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Optimal design of a hysteretic vibration absorber using fixed-points theory

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H_∞ optimum parameters of a dynamic vibration absorber (DVA) using a hysteretic or structural damping element have been derived analytically for suppressing vibration of a single degree-of-freedom (SDOF) system excited by harmonic forces or due to ground motions. Although the frequency response function of the traditional DVA (TDVA) with viscous damping may be converted to that of the hysteretic DVA (HDVA) using the equivalent viscous damping coefficient, it is found that the two frequency response functions are not equivalent after the optimization process. Therefore, the optimum parameters of the HDVA are derived using the fixed-points theory rather than converted directly from the TDVA model. The analytical results show that the optimized hysteretic vibration absorber can provide a similar vibration reduction effect as the optimized traditional dynamic vibration absorber at the resonance of a SDOF primary vibrating system. Advantages as well as the limitations of the fixed-points theory for the H_∞ optimization of the hysteretic dynamic vibration absorbers using the fixed-points theory are discussed.
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I. INTRODUCTION

The traditional dynamic vibration absorber (TDVA) as illustrated in Fig. 1(a) is a passive device used to reduce vibration of the primary system. It uses a viscous damping element to damp down the vibration of the primary system mostly at the pre-tuned frequency. Considerable research has been carried out to derive analytically the optimum parameters of the traditional dynamic vibration absorber (DVA). Ormondroyd and Den Hartog[1] showed that the DVA has an optimum damping value for the minimization of the resonant amplitude response of the single degree-of-freedom (SDOF) system. The optimum damping and optimum tuning frequency were derived by Brock[2] and Hahnkamn,[3] respectively. These formulations can be deduced using the fixed-points theory of Den Hartog,[4] which stated that all frequency response curves of the primary mass pass through two invariant points regardless of the amount of the viscous damping. The optimal frequency and damping ratios of the traditional DVA for the undamped SDOF primary system based on the fixed-points theory were very good approximations of the exact values derived by Nishihara and Asami.[5] Optimization of the frequency and damping parameters of the other designs of DVA using a viscous damper based on the fixed-points theory were undertaken by other researchers[6–10] to minimize the resonant vibration amplitude of single and multi degree-of-freedom systems.

Viscous damping and hysteretic damping are different types of damping models. While the viscous damping force is proportional to the velocity, the hysteretic damping force is proportional to the displacement multiplied by complex number i. The viscous damping assumption used in many vibration analyses was chosen mainly for mathematical convenience. On the other hand, hysteretic damping, based on the concept of a complex modulus, can often be effectively utilized in the calculation of the steady state response of vibrating structures.

Hysteretic dynamic vibration absorber (HDVA) is a tuned mass damper that uses hysteretic or frictional damping to dissipate the energy of oscillations in a vibrating structure as illustrated in Fig. 1(b). A well-known application of HDVA is the Stockbridge damper, which is commonly used to suppress wind-induced vibrations on slender structures such as overhead electric power transmission lines and long cantilever signs.[11] The importance of the hysteretic damping model for the absorber is in designing the practical absorber. The complex modulus for material damping can be employed to the absorber structure whatever the configuration of the absorber is and the hysteretic loss factor of the absorber can be obtained by direct measurement with a suitable experiment.[12] However, there is no standard or easy method to derive an optimized design of the hysteretic dynamic vibration absorber in order to minimize the resonant vibration of the primary system.

A common way to relate hysteretic damping to viscous damping for harmonic forced vibrations is to use the equivalent viscous damping coefficient,[13] $c_{eq} = k\eta/\omega$ or in dimensionless form written as $\zeta_{eq} = \gamma\eta/(2\lambda)$, where the definition of the symbols can be found in Sec. II. Although the frequency response function (FRF) of the HDVA may be converted to that of the TDVA using the equivalent viscous damping coefficient, it will be shown that in the latter part of Sec. III that the two frequency response functions are not equivalent after the optimization process. It is, therefore, better to derive the fixed points as well as the optimum

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parameters of the HDVA (model B) using the fixed-points theory[4] than to convert directly the optimum parameters from the TDVA model to obtain those of the HDVA model.

The fixed-points theory for the optimization of traditional DVA design is briefly reviewed in Sec. II. The mathematical model of the proposed HDVA is formulated in Sec. III. The fixed points in the frequency response spectra of the primary mass of the hysteretic dynamic absorber with different damping ratios are found and the optimum tuning frequency and damping ratios of the HDVA for the minimization of the resonant vibration amplitude of the primary mass are then derived using the fixed-points theory. To the author's knowledge, this is the first research report on the H_∞ design optimization of a HDVA using the fixed-points theory.

II. THE TRADITIONAL DYNAMIC VIBRATION ABSORBER (MODEL A)

A schematic diagram of the traditional damped dynamic vibration absorber incorporate with a viscous damper is shown in Fig. 1(a). This vibration absorber denoted by model A is attached to a SDOF undamped primary system. Vibration of mass M is excited by harmonic force $f = F\sin\omega t$ or due to ground motion $y = Y\sin\omega t$. The amplitude ratio $|H_A(\lambda)|$ can be derived as[4,8]

$$|H_A(\lambda)| = \left|\frac{X_1}{F/K}\right|_A = \left|\frac{X_1}{Y}\right|_A = \sqrt{\frac{(\gamma^2-\lambda^2)^2+(2\gamma\lambda\zeta)^2}{[(1-\lambda^2)(\gamma^2-\lambda^2)-\mu\gamma^2\lambda^2]^2+[2\gamma\lambda\zeta(1-\lambda^2-\mu\lambda^2)]^2}}, \quad (1)$$

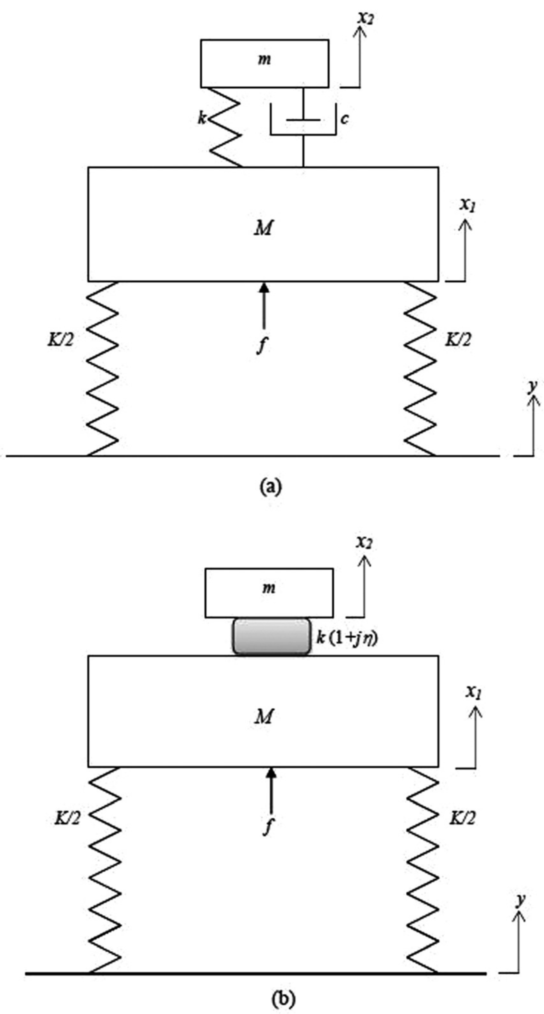

FIG. 1. A damped dynamic vibration absorber as an auxiliary mass-spring-damper (m-k-c) system attached to a primary system (M-K) (a) model A: traditional design of the absorber (Ref. 1), (b) model B: the proposed hysteretic dynamic vibration absorber of the absorber for suppressing the vibration of the mass M excited by a harmonic force f or due to ground motion y.

where $\mu = m/M$, $\omega_a = \sqrt{k/m}$, $\omega_n = \sqrt{K/M}$, $\gamma = \omega_a/\omega_n$, $\lambda = \omega/\omega_n$, $\zeta = c/(2\sqrt{mk})$, and X_1 is the vibration amplitude of the primary mass M.

The objective function of the H_∞ optimization is to minimize the maximum amplitude ratio of the response of the primary system to the excitation force. It may be expressed mathematically as

$$\max(|H_A(\lambda, \gamma_{opt\_A}, \zeta_{opt\_A})|) = \min(\max_{\gamma,\zeta} |H_A(\lambda)|). \quad (2)$$

The procedure for derivation of the optimum tuning frequency and damping ratios of the absorber are referred to the fixed-points theory of Den Hartog.[4] The optimum design parameters of this absorber are given by the following equations:

$$\gamma_{opt\_A} = \frac{k/m}{K/M} = \frac{1}{1+\mu}, \quad (3)$$

$$\zeta_{opt\_A} = \frac{c}{2\sqrt{mk}} = \sqrt{\frac{3\mu}{8(1+\mu)}}. \quad (4)$$

An approximate value of the maximal amplitude ratio derived by Den Hartog[1] is written as

$$|H_A(\lambda)|_{max} = \sqrt{\frac{2+\mu}{\mu}}. \quad (5)$$

III. HYSTERETIC DYNAMIC VIBRATION ABSORBER (MODEL B)

The hysteretic vibration absorber denoted by the model B has a single mass connected to the primary structure through a hysteretic or frictional damping device, as shown in Fig. 1(b). The stiffness of the hysteretic damper is modeled by the complex stiffness and is given by

$$k_s = k(1 + i\eta), \tag{6}$$

where η is the hysteretic damping ratio which is the fraction of energy lost in each cycle of the vibration. The governing equations of the hysteretic vibration absorber in the frequency domain may be written as

Case 1: Vibration due to harmonic force excitation ($f = F \sin \omega t$, $y = 0$):

$$-MX_1\omega^2 + KX_1 + k(1 + i\eta)(X_1 - X_2) = F(\omega), \tag{7a}$$

$$-mX_2\omega^2 + k(1 + i\eta)(X_2 - X_1) = 0. \tag{7b}$$

Case 2: Vibration due to ground motion ($f = 0$, $y = Y \sin \omega t$):

$$-MX_1\omega^2 + K(X_1 - Y) + k(1 + i\eta)(X_1 - X_2) = 0, \tag{8a}$$

$$-mX_2\omega^2 + k(1 + i\eta)(X_2 - X_1) = 0, \tag{8b}$$

where X_1 and X_2 are the vibration magnitudes of the primary mass M and hysteretic absorber m, respectively. Solving Eqs. (7a) and (7b) for case 1, and Eqs. (8a) and (8b) for case 2 yields

$$\text{Case 1}: \quad X_1 = \frac{[k(1 + i\eta) - m\omega^2]F}{k(1 + i\eta)(K - M\omega^2) - [K - M\omega^2 + k(1 + i\eta)]m\omega^2}, \tag{9a}$$

$$\text{Case 2}: \quad X_1 = \frac{[k(1 + i\eta) - m\omega^2]KY}{k(1 + i\eta)(K - M\omega^2) - [K - M\omega^2 + k(1 + i\eta)]m\omega^2}, \tag{9b}$$

$$\text{Cases 1 and 2}: \quad X_2 = \frac{k(1 + i\eta)X_1}{k(1 + i\eta) - m\omega^2}. \tag{10}$$

The amplitude ratio of the primary system in both cases 1 and 2 may be written as

$$|H_B(\omega)| = \left|\frac{X_1}{F/K}\right|_B = \left|\frac{X_1}{Y}\right|_B = \sqrt{\frac{K^2(k - m\omega^2)^2 + K^2(k\eta)^2}{[(K - M\omega^2)(k - m\omega^2) - km\omega^2]^2 + [k\eta(K - M\omega^2 - m\omega^2)]^2}}. \tag{11}$$

Equation (11) may be expressed in the dimensionless form written as

$$|H_B(\lambda)| = \frac{\sqrt{(\gamma^2 - \lambda^2)^2 + (\gamma^2)^2\eta^2}}{\sqrt{[(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2]^2 + [\gamma^2(1 - \lambda^2 - \mu\lambda^2)]^2\eta^2}}, \tag{12}$$

where $\mu = m/M$, $\omega_a = \sqrt{k/m}$, $\omega_n = \sqrt{K/M}$, $\gamma = \omega_a/\omega_n$, and $\lambda = \omega/\omega_n$.

It can be observed by comparing Eqs. (12) and (1) that the FRF of the primary system of the HDVA is similar but not the same as that of the TDVA. The H_∞ optimization of the HDVA (model B) based on the fixed-points theory is presented next.

The purpose of the H_∞ optimization of the model B is to optimize the parameters of hysteretic damping material in order to minimize the maximum amplitude ratio of the response of the primary mass M to the excitation. It may be expressed mathematically as

$$\max(|H_B(\lambda, \gamma_{\text{opt\_}B}, \eta_{\text{opt\_}B})|) = \min(\max_{\gamma,\eta} |H_B(\lambda)|). \tag{13}$$

Equation (12) may be rewritten as

$$|H_B(\lambda)| = \sqrt{\frac{A + B\eta^2}{C + D\eta^2}}, \tag{14}$$

where $A = (\gamma^2 - \lambda^2)^2$, $B = (\gamma^2)^2$, $C = [(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2]^2$, and $D = [\gamma^2(1 - \lambda^2 - \mu\lambda^2)]^2$.

The amplitude ratios of the primary mass M, $|H_B(\lambda)|$, are calculated using Eq. (12) with tuning frequency ratio $\gamma = 1$ and damping ratios $\eta = 0.1$, 0.2, and 0.3, respectively, and plotted in Fig. 2 to show the existence of the fixed points in the frequency spectra of mass M. To find these two fixed points P and Q, we consider the frequency response curves for $\eta = 0$ and $\eta = \infty$. The curves for $\eta = 0$ and $\eta = \infty$ and other real values of η would pass through the fixed points P and Q. This can be expressed mathematically as

$$\frac{A}{C} = \frac{B}{D} = \frac{A + B\eta^2}{C + D\eta^2}. \tag{15}$$

Substituting $\eta = 0$ into Eq. (11), we may write

$$|H_B(\lambda)|_{\eta=0} = \left|\frac{\gamma^2 - \lambda^2}{(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2}\right| = \left(\frac{A}{C}\right)^{1/2}. \tag{16}$$

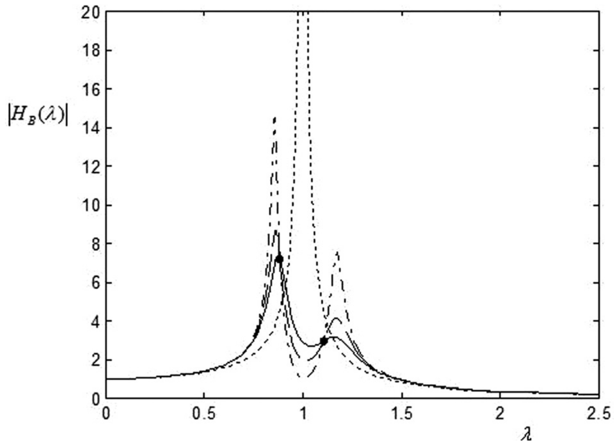

FIG. 2. Frequency response curves of the primary mass M of the hysteretic damping absorber (model B) with $\mu = 0.1$ and $\gamma = 1$ before adding HVDA (- - - - - - - -), and after adding HVDA with $\eta = 0.1$ (–·–·–), 0.2 (– – – –), 0.3 (———). Fixed points are marked with (•).

Substituting $\eta = \infty$ into Eq. (12), we may write

$$|H_B(\lambda)|_{\eta=\infty} = \left|\frac{1}{1-\lambda^2-\mu\lambda^2}\right| = \left(\frac{B}{D}\right)^{1/2}. \tag{17}$$

Using Eqs. (15), (16) and (17), we may write

$$\left(\frac{\gamma^2-\lambda^2}{(1-\lambda^2)(\gamma^2-\lambda^2)-\mu\gamma^2\lambda^2}\right)^2 = \left(\frac{1}{1-\lambda^2-\mu\lambda^2}\right)^2. \tag{18}$$

Taking square root on both sides of Eq. (18) and consider the responses at $\eta = 0$ and $\eta = \infty$ at opposite phases,[8] we may write

$$\frac{\gamma^2-\lambda^2}{(1-\lambda^2)(\gamma^2-\lambda^2)-\mu\gamma^2\lambda^2} = \frac{-1}{1-\lambda^2-\mu\lambda^2}. \tag{19}$$

Equation (19) may be rewritten as

$$(2+\mu)\lambda^4 - 2\lambda^2(1+\gamma^2+\mu\gamma^2) + 2\gamma^2 = 0. \tag{20}$$

The two roots of Eq. (20) expressed as λ_P and λ_Q may be written as

$$\lambda_P = \sqrt{\frac{1+(1+\mu)\gamma^2 - \sqrt{1-2\gamma^2+(1+\mu)^2\gamma^4}}{2+\mu}}, \tag{21}$$

$$\lambda_Q = \sqrt{\frac{1+(1+\mu)\gamma^2 + \sqrt{1-2\gamma^2+(1+\mu)^2\gamma^4}}{2+\mu}}, \tag{22}$$

λ_P and λ_Q in Eqs. (21) and (22) have real positive values if μ and γ are also real positive values. This shows that Eq. (15) has solutions of λ_P and λ_Q after μ and γ are chosen for the HDVA. λ_P and λ_Q are the non-dimensionless frequencies of the fixed points in this case and they are same as those of the traditional DVA (model A). The frequency response amplitudes of mass M at λ_P and λ_Q can be derived by substituting Eqs. (21) and (22), respectively, into Eq. (17) and written, respectively, as

$$|H_B(\lambda_P)| = \left|\frac{1}{1-\lambda_P{}^2-\mu\lambda_P{}^2}\right| = \frac{2+\mu}{1-\gamma^2(1+\mu)^2+(1+\mu)\sqrt{1-2\gamma^2+(1+\mu)^2\gamma^4}}, \tag{23}$$

$$|H_B(\lambda_Q)| = \left|\frac{1}{1-\lambda_Q^2-\mu\lambda_Q^2}\right| = -\frac{2+\mu}{1-\gamma^2(1+\mu)^2-(1+\mu)\sqrt{1-2\gamma^2+(1+\mu)^2\gamma^4}}. \tag{24}$$

The optimum tuning frequency can be derived by equating $|H_B(\lambda_P)|$ and $|H_B(\lambda_Q)|$ using Eqs. (23) and (24) and simplified as

$$\gamma_{\text{opt}\_B} = \frac{1}{1+\mu}. \tag{25}$$

Equations (3) and (25) show that the optimum tuning frequencies of the traditional DVA (model A) and the hysteretic DVA (model B) are the same. The frequency ratios of the stationary points, λ_P and λ_Q can be obtained by substituting Eq. (25) into Eqs. (21) and (22) and can be written as

$$\lambda_P^2 = \frac{\sqrt{2+\mu}-\sqrt{\mu}}{(1+\mu)\sqrt{2+\mu}}, \tag{26}$$

$$\lambda_Q^2 = \frac{\sqrt{2+\mu}+\sqrt{\mu}}{(1+\mu)\sqrt{2+\mu}}. \tag{27}$$

The response amplitude of the mass M at the stationary points can be derived by substituting Eq. (26) into Eq. (23) and Eq. (27) into Eq. (24) and can be written as

$$|H_B|_{P,Q} = \sqrt{\frac{2+\mu}{\mu}}. \tag{28}$$

To determine the optimum damping in order to make points P and Q to be the maximum points on the response curve of mass M, it requires zero slopes at the two stationary points, P and Q. We may therefore write

$$\left.\frac{\partial|H_B(\lambda)|^2}{\partial\lambda^2}\right|_{\lambda=\lambda_P,\lambda_Q} = 0. \tag{29}$$

Rewrite Eq. (12) as

$$|H_B(\lambda)|^2 = \frac{S}{T}, \tag{30}$$

where $S = K^2(k - m\omega^2)^2 + K^2(k\eta)^2$ and (31)

$$T = [(K - M\omega^2)(k - m\omega^2) - km\omega^2]^2 + [k\eta(\omega)(K - M\omega^2 - m\omega^2)]^2. \tag{32}$$

If $[\partial|H_B(\lambda)|^2]/\partial\lambda^2 = 0$ then we may write

$$\frac{\partial}{\partial\lambda^2}\left(\frac{S}{T}\right) = \left(\frac{S'T - ST'}{T^2}\right) = 0, \tag{33}$$

where $S' = \partial S/\partial\lambda^2$ and $T' = \partial T/\partial\lambda^2$.

Using Eq. (33), we may write

$$S'T - ST' = 0, \tag{34}$$

S and T may be rewritten in the nondimensional form as

$$S = (\gamma^2 - \lambda^2)^2 + (\gamma^2)^2\eta^2 \text{ and} \tag{35}$$

$$T = [(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2]^2 + [\gamma^2(1 - \lambda^2 - \mu\lambda^2)]^2\eta^2. \tag{36}$$

Differentiating Eqs. (35) and (36) with respect to λ^2 and then substituting them back to Eq. (34), we may write

$$X\eta^2 + Y = 0, \tag{37}$$

where

$$X = \gamma^4(1 - \lambda^2 - \mu\lambda^2)(1 + \mu) \text{ and} \tag{38}$$

$$Y = -(\gamma^2 - \lambda^2)(1 - \lambda^2 - \mu\lambda^2)^2 + (1 - 2\lambda^2 + \gamma^2 + \mu\gamma^2)[(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu\lambda^2\gamma^2]. \tag{39}$$

The optimum damping at the fixed points P and Q may be derived using Eqs. (25)–(27) and Eqs. (37)–(39) and written, respectively, as

$$\eta_P{}^2 = -\left.\frac{Y}{X}\right|_{\lambda^2=\lambda_P{}^2,\,\gamma=\gamma_{\text{opt\_}B}} = \frac{3\mu}{2} + \frac{\mu^2}{2(2+\mu)} - 4\mu\sqrt{\frac{\mu}{2+\mu}}, \tag{40}$$

$$\eta_Q{}^2 = -\left.\frac{Y}{X}\right|_{\lambda^2=\lambda_Q{}^2,\,\gamma=\gamma_{\text{opt\_}B}} = \frac{3\mu}{2} + \frac{\mu^2}{2(2+\mu)} + 4\mu\sqrt{\frac{\mu}{2+\mu}}. \tag{41}$$

Taking the average of $\eta_P{}^2$ and $\eta_Q{}^2$ produces

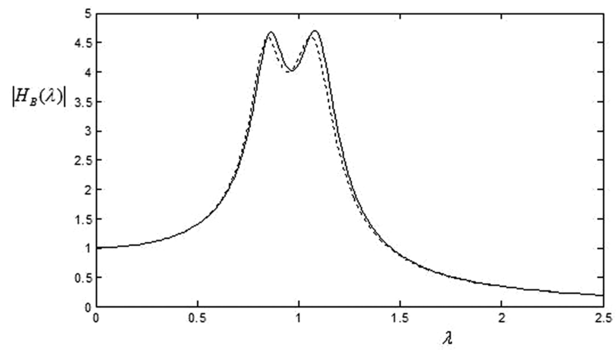

FIG. 3. Frequency response curves of the primary mass M of the optimized HDVA (model B, ———), and the traditional DVA (model A, - - - - - - - -). $\mu = 0.1$.

$$\eta_{\text{opt\_}B} = \sqrt{\frac{3\mu}{2} + \frac{\mu^2}{2(2+\mu)}}. \tag{42}$$

The resonant amplitude response of the mass M of model B with optimum tuning frequency $\gamma_{\text{opt\_}B}$ and damping $\eta_{\text{opt\_}B}$ may be approximately written using Eq. (28) as

$$|H_B|_{\max} = \sqrt{\frac{2+\mu}{\mu}}. \tag{43}$$

Equations (5) and (43) show that approximate maximum amplitude ratios of the optimized traditional DVA (model A) and the optimized hysteretic DVA (model B) are the same. The frequency responses of the HDVA are plotted in Fig. 3 for illustration. The typical double equal peaks response of the optimized HDVA can be seen in the figure. The peak vibration response in Fig. 3 is much lower than those in Fig. 2. This shows the usefulness of the optimization of the HDVA. The frequency response curve of the optimized model A is also plotted in Fig. 3 for comparison. It is observed that the peak response of model B is slightly higher than that of model A. This deviation from the theoretical prediction is explained next.

While the hysteretic damping η is assumed to be constant with vibration frequency, the equivalent viscous damping ratio $\zeta_{eq} = \gamma\eta/(2\lambda)$ is a function of vibration frequency λ. The FRF of model B could be converted to that of model A using the equivalent viscous damping coefficient, $c_{eq} = k\eta/\omega$. Equation (12) can be obtained by substituting $\zeta_{eq} = \gamma\eta\,/\,(2\lambda)$ into Eq. (1). Therefore, both model A and model B have the same locations of the fixed points λ_P and λ_Q in their respective frequency spectrum as well as the optimum tuning frequency ratio $\gamma_{\text{opt\_}A} = \gamma_{\text{opt\_}B} = 1/(1+\mu)$. However, the two frequency response functions are not equivalent after the optimization process. This is shown by the plots of the two FRFs in Fig. 3. There are two reasons for the differences of the two FRFs after optimization. The first one is that the optimum damping in the two cases are not exactly equivalent. Converting the optimum damping in model A, $\zeta_{\text{opt\_}A} = \sqrt{3\mu/[8(1+\mu)]}$ to the equivalent hysteretic damping by using the formula $\zeta_{eq} = \gamma\eta/2\lambda$ then we will derive $\eta_{\text{opt\_}B} = \sqrt{3\mu/2}$. This is due to the approximation taken when optimum damping is derived by the fixed-points theory as shown in Eqs. (40)–(42). The second reason is that λ_P and λ_Q are used to derive the optimum damping $\eta_{\text{opt\_}B}$ in Eqs. (40)–(42). The equivalent viscous damping ratio ζ_{eq} of the optimum damping $\eta_{\text{opt\_}B}$ would not depend on the frequency λ and therefore the two respective spectra of the optimized TDVA and the optimized HDVA are not the same as shown in Fig. 3. The difference between the peak values of the two response curves in Fig. 3 is about 2%. This may be a limitation of the fixed-points theory. Exact optimization of the TDVA has been reported[5] but the mathematical expressions of the optimized parameters are complicated and not convenient to be applied in practice. The major advantage of using the fixed-points theory is that it can derive simple approximate expression of the optimum damping value of the HDVA which may be useful in many engineering applications.

IV. CONCLUSION

The H_∞ optimum tuning frequency and damping of a hysteretic dynamic vibration absorber are derived for a SDOF vibrating system excited by harmonic forces or due to ground motions. Although the FRF of the TDVA with viscous damping may be converted to that of the HDVA using the equivalent viscous damping coefficient, it is found that the two frequency response functions are not equivalent after the optimization process. Fixed-points theory is applied to derive the optimum design parameters of the HDVA instead of converting directly the optimum parameters from the TDVA model to those of the HDVA model. The optimum tuning frequency is derived to be the same as that of the traditional DVA but the optimum damping ratios in the two cases are different. The optimized HDVA has similar vibration suppression capability as the traditional DVA and, therefore, it may be used as an alternative to the traditional DVA. Advantages as well as the limitations of the fixed-points theory for the H_∞ optimization of the hysteretic dynamic vibration absorbers using the fixed-points theory are discussed. This theoretical analysis can improve our understanding of the vibration suppression performance of the proposed HDVA and help to achieve better design of the HDVA.

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