + \Delta\sigma'_{z1,i})
\end{aligned} \tag{18}$$

Thus, the new pre-consolidation pressure in stage 2 can be obtained:

$$\sigma'_{zp2,i} = 10^{\left\{ \varepsilon_{z1,t1,i} - \varepsilon_{zp,i} \right\} \frac{V}{(C_c - C_e)}} \times (\sigma'_{z0,i} + \Delta\sigma'_{z1,i})^{\frac{C_e}{(C_c - C_e)}} \times \sigma'_{zp,i}^{\frac{C_c}{(C_c - C_e)}} \tag{19}$$

Similarly, the pre-consolidation pressure of each sublayer in stage 2 is also valid for NCS. The new pre-consolidation pressure is related to the final effect stress,  $(\sigma'_{z0,i} + \Delta\sigma'_{z1,i})$ , pre-consolidation pressure,  $\sigma'_{zp,i}$ , of stage 1 loading, and consolidation duration,  $t_l$ , of previous loading stage.

As shown in Figure 2, the stress state of soils in sublayer under stage 2 in multi-staged loading is from point 1 to point 2 for both OCS and NCS. Similarly, the total consolidation settlement of each sublayer can be calculated as follows:

$$S_{f2,i} = \varepsilon_{f2,i} H_i = \frac{C_e}{V} \log \left( \frac{\sigma'_{z0,i} + \Delta\sigma'_{z1,i} + \Delta\sigma'_{z2,i}}{\sigma'_{z0,i} + \Delta\sigma'_{z1,i}} \right) H_i \quad \text{for OCS} \tag{20}$$

$$S_{f2,i} = \varepsilon_{f2,i} H_i = \left\{ \frac{C_e}{V} \log \left( \frac{\sigma'_{zp2,i}}{\sigma'_{z0,i} + \Delta\sigma'_{z1,i}} \right) + \frac{C_c}{V} \log \left( \frac{\sigma'_{z0,i} + \Delta\sigma'_{z1,i} + \Delta\sigma'_{z2,i}}{\sigma'_{zp2,i}} \right) \right\} H_i \quad \text{for NCS} \tag{21}$$

The coefficient of volume compressibility of sublayer under the stage 2 loading,  $m_{v2}$ , the

217 total consolidation settlement,  $S_{consolidation-2}$ , is calculated similarly to Eqs. (4), (5), and (6).

218

### 219 2.2.2 The Calculation of Creep Settlement

220 Eqs. (9), (10), and (11) can be utilized to calculate the creep settlement under stage 2 loading.

221 The only difference is that the determination of  $\Delta t_{e2,i}$ :

$$222 \quad \Delta t_{e2,i} = t_o \times 10^{\left( (\varepsilon_{f2,i} - \varepsilon_{zp2,i}) \frac{V}{C_{ae}} \right) \left( \frac{\sigma'_{z0,i} + \Delta \sigma'_{z1,i} + \Delta \sigma'_{z2,i}}{\sigma'_{zp2,i}} \right) \frac{C_c}{C_{ae}}} - t_o \quad (22)$$

223 The value of  $\Delta t_{e2,i}$  should be substituted into Eqs. (8), (10), and (16) to calculate the creep

224 settlement and delayed creep settlement under stage 2 loading. It should be noted that the

225 starting point of creep compression in stage 2 loading is from the day of  $t_1 + t_{c2}/2$ , as

226 illustrated in Figure 1. Thus, the “equivalent time” for the creep compression in stage 2 is

227 determined as:

$$228 \quad t_{e,i} = t - t_1 - t_{c2}/2 - t_o + \Delta t_{e2,i} \quad \text{for OCS} \quad (23)$$

$$229 \quad t_e = t - t_1 - t_{c2}/2 - t_o \quad \text{for NCS} \quad (24)$$

230 The process of calculating the consolidation settlement under stage 3 loading or stage 4

231 loading is similar to that of calculation under stage 2 loading. Thus, the details are not

232 repeated here.

233

### 234 2.3 The Hypothesis A Method for Consolidation Settlement of Soil Layer

235 A simplified method based on Hypothesis A is utilized in this study for the calculation of the

236 total consolidation settlement  $S_{totalA}$  in the field:

$$237 \quad S_{totalA} = S_{primary} + S_{secondary} \\ = \begin{cases} U_z S_f & \text{for } t < t_{EOP,field} \\ U_z S_f + \frac{C_{ae}}{V} \log \left( \frac{t}{t_{EOP,field}} \right) H & \text{for } t \geq t_{EOP,field} \end{cases} \quad (25)$$

where  $S_{\text{primary}}$  is the “primary” consolidation settlement at time  $t$  and, taking stage 1 loading as the example, it is equal to  $S_{\text{consolidation-1}}$  in Eq. (7),  $t_{\text{EOP,field}}$  represents the end of “primary” consolidation in the field, and it is usually taken as the time for  $U_z = 98\%$ . For the over-consolidated soils, the “secondary consolidation” settlement is not considered in this study.

### 3. CASES OF CLAYEY SOILS SUBJECTED TO MULTI-STAGED LOADING AND ITS VERIFICATION

In this part, we selected one typical clayey soil from *Berthierville* test site with different stress-strain states as the examples to analyze the consolidation settlement including creep. The typical parameter values are listed in Table 1, which is reported by Kim and Leroueil (2001). The FE software (Plaxis 2D, 2015 version) was utilized as the reliable reference to evaluate the performance of the new simplified method because FEM could solve the coupling equations of consolidation and creep using Newton-Raphson method (Yin and Feng, 2017; Feng and Yin, 2017, 2018). Relative error ( $\xi_{\text{totalB},t}$ ) is defined to assess the performance of the new simplified method with the FE simulations:

$$\xi_{\text{totalB},t} = \frac{|S_{\text{totalB},t} - S_{\text{FE},t}|}{S_{\text{FE},t}} \times 100\% \quad (26)$$

where  $S_{\text{FE},t}$  is the settlement simulated by FE method with the fully coupled analysis of consolidation and creep compression at a certain time,  $S_{\text{totalB},t}$  is the calculated consolidation settlement from the new simplified method.  $\xi_{\text{totalA},t}$  is similarly defined to examine the accuracy of the simple method based on Hypothesis A.

As addressed in Figure 1, the multi-staged loading includes the loading stages, loading durations, and construction periods. In practice, the loading stage depends on the geological

conditions in the field, normally varying from one to five stages, therefore, we take the loading stage from two to four stages and the loading duration for each stage loading from one month to ten years. The construction period is closely related to the project construction, whose period for each stage loading is controlled within 1 year. To illustrate the feasibility of the new simplified method for different stress-strain state of soft soil, three different OCRs (OCR=1, 1.5, and 2) are considered in this study.

### 3.1 Cases Description and Finite Element Simulation

Table 2 lists all the cases including different OCRs, different construction periods ( $t_{cl}$ =30 days and 365 days), different consolidation durations ( $t_1$ =365 days, 730 days, and 3650 days), and different multi-staged applied loadings ( $j$ =2, 3, and 4) are considered in this study to illustrate the influence of multi-staged loading on the consolidation settlement of soil layers with different stress-strain states. All these cases of soil layers were simulated by Plaxis software, as shown in Figure 3. The *plane strain* model type was set and 15-noded element and fine mesh was chosen in the model simulation. For the boundary condition, the top was set as drained and the bottom was set as impermeable. The default  $K_0$  condition, which has been proved to be suitable for normally consolidated soil state, was utilized in this study. Soft Soil Creep (SSC) model is widely used to analyze the settlement of soft soils exhibiting creep (Stolle, *et al.*, 1999), and is also adopted in this study. The multi-staged loading was applied, as listed in Table 2, and total duration was 36500 days to make sure the consolidation was completed for all simulated cases.

### 3.2 Calculation Procedures of New Simplified Hypothesis B Method and Hypothesis A Method

#### 3.2.1 For the Calculation of Stage 1 Loading

The total soil layer was firstly divided into sublayers with a thickness of 0.5m for each. Initial and final effective stresses of each sublayer can be obtained as below:

$$\sigma'_{z0,i} = (\gamma_{soil} - \gamma_w) z_i \quad (26)$$

$$\sigma'_{z1,i} = \sigma'_{z0,i} + \Delta\sigma'_{z1,i} \quad (27)$$

$$\sigma'_{zp,i} = \sigma'_{z0,i} \times OCR \quad (28)$$

where  $\gamma_{soil}$  represents the unit weight of soils,  $\gamma_w$  is water unit weight, taken as  $9.81 \text{ kN/m}^3$ ,  $z_i$  is the middle depth of  $i$ -th sublayer,  $\Delta\sigma'_{z1,i}$  is the vertical stress increment of stage 1 loading, and OCR is the value of over-consolidation ratio (details can be referred to Manual of Plaxis, 2015 version). Then, the stress state of each sublayer can be determined by comparing the pre-consolidation pressure with the final effective stress, i.e. Eq. (2) was used for the sublayer in OCS, and Eq. (3) was utilized for the sublayer in NCS. Afterwards, the consolidation settlement was computed by Eqs. (4) ~ (7).

Subsequently, Eqs. (10), (12), and (13) were adopted to calculate the final creep strain of stage 1 loading and Eq. (16) to obtain the delayed creep strain for the sublayer in OCS. Similarly, when the sublayer is in NCS, Eqs. (11), (14) and Eq. (17) were used for final creep strain and delayed creep strain, respectively. Next, the creep settlement can be calculated with the help of Eqs. (9), (15), and (8). It should be noted that the creep strain will stay at the exact value of  $\varepsilon_{creep,t1,i}$  after the time becomes larger than  $t_1$  for the stage 1 loading.

### 3.2.2 For the Calculation of Stage 2, 3, or 4 Loading

The initial effective stress of each sublayer is the final effective stress of stage 1 loading,  $\sigma'_{z1,i}$ . And the final effective stress of each sublayer was updated for stage 2 loading, expressed as:

$$\sigma'_{z2,i} = \sigma'_{z1,i} + \Delta\sigma'_{z2,i} \quad (29)$$

Importantly, the pre-consolidation pressure in stage 2 loading was also re-determined using Eq. (18). Next, the sequences of consolidation settlement and creep settlement calculation were repeated. Table 3 lists a summary of calculation results in Case *III* with OCR=2 including stage 1 loading and stage 2 loading for a reference. For the calculation of settlement under stage 3 and 4 loadings, all the key points were the same as those in stage 2 loading.

Lastly, the total consolidation settlement was obtained by summing the consolidation settlement and creep compression under all the staged loadings, using Eq. (1). [A schematic diagram of the calculations for the new simplified method is provided in Figure 4.](#)

### 3.3 Verification of the New Simplified Method by Finite Element Analysis

The calculation results using the new simplified method and Hypothesis A method are compared with finite element simulations for all the cases described above. The certain time at  $t = 36500$  days is set as the special time points to examine the accuracy and performance of the simplified method as well as Hypothesis A.

#### 3.3.1 Consolidation durations effect

Figures 5, 6, and 7 display the comparison between the calculation results from the new simplified method and finite element simulation results of 4m soil layers subjected to two-staged instant loading (Case *I*) for OCR=1, 1.5, and 2, respectively. The construction period of both stage 1 and stage 2 loadings is 0.1 day. The consolidation duration of stage 1 loading,  $t_1$ , varies from 365 days, 730 days, to 3650 days.

As shown in Figure 5, under stage 1 and 2 loadings, Hypothesis A method obviously

underestimates the total settlement for all three different consolidation durations when  $OCR=1$ . Comparatively, the calculated results of the new simplified method are very close to the simulation results of Plaxis due to the updated pre-consolidation pressure using the “equivalent time” concept for each sublayer in stage 2 loading (in Eq. (19)). This, further, indicates that the new simplified method can correctly determine the total consolidation settlement including creep compression. The same observation is found in Figures 6 and 7, which compare the calculation results of the new simplified method, Hypothesis A method, and finite element modelling results for  $OCR=1.5$  and 2. Taking the finite element simulations as the references, the values of *relative error* for Hypothesis A method and the new simplified method are calculated and listed in Table 4. It is found that all *relative errors* of this new simplified method are within 5.7%, which is satisfactory in the geotechnical design. This means that the influence of consolidation duration of the staged loading is correctly considered in this new simplified method. Correspondingly, the values of *relative error* for Hypothesis A method in Case I are from 23.3% to 43.1%.

### 3.3.2 Construction period effect

The consolidation settlements using the new simplified method and Hypothesis A method are compared with the finite element simulations for the soft soils subjected to three different two-staged ramp loadings (Case II listed in Table 2) in Figures 8, 9, and 10.

The consolidation duration of stage 1 loading is 3650 days. Similarly, the calculated results of the new simplified method are very close to the finite element results under both stage 1 loading and stage 2 loading. The obvious underestimations of Hypothesis A method in stage 2 loading are found for all cases. As listed in Table 4, *relative errors* of the new simplified method, in this case, are within 0.9% ~ 5.1%, while those of Hypothesis A method vary from



29.8% to 43.1%. All the comparisons of Figures 8, 9, and 10 indicate that the new simplified method can correctly capture the creep compression in the construction stage and it possesses advantages in applying for the soil layers subjected to the ramp loading under both OCS and NCS.

### 3.3.3 Multi-staged applied loadings effect

In this part, the soft soil layers subjected to two-staged, three-staged, and four-staged ramp loadings are investigated and the calculated results are compared with the finite element modelling results, as plotted in Figures 11, 12, and 13.

Obviously, there is a good agreement between the results from the new simplified method and finite element simulations, whereas an underestimation of Hypothesis A method is found. The underestimation of Hypothesis A method is the reason that the creep is only considered after the consolidation stage. In the FEM, the creep is only related to the effective stress and the coupled equations of consolidation and creep are solved by using Newton-Raphson method. In the new simplified method, there is a bit overestimation of the total settlement after the consolidation stage, comparing with the FE simulations. This is the reason that the new simplified method is directly calculated from the final effective stress by simply taking a parameter of  $\alpha$ . Table 4 lists the representative values of the *relative error* using Eq. (25). The *relative errors* of the new simplified method are in the range of 3.8% ~ 10.9%. Again, Hypothesis A method underestimates the settlement by 25.1%~51.2%. In the Case IV and Case V, this new simplified method can reasonably determine more than 2 staged loading in the calculation (as shown in Figures 12 and 13). Thus, the results comparison and *relative errors* demonstrate that the new simplified method can be used to represent the finite element modelling of consolidation settlement of soils under the multi-staged ramp loading.

#### 4. THE APPLICATION OF NEW SIMPLIFIED METHOD IN BERTHIERVILLE TEST SITE

In this section, the new simplified method was applied in the *Berthierville* test site to calculate the settlement during the consolidation of the clayey soil layer subjected to three-staged loading.

##### 4.1 Field Condition of *Berthierville* Test Site

As reported by Kim and Leroueil (2001), the creep oedometer tests and the constant rate of strain (CRS) tests were conducted to determine the values of the creep coefficient, compressibility index, and rebounding index. Based on the back-calculation, the parameter values of new simplified method are listed in Table 1. For the creep oedometer test, the initial effective stress is 39 kPa and the pre-consolidation pressure is 64 kPa. Figure 14 compares the measured data of creep oedometer test and the calculated results using the new simplified method, which confirms that the parameter values are credible. The coefficient of consolidation corresponding to each applied stress is also shown in Figure 14.

In *Berthierville* test site, there are mainly three soil layers: the top layer is a 2.3 m thick sand layer; the middle layer is a 3.2 thick clayey soil layer, whose values of the parameter are listed in Table 1; underneath the clayey layer, there are a few meters of the sand layer. Three-staged ramp loading was added by surcharging the fill material on the sandy layer: the first two days, the surcharge loading was 10 kPa; then the surcharge loading was increased to 39 kPa within 2 days; after twenty days of the construction, a heavy rain occurred and the surcharge loading was increased to 44 kPa (Kim and Leroueil, 2001). The groundwater level is 0.9 m above the clayey soil layer. According to the initial effective stress and measured pre-consolidation

pressure from Kim and Leroueil (2001), OCR of clayey soil layer is taken as 1.33. Neglecting the compression of the sand soil layer, the condition of *Berthierville* test site is a typical case of soft soil (3.2 m) subjected to multi-staged loading, thus the new simplified method is used to calculate the consolidation settlement and its result is compared with the measured data.

The FEM was established for the *Berthierville* test site based on the information of clayey soil layer with 3.2 m. The SSC model is used for the clayey soil and the values are listed in Table 1. The top sandy soil layer with 2.3 m in thickness was also modelled and it is set as drained condition. The bulk density of the sandy soil is  $17.5 \text{ kN/m}^3$ . Details could be referred in Section 3.1. It should be noted that the bottom of the clayey soil layer is set as drained because there is sandy soil layer below the soft soil. The loading was applied in three stages according to the above description.

#### **4.2 The New Simplified Method Calculation and Result Discussion**

Dividing the clayey soil layer into eight sublayers with 0.4 m thickness, the calculation procedures are similar to those in Section 3.2 (details are not repeated). There are two essentials in the new simplified method calculation: (1) the vertical effective stress due to gravity of sand layer is estimated as 32.5 kPa and contributes to the initial effective stress of the clayey soil layer; (2) there is a nonlinear relationship between the permeability and void ratio, Kim and Leroueil (2001) suggested that a parameter of permeability and void ratio change to describe the variation of the hydraulic conductivity:

$$\log k_z = \log k_{zo} - (e_o - e)/C_k \quad (30)$$

where  $k_{zo}$  is the permeability with respect to the initial void ratio  $e_o$ ,  $C_k$  is the slope of the  $e - \log k_z$ , taken as 0.865. For the calculation of the new simplified method, the value of permeability is updated using Eq. (30) based on the change of void ratio of clayey soil under

each stage loading.

Figure 14 shows the comparison of calculated settlement using the new simplified method and measured data in the field. It is found that the calculated settlement agrees fair well with those measured in the test site, which provides a good representation of a clayey soil layer subjected to multi-staged loading. The high-quality laboratory tests provide the credible parameter values in the calculation. Recently, the advanced analysis of the creep parameter based on the optimization techniques has been reported by Zhou et al. (2018), Yin et al. (2018) *etc.* It is recommended that those advanced techniques could be utilized in the new simplified method to predict the accurate settlement.

## 5. Conclusions

A new simplified method was developed to calculate the consolidation settlement of a soil layer subjected to the multi-staged loading. In this method, the creep settlement occurs during and after the consolidation. The equations of de-coupled and delayed creep settlement and new pre-consolidation pressure in the multi-staged loading were derived in the calculation. The finite element programs using an Elastic Visco-Plastic model, as a fully coupled Hypothesis B method, were used to verify this new simplified method and Hypothesis A method. Main findings and conclusions are drawn based on the derived equations and comparison of results as follows:

- (a) The calculation of creep compression in the new simplified Hypothesis B method was modified by replacing the “secondary consolidation” settlement with the delayed creep settlement. The delayed creep settlement can be calculated for the soils under both over-consolidated state and normally consolidated state.
- (b) For one typical soil layer under different multi-staged ramp loadings, it is demonstrated

that results of the new simplified Hypothesis B method are very close to the finite element modelling results with *relative errors* within 10.9%, which is fully complied with the requirement in engineering design.

(c) Hypothesis A method generally gives a poor consolidation settlement prediction compared to finite element simulations, especially for the long-term settlement. Thus, Hypothesis A method may not be competent in predicting the long-term consolidation settlement of soils subjected to multi-staged loading.

(d) There is a good agreement between the calculated settlement of the new simplified method and measured data in the *Berthierville* test site, which demonstrates the feasibility of this new simplified method.

To date, the new simplified method has been successfully applied in the calculation of consolidation settlement of soft soil with creep under a general multi-staged loading. However, there are still some limitations to be concerned, such as: (a) the soft ground improved by stone columns (SC), deep cement mixing (DCM) columns, *etc.*; (b) the long-term nonlinear creep behavior of soft soils (Yin, 1999; Feng *et al.*, 2017); (c) the settlement is only suitable for 1-D compression condition, which may encounter the embarrassment when it is used in the large strain deformation such as self-consolidation condition, vacuum preloading (Feng, et al., 2019; Ni et al., 2019; Tian, et al., 2019). In the future, further studies in the improved soft ground, long-term nonlinear creep settlement in the field applications, *etc.* are going to be put into practice.

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485    No potential conflict of interest was reported by the authors.

486

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