Input- vs. Output-based Farm Subsidies in Developing Economies: Farmer Welfare and Income Inequality

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Abstract

To alleviate farmer poverty in developing economies, two common farmer subsidy schemes are either *input-based that intends to reduce farmers'* input purchasing costs or *output-based that aims to lower farmers'* output processing costs. By analyzing a stylized model that captures yield heterogeneity across farmers who engage in quantity competition, we find that both schemes can improve farmers' income. However, these two schemes generate different effects. First, the input-based subsidy scheme narrows the income gap between farmers, but the output-based scheme widens this gap. Second, the output-based subsidy scheme outperforms the input-based subsidy scheme in terms of total farmer income and farmer productivity. Overall, we find that low-yield farmers prefer input-based subsidies, while high-yield farmers prefer output-based subsidies. These results continue to hold even when the farmer's yield rate is uncertain.

Keywords: Socially responsible operations, Farm subsidies, Income inequality.

1 Introduction

Farmer poverty is a serious concern in developing countries. In India, 50% of workforce participates in the agriculture sector, but the average income of an Indian farmer is below US$5 per day (Sodhi and Tang 2014). Also, more than 40 million small-scale farmers in China live below the national poverty line (Zhao 2018). Two major obstacles in these regions are high input purchase costs and output processing costs. First, due to low income and high distribution costs in developing countries, quality fertilizers and seeds are often not affordable (Kwa 2001, Mare et al. 2010). Without affordable inputs, farmer income in India has declined over the years (Jayan 2017). Second, due to poor infrastructure (roads, storage facilities, etc.) in developing countries, farmers incur high output processing (e.g., harvest, transportation, post-harvest handling, etc.) costs. Gedaref (2017) reported that the harvest cost increased by four times in Sudan, which led to significant losses among the farmers.

The above observations suggest that it is important to help farmers to reduce their input purchasing costs of seeds and fertilizers or/and output processing costs of transportation and post-harvest handling (Sodhi and Tang 2011). Two commonly observed farm subsidy programs in developing countries are either *input-* or *output-based* subsidy schemes. The input-based subsidies reduce the purchase cost of certain inputs such as fertilizers and seeds. For example, the governments in Mali, Ghana and Nigeria provide fertilizers to smallholder farmers at a discounted price (Jayne and Rashid 2013, and Wiggins and Brooks 2010). In India, the government deposits appropriate subsidies into farmers' bank accounts after they purchased the seeds at market price (Prasad 2016). Unlike the input-based scheme, the output-based subsidy scheme is intended to reduce different output processing costs. For example, the Indian government offers the transportation subsidy to defray farmers' transportation cost of their outputs (Roy 2018), while Thai rice farmers receive a storage cost subsidy of 1,500 baht per metric ton (Gain Report 2017). Governments in other developing countries such as China, Brazil, Ukraine and Turkey also implement various output-based subsidy programs (OECD 2009).

Despite the popularity of various input- and output-based subsidy schemes, their impact on farmer welfare, productivity and the income gap are not well understood, especially when farmers have different yield rates due to different farming environments: access to irrigation water, soil composition, farming experience, etc. (World Bank 2012). The existing empirical evidence about the implications of these two schemes have been mixed. Some advocates argue that input-based subsidies can reduce the income inequality (Darko 2015) and alleviate poverty (Dorward and Chirwa 2011), but Blarcom et al. (1993) argue the opposite. Regarding output-based subsidies, Chinyamakobvu (2012) argues that output-based subsidies can increase productivity, while Ahmed (2011) shows that output-based subsidies for transportation can be detrimental to farmers due to intensified competition.

To gain a better understanding about the implications of the input- and output-based subsidy schemes in terms of farmer welfare, productivity and the income gap, we develop a two-stage model in which the government first determines the subsidy scheme (i.e., input-based or output-based) and the corresponding subsidy level with an aim to maximize farmers' welfare. Given the subsidy program, different farmers with different yield rates determine their planting quantities as they engage in quantity competition.[1] We first analyze the case when farmers' different yield rates are deterministic. Our equilibrium analysis reveals that both input- and output-based subsidy schemes can improve the income of each farmer. However, we note these two schemes generate different effects:

1. The input-based subsidy narrows the income gap between the farmers, but the output-based subsidy widens this gap.

2. The output-based scheme outperforms the input-based subsidy scheme in terms of total farmer income as well as farmer productivity.

A further comparison of equilibrium outcomes reveals that high- and low-yield farmers hold opposite preferences: low-yield farmers prefer input-based subsidies while high-yield farmers prefer output-based subsidies. Therefore, the selection of a particular subsidy scheme will depend on the government's ultimate goal. We also consider two extensions. First, we study a "combined" subsidy scheme under which the government offers both input- and output-based subsidies (instead of one of them). From the perspective of the farmers' total income, we find that input- and output-based subsidies are "complementary". However, from the perspective of the income gap between farmers, these two schemes are "substitutes". Also, we find that it is sufficient to offer only one of the subsidy schemes but not both. Second, we examine the case when farmers' yield rates are uncertain. We show that our key results obtained for the deterministic yield case continue to hold. We also find that reducing the yield uncertainty can improve the total farmer income but at the expense of widening the income gap.

This chapter is organized as follows. Section 2 briefly reviews the relevant literature. Section 3 describes the base model. The equilibrium analysis associated with these two subsidy schemes is presented in Section 4. In Section 5, we compare the performance of the two subsidy schemes. We extend our base model in Section 6. Concluding remarks are provided in Section 7.

2 Literature Review

Our study belongs to an emerging research stream that deals with socially responsible operations in developing countries. Various researchers examine different mechanisms for alleviating farmer poverty. Due to poor infrastructure for transportation and information access,

[1] The Cournot quantity competition is an appropriate representation of various agricultural product markets such as malting barley and banana (Deodhar and Sheldon 1996, Dong et al. 2006).

farmers face high input purchasing and output processing costs and lack proper information to make their selling decisions. To overcome these challenges, Sodhi and Tang (2014) propose direct purchase from and disseminating market information to farmers. An et al. (2015) examine the implications of farm cooperatives that can help affiliated farmers to reduce purchase costs (via aggregation), improve the process yield (via mutual learning), and reduce selling costs (through disintermediation).

More recently, various researchers examine the value of market information as well as farming advisory information. Chen et al. (2013) discover an economic incentive for ITC to provide advisory information to farmers even though these farmers are outside the ITC network. In a similar vein, Chen et al. (2015) examine the strategic behavior for expert farmers who can decide on the knowledge level to share with other farmers over a support hotline in India. Instead of providing private information to farmers through ITC network or hotline, Chen and Tang (2015) examine whether the government should share public information with farmers who may be endowed with private knowledge. They find that in the presence of private knowledge, providing public information can be detrimental to farmers due to the unintended herd effect. Tang et al. (2015) study whether farmers should adopt market information or agricultural advice when making production planning decisions. They show that the market information is beneficial to farmers; however, the value of the agricultural advice depends on the upfront investment cost. Liao et al. (2017) examine the case when farmers can select the crop to grow and determine the market to sell in. They show that information provision may not improve the farmers' total welfare. He et al. (2018) examine the farmers' incentive to join the informational coalitions in the presence of both private and public market information.

Recently, some researchers begin to investigate different agricultural supply chains. For example, Hsu et al. (2017) compare three business models in a milk supply chain consisting of a social enterprise and dairy farmers. They find that the social enterprise prefers the partnership model over both the conventional decentralized model and the independent integrated model when the market size of dairy products is intermediate. Hu et al. (2017) study the impact of strategic farmers on the price fluctuation in agricultural markets. They find that the strategic farmers' self-interest can reduce price volatility and benefit all farmers. Zhang and Swaminathan (2018) construct a finite-horizon stochastic dynamic program to investigate smallholder farmers' optimal seeding policy under rainfall uncertainty. They show that each farmer should plant only when the seed amount is larger than a threshold. They find that the adoption of the optimal time dependent threshold-type planting schedule could help mitigate the risk of yield drop due to severe climate conditions.

Unlike the aforementioned studies, here we focus on the agricultural subsidies. In this research stream, Akkaya et al. (2016a) investigate the impact of the government interventions such as tax reductions and farm subsidies on the farmers' adoption of sustainable farming practice. In a similar vein, Akkaya et al. (2016b) consider the impact of three types of government interventions– price support, cost support, and yield enhancement– on farmer income, consumer surplus, and government spending. They find that when the total budget is public information, the price support and the cost support yield the same performance. Guda et al. (2016) examine a guaranteed support price scheme under which the government purchases the crop at the predetermined price and sells them to the poor customers at a discounted price. Alizamir et al. (2017) compare the price loss coverage (PLC) program ((i.e., subsidizing farmers when the market price is below a certain threshold) with the agriculture risk coverage (ARC) program (i.e., subsidizing farmers when farmers' revenue is below a certain threshold). They find that the PLC program dominates the ARC program in terms of the farmer revenue, the consumer surplus, and the government's cost for a large range of parameter values. Different from the above-mentioned studies, we make an initial attempt to examine the implications of the input- and output-based subsidy schemes on the farmer income and income inequality.

3 Model Preliminaries

We now present a parsimonious model that captures the underlying issues of our study. Consider a situation when two farmers (or cooperatives) who grow a single commodity crop and sell it in a local market.[2] Due to the heterogeneity in the endowed resources (water sources, soil quality, farming knowledge), we shall consider the case when the yield rate z_i is farmer-specific. (In the base model, we assume that z_i is deterministic but we shall extend our analysis in Section 6 to deal with yield uncertainty.) Without loss of generality, we shall assume that $0 < z_1 < z_2 < 1$. For ease of reference, we shall refer to farmer 1 (farmer 2, respectively) as the low-yield (high-yield, respectively) farmer.

3.1 Farmers' Planning Problem

At the beginning of a planting season, each farmer i, $i = 1, 2$, decides on his (input) planting quantity q_i, which will generate an output quantity $z_i q_i$. Hence, the total sales quantity during the harvest season is equal to $\Sigma_{i=1}^{2} z_i q_i$. For a commodity crop, it is reasonable to

[2]Our main insights continue to hold when there are more than 2 farmers.

assume that farmers engage in Cournot competition so that the market price P satisfies [3]

$$P = m - z_1 q_1 - z_2 q_2, \tag{1}$$

where m represents the market potential.

We assume that both farmers incur identical "input" unit purchasing cost c that covers the cost of seeds, fertilizers, and labor. We also assume that both farmers incur identical "output" unit processing cost t that covers the transportation and storage costs. Therefore, given the input cost c, the output cost t, and the market price P (stated in (1)), each farmer i selects his planting quantity q_i to maximize his expected income $\pi_i(q_i)$, where:

$$\pi_i(q_i) = (P - t)z_i q_i - c q_i = (a - z_i q_i - z_j q_j) z_i q_i - c q_i, \text{ and } a \equiv m - t, i, j = 1, 2, i \neq j. \tag{2}$$

By considering the best response functions of both farmers simultaneously, we can determine the planting quantities in equilibrium. To ensure that in equilibrium the planting quantity for each farmer is positive so that we can examine the issue of income gap between farmers, we make the following two assumptions:

Assumption 1. *The market potential m is sufficiently high so that $m > c \cdot (\sum_{i=1}^{2} \frac{1}{z_i}) + t$.*

Assumption 2. *The yield rate of farmer 2 is moderately higher than farmer 1 so that $z_1 < z_2 < 2z_1$.*

Assumption 1 ensures that the market potential m is sufficiently high so that the market price P given in (1) is attractive enough to entice both farmers to plant a positive amount. Assumption 2 holds when the variation of yield rates across farmers are moderate. There is empirical evidence that supports Assumption 2: Filho et al. (2010) show that the variation of yield rates across all farmers in Brazil is moderate – the coefficient of variation of the yield rates across all Brazilian farmers is less 60%. For each of the four crops (maize, rice, wheat and soybean), Ray et al. (2015) show that the coefficient of variation of the yield rates across the world is less than 50%.

Let us define the following two terms that will enable us to simplify the exposition:

$$K = \sum_{i=1}^{2} \frac{1}{z_i} \text{ and } V = \frac{1}{z_1} - \frac{1}{z_2}. \tag{3}$$

By interpreting $1/z_i$ as the "inefficiency" of farmer i, we can interpret K as the "total inefficiency" of both farmers and V as the "inefficiency disparity" between farmers. By considering K and V as defined in (3), Assumptions 1 and 2 can be simplified as $a \equiv m - t > cK$ and $K > 3V$, respectively.

[3]The reader is referred to Chen and Tang (2015) for a detailed discussion about the existence of the Cournot competition in the agricultural economics.

3.2 Output-based and Input-based Subsidy Schemes

As articulated in the Introduction section, we shall analyze two subsidy schemes:

1. The *Input-based Subsidy* scheme intends to defray the input purchasing cost: the government offers a subsidy δ for each input unit (e.g., seeds, fertilizers, etc.) so that the effective unit planting cost is reduced from c to $c - \delta$, where $\delta \in [0, c]$.

2. The *Output-based Subsidy* scheme aims to reduce the output processing cost: the government offers a subsidy θ for each output unit (e.g., transportation, storage, etc.) so that the effective unit processing cost is reduced from t to $t - \theta$, where $\theta \in [0, t]$.

In the base model, we consider the case in which the government offers either an input-based subsidy or an output-based subsidy. However, in a later section, we shall extend our analysis to the case where a combination of both subsidy schemes is offered.

3.3 The Government's Subsidy Scheme

From the government's perspective, it will determine the subsidy level (i.e., δ or θ) to maximize the farmer welfare subject to an earmarked budget B. Here, due to the uniqueness of the agricultural industry, the "farmer welfare" under our setting takes both the farmers' total income $\pi_1(\cdot) + \pi_2(\cdot)$ and the income gap $|\pi_2(\cdot) - \pi_1(\cdot)|$ into consideration, where $\pi_i(.)$ is given in (2). In our context, the farmer welfare denoted as $\Pi(\cdot)$ can be expressed as

$$\Pi(\cdot) = (\pi_1(\cdot) + \pi_2(\cdot)) - \alpha \cdot |\pi_2(\cdot) - \pi_1(\cdot)|, \tag{4}$$

where the parameter $\alpha \in [0, 1]$ captures the extent to which the government cares about the income gap between farmers.

The sequence of events is defined as follows. First, given an earmarked budget B, the government decides which subsidy scheme to adopt and the corresponding subsidy level; i.e., either the input-based unit subsidy δ under the input-based subsidy scheme or the output-based unit subsidy θ under the output-based subsidy scheme. Next, given a particular subsidy scheme and the corresponding subsidy level (i.e., δ or θ), each farmer i ($i = 1, 2$) decides the planting quantity q_i, and sells the harvest quantity $z_i q_i$ in the market according to the market price P as given in (1). The farmers' income is then realized. Below, we use the backward induction to determine the optimal subsidy level under each scheme.

4 Equilibrium Analysis

4.1 Input-based Subsidy Scheme

We now analyze the equilibrium outcome associated with the input-based subsidy scheme (δ) via the backward induction. First, we determine the planting quantities of both farmers for any given unit subsidy δ. Anticipating the farmers' equilibrium planting quantities, we then derive the government's optimal subsidy level decision.

4.1.1 The farmers' planting decisions

Given the subsidy δ per unit of the planting quantity, farmer i's unit planting cost is reduced from c to $c - \delta$. By letting $a = m - t$, farmer i's expected income $\pi_i(.)$ as given in (2) can be expressed as

$$\pi_i(q_i) = (a - z_i q_i - z_j q_j) z_i q_i - (c - \delta) q_i. \tag{5}$$

Based on the first-order condition, farmer i's best response function for any given q_j can be derived as

$$q_i(q_j) = \frac{(a - z_j q_j) z_i - c + \delta}{2 z_i^2}, \quad i, j = 1, 2, \text{ and } i \neq j. \tag{6}$$

Since $dq_i(q_j)/d\delta = 1/2z_i^2 > 0$, we can conclude that farmer i plants more when the government increases the input-based unit subsidy δ. By considering the best response functions stated in (6) simultaneously for both farmers, we get:

Proposition 1. *For any given input-based subsidy δ, the planting quantity $\tilde{q}_i(\delta)$ and the corresponding income $\tilde{\pi}_i(\delta)$ associated with farmer i satisfy:*

$$\tilde{q}_i(\delta) = \frac{a + (c - \delta) \cdot K}{3 z_i} - \frac{c - \delta}{z_i^2} > 0 \quad \textit{and} \quad \tilde{\pi}_i(\delta) = \left(z_i \cdot \tilde{q}_i(\delta)\right)^2, i = 1, 2. \tag{7}$$

Furthermore, $\tilde{q}_i(\delta)$ and $\tilde{\pi}_i(\delta)$ possess the following properties:

(a) The planting quantity $\tilde{q}_i(\delta)$ and the harvest quantity $z_i \tilde{q}_i(\delta)$ of farmer i are increasing in δ, while his expected income $\tilde{\pi}_i(\delta)$ is increasing and convex in δ.

(b) Both the output (harvest) quantity and the expected income of the high-yield farmer 2 are larger than those of the low-yield farmer 1; i.e., $z_2 \tilde{q}_2(\delta) > z_1 \tilde{q}_1(\delta)$ and $\tilde{\pi}_2(\delta) > \tilde{\pi}_1(\delta)$.

(c) The income gap between the high-yield farmer 2 and the low-yield farmer 1 (i.e., $\tilde{\pi}_2(\delta) - \tilde{\pi}_1(\delta)$) is decreasing in δ.

Statement (a) of Proposition 1 reveals that by defraying the unit planting cost from c to $c-\delta$, the input-based subsidy provides incentives for both farmers to plant more and harvest more, which in turn causes the market price P to decrease. However, the overall effect of the input-based subsidy is that it can help each farmer to improve income. This implies that under the input-based subsidy scheme, the benefit of defraying the farmer's planting cost outweighs the loss caused by the declining market price P. Statement (b) of Proposition 1 shows that the yield advantage enables the high-yield farmer 2 to harvest more and earn more than the low-yield farmer 1, resulting in the income inequality between the farmers. Despite the fact that there exists income inequality, statement (c) reveals that the income gap $\tilde{\pi}_2(\delta) - \tilde{\pi}_1(\delta)$ is decreasing in the unit subsidy δ. This indicates that *the input-based subsidy scheme can help reduce the income inequality between the heterogenous farmers.* The underlying reason is due to the fact that the cost savings per unit of the output quantity is higher for the low-yield farmer because $\delta/z_1 > \delta/z_2$. In summary, Proposition 1 has the following implication.

Insight 1. An increase of the input-based unit subsidy δ improves the income for both farmers while reducing the income gap.

4.1.2 The government's input-based subsidy level decision

Anticipating the farmers' equilibrium planting decisions stated in Proposition 1, the government determines the input-based unit subsidy δ to maximize the farmer welfare $\tilde{\Pi}(\delta)$ given in (4) subject to an earmarked budget constraint B. By noting from Proposition 1 that $\tilde{\pi}_2(\delta) > \tilde{\pi}_1(\delta)$, the government's problem can be rewritten as:

$$\max_{\delta \le c} \tilde{\Pi}(\delta) = (1+\alpha)\tilde{\pi}_1(\delta) + (1-\alpha)\tilde{\pi}_2(\delta), \tag{8}$$

$$s.t. \quad \delta \cdot (\tilde{q}_1(\delta) + \tilde{q}_2(\delta)) \le B, \tag{9}$$

where (9) is the budget constraint associated with the input-based subsidy scheme, and $\tilde{q}_i(\delta)$ and $\tilde{\pi}_i(\delta)$ are given in (7), $i = 1, 2$.

By applying statement (a) of Proposition 1, it is immediate that both the farmer welfare $\tilde{\Pi}(\delta)$ and the total input-based subsidy cost (i.e., the left-hand side of (9)) are increasing with the subsidy level δ. Hence, the budget constraint (9) is binding.

Proposition 2. *If $B \le acK/3$, then the optimal input-based subsidy level δ^* is*

$$\delta^* = \frac{-aK + 3c\left(\frac{K^2}{6} + \frac{V^2}{2}\right) + \sqrt{\left(aK - 3c\left(\frac{K^2}{6} + \frac{V^2}{2}\right)\right)^2 + 36\left(\frac{K^2}{6} + \frac{V^2}{2}\right)B}}{6\left(\frac{K^2}{6} + \frac{V^2}{2}\right)} > 0. \tag{10}$$

Furthermore, δ^* decreases in the total inefficiency K but increases in the inefficiency disparity V.[4]

It follows from Propositions 1 and 2 and Assumption 1 (i.e., $a > cK$) that the total planting quantity ($i.e.$, $\tilde{q}_1(\delta^*)+\tilde{q}_2(\delta^*) = (2a-(c-\delta^*)\cdot K)K/6-V^2\cdot(c-\delta^*)/2$) is increasing in K and decreasing in V, where K and V are given in (3). Thus, when the "inefficiency disparity" V is fixed, as the "total inefficiency" K increases (i.e., as the yield rate decreases), both farmers will grow more and the government needs to reduce the optimal unit subsidy δ^* to ensure the budget constraint remains binding. In the same vein, when the "total inefficiency" K is fixed, as the "inefficiency disparity" V increases (i.e., the yield rates between farmers vary more), the yield rates of both farmers diverge further. In this case, the government can increase the unit subsidy δ^* to induce the farmers to plant more without violating the budget constraint.

So far, we have shown that the input-based subsidy scheme is an effective scheme to entice farmers to plant more, harvest more, and earn more. At the same time, through Proposition 1, we have also shown that the adoption of the input-based subsidy scheme can help reduce the income gap. Will these results hold for the output-based subsidy scheme? We shall investigate this issue next.

4.2 Output-based Subsidy Scheme

We now consider the output-based subsidy scheme θ. For each unit of planting quantity, the high-yield farmer 2 receives more output subsidies than the low-yield farmer 1 because $\theta z_1 < \theta z_2$. However, under the input-based subsidy scheme, both farmers receive the same subsidy δ for each unit of planing quantity. As will be shown later, this observation plays a key role in understanding the difference between the two subsidy schemes.

4.2.1 The farmer's planting decision

For any output-based unit subsidy θ, the income associated with each harvest quantity of farmer i is $P - (t - \theta)$, where $P = m - z_i q_i - z_j q_j$ is given in (1). By letting $a = m - t$, the expected income $\pi_i(.)$ of farmer i given in (2) can be rewritten as:

$$\pi_i(q_i) = (a + \theta - z_i q_i - z_j q_j)\, z_i q_i - c q_i,\ \ i, j = 1, 2,\ \text{and}\ i \neq j. \tag{11}$$

[4]Note that the optimal input-based subsidy level δ^* is increasing in B and when $B = acK/3$, $\delta^* = c$. As it is never in the best interest of the government to over-subsidize the farmers (i.e., $\delta^* \leq c$ is required), the government's budget cannot be too large so that $B \leq acK/3$.

By considering the first-order condition, we can obtain the best response function of farmer i as:

$$q_i = \frac{(a+\theta - z_j q_j) z_i - c}{2z_i^2}, \ i,j = 1,2, \text{ and } i \neq j. \tag{12}$$

By considering the best response functions simultaneously, we get:

Proposition 3. *For any given output-based unit subsidy θ, the planting quantity $\tilde{q}_i(\theta)$ and the corresponding income $\tilde{\pi}_i(\theta)$ associated with farmer i satisfy:*

$$\tilde{q}_i(\theta) \quad = \quad \frac{a+\theta+cK}{3z_i} - \frac{c}{z_i^2} > 0 \quad and \quad \tilde{\pi}_i(\theta) = \left(z_i \cdot \tilde{q}_i(\theta)\right)^2, i = 1,2. \tag{13}$$

Furthermore, $\tilde{q}_i(\theta)$ and $\tilde{\pi}_i(\theta)$ possess the following properties:

(a) The planting quantity $\tilde{q}_i(\theta)$ and the harvest quantity $z_i\tilde{q}_i(\theta)$ of farmer i are increasing in the output-based unit subsidy θ, while his expected income $\tilde{\pi}_i(\theta)$ is increasing and convex in θ.

(b) Both the output quantity and the expected income of the high-yield farmer 2 are larger than those of the low-yield farmer 1; i.e., $z_2\tilde{q}_2(\theta) > z_1\tilde{q}_1(\theta)$ and $\tilde{\pi}_2(\theta) > \tilde{\pi}_1(\theta)$.

(c) The income gap between the high-yield farmer 2 and the low-yield farmer 1 (i.e., $\tilde{\pi}_2(\theta) - \tilde{\pi}_1(\theta)$) increases in θ.

Statement (a) of Proposition 3 shows that a higher output-based unit subsidy θ motivates both farmers to plant more, harvest more output, and obtain a higher income. Statement (b) reveals that the yield advantage enables the high-yield farmer 2 to harvests more and earns more than the low-yield farmer 1, again resulting in income inequality. These results are consistent with those under the input-based subsidy scheme stated in Proposition 1. However, statement (c) shows that an increase of the output-based unit subsidy θ further widens the income gap between the two farmers, which is the opposite of Proposition 1. This opposite result is due to the fact that under the output-based subsidy scheme, the high-yield farmer 2 receives a higher subsidy than the low-yield farmer 1 when planting one unit of the crop (i.e., $\theta z_1 < \theta z_2$). Hence, the output-based subsidy scheme favors the high-yield farmer. In summary, Proposition 3 enables us to obtain the following implication.

Insight 2. An increase of the output-based unit subsidy improves the income of both high- and low-yield farmers, but it widens the income gap.

Overall, Insights 1 and 2 reveal that, although both the input- and output-based subsidy schemes can increase the farmers' income, the input-based subsidy scheme seems *fairer* than the output-based one. This is in light of the fact that the adoption of the input-based (output-based, respectively) subsidy scheme can reduce (widen, respectively) the farmers' income gap.

4.2.2 The government's output-based subsidy level decision

Anticipating the farmers' planting quantity decisions stated in (13), the government determines the output-based unit subsidy θ to maximize the farmer welfare $\tilde{\Pi}(\theta)$ given in (4), which takes both the farmers' total income and the income gap into consideration subject to a limited budget B. According to Proposition 3, $\tilde{\pi}_2(\theta) > \tilde{\pi}_1(\theta)$. Hence, the government decides the optimal subsidy level θ to maximize the following problem:

$$\max_{\theta \leq t} \tilde{\Pi}(\theta) = (1+\alpha) \cdot \tilde{\pi}_1(\theta) + (1-\alpha) \cdot \tilde{\pi}_2(\theta), \tag{14}$$

$$s.t. \quad \theta \cdot (z_1 \tilde{q}_1(\theta) + z_2 \tilde{q}_2(\theta)) \leq B, \tag{15}$$

where the left-hand side of the budget constraint (15) represents the total amount of the output-based subsidy the government provides. Based on Proposition 3, both the farmer welfare $\tilde{\Pi}(\theta)$ and the total output-based subsidy cost (the left hand side of (15)) are increasing in the unit subsidy θ. This observation implies that the budget constraint (15) is binding.

Proposition 4. *If $B \leq t \cdot (2m - cK)/3$, then the optimal subsidy per unit of the harvest quantity θ^* is*

$$\theta^* = \frac{-(2a - cK) + \sqrt{(2a - cK)^2 + 24B}}{4}. \tag{16}$$

Moreover, θ^* increases in the total inefficiency K but is independent of the inefficiency disparity V.[5]

Proposition 4 has the following implications. From (13), we can easily show that the total harvest quantity $z_1\tilde{q}_1(\theta^*) + z_2\tilde{q}_2(\theta^*) = (2(a + \theta^*) - cK)/3$ is increasing in K and independent of V. Consequently, when the "inefficiency disparity" V is fixed, as the "total inefficiency" K increases (i.e., as the yield rate decreases), each farmer's output decreases so that the government can afford to increase the output-based unit subsidy θ to ensure that the budget constraint remains binding. However, when the "total inefficiency" K is fixed, the optimal output-based subsidy level θ^* shall be independent of V.

A closer look at Propositions 2 and 4 implies that when the farmers are less efficient (reflected by a larger K), they may prefer the output-based subsidy scheme over the input-based one; while when the farmers are very heterogenous in their growing abilities (represented by a larger V), they may prefer the input-based subsidy scheme over the output-based one.

[5] Note that θ^* is increasing in B and when $B = t \cdot (2m - cK)/3$, $\theta^* = t$. To avoid over-subsidizing the farmers (i.e., $\theta^* \leq t$ is required), the government's budget cannot be too large (i.e., $B \leq t \cdot (2m - cK)/3$). Combining Propositions 2 and 4, to ensure that $\delta^* \leq c$ and $\theta^* \leq t$, we assume that $B \leq \min\{acK/3, t \cdot (2m - cK)/3\}$ hereafter.

5 Input-based versus Output-based Subsidy Schemes

So far, we have obtained the equilibrium outcomes associated with both the input- and output-based subsidy schemes. Next, we shall compare the performance of these two subsidy schemes in terms of farmer i's income $\tilde{\pi}_i(\cdot)$ and harvest quantity $z_i\tilde{q}_i(\cdot)$, $i = 1, 2$ as well as the farmers' total income $\tilde{\pi}_1(\cdot) + \tilde{\pi}_2(\cdot)$ and the income gap $\tilde{\pi}_2(\cdot) - \tilde{\pi}_1(\cdot)$.

Proposition 5. *The performance associated with the input-based versus the output-based subsidy schemes can be described as follows:*

(a) The planting quantity, the harvest quantity and the expected income of the low-yield farmer 1 *(high-yield farmer* 2*, respectively) are higher (lower, respectively) under the input-based subsidy scheme than under the output-based subsidy scheme.*

(b) The farmers' total income is lower under the input-based subsidy scheme than that under the output-based subsidy scheme: $\tilde{\pi}_1(\delta^*) + \tilde{\pi}_2(\delta^*) < \tilde{\pi}_1(\theta^*) + \tilde{\pi}_2(\theta^*)$.

(c) The income gap is narrower under the input-based subsidy scheme than that under the output-based subsidy scheme: $\tilde{\pi}_2(\delta^*) - \tilde{\pi}_1(\delta^*) < \tilde{\pi}_2(\theta^*) - \tilde{\pi}_1(\theta^*)$.

Proposition 5 implies that the high- and low-yield farmers favor different subsidy schemes: the low-yield farmer prefers the input-based subsidy scheme δ^*, while the high-yield farmer prefers the output-based subsidy scheme θ^*. This is because the input-based subsidy δ enables the low-yield farmer 1 to obtain higher cost savings per unit of output because $\delta/z_1 > \delta/z_2$. The output-based subsidy θ enables the high-yield farmer 2 to collect more subsidy per unit of the planting quantity than the low-yield farmer 1 because $\theta z_2 > \theta z_1$, thereby enhancing the yield advantage of the high-yield farmer relative to the low-yield one. Furthermore, because the low-yield farmer 1 produces more while the high-yield farmer 2 produces less under the input-based subsidy scheme than under the output-based subsidy scheme, the farmers' total income under the input-based subsidy scheme is lower than that under the output-based subsidy scheme (i.e., $\tilde{\pi}_1(\delta^*) + \tilde{\pi}_2(\delta^*) < \tilde{\pi}_1(\theta^*) + \tilde{\pi}_2(\theta^*)$). In addition, compared with the input-based subsidy scheme, although the output-based subsidy scheme is more effective in improving the farmers' total income, it also widens the income gap. These results are the natural consequences of Propositions 1 and 3. Below, we summarize the main implications obtained from Proposition 5.

Insight 3(a). The high-yield farmer prefers the output-based subsidy scheme, while the low-yield farmer favors the input-based subsidy scheme.

Insight 3(b). The input-based subsidy scheme is more effective in reducing the income gap, while the output-based subsidy scheme is more effective in improving the farmers' total income.

In practice, due to the different endowed resources such as water availability and the soil type, farmers in different regions may exhibit different yield rates. Insight 3(a) implies that it is better to implement the input-based subsidy scheme in the regions where the yield rate is low, while the output-based subsidy scheme is more suitable to be implemented in a region where the yield rate is high. Furthermore, Insight 3(b) reveals that which subsidy scheme to adopt depends on the government's ultimate goal: improving the farmers' total income versus reducing the income gap. These observations yield:

Proposition 6. *The farmer welfare under the input-based subsidy scheme is higher than that under the output-based subsidy scheme (i.e., $\tilde{\Pi}(\delta^*) > \tilde{\Pi}(\theta^*)$) if and only if α, the weight associated with the income gap, is larger than a threshold $\hat{\alpha}$, where $\hat{\alpha} = \frac{\tilde{\pi}_1(\theta^*)+\tilde{\pi}_2(\theta^*)-(\tilde{\pi}_1(\delta^*)+\tilde{\pi}_2(\delta^*))}{\tilde{\pi}_2(\theta^*)-\tilde{\pi}_1(\theta^*)-(\tilde{\pi}_2(\delta^*)-\tilde{\pi}_1(\delta^*))}$.*

Note that both the optimal input-based unit subsidy δ^* (given in (10)) and the optimal output-based unit subsidy θ^* (given in (16)) are independent of the weight parameter α. Combining this with Insight 3(a) indicates that when the government puts a high priority on reducing the income inequality between the farmers (i.e., $\alpha > \hat{\alpha}$), it should adopt the input-based subsidy scheme rather than the output-based subsidy scheme.

6 Discussion

We now extend our analysis by considering the following three settings. First, we examine the impact of different subsidy schemes on the total production quantity and the aggregate yield rate. Second, we shall consider a "combined" subsidy scheme under which the government provides both the input- and output-based subsidies. Third, we extend our analysis to the case when the farmer's yield rate is uncertain.

6.1 The Impact of Subsidy Schemes on the Aggregate Level Performance

We now examine the impact of the input- and output-based subsidy schemes on the following aggregate level performance measures:

- Aggregate planting quantity $\tilde{I}(\cdot) := \tilde{q}_1(\cdot) + \tilde{q}_2(\cdot)$;

- Aggregate harvest quantity $\tilde{Q}(\cdot) := z_1\tilde{q}_1(\cdot) + z_2\tilde{q}_2(\cdot)$; and
- Aggregate yield rate $\tilde{Y}(\cdot) := \tilde{Q}(\cdot)/\tilde{I}(\cdot)$,

By using the same approach as before, we get:

Proposition 7. *The impact of the two subsidy schemes on the aggregate farmer performance are as follows.*

(a) Under the input-based subsidy scheme δ, both the aggregate planting quantity $\tilde{I}(\delta)$ and the aggregate harvest quantity $\tilde{Q}(\delta)$ are increasing in δ, while the aggregate yield rate $\tilde{Y}(\delta)$ is decreasing in δ.

(b) Under the output-based subsidy scheme θ, both the aggregate planting quantity $\tilde{I}(\theta)$ and the aggregate harvest quantity $\tilde{Q}(\theta)$ are increasing in θ, while the aggregate yield rate $\tilde{Y}(\theta)$ is decreasing in θ.

(c) Both the aggregate planting quantity and the aggregate harvest quantity are larger under the input-based subsidy scheme than that under the output-based subsidy scheme, respectively (i.e., $\tilde{I}(\delta^*) > \tilde{I}(\theta^*)$ and $\tilde{Q}(\delta^*) > \tilde{Q}(\theta^*)$). The aggregate yield rate is higher under the output-based subsidy scheme than that under the input-based subsidy scheme (i.e., $\tilde{Y}(\theta^*) > \tilde{Y}(\delta^*)$).

Proposition 7 shows that both the input- and output-based subsidy schemes can increase the aggregate planting and harvest quantities $\tilde{I}$ and $\tilde{Q}$. However, both subsidy schemes reduce the system production efficiency in terms of the aggregate yield rate; i.e., $\tilde{Y}(\delta)$ and $\tilde{Y}(\theta)$ are decreasing in δ and θ, respectively. By comparing the equilibrium outcomes under the two subsidy schemes, statement (c) of Proposition 7 indicates that both the aggregate planting quantity and the aggregate harvest quantity under the input-based subsidy scheme are larger than those under the output-based subsidy scheme, respectively. This is because a unit increase in the input-based subsidy can enable the farmer i to obtain cost savings $1/z_i$ per output unit, while the cost savings associated with a unit increase in the output-based subsidy is just a unit per harvest quantity. Therefore, the input-based subsidy provides a stronger stimulus to increase the farmers' planting and harvest quantities. However, because the low-yield farmer plants more under the input-based subsidy scheme than under the output-based subsidy scheme (see Proposition 5), the aggregate yield rate is lower under the input-based subsidy scheme (i.e., $\tilde{Y}(\delta^*) < \tilde{Y}(\theta^*)$). In summary, the output-based subsidy scheme is more efficient in terms of the aggregate yield rate; and the input-based subsidy scheme is more effective in enticing farmers to plant more and harvest more.

6.2 A Combined Subsidy Scheme

We now examine a combined subsidy scheme under which the government provides both the input- and output-based subsidies so that each farmer receives δ per unit of the planting quantity and obtains θ per unit of harvest quantity.

6.2.1 The farmers' planting decisions

Under a combined subsidy scheme (δ, θ), the input cost is reduced from c to $c - \delta$ and the output cost is reduced from t to $t - \theta$. In this case, the expected income $\pi_i(.)$ given in (2) associated with farmer i can be expressed as:

$$\pi_i(q_i) = (a + \theta - z_i q_i - z_j q_j) z_i q_i - (c - \delta) q_i, i, j = 1, 2, i \neq j, \tag{17}$$

where $a = m - t$. By considering the first-order condition, we can easily derive farmer i's best response function as follows:

$$q_i(q_j) = \frac{(a + \theta - z_j q_j) z_i - c + \delta}{2 z_i^2}, \ i, j = 1, 2, \text{ and } i \neq j. \tag{18}$$

Solving these best response functions simultaneously, we obtain the following equilibrium planting quantity and expected income of farmer i:

$$\tilde{q}_i(\delta, \theta) \ = \ \frac{a + \theta + (c - \delta) \cdot K}{3 z_i} - \frac{c - \delta}{z_i^2}, \quad \text{and} \quad \tilde{\pi}_i(\delta, \theta) = (z_i \cdot \tilde{q}_i(\delta, \theta))^2, i = 1, 2. \tag{19}$$

Note that by letting either $a^{'} \equiv a + \theta$ or $c^{'} \equiv c - \delta$, the above quantities are equivalent to their counterparts associated with the input- or the output-based subsidy scheme, respectively. This implies that the results given in Propositions 1 and 3 still hold under the combined subsidy scheme. To avoid repetition, we omit the details here. Next, we characterize the interactions between the input- and output-based subsidies regarding the above equilibrium outcomes.

Proposition 8. *Under the combined subsidy scheme, both the expected income of farmer i, $\tilde{\pi}_i(\delta, \theta)$, $i = 1, 2$ and the farmers' total income, $\tilde{\pi}_1(\delta, \theta) + \tilde{\pi}_2(\delta, \theta)$, are "supermodular" in the subsidy levels (δ, θ) while the income gap, $\tilde{\pi}_2(\delta, \theta) - \tilde{\pi}_1(\delta, \theta)$, is "submodular" in (δ, θ).*

Proposition 8 implies that the effects of the input- and output-based subsidies on both the income of each farmer and the total farmer income are complementary with each other due to supermodularity. However, their effects on the income gap $\tilde{\pi}_2(\delta, \theta) - \tilde{\pi}_1(\delta, \theta)$ are substitutes due to submodularity. Recall that compared with the input-based subsidy scheme, the output-based subsidy scheme leads to a higher total income but a larger income gap. Proposition 8 reveals that under the combined subsidy scheme, increasing the input-based subsidy level can enhance the effect of the output-based subsidy on the farmers' total income on the one hand while mitigating its impact on the income gap on the other hand.

6.2.2 The government's subsidy level decision

Anticipating farmers' planting quantity decisions as stated in (19), the government determines the subsidy levels δ and θ to maximize the farmer welfare $\tilde{\Pi}(\delta,\theta)$ given in (4). Because the government subsidizes δ per unit of the planting quantity and θ per unit of the harvest quantity, the total amount of subsidy under the combined subsidy scheme can be derived as $\delta \cdot (\tilde{q}_1(\delta,\theta) + \tilde{q}_2(\delta,\theta)) + \theta \cdot (z_1\tilde{q}_1(\delta,\theta) + z_2\tilde{q}_2(\delta,\theta))$. By taking the budget constraint into account, the government solves the following problem:

$$\max_{\delta,\theta} \tilde{\Pi}(\delta,\theta) = ((\tilde{\pi}_1(\delta,\theta) + \tilde{\pi}_2(\delta,\theta)) - \alpha(\tilde{\pi}_2(\delta,\theta) - \tilde{\pi}_1(\delta,\theta)), \tag{20}$$

$$s.t. \quad \delta \cdot (\tilde{q}_1(\delta,\theta) + \tilde{q}_2(\delta,\theta)) + \theta \cdot (z_1\tilde{q}_1(\delta,\theta) + z_2\tilde{q}_2(\delta,\theta)) \leq B. \tag{21}$$

It follows immediately from Proposition 8 that the objective function given in (20) is supermodular. Also, by applying statement (a) as stated in Propositions 1 and 3, we can conclude that the subsidy expense; i.e., the left hand side of the budget constraint as given in (21), is increasing in δ and θ. By using these two observations, we can show that the budget constraint is binding. Hence, we get:

Proposition 9. *Under the combined subsidy scheme, the budget constraint* (21) *is binding. When $\alpha \geq \hat{\alpha}$ (where $\hat{\alpha}$ is given in Proposition 6), the optimal subsidy per unit of the planting quantity δ_m^* satisfies* (10) *while the optimal subsidy per unit of the harvest quantity $\theta_m^* = 0$. When $\alpha < \hat{\alpha}$, the optimal subsidy per unit of the harvest quantity θ_m^* satisfies* (16) *while the optimal subsidy per unit of the planting quantity $\delta_m^* = 0$.*

Although the effects of the input- and output-based subsidies on the farmer welfare are complementary to each other, Proposition 9 indicates that it is never optimal to provide both the input- and output-based subsidies. Recall that the optimal subsidy levels shall lead to the binding of the budget constraint. And based on (19), we can easily show that the total amount of subsidies $\delta \cdot (\tilde{q}_1(\delta,\theta) + \tilde{q}_2(\delta,\theta)) + \theta \cdot (z_1\tilde{q}_1(\delta,\theta) + z_2\tilde{q}_2(\delta,\theta))$ is convex in both the input- and output-based unit subsidies δ and θ. This implies that in equilibrium, one of the subsidy levels (either δ_m^* or θ_m^*) shall be degenerated to zero. Also, Proposition 9 confirms that the government shall provide the input-based subsidy if and only if its concern about the farmers' income inequality is high enough (i.e., when $\alpha > \hat{\alpha}$).

6.3 When the Yield Rate is Uncertain

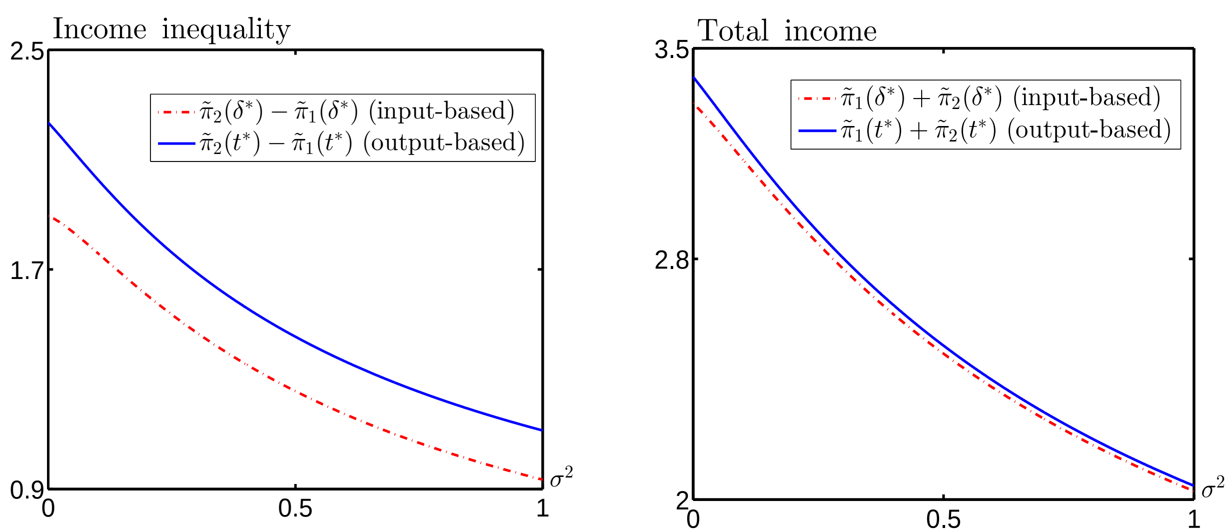
We now extend our analysis to the case in which the farmer's yield rate is uncertain. Specifically, we assume that the yield rate of farmer i, z_i, $i = 1, 2$, is a random variable with mean $E(z_i) = \mu_i$ and variance $Var(z_i) = \sigma_i^2$, where $0 < \mu_1 < \mu_2 < 1$. To simplify our analysis, following Tang et al. (2015), we assume that $\sigma_1 = \sigma_2 = \sigma$ so that the yield rate of farmer 2 is stochastically larger than that of farmer 1. We shall refer to farmer 2 as the high-yield farmer and farmer 1 as the low-yield farmer. Analogously, we define $K = 1/\mu_1 + 1/\mu_2$ and $V = 1/\mu_1 - 1/\mu_2$ in this section. Furthermore, to ensure that both farmers plant a positive amount, we assume that both Assumptions 1 and 2 still hold here; that is, $a = m - t > cK$ and $K > 3V$. By noting that $P = m - z_i q_i - z_j q_j$, $a = m - t$, and $E[z_i^2] = \mu_i^2 + \sigma^2$, $i, j = 1, 2$, and $i \neq j$, farmer i's expected income under input- and output-based subsidy schemes are as follows:

$$\pi_i^{Input}(q_i) = E\{(P-t)z_i q_i - (c-\delta)q_i\} = a\mu_i q_i - (\mu_i^2 + \sigma^2)q_i^2 - \mu_i \mu_j q_i q_j - (c-\delta)q_i,$$
$$\pi_i^{Output}(q_i) = E\{(P-(t-\theta))z_i q_i - cq_i\} = (a+\theta)\mu_i q_i - (\mu_i^2 + \sigma^2)q_i^2 - \mu_i \mu_j q_i q_j - cq_i. \quad (22)$$

Table 1: List of Results when the Yield Rates Are Uncertain

	Equilibrium Outcomes
Input-based Subsidy Scheme	1. $\tilde{q}_i(\delta)$, $\mu_i\tilde{q}_i(\delta)$, $\tilde{\pi}_i(\delta)$, and $\tilde{\pi}_1(\delta)+\tilde{\pi}_2(\delta)$ are all increasing in the input-based unit subsidy δ;
	2. $\mu_1\tilde{q}_1(\delta) < \mu_2\tilde{q}_2(\delta)$ and $\tilde{\pi}_1(\delta) < \tilde{\pi}_2(\delta)$;
	3. When $\sigma^2 \leq \frac{(a-cK)\mu_1\mu_2}{a}$, $\tilde{\pi}_2(\delta) - \tilde{\pi}_1(\delta)$ is decreasing in δ; otherwise, it is first increasing and then decreasing in δ.
Output-based Subsidy Scheme	1. $\tilde{q}_i(\theta)$, $\mu_i\tilde{q}_i(\theta)$, $\tilde{\pi}_i(\theta)$, and $\tilde{\pi}_1(\theta)+\tilde{\pi}_2(\theta)$ are all increasing in the output-based unit subsidy θ;
	2. $\mu_1\tilde{q}_1(\theta) < \mu_2\tilde{q}_2(\theta)$ and $\tilde{\pi}_1(\theta) < \tilde{\pi}_2(\theta)$;
	3. $\tilde{\pi}_2(\theta) - \tilde{\pi}_1(\theta)$ is increasing in θ.
Equilibrium Outcome Comparison	1. $\tilde{q}_1(\delta^*) > \tilde{q}_1(\theta^*)$, $\mu_1\tilde{q}_1(\delta^*) > \mu_1\tilde{q}_1(\theta^*)$, and $\tilde{\pi}_1(\delta^*) > \tilde{\pi}_1(\theta^*)$;
	2. $\tilde{q}_2(\theta^*) > \tilde{q}_2(\delta^*)$, $\mu_2\tilde{q}_2(\theta^*) > \mu_2\tilde{q}_2(\delta^*)$, and $\tilde{\pi}_2(\theta^*) > \tilde{\pi}_2(\delta^*)$;
	3. $\tilde{\pi}_1(\theta^*) + \tilde{\pi}_2(\theta^*) > \tilde{\pi}_1(\delta^*) + \tilde{\pi}_2(\delta^*)$ and $\tilde{\pi}_2(\theta^*) - \tilde{\pi}_1(\theta^*) > \tilde{\pi}_2(\delta^*) - \tilde{\pi}_1(\delta^*)$;
	4. $\tilde{\pi}_1(\delta^*) - \tilde{\pi}_1(\theta^*)$, $\tilde{\pi}_2(\theta^*) - \tilde{\pi}_2(\delta^*)$, $\tilde{\pi}_1(\theta^*) + \tilde{\pi}_2(\theta^*) - (\tilde{\pi}_1(\delta^*) + \tilde{\pi}_2(\delta^*))$, and $\tilde{\pi}_2(\theta^*) - \tilde{\pi}_1(\theta^*) - (\tilde{\pi}_2(\delta^*) - \tilde{\pi}_1(\delta^*))$ are all increasing in the budget B;
	5. $\tilde{\Pi}(\delta^*) > \tilde{\Pi}(\theta^*)$ if and only if α is larger than a threshold $\breve{\alpha}$.

To avoid repetition, here we omit the detailed analysis. Table 1 summarizes the results when the farmers' yield rates are uncertain. It can be easily shown that the main insights 1, 2, 3(a) and 3(b) under the base model continue to hold here when the farmer's yield rate is uncertain. Specifically, compared to the output-based subsidy scheme, the input-based subsidy scheme is preferred by the low-yield farmer 1 and less preferred by the high-yield farmer 2. Again, the output-based subsidy scheme outperforms the input-based subsidy scheme in terms of farmers' total income, while the input-based subsidy scheme dominates the output-based subsidy scheme in terms of lower income gap. Thus, the government prefers the input-based subsidy scheme if and only if it has a high concern over the farmers' income inequality (i.e., $\alpha > \breve{\alpha}$). However, relative to the base model, we obtain a different result: increasing the input-based unit subsidy δ may not reduce the income gap, especially when the yield uncertainty is relatively high and the subsidy level δ is low. This is because a higher yield uncertainty may cause both farmers to produce less, which results in a higher market price. Due to his yield advantage, the high-yield farmer 2 then generates more expected revenue per unit of the planting quantity (i.e., $\mu_i \cdot P$) than the low-yield farmer 1, as the market price increases. Meanwhile, a higher input-based unit subsidy reduces the low-yield farmer 1's cost per unit of the harvest quantity more than that of the high-yield farmer 2 (i.e., $\delta/\mu_1 > \delta/\mu_2$), and thus benefits the low-yield farmer more than the high-yield one. Consequently, when the yield uncertainty is high and the input-based subsidy level is low, the input-based subsidy scheme may benefit the high-yield farmer more than the low-yield one, leading to the increase of the income inequality in δ.



(a) Income gap under the two schemes

(b) Farmers' total income under the two schemes

Figure 1: The Impact of the Yield Uncertainty on the Total Farmer Income and the Income Inequality under the Two Subsidy Schemes: $\mu_1 = 0.5$, $\mu_2 = 0.7$, $a = 6$, $c = 1.6$, $B = 1$.

Next, we numerically examine the impact of the farmers' yield uncertainty σ^2 on the equilibrium outcomes (Figure 1). Figures 1 reveals that as the yield uncertainty σ^2 increases, both the farmers' total income and the income gap between farmers will decrease under both subsidy schemes. This is because as the yield uncertainty (via σ^2) increases, both farmers will plant less because the farmer's expected income given in (22) is decreasing in σ^2 under both schemes. As both farmers reduce their planting quantities, both farmers' incomes drop but the high-yield farmer's income will drop faster because of a bigger drop in its output as σ^2 increases. Consequently, the income gap reduces as σ^2 increases. In practice, in order to improve farmers' total income, many developing countries such as India help reduce the farmers' yield uncertainty by providing the timely weather information to farmers (Rathore and Chattopadhyay 2016). Our numerical results imply that reducing the farmers' yield uncertainty can indeed improve the farmers' total income, but it may widen the income gap.

7 Conclusion

Motivated by mixed empirical evidence about the effects of the input- and output-based subsidy schemes, we analytically examine the performance of these two schemes in terms of individual farmer income, the total farmer income and income gap. We have shown that, while both subsidies can improve the income of each farmer, their impact on the income inequality is opposite: a higher input-based unit subsidy can reduce the income gap, while a higher output-based unit subsidy widens such gap. Interestingly, the high- and low-yield farmers hold different preferences toward the two schemes: the input-based subsidy scheme is preferred by the low-yield farmer, while the output-based subsidy scheme is preferred by the high-yield farmer. We show that both the input- and output-based subsidy schemes have their own unique advantages: the output-based subsidy is more effective in improving the farmers' total income and production efficiency, while the input-based subsidy is more efficient in reducing the income gap and increasing the total production quantity. Therefore, the government should implement the input-based subsidy scheme only when it has significant concern over the income gap.

When the government provides both types of subsidies, we have shown that the input-based subsidy can enhance the effect of the output-based subsidy on the farmers' total income (as complements) but dampen its effect on the income gap (as substitutes). We then show that it is never optimal for the government to provide both types of subsidies. Last, when the farmer's yield rate is uncertain, we have shown that the main insights when the yield rates of farmers are predetermined continue to hold in this case. We also numerically show that reducing the yield uncertainty can improve the farmers' total income but it can widen the income gap.

Our study is an initial attempt to examine the implications of the input- and output-based subsidy schemes on the farmer income and income gap. There are many research opportunities for further examination. For example, in this study, we consider the impact of the input- and output-based subsidy schemes on the individual farmers. However, it may be of interest to consider their impact on the agricultural supply chain that involves the smallholder farmers, the middlemen and the agricultural companies. Here, one potential research question is to examine the preference of the different stakeholders along the agricultural supply chain over the two subsidy schemes. In this study, we consider that the farmers grow only one crop. However, in practice, there exist multiple crops that the farmers can plant. Under such setting, the decisions of both the government and the farmers are much more complicated: the farmers shall determine which crop to plant while the government shall determine which crop to subsidize and what type of subsidy scheme to adopt. We leave this analysis as the future research.

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