

Gabor Feature Based Discriminative Dictionary Learning for Period Order Detection in Fringe Projection Profilometry

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Abstract— Fringe projection profilometry (FPP) is a popular method for accurate 3D model reconstruction. In a typical FPP process, a tedious phase unwrapping procedure is often needed to obtain the true phase information of the captured fringe images. However, conventional phase unwrapping algorithms often suffer from the ambiguity problems when the scene contains occlusions or sudden jumps in object's height profile. In this paper, we propose an efficient decoding strategy to overcome the ambiguity problems. A novel coded fringe pattern is employed and a Gabor feature based discriminative dictionary is used to estimate the unknown period order of the wrapped phase. Experimental results show that the proposed method can achieve an accurate 3D reconstruction of objects' model in various adverse conditions where traditional FPP methods often fail to perform.

I. INTRODUCTION

Optical three-dimensional (3D) scanning is an increasingly important and active research area in computer vision and image processing. It offers non-contact measurements of the 3D information of objects that can be used in many applications such as industrial modelling and inspection [1, 2], 3D scene reconstruction [3, 4], and 3D face scanning [5], etc. Among various optical 3D scanning systems, the fringe projection profilometry (FPP) method is widely used because it can provide a fast, high resolution, and full-field measurement of the 3D information of objects using only a relatively low cost camera and projector. In a typical FPP setup, structured light patterns are first projected onto the target object. Due to the height profile of the object, the light patterns will be deformed as shown on the object's surface. The deformed light patterns will then be captured by a camera. By analyzing the deformation of the light patterns in the captured images, the height profile and in turn the 3D model of the object can be reconstructed.

According to the periodicity, traditional FPP methods can be divided into two groups, namely, the aperiodic FPP and periodic FPP. The aperiodic FPP requires the projection of a code pattern or a set of code patterns unto the object [8]. The patterns are specially designed to carry a unique set of codewords. Examples of the aperiodic fringe pattern is the graycode pattern [9, 10] and the De Bruijn pattern [11, 12]. Although the code patterns can give direct information of the objects' 3D model, they can suffer from the interference due to the ambient illumination.

In this paper, we focus on the periodic FPP [13-15]. The periodic FPP offers a salient advantage of being resistant to the ambient illumination. The key factor is the use of the phase of the periodic sinusoids (fringes) which is less likely to be influenced by distortions from various sources such as, object's texture, global illumination, ambient light, etc. However, traditional FPP methods can only obtain the modulo- 2π or the wrapped phase information of the fringes [6, 7]. To solve this problem, traditional FPP methods adopt a phase unwrapping procedure. It assumes that the true phase difference between two neighbor pixels is less than π . This assumption is known as the Itoh condition [16]. Based on such assumption, the true phase can be obtained from the wrapped phase by integrating the wrapped phase differences. However, such assumption is often not valid in practice particularly when the image scene contains occlusions or sudden jumps in object's height profile.

Recently many research works have been conducted to look for the solution to this problem. They use multiple cameras [3, 18], multi-wavelength fringe patterns [19], fringe pattern with additional information, such as colours [20], markers [7, 21-26], more frequencies [27, 28], extra patterns [1, 10, 29, 30], etc. These approaches either use additional hardware [3, 18] or need to project additional fringe patterns [1, 10, 29, 30]. Besides, some of these approaches can only perform FPP with fringe images of simple scene, e.g. scenes contain only a single isolated object, a mono-colour object, etc. Furthermore, the estimation in these approaches is often performed in a heuristic way such as using the correlations with predetermined referenced coefficients [21, 22].

In this paper, a new phase unwrapping strategy is proposed. We employ a novel strategy which embeds to the fringe pattern some code patterns that indicate the true phase information. More specifically, for each period of the fringe pattern, a unique codeword (period order number) is assigned to a set of pixels and form a code pattern. The code patterns are specifically designed to fulfil two requirements: 1) they have different morphological structures from the original fringe pattern; and 2) the codewords they encoded are uniquely represented by the texture properties of the patterns, such as, orientations and scales. The first condition is essential to accommodate the separation of the fringe pattern and code patterns in the decoding stage. We achieve this by

using the morphological component analysis (MCA) method [37]. The second condition is to ensure that the codewords can be accurately detected and the period order can be estimated using a discriminative dictionary.

This paper is organized as follows. We first present in Section 2 an overview on the FPP method. In Section 3, the proposed coded fringe pattern is discussed. The proposed period order estimation using the Gabor feature based discriminative dictionary learning method is explained in Section 4. Finally the experimental results and conclusions are presented in Section 5 and 6, respectively.

II. PROPOSED FPP FRAMEWORK

For FPP, the phase shifting profilometry (PSP) method is commonly employed. In PSP, three fringe patterns with fixed phase shifts are projected onto the target object sequentially. These fringe patterns are deformed due to the height profile of the object. The deformed patterns will then be captured by a camera. The captured fringe images can be mathematically expressed as follows:

$$\begin{aligned} I_1 &= a + b \cos(\phi - 2\pi/3) \\ I_2 &= a + b \cos(\phi) \\ I_3 &= a + b \cos(\phi + 2\pi/3) \end{aligned} \quad (1)$$

where I_1, I_2 , and I_3 are the three fringe images; a is the DC bias; b is the amplitude of the fringes; and ϕ is the phase modulated by the depth (or the height profile) of the object. Note that the 3 fringes have a constant phase shift of $2\pi/3$. Thus the phase information ϕ in (1) can be evaluated from the 3 fringe images by,

$$\hat{\phi} = \arctan \left[\frac{\sqrt{3}I_1 - I_3}{2I_2 - I_1 - I_3} \right] \quad (2)$$

The depth information of a scene is directly related to the absolute phase ϕ . However (2) gives only the wrapped phase $\hat{\phi}$ with limited value ranging from $-\pi$ and π . As mentioned above, traditionally the true phase ϕ is estimated from $\hat{\phi}$ by phase unwrapping procedures based on the Itoh condition. However, they will fail if some fringes are missing due to the aforementioned reasons.

In fact, the true phase ϕ is related to the wrapped phase $\hat{\phi}$ by $\phi = \hat{\phi} + k2\pi$, where k is the period order or the so-called K-map. If we know the k-value for all $\hat{\phi}$, the phase unwrapping problem can be automatically solved. Hence rather than passively estimating the true phase ϕ from the wrapped phase $\hat{\phi}$, we propose to actively embed the k-value into the fringe patterns. After the fringe images are obtained, we propose to decode the k-values by using a sparse coding method with dictionary learned with a novel Gabor feature based discriminative dictionary learning algorithm. The period order information can then be used to evaluate the true phase ϕ . An overview of the proposed decoding algorithm is shown in Fig. 1. It contains an offline dictionary learning stage and an online classification stage. They will be explained in more detail in Section IV.

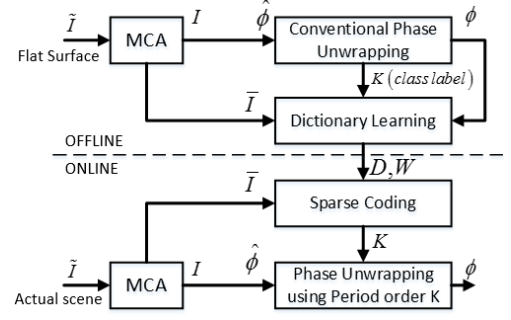


Fig. 1. Proposed FPP framework

III. PERIOD ORDER ENCODED FRINGE PATTERN

As mentioned above, the period order or the k-value is encoded and embedded into the fringe pattern before projecting to the target object. Hence a period order encoded fringe pattern should consist of both the code patterns which carry the period order information and a fringe pattern. Mathematically, an encoded fringe pattern can be expressed as,

$$\tilde{I}_i = I_i + \bar{I} \quad (3)$$

where $\{I_i\}_{i=1,2,3}$ is the three phase shifting fringe patterns. \bar{I} denotes the code patterns which can be written as,

$$\begin{aligned} \bar{I} &= M(k(\phi)) \\ k: \mathbb{R} &\rightarrow \mathbb{Z}^+ \\ \phi &\rightarrow \left\lfloor \frac{\phi + \pi}{2\pi} - \pi \right\rfloor \end{aligned} \quad (4)$$

where M is the encoding function for each k-value; and $\lfloor \cdot \rfloor$ is the floor function to the closest smaller integer number.

To generate the code patterns \bar{I} , we employ a set of textons, which are some predefined image patches that have different morphological structures from the fringe pattern. Fig. 2 shows a set of five unique textons (5×5 pixels). Each has a unique orientation and scale. Given a patch $p \subset \bar{I}$ and its associated period order k_p , we define the encoding function M of a patch as follows:

$$p = M(k_p) = \mathcal{T}_k^p \quad (5)$$



Fig. 2. Some of the binary textons (5×5 pixel) used to generate the code patterns.

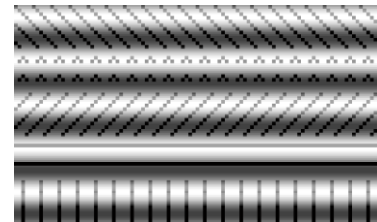


Fig. 3. A period order encoded fringe pattern whose code patterns are generated using the textons in Fig. 2.



Fig. 4. A subset of the elements in the dictionary of 6-level 6-directional Gabor features (patch size: 6x6) generated from the 16x16 patch of the encoded fringe pattern. For each k value, the total number of dictionary elements is 32. The dictionary elements in each row provide a sparse representation for a particular code pattern.

where \mathcal{T}_k^p is a unit texton to represent a period order k . An example of the encoded fringe pattern is illustrated in Fig. 3. Note that the design of the textons is not arbitrary. They should uniquely represent all k -values by means of their orientation and scale.

IV. PERIOD ORDER ESTIMATION

The standard procedure to decode a given code pattern is by measuring the correlation between the code pattern and the referenced data set [22]. It requires a search of the minimum distance between the code pattern and a set of reference codes, which can take a long computation time. Besides, the accuracy can be affected by the artefacts in the image, such as noise. Recently the discriminative dictionary learning methods are widely investigated. They can generate a set of dictionaries that can be extremely discriminative to particular patterns [31, 32]. This provides not only a more formal way to classify a given code pattern but also a faster performance through the sparse coding technique.

In this paper, the discriminative dictionary learning method is adopted in the proposed period order decoding algorithm. As shown in Fig. 1, in the online stage, the period order estimation is done via a sparse coding step and needs a discriminative dictionary D , which is learned in the offline stage. Given the class labels of the training data, a discriminative dictionary can be obtained using various discriminative dictionary learning approaches, such as the LC-KSVD [32, 33]. When applying to the proposed FPP system, these class labels, which are the period orders of the training code patterns, are obtained during the calibration stage (offline), where they are known for all pixels. The details of the dictionary learning and the sparse coding are presented in the following subsections.

A. Dictionary Learning

To prepare for the dictionary learning process, a large number of high quality image patches with known period order needs to be obtained from the training code patterns. To achieve this, we project a set of training fringe patterns with embedded code patterns onto a flat surface such that all parts of the fringe patterns can be clearly shown and captured by the camera. With the captured fringe images, an MCA is carried out as shown in Fig. 1 to separate the fringe pattern and the code patterns. Then the training image patches are

selected from the code patterns and their period order k can be obtained.

Now let us define $P^k = \{p_i^k\}_{i=1, \dots, N^k}$ and $Z^k = \{\xi_i | \xi_i = \mathcal{F}(p_i^k)\}_{i=1, \dots, N^k}$ as a patch set and a patch feature set respectively. Here $\mathcal{F}(\cdot)$ is a feature extraction function of a patch, and N^k is the number of the training patches for each period order k in the set P^k . For each period order k , 256 patches ($N^k = 256$) are selected randomly from the code patterns. Hence with period order $k = 1, \dots, K$, the total number of training sets is $N^k K$ patches.

For the feature extraction, we adopt the Gabor kernel. By convolving a patch p with a Gabor kernel of J scales, $j=1 \dots J$ and N_m orientations, $\theta = \{\theta_1, \dots, \theta_{N_m}\}$, a vector of Gabor features can be obtained. Note that the vectors are in the form of complex numbers which can be written as, $\mathcal{G}_j^m = r_j^m e^{i\theta_j^m} = p * \mathcal{G}_j^m$, where $*$ denotes the convolution operator. We define the feature extraction function $\mathcal{F}(\cdot)$ of a patch p as,

$$\mathcal{F}(p_i^k) = [s(|\mathcal{G}_1^{\theta_1}|), \dots, s(|\mathcal{G}_{N_m}^{\theta_{N_m}}|), \dots, s(|\mathcal{G}_1^{\theta_1}|), \dots, s(|\mathcal{G}_{N_m}^{\theta_{N_m}}|)]^T \quad (6)$$

where $S(x)$ is the mean of vector x . We observe that by representing an image patch with Gabor features of at least 3 levels and 6 orientations, a good estimation of the smooth region of the wrapped phase can be achieved.

To generate a discriminative dictionary D , the label consistent K-SVD (LC-KSVD) algorithm (version 1) [32] is employed. In LC-KSVD, a dictionary is learned by minimizing the following objective function [32],

$$\arg \min_{D, A, \Gamma} \|Z - D\Gamma\|_2^2 + \lambda_q \|Q - A\Gamma\|_2^2 \text{ s.t. } \forall i, \|\gamma_i\|_0 \leq T \quad (7)$$

where $Z = [\xi_1^1, \xi_2^1, \dots, \xi_{N^k K}^K] \in \mathbb{R}^{N_m \times N^k K}$ is the training feature set; $\|Q - A\Gamma\|_2^2$ is the term to enforce the sub-dictionaries in D to discriminate the input according to its class; λ_q is a constant to control the contribution of the corresponding term and $Q \in \mathbb{R}^{N^k K \times N^k K}$ is the discriminative binary matrix which has a value determined by the period order k ,

$$Q(i, j) = \begin{cases} 1 & \text{if } (k-1)N^k \leq i \leq kN^k \text{ and} \\ & (k-1)N^k \leq j \leq kN^k \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

Note that the matrix D and A are initialized using the discrete cosine transform (DCT) basis. In (7), Γ is the trained sparse representation of Z . We can further use it to generate the period order number (class label) by defining a linear classifier W , which can be obtained by solving the following ridge regression problem:

$$\arg \min_W \|H - W\Gamma\|_2^2 + \lambda_w \|W\|^2 \quad (9)$$

where λ_w is a constant to control the contribution of the corresponding term and $H = [h_1 \dots h_K] \in \mathbb{R}^{K \times N^k K}$ is the class

label (period order) of Z . $h_i = [0, \dots, 1, \dots, 0]^T \in \mathbb{R}^K$ is the period order corresponding to the input patch feature set ξ_i . (9) has a close form solution as follows:

$$W = H\Gamma^T(\Gamma\Gamma^T + \lambda I)^{-1} \quad (10)$$

where I is the identity matrix.

In the proposed method, 64 dictionaries for each period order are constructed using the patch feature set Z as the training set with 9 classes in total. The total number of elements is 576. A subset of the discriminative dictionary D is shown in Fig. 4. In the figure, each row of the elements provides a sparse representation for a particular period order.

B. Sparse Coding Step

When the dictionary D and the linear classifier W are learned, we can use them in the online stage for decoding the period order from the encoded fringe images. To do so, the captured encoded fringe images are first sent to an MCA process as shown in Fig. 1 to separate the code patterns from the fringe pattern. Image patches are then randomly selected from the good pixels of the code patterns. By good pixels, we refer to those do not belong to the background area, have a good quality value, and lie on the smooth area of the wrapped phase. The quality of a pixel can be obtained from a quality map that is determined using the approach as in [38]. Given a patch feature $\xi = \mathcal{F}(p)$ of the code patterns, the dictionary D and the linear classifier W , the sparse coding step gives a sparse representation γ of the patch feature ξ by solving the following minimization problem using the orthogonal matching pursuits (OMP) approach [34],

$$\hat{\gamma} = \arg \min_{\gamma} \|\xi - D\gamma\|_2^2 \quad s.t. \quad \forall i, \|\gamma\|_0 \leq T_0 \quad (11)$$

where the period order of the patch p can be estimated by,

$$\hat{k} = \max_{idx}(W\hat{\gamma}) \quad (12)$$

where $\max_{idx}(\cdot)$ returns the index of the coefficient in the vector $W\hat{\gamma}$ of which the value is the maximum. The results are then used to guide a multi-level quality guided phase unwrapping algorithm to obtain the true phase. More specifically, the scanline algorithm is adopted to unwrap the wrapped phase $\hat{\phi}$ in two directions, up and down, until reaching either the boundary of the image, the bad quality map, or the background area. Once it is finished, this process is repeated until all pixels are unwrapped.

V. EXPERIMENTAL RESULTS

To verify the actual performance of the proposed strategy in practical working environment, we implement the proposed algorithm in a real FPP hardware setup and compare with different conventional FPP approaches. The hardware setup consists of a digital projector and a digital camera. The camera is equipped with a 22.2 x 14.8mm CMOS sensor and a 17-50mm lens whereas the projector has a contrast ratio of 2000:1 and a light output of 3300ANSI lumens. Both devices are placed at a distance of 700mm – 1200mm from the target

object and are connected to a personal computer with a 3.4GHz CPU and 16GB RAM for image processing. All programs are developed in the MATLAB environment.

TABLE I. Comparison in terms of the phase error (in $\times 10^{-5}$ Radians) for various total numbers of periods

Number of periods	PSP + Goldstein	PSP-Speckle	Proposed method
8	0.2541	1.1510	0.1788
12	0.1478	0.5567	0.1185
46	0.0038	0.0080	0.0036

In the first experiment, we verify the performance of the proposed method when measuring the 3D model of a simple object. We use a flat board with size 500mm x 400mm as the target object in the experiment. The proposed algorithm is compared with two traditional approaches including: three step phase shifting profilometry with the Goldstein phase unwrapping algorithm (PSP+Goldstein) [35, 36], and PSP with speckles (PSP-Speckle) [22]. The PSP+Goldstein method is popularly used nowadays and the PSP-Speckle was newly proposed to tackle the phase unwrapping problem in FPP. Table I shows the comparison results. They show that when measuring simple objects, the proposed method can obtain an accurate measurement comparable to (if not better than) the conventional PSP method. Unlike PSP-Speckle, the proposed method does not introduce additional distortions due to the embedded code patterns.

In the second experiment, we verify the performance of the proposed method when measuring the 3D model of objects with strong global illumination and sudden jumps in height profile. In the experiment, a jar is used as the target object. The jar is illuminated by a strong light source. Since the jar is placed in the middle of the table, it has a sudden jump in height profile as compared with the background. With such object, we compare the performance of the proposed algorithm with the conventional PSP+Goldstein [35, 36] and PSP-Speckle [22]. The result of the comparison is illustrated in **Error! Reference source not found.** As can be seen in the figure, the PSP+Goldstein method generates incorrect depth information (**Error! Reference source not found.a**) since it does not have the period order information, and the Goldstein phase unwrapping algorithm fails to perform since the object has sudden jumps in height profile. Although the PSP+Speckle method can recover the depth information, the embedded speckles introduce artifacts to the reconstructed 3D model as can be seen in **Error! Reference source not found.b** (zoom version). Meanwhile the proposed algorithm can reconstruct the 3D model of the object accurately as shown in **Error! Reference source not found.c** and is similar to the ground truth as shown in **Error! Reference source not found.d**.

VI. CONCLUSIONS

In this paper, we presented a novel Gabor feature based discriminative dictionary learning method for decoding the period order information in the fringe pattern used for the FPP.

The proposed approach solved the ambiguity problem when evaluating the true phase information in the FPP. It is achieved by first embedding the code patterns which carry the period order information to the fringe pattern. They are then extracted by an MCA procedure and decoded by a discriminative dictionary learned by the LC-KSVD algorithm. Based on the decoded period order information, the true phase can be obtained using a multi-level quality guided phase unwrapping algorithm. The proposed algorithm is robust to operate in various adverse conditions, such as the target object has sudden jumps in height profile or is illuminated strongly by a global light source. Experimental results have verified the robustness of the proposed algorithm as compared with other traditional FPP methods.

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