Supplying Masks to Combat Respiratory Diseases: Safety Index, Welfare and Government Involvement

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Abstract Coronavirus (COVID-19) is a highly infectious respiratory disease whose spread can be effectively curbed by wearing facial masks, especially N95 and surgical masks. In this paper, we focus on examining how masks should be effectively managed and sold during a health crisis. We examine the interaction between N95 and surgical masks. We propose the safety index to measure the risk of respiratory diseases. We develop a stylised game-theoretical model to evaluate the impacts of producing and selling both of these masks on supply chain profits, safety index and consumer and social welfare. Firstly, we find that as the infection probability without protection (IPWP) increases, both the retail price and demand for these masks will increase. When the IPWP is sufficiently low, those consumers who want to purchase masks are more likely to purchase N95 masks, but when the IPWP increases, surgical masks are more popular amongst consumers. Secondly, we develop a safety index that indicates the effectiveness of using masks in preventing respiratory disease infection. This index is especially crucial in cases where the IPWP is moderate; in other words, recommending to wear masks is particularly important when the IPWP is moderate. We also examine the impacts of government involvement in handling the outbreak of respiratory diseases. We find that providing consumer subsidies and promoting the social mask enterprise can effectively combat respiratory diseases under different conditions. Our results can be used for combating COVID-19 and preparing for future health crises.

Keywords: COVID-19, consumer subsidy, mask supply chain, safety index, social welfare.

1. Introduction

Coronavirus (COVID-19) has spread to all corners of the world in 2020 and 2021. Worldometers data show that COVID-19 has resulted in more than 157 million infections and 3.2 million deaths as of May 2021. Given the high infection rate of COVID-19, the number of infections and COVID-related deaths continue to increase, thereby disrupting the normal lives of people and the operations of national economies. The transmission of COVID-19 from people who do not display any symptoms has been supported by plenty of evidence (WHO 2020). People can easily catch the disease from its carriers without wearing any protective gear (Kaplan 2020). Governments all over the world have shown great concern about the consequences of COVID-19 and have spent much effort in slowing down or curbing its spread (Ivanov 2020a). These governments regularly publish reports on the COVID-19 infection rate in their respective jurisdictions. This infection rate is calculated based on daily information, such as the number of confirmed cases and number of deaths. For example, a report from the UK shows that in 2020, the COVID infection rate in England reached 1.2% on November 21 and 0.9% on November 28. Such infection rate can refer to the probability for an ordinary person to be infected with COVID-19 without using any protection. Without taking protective measures, a single confirmed case can easily lead to a widespread infection in high-density areas.

The key objective of governments under this scenario is to reduce the infection rate of COVID-19 and completely eliminate its spread. Using masks has been strongly advocated by the World Health Organization (WHO) as a way for infected people to stop spreading the virus any further (CDC 2020); accordingly, wearing masks later became mandatory in public areas, particularly in areas where social distancing is difficult to maintain (Mandal 2020).

The effectiveness of wearing masks in curbing the spread of diseases has already been proven in the past respiratory disease crises. In 2003, the facemasks were recommended to use by Centers for Disease Control

(CDC) for preventing transmission of Severe Acute Respiratory Syndrome (SARS) (CDC 2005). This recommendation was continued in Influenza A(H1N1) Virus in 2009 (CDC 2009) and Ebola in 2014 (WHO 2014). The spread of respiratory diseases is inevitable. When we overcome the existing virus (e.g. COVID-19), a new virus may be born and coming. We could not avoid the risk of respiratory diseases. However, wearing masks is always effective no matter what new virus is challenging our lives. In this paper, our focal point is how masks should be effectively managed and sold during a health crisis. Table 1 summarizes the major respiratory diseases in the recent two decades and the associated recommendation of using masks.

Table 1. Recommendation of wearing masks in curbing the spread of respiratory diseases

Major Respiratory Diseases	Recommendation of using face masks
SARS (2003)	The transmission of SARS appears caused by direct contacting with infectious material, comprising diffusion of large respiratory droplets and through the airborne route. Hence, CDC has recommended the use of N95 masks to combat SARS (CDC 2003)
H1N1 (2009)	Persons at increased risk of severe illness from H1N1, facial mask or respirator are recommended (CDC 2009)
Ebola (2014-2016)	Infection in human communities is sustained through person-to-person contact, Personal Protective Equipments (PPE) e.g. gloves, mask, gown are most important in preventing transmission of the Ebola virus. (WHO 2014)

Facial masks mainly include surgical (also known as medical) and N95/KN95 masks[1]. These two mask types show differences in their quality and performance. Specifically, CDC shows that surgical masks cannot offer a 100% protection against viruses because the wearers can still inhale small airborne particles (Grainger 2020). By contrast, N95 masks can filter at least 95% of airborne particles, thereby offering greater protection to their wearers compared with surgical masks (Grainger 2020). Therefore, N95 masks are generally more reliable in preventing respiratory diseases compared with surgical masks. However, N95 masks are more expensive than surgical masks. For example, in March 2020 (the early stage of COVID-19 outbreak in Europe),

[1] We focus on surgical and N95 masks and ignore cloth masks and elastomeric respirators because (1) surgical and N95 masks are recommended by WHO and can be easily found in the market (WHO 2020; Nierenberg 2020), (2) cloth masks cannot prevent wearers from transmitting COVID-19 to others (Grainger 2020) and (3) elastomeric respirators are not commonly used in public.

N95 masks in Germany cost 17 Euro per piece, whereas surgical masks only cost 1 Euro per piece[2]. This large price difference also exists in China, the U.S., UK, and other countries during the outbreak of COVID-19. Suppose that the inventories of both N95 and surgical masks are available in the market, consumers will purchase N95 or surgical masks based on the price and quality differences in terms of the risk of respiratory diseases.

We build a safety index to reflect the probability for three types of consumers—those who buy N95 masks, those who buy surgical masks and those who do not buy anything at all—of transmitting respiratory diseases. A high safety index corresponds to a low infection probability after wearing masks. Governments can improve this safety index by encouraging people to buy and use surgical masks. In addition to mandating the wearing of masks in public areas, governments also provide consumer subsidies to reduce the prices of these masks. For example, during the peak of COVID-19 in February 2020, surgical masks in Changji, China were priced at 2.5 RMB a piece, of which 1.5 RMB is subsidised by the local government (Changji Government News 2020). Whilst such subsidy scheme has also been adopted in many countries, whether or not consumer subsidies can help reduce COVID-19 infection risk and improve social welfare remains unclear. Moreover, during the COVID-19 outbreak, some governments, such as that of Singapore, have procured masks globally and sold them through government-owned organisations (Li and Heijmans 2020).

In this paper, we examine the interaction between surgical and N95 masks in the market during the outbreak of respiratory diseases. We consider a scenario where consumers can choose to purchase either surgical or N95 masks. Consistent with real practice, wearing an N95 mask offers better protection than wearing a surgical one. We also explore the effect of consumer subsidies on mask purchase. Our research questions (RQs) are as follows:

[2] Data were collected from a pharmacy store in Munich, Germany in March 2020, during which Germany reached its first peak in COVID-19 cases.

RQ1: How do surgical and N95 masks interact in the market during the outbreak of respiratory diseases?

RQ2: How does supply chain integration affect infection risk?

RQ3: Should the consumer subsidy and social mask enterprise be promoted during the outbreak of respiratory diseases? How does such government involvement influence infection risk and social welfare?

Our major contributions are as follows. Firstly, we find that as the infection probability without protection (IPWP) increases, both the retail price and sales quantity of masks will increase because when the IPWP is high, consumers become willing to purchase masks regardless of their price for the sake of protecting themselves from the virus. Meanwhile, when the IPWP is low, those consumers who are willing to buy masks are more likely to purchase N95 masks than surgical ones (which is the case during the early stages of the COVID-19 pandemic). Specifically, consumers who want to buy masks are very concerned about being infected and are therefore willing to pay more for N95 masks due to their higher degree of protection as opposed to surgical masks. In this case, surgical masks only become more popular than N95 masks when the IPWP increases because surgical masks are more affordable.

Secondly, we find that horizontal integration can generate high channel profits when the production cost difference between N95 and surgical masks is sufficiently small. With horizontal integration, safety index and consumer welfare are lower. Under this circumstance, social welfare is higher if the IPWP is sufficiently low. We construct a safety index to reflect how using masks can prevent wearers from being infected by the respiratory diseases. This safety index is high when the IPWP is either relatively high or low because a low IPWP corresponds to a minimum risk of being infected, whereas a high IPWP corresponds to a large number of consumers buying and wearing masks regardless of their price as they begin to pay more attention to their safety and health. When the IPWP is moderate, consumers become concerned about being infected even if some of them refuse to wear masks. Therefore, encouraging people to wear masks is very necessary when the IPWP is moderate. We also find that during the outbreak of respiratory diseases, a horizontal integration is not helpful in improving both the safety index and consumer welfare but may help improve social welfare. Thus, if the government focuses on the safety index and consumer welfare, recommending masks to sell through the integrated seller is not preferable, but if the government focuses on social welfare, it is desirable to sell through an integrated seller.

Thirdly, we examine the impacts of government involvement during the outbreak of respiratory diseases. We propose that the government can provide consumer subsidies and promote the social mask enterprise for social welfare maximisation. A consumer subsidy scheme can effectively promote consumer and social welfare. Interestingly, we find that such a subsidy scheme plays a positive role in increasing the safety index when the IPWP is relatively high. As a result, consumers have a higher incentive to buy masks when a consumer subsidy is provided. Meanwhile, when masks are being sold by the social mask enterprise, consumers prefer buying N95 masks than surgical ones if the IPWP is relatively low. Therefore, government involvement plays a meaningful role in recovering both consumer and social welfare. Governments may either provide a consumer subsidy scheme or sell masks through the social mask enterprise because these two approaches have equivalent effects on the safety index, consumer welfare and social welfare. However, comparing these two social maximization approaches, providing consumer subsidies is more profitable for the seller because the seller can charge higher prices.

The rest of this paper is organised as follows. Section 2 reviews the related literature. Section 3 develops the analytical model with surgical and N95 masks. Section 4 examines horizontal integration. Section 5 evaluates the impacts of government involvement. Section 6 concludes the paper and provides directions for future research. All proofs are relegated to the Appendix, and the important parameters and thresholds are presented at the end of each proof.

2. Literature review

Our paper considers three domains, namely, (1) operations and production in COVID-19, (2) government involvement in supply chains and (3) healthcare supply chain management.

The COVID-19 outbreak has significantly influenced global supply chain management (Queiroz et al. 2020; Ivanov 2020b; Ivanov and Dolgui 2020b). Ivanov (2020a) evaluates the short- and long-term effects of outbreaks on the supply chain and identifies lead time, speed of epidemic propagation and supply chain disruption duration as key influential factors. Ivanov and Dolgui (2020a) propose several operational research methods to address the ripple effect of the COVID-19 outbreak on supply chains. Choi (2020) evaluates logistics transformation during the COVID-19 outbreak by focusing on 'bring-service-near-your-home' mobile service operations and find that government subsidies, such as the fixed cost and operations cost subsidies, support service realisation. Nikolopoulos et al. (2020) propose a forecasting and planning model for measuring COVID-19 growth rate, supply chain disruption and government decisions during the COVID-19 outbreak. By using real data from the US, India, UK, Germany and Singapore, they identify government intervention as a key factor in reducing COVID-19 growth rate. Ivanov (2021a) examines supply chain adaptive behaviours selection and supply chain management during the pandemic and post-pandemic recovery. The author finds that supply chain coordination could help the supply chain recover from disruption tails. Ivanov (2021b) examines how the outbreak of COVID-19 affects viability and adaptation. Ruel et al. (2021) study the impacts of supply chain viability on operations management during the COVID-19 pandemic. Aldrighetti et al. (2021) conducts a systematic literature review about supply chain network design in terms of disruption risk modelling. In this paper, we model the real situation of using masks during the respiratory disease outbreak. To the best of our knowledge, this paper is the first to evaluate how mask selection and government involvement contribute to addressing the challenges brought about by the pandemic. Our results can not only apply to address the problem of mask supply during the COVID-19, but also prepare for the future respiratory diseases.

Combating the health crises like COVID-19 needs citizens' cooperation and government involvement. In the production and operations management literature, government involvement has been widely examined. Li et al. (2020) study the green government subsidies for innovations and products in supply chains and find that innovation subsidies are more efficient than product subsidies. Shao et al. (2017) examine the optimal subsidy incentive scheme for the government to promote the adoption of electric vehicles. The government's objective is social welfare maximization. Bian and Zhao (2020) study how the government imposes environmental policy tax and subsidy to realize social welfare maximization. Zhang et al. (2020) explore the optimal remanufacturing regulatory for social welfare maximization. Li et al. (2020) evaluate how the government manages environmental sustainability and realizes social welfare maximization by designing the optimal tax and subsidy strategies. Yang et al. (2020) find that the government subsidy can not only realize social welfare maximization but also reduce competition and alleviate firms; financial burden for technology investment. He et al. (2019) study the impacts of government subsidy on channel structure and pricing decisions.

In this paper, we examine government involvement from two perspectives. One of these perspectives focuses on the consumer subsidies for selling masks. Consumer subsidies are used to promote products like environmental products and vaccines. High price is an important factor that hinders consumers from purchasing products. If governments want to encourage consumers to buy, then they can provide consumer subsidies. Bian et al. (2020) compare the consumer and manufacturer subsidies for green products and find that consumer subsidies can gain higher social welfare than manufacturer subsidies. Yu et al. (2018) evaluate how both consumer and manufacturer subsidies improve consumer welfare and find that the efficiency of subsidies depends on the pricing strategies and attitudes of the government. Chemama et al. (2019) examine the optimal consumer subsidy under uncertain demands and find that a flexible subsidy policy is more expensive than a fixed subsidy on average and that a seller generally aims to maximise social welfare. Providing consumer subsidies is consistent with the real practice where governments, such as that of China, provide consumer subsidies for mask purchases to encourage consumers to wear mask in preventing transmission of the COVID-19.

The other perspective of government involvement is related to the social welfare maximisation-oriented sellers, which have been widely examined in the literature of supply chain management (Benjaafar et al. 2019; Hassin and Roet–Green 2017). Tang et al. (2015) investigate how the government provide market information and agricultural advice for farmers to maximize social welfare. Hassin and Roet–Green (2017) investigate the impacts of inspection cost on social welfare. They find the optimal solution for inspection under social welfare maximization. Benjaafar et al. (2019) compare the objectives of profit-maximizing and social-welfare maximizing on on-demand platforms. They find these two objectives play different role in collaborative consumption. Our paper also examines social welfare maximization because this is the true objective for the government during the COVID-19 outbreak. We compare these two perspectives and identify which of them reigns supreme during the COVID-19 outbreak.

Healthcare supply chain management has recently emerged as a hot research topic (Zhao et al. 2012). Arifoğlu et al. (2012) study the influenza vaccine supply chain with yield uncertainty and find that the incentives of manufacturers can increase the equilibrium demand of these vaccines. Adida et al. (2013) examine the operational issues in using vaccines to prevent the spread of infectious diseases and find that consumer subsidies can effectively achieve a socially optimal level. Yu et al. (2020) study the optimal subsidy programme in a three-echelon supply chain consisting of manufacturers, retailers and consumers and find that manufacturer, retailer and consumer subsidies do not differ if the total subsidy for all supply chain members is optimal. Arifoğlu and

Tang (2020) develop a decentralised flu vaccine supply chain by considering an uncertain production yield and find that the vaccination incentive for consumers can encourage them to purchase but does not increase product availability. Zhao and Kim (2021) investigate the impacts of personal protective equipment on combating the pandemic. They focus on adaptive behaviour of clothing companies from design and development to distribution after the crises subside. We are concerned about the selection of mask usage and the effective approach to combat the COVID-19 by using masks. In this paper, we consider a mask supply chain that aims to reduce the infection risk and improve social welfare during respiratory diseases (e.g., COVID-19) outbreak.

3. Model

The market consists of two firms that sell different types of masks with various infection probabilities. One firm sells N95 masks (n), which may provide wearers with a θ_n probability of catching respiratory diseases, and the other firm sells surgical masks (s), which provide wearers with a θ_s probability of infection. In reality, N95 masks offer wearers with nearly full protection from virus, whereas surgical masks are not as good as N95 masks in preventing infection. Therefore, we assume $\theta_s > \theta_n > 0$. We consider that consumers have three choices with regard to their consumption of masks during the respiratory disease outbreak, that is, these consumers may 1) purchase an N95 mask, 2) purchase a surgical mask and 3) purchase nothing. If consumers do not purchase masks, then they will face a probability θ of catching the virus. In this paper, we define θ as the IPWP and assume $\theta_s < \theta < 1$. In other words, if consumers do not wear masks, then they are likely to be infected. The infection rate is obtained from the public data being released by governments every day during the COVID-19 outbreak, including the number of cases and COVID-19 related deaths. These reports are being published by all countries affected by COVID-19.

The willingness of consumers to pay for masks is denoted by v, which follows a uniform distribution $v \in$

$[0, 1]$. We assume that each consumer purchases at most one unit of mask. The market size is denoted by M and is assumed to be equal to 1 for simplicity. Consumers are sensitive to the price and quality of masks. Mask quality may be measured by the probability of the wearer to be infected by respiratory diseases. The retail prices of N95 and surgical masks are denoted by p_n and p_s, respectively. Consumers are more willing to purchase masks that offer them a low infection rate. We consider the consumer utilities regarding N95 and surgical masks as follows.

$$U_n = (1 - \theta_n)v - p_n \text{ and}$$

$$U_s = (1 - \theta_s)v - p_s.$$

Recall that if consumers do not buy any mask, then they will face a probability θ of catching the virus. Therefore, their utility is $-\theta$. Consumers decide whether or not to purchase a mask based on their utilities. When $U_n > \max\{U_s, -\theta\}$, the consumer will purchase a N95 mask; when $U_s > \max\{U_n, -\theta\}$, the consumer will purchase a surgical mask; otherwise, the consumer will buy neither an N95 nor a surgical mask. By deriving the consumer utilities related to purchasing N95 masks, surgical masks and nothing, the demand functions of N95 and surgical masks can be expressed as $D_n = 1 - \frac{p_n - p_s}{\theta_s - \theta_n}$ and $D_s = \frac{p_n - p_s}{\theta_s - \theta_n} - \frac{p_s - \theta}{1 - \theta_s}$, respectively. The consumers $(1 - D_n - D_s) = \frac{p_s - \theta}{1 - \theta_s}$ will buy nothing. To ensure that the market demand for N95 and surgical masks are positive, we assume $D_n > 0$ and $D_s > 0$.

We consider that the production costs of N95 and surgical masks are c_n and c_s, where $c_n > c_s > 0$. The profits of firm *n* and *s* are

$$\pi_n(p_n) = (p_n - c_n)D_n \text{ and}$$

$$\pi_s(p_s) = (p_s - c_s)D_s.$$

By maximising the profits of these firms by deciding the prices p_n and p_s simultaneously, the optimal prices of N95 and surgical masks are $p_n^* = \frac{(2c_n + c_s)(1 - \theta_n) + (2 + \theta - 2\theta_n)(\theta_s - \theta_n)}{3 + \theta_s - 4\theta_n}$ and $p_s^* =$

$\frac{2c_s(1-\theta_n)+(1+2\theta-\theta_s)(\theta_s-\theta_n)+c_n(1-\theta_s)}{3+\theta_s-4\theta_n}$, respectively. Substituting the optimal prices into the market demand of N95 and surgical masks and profit functions yields

$$D_n^* = \frac{c_s(1-\theta_n)-(2+\theta-2\theta_n)(\theta_n-\theta_s)+c_n(2\theta_n-1-\theta_s)}{4\theta_n^2+\theta_s(3+\theta_s)-\theta_n(3+5\theta_s)}, \quad (3.1)$$

$$D_s^* = \frac{(1-\theta_n)[c_s(2\theta_n-1-\theta_s)+c_n(1-\theta_s)+(1+2\theta-\theta_s)(\theta_s-\theta_n)]}{(1-\theta_s)[4\theta_n^2+\theta_s(3+\theta_s)-\theta_n(3+5\theta_s)]}, \quad (3.2)$$

$$\pi_n^* = \frac{[c_s(\theta_n-1)+(2+\theta-2\theta_n)(\theta_n-\theta_s)+c_n(1-2\theta_n+\theta_s)]^2}{(\theta_s-\theta_n)(3-4\theta_n+\theta_s)^2} \text{ and} \quad (3.3)$$

$$\pi_s^* = \frac{(1-\theta_n)[c_s(2\theta_n-\theta_s-1)+c_n(1-\theta_s)-(1+2\theta-\theta_s)(\theta_n-\theta_s)]^2}{(1-\theta_s)(\theta_s-\theta_n)(3-4\theta_n+\theta_s)^2}. \quad (3.4)$$

Following Kaplan (2020), the probability of no virus transmission is observed when infected persons do not attend an event or when some infected persons attend such event but fail to transmit infection to others. We assume that the probability of no virus transmission is $Pr\{No\ Transmission\} = (1-\gamma)^M \approx (1-\gamma M)$, where each consumer has a probability γ of being infected with market size M. Given that market size is assumed to be 1, the probability of no infection amongst consumers who buy N95 masks is $D_n(1-\theta_n)^M \approx D_n(1-M\theta_n) = D_n(1-\theta_n)$, the probability of no infection amongst consumers who buy surgical masks is $D_s(1-\theta_s)^M \approx D_s(1-M\theta_s) = D_s(1-\theta_s)$ and the probability of no infection amongst consumers who are not wearing any masks is $(1-D_n-D_s)(1-\theta)^M \approx (1-D_n-D_s)(1-M\theta) = (1-D_n-D_s)(1-\theta)$.

Protecting users from the virus and other contagious diseases is the key purpose of wearing masks. To avoid infection, governments encourage their citizens to wear masks in public. We build a safety index SI to evaluate the efficiency of wearing masks. This index, which is expressed as $SI = D_n(1-\theta_n) + D_s(1-\theta_s) + (1-D_n-D_s)(1-\theta)$, consists of no infection amongst consumers who buy N95, buy surgical masks and buy nothing. A high safety index corresponds to a low IPWP after wearing masks. Therefore, using masks increases the safety index. We also propose consumer welfare (CW), which consists of consumer welfare from buying N95, buying surgical masks and buying nothing. Conventionally, social welfare (SW) consists of the profits of firms and consumer welfare. The safety index is also an important element of social welfare during the outbreak of respiratory diseases. For example, during the COVID-19 pandemic, all of the countries emphasize the effective ways to reduce the risk of COVID-19, i.e., improving the safety index. Therefore, social welfare includes the safety index, that is, $SW = (\pi_n + \pi_s) + CW + SI$. By substituting the optimal prices and the market demand in Equations (3.1) and (3.2), we obtain

$$SI^* = \frac{[(1-\theta)(c_s-\theta)-(\theta-\theta_n)(1-\theta_s)](6-8\theta_n+2\theta_s)+2(1-\theta_s)(6+3\theta-2c_n-c_s-6\theta_n)(1+\theta-2\theta_n)}{4\Phi(1-\theta_s)}, \tag{3.5}$$

$$CW^* = \frac{\{[9(\theta-c_s)^2-72\theta(1-\theta_s)]\Phi^2+\Upsilon(1-\theta_s)(3\theta-30+2c_n-5c_s+30\theta_n)\}(\theta_s-\theta_n)+24(1-\theta_n)\Upsilon^2+4(1-\theta_s)(c_n-c_s)^2\Phi^2}{72\Phi^2(\theta_s-\theta_n)(1-\theta_s)} \text{ and} \tag{3.6}$$

$$SW^* = \frac{(1-\theta_s)[c_s(1-\theta_n)-(2+\theta-2\theta_n)(\theta_n-\theta_s)-\Omega c_n]^2+(1-\theta_n)[c_n(1-\theta_s)+\Omega(\theta_s-c_s-\theta_n)]^2}{\Phi^2(1-\theta_s)(\theta_s-\theta_n)} + SI^* + CW^*, \tag{3.7}$$

where $\Phi = 3 - 4\theta_n + \theta_s$, $\Upsilon = (6 + 3\theta - 2c_n - c_s - 6\theta_n)$, $\Omega = 1 - 2\theta_n + \theta_s$.

Proposition 1.

(i) $p_n^* > p_s^*$.

(ii) p_n^*, p_s^*, D_n^*, D_s^*, π_n^* *and* π_s^* *are increasing with* θ.

(iii) $D_n^* > D_s^*$ *if and only if* θ *is sufficiently low, and* $D_n^* - D_s^*$ *is decreasing in* θ.

Proposition 1 is consistent with the real practice during the COVID-19 outbreak. N95 masks are more expensive than surgical ones because of their better quality. For example, the prices of N95 and surgical masks during the first wave of the virus in Europe from March to April 2020 were at least 50% higher than their levels in February. Specifically, N95 masks cost 17 Euro per piece, whereas surgical masks cost 1 Euro. The WHO recommends citizens to wear masks in public after whisper of COVID-19 (WHO 2020). Therefore, to protect themselves from the virus, an increasing number of consumers will purchase masks, which drive up the prices of these materials along with their sales quantity. In other words, a high infection rate corresponds to high sales and prices. This result contradicts the arguments of standard economic theory, which posits that sales quantity is negatively influenced by price. However, this result is consistent with the actual situation of the COVID-19 pandemic. Specifically, although the prices of masks continuously increase, consumers continue to purchase them when the IPWP increases. Moreover, masks retailers gain much profit during the pandemic. The high sales quantities and retail prices of N95 and surgical masks lead to increased profits for firms.

Interestingly, Proposition 1(iii) argues that when the IPWP is sufficiently low, consumers who want to buy masks are more likely to purchase N95 masks than surgical ones. This argument holds true during the early stage of the pandemic, that is, the IPWP is relatively low. Those consumers who want to buy masks are very concerned about being infected, hence driving their willingness to pay more for N95 masks. However, when the IPWP is high, masks become a necessity and consumers become highly price sensitive, thereby driving their decision to purchase surgical masks instead of N95 ones. Surgical masks can serve a better purpose of reducing the risk of being infected during the COVID-19 outbreak because they are more affordable than N95 masks. In practices, China purchased and supplied the large quantity of N95 in January 2020 locally and globally (Li and Li, 2020). This helps control and slow down the spread of COVID-19 in China afterwards. In the peak period of transmission, China was supplying the sufficient inventory of surgical masks which had sold over 13 billion in Tmall until April 2020 (Li and Li 2020). This helps eliminate the spread of COVID-19 in China after May.

Proposition 2. *(i)* CW^* *is decreasing in* θ. *(ii)* SI^* *and* SW^* *are convex in* θ.

Proposition 2(i) suggests that a high IPWP negatively affects consumer welfare. This result is consistent with the reality given the disastrous effects of COVID-19 on people's lives. From Proposition 1, we can see that when the IPWP θ is high, the demand and price of masks are also high. With a high IPWP, the retail price of masks increases, which is detrimental to consumer welfare.

The safety index exhibits a U-shaped relationship with IPWP, thereby suggesting that when the IPWP is relatively low, the risk of infection is relatively low even though only a small number of consumers are buying and wearing masks. Meanwhile, when the IPWP is relatively high, a large number of consumers start to buy and wear masks. The potential risk of being infectious is also low in this case because many consumers have protective equipment. Wearing masks can effectively prevent and control cross infection. Therefore, when the IPWP is very high, consumers naturally pay attention to their safety and health, and hence will buy and wear masks regardless of their prices.

The potential risk of being infected is high when the IPWP is moderate. The safety index is critically high in cases where many consumers still do not buy and wear masks. Our results are consistent with real practice. The carefulness of consumers is the major driver of reduction in the safety index. Our result indicates that when the IPWP is moderate, consumers should be encouraged to wear masks. Therefore, our results imply that when the risk of COVID-19 is increasing (i.e., the number of infections is climbing), the government should impose a mandatory mask-wearing policy. Many countries such as China, Singapore, and Japan mandedately require people to wear masks in the public (Mandal 2020). These countries had properly managed the spread of virus. However, if the governments do not provide mandatory policy. The situation of virus spread will be challenging. For example, India government did not mandedately requrie people to wear masks in public events in April 2021. The situation of virus spread was relatively out of control afterwards (Schemm et al. 2021).

The sellers' profits increase in the IPWP, whereas consumer welfare decreases in the IPWP. After combining the effects of the safety index, social welfare initially decreases and then increases in the IPWP. This result implies that, on the one hand, when the IPWP increases, consumers are endangering themselves by not wearing masks. As a result, both consumer welfare and safety index decrease. On the other hand, with a high IPWP, consumers become more willing to purchase masks. As a result, both profits and social welfare increase.

4. Horizontal integration during the outbreak of respiratory diseases

In the previous section, we consider two independent firms selling N95 and surgical masks. Meanwhile, in this section, we consider that an integrated firm sells both N95 and surgical masks simultaneously. This assumption is also realistic because the well-established mask producers (e.g., 3M) sell both types of masks, but some companies produce only surgical masks (due to the technology limitation to produce N95) during the COVID-19 pandemic.

We use Π to represent the profit of the integrated firm and $-$ to represent the optimal solutions of this integrated firm. The profit function of the supply chain is $\Pi(\bar{p}_n, \bar{p}_s) = (\bar{p}_n - c_n)\overline{D}_n + (\bar{p}_s - c_s)\overline{D}_s$. By maximising the supply chain's profit, we obtain the optimal prices for N95 and surgical masks as $\bar{p}_n^* = \frac{1}{2}(1 + \theta + c_n - \theta_n)$ and $\bar{p}_s^* = \frac{1}{2}(1 + \theta + c_s - \theta_s)$, respectively. Substituting the optimal prices into the market demand for N95 and surgical masks and profit functions of firms $\bar{\pi}_n^* = (\bar{p}_n - c_n)\overline{D}_n^*$, and $\bar{\pi}_s^* = (\bar{p}_s^* - c_s)\overline{D}_s^*$ yields

$$\overline{D}_n^* = \frac{c_s - c_n - \theta_n + \theta_s}{2(\theta_s - \theta_n)}, \tag{4.1}$$

$$\overline{D}_s^* = \frac{c_n - c_s - \theta\theta_n + c_s\theta_n + \theta\theta_s - c_n\theta_s}{2(1-\theta_s)(\theta_s - \theta_n)}, \tag{4.2}$$

$$\bar{\pi}_n^* = \frac{(c_n + \theta_n - 1 - \theta)(c_n - c_s + \theta_n - \theta_s)}{4(\theta_s - \theta_n)}, \tag{4.3}$$

$$\bar{\pi}_s^* = \frac{(1 + \theta - c_s - \theta_s)(c_n - c_s - \theta\theta_n + c_s\theta_n + \theta\theta_s - c_n\theta_s)}{4(1-\theta_s)(\theta_s - \theta_n)} \text{ and} \tag{4.4}$$

$$\Pi^* = \frac{c_s^2(1-\theta_n) - c_n(2-c_n)(1-\theta_s)(c_s - \theta_n + \theta_s) + (\theta_n - \theta_s)[2\theta c_s + \theta_n + (6\theta - 1 + \theta_n)(1-\theta_s)]}{4(1-\theta_s)(\theta_s - \theta_n)}. \tag{4.5}$$

Proposition 3. *(i)* $p_n^* < \bar{p}_n^*,\ p_s^* < \bar{p}_s^*$. *(ii)* $D_n^* > \overline{D}_n^*,\ D_s^* > \overline{D}_s^*$.

We find that horizontal integration increases the retail price of N95 and surgical masks yet reduces their demand. These results are consistent with the literature on horizontal integration in supply chains (Xu et al. 2019), that is, horizontal integration can mitigate market competition and drive performance improvements amongst firms. In other words, a market competition induces firms to sell products at low prices to increase their market potential.

Proposition 4.

(i) When the production cost of N95 masks is sufficiently high and that of surgical masks is sufficiently low, $\pi_n^* \geq \bar{\pi}_n^*$ *and* $\pi_s^* < \bar{\pi}_s^*$*; when the production cost of N95 is sufficiently low and that of surgical masks is sufficiently high,* $\pi_n^* < \bar{\pi}_n^*$ *and* $\pi_s^* \geq \bar{\pi}_s^*$*.*

(ii) When the production cost difference between N95 and surgical masks is small, $\pi_n^* + \pi_s^* < \Pi^*$*.*

Proposition 4(i) suggests that horizontal integration can improve the total channel profit from selling both N95 and surgical masks when the production cost difference between these masks is small mainly because integration can mitigate cannibalisation and reduce retail competition when the production costs are relatively equal. However, the profit from selling N95 masks is influenced not only by the price and demand for such products but also by the price and demand for surgical masks. If we look at the individual profit from selling one specific mask as shown in Proposition 4(ii), we can see that the differences in production cost play an important role in channel profitability. Intuitively, the production cost of N95 masks is higher than that of surgical masks. When the production cost of N95 masks is sufficiently high and that of surgical masks is sufficiently low, the production cost difference between these two types of masks is large, that is, surgical masks are price competitive in the market. By contrast, when the production cost of N95 masks is sufficiently low and that of surgical masks is sufficiently high, the production cost difference between these masks is low, hence indicating a sufficiently fierce competition between them. Consumers may be more willing to purchase an N95 mask given its highly competitive price in the market due to the retail competition effect, which can increase economic benefits when a large price advantage is present in the market.

In horizontal integration, the safety index, consumer welfare and social welfare are

$$\overline{SI}^* = \frac{(c_s-\theta)(1-\theta)-(\theta_n+c_n-2)(1-\theta_s)}{2(1-\theta_s)}, \quad (4.6)$$

$$\overline{CW}^* = \frac{c_s^2(1-\theta_n)-c_n(2-c_n)(1-\theta_s)(c_s-\theta_n+\theta_s)+(\theta_n-\theta_s)[\theta(2c_s-\theta)+(6\theta-1+\theta_n)(1-\theta_s)]}{8(1-\theta_s)(\theta_s-\theta_n)} \text{ and} \quad (4.7)$$

$$\overline{SW}^* = \frac{c_s^2(1-\theta_n)+(\theta_n-\theta_s)[(1+\theta)^2-2\theta c_s-\theta_n-(1+2\theta-\theta_n)\theta_s]+c_n(1-\theta_s)[2(c_s-\theta_n+\theta_s)-c_n]}{8(1-\theta_s)(\theta_s-\theta_n)} + \overline{SI}^* + \overline{CW}^*. \quad (4.8)$$

Proposition 5.

(i) $SI^* > \overline{SI}^*$ *and* $CW^* > \overline{CW}^*$.

(ii) $SW^* > \overline{SW}^*$ *if* $\theta > \theta_2$; *otherwise*, $SW^* \leq \overline{SW}^*$.

Proposition 5(i) suggests that a high demand for masks corresponds to high safety index and consumer welfare. With horizontal integration, the supply chain increases its retail prices, thereby reducing the incentive for consumers to purchase masks and subsequently reducing the safety index and consumer welfare. This result implies that horizontal integration does not effectively improve safety index and consumer welfare. This case is especially important during the COVID-19 pandemic because horizontal integration increases the prices of N95 and surgical masks yet reduces their sales quantity (as shown in Proposition 3).

Proposition 5(ii) suggests that when the IPWP is sufficiently low, social welfare can perform better with integration, whereas when the IPWP is high, social welfare with horizontal integration is lower than that with non-integration. Surprisingly, the results for social welfare contradict those for safety index and consumer welfare in terms of horizontal integration. Specifically, when the IPWP is low, horizontal integration can effectively improve social welfare because of the large prices and profits of masks, which can cover the gains of safety index and consumer welfare from non-integration. Our results provide important insights into supply chain integration during the COVID-19 outbreak. From the perspectives of the safety index and consumer welfare, horizontal integration is not a wise strategy. From the social welfare perspective, horizontal integration is only effective when the IPWP is sufficiently low. Previous studies show that horizontal integration can effectively improve supply chain performance (Orsdemir et al. 2019; Xu et al. 2019; Yang et al. 2017). However, we find that during the COVID-19 pandemic, horizontal integration is not as good as non-integration in improving safety index and consumer welfare but may benefit the improvement of social welfare. Thus, to improve safety index and consumer welfare, the government should not promote the mask producers (e.g., 3M) which sell both types of masks during the COVID-19. Instead, the government should promote the mask producers which sell surgical masks only. For example, promoting companies which produce surgical masks only during the COVID-19 pandemic can improve safety index and consumer welfare.

5. Government involvement during the COVID-19 outbreak

Governments play an important role in preventing and controlling the spread of respiratory diseases (e.g., SARS, H1N1, and COVID-19). Governments all over the world have adopted various schemes to decelerate the spread of the virus. For example, the governments of many countries, such as China, offer subsidies to consumers to encourage mask purchase (Changji Government News 2020). Meanwhile, the government of Singapore plays the role of a mask seller (Li and Heijmans 2020). In this section, we compare the effects of government involvement during the spread of respiratory diseases.

Offering Consumer Subsidies

The government offers subsidies to those consumers who are purchasing either N95 or surgical masks. Firstly, the government aims to maximise social welfare by deciding the amount of subsidies (τ_n and τ_s for N95 and surgical masks, respectively) to be given to consumers when buying these masks. Secondly, firms maximise their profits by deciding the prices p_n and p_s simultaneously. We use GS to represent the government offering subsidies to consumers. The consumer utilities of N95 and surgical masks are defined as

$$U_n^{GS} = (1-\theta_n)v - p_n^{GS} + \tau_n, \text{ and}$$

$$U_s^{GS} = (1-\theta_s)v - p_s^{GS} + \tau_s.$$

Following the approach described in the above sections, when $U_n^{GS} > \max\{U_s^{GS}, -\theta\}$, the consumer will purchase an N95 mask; when $U_s^{GS} > \max\{U_n^{GS}, -\theta\}$, the consumer will purchase a surgical mask; otherwise, the consumer will buy nothing. The market demands for N95 and surgical masks are $D_n^{GS} = 1 - \frac{p_n^{GS} - p_s^{GS} - \tau_n + \tau_s}{\theta_s - \theta_n}$

and $D_s^{GS} = \frac{p_n^{GS} - p_s^{GS} - \tau_n + \tau_s}{\theta_s - \theta_n} - \frac{p_s^{GS} - \theta - \tau_s}{1 - \theta_s}$, respectively, and the profits of firms *n* and *s* are

$$\pi_n^{GS}(p_n^{GS}) = (p_n^{GS} - c_n)D_n^{GS} \text{ and}$$

$$\text{and } \pi_s^{GS}(p_s^{GS}) = (p_s^{GS} - c_s)D_s^{GS}.$$

Social welfare in this case is defined as $SW(\tau_n, \tau_s) = (\pi_n^{GS} + \pi_s^{GS}) + CW^{GS} + SI^{GS} - (\tau_n D_n^{GS} + \tau_s D_s^{GS})$, where $(\tau_n D_n^{GS} + \tau_s D_s^{GS})$ is the subsidy offered by the government to consumers.

By maximising social welfare, the optimal subsidies offered to consumers who purchase N95 and surgical masks are $\tau_n^* = \theta - c_n + c_s - 3\theta_n + 2\theta_s$ and $\tau_s^* = \frac{\theta - c_s(1-\theta_n) + \theta_n - 3\theta\theta_n + c_n(1-\theta_s) - (2 - 2\theta - \theta_n)\theta_s}{1 - \theta_n}$, respectively. To maximise profits, the optimal prices of N95 and surgical masks are $p_n^{GS*} = c_s - 2\theta_n + 2\theta_s$ and $p_s^{GS*} = \frac{(1-2\theta)(\theta_n - \theta_s) + c_n(1-\theta_s)}{1-\theta_n}$, respectively. Substituting the optimal subsidies and prices into the demand for N95 and surgical masks and the profit functions yields

$$D_n^{GS*} = \frac{c_s - c_n - 2\theta_n + 2\theta_s}{\theta_s - \theta_n}, \tag{5.1}$$

$$D_s^{GS*} = \frac{c_n - c_s + \theta_n(1 - 2\theta + c_s) - \theta_s(1 - 2\theta + c_n)}{(1-\theta_s)(\theta_s - \theta_n)}, \tag{5.2}$$

$$\pi_n^{GS*} = \frac{(c_n - c_s + 2\theta_n - 2\theta_s)^2}{\theta_s - \theta_n}, \tag{5.3}$$

$$\pi_s^{GS*} = \frac{[c_n - c_s + \theta_n(1 - 2\theta + c_s) - \theta_s(1 - 2\theta + c_n)]^2}{(1-\theta_s)(1-\theta_n)(\theta_s - \theta_n)}, \tag{5.4}$$

$$SI^{GS*} = \frac{1 + 2\theta^2 - 2\theta_n + c_s(1-\theta) - c_n(1-\theta_s) + \theta_s(2\theta_n - 3\theta)}{1-\theta_s}, \tag{5.5}$$

$$CW^{GS*} = \frac{c_s[2(2\theta-1)(\theta_n - \theta_s) + c_s(1-\theta_n)] - c_n(1-\theta_s)[2(c_s - 2\theta_n + 2\theta_s) - c_n] - (\theta_n - \theta_s)[1 + 2\theta(1 + 2\theta - 3\theta_s) - 4\theta_n(1-\theta_s)]}{2(1-\theta_s)(\theta_s - \theta_n)}, \tag{5.6}$$

$$SW^{GS*} = \frac{(2\theta_n + c_n)(1-\theta_s) + \theta_s(4\theta - 1) - \theta(1 + 2\theta) - c_s(1-\theta)}{1-\theta_s} + SI^{GS*} + CW^{GS*}. \tag{5.7}$$

Proposition 6.

(i) For the prices of N95 and surgical masks, (a) $p_n^* < p_n^{GS*}$*; (b) If* $\theta > \theta_3$*, then* $p_s^* < p_s^{GS*}$*; otherwise,* $p_s^* \geq p_s^{GS*}$.

(ii) For the sales quantities of N95 and surgical masks, (a) $D_n^* < D_n^{GS*}$*; (b) If* $\theta > \theta_3$*, then* $D_s^* < D_s^{GS*}$*; otherwise,* $D_s^* \geq D_s^{GS*}$.

When the government offers subsidies to consumers, the retail price of N95 masks increases. Compared with the case where a consumer subsidy is not provided, if the IPWP is relatively high, then the retail price of surgical masks is high when consumer subsidies are issued. However, if the IPWP is relatively low, then the retail price of surgical masks is also low when consumer subsidies are issued. If the government offers a subsidy to those consumers who buy masks, then these consumers become willing to purchase high-quality masks. Therefore, N95 masks are more desirable than surgical masks in this case.

Subsidising consumers can generate a higher sales quantity for N95 masks than for surgical masks. This result is intuitive because providing consumer subsidies implies that consumers can spend less in purchasing products (Yu et al. 2018). If the IPWP is relatively high, subsidising consumers can also increase the sales quantity of surgical masks. If the IPWP is relatively low, then consumers have a lower incentive to purchase surgical masks when the government offers subsidies. The sales quantities of N95 and surgical masks are influenced by different factors, such as price and IPWP.

Proposition 7.

(i) $\pi_n^* < \pi_n^{GS*}$.

(ii) *If* $\theta > \theta_3$, *then* $\pi_s^* < \pi_s^{GS*}$; *otherwise,* $\pi_s^* \geq \pi_s^{GS*}$.

N95 mask sellers benefit from the government subsidy scheme. Proposition 7(ii) confirms that under certain conditions, if the IPWP is relatively high, then selling surgical masks is more desirable when government subsidies are available than the case that they are not provided. Meanwhile, if the IPWP is relatively low, then selling surgical masks is more desirable when such subsidies are not offered than the case that they are provided.

Proposition 8.

(i) *If* $\theta > \theta_4$, *then* $SI^* < SI^{GS*}$; *otherwise,* $SI^* \geq SI^{GS*}$.

(ii) $CW^{GS*} > CW^*$ *and* $SW^{GS*} > SW^*$.

If the IPWP is relatively high, then subsidising consumers will increase the safety index. Under this circumstance, if the government wants to increase the safety index, then they should provide consumers with a subsidy. Our result provides important insights into how the government prevents and controls the spread of COVID-19 even though the IPWP is critically high. Moreover, if the IPWP is relatively low, then subsidising consumers is not necessary because doing so would not improve the safety index. Our result confirms that consumer subsidies can effectively improve the safety index if the IPWP is relatively high. Given that a relatively high IPWP will threaten community security, the government should offer subsidies to consumers and improve the safety index.

Consumer and social welfare can benefit from government subsidies. This result provides important insights for governments who want to improve consumer and social welfare during the COVID-19 outbreak.

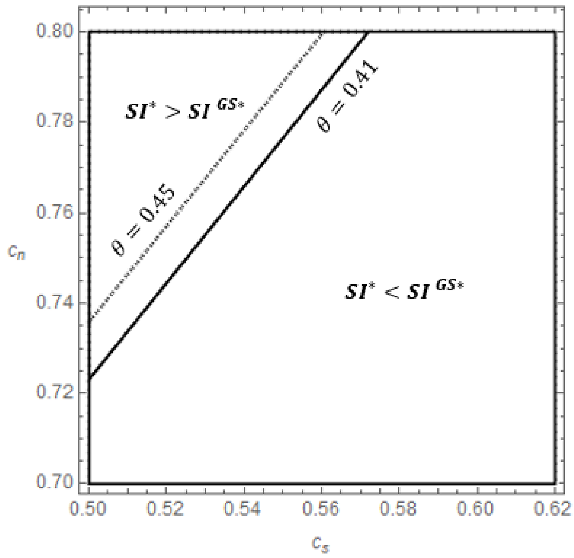

Figure 1. Safety index with respect to the production costs of N95 and surgical masks.

We let $\theta_n = 0.3$ and $\theta_s = 0.4$ in Figure 1 and find that the subsidy scheme increases the safety index if the production cost of N95 masks is sufficiently low and that of surgical masks is sufficiently high. With a sufficiently high production cost, N95 masks have a sufficiently high price and, in turn, a very low sales quantity. Meanwhile, when the production cost of surgical masks is sufficiently low, their price will be sufficiently low, whereas their sales quantity will be very high. Although consumer subsidies can partially compensate for the prices of N95 masks, they cannot improve the safety index because this index considers the total prevention of consumers who buy N95, buy surgical masks and buy nothing. Consumers can receive subsidies when purchasing either N95 or surgical masks. However, when the production costs of N95 and surgical masks are relatively similar, the consumer subsidy scheme can effectively improve the safety index. When the IPWP tends to be high, the production cost difference between these two masks is high. Offering subsidies will be detrimental to the safety index because they may fail to improve the demand for surgical masks.

Promoting the Social Mask Enterprise

We consider a scenario where the mask enterprise is socialised, that is, this enterprise aims to maximise social welfare. This scenario is realistic in cases where the mask enterprise is owned by the government, which holds true during the COVID-19 outbreak because many government-owned (GO) companies (equal to social enterprise) have started to sell masks in the market (Li and Heijmans 2020). GO firms aim for social welfare maximisation by deciding the optimal retail prices of N95 and surgical masks. We use GO to refer to this case. The social welfare for a GO firm is

$$SW^{GO}(p_n^{GO}, p_s^{GO}) = (\pi_n^{GO} + \pi_s^{GO}) + CW^{GO} + SI^{GO}.$$

The optimal prices of N95 and surgical masks can be derived as $p_n^{GO*} = c_n + \theta_n - \theta$ and $p_s^{GO*} = c_s + \theta_s - \theta$, respectively. Substituting the optimal prices into the market demand for N95 and surgical masks and profit functions yields

$$D_n^{GO*} = \frac{c_s - c_n - 2\theta_n + 2\theta_s}{\theta_s - \theta_n}, \tag{5.8}$$

$$D_s^{GO*} = \frac{c_n - c_s + \theta_n(1 - 2\theta + c_s) - \theta_s(1 - 2\theta + c_n)}{(\theta_s - \theta_n)(1 - \theta_s)}, \tag{5.9}$$

$$\pi_n^{GO*} = \frac{(\theta - \theta_n)(c_s - c_n - 2\theta_n + 2\theta_s)}{\theta_s - \theta_n}, \tag{5.10}$$

$$\pi_s^{GO*} = \frac{(\theta_s - \theta)[c_n - c_s + \theta_n(1 - 2\theta + c_s) - \theta_s(1 - 2\theta + c_n)]}{(1 - \theta_s)(\theta_s - \theta_n)}, \tag{5.11}$$

$$SI^{GO*} = \frac{1+2\theta^2-2\theta_n+c_s(1-\theta)-c_n(1-\theta_s)+\theta_s(2\theta_n-3\theta)}{1-\theta_s}, \tag{5.12}$$

$$CW^{GO*} = \frac{c_s[2(2\theta-1)(\theta_n-\theta_s)+c_s(1-\theta_n)]-c_n(1-\theta_s)[2(c_s-2\theta_n+2\theta_s)-c_n]-(\theta_n-\theta_s)[1+2\theta(1+2\theta-3\theta_s)-4\theta_n(1-\theta_s)]}{2(\theta_s-\theta_n)(1-\theta_s)}, \tag{5.13}$$

$$SW^{GO*} = \frac{(2\theta_n+c_n)(1-\theta_s)+\theta_s(4\theta-1)-\theta(1+2\theta)-c_s(1-\theta)}{1-\theta_s} + SI^{GO*} + CW^{GO*}. \tag{5.14}$$

Proposition 9.

(i) $p_n^{GO*} < c_n < \bar{p}_n^*$, *and* $p_s^{GO*} < c_s < \bar{p}_s^*$.

(ii) $D_n^{GO*} > \overline{D}_n^*$.

(iii) If $\theta < \theta_5$, *then* $D_s^{GO*} < \overline{D}_s^*$; *otherwise,* $D_s^{GO*} \geq \overline{D}_s^*$.

Surprisingly, when the masks are provided by the social mask enterprise, their prices are lower than their production costs. In other words, to maximise social welfare, a social mask enterprise will sacrifice its profit to encourage more purchases from consumers. With sufficiently low prices, the market demand for these masks is naturally high. This case holds true for N95 masks. The social mask enterprise will have the higher market demand of N95 masks than that of the profit maximisation enterprise. Proposition 9(iii) suggests that when the IPWP is sufficiently low, the demand for surgical masks from a social mask enterprise is lower than that from a profit-oriented seller. When the prices of N95 and surgical masks are sufficiently low, consumers are more willing to buy N95 masks because of their better quality compared with surgical masks. Consumers will select masks based on their price and quality. Eventually, they may or may not choose to purchase surgical masks.

Proposition 10. *(i)* $\pi_n^{GO*} < \bar{\pi}_n^*$, $\pi_s^{GO*} < \bar{\pi}_s^*$, *and* $\pi_n^{GO*} + \pi_s^{GO*} < \bar{\Pi}^*$. *(ii)* $SI^{GO*} > \overline{SI}^*$, $CW^{GO*} > \overline{CW}^*$, *and* $SW^{GO*} > \overline{SW}^*$.

Proposition 10 captures a scenario where profit maximisation is more desirable than social welfare maximisation given that the retail price of N95 and surgical masks is lower in the case where the masks are sold by the social mask enterprise instead of profit-oriented firms. The social mask enterprise generates a high demand for N95 masks, thereby reducing the demand for surgical masks if the IPWP is relatively low.

Our results show that promoting the social mask enterprise can increase the safety index because they can attract a larger number of N95 and surgical mask buyers when the IPWP is high. Pursuing social welfare maximisation is more beneficial than pursuing profit maximisation to the safety index. As the IPWP increases, using the social mask enterprise is more beneficial to consumer and social welfare compared with horizontal integration. These findings suggest that during the COVID-19 pandemic, the government can develop a social mask enterprise to improve the safety index, consumer welfare and social welfare.

Proposition 11.

(i) $p_n^{GO*} < p_n^{GS*}$, and $p_s^{GO*} < p_s^{GS*}$.

(ii) $\pi_n^{GO*} < \pi_n^{GS*}$, and $\pi_s^{GO*} < \pi_s^{GS*}$.

(iii) $D_n^{GO*} = D_n^{GS*},\ D_s^{GO*} = D_s^{GS*},\ SI^{GO*} = SI^{GS*},\ CW^{GO*} = CW^{GS*}, and\ SW^{GO*} = SW^{GS*}$.

The profits of selling N95 and surgical masks are higher under the case where the government offers consumer subsidies than those under the case where a social mask enterprise provides masks to consumers. This case is partially due to the high retail prices of N95 and surgical masks in the case of offering consumer subsidies. Moreover, the sales quantities of N95 and surgical masks, safety index, consumer welfare and social welfare under the case where government subsidies are offered to consumers are equal to those under the case where a social mask enterprise provides masks to consumers. These results imply that the efficiencies of the social mask enterprise and consumer subsidies in improving the safety index, consumer welfare and social welfare are equivalent. If governments are unable to sell masks through the social mask enterprise, then they can issue consumer subsidies instead. By contrast, if the government is unable to provide consumer subsidies, then they can develop a social mask enterprise whose objective is to maximise social welfare. However, from the seller's profit perspective, providing consumer subsidies is more desirable because the seller can charge higher prices.

6. Conclusions

In this paper, we focus on examining how masks should be effectively managed and sold during a health crisis. We examine the interaction between surgical and N95 masks in the market during the spread of respiratory diseases. We assume that consumers can choose between surgical and N95 masks. Consistent with real practice, wearing an N95 mask offers better protection against respiratory diseases than wearing a surgical mask. To encourage people to buy and use surgical masks during the outbreak of respiratory diseases, governments have mandated the wearing of masks in public areas and provided consumers with subsidies to purchase these masks at reduced prices. We explore the value of these subsidies in encouraging mask purchase. Moreover, we explore the value of promoting the social mask enterprise to combat respiratory diseases.

The presence of respiratory diseases is inevitable. Our paper mainly focuses on examining how masks should be effectively managed and sold during a health crisis. We show suggestions and real practices for combating COVID-19 in different stages of virus spread in Table 2 and Managerial insights with government involvement during the COVID-19 in Table 3. Our results provide important insights on combating COVID-19 and future respiratory diseases.

Table 2. Suggestions and real practices for combating the COVID-19 in different stages of virus spread

Stages of virus spread	Suggestions	Real practices
Peak (High IPWP)	Supplying the sufficient inventory of surgical masks	China is supplying the sufficient inventory of surgical masks which has sold over 13 billion in Tmall until April 2020 (Li and Li 2020). This help eliminate the spread of COVID-19 in China after May.
Climbing (Moderate IPWP)	Strongly Recommending citizens to use masks	WHO advises people to wear masks (WHO 2020). Many countries such as China, Singapore, and Japan mandedately require people to wear masks in the public (Mandal 2020). These countries had properly managed the spread of virus. India government did not mandedately requrie people to wear masks in public events in April 2021. The situation of virus spread was relatively out of control afterwards (Schemm et al. 2021).
After whisper (Low IPWP)	Supplying the sufficient inventory of N95	China purchased and supplied the large quantity of N95 in January 2020 (Li and Li, 2020). This helps control and slow down the spread of COVID-19 in China afterwards.

Table 3. Managerial insights with government involvement during the COVID-19

Government involvement	**Impacts**				**Managerial insights during the COVID-19**
	Profits	SI	CW	SW	
Government suggests horizontal integration	+ (Small cost difference)	−	−	+ (Low IPWP)	To improve safety index and consumer welfare, the government should not promote the mask producers (e.g., 3M) which sell both types of masks during the COVID-19. Instead, the government should promote the mask producers which sell surgical masks only. This kind of producers is existed because producing N95 has to meet the technology standard and get the licenses, not all producers have the permission to produce N95.
	− (Large cost difference)			− (High IPWP)	
Government provides the consumer subsidy	+ (High IPWP)	+ (High IPWP)	+	+	Providing the consumer subsidy can realize 4-wins (firms, safety, consumer, and society) when the IPWP is sufficiently large. In practice, the governments of many countries, such as China, offer subsidies to consumers to encourage mask purchase when the risk of COVID is high (Changji Government News 2020).
	− (Low IPWP)	− (Low IPWP)			
Government promotes the social mask enterprise	−	+	+	+	If the government cares about the safety index, consumer and social welfare, promoting the social mask enterprise is very effective. For example, in January 2020, Shanghai local government request some government-owned pharmacy to provide masks at a lower retail price (SMHC 2020)

Note. "+" implies the positive effect, "−" the negative effect".

Our findings provide several directions for future research. Firstly, we find that the government should invest in consumer subsidies and developing the social mask enterprise during the COVID-19 pandemic. However, some developing countries have limited resources and budget to adopt such schemes. Therefore, future studies may examine the effects of budget constraints on curbing COVID-19 with disruption risk (Li et al. 2020). Secondly, we consider a market where both N95 and surgical masks are being sold. In some countries such as the US, supplying N95 masks for frontline healthcare workers is crucial due to frequent occurrences of supply shortages. Future studies may explore the impacts of product shortage on the safety index and social welfare (Huang et al. 2018; Dolgui et al. 2019; Xu and Duan 2020). Thirdly, masks may have counterfeiting problem in the market. How disruptive technologies, such as blockchains, can be used to eliminate counterfeiting incidents can be investigated within the Industrial 4.0 framework (Dolgui et al. 2018; Dolgui et al. 2020a; Dolgui et al. 2020b; Ivanov et al. 2020).

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Appendix - All Proofs

Proof of Proposition 1.

In Proposition 1, we focus on the interplay between N95 and surgical masks. We subtract the retail price of surgical masks from that of N95 masks and obtain $p_n^* - p_s^* = \frac{c_n\Omega - c_s\lambda_2 + \lambda_4(\Omega-\theta)}{\Phi} > 0$. We are interested in the competition between N95 and surgical masks; therefore, we assume that $D_n^* > 0$, $D_s^* > 0$, $p_s^* > \theta$. Given $0 < \theta_n < \theta_s < \theta < 1$ and $c_n > c_s > 0$, we can verify that $\frac{\partial p_n^*}{\partial \theta} = \frac{\lambda_4}{\Phi} > 0$, $\frac{\partial p_s^*}{\partial \theta} = \frac{2\lambda_4}{\Phi} > 0$, $\frac{\partial D_n^*}{\partial \theta} = \frac{1}{\Phi} > 0$ and $\frac{\partial D_s^*}{\partial \theta} = \frac{2\lambda_2}{\lambda_3\Phi} > 0$. Moreover, we find $\frac{\partial \pi_n^*}{\partial \theta} = \frac{2[c_s\lambda_2 + (2+\theta-2\theta_n)\lambda_4 - c_n\Omega]}{\Phi^2} > 0$ and $\frac{\partial \pi_s^*}{\partial \theta} = \frac{4\lambda_2[(1+2\theta-\theta_s)\lambda_4 - c_n\lambda_3 - c_s\Omega]}{\lambda_3\Phi^2} > 0$.

We compare the quantities of N95 and surgical masks and find $D_n^* - D_s^* = \frac{\Upsilon - [3(\theta - c_s)\lambda_4 + 4(c_n - c_s)\lambda_3]\Phi}{6\lambda_3\lambda_4\Phi}$. If $\theta < \theta_1$, then consumers prefer N95 masks, that is, $D_n^* > D_s^*$; otherwise, surgical masks are more desirable, that is, $D_n^* \le D_s^*$. By taking the first-order derivative of the demand differentiation of N95 and surgical masks, we find that $\frac{\partial(D_n^* - D_s^*)}{\partial \theta} = \frac{4\theta_n - 2 - 2\theta_s}{2(1-\theta_s)\Phi} < 0$ for $0 < \theta_n < \theta_s < 1$. ■

Proof of Proposition 2.

By taking the first-order derivative of CW^*, we find $\frac{\partial CW^*}{\partial \theta} = \frac{(\theta - c_s - 4 - 4\theta_s)\Phi^2 + 8\Upsilon\lambda_2 - (4 - \theta + c_s - 4\theta_n)\Phi}{4\Phi^2} < 0$. It can be readily shown that consumer welfare is decreasing in θ. By taking the second-order derivative of SI^* and SW^*, with respect to θ, we obtain $\frac{\partial^2 SI^*}{\partial \theta^2} = \frac{2(3 - 2\theta_n - \theta_s)}{(1-\theta_s)\Phi} > 0$ and $\frac{\partial^2 SW^*}{\partial \theta^2} = \frac{27 + 28\theta_n^2 + (5 - 4\theta_s)\theta_s - \theta_n(59 - 3\theta_s)}{(1-\theta_s)\Phi^2} > 0$. As a result, SI^* and SW^* are convex in θ. ■

Proof of Proposition 3.

With horizontal integration, we also focus on the competition between N95 and surgical masks. Thus, we assume $\overline{D}_n^* > 0$, $\overline{D}_s^* > 0$, and $\overline{p}_s^* > \theta$. We compare the prices and demand for masks of non-integration with that for masks of integration and find $p_n^* - \overline{p}_n^* = \frac{3 + 3\theta - 2c_s\lambda_2 - \theta_n(3+2\theta) - c_n\lambda_3 - (3 + \theta - 3\theta_n)\theta_s}{8\theta_n - 2(3+\theta_s)} < 0$, $p_s^* - \overline{p}_s^* = \frac{(2c_n + c_s + 2\theta_n + \theta_s - 3 - 3\theta)\lambda_3}{6 - 8\theta_n + 2\theta_s} < 0$, $D_n^* - \overline{D}_n^* = \frac{c_n\lambda_3 - c_s(1 - 2\theta_n + \theta_s) + (1 + 2\theta - \theta_s)\lambda_4}{2(\theta_s - \theta_n)\Phi} > 0$ and $D_s^* - \overline{D}_s^* = \frac{c_s(1-\theta_n) + (2 + \theta - 2\theta_n)\lambda_4 - c_n\Upsilon}{2(\theta_s - \theta_n)\Phi} > 0$. Clearly, a higher retail price generates a lower demand for masks with integration.

■

Proof of Proposition 4.

From Equations (3.3) and (4.3), we subtract the profits of integration from those of non-integration and obtain

$$\pi_n^* - \bar{\pi}_n^* = \frac{\Phi^2(1+\theta-c_n-\theta_n)(c_n-c_s+\theta_n-\theta_s)-4\lambda_4[c_s\lambda_2+(2+\theta-2\theta_n)\lambda_4-c_n(1-2\theta_n+\theta_s)]^2}{4\lambda_4\Phi^2}.$$

If $c_n \geq \eta$, then $\pi_n^* \geq \bar{\pi}_n^*$; otherwise, $\pi_n^* < \bar{\pi}_n^*$.

From Equations (3.4) and (4.4), we have

$$\pi_s^* - \bar{\pi}_s^* = \frac{4\lambda_2[c_n\lambda_3-c_s(1-2\theta_n+\theta_s)+(1+2\theta-\theta_s)(\theta_s-\theta_n)]^2+\Phi^2(1+\theta-c_s-\theta_s)(c_n-c_s-\theta\theta_n+c_s\theta_n+\theta\theta_s-c_n\theta_s)}{4\lambda_3\lambda_4\Phi^2}.$$

If $c_s \geq \zeta$, then $\pi_s^* \geq \bar{\pi}_s^*$; otherwise, $\pi_s^* < \bar{\pi}_s^*$.

By combining the conditions in Propositions 1 and 3, we find that $\pi_n^* + \pi_s^* - \Pi^* = \frac{\Phi^2(1+\theta-c_n-\theta_n)(\Delta-\lambda_4)-4\lambda_4[c_s\lambda_2+(2+\theta-2\theta_n)\lambda_4-c_n\Omega]^2+4\lambda_2[c_n\lambda_3-c_s\Omega+(1+2\theta-\theta_s)\lambda_4]^2+\Phi^2(1+\theta-c_s-\theta_s)(\lambda_3c_n-\lambda_2c_s+\theta\lambda_4}{4\lambda_3\lambda_4\Phi^2} < 0.$

■

Proof of Proposition 5.

Using the same conditions used in Proposition 3, we compare safety index and consumer welfare in non-integration with those in integration. We obtain

$$SI^* - \overline{SI}^* = \frac{3\theta+3\theta^2-c_s(1+\theta-2\theta_n)-\theta_n-4\theta\theta_n+c_n(1-2\theta+\theta_s)-(2+2\theta-3\theta_n)\theta_s}{2\Phi} > 0 \text{ and}$$

$$CW^* - \overline{CW}^* = \frac{\Phi^2[9(1+2\theta-2c_n-\theta_n)\lambda_4-5\Delta^2]+\Upsilon\lambda_4[24\Upsilon\lambda_2+\Phi(3\theta-30+2c_n-5c_s+30\theta_n)]}{72\Phi^2\lambda_4} > 0.$$

The difference between social welfare for non-integration and that for integration is

$$SW^* - \overline{SW}^* = \frac{\Phi^2[9\lambda_4(10c_n+23\theta_n-19-14\theta)-7(c_n-c_s)^2]-\lambda_4\Upsilon[24\Upsilon\lambda_2-\Phi(138+69\theta-26c_n-7c_s-174\theta_n)]}{72\Phi^2\lambda_4}.$$

If $\theta > \theta_2$, then $SW^* > \overline{SW}^*$; otherwise, $SW^* \leq \overline{SW}^*$.

It is readily shown that the integration of a firm reduces safety index and consumer welfare yet increases social welfare if the IPWP is relatively low. ■

Proof of Propositions 6 and 7.

Given that $D_n^{GS*} > 0$, $D_s^{GS*} > 0$ and $p_s^{GS*} > \theta$ are assumed, we compare the retail prices, demand and profits of N95 masks with a subsidy scheme with those without a subsidy scheme and find

$$p_n^* - p_n^{GS*} = \frac{2c_n\lambda_2-(4-\theta-6\theta_n+2\theta_s)\lambda_4-c_s(2-3\theta_n+\theta_s)}{\Phi} < 0,$$

$$D_n^* - D_n^{GS*} = \frac{2c_n\lambda_2-(4-\theta-6\theta_n+2\theta_s)\lambda_4-c_s(2-3\theta_n+\theta_s)}{(\theta_s-\theta_n)\Phi} < 0 \text{ and}$$

$$\pi_n^* - \pi_n^{GS*} = \frac{36\Phi^2(c_n-c_s+\theta_n-\theta_s)\lambda_4-8(c_n-c_s)^2-\Upsilon\lambda_4[3\Upsilon\lambda_2-\Phi(\Upsilon-2c_n)]}{9(\theta_s-\theta_n)\Phi^2} < 0.$$

According to the optimal solution of firm s, the difference between the retail price, demand and profits of surgical masks with a subsidy and those without a subsidy are

$$p_s^* - p_s^{GS*} = \frac{2c_s{\lambda_2}^2-c_n(2-3\theta_n+\theta_s)\lambda_3+\lambda_4[4-4\theta-2\theta\theta_s-\theta_n(5-6\theta-\theta_s)]}{(1-\theta_n)\Phi},$$

$$D_s^* - D_s^{GS*} = \frac{2c_s{\lambda_2}^2-c_n(2-3\theta_n+\theta_s)\lambda_3+\lambda_4[4-4\theta-2\theta\theta_s-\theta_n(5-6\theta-\theta_s)]}{\lambda_3\lambda_4\Phi} \text{ and}$$

$$\pi_s^* - \pi_s^{GS*} = \frac{4\lambda_2[c_s(2\theta_n-1-\theta_s)+c_n\lambda_3+(1+2\theta-\theta_s)\lambda_4]^2+\Phi^2(1+\theta-c_s-\theta_s)(\lambda_3c_n-\lambda_2c_s+\theta\lambda_4)}{4\lambda_3\lambda_4\Phi^2}.$$

If $\theta > \theta_3$, then $p_s^* < p_s^{GS*}$, $D_s^* < D_s^{GS*}$ and $\pi_s^* < \pi_s^{GS*}$; otherwise, $p_s^* \geq p_s^{GS*}$, $D_s^* \geq D_s^{GS*}$ and $\pi_s^* \geq \pi_s^{GS*}$. ■

Proof of Proposition 8.

Using the above conditions, we obtain $SI^* - SI^{GS*} = \frac{\Phi[(c_n+3\theta_n-4\theta)-\lambda_1(2-3\theta+c_s)]+\Upsilon\lambda_3(1+\theta-2\theta_n)}{2\Phi\lambda_3}$. Thus, if $\theta > \theta_4$, then $SI^* < SI^{GS*}$; otherwise, $SI^* \geq SI^{GS*}$.

Using the same approach, we derive

$$CW^* - CW^{GS*} = \frac{\Phi^2[144(c_n+\theta_n-2\theta)-32\Delta^2\lambda_3-9\lambda_4(2-5\theta+3c_s)(2-3\theta+c_s)]+\Upsilon\lambda_3\lambda_4[24\lambda_2\Upsilon-\Phi(5\Upsilon+12\theta+8c_n)]}{72\lambda_3\lambda_4\Phi^2} < 0 \text{ and}$$

$$SW^* - SW^{GS*} = \frac{\lambda_4\{144\Phi^2(c_n+2\theta_n-1-2\theta)-\Upsilon[\Upsilon(24\lambda_2+23\Phi)+(40c_n+46c_s+36\theta_n)]\}-\Phi^2[16\lambda_3\Delta^2+9\lambda_4(2-3\theta+c_s)^2]}{72\lambda_4\Phi^2} < 0.$$

Clearly, the subsidy scheme increases consumer and social welfare. ■

Proof of Proposition 9.

From the optimal solutions of GO and firm integration, we find $p_n^{GO*} < c_n < \bar{p}_n^*$. Clearly, $p_n^{GO*} - \bar{p}_n^* = \frac{1}{2}(c_n - 1 - 3\theta + 3\theta_n) < 0$. Using the same approach, given that $p_s^{GO*} < c_s < \bar{p}_s^*$, we obtain $p_s^{GO*} - \bar{p}_s^* = \frac{1}{2}(c_s - 1 - 3\theta + 3\theta_s) < 0$.

From Equations (4.1) and (5.8), we verify that $D_n^{GO*} - \overline{D}_n^* = \frac{c_n\lambda_3-c_s\lambda_2-(2-3\theta)\lambda_4}{2\lambda_3\lambda_4} > 0$.

From Equations (4.2) and (5.9), we obtain

$$D_s^{GO*} - \overline{D}_s^* = \frac{c_n(1+5\theta+c_s-c_n-6\theta_n+\theta_s)-c_s(1+5\theta-5\theta_n)-(1+9\theta-9\theta_n)\lambda_4}{4\lambda_4}.$$

If $\theta < \theta_5$, then $D_s^{GO*} < \overline{D}_s^*$; otherwise, $D_s^{GO*} \geq \overline{D}_s^*$. ■

Proof of Proposition 10.

Given that $p_n^{GO*} - c_n < 0$, we have $\pi_n^{GO*} < 0$. Then, it is intuitive to find that $\pi_n^{GO*} < \bar{\pi}_n^*$. By using the same approach, we obtain $\pi_s^{GO*} < \bar{\pi}_s^*$, and $\pi_n^{GO*} + \pi_s^{GO*} < \bar{\Pi}^*$.

Given $D_n^{GO*} > 0$, $\bar{D}_n^* > 0$, $D_s^{GO*} > 0$, $\bar{D}_s^* > 0$, $p_s^{GO*} > \theta$ and $\bar{p}_s^* > \theta$, we compare the safety index, consumer welfare and social welfare of the social mask enterprise to those of the integrated firm.

$$SI^{GO*} - \overline{SI}^* = \frac{c_n - \theta(1+3\theta) - \lambda_1 c_s + 3\theta_n - (2-6\theta)\theta_s - (c_n+3\theta_n)\theta_s}{2\lambda_2} > 0,$$

$$CW^{GO*} - \overline{CW}^* = \frac{(2-5\theta+3c_s)(2-3\theta+c_s)\lambda_4 + 3(c_n-c_s)^2\lambda_4 - (1-30\theta+14c_n+15\theta_n)\lambda_3\lambda_4}{8\lambda_2\lambda_4} > 0 \text{ and}$$

$$SW^{GO*} - \overline{SW}^* = \frac{(2-3\theta+c_s)^2 + \Delta^2 - \lambda_3\lambda_4(3-18\theta+6c_n+9\theta_n)}{8\lambda_3\lambda_4} > 0. \qquad \blacksquare$$

Proof of Proposition 11.

Obviously, $p_n^{GO*} < c_n < p_n^{GS*}$, $p_s^{GO*} < c_s < p_s^{GS*}$. Therefore, we obtain $D_n^{GO*} = D_n^{GS*}$, $D_s^{GO*} = D_s^{GS*}$. By comparing the profits between the two strategies of the government, we find that $\pi_n^{GO*} < \pi_n^{GS*}$ and $\pi_s^{GO*} < \pi_s^{GS*}$.

From Equations (5.5–5.7) and (5.12–5.14), it is straightforward to show that $SI^{GO*} = SI^{GS*}$, $CW^{GO*} = CW^{GS*}$ and $SW^{GO*} = SW^{GS*}$. $\blacksquare$

Note.

We denote $\Delta = c_n - c_s$, $\lambda_1 = 1-\theta$, $\lambda_2 = 1-\theta_n$, $\lambda_3 = 1-\theta_s$, $\lambda_4 = \theta_s - \theta_n$, $\Phi = 3 - 4\theta_n + \theta_s$, $\Upsilon = (6 + 3\theta - 2c_n - c_s - 6\theta_n)$, $\Omega = 1 - 2\theta_n + \theta_s$,

$$\theta_1 = \frac{\lambda_2(2c_s\lambda_2 + \lambda_3\lambda_4) - c_n(2-3\theta_n+\theta_s)\lambda_3}{\lambda_4\Omega},$$

$$\theta_2 = \frac{\sqrt{\Upsilon\lambda_4\{35\Delta^2 - \left(36-170c_n+64c_n^2+98c_s-88c_nc_s+60c_s^2\right)\theta_n + (87-192c_n+200c_s-144\theta_n)\theta_n^2 \; \theta_s[174\theta_n(1-2\theta_n) - 36 - 29c_n^2 - 25c_s^2 + c_s(270\theta_n-98-70\theta_s) + (242\theta_n-87)\theta_s - 49\theta_s^2 + 2c_n(85+9c_s-127\theta_n+31\theta_s)]\}}}{(45-68\theta_n+23\theta_s)\lambda_3\lambda_4} + \frac{76\theta_n - 27 - 48\theta_n^2 - 4(1+\theta_n)\theta_s + 7\theta_s^2 + 4c_n(5-8\theta_n+3\theta_s) + c_s(7-12\theta_n+5\theta_s)}{45-68\theta_n+23\theta_s},$$

$$\theta_3 = \frac{2c_s - 2c_n - (4-2c_n+3c_s-6\theta_n)\theta_n + (4+c_s-8\theta_n+2\theta_s)\theta_s}{\lambda_4},$$

$$\theta_4 = \frac{\sqrt{12\Omega\{4\lambda_4 + \lambda_3\left[c_n(2\lambda_2+\theta_s) - 3\theta_n(2\theta_n-\theta_s) - c_s\left(2+\theta_n(\theta_s-3)\right)\right]\} + (c_s\Omega + 2\theta_s(3-5\theta_n+2\theta_s) - c_n\lambda_3)^2} + c_s\Omega + 2\theta_s(3-5\theta_n+2\theta_s) - c_n\lambda_3}{6\Omega},$$

$$\theta_5 = \frac{c_s - c_n - 2\theta_n - c_s\theta_n + 2\theta_s + c_n\theta_s}{3\lambda_4},$$

$$\eta = \frac{\lambda_3\left[9(1+\theta)+16\theta_n^2+c_s\lambda_3+\theta_s(8+7\theta-\theta_s)-2\theta_n(13+8\theta+3\theta_s)\right]+\Phi(1+\theta-c_s-\theta_s)\sqrt{(9-8\theta_n-\theta_s)\lambda_3}}{2(5-8\theta_n+3\theta_s)\lambda_3},$$

$$\zeta = \frac{9+9\theta+c_n-(25+26\theta)\theta_n+16(1+\theta)\theta_n^2+(8\theta-2c_n+18\theta_n-6\theta\theta_n-16\theta_n^2-2)\theta_s-(7\lambda_2+\theta-c_n)\theta_s^2-(\lambda_2+\theta-c_n)\Upsilon\sqrt{9-8\lambda_3\theta_n-10\theta_s+\theta_s^2}}{2(5-13\theta_n+8\theta_n^2+3\theta_s-3\theta_n\theta_s)}.$$