To: Structural Health Monitoring

Multi-rate data fusion to dynamic displacement measurement of beam-like supertall structures using acceleration and strain sensors

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ABSTRACT

Accurate measurement of dynamic displacement is important to the structural health monitoring and safety assessment of supertall structures. However, the displacement of a supertall structure is difficult to be accurately measured using the conventional methods because they are either inaccurate or inconvenient to be set up in practice. This study provides an accurate and economical method to measure dynamic displacement of supertall structures accurately by fusing acceleration and strain data, which are generally available in the structural health monitoring system. Dynamic displacement is first derived from the measured longitudinal strains based on geometric deformation without requiring mode shapes. An optimization technique is utilized to optimize the deployment of strain sensors for higher accuracy of the strain-derived displacement. The strain-derived displacement is then combined with measured acceleration via a multi-rate Kalman filtering approach. Applications to a numerical supertall structure and a laboratory cantilever beam verify that the proposed method accurately estimates displacement including both high-frequency and pseudo-static components, under different noise cases and sampling rates. A full-scale field test on the 600 m-high Canton Tower is implemented to validate the applicability of the proposed method to real supertall structures. Error analysis demonstrates that the data fusion displacement has higher accuracy than the GPS-measured displacement in the time and frequency domains.

Keywords: structural health monitoring, dynamic displacement, supertall structure, multi-rate Kalman filtering, data fusion, geometric deformation

1. Introduction

Dynamic displacement of a supertall structure is an intuitive response that results from external loads, such as winds and earthquakes[1]. Since the dynamic displacement is directly related to the structural deformation, it provides useful information for structural health monitoring (SHM) and condition assessment[2,3]. Different from normal buildings and bridges, supertall structures suffer risks of error accumulation, i.e. a small error of the displacement measurement at the bottom will cause large measurement error at the top of the structure. Therefore, the accurate measurement of structural displacement is an important topic in SHM, deformation control and damage detection of supertall structures[4,5].

In the past decades, significant efforts have been made toward the development of dynamic displacement measurement. Structural dynamic displacement is generally measured directly or indirectly. With direct methods, structural displacement is directly measured by devices such as linear variable differential transducers (LVDTs), global position systems (GPSs) and vision-based systems. Moreu et al.[6] assessed bridge conditions under traffic using displacement measured by LVDTs, which is required to contact with the target structure and firm supports. GPSs have been widely used in the SHM of supertall structures in the past decades[7]. The applications of GPSs are limited when high accuracy of displacement is required. Vision-based methods generally measure structural displacement at multiple points simultaneously, whereas the measurement accuracy is dependent on a good visual condition[8,9]. These direct methods are sometimes difficult to be applied to supertall structures. For example, it is usually difficult to set a support at the tall buildings for the LVDTs. The displacement measured from GPSs is not sufficiently accurate, and the invisibility caused by occlusion of the scaffold limits the implementation of the vision-based methods.

Indirect methods estimate the displacement by the transformation from acceleration[10] or strain[11,12,13]. Acceleration is usually accurate at the high-frequency component with high sampling rates, and many researchers obtain structural dynamic displacement via double integration of measured acceleration data[14]. The error that accumulates from the double integration causes base-line drift, especially for long-term SHM[15]. Another limit of the acceleration-based approach is that the acceleration cannot estimate the pseudo-static displacement[16]. Strain responses have also been used to calculate displacement via the strain–displacement relations. Wang et al.[17] used the strain-based mode shape to measure the dynamic displacement of a simply supported beam structure. Shin et al.[18] proposed a displacement measurement method for a simply supported beam using FBG strain sensors based on mode superposition. These strain-based methods require accurate mode shapes of the structures, which are inaccessible and time consuming for large-scale supertall structures.

Recently, some researchers have combined the direct and indirect methods to improve the accuracy of dynamic displacement measurement. Smyth and Wu[1] developed a multi-rate Kalman filtering method that calculated displacement accurately by fusing measured displacement with acceleration. This method also has a merit of fusing acceleration and displacement with different sampling rates[19]. Kim et al.[20,21] improved the accuracy of displacement measurements by fusing the displacement measured by a terrestrial laser scanner with the velocity measured by a laser Doppler vibrometer. They also proposed a two-stage Kalman filtering algorithm to acquire dynamic displacement by combining high-sampling acceleration and low-sampling displacement data[22]. The above data fusion methods combine the direct and indirect methods, whereas the implementation of the direct methods are limited by the visibility and installation conditions for supertall structures. Data fusion of different indirect methods has also been conducted for displacement measurements. Park et al.[23,24] fused mode shape-based displacement with acceleration to monitor dynamic displacement of a simply supported bridges through an extension finite impulse response filter. Cho et al.[25] proposed a Kalman filtering method that combines acceleration and mode shape-based displacement to obtain dynamic displacement of bridges. Those data fusion methods require mode shapes and focus on displacement estimation of bridges, which are different from supertall structures in structural configuration, boundary conditions and loading forms. Therefore, those data fusion methods to bridges are limited to be used in supertall structures.

At present, there is still no efficient and accurate method to measure the dynamic displacement of supertall structures. This study develops a displacement estimation method for a supertall structure by fusing strain-derived displacement and acceleration via Kalman filtering. Different from previous researches, this study derives the strain–displacement formulas directly from the structural geometric deformation without requiring mode shapes, which is more accessible to practical supertall structures. In addition, a technique is used to optimize sensor deployment by minimizing the error of the strain-derived displacement. The proposed strain–displacement formulas are also capable to obtain the structural displacement at different heights of the structure. Accelerometer and strain sensors are commonly installed in SHM system to measure structural acceleration and strain[26], therefore the proposed method is performed on standard SHM system without additional instruments. The proposed method accurately estimates dynamic displacement including high-frequency and pseudo-static components by fusing high-sampling rate acceleration and low-sampling rate strain via a multi-rate Kalman filtering. A smoothing process is used to further improve the accuracy of the multi-rate data fusion displacement. The accuracy of the proposed data fusion method is verified through a simulation of a supertall structure and a laboratory test of a cantilever beam. Finally, the performance and applicability of the proposed method to practical structure is validated by a field test on the 600 m-high Canton Tower.

2. Displacement measurement from geometric deformation

In this section, the horizontal dynamic displacement is first estimated from the longitudinal strain. Different from the mode shape-based approaches[17,18,25], the proposed approach derives displacement directly from the structural geometric deformation, which is generally available for supertall structures, such as Canton Tower[26] and Shanghai Tower[27]. Additionally, an optimization technique is proposed to obtain the optimal arrangement of strain sensors.

2.1 Derivation of strain-derived displacement

A supertall structure has a large slenderness ratio and the bottom is fixed, which can thus be simulated with a cantilever beam[26]. Therefore, a cantilever beam model is used here to derive the strain-based displacement formulas (Figure 1). The beam model is divided into n sub-elements, and the i-th sub-element connects Point $i-1$ and Point i. The strain sensors are located at $n+1$ Points (from Point 0 to Point n) along the beam. Each sub-element is divided into a number of micro-units. Vibration of the beam-like structure generates bending deformation, which leads to tension or compression strain along the longitudinal direction on the two sides of the beam. The tension strain is regarded as positive and the compression strain is negative. The strain of a micro-unit is measured in practice. The deformation of a micro-unit is an angular rotation resulting in the strain difference on the two sides of the micro-unit. The horizontal displacement of a sub-element is integrated from the deformations of all micro-units within the sub-element. Finally, the horizontal displacement of Point n is calculated by summing up the horizontal displacements of all sub-elements below Point n. Accordingly, the strain-derived displacement is derived from the longitudinal strain data by the following three steps.

The first step is to calculate the angular rotation of the micro-unit. As shown in Figure 2, the bending deformation of the micro-unit leads to the strain difference on the two sides of the beam, and the angular rotation is calculated by the strain difference as

$$d\theta = \frac{\left(\varepsilon_l - \varepsilon_r\right) dh}{s} = \frac{\Delta\varepsilon dh}{s} \tag{1}$$

where $d\theta$ is the angular rotation, ε_l and ε_r are longitudinal strains at the left and right sides of the micro-unit, respectively. $\Delta\varepsilon = \varepsilon_l - \varepsilon_r$ is the strain difference on the two sides of the micro-unit. s is the width of the micro-unit in the horizontal direction, and dh is the length of the micro-unit in the longitudinal direction.

The second step is to calculate the horizontal displacement of the sub-element by integrating the deformations of the micro-units. In this step, the strain difference and bending moment diagrams of the beam are used to calculate the displacement of the sub-element through diagram multiplication[28]. Figure 3 shows the strain difference diagram acquired from the deformations of the micro-units and the bending moment diagram obtained by loading a unit horizontal force at Point n. The bending moment at Point i is the multiplication of the unit force with the arm of the unit force to Point i, i.e., the distance between the unit force and the Point i.

$$\bar{M}_i = f_{unit} \sum_{j=i+1}^{n} h_j = \sum_{j=i+1}^{n} h_j \tag{2}$$

where $\bar{M}_i$ is the bending moment at Point *i* and h_j is the length of the sub-elements above Point *i*. The unit force f_{unit} = 1. Concerning on the *i*-th sub-element plotted in Figure 4, the horizontal displacement of the *i*-th sub-element is an integration of the deformation of micro-units within the sub-element. Based on the assumption that the strain variation between Point *i*−1 and Point *i* is linear[26], the integration is presented by the multiplication of the areas of the strain difference with the corresponding bending moment[28]. To simplify the multiplication, the strain difference can be divided into a triangle and a rectangle, with areas of A_1 and A_2 (Figure 4(a)). The corresponding bending moments of the triangle and rectangle are y_1 and y_2, respectively (Figure 4(b)). The calculation of the displacement of the *i*-th sub-element is presented as

$$\Delta x_i = \int_0^{h_i} \bar{M} d\theta = \frac{1}{s}\int_0^{h_i} \bar{M}\Delta\varepsilon dh = \frac{1}{s}(A_1 y_1 + A_2 y_2) \quad (3)$$

where Δx_i is the horizontal displacement of the *i*-th sub-element. A_1, A_2, y_1 and y_2 in equation (3) are presented as

$$A_1 = \frac{(\Delta\varepsilon_{i-1} - \Delta\varepsilon_i)h_i}{2},\quad A_2 = \Delta\varepsilon_i h_i,\quad y_1 = \frac{(2\bar{M}_{i-1} + \bar{M}_i)}{3},\quad y_2 = \frac{(\bar{M}_{i-1} + \bar{M}_i)}{2} \quad (4)$$

Substituting equations (4) into (3) leads to

$$\Delta x_i = \frac{h_i}{6s_i}(2\bar{M}_{i-1}\Delta\varepsilon_{i-1} + \bar{M}_{i-1}\Delta\varepsilon_i + \bar{M}_i\Delta\varepsilon_{i-1} + 2\bar{M}_i\Delta\varepsilon_i) \quad (5)$$

Consequently, the third step is to calculate the horizontal displacement of Point *n* by the integration of the displacements of all sub-elements below Point *n* as

$$x_{sd} = \frac{1}{6}\sum_{i=1}^{n}\left[\frac{h_i}{s_i}(2\bar{M}_{i-1}\Delta\varepsilon_{i-1} + \bar{M}_{i-1}\Delta\varepsilon_i + \bar{M}_i\Delta\varepsilon_{i-1} + 2\bar{M}_i\Delta\varepsilon_i)\right] \tag{6}$$

where x_{sd} is the strain-derived displacement of Point *n*. Hereinafter, the subscript *sd* means the items derived from strain. The horizontal displacement of the supertall structure at any heights can also be achieved by altering the location of the unit force to the required Point. In order to simplify the derivation of the strain–displacement formulas, the strain sensors are assumed to be aligned along the structure. The distribution of the sensors and the magnitude of strain may affect the accuracy of the strain-derived displacement, which will be investigated in the case studies. In practice, errors from the alignment deviation can be compensated by piecewise linear interpolation technique[29] and least squares method[30].

2.2 Optimization of sensor deployment

The quantity and location of the strain sensors would affect the accuracy of the strain-derived displacement. Therefore, hence the sensor deployment should be optimized for achieving higher more accurate displacement fusionaccuracy. The strain-derived displacement in equation (6) is a multivariate function, where the variables $\Delta\varepsilon_i$ (i = 1, 2,…, n) are independent to each other. Consequently, the error of the strain-derived displacement is can be quantified by the standard deviation of the measurement error in each strain sensor, that is,

$$\sigma_{sd} = \sqrt{\left(\frac{\partial x_{sd}}{\partial \Delta\varepsilon_1}\right)^2 \sigma^2_{\Delta\varepsilon_1} + \cdots\cdots + \left(\frac{\partial x_{sd}}{\partial \Delta\varepsilon_n}\right)^2 \sigma^2_{\Delta\varepsilon_n}} \qquad (7)$$

where σ_{sd} is the standard deviation~~ that quantifies the error of the strain-derived displacement~~. In equation (7), $\sigma^2_{\Delta\varepsilon_i} = \sigma^2_{\varepsilon_{li}} + \sigma^2_{\varepsilon_{ri}}$, where $\sigma_{\varepsilon_{li}}$ and $\sigma_{\varepsilon_{ri}}$ are the standard deviations of the measurement error of strain sensors at Point *i*. Substituting equation (6) into equation (7) leads to

$$\sigma_{sd} = \frac{1}{6s}\sqrt{h_1^2(3H - h_1)^2 \sigma^2_{\Delta\varepsilon_1} + \sum_{i=1}^{n-1}[h_i(h_i + 3\sum_{j=i+1}^{n} h_j) + h_{i+1}(3\sum_{j=i+1}^{n} h_j - h_{i+1})]^2 \sigma^2_{\Delta\varepsilon_i} + h_n^4 \sigma^2_{\Delta\varepsilon_n}} \qquad (8)$$

Since *H* and $\sigma^2_{\Delta\varepsilon_i}$ are known and σ_{sd} is a multivariate function of h_i ($i = 1, 2\ldots, n$), the optimal deployment of the strain sensors can be obtained by ~~solving the following optimization problem to~~ minimizing ~~e~~ the standard deviation of the strain-derived displacement.

$$f(h_j) = \min \sigma_{sd}(h_j)$$
$$\text{s.t.} \quad \begin{cases} \sum_{j=1}^{n} h_j = H \\ h_{low} \le h_j \le h_{up} \end{cases} \qquad (9)$$

where ~~$f(h_j)$ and s.t. are the objective function and constraints, respectively.~~ h_{low} and h_{up} are the lower and upper ~~limit~~ bounds of h_i. ~~Besides, the constraints can be expanded according to the practical condition. In consequence,~~ T~~t~~he above objective function can be solved via nonlinear programming methods[32].

3. Displacement measurement using Kalman filtering

This section develops the Kalman filtering algorithm to fuse the strain-derived displacement derived in Section 2 and measured acceleration to estimate dynamic displacement accurately. Note that the displacement and acceleration construct the state equation of the state-space model in Kalman filtering algorithm[1,19,24]. The Kalman filtering algorithm includes two components: (i) the state-space model and (ii) the recursive filtering algorithm. The state-space model constructs the relationship between the inputs (acceleration and strain) and the output (displacement) in the discrete time domain. The recursive filtering algorithm is an optimal estimator, which estimates optimal displacement based on the state-space model via the minimum root-mean square principle[27].

3.1 State-space model

Denote the displacement, velocity and acceleration of the structure at time step $k-1$ as $x(k-1)$, $\dot{x}(k-1)$ and $\ddot{x}(k-1)$, respectively. The displacement at time step k is derived from the displacement, velocity, acceleration and a system noise at time step $k-1$ as

$$x(k) = x(k-1) + \dot{x}(k-1)\Delta t + \frac{1}{2}\ddot{x}(k-1)\Delta t^2 + w_d(k-1) \tag{10}$$

where $w_d(k-1)$ is the system noise[18] introduced in displacement and Δt is the time interval between adjacent time steps. The velocity at time step k is calculated from the velocity and acceleration at time step $k-1$ as

$$\dot{x}(k) = \dot{x}(k-1) + \ddot{x}(k-1)\Delta t + w_v(k-1) \tag{11}$$

where $w_v(k-1)$ is the system noise introduced in velocity. The system noises $w_d(k-1)$

and $w_v(k-1)$ are assumed as zero-mean Gaussian white noise[19]. The measured displacement is written as the summation of the real displacement and measurement noise as

$$x_m(k) = x(k) + v(k) \tag{12}$$

where $x_m(k)$ is the measured displacement that identifies with $x_{sd}(k)$ in this study. $v(k)$ is the measurement noise caused by measurement error of the sensor, which is also assumed as zero-mean Gaussian white noise[19].

To build the state-space model, the displacement and velocity are defined as state variables to construct the state vector as

$$\mathbf{X}(k) = \begin{bmatrix} x(k) & \dot{x}(k) \end{bmatrix}^T \tag{13}$$

where the superscript T is the transpose of a vector. Combining equations (10)—(13), the state-space model for the data fusion of acceleration and strain-derived displacement is expressed as

$$\mathbf{X}(k) = \mathbf{AX}(k-1) + \mathbf{B}\ddot{x}(k-1) + \mathbf{w}(k-1) \tag{14}$$

$$x_{sd}(k) = \mathbf{HX}(k) + v(k) \tag{15}$$

where the matrices **A**, **B**, **H** and **w**(k-1) in equations (14) and (15) are denoted as

$$\mathbf{A} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} (\Delta t)^2/2 \\ \Delta t \end{bmatrix},\ \mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix},\ \mathbf{w}(k-1) = \begin{bmatrix} w_d(k-1) \\ w_v(k-1) \end{bmatrix} \tag{16}$$

The system noise vector $\mathbf{w}(k-1)$ and the measurement noise $v(k)$ are Gaussian white noise with the covariance matrix of **Q** and **R** respectively as

$$\mathbf{Q} = q\begin{bmatrix} (\Delta t)^3/3 & (\Delta t)^2/2 \\ (\Delta t)^2/2 & \Delta t \end{bmatrix}, \quad \mathbf{R} = \frac{r}{\Delta t} \tag{17}$$

where q and r are the variance of acceleration noise and displacement noise, respectively.

3.2 Recursive filtering algorithm

The accelerometer is installed at the identical location of the required displacement. The measured acceleration fuses the strain-derived displacement via the recursive filtering to estimate the required displacement. For instance, if the displacement at the top of the structure is estimated, an accelerometer is required to be installed at the top. The recursive filtering algorithm has two processes, namely, estimating and updating processes. In the estimating process, the state vector at time step k is predicted from the filtered state vector and measured acceleration at time step k−1:

$$\bar{\mathbf{X}}(k) = \mathbf{A}\hat{\mathbf{X}}(k-1) + \mathbf{B}\ddot{x}_m(k-1) \tag{18}$$

where $\bar{\mathbf{X}}(k)$, $\hat{\mathbf{X}}(k-1)$ and $\ddot{x}_m(k-1)$ are the predicted state vector, the filtered state vector and the measured acceleration, respectively. The covariance matrix is an error evaluation of the prediction, which is calculated by

$$\bar{\mathbf{P}}(k) = \mathbf{A}\hat{\mathbf{P}}(k-1)\mathbf{A}^T + \mathbf{Q} \tag{19}$$

where $\bar{\mathbf{P}}(k)$ is the covariance matrix of the prediction error and $\hat{\mathbf{P}}(k-1)$ is the covariance matrix of the filtering error.

In the updating process, the strain-derived displacement in equation (6) corrects the predicted displacement in equation (18). The correction is a weighted data fusion based on the minimum mean-square error principle, which is represented as

$$\hat{\mathbf{X}}(k) = \overline{\mathbf{X}}(k) + \mathbf{K}(k)\left[x_{sd}(k) - \mathbf{H}\overline{\mathbf{X}}(k)\right] \tag{20}$$

In consequence, the displacement is expressed by a data fusion of measured acceleration and strains. The Kalman gain $\mathbf{K}(k)$ serves as an optimal weight between the predicted displacement and the strain-derived displacement. It is calculated by

$$\mathbf{K}(k) = \overline{\mathbf{P}}(k)\mathbf{H}^T\left[\mathbf{H}\overline{\mathbf{P}}(k)\mathbf{H}^T + \mathbf{R}\right]^{-1} \tag{21}$$

The covariance matrix of the filtered estimation is calculated by

$$\hat{\mathbf{P}}(k) = \left[\mathbf{I} - \mathbf{K}(k)\mathbf{H}^T\right]\overline{\mathbf{P}}(k) \tag{22}$$

The target of the filtering algorithm is to obtain the optimal displacement by minimizing the trace of the estimation covariance matrix $\hat{\mathbf{P}}(k)$. The minimum estimation covariance is obtained when the Kalman gain is equal to the optimal value; therefore, the displacement estimation converges to the real displacement. Figure 5 shows the framework of the proposed Kalman filtering approach presented above. The displacement is predicted in the estimating process, and then is fused with the strain-derived displacement in the updating process. The two processes of the Kalman filtering repeat alternatively at every time step to obtain the real-time optimal displacement.

The accuracy of the estimated displacement depends on the selection of q and r, thus the noise variances need to be determined ahead before the operation of the Kalman filtering. In this study, a maximum likelihood estimation approach[33] is utilized to determine the noise parameter for the simulation signals and the measurement data. The standard deviation of the measured acceleration and the strain-derived displacement are regarded as the initial values for estimation of parameter q and r.

3.3 Multi-rate Kalman filtering with smoothing

A multi-rate Kalman filtering approach is required when acceleration and strain are measured at different sampling rates or when the strain data are intermittent. In practice, acceleration data usually have a higher sampling rate than strain data. The sampling rates of acceleration and strain data are denoted as f_a and f_s, respectively. Denote $D = f_a/f_s$ as the ratio of sampling rates hereinafter.

Figure 6 indicates the flowchart of the multi-rate Kalman filtering. In the multi-rate Kalman filtering, the estimating process is performed solely when acceleration data are available while strain data are absent. In this situation, the updating processes are absent and thus the filtered displacement is equivalent to the predicted displacement. The recursive filtering algorithm at time step k is thus simplified as

$$\hat{\mathbf{X}}(k) = \overline{\mathbf{X}}(k) = \mathbf{A}\hat{\mathbf{X}}(k-1) + \mathbf{B}\ddot{x}_m(k-1) \quad (23)$$

$$\hat{\mathbf{P}}(k) = \overline{\mathbf{P}}(k) = \mathbf{A}\hat{\mathbf{P}}(k-1)\mathbf{A}^T + \mathbf{Q} \quad (24)$$

Meanwhile, the updating process works together with the estimating process when both acceleration and strain data are available. The absence of the updating process leads to error of the multi-rate fused displacement, and the error increases as the difference of sampling rates increases. Thus, Kalman smoothing is introduced to diminish the displacement error. There are three kinds of Kalman smoothing algorithms, including the fixed-point smoothing, fixed-lag smoothing and fixed-interval smoothing[20]. Fixed-point smoothing amends the state vectors at a specific point using the entire sequence of measurements. Fixed-lag smoothing corrects the state vectors at a fixed time lag from the current observation process. Fixed-interval smoothing corrects the state vectors at a fixed time interval, and it can also be used as fixed-lag smoothing. The smoothing used in the proposed multi-rate Kalman filtering approach is a fixed-interval smoothing based on the Rauch–Tung–Striebel algorithm[1]. In fact, the fixed-interval smoothing is an inverse filtering process that starts from the current time step and filters backward at a fixed interval of time steps. The backward steps for the smoothing should be big enough to diminish the displacement error due to the absence of the updating process. For example, the number of backward steps is set as 10 when $D = 10$. The smoothed displacement is obtained by

$$\tilde{\mathbf{X}}(k^*) = \hat{\mathbf{X}}(k^*) + \mathbf{G}(k^*) \cdot \left[\tilde{\mathbf{X}}(k^*+1) - \overline{\mathbf{X}}(k^*+1) \right] \tag{25}$$

where $\tilde{\mathbf{X}}(k^*)$ is the smoothed state vector, and $k^* = k+p-1, k+p-2, \ldots k$ (p is a round number of f_a/f_s). $\mathbf{G}(k^*)$ is the smoothing gain calculated by

$$\mathbf{G}(k^*) = \hat{\mathbf{P}}(k^*)\mathbf{A}^T\overline{\mathbf{P}}(k^*+1)^{-1} \tag{26}$$

In real-life measurement, the multi-rate data fusion with smoothing technique is also advantageous to deal with the drift of the acceleration data. The drift of the acceleration data is reduced not only in the updating process by the strain-derived displacement, but also in the smoothing process by the filtered displacement.

3.4 Accuracy evaluation

The accuracy of the estimated displacement is evaluated using the normalised root mean square error (NRMSE) as

$$\mathrm{NRMSE} = \sqrt{\sum_{k=1}^{N}\left(x_{est}(k) - x_{ref}(k)\right)^2} \Bigg/ \sqrt{\sum_{k=1}^{N}\left(x_{ref}(k)\right)^2} \times 100\% \tag{27}$$

where $x_{est}(k)$ is the displacement estimated from strain or data fusion, and $x_{ref}(k)$ denotes the reference value.

In case of reference of displacement is difficult or unable to acquire, the standard deviation of the data fusion displacement is calculated to quantify its accuracy. By considering the state space model in Section 3.1, the standard deviation of the data fusion displacement is calculated by

$$\sigma_{fusion} = \sqrt{\sigma_{sd}^2 + \left(\frac{(\Delta t)^2}{2}\right)^2 \sigma_{acc}^2} \tag{28}$$

where the σ_{sd} and σ_{acc} are standard deviation of the strain-derived displacement and acceleration, respectively.

4. The simulation of Wuhan Yangtze Navigation Centre

Wuhan Yangtze Navigation Centre is a 66-floor frame-core wall supertall structure with a height of 335 m[34,35]. Figure 7 shows the structure and its finite element model. The cross section of the structure is a square with an outer frame of 50 m × 50 m and the inner core wall of 30 m × 30 m (Figure 8). The excitation imposed on the top floor comprises high-frequency and pseudo-static ingredients for generating dynamic and pseudo-static displacements. A sampling rate of 100 Hz is used in the simulation. The displacement response calculated from the numerical model via dynamic analysis is regarded as reference. The horizontal dynamic displacement is estimated by the proposed method, and then compared with the reference.

The displacement at Floor 66 is first calculated by the strain data using the method described in Section 2. According to the locations of the strain measurement points, the model of Wuhan Yangtze Navigation Centre is divided into sub-elements, and each measured storey is regarded as a micro-unit. The strains on two sides of the micro-units are measured to acquire the strain differences. The angular rotation of a micro-unit is calculated by the strain difference via equation (1). By the diagram multiplication, the horizontal displacement of a sub-element is obtained by integrating the angular rotation of micro-units via equation (5). Afterwards, the horizontal displacement at Floor 66 is obtained by integrating the displacements of all sub-elements via equation (6).

To investigate the influence of the sensor quantity on the accuracy of the strain-derived displacement, Table 1 summarises the NRMSEs of the strain-derived displacement of four different quantity schemes. The NRMSE of the strain-derived displacement is around 5% for schemes 1–3, while the NRMSE raises to 7.22% for scheme 4. Therefore, at least four strain sensors are thus required for a successful displacement estimation. Next, the sensor deployment is optimized via nonlinear programming proposed in Section 2.2. In this simulation, the standard deviations of each point are assumed to be equal at 1 $\mu\varepsilon$. The optimal h_i (i=1, 2, 3) estimated via equation (9) are 25.5 m, 144.5 m and 165 m, respectively. Therefore, the strain sensors are suggested to be located at 0 m, 25.5 m, 170.0 m and 335 m. In this case, the NRMSE of the strain-derived displacement is 5.02%, which shows a higher accuracy than that of scheme 3 (5.18%). This sensor deployment is selected to verify the accuracy of the data fusion displacement.

Afterwards, the strain-derived displacement and the acceleration are fused via the proposed Kalman filtering approach. In this simulation, the noise parameters estimated via the maximum likelihood estimation approach are q = 0.042 m^2/s^3 and r = 0.0023 $m^2 \cdot s$. The displacement of Floor 66 is predicted via equation (18). The predicted displacement is then fused with the strain-derived displacement to obtain the optimal displacement. Acceleration-based displacement estimation proposed by Lee et al.[32] is used for comparison. Figure 9 compares the displacements of Floor 66 estimated by the acceleration, strain and data fusion methods. The acceleration-derived displacement has large errors in recovering the pseudo-static displacement. The strain-derived displacement is close to the reference, and the data fusion displacement matches the reference better. The NRMSE of the data fusion displacement of Floors 66 is 3.64%, which is more accurate than the strain-derived displacements. The confidence level of the data fusion displacement consists of a mean value and a covariance. The filtered displacement is the mean value. The square root of the upper diagonal element of the covariance matrix in equation (22) is the covariance, which indicates the uncertainty of the filtered displacement. The uncertainty of the data fusion displacement is ±1.2 mm.

Gaussian white noises of 5%–20% are added to the strain and acceleration data to investigate the influence of measurement noise on the accuracy of the data fusion method. Table 2 compares the NRMSEs of the strain-derived and data fusion displacements at different noise levels. The NRMSEs of the acceleration-derived displacement with different noises are around 65.5%, which are relatively larger than other displacements. This is because the acceleration-derived displacement fails to estimate the pseudo-static displacement. The NRMSEs of the strain-derived displacement increases from 5.03% to 8.72% as the noise increases, while the NRMSEs of the data fusion displacement increases slightly from 4.06% to 5.05%.

Therefore, the data fusion method is more robust to measurement noise than the strain-derived method.

In practice, the sampling rate of acceleration data is generally higher than that of strain data. Therefore, the multi-rate Kalman filtering approach with smoothing proposed in Section 3.3 is used to fuse the acceleration data and strain-derived displacement with different sampling rates. The sampling rate of acceleration is set to 100 Hz, and the sampling rates of strain are 10, 2, 1 and 0.5 Hz, i.e. D = 10, 50, 100 and 200, respectively. A 10% Gaussian white noise is used to simulate a reasonably working noise in practical environment[1]. Figure 10 compares the displacements obtained by the strain-derived method and the multi-rate data fusion approach when D = 200. The strain-derived displacement contains only a few displacement points and fails to capture the high-frequency component of the dynamic displacement. The acceleration-derived displacement fails to estimate the pseudo-static component, which leads to large error. The multi-rate data fusion displacement approximates the exact displacement with slight error. The slight error comes from the absence of updating process during the time interval of strain data. The smoothing technique diminishes the error, and the smoothed displacement is in good agreement with the reference. As shown in Table 3, the NRMSE increases slightly from 3.23% to 5.63% as D increases from 10 to 100, and then rises to 8.37% as D is 200, which remains accurate. Consequently, the proposed multi-rate Kalman filtering and smoothing technique fuses high-frequency acceleration and low-frequency strain to accurately obtain dynamic displacement including both high-frequency and pseudo-static components.

5. Experiment on a cantilever beam

A steel cantilever beam (Figure 11) is tested in the laboratory to validate the accuracy of the proposed displacement measurement method. The beam is 800 mm long with a rectangular cross section of 50 mm × 5 mm. The material constants of the beam are Young's modulus = 210 GPa and Poisson's ratio = 0.3. Ten resistance-type strain sensors are installed on two sides of the beam (node 0 to 9), with an equal distance of 80 mm between adjacent sensors. Acceleration and horizontal displacement data of Point 1 are measured by accelerometer and laser displacement meter, respectively. The location of the accelerometer and laser displacement meter is then moved to Point 2 for measurement. The laser-measured displacement is regarded as reference. The structure is excited by two loading cases. Case 1 is an acceleration excitation generated by an exciter that contacts the beam via a steel bar. Case 2 is an excitation that includes high-frequency and pseudo-static components. The high-frequency component is generated by an exciter similar to Case 1, and the pseudo-static component is generated by manual push. This loading type is selected to highlight the merit of the proposed strain-derived method in estimating pseudo-static displacement. The displacement is first derived by from the strain and then fused with the acceleration by the Kalman filtering approach. The noise parameters used in the experiment are as follows: q =

2.51 mm^2/s^3 and r = 0.07 $mm^2 \cdot s$ for Case 1 and q = 1.71 mm^2/s^3 and r = 0.13 $mm^2 \cdot s$ for Case 2.

An investigation about the minimum quantity of strain sensors is provided before the optimization of sensor deployment. As shown in Table 6, the quantity of the sensors has slight influence on the accuracy of the data fusion displacement when the quantity exceeds four. Therefore, five strain sensors are sufficient to estimate displacement accurately in the experiment. The optimal deployment of the five sensors is then determined by the technique proposed in Section 2.2. The standard deviations of each strain sensor are acquired according to the strain data under ambient excitation. The result shows that the optimal locations of the five sensors are 0, 8, 16, 40 and 70 cm, respectively. Therefore, the strain sensors are installed at node 0, 1, 2, 5 and 9 (Figure 11(b)) to derive the displacement in the experiment.

Figure 12 shows that the data fusion displacement is more accurate than the strain-derived and acceleration-derived displacements. Figure 13 shows that the acceleration-derived displacement fails to estimate the pseudo-static component. The strain-derived displacement approximates the reference, and the data fusion displacement is the most accurate one. Table 4 shows that the average NRMSEs are 56.05%, 8.16% and 4.85% for the acceleration-derived, strain-derived and data fusion displacements, respectively. The reason why the acceleration-derived displacement has relatively larger error than those of other displacements is that the acceleration-derived displacement fails to estimate the pseudo-static displacement. The proposed data fusion method accurately estimate displacement when both the high-frequency and pseudo-static components are included. The uncertainties of the data fusion displacements are ± 0.04 mm for Case 1 and ± 0.06 mm for Case 2 according to the covariance matrix via equation (22).

The effect of the strain magnitude is also analyzed to evaluate the proposed data fusion approach with small strain amplitudes. An overlapping time window with a size of 0.25 s that moves forward in an increment of 0.005 s is used to calculate the error. Figure 14 shows that the NRMSEs at the beginning are about 3% for Case 1 and 0.8% for Case 2 (Figure 14(a)). Note that the error magnitudes drop lower than 0.4% for Case 1 and lower than 0.3% for Case 2 in the rest period (Figure 14(b)). The reason for the relatively large error at the beginning stage is that the beam is unexcited and the strain magnitude is close to zero. These observations indicate that a smaller strain amplitude suffers larger error magnitude, and the error of the data fusion displacement maintains at a low level.

Afterwards, the accuracy of the data fusion displacement with different sampling rates of the acceleration and strain data is investigated. The acceleration is sampled at 200 Hz, and the strain is sampled at 40, 20, 10 and 5 Hz (i.e. D = 5, 10, 20 and 40). The smoothing technique presented in Section 3.3 is used in the multi-rate data fusion to reduce the error of the displacement caused by the absence of updating processes. Figure 15 compares the displacement obtained using only 10 Hz strain data and a fusion of 200 Hz acceleration and 10 Hz strain data for the two loading cases. The strain-derived displacement only includes a low-frequency component of the real dynamic displacement due to the low sampling rate of strain. The multi-rate data fusion supplements sufficient data points to obtain the dynamic displacement, which is in good agreement with the reference. As shown in Table 5, the NRMSE of the displacement in Case 2 increases significantly from 7.17% to 17.86% due to the low sampling rate of strain and the pseudo-static displacement. On the other hand, the NRMSE of the smoothed displacement in Case 1 rises slightly from 7.67% to 8.21%, as D rises from 5 to 40. Therefore, the proposed multi-rate data fusion with smoothing technique is capable to estimate the displacement accurately although considerable strain data are missing.

6. Field experiment on a supertall structure

Validation of the proposed data fusion approach in field test is implemented on the 600 m-high Canton Tower (Figure 16) in Guangzhou, China. The Canton Tower is a typical tube-in-tube supertall structure with a 454 m-high main tower and a 146 m-high steel antenna on the top of the main tower[36,37]. The main tower consists of an oval reinforced concrete inner tube and an oval steel outer tube. The oval inner tube has a constant size of 14 × 17 m, and the outer tube has a gradually changing size along the height. A sophisticated long-term SHM system has been established in the tower since the construction period and become a SHM benchmark to supertall structures[38,39,40,41]. After attempts and comparisons of different approaches, GPS system is chosen as a practical way to monitor structural displacement in the long term. The accuracy of the GPS-measured displacement is sometimes not sufficient in extreme weathers such as storm and typhoon. As shown in Figure 16, a GPS system and an anemometer were installed at the top of the main tower (454 m). Strain sensors were installed at four faces of the inner tube at 12 sections (Figure 16(b)). Each section has four strain sensors: two for the longitudinal strain along the long-axis and the other two for the short-axis (Figure 16(c)). Uni-axial accelerometers installed at 446 m are used to measure the horizontal acceleration along the long-axis and the short-axis of the inner tube. The proposed method uses the sensors of the original SHM system in Canton Tower to accurately obtain dynamic displacement without additional instruments required.

The structural responses of the Canton Tower during typhoon Usagi[34], which passed through Guangzhou on 22 September 2013 are used for the field validation of the proposed data fusion method. The maximum 10-min mean wind speed during the day was 22 m/s. With an optimization of sensor deployment that similar to the simulation and experiment, strain sensors at six sections are selected for the strain-derived displacement. The selected sections are No. 1, 2, 4, 6, 8 and 12 section shown in Figure

16(b). The longitudinal strains of Point 2 and Point 4 at these six levels are utilized to derive the horizontal displacement of the top of the main tower along the short-axis of the oval section. The horizontal acceleration measured at 446 m is fused with the strain-derived displacement. The sampling rates of strain, acceleration and GPS are 0.2 Hz, 5 Hz and 1 Hz, respectively.

The first step is to identify the noise parameters of strain and acceleration data by calculating the standard deviation[26] and maximum likelihood estimation[33]. The standard deviations of strain and acceleration are estimated directly from the measured data, and the standard deviation of the strain-derived displacement is calculated via equation (8). The standard deviations are then used as the initial values of the parameter estimation. The noise parameters for the proposed Kalman filtering are q = 8.59 mm^2/s^3 and r = 0.108 $mm^2 \cdot s$.

The second step is to fuse the strain-derived displacement and acceleration in horizontal direction via the proposed multi-rate Kalman filter algorithm. Figure 18 compares different displacements obtained from strain, acceleration, data fusion and GPS. The data points of the strain-derived displacement are close to the GPS-measured displacement, but they cannot estimate the overall characteristics of the dynamic displacement due to the low-sampling rate (0.2 Hz). The acceleration-derived displacement estimated via the FIR filter[32] has errors and high-frequency noise, especially at the peaks and valleys. The data fusion displacement agrees well with the GPS-measured displacement.

In field monitoring, the displacement from GPS is adopted for comparison. The accuracy of GPS measurement relies on the atmosphere, satellite geometry and multi-path[34]. The weather condition during a typhoon reduces the accuracy of the GPS-measured displacement. According to equation (28), the standard deviation of the data fusion displacement is 10.81 mm, which is better than that of the GPS measurements, whose error is about a few centimetres in field measurement[26, 34].

The fast Fourier transform is performed on the data fusion displacement and the GPS-measured displacement. Figure 19 compares the frequencies identified from the data fusion displacement, GPS-measured displacement and acceleration. A theoretical modal analysis[34] indicates that the first three modal frequencies of the Canton Tower are 0.095 Hz, 0.139 Hz and 0.347 Hz, respectively. Therefore, the sampling rates of the acceleration and GPS are sufficient to capture the first three modal frequencies of the Canton Tower. As shown in Figure 19, the first modal frequency of the three displacements are close to the theoretical value. The acceleration and data fusion displacements accurately identify the second modal frequency at 0.1367 Hz, whereas the frequency from GPS differs from the theoretical one with a 10% error. In addition, the acceleration-derived and data fusion displacements detect the third modal frequency, whereas the GPS fails to do. Consequently, the accuracy of the data fusion displacement is higher than that of the GPS-measured displacement both in the time and frequency domains.

7. Conclusions

A new and practical dynamic displacement measurement approach is developed for beam-like supertall structures by fusing the measured acceleration and strain. Accelerometers and strain sensors are regularly included in standard SHM system of supertall structures, so the proposed method does not require additional instruments. The horizontal dynamic displacement is first derived from longitudinal strain based on geometric deformation instead of mode shapes, and the accuracy of displacement is not affected by the model error. An optimization technique is proposed to obtain the optimal deployment of the strain sensors to improve the accuracy of the strain-derived displacement. These merits make the proposed displacement measurement method of a supertall structure efficient and economical. The proposed multi-rate Kalman filtering approach combines high-frequency acceleration and low-frequency strain to accurately obtain dynamic displacement including both high-frequency and pseudo-static components, which is significant to practical supertall structures.

The simulation of a supertall structure and the laboratory beam experiment verify that the proposed strain-derived method is capable to estimate nonzero mean dynamic displacement with pseudo-static component. The proposed Kalman filtering approach considerably improves the accuracy of the strain-derived displacement by reducing the measurement noise. The influences of noise and the distribution of strain sensors are investigated. Results show that the proposed data fusion method accurately estimates displacement in high noise cases. The strain sensors should be installed more intensively at the lower floors of a supertall structure than at the upper floors. The effect of strain amplitude to the error is also investigated. Smaller strain amplitude has larger error, while the error of the data fusion displacement maintains at a low level. In addition, the multi-rate Kalman filtering with smoothing is effective for acquiring dynamic displacement accurately although the sampling rate of the strain data is low. Finally, the field test on the 600 m-high Canton Tower demonstrates the applicability and accuracy of the proposed multi-rate Kalman filtering approach to practical supertall structures.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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