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Using Blockchain to Improve Buffer-Stock-Sharing and Combat Cheating Behaviors under Virtual Pooling[1]

Abstract: Blockchain is a disruptive technology which is crucial for business operations. In this paper, we analytically explore how two manufacturers can achieve efficient buffer stock sharing using the blockchain technology (BCT). We first build an analytical basic model with a deterministic lead time for material replenishment and quantify the benefit of adopting a buffer stock sharing scheme. In the absence of BCT, we demonstrate the natural occurrence of a cheating problem. We analytically derive the overall value of blockchain technology (OVBCT) for the buffer stock sharing scheme and highlight the conditions under which it is increasing or decreasing in demand uncertainty. We also show how the buffer stock service level can be improved with the use of BCT. To show robustness of the analytical findings, several extended cases are explored. Novel buffer stock division rules are then generated under the proposed buffer stock sharing scheme which makes the alliance profitable. In addition, an *n*-manufacturers alliance is further analytically explored. We find that the core findings from the basic model continue to hold in the extended models. Finally, we establish the conditions for achieving Pareto improvement with the use of BCT by considering the logistics services adopted.

Keywords: Manufacturing, sharing economy, blockchain technology, virtual pooling, buffer stock sharing.

1. Introduction

1.1. Background and Motivation

In the sharing economy, individuals as well as companies are considering to share resources to enhance their operations efficiency (Zhang et al. 2021). Traditionally, it is known in inventory management that two manufacturers can cooperate and adopt “virtual pooling” by sharing buffer stocks so that both of them can reduce the amount of required buffer stock to achieve the desired level of materials inventory service level. To be specific, suppose that there are two manufacturers producing products using the same material, called Material A. Owing to uncertainties of demand and a substantial replenishment lead time for Material A, both manufacturers keep buffer stocks for Material A. Upon negotiation and discussion, the two manufacturers agree to join a cooperative virtual pooling scheme (called the “cooperative virtual pooling buffer stock sharing scheme” in this paper) in which when one manufacturer is running out of Material A, it can seek help from the other manufacturer to see if any inventory of Material A can be shared. This mechanism operates in a reciprocal way under a “trust” system. If both manufacturers faithfully participate in this scheme, it is known that both of them are benefited with a lower burden of buffer stock and hence the respective inventory cost, without

[1] “The authors sincerely thank the editor and reviewers for their very helpful and important comments, especially during the very difficult time with COVID19 pandemic. They feel very grateful and hope all editors and reviewers stay safe from the virus.”

sacrificing inventory service level. In the real world, inventory (such as buffer stock) sharing is commonly seen. Well-known examples are reported in Zhao et al. (2020) which include manufacturers in machineries, fashion apparel (Dong and Rudi 2004), computers, electronics devices (Alfaro and Corbett 2003), and automobile. In fact, even for wholesale and retail operations, we also see reported cases in inventory sharing.

However, cheating problems may arise in which the participating manufacturers tell lies in providing false figures in favour of themselves (but may hurt others) in this scheme. As a result, real world implementation of this buffer stock sharing scheme is less common than it deserves. Note that the buffer stock approach is not just confined to manufacturing inventory, it can be retail inventory or even service capacity. For instance, shipping companies usually reserve extra capacities to deal with uncertain demand and the respective extra capacities are also "buffer stocks". In this paper, we do not limit our analysis to any specific industry or type of buffer stock even though we mainly use the manufacturing case (and hence related to procurement policies (Hu and Qi 2018)) as the illustration for the sake of simplicity.

Information sharing is a critical topic in business operations. Nowadays, with the advance of blockchain technology (BCT), information can be shared efficiently with a very low expense. Most importantly, information is secure, trustworthy and permanent with the use of blockchain (Chod et al., 2020; Du et al. 2020). Thus, cheating problems with the participating manufacturers telling lies will be avoided. Moreover, BCT supports the digital fast payment by cryptocurrency, and smart contracting mechanism in which contracting operations are made automatic and accurate. In inventory management, digital fast payment and smart contracting both help to reduce lead time which is a critical factor for determining the amount of required buffer stock. Table 1 shows the features of BCT which are relevant to the buffer stock sharing scheme. Note that for applications in supply chain and operations management, the BCT commonly refers to the permissioned private blockchain, which is different from the public blockchain (like the one for bitcoins). The major differences include the fact that information stored in the permissioned private blockchain is not going to be shown to everybody in the public. The consensus algorithm is usually un-incentivized.

Table 1. Features of BCT for the buffer stock sharing scheme that we explore in this paper.

Features	Details
Permanent history	The historical data are permanently stored which can be checked by all permissioned relevant parties. This helps to overcome the cheating problem of lies telling with fabricated data.
Trustworthy data	Under the decentralized and distributed setting with its built-in consensus algorithm, BCT keeps secure and trustworthy data. This helps to establish trust and avoid cheating.
Smart contracting	BCT can automate the order contracting process. This shortens replenishment lead time and facilitates inventory sharing.
Efficient data sharing	Data can be shared with a very low expense, even for big data (Choi et al. 2018).
Virtual currency for transactions	Facilitate financial transactions with a shorter processing time.

As a remark, when we talk about overcoming the cheating problems on lies telling, the economics literature has a lot of proposals. However, even if the proposals work perfectly, they only deal with cheating but not the other aspects (such as shortening the lead time) that BCT can bring. Moreover, as we will discuss later on, the use of BCT is super simple which makes it a very practical measure. Furthermore, if we compare BCT with RFID-ERP kind of more matured technologies for inventory management, we see differences and similarities (Hastig and Sodhi, 2020; Shen et al. 2021; Wang et al. 2021; Wustmans et al., 2021). It is true that RFID-ERP can also help facilitate information sharing. However, the data points of RFID-ERP systems can still be erased or amended easily whereas BCT disallows this function (or at least, make it very difficult to do so) (Lockl et al., 2020; Rahman et al., 2020; Rehman et al., 2020). This is critical to deal with the cheating problem that we will discuss later on. BCT also has smart contracting and virtual currency functions, which are absent from the RFID-ERP based system. Last but not least, ERP is a centralized system and BCT is decentralized. With the advance of information technologies, the shortcomings of decentralized systems have been well-tackled and hence the decentralized BCT functions very well compared to the centralized ERP systems in terms of maintaining data quality and consistency.

To the best of our knowledge, BCT is the only technology currently available in the work which can achieve all the features as stated in Table 1 simultaneously. In this paper, by exploring the benefits brought by these features, we highlight the value of BCT for buffer stock sharing schemes in a supply chain.

1.2. Research Questions and Major Findings

Motivated by the emergence of BCT and the importance of buffer stock sharing schemes in operations management, we follow the buffer stock literature and analytically examine the roles and value of BCT. We first consider the buffer stock sharing problem in a supply chain with a single supplier and two manufacturers for a single replenishment period (see Figure 1). At the beginning of a replenishment period, both manufacturers order the replenishment quantities of the same material A. Owing to the uncertainties of demand and the substantial replenishment lead time, the two manufacturers agree to join a cooperative virtual pooling buffer stock sharing scheme. Under the buffer stock sharing scheme, within the replenishment period, if one manufacturer is running out of Material A, it can seek help from the other manufacturer to see if any inventory of Material A can be shared. We also consider other generalized cases in the extended analyses. To be specific, we aim to address the following research questions.

1. For the buffer stock sharing scheme, will the cheating problem appear and will its occurrence imply that the buffer stock sharing scheme may do more harm than good?
2. What is the value of BCT in buffer stock sharing scheme (including fight the cheating problem and shortening lead time)? If the manufacturer plans to improve buffer stock service level using BCT, what is the amount of improvement if the manufacturer has the same budget as before?
3. For robustness checking, will the main results continue to hold under settings with, e.g., lead times being stochastic, and lead times and service levels being different between manufacturers?
4. For the upstream supplier, will the downstream application of BCT benefit or hurt it? If the downstream manufacturers choose premier logistics services before adopting BCT, what is the impact of switching to a cheaper logistics service after the implementation of BCT on both the downstream manufacturers and

upstream supplier? Will Pareto improvement be possible?

Addressing the above research questions yields various interesting findings. Under the basic model, we illustrate the occurrence of cheating in which one manufacturer tells lies regarding its demand parameter and can enjoy a reduction of the amount of buffer stock it needs to keep. Moreover, comparing between the "faithful participation" and "cheating" cases of the buffer stock sharing scheme, the dishonest manufacturer is always benefited whereas the honest manufacturer's benefit is smaller when the cheating case occurs. It is interesting to note that compared to the no cooperation scenario, the honest manufacturer will never suffer a loss and it will be strictly benefited (if the correlation coefficient of the two manufacturers' demands is not equal to 1 [which is the common case in practice]) even if the cheating problem arises. It shows that despite the potential occurrence of cheating, the negative impact brought by the cheating is basically no larger than the benefit generated from the buffer stock sharing scheme. To tackle the cheating problem and further shorten lead time, we propose the use of BCT and prove that they can help. We then derive the overall value of BCT (OVBCT) and uncover that a higher correlation coefficient of the two markets' demands would lead to a drop of OVBCT for both manufacturers. Besides, we reveal that the rate of inventory reduction achieved by the buffer stock sharing scheme is negatively related to the demand correlation coefficient. Therefore, under the case with smaller demand correlation coefficients, manufacturers are encouraged to establish the buffer stock sharing scheme with/without BCT. If the market demand uncertainty faced by a manufacturer increases, the effect on OVBCT depends on the relative size of demand uncertainties of the two markets. This result highlights the importance of noting the relative size of demand uncertainties of the two manufacturers' markets before realizing the impacts brought by a change of demand uncertainty towards the value of BCT. For those manufacturers who want to use the same buffer stock's budget to achieve a higher inventory service level, we prove that the BCT mediated buffer stock sharing scheme can always lead to an improved inventory service level. To show the robustness of research findings derived from the basic model, we consider the extended models for the cases with stochastic lead times, different lead times, different service levels, and multiple ($n > 2$) manufacturers participation, and prove that the qualitative results obtained in the basic model are still valid. Moreover, we analytically show the diverse impacts of the downstream utilization of BCT on the upstream supplier. Specifically, if the manufacturers choose to reduce their buffer stock levels, the upstream supplier always suffers due to a decrease in profit, while Pareto improvement could be achieved if the manufacturers decide to improve their service levels. Finally, we show that the blockchain's ability in reducing lead time and the demand correlation coefficient between the two manufacturers have opposite impacts on the supply chain members (i.e., the supplier and the manufacturers). Furthermore, the condition to achieve Pareto improvement is identified with the consideration of logistics services.

1.3. Contribution Statements and Organization

To the best of our knowledge, this paper is the first study in the literature which highlights the value of BCT for the buffer stock sharing scheme. It naturally lies in the interface of operations management (OM) and information systems (Kumar et al. 2018; Hu et al. 2017). Applications of BCT to facilitate buffer stock sharing, overcome cheating problem, and shorten lead time are explored. Many novel findings are identified and insights are generated. All results are theoretically proven and shown in closed-form. Since the buffer stock sharing scheme and BCT are practical measures, this paper not only contributes to the related literature but it also provides important guidance to operations managers regarding the value of BCT in buffer stock sharing.

This paper is arranged as follows. After presenting the background, research motivation, research questions and contribution in Section 1, we report the literature review in Section 2. Then, we follow the literature and build the buffer stock basic model in Section 3. We discuss the values of BCT in Section 4. We explore the buffer stock service improvement in Section 5. We conduct analyses in the extended models with stochastic lead times, different lead times, different service levels, and multiple manufacturers in Section 6. We study the impact on upstream suppliers and Pareto improvement conditions in Section 7. We conclude with discussions on managerial insights and future studies in Section 8. All technical proofs of this paper are placed in the online supplementary appendix.

2. Related Literature

2.1. Buffer Stock Management

Buffer stock, also known as safety stock, is a very important concept and practice in inventory management. The goal of keeping buffer stock is to avoid stockout over the replenishment cycle. The classic buffer stock model can be found in the pioneering text by Feller (1960) and virtually all major textbooks in operations management (OM) nowadays. Operations managers commonly know the "buffer stock formula" and hence prior research has focused on providing practical tools such as simple tables to assist the estimation of buffer stocks (Aucamp and Barringer 1987). In addition to the fundamental function of protecting the company from inventory stockout, the buffer stock concept also has been applied in different contexts. For instance, Sridharan and LaForge (1989) study the impacts of buffer stock on production schedule's stability and cost. The authors develop an intelligent buffer stock policy to improve production scheduling with respect to the inventory service target. Van der Laan et al. (2016) study the demand forecasting and inventory planning process for humanitarian logistics. They identify the critical factors which affect the operations performance and explore the role played by demand uncertainty and buffer stock. De Treville et al. (2017) explore production in a high-cost environment with time-insensitive products. The authors propose the strategic use of buffer capacity for production flexibly.

There is no doubt that buffer stock is critically important but keeping buffer stock is expensive. One way to reduce buffer stock without a sacrifice of cost or inventory service level is by centralization of inventory (pooling), or buffer stock sharing (also called virtual pooling). For example, Eppen (1979) explores the inventory pooling effect on inventory reduction by centralization. Inderfurth (1995) examines buffer stock planning and optimization with multiple products when their demands are correlated. Alfaro and Corbett (2003) study the inventory pooling problem and uncover its value in different scenarios. Benjaafar et al. (2005) study the benefits of pooling for the case when supply lead times are generated by a capacitated production system.

Corbett and Rajaram (2006) study and generalize the inventory pooling problem to the case without assuming normally distributed demand. Berman et al. (2011) investigate the significance of risk pooling in a multiple-location newsboy setting. Mak and Shen (2014) study inventory pooling by a robust stochastic optimization approach. Note that in the standard textbook buffer stock models, the normally distributed iid[2] demands and/or iid lead times are considered. Despite the fact that these assumptions are far from perfect, we also follow them in developing our models because (i) they are commonly known in practice and in academia. Built on them, our results can be directly compared to them and easier to understand by OM people (including managers); (ii) we also want to analytically highlight the role played by the mean and standard deviation of demand (Wei and Choi 2010). Hence, the normal distribution is the choice we take in this paper.

2.2. Values of BCT

BCT has been known as a prominent technological advance which can help different business operations (Farshidi et al., 2020; Geneiatakis et al., 2020; Olsen & Tomlin, 2019 Whitaker & Kräussl, 2020; Yu et al., 2020). In the literature, the background and main features of the blockchain have been examined by Iansiti and Lakhani (2017). The role of BCT in reducing operations cost and enhancing information sharing are proposed by Michelman (2017). As one of the started projects by industrial giants like Walmart and JD.com, BCT are known to help with food supply chain management. Motivated by this industrial observation, Shi and Choi (2018) analytically study how BCT can improve operations in food supply chains. The authors focus on studying the information visibility and traceability aspects that BCT can help. Babich and Hilary (2018) introduce the features of BCT and discuss from various perspectives how BCT can be applicable to OM. The authors highlight a few OM areas, such as the use of information, supply chain risk management, contract automation, where BCT can contribute. Moreover, Chod et al. (2018) explore how blockchain can be used to signal a company's operational capabilities. The authors find that blockchain can help companies to secure favourable terms in finance with a reduced signalling cost. Choi (2020) examines the initial coin offering problem for a product development project. The authors reveals how blockchain helps and the corresponding benefit. Most recently, Choi et al. (2020) study the value of BCT in on-demand service platforms. The authors highlight the function of BCT in providing accurate information for risk analysis. Besides, Guo et al. (2020) examine how blockchain can be implemented to enhance environmental sustainability in apparel supply chains. Chod et al. (2020) evaluate the financial benefits brought by blockchain in enhancing supply chain transparency. The authors propose that blockchain is able to secure appealing financing terms with a lower signaling cost. Most recently, Pun et al. (2021) examine the effectiveness of blockchain to fight against "deceptive" counterfeits, in which the government can provide a subsidy to encourage the blockchain adoption. Pun et al. (2021) analytically show how the government subsidy can benefit both consumers and the whole society. Similar to the above studies, this paper also explores the application of BCT in OM. However, different from them, we focus on buffer stock management and highlighting the value of BCT. To the best of our knowledge, this is the first paper in the OM

[2] Following literature, "iid" is utilized to stand for "independent and identically distributed" in this paper.

literature which analytically explores the use of BCT for buffer stock management. Novel managerial insights are generated which provide important guidance to operations managers.

3. Basic Model: Preliminaries

To clearly present the idea of using BCT for buffer stock management, we start with a very simple basic model and we will generalize and extend it to the much more complicated setting in the subsequent sections. We follow the standard buffer stock literature and build the following basic model in the presence of two symmetric manufacturers getting supplies of a common material, Material A, from the same supplier[3]. In the basic model, to clearly show the picture and introduce the topic, we go for the simplest model case where lead time is assumed to be fixed and it covers L periods. Both manufacturers will decide their replenishment quantities (i.e., the expected amount of the demand and the buffer stock) at the beginning of the replenishment period under the buffer stock sharing scheme, and make orders from the same supplier at the same time. For Manufacturer 1, the demand per period that it faces, denoted by x_1, is independently identically normally distributed with a mean μ_1 and standard deviation σ_1. The inverse cumulative distribution function of standard normal distribution is denoted as $\Phi^{-1}(\cdot)$. Similarly, for Manufacturer 2, the demand per period that it faces, denoted by x_2, is independently identically normally distributed with a mean μ_2 and standard deviation σ_2. Note that $\mu_1, \mu_2, \sigma_1, \sigma_2 > 0$. We consider the case when x_1 and x_2 are correlated with the coefficient of correlation ρ, where by definition $-1 \le \rho \le 1$. Following the industrial norm, both manufacturers aim to achieve a buffer stock service level α, where $0.5 < \alpha < 1$. We do not consider the case when the buffer stock service level is ridiculously low (i.e. $\alpha \le 0.5$) because it will violate the main goal of keeping buffer stock – avoiding stockout. As a remark, even though we employ the term "manufacturers" in this paper's model development and analysis, the findings derived are not limited to manufacturers. They are applicable to retailers and even service providers as what we commented in Section 1. For the cost of holding the buffer stock, we consider each unit of buffer stock will incur a cost θ per a certain duration of time (e.g., a year). The summary of the major notation and subscripts used in the basic model (as well as in the extensions of the model) can be found in Table 2.

As a standard result, if the two manufacturers plan their buffer stocks separately (i.e. the "no cooperation scenario"), the required buffer stocks for Manufacturers 1 and 2 are respectively given as follows:

$$BS_1 = \sqrt{L}\sigma_1\Phi^{-1}(\alpha), \quad (1)$$

$$BS_2 = \sqrt{L}\sigma_2\Phi^{-1}(\alpha). \quad (2)$$

Define:

$$\sigma_{1+2} = \sqrt{\sigma_1^{\,2} + \sigma_2^{\,2} + 2\rho\sigma_1\sigma_2}, \quad (3)$$

[3] Note that in the OR literature, using two players in conducting analytical investigation is common (e.g., see Leng and Zhu 2009). In Section 6.2, we extend the analysis to the case with $n > 2$ manufacturers.

$$\Omega = \frac{\sqrt{{\sigma_1}^2 + {\sigma_2}^2 + 2\rho\sigma_1\sigma_2}}{\sigma_1 + \sigma_2}. \tag{4}$$

Note that σ_{1+2} is the standard deviation of the aggregated demands of the two markets faced by the two manufacturers. Ω is an important parameter to help define the buffer stock for each manufacturer under the buffer stock sharing scheme. Moreover, $0 < \Omega \le 1$[4] .

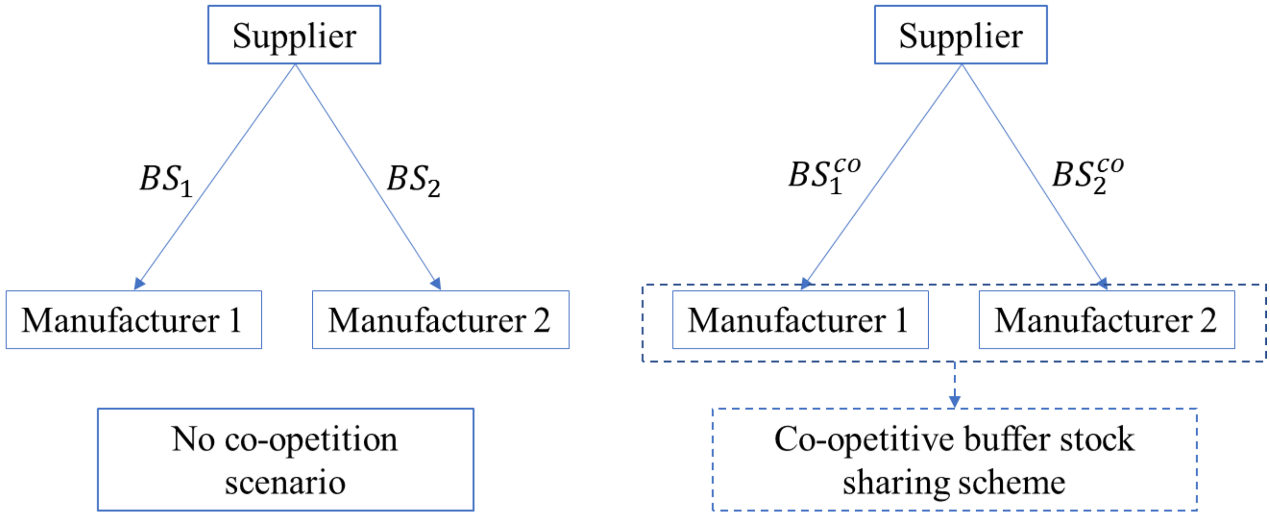

Figure 1. The two scenarios considered in this paper.

Now, suppose that the two manufacturers join a buffer stock sharing scheme (which is as mentioned in Section 1). For $i = (1, 2)$, under the basic model: (a) If both manufacturers are honest and faithfully joining the buffer stock sharing scheme, the required amount of buffer stock for Manufacturer *i* is:

$$BS_i^{CO} = \sqrt{L}\sigma_i\Omega\Phi^{-1}(\alpha), \tag{5}$$

and we have: $BS_i^{CO} < BS_i$,when $\rho \neq 1$.

The above finding indicates the classic result that the buffer stock sharing scheme can lead to a reduction of the amount of required buffer stock when all manufacturers faithfully participate in the scheme. From Eq, (5), it is seen that $(1-\Omega)$ actually indicates the rate of inventory reduction. A smaller Ω leads to a larger buffer stock saving for both manufacturers through the sharing scheme. Besides, Ω is positively related to the coefficient of correlation ρ between the demands of the two participating manufacturers, which implies that the buffer stock sharing scheme can realize more benefits if ρ is smaller. Accordingly, the manufacturers with smaller demand correlation coefficients are more encouraged to launch such kind of buffer stock sharing scheme. Figure 1 depicts the two scenarios considered in this paper.

We now explore whether the manufacturers are tempted to tell lies and create a cheating problem. To be

[4] Note that $\Omega = 0$ ($\sigma_{1+2} = 0$) only holds when $\rho = -1$ and $\sigma_1 = \sigma_2$. To focus on the interesting situation when the buffer stock sharing scheme could help reduce the risk of demand uncertainty, the special case where $\Omega = 0$ ($\sigma_{1+2} = 0$) is excluded from the analysis.

specific, without loss of generality, let's imagine that Manufacturer 1 is honest whereas Manufacturer 2 is dishonest. In the information sharing scenario, Manufacturer 2 may claim with fabricated data that after calculation, the correlation coefficient is $\hat{\rho}$, where $\hat{\rho} > \rho$. Note that we consider the correlation coefficient as the way for Manufacturer 2 to tell lies is because it is relatively difficult to be discovered by Manufacturer 1 if there is no BCT in place. Of course, this type of cheating problems may arise by other means, e.g., Manufacturer 2 can tell lies regarding its demand variance[5]. Similar results will be found.

With $\hat{\rho}$, we have the revised aggregated market demand uncertainty:

$$\hat{\sigma}_{1+2} = \sqrt{\sigma_1^{\,2} + \sigma_2^{\,2} + 2\hat{\rho}\sigma_1\sigma_2}\,, \tag{6}$$

$$\hat{\Omega} = \frac{\sqrt{\sigma_1^{\,2} + \sigma_2^{\,2} + 2\hat{\rho}\sigma_1\sigma_2}}{\sigma_1 + \sigma_2}\,, \tag{7}$$

$$BS_{1,MH}^{CO} = \sqrt{L}\sigma_1\hat{\Omega}\Phi^{-1}(\alpha)\,. \tag{8}$$

In this case, we define the following terms and then present Proposition 1.

$$\Delta\hat{\sigma} = \frac{\sigma_1(\hat{\sigma}_{1+2} - \sigma_{1+2})}{\sigma_1 + \sigma_2}\,, \tag{9}$$

$$\Delta K = \sqrt{L}(\Delta\hat{\sigma})\Phi^{-1}(\alpha)\,. \tag{10}$$

Here, $\Delta\hat{\sigma}$ represents the difference of demand variances after and before the cheating behavior of the dishonest manufacturer 2, while ΔK represents the extra buffer stock undertaken by the honest manufacturer 1 if the cheating behavior of Manufacturer 2 happens.

Proposition 1 (Cheating). *Under the basic model, if Manufacturer 2 tells lies on the correlation coefficient and Manufacturer 1 simply follows: (a) Manufacturer 1 has to keep* ΔK *units more of buffer stock, and (b) Manufacturer 2 can save* ΔK *units of buffer stock under the buffer stock sharing scheme.*

Note that, for both manufacturers, they aim to keep the total buffer stock cost as low as possible while achieving the target inventory service level α. In other words, the objective function of each manufacturer is to minimize the total buffer stock cost, which is equivalent to holding the minimum amount of buffer stock under the constraint of meeting the inventory service level requirement. Proposition 1 indicates that there is a real and strong incentive for the manufacturer to tell lies and create the cheating problem. To be specific, as shown in Proposition 1, Manufacturer 2 (who tells lies) will benefit from holding less buffer stock whereas the burden is passed to Manufacturer 1 who is honest. This creates a fairness issue which directly affects the implementation of the buffer stock sharing scheme. Moreover, note that (1-$\hat{\Omega}$) now represents the fabricated rate of inventory reduction. Therefore, if the dishonest Manufacturer 2 tells a bigger lie about the correlation

[5] For the situation when Manufacturer 2 tells lie regarding its demand variance, the similar result as what Proposition 1 shows can be derived. Please refer to Proposition 2A in Online Supplementary Appendix A1 for the details.

coefficient (i.e., a larger $\hat{\rho}$), the honest Manufacturer 1 needs to bear more buffer stock. Since Manufacturer 2 has an incentive to tell lies and it is difficult to verify (without the use of technology such as blockchain), one important question arises: Will such kind of cheating problems make the buffer stock sharing scheme a bad measure which does more harm than good? Proposition 2 shows the result.

Proposition 2. *Under the basic model, comparing between the "faithful participation" and "cheating" cases of the buffer stock sharing scheme, Manufacturer 2 (the dishonest manufacturer) is always benefited by amplifying ρ whereas Manufacturer 1 (the honest manufacturer)'s benefit is smaller when the cheating case occurs. However, compared to the no cooperation scenario, Manufacturer 1 will never suffer a loss and it will be strictly benefited (when $\rho \neq 1$) even if the cheating problem arises.*

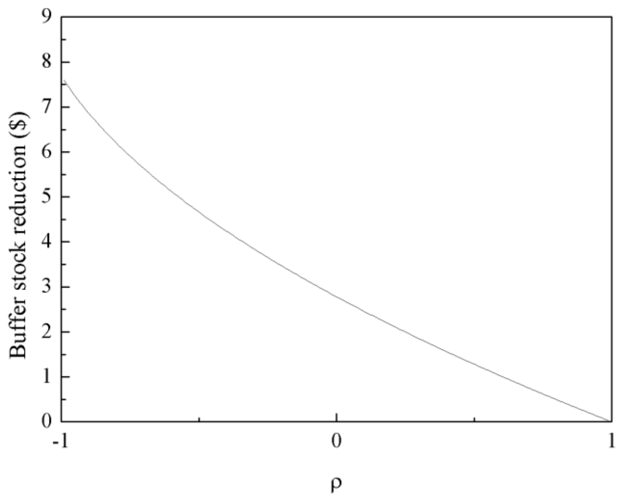

Figure 2. The negative relationship between the benefit of the sharing scheme and the demand correlation between the manufacturers.

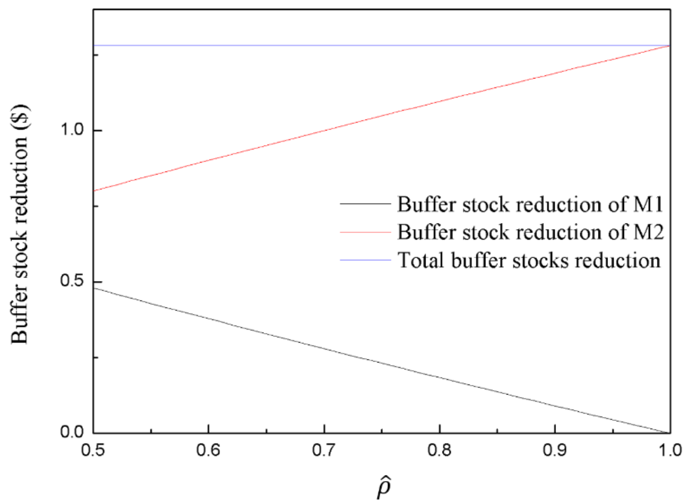

Figure 3. The impacts of cheating behavior on the buffer stock reduction under the sharing scheme[6].

Proposition 2 is an important result. It shows that despite the potential occurrence of cheating, the negative impact brought by the cheating is basically no larger than the benefit derived from the buffer stock sharing scheme. To better show the results of Proposition 2, we include a numerical example and the results are shown in Figures 2 and 3. To be specific, from Figures 2 and 3, we can see that no matter cheating is present or not, the sharing scheme always brings positive benefit to the whole system (when $\rho \neq 1$). As such, from the operations efficiency perspective, manufacturers should still seriously consider the scheme even if the cheating problem may occur. However, it doesn't mean the cheating problem is "fine" because it creates two problems: (i) The unfairness issue in which the honest manufacturer suffers a loss and the dishonest manufacturer takes advantage, and (ii) if the honest manufacturer feels bad and also turns into "dishonest", the scheme may collapse. Thus, we still need to think about a way to overcome the cheating problem of this type.

4. Values of BCT

In Section 3, we analytically show the occurrence of the cheating problem and the corresponding impact. In this section, we demonstrate the value of BCT in overcoming the cheating problems and also reducing the lead time.

First of all, suppose that now, Manufacturer 1 and Manufacturer 2 vote for BCT[7]. Then, they share the real time data regarding demand to the BCT supported system. Then, owing to the features of BCT, we know that it keeps information in a permanent manner and also disallows editing or changing of previous data without the consent of all related parties. As such, if the two manufacturers are asked to participate in the buffer stock sharing scheme by offering data supported by the BCT, it is basically impossible for them to tell lies on the data and hence the cheating problem we proposed in Section 3 will not occur. As such, using BCT, we can effectively overcome the cheating issue. By comparing between $BS_{1,MH}^{CO}$ and BS_1^{CO}, we define the value of BCT for avoiding cheatings as follows:

$$VBCT_{1,MH} = (BS_{1,MH}^{CO} - BS_1^{CO})\theta \tag{11}$$

$$= \frac{\sqrt{L}\theta\sigma_1(\hat{\sigma}_{1+2} - \sigma_{1+2})\Phi^{-1}(\alpha)}{\sigma_1 + \sigma_2}, \tag{12}$$

It is easy to find that $VBCT_{1,MH}$ is a function directly proportional to $(\hat{\sigma}_{1+2} - \sigma_{1+2})$, it means that if the dishonest manufacturer tells bigger lies which make $\hat{\sigma}_{1+2}$ larger, the effect of BCT will also be higher. Since $VBCT_{1,MH}$ is always positive, it is basically guaranteed that the manufacturers will be better off with the

[6] The illustrated example is conducted by the following numerical settings: $\alpha = 0.9, L = 1, \theta = 1, \sigma_1 = 3, \sigma_2 = 5, \rho = 0.5$.

[7] In our analysis, we do not include an additional blockchain cost because: (i) For inventory management, blockchain is mainly an infrastructure technology (just like another kind of ERP, etc.), using it mainly involves a big initial investment which means the sunk cost. Besides, the per unit cost of blockchain application is negligibly small. (ii) By estimating the value generated by using blockchain without considering blockchain's cost can yield the benefit of using blockchain, engineering managers can then check to see if the value is sufficiently large to justify the investment, etc. (iii) In the literature, some studies also focus on the benefits brought by blockchain in supply chain management, while ignoring the implementation cost, like Choi et al. (2019) and Choi et al. (2020).

deployment of BCT to deal with cheating and whether the investment is well-justified can be decided by checking the respective benefit realized, as shown in (12).

BCT are known to facilitate smart contracting, which refers to the situation when all business contractual arrangements can be done automatically in the most efficient manner. In our problem domain, it means the replenishment can be made in a speedier manner. Moreover, BCT also support virtual currency. If both manufacturers and their supplier are adopters of the virtual currency, this can further facilitate transactions and streamline the respective financial processes. Thus, lead time is reduced. Note that lead time reduction is very critical. In the following model, we just look at the traditional lead time reduction model when demand distribution is unchanged[8].

Suppose that the original lead time L is reduced to be L_{BCT}:

$$L_{BCT} = \gamma L, \tag{13}$$

where $0 < \gamma < 1$.

From (5), we have the analytical expression for $BS_i^{CO} = \sqrt{L}\sigma_i \Omega \Phi^{-1}(\alpha)$. Thus, we can define the amount of required buffer stock under the buffer stock sharing scheme with reduced lead time as follows:

$$BS_{i,LTR}^{CO} = \sqrt{\gamma L}\sigma_i \Omega \Phi^{-1}(\alpha). \tag{14}$$

We can derive the closed-form analytical expression of the value of BCT derived from lead time reduction in the following:

$$VBCT_{i,LTR} = \sqrt{L}(1-\sqrt{\gamma})\sigma_i \Omega \Phi^{-1}(\alpha)\theta > 0, \text{ for } i = (1, 2). \tag{15}$$

Undoubtedly, if the achieved lead reduction is higher, i.e. γ is smaller, then $VBCT_{i,LTR}$ will be higher, as what we observe from (15). Besides, it is also straightforward that the value of BCT derived from lead time reduction increases along with parameter Ω.

With the results derived so far, we can proceed to show the overall value of BCT (OVBCT) as shown in Eq. (17), i.e., the sum of the cost reductions induced by the faithful participation of both manufacturers under the buffer stock sharing scheme and the reduced lead time, respectively. Define the following and we have Lemma 1.

$$VBCT_{i,CO} = (BS_i - BS_i^{CO})\theta, \tag{16}$$

$$OVBCT_i = VBCT_{i,CO} + VBCT_{i,LTR}. \tag{17}$$

Proposition 3. $OVBCT_i = \sqrt{L}(\sigma_i \Phi^{-1}(\alpha)\theta)(1-\Omega\sqrt{\gamma})$, for $i = (1, 2)$. (18)

Proposition 3 shows the analytical expression of the overall value brought by BCT for the buffer stock sharing scheme, which covers the trustworthy information sharing and faithful participation of both manufacturers, as well as the reduction of lead time.

8 See Biçer et al. (2018) for the significance of lead time reduction for supply chains in the presence of demand jumps.

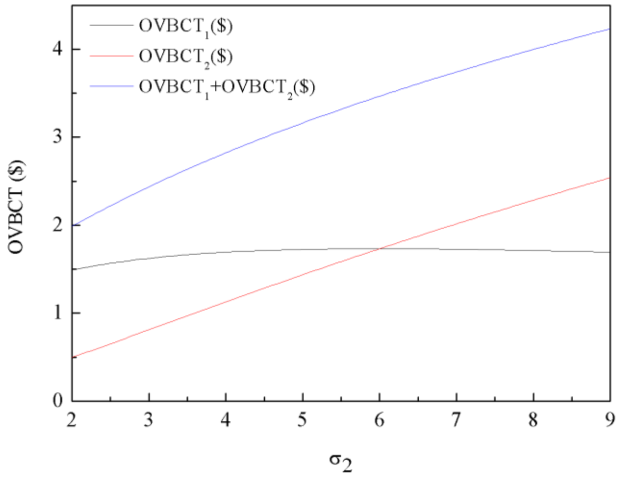

Figure 4. The overall value of blockchain technology of the individual participant and the whole system under the buffer stock sharing scheme[9].

After deriving OVBCT in Proposition 3, we conduct an analytical sensitivity analysis towards $OVBCT_i$. Table 3 (Appendix) summarizes the results and many findings are intuitive. To be specific, from the sensitivity analysis findings (see Table 3), we have a few interesting observations. First, a higher correlation coefficient of the two markets' demands (which implies a smaller rate of inventory reduction (1-Ω)) would lead to a drop of $OVBCT_i$ for both manufacturers. Accordingly, it is concluded that a larger correlation coefficient between the demands of the two participants will impair the overall benefits brought by the blockchain technology. Second, for the effects brought by demand uncertainties, the findings are less straightforward than expected (see Figure 4). To be specific, for Manufacturer *i*, if σ_i increases, its effect to the manufacturer partner (i.e. Manufacturer *j*) is neat and it depends on the relative size of demand uncertainty. For example, if $\sigma_i > \sigma_j$, increasing σ_i will yield a drop of $OVBCT_j$; the effect is reverted if $\sigma_i < \sigma_j$.

For the impact of increasing σ_i *to* $OVBCT_i$, the situation is clean as $OVBCT_i$ increases with σ_i. Due to the symmetry, if either of the manufacturers' demand uncertainty increases, the whole OVBCT of the buffer stock sharing system will increase. For illustration, Figure 4 displays the case when σ_2 is increasing, compared to σ_1. Given $\sigma_1 = 6$, $OVBCT_1$ is increasing first when $\sigma_2 \leq \sigma_1$, but decreasing then when $\sigma_2 > \sigma_1$. However, $OVBCT_2$ and the whole OVBCT are increasing in σ_2.

5. Buffer Stock Service Level Improvement

With the implementation of BCT supported buffer stock sharing scheme, the manufacturers can save budget for buffer stock while maintaining the originally set buffer stock service level. However, for some manufacturers,

[9] The illustrated example is conducted by the following numerical settings: $\alpha = 0.9, L = 1, \theta = 1, \sigma_1 = 6, \rho = 0.5, r = 0.8$.

saving budget is not the key concern. Instead, with the same budget, they may wish to improve inventory service level.

Among various reasons, improving inventory service level is in general a positive move and usually regarded as a "quality" improvement of the operations because the chance of having "stockout" is lower. This is especially an important issue if the manufacturer is developing in the competitive market. Having a high inventory service level is a symbol of operations success as well as a competitive edge for the manufacturer.

However, there is no free lunch in the world and maintaining a high inventory service level is uneasy. It usually requires a higher cost and also substantial operations changes. In this paper, the operations efficiency is enhanced substantially by the establishment of the buffer stock sharing scheme and the deployment of BCT. Thus, if the manufacturers do not aim to reduce buffer stock budget, they could significantly improve inventory service level.

In our basic model, the initial buffer stock budget for Manufacturer *i* is (P.S.: From (1) and (2)):

$$\Pi_i = \theta[\sqrt{L}\sigma_i\Phi^{-1}(\alpha)], \; i=(1,2). \tag{19}$$

With the amount of available buffer stock budget as shown in (19), the manufacturers can achieve a higher buffer stock service level $\alpha_{New,BCT}^{CO}$ and the results are summarized in Lemma 2.

Lemma 2. *Under the basic model, for both Manufacturers 1 and 2, if they use the initially available buffer stock budget to improve buffer stock service level, the new inventory service level is the same for both manufacturers and is given by:*

$$\alpha_{New,BCT}^{CO} = \Phi\left(\frac{\Phi^{-1}(\alpha)}{\Omega\sqrt{\gamma}}\right), \tag{20}$$

and $\alpha_{New,BCT}^{CO} > \alpha$.

Note that in Lemma 2, since $0 < \Omega\sqrt{\gamma} < 1$, we have: $\alpha_{New,BCT}^{CO} > \alpha$, which indicates that with the same budget, the manufacturers definitely can achieve a higher inventory service level after adopting the buffer stock sharing scheme and the use of BCT. The service level improvement is especially prominent if (i) Ω is small (that is, the rate of inventory reduction ($1-\Omega$) is larger), or (ii) γ is small. For (i), it reflects the fact that demand aggregation yields a more significant result when the demand correlation coefficient of the two manufacturers is smaller; for (ii), it means the amount of lead time reduction brought by BCT is more substantial. All these findings are solid and neat.

We have conducted the sensitivity analysis results towards $\alpha_{New,BCT}^{CO}$ (see Table 4 in Appendix). Similar to the findings in Section 4, note that for the effect brought by demand uncertainty, it depends on the relative size of both markets' demand uncertainties. If the two markets' demand uncertainties are the same (i.e. $\sigma_1 = \sigma_2$), varying the demand uncertainties of both markets will not affect $\alpha_{New,BCT}^{CO}$.

6. Robustness Checking

6.1. Stochastic Lead Time

In the basic model, we consider the situation when the lead time L is fixed. While in general, lead time is not fixed as there is supply uncertainty (Li et al. 2017). In the extended model of this sub-section, we explore the case when lead time is not fixed and it is an iid normally distributed random variable with a mean μ_L and standard deviation σ_L ($\mu_L, \sigma_L > 0$). Note that this "normally distributed" buffer stock problem with stochastic lead time is classical and we adopt it for two reasons: (i) The results are comparable to the existing literature. (ii) Closed-form analytical results are obtained. We do note the limitations of this model and refer readers to Eppen and Martin (1988) for more discussions.

If the two manufacturers plan their buffer stocks separately (i.e. the "no cooperation scenario"), the required buffer stocks for Manufacturers 1 and 2 are respectively given in the following:

$$BS_{1,UCL} = \sigma_1^{UCL}\Phi^{-1}(\alpha), \tag{21}$$

$$BS_{2,UCL} = \sigma_2^{UCL}\Phi^{-1}(\alpha), \tag{22}$$

where $\sigma_i^{UCL} = \sqrt{\mu_L \sigma_i^{\,2} + \mu_i^{\,2}\sigma_L^{\,2}}$. (23)

Now, suppose that the two manufacturers join a buffer stock sharing scheme, which is supported by the BCT. Define:

$$\sigma_\Sigma^{UCL} = \sqrt{\mu_L \sigma_{1+2}^2 + (\mu_1+\mu_2)^2 \sigma_L^{\,2}}, \tag{24}$$

$$BS_\Sigma^{UCL} = \sigma_\Sigma^{UCL}\Phi^{-1}(\alpha), \tag{25}$$

Then, the required buffer stocks for Manufacturers 1 and 2 are respectively:

$$BS_{1,UCL}^{CO} = \left(\frac{\sigma_1^{UCL}}{\sigma_1^{UCL}+\sigma_2^{UCL}}\right)\sigma_\Sigma^{UCL}\Phi^{-1}(\alpha), \tag{26}$$

$$BS_{2,UCL}^{CO} = \left(\frac{\sigma_2^{UCL}}{\sigma_1^{UCL}+\sigma_2^{UCL}}\right)\sigma_\Sigma^{UCL}\Phi^{-1}(\alpha), \tag{27}$$

Define:

$$\Omega_{UCL} = \left(\frac{\sigma_\Sigma^{UCL}}{\sigma_1^{UCL}+\sigma_2^{UCL}}\right), \text{ where } 0 < \Omega_{UCL} \le 1 \ \ (\Omega_{UCL} = 1 \text{ only when } \rho = 1 \text{ and } \mu_1\sigma_2 = \mu_2\sigma_1). \tag{28}$$

We have Lemma 3.

Lemma 3. *For $i = (1, 2)$, under the extended model: (a) If both manufacturers are honest and they faithfully join the buffer stock sharing scheme, the required amount of buffer stock for Manufacturer i is:*

$$BS_{i,UCL}^{CO} = \sigma_i^{UCL}\Omega_{UCL}\Phi^{-1}(\alpha). \tag{29}$$

(b) $BS_{i,UCL}^{CO} < BS_{i,UCL}$, *when* $\rho \ne 1$ *or* $\mu_1\sigma_2 \ne \mu_2\sigma_1$.

If we take a look at Proposition 1 and Lemma 3, we see that the results in both the basic model and the extended model are similar. In particular, the amount of required buffer stocks under the sharing scheme for both models exhibit the very similar form. Note that Ω_{UCL} functions similarly to Ω in the basic model, and (1-Ω_{UCL}) indicates the rate of inventory reduction under stochastic lead time. Ω_{UCL} is also positively related to the coefficient of correlation ρ between the demands of the two participants.

Define:

$$\sigma_{i,LTR}^{UCL} = \sqrt{\gamma\mu_L\sigma_i^{\ 2} + \gamma^2\mu_i^{\ 2}\sigma_L^{\ 2}}\ ,\ \ \sigma_{i,LTR}^{UCL} < \sigma_i^{UCL}\ , \tag{30}$$

$$\sigma_{\Sigma,LTR}^{UCL} = \sqrt{\mu_L\gamma\sigma_{1+2}^2 + \gamma^2(\mu_1+\mu_2)^2\sigma_L^{\ 2}}\ , \tag{31}$$

$$\Omega_{UCL}^{LTR} = \left(\frac{\sigma_{\Sigma,LTR}^{UCL}}{\sigma_{1,LTR}^{UCL} + \sigma_{2,LTR}^{UCL}}\right),\ \ 0 < \Omega_{UCL}^{LTR} \le 1\ . \tag{32}$$

Similar to the basic model, we can derive and explore the value of BCT as well as the enhanced buffer stock service level as what we did in Sections 4 and 5, respectively. We summarize the results as follows:

$$OVBCT_i^{UCL} = (\sigma_i^{UCL} - \sigma_{i,LTR}^{UCL}\Omega_{UCL}^{LTR})\Phi^{-1}(\alpha)\theta\ ,\ \text{for}\ \ i = (1, 2)\ . \tag{33}$$

Now, for both Manufacturers 1 and 2, if they use the initially available buffer stock budget to improve buffer stock service level, the new inventory service level is the same for both manufacturers and is given by:

$$\alpha_{New,BCT}^{UCL,CO} = \Phi\left(\frac{\sigma_i^{UCL}\Phi^{-1}(\alpha)}{\sigma_{i,LTR}^{UCL}\Omega_{UCL}^{LTR}}\right), \tag{34}$$

and $\alpha_{New,BCT}^{UCL,CO} > \alpha$.

From (33) and (34), it is obvious that the findings and qualitative managerial insights under the basic model continue to hold in this extended model with stochastic lead time, in which BCT can bring a substantial positive value (either reduction of inventory cost or improvement of inventory service level) to support implementation of the buffer stock sharing scheme. Our core findings derived from the basic model are hence robust with respect to whether lead time is fixed or stochastic.

6.2. Multiple *n*-Manufacturers

In the basic model, we consider the situation when only two manufacturers participate into the BCT mediated buffer stock sharing scheme. Undoubtedly, the deployment of BCT will stimulate more and more manufacturers, especially the medium and small size companies, to join in the buffer stock sharing scheme. Intuitively, more participants bring more benefits. Here the question is that: Is it always better to let a "new" manufacturer join in the existing buffer stock sharing alliance? When will the BCT mediated buffer stock sharing scheme attain its optimal value with respect to the number of participating manufacturers? To answer the above questions, we explore a more general case in which *n* manufacturers may participate in the BCT mediated buffer stock sharing scheme. The closed-form analytical results are obtained to verify the value of BCT.

As a standard result, if the *n* manufacturers plan their buffer stocks separately (i.e. the "no cooperation scenario"), the required total amount of buffer stocks for the *n* manufacturers is given as follows:

$$\sum_{i=1}^{n} BS_i = \sum_{i=1}^{n} \sqrt{L}\sigma_i \Phi^{-1}(\alpha). \tag{36}$$

Define the standard deviation of the aggregated demands of the *n* markets faced by the *n* manufacturers as follows:

$$\sigma_{\sum_{i=1}^{n}} = \sqrt{\sum_{i=1}^{n} \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{i,j} \sigma_i \sigma_j}, \tag{37}$$

where, $\rho_{i,j}$ is the coefficient of correlation between the demands per period of Manufacturer *i* and *j (i, j=1, 2, ..., n*; $i \neq j$).

Now, suppose that the *n* manufacturers join a buffer stock sharing scheme, similar to the basic model, the total amount of required buffer stocks that the *n* manufacturers need to hold is summarized in Proposition 4.

Proposition 4 (Faithful participation among *n* manufacturers). *Under the extended model in which n manufacturers are involved, (a) If the n manufacturers are honest and they faithfully join the buffer stock sharing scheme, the total required amount of buffer stock for this n-manufacturer alliance is:*

$$BS^{co}_{\sum_{i=1}^{n}} = \sqrt{L}\sigma_{\sum_{i=1}^{n}} \Phi^{-1}(\alpha). \tag{38}$$

Define:

$$D_n = \sum_{i=1}^{n} \sigma_i - \sigma_{\sum_{i=1}^{n}}. \tag{39}$$

$$D^{BS}_{\sum_{i=1}^{n}} = \sum_{i=1}^{n} BS_i - BS^{co}_{\sum_{i=1}^{n}} = \sqrt{L} D_n \Phi^{-1}(\alpha). \tag{40}$$

Obviously, $D_n > 0$, if $\exists i, j \in (1,2,...,n)$ and $i \neq j$, $\rho_{i,j} \neq 1$. Thus, we have the following proposition.

Proposition 5 (Buffer stock reduction). $D^{BS}_{\sum_{i=1}^{n}} > 0$, *when* $\exists i, j \in (1,2,...,n)$ *and* $i \neq j$, $\rho_{i,j} \neq 1$. (41)

Proposition 5 indicates the general result that the buffer stock sharing scheme can always benefit the *n*-manufacturers alliance regarding the reduction of the amount of required buffer stock, especially when all manufacturers faithfully join the scheme. It is interesting to explore, if *n*-manufacturers alliance is already established, whether more manufacturers participate in will always improve the existing alliance. Firstly, we can derive the overall value of BCT (OVBCT) for the buffer stock sharing scheme among *n* manufacturers as follows:

$$OVBCT_{\sum_{i=1}^{n}} = \sum_{i=1}^{n} BS_i - \sqrt{\gamma} BS^{co}_{\sum_{i=1}^{n}} = (\sum_{i=1}^{n} \sigma_i - \sqrt{\gamma}\sigma_{\sum_{i=1}^{n}})\sqrt{L}\Phi^{-1}(\alpha). \tag{42}$$

Observe that (42) shows the analytical expression of the overall value brought by BCT for the *n*-manufacturers buffer stock sharing scheme, which covers the trustworthy information sharing and faithful participation of all the *n* manufacturers, as well as the reduction of lead time. Intuitively, more participants incur more benefits. The deployment of the BCT facilitates the establishment of faithful sharing scheme and incentivizes more manufacturers to join. It is interesting to investigate the condition under which the expansion of the existing sharing alliance will benefit most. To verify this, we first present Lemma 4.

Lemma 4. *For the n-manufacturers buffer stock sharing alliance (where* $n \geq 2$*): For i = 1, 2, …, n,*
(a) the further reduction on the standard deviation of the combined market's demands achieved by the entry of n_{th} manufacturer is positive (i.e. $\Delta_{n+1} = D_{n+1} - D_n > 0$*) and*
(b) Δ_{n+1} *can attain its largest value, if and only if* $\rho_{i,n+1} < \min(\rho_{i,m} \mid m \neq i, m = 1,2,...,n)$.

Lemma 4 verifies that, when the coefficient of correlation $\rho_{i,n+1}$ between the market demands of the (n+1)th manufacturer and Manufacturer i $(i = 1,2,...,n)$ is smaller than the coefficients of correlation $\rho_{i,j}\,(j = 1,2,...,n,\ j \neq i)$ between market demands of the other $(n-1)$ manufacturers and Manufacturer i, more reduction in terms of the standard deviation of the aggregated market demands can be achieved when the (n+*1*)-manufacturers alliance can be formed. In other words, under the condition of $\rho_{i,n+1} < \min(\rho_{i,m} \mid m \neq i, m = 1,2,...,n),$ the risk from uncertain market demands of the (n+1) manufacturers can be further mitigated through expanding the alliance. In addition, the reduction regarding the aggregated standard deviation of the total (n+1) market demands can reach its largest value (see Lemma 4(b)). Hence, we have the following proposition in terms of the total reduction of the amount of required buffer stock of the new (n+1)-manufacturer alliance.

Proposition 6 (Alliance member expansion). *For a given n-manufacturers buffer stock sharing alliance, the new entry of the* $(n+1)th$ *manufacturer can benefit the alliance most (i.e., the additional buffer stock reduction amount* $\Delta_{n+1}^{BS} > 0$ *and it can attain its largest value) if and only if the correlation coefficients between the market demands of the* $(n+1)th$ *manufacturer and the other n manufacturers satisfy the condition that* $\rho_{i,n+1} < \min(\rho_{i,m} \mid m \neq i, m = 1,2,...,n),$ *where i = 1, 2, …, n,* $n \geq 2$.

Proposition 6 indicates that, when the correlations between the market demands of the $(n+1)th$ manufacturer and the other n manufacturers are smaller than the correlations among any two of the n-manufacturers alliance, more reduction regarding the amount of the required buffer stock (i.e, $\Delta_{n+1}^{BS} > 0$) can be achieved by expanding the current n-manufacturers alliance into (n+1)-manufacturers sharing alliance. In particular, the buffer stock reduction amount (Δ_{n+1}^{BS}) achieved by the new entry of the $(n+1)th$ manufacturer to the existing n-manufacturers sharing alliance can attain its largest value. To be specific, the entry of "manufacturers with low-correlated market demands" can always bring more benefit to the existing alliance, compared to the entry of "manufacturers with high-correlated market demands", and further improvement on the cost efficiency (i.e., less buffer stock, higher service level) will be achieved under the new alliance expansion.

As a remark, even though we employ the term "manufacturers" in this paper's model development and analysis, the findings derived are not limited to manufacturers. They are applicable to retailers and even service providers as what we commented in Section 1.

6.3. Different Lead Times

In the basic model, we consider the situation when the replenishment lead times of two manufacturers are the same, i.e., $L_1 = L_2 = L$. In the extended model here, we examine the case when the replenishment lead times are different, i.e., $L_1 \neq L_2$. We investigate two scenarios where the lead times are fixed and stochastic, respectively. Closed-form analytical results are obtained, which verify the robustness of our research findings derived from the basic model.

6.3.1. Fixed Lead Times

In this subsection, we discuss the situation in which $L_1 \neq L_2$, and L_i (i = 1,2) is fixed. Similar to the basic model, if the two manufacturers plan their buffer stocks separately (i.e. the "no cooperation scenario"), the required buffer stock for Manufacturers i (i = 1, 2) is given as follows:

$$\widetilde{BS}_i = \sqrt{L_i}\sigma_i\Phi^{-1}(\alpha). \tag{43}$$

In this case, the standard deviation of the aggregated demands of the two markets faced by the two manufacturers is formulated below:

$$\tilde{\sigma}_{1+2} = \sqrt{L_1\sigma_1^2 + L_2\sigma_2^2 + 2\rho\sigma_1\sigma_2\, min(L_1, L_2)}. \tag{44}$$

Define:

$$\widetilde{\Omega} = \frac{\tilde{\sigma}_{1+2}}{\sqrt{L_1}\sigma_1+\sqrt{L_2}\sigma_2}, 0 < \widetilde{\Omega} \leq 1. \tag{45}$$

Now, suppose that both manufacturers are honest and they faithfully join the buffer stock sharing scheme, similar to the basic model, the amount of required buffer stock $\widetilde{BS}_i^{co}$, i = (1,2), that each manufacturer needs to hold is shown as follows:

$$\widetilde{BS}_i^{co} = \sqrt{L_i}\sigma_i\widetilde{\Omega}\Phi^{-1}(\alpha), \tag{46}$$

and $\widetilde{BS}_i^{co} < \widetilde{BS}_i$, when $\rho \neq 1$.

Compared with Proposition 1, we see that the results in both the basic model and this extended model are similar. In particular, the amount of required buffer stocks under the sharing scheme for both models exhibit the very similar form. However, the new "division ratio" of the buffer stock for Manufacturer i, (i = 1,2), under the buffer stock sharing scheme, becomes:

$$\frac{\sqrt{L_i}\sigma_i}{\sqrt{L_1}\sigma_1+\sqrt{L_2}\sigma_2}. \tag{47}$$

It is interesting to note that the replenishment lead time plays a role in the buffer stock division as well. Given the standard deviations of the market demand of the two manufacturers during per unit duration of time, if Manufacturer 1 has a longer replenishment lead time, the buffer stock undertaken by Manufacturer 1 is supposed to be increased.

Define:

$$L_{i,BCT} = \gamma L_i, \text{ where } i = 1,2,\ 0<\gamma<1. \tag{48}$$

$$\widetilde{BS}_{i,LTR}^{CO} = \sqrt{\gamma L_i}\sigma_i\widetilde{\Omega}\Phi^{-1}(\alpha). \tag{49}$$

Similar to the basic model, we can derive and explore the value of BCT as well as the enhanced buffer stock service level as what we did in Sections 4 and 5, respectively. We summarize the results as follows:

$$\widetilde{OVBCT}_{i,LTR} = \sqrt{L_i}(1 - \widetilde{\Omega}\sqrt{\gamma})\sigma_i\Phi^{-1}(\alpha)\theta, \text{ for } i = (1,2). \quad (50)$$

In addition, under the extended model in which $L_1 \neq L_2$, for both Manufacturers 1 and 2, if they use the initially available buffer stock budget to improve buffer stock service level, the new inventory service level is the same for both manufacturers and is given by:

$$\tilde{\alpha}_{New,BCT}^{co} = \Phi(\frac{\Phi^{-1}(\alpha)}{\widetilde{\Omega}\sqrt{\gamma}}), \quad (51)$$

and $\tilde{\alpha}_{New,BCT}^{co} > \alpha$.

From (50) and (51), it is obvious that the findings and qualitative managerial insights under the basic model continue to hold in this extended model when the replenishment lead times are different. BCT can bring a substantial positive value (either reduction of inventory cost or improvement of inventory service level) to support implementation of the buffer stock sharing scheme. Our core findings derived from the basic model remain robust.

6.3.2. Stochastic Lead Times

In this subsection, we discuss the situation in which $L_1 \neq L_2$, and each L_i (i = 1,2) is random, following the normal distribution with mean μ_{L_i} and variance $\sigma_{L_i}^2$. If the two manufacturers plan their buffer stocks separately (i.e. the "no cooperation scenario"), the required buffer stock for Manufacturers *i* (*i* = 1, 2) is given in the following:

$$\widetilde{BS}_{i,UCL} = \tilde{\sigma}_i^{UCL}\Phi^{-1}(\alpha), \quad (52)$$

where $\tilde{\sigma}_i^{UCL} = \sqrt{\mu_{L_i}\sigma_i{}^2 + \mu_i{}^2\sigma_{L_i}{}^2}$. (53)

Under the buffer stock sharing scheme, which is supported by the BCT, we have the standard deviation of the aggregated demands of the two markets faced by the two manufacturers as follows:

$$\tilde{\sigma}_{\Sigma}^{UCL} = \sqrt{(\tilde{\sigma}_1^{UCL})^2 + (\tilde{\sigma}_2^{UCL})^2 + 2\rho\sigma_1\sigma_2[\mu_{L_1}p + \mu_{L_2}(1-p)]}, \quad (54)$$

where, $p = \Phi^{-1}\left(\frac{\mu_{L_2} - \mu_{L_1}}{\sqrt{\sigma_{L_1}^2 + \sigma_{L_2}^2}}\right)$.

Define the aggregated buffer stock as:

$$\widetilde{BS}_{\Sigma}^{UCL} = \tilde{\sigma}_{\Sigma}^{UCL}\Phi^{-1}(\alpha) \quad . \quad (55)$$

Let $\widetilde{\Omega}^{UCL} = \frac{\tilde{\sigma}_{\Sigma}^{UCL}}{\tilde{\sigma}_1^{UCL} + \tilde{\sigma}_2^{UCL}}$, $(0 < \widetilde{\Omega}^{UCL} \leq 1$, when $\rho \leq \frac{\tilde{\sigma}_1^{UCL}\tilde{\sigma}_2^{UCL}}{\sigma_1\sigma_2[\mu_{L_1}p + \mu_{L_2}(1-p)]})$; (56)

then, the required amounts of buffer stocks for Manufacturers 1 and 2 are, respectively:

$$\widetilde{BS}_{1,UCL}^{CO} = \left(\frac{\tilde{\sigma}_1^{UCL}}{\tilde{\sigma}_1^{UCL} + \tilde{\sigma}_2^{UCL}}\right)\tilde{\sigma}_{\Sigma}^{UCL}\Phi^{-1}(\alpha) = \tilde{\sigma}_1^{UCL}\widetilde{\Omega}^{UCL}\Phi^{-1}(\alpha), \quad (57)$$

$$\widetilde{BS}_{2,UCL}^{CO} = \left(\frac{\tilde{\sigma}_2^{UCL}}{\tilde{\sigma}_1^{UCL}+\tilde{\sigma}_2^{UCL}}\right)\tilde{\sigma}_{\Sigma}^{UCL}\Phi^{-1}(\alpha) = \tilde{\sigma}_2^{UCL}\tilde{\Omega}^{UCL}\Phi^{-1}(\alpha). \quad (58)$$

Therefore, under this extended model in which $L_1 \neq L_2$, and L_i is stochastic and follows a normal distribution: $N\left(\mu_{L_i}, \sigma_{L_i}^2\right)$ (i = 1,2), if both manufacturers are honest and they faithfully join the buffer stock sharing scheme, the required amount of buffer stock for Manufacturer i is:

$$\widetilde{BS}_{i,UCL}^{CO} = \tilde{\sigma}_i^{UCL}\tilde{\Omega}^{UCL}\Phi^{-1}(\alpha), \text{ and} \quad (59)$$

$$\widetilde{BS}_{i,UCL}^{CO} < \widetilde{BS}_{i,UCL} \text{ when } \rho < \frac{\tilde{\sigma}_1^{UCL}\tilde{\sigma}_2^{UCL}}{\sigma_1\sigma_2[\mu_{L_1}p+\mu_{L_2}(1-p)]}.$$

Compared with Proposition 1, we see that, except for the condition that makes the inequality hold, the results and expression forms in both the basic model and the current extended model are similar.

Define $L_{i,BCT} = \gamma L_i$, where i = 1,2, $0 < \gamma < 1$, then we have,

$$\tilde{\sigma}_{i,LTR}^{UCL} = \sqrt{\gamma\mu_{L_i}\sigma_i{}^2 + \gamma^2\mu_i{}^2\sigma_{L_i}{}^2},\ \tilde{\sigma}_{i,LTR}^{UCL} < \tilde{\sigma}_i^{UCL}. \quad (60)$$

The standard deviation of the aggregated demands of the two markets faced by the two manufacturers during the stochastic lead times γL_1 and γL_2 is:

$$\tilde{\sigma}_{\Sigma,LTR}^{UCL} = \sqrt{(\tilde{\sigma}_{1,LTR}^{UCL})^2 + (\tilde{\sigma}_{2,LTR}^{UCL})^2 + 2\gamma\rho\sigma_1\sigma_2[\mu_{L_1}p + \mu_{L_2}(1-p)]}. \quad (61)$$

Define:

$$\tilde{\Omega}_{UCL}^{LTR} = \left(\frac{\tilde{\sigma}_{\Sigma,LTR}^{UCL}}{\tilde{\sigma}_{1,LTR}^{UCL}+\tilde{\sigma}_{2,LTR}^{UCL}}\right), \left(0 < \tilde{\Omega}_{UCL}^{LTR} \leq 1, \text{ when } \rho \leq \frac{\tilde{\sigma}_{1,LTR}^{UCL}\tilde{\sigma}_{2,LTR}^{UCL}}{\sigma_1\sigma_2\gamma[\mu_{L_1}p+\mu_{L_2}(1-p)]}\right). \quad (62)$$

Similar to the basic model, we can derive the value of BCT $O\widehat{VBCT}_i^{UCL}$ as well as the enhanced buffer stock service level $\tilde{\alpha}_{New,BCT}^{UCL,CO}$ under the general case in which the lead times are different and stochastic. We summarize the results as follows:

$$O\widehat{VBCT}_i^{UCL} = \left(\tilde{\sigma}_i^{UCL} - \tilde{\sigma}_{i,LTR}^{UCL}\tilde{\Omega}_{UCL}^{LTR}\right)\Phi^{-1}(\alpha)\theta, \text{ for } i = 1, 2. \quad (63)$$

$$\tilde{\alpha}_{New,BCT}^{UCL,CO} = \Phi\left(\frac{\tilde{\sigma}_i^{UCL}\Phi^{-1}(\alpha)}{\tilde{\sigma}_{i,LTR}^{UCL}\tilde{\Omega}_{UCL}^{LTR}}\right), \textit{ and } \tilde{\alpha}_{New,BCT}^{UCL,CO} > \alpha. \quad (64)$$

As a remark, from (63) and (64), it is obvious that the findings and qualitative managerial insights under the basic model continue to hold. Besides, observe that the impact brought by the rates of inventory reduction under different lead times (both fixed and stochastic scenarios) is similar to the one under the basic model, which validates the robustness of our major findings.

6.4. Different Service Levels

In the basic model, we consider the situation when the inventory service levels of two manufacturers are the same, i.e., $\alpha_1 = \alpha_2 = \alpha$. In this extended model, we further investigate the impact of different service levels of the manufacturers on the BCT mediated buffer stock sharing scheme. We relax the original assumption and let the service level be different, i.e., $\alpha_1 \neq \alpha_2$. Without loss of generality, we assume that $\alpha_1 > \alpha_2$. In addition, we investigate two scenarios in which the replenishment lead time is fixed and stochastic, respectively. Closed-form analytical results are obtained to verify the robustness of our research findings derived from the basic model.

6.4.1. Fixed Lead Time

In this subsection, we discuss the situation in which $\alpha_1 \neq \alpha_2$, and L is fixed. Similar to the basic model, if the two manufacturers plan their buffer stocks separately (i.e. the "no cooperation scenario"), the required buffer stock for Manufacturers i (i = 1, 2) is given as follows:

$$\widehat{BS}_i = \sqrt{L}\sigma_i\Phi^{-1}(\alpha_i). \tag{65}$$

Define:

$$\Phi^{-1}(\alpha_\Sigma) = \sqrt{\Phi^{-1}(\alpha_1)\Phi^{-1}(\alpha_2)}. \tag{66}$$

Here, to make the problem tractable and obtain closed-form managerial insights, we apply the geometric mean to calculate the aggregated service level of the two manufacturers. Then, the aggregated buffer stock under the BCT mediated buffer stock sharing scheme is:

$$\widehat{BS}_\Sigma^{co} = \sqrt{L}\sigma_{1+2}\Phi^{-1}(\alpha_\Sigma). \tag{67}$$

The required buffer stocks for Manufacturer i is:

$$\widehat{BS}_i^{co} = \frac{\sigma_i\Phi^{-1}(\alpha_i)}{\sigma_1\Phi^{-1}(\alpha_1)+\sigma_2\Phi^{-1}(\alpha_2)}\sqrt{L}\sigma_{1+2}\Phi^{-1}(\alpha_\Sigma). \tag{68}$$

Define:

$$\widehat{\Omega} = \frac{\sigma_{1+2}\Phi^{-1}(\alpha_\Sigma)}{\sigma_1\Phi^{-1}(\alpha_1)+\sigma_2\Phi^{-1}(\alpha_2)}, \tag{69}$$

where $0 < \widehat{\Omega} \leq 1$, when $\sigma_1^2\Phi^{-1}(\alpha_1) - \sigma_2^2\Phi^{-1}(\alpha_2) \geq -\frac{2(1-\rho)\sigma_1\sigma_2\Phi^{-1}(\alpha_1)\Phi^{-1}(\alpha_2)}{\Phi^{-1}(\alpha_1)-\Phi^{-1}(\alpha_2)})$.

Therefore, for $i = (1, 2)$, under the extended model in which $\alpha_1 \neq \alpha_2$, if both manufacturers are honest and they faithfully join the buffer stock sharing scheme, the required amount of buffer stock for Manufacturer i is:

$$\widehat{BS}_{\mathrm{i}}^{co} = \sqrt{L}\sigma_i\widehat{\Omega}\Phi^{-1}(\alpha_i),$$

and $\widehat{BS}_i^{co} < \widehat{BS}_i$, when $\sigma_1^2\Phi^{-1}(\alpha_1) - \sigma_2^2\Phi^{-1}(\alpha_2) > -\frac{2(1-\rho)\sigma_1\sigma_2\Phi^{-1}(\alpha_1)\Phi^{-1}(\alpha_2)}{\Phi^{-1}(\alpha_1)-\Phi^{-1}(\alpha_2)}$. (70)

Compared with Proposition 1, we see that the results in both the basic model and the extended model are similar. In particular, the amount of required buffer stocks under the sharing scheme for both models exhibit the very similar form. However, the condition that makes the inequality in (70) hold becomes complex. It indicates that the buffer stock sharing scheme can always bring a reduction of the amount of required buffer stock when $\sigma_1 > \sigma_2$ with $\alpha_1 > \alpha_2$. While, the situation may not hold when $\sigma_1 << \sigma_2$with $\alpha_1 > \alpha_2$. In addition, the new division ratio of the buffer stock for Manufacturer i (i = 1,2), under the buffer stock sharing scheme, becomes:

$$\frac{\sigma_i\Phi^{-1}(\alpha_i)}{\sigma_1\Phi^{-1}(\alpha_1)+\sigma_2\Phi^{-1}(\alpha_2)}. \tag{71}$$

It is interesting to note that the service level takes part in the buffer stock division as well. Intuitively, given the standard deviations of the market demands of the two manufacturers during per unit duration of time, if Manufacturer 1 has a higher service level, the proportion of buffer stock undertaken by Manufacturer 1 is supposed to be increased.

Define:

$L_{i,BCT} = \gamma L_i$, where i = 1,2, $0 < \gamma < 1$. (72)

$\widehat{BS}_{i,LTR}^{co} = \sqrt{\gamma L}\sigma_i \widehat{\Omega}\Phi^{-1}(\alpha_i)$. (73)

Similar to the basic model, we can derive and explore the value of BCT $\widehat{OVBCT}_i$ as well as the enhanced buffer stock service level $\hat{\alpha}_{i,New,BCT}^{CO}$ as follows:

$\widehat{OVBCT}_i = \sqrt{L}(\sigma_i \Phi^{-1}(\alpha_i)\theta)(1 - \widehat{\Omega}\sqrt{\gamma})$, for $i = (1, 2)$. (74)

Under this extended model in which $\alpha_1 \neq \alpha_2$, for both Manufacturers 1 and 2, if they use the initially available buffer stock budget to improve buffer stock service level, the new inventory service level for each manufacturer is different from each other. It is given by:

$\hat{\alpha}_{i,New,BCT}^{co} = \Phi\left(\frac{\Phi^{-1}(\alpha_i)}{\widehat{\Omega}\sqrt{\gamma}}\right)$, for $i = (1,2)$, (75)

and $\hat{\alpha}_{i,New,BCT}^{co} > \alpha_i$.

From (74) and (75), we can see that the findings and qualitative managerial insights under the basic model continue to hold in this extended model when the service levels of two manufacturers are different. BCT can bring a substantial positive value (either reduction of inventory cost or improvement of inventory service level) to support implementation of the buffer stock sharing scheme, except the extreme case when $\sigma_1 << \sigma_2$ with $\alpha_1 > \alpha_2$. More investigation of the extreme case is conducted in Subsection 6.4.2. Therefore, except for the extreme case, our core findings derived from the basic model are robust.

6.4.2. Stochastic Lead Time

In this subsection, we discuss the situation in which $\alpha_1 \neq \alpha_2$, and L is a random variable, following the normal distribution with mean μ_L and variance σ_L^2. If the two manufacturers plan their buffer stocks separately (i.e. the "no cooperation scenario"), the required amount of buffer stock for Manufacturers i (i = 1, 2) is given in the following:

$\widehat{BS}_{i,UCL} = \sigma_i^{UCL}\Phi^{-1}(\alpha_i)$, for i = (1,2). (76)

Define the aggregated buffer stock as:

$\widehat{BS}_{\Sigma}^{UCL} = \sigma_{\Sigma}^{UCL}\Phi^{-1}(\alpha_{\Sigma})$. (77)

Then, the required buffer stocks for Manufacturers i (i = 1,2):

$\widehat{BS}_{i,UCL}^{CO} = \left(\frac{\sigma_i^{UCL}\Phi^{-1}(\alpha_i)}{\sigma_1^{UCL}\Phi^{-1}(\alpha_1)+\sigma_2^{UCL}\Phi^{-1}(\alpha_2)}\right)\sigma_{\Sigma}^{UCL}\Phi^{-1}(\alpha_{\Sigma})$. (78)

Define:

$\widehat{\Omega}_{UCL} = \frac{\sigma_{\Sigma}^{UCL}\Phi^{-1}(\alpha_{\Sigma})}{\sigma_1^{UCL}\Phi^{-1}(\alpha_1)+\sigma_2^{UCL}\Phi^{-1}(\alpha_2)}$, (79)

$0 < \widehat{\Omega}_{UCL} \leq 1$, when

$$(\sigma_1^{UCL})^2\Phi^{-1}(\alpha_1) - (\sigma_2^{UCL})^2\Phi^{-1}(\alpha_2) \geq -\frac{2(\sigma_1^{UCL}\sigma_2^{UCL} - \rho\sigma_1\sigma_2\mu_L - \mu_1\mu_2\sigma_L^2)\Phi^{-1}(\alpha_1)\Phi^{-1}(\alpha_2)}{\Phi^{-1}(\alpha_1) - \Phi^{-1}(\alpha_2)}.$$

As a result, for $i = (1, 2)$, under the extended model in which $\alpha_1 \neq \alpha_2$ and L is stochastic and follows $N(\mu_L, \sigma_L^2)$: (a) If both manufacturers are honest and they faithfully join the buffer stock sharing scheme, the required amount of buffer stock for Manufacturer *i* (*i* = 1, 2) is:

$\widehat{BS}_{i,UCL}^{CO} = \sigma_i^{UCL}\widehat{\Omega}_{UCL}\Phi^{-1}(\alpha_i)$, and

$$\widehat{BS}_{i,UCL}^{CO} < \widehat{BS}_{i,UCL}, \tag{80}$$

when $(\sigma_1^{UCL})^2\Phi^{-1}(\alpha_1) - (\sigma_2^{UCL})^2\Phi^{-1}(\alpha_2) > -\frac{2(\sigma_1^{UCL}\sigma_2^{UCL} - \rho\sigma_1\sigma_2\mu_L - \mu_1\mu_2\sigma_L^2)\Phi^{-1}(\alpha_1)\Phi^{-1}(\alpha_2)}{\Phi^{-1}(\alpha_1) - \Phi^{-1}(\alpha_2)}$).

In this case, the new division ratio of the buffer stock for Manufacturer *i* (*i* = 1,2), under the buffer stock sharing scheme becomes:

$$\frac{\sigma_i^{UCL}\Phi^{-1}(\alpha_i)}{\sigma_1^{UCL}\Phi^{-1}(\alpha_1) + \sigma_2^{UCL}\Phi^{-1}(\alpha_2)}. \tag{81}$$

As we can see, the results in both the basic model and the current extended model are similar. However, the condition that makes the inequality in (80) hold becomes more complex when the service levels are different with stochastic lead times. It indicates that the buffer stock sharing scheme can lead to a reduction of the amount of required buffer stock when $\sigma_1^{UCL} > \sigma_2^{UCL}$ with $\alpha_1 > \alpha_2$. It is important to note that, the situation may not hold when $\sigma_1^{UCL} << \sigma_2^{UCL}$ with $\alpha_1 > \alpha_2$.

Similar to the previous analysis in Subsection 6.4.1, the service level takes part in the buffer stock division as well under stochastic lead time scenario. Given the standard deviations of the market demand of the two manufacturers during the stochastic lead time replenishment cycles, if Manufacturer 1 has a higher service level, intuitively the proportion of buffer stock held by Manufacturer 1 is supposed to be increased.

Define:

$$\widehat{\Omega}_{UCL}^{LTR} = \frac{\sigma_{\Sigma,LTR}^{UCL}\Phi^{-1}(\alpha_\Sigma)}{\sigma_{1,LTR}^{UCL}\Phi^{-1}(\alpha_1) + \sigma_{2,LTR}^{UCL}\Phi^{-1}(\alpha_2)}. \tag{82}$$

where, $0 < \widehat{\Omega}_{UCL}^{LTR} \leq 1$,

when $$(\sigma_{1,LTR}^{UCL})^2\Phi^{-1}(\alpha_1) - (\sigma_{2,LTR}^{UCL})^2\Phi^{-1}(\alpha_2) \geq -\frac{2(\sigma_{1,LTR}^{UCL}\sigma_{2,LTR}^{UCL} - \rho\gamma\sigma_1\sigma_2\mu_L - \mu_1\mu_2\gamma^2\sigma_L^2)\Phi^{-1}(\alpha_1)\Phi^{-1}(\alpha_2)}{\Phi^{-1}(\alpha_1) - \Phi^{-1}(\alpha_2)}.$$

Similarly, we can derive the value of BCT $O\widehat{VBCT}_i^{UCL}$ as well as the enhanced buffer stock service level $\hat{\alpha}_{i,New,BCT}^{UCL,CO}$ under the general case in which the service levels are different and lead time is stochastic. To summarize, we have the following analytical expressions:

$$O\widehat{VBCT}_i^{UCL} = \left(\sigma_i^{UCL} - \sigma_{i,LTR}^{UCL}\widehat{\Omega}_{UCL}^{LTR}\right)\Phi^{-1}(\alpha_i)\theta. \tag{83}$$

$$\hat{\alpha}_{i,New,BCT}^{UCL,CO} = \Phi\left(\frac{\sigma_i^{UCL}\Phi^{-1}(\alpha_i)}{\sigma_{i,LTR}^{UCL}\widehat{\Omega}_{UCL}^{LTR}}\right), \text{ and } \hat{\alpha}_{i,New,BCT}^{UCL,CO} > \alpha_i. \tag{84}$$

To identify the general case in which the BCT mediated buffer stock sharing scheme can always bring benefits under different service levels, we have the following lemma and proposition.

Define $\Delta_\alpha = \alpha_1 - \alpha_2 > 0$ and

$$K = \frac{\sqrt{((\sigma_{1,LTR}^{UCL} - \sigma_{2,LTR}^{UCL})^2 + 2\rho\gamma\sigma_1\sigma_2\mu_L + 2\mu_1\mu_2\gamma^2\sigma_L^2)^2 - 4(\sigma_{1,LTR}^{UCL})^2(\sigma_{2,LTR}^{UCL})^2} + (\sigma_{1,LTR}^{UCL} - \sigma_{2,LTR}^{UCL})^2 + 2\rho\gamma\sigma_1\sigma_2\mu_L + 2\mu_1\mu_2\gamma^2\sigma_L^2}{2(\sigma_{1,LTR}^{UCL})^2}.$$

Lemma 5. *If* $\Delta_\alpha > \Phi\left(K \cdot \Phi^{-1}(\alpha_2)\right) - \alpha_2$, *we have* $0 < \widehat{\Omega}_{UCL}^{LTR} < 1$, *thus* $\widehat{OVBCT}_i^{UCL} > 0$ *always holds.*

Note that K is an increasing function of the demand standard deviation of Manufacturer 2 during the stochastic replenishment lead time L, i.e., $\sigma_{2,LTR}^{UCL}$. Thus, from Lemma 5, we have Proposition 7.

Proposition 7 (Buffer stock balancing). *For* $i = (1, 2)$, *under the extended model in which* $\alpha_1 > \alpha_2$ *and* L *is stochastic and follows* $N(\mu_L, \sigma_L^2)$: *If both manufacturers are honest and faithfully join the buffer stock sharing scheme, Manufacturer 1 with a low demand uncertainty level will set the service level higher enough (i.e.,* $\alpha_1 > \Phi\left(K \cdot \Phi^{-1}(\alpha_2)\right)$) *to complement the high buffer stock amount incurred by the high demand uncertainty level of Manufacturer 2 (e.g.* $\sigma_{2,LTR}^{UCL} > 2\sigma_{1,LTR}^{UCL}$ *), so that the mutual benefit by buffer stock reduction is guaranteed under the sharing scheme* (i.e., $\widehat{OVBCT}_i^{UCL} > 0$).

Proposition 7 indicates that, for the two manufacturers, when one faces a low demand standard deviation and the other faces a much higher demand standard deviation (e.g. $\sigma_{2,LTR}^{UCL} > 2\sigma_{1,LTR}^{UCL}$), the manufacturer who faces a low demand deviation but targets for a high service level will target a service level sufficiently high so that both manufacturers will benefit from the sharing scheme by buffer stock reduction; otherwise, both of the manufacturers get limited benefit from the buffer stock sharing scheme and may even suffer a loss under the sharing scheme brought by the big gap between the market demand deviations of two manufacturers. In particular, the gap becomes much wider under the stochastic environment. Therefore, it is unwise to make an alliance between the "low-expected-demand low-demand-uncertainty" (LL) manufacturer and the "high-expected-demand high-demand-uncertainty" (HH) manufacturer, unless the LL manufacturer aims at achieving a sufficiently high service level. Proposition 7 in fact verifies the general case, which also holds for the case when the lead time is fixed.

Through the above discussion, we find that in most cases, the findings and qualitative managerial insights under the basic model (e.g., the impact of the rate of inventory reduction) continue to hold in the extended models when the service levels are different (for both fixed and stochastic lead time cases), in which BCT can bring a substantial positive value for both of the alliance members (either reduction of inventory cost or improvement of inventory service level) to support implementation of the buffer stock sharing scheme. Therefore, except for the extreme case that $\sigma_2 >> \alpha_1$, our core findings derived from the basic model remain to hold.

7. Further Extended Analyses

7.1. Impacts on Upstream Supplier

In the above basic model and extended models, we have examined the robust advantages of BCT in facilitating fair and beneficial buffer stock sharing in a horizontal setting, especially on overcoming the cheating problem and the shortening of lead time. Therefore, with the application of BCT, manufacturers could be benefited from either reduced buffer stock levels or improved service levels. However, from the perspective of upstream suppliers, whether the downstream utilization of BCT benefits or hurts them is unknown. Therefore, in this section, we extend our analysis to study the supply chain (Luo et al. 2014; Shi et al. 2014; Xu et al. 2015; Li et al. 2019) and uncover impacts of BCT on the supplier side that depend on the different strategies adopted by the manufacturers (i.e., reducing buffer stock levels or improving service levels).

7.1.1 Reducing Buffer Stock

First, we consider the case when the manufacturers choose to reduce their buffer stock levels with the assistance of BCT.

Consider that the supplier offers Material A to the manufacturers at a unit profit margin *p* (*p*>0). Through the analyses in Sections 3, 4, and 6.1, we could obtain the buffer stock levels required by Manufacturer *i* under deterministic or stochastic lead time both with and without the use of BCT. Therefore, we summarize the reduction in supplier's profit due to the decrease in buffer stock levels of the manufacturers in Proposition 8 as follows.

Proposition 8. *When the manufacturers choose to reduce their buffer stock levels through the application of blockchain facilitated buffer stock sharing scheme, the total profit of the supplier is reduced by:*

(a) Fixed lead time: $RP_{sum,LTR}^{CO} = p\Phi^{-1}(\alpha)\sqrt{L}[(\sigma_1+\sigma_2)-\Omega\sqrt{\gamma}(\sigma_1+\sigma_2)]$, ($RP_{sum,LTR}^{CO}>0$, $\frac{\partial RP_{sum,LTR}^{CO}}{\partial\gamma}<0$) (85)

(b) Stochastic lead time: $RP_{sum,LTR}^{UCL,CO} = p\Phi^{-1}(\alpha)[\sigma_1^{UCL}+\sigma_2^{UCL}-\Omega_{UCL}^{LTR}(\sigma_{1,LTR}^{UCL}+\sigma_{2,LTR}^{UCL})]$, ($RP_{sum,LTR}^{UCL,CO}>0$, $\frac{\partial RP_{sum,LTR}^{UCL,CO}}{\partial\gamma}<0$). (86)

Proposition 8 shows the closed-form analytical expressions of the overall profit reduction of the supplier ($RP_{sum,LTR}^{CO}$ and $RP_{sum,LTR}^{UCL,CO}$), and indicates the result that the BCT mediated buffer stock sharing scheme will always lead to a profit loss for the supplier if the manufacturers choose to reduce their buffer stock levels no matter whether lead time is fixed or stochastic. This is caused by the resulting decline in manufacturers' ordering quantities of Material A. Obviously, the profit loss of the supplier is increasing with its profit margin *p*. In addition, it is observed that the profit reduction is directly proportional to the manufacturer's buffer stock service level α . This is intuitive that when a manufacturer requires a higher service level, the benefits for the manufacturer brought by the BCT would increase (i.e., a sharper reduction in the required buffer stock level), which leads to greater suffering for the supplier. Moreover, it is interesting to note that a more significant blockchain-facilitated reduction of lead time (i.e., γ decreases), which helps cut down the buffer stock for manufacturers, would impair the gains of the supplier more significantly.

Consequently, in this section, we analytically show that the downstream application of BCT hurts the upstream supplier due to a decrease in profit, if the downstream agents (i.e., manufacturers) choose to benefit itself through reducing buffer stock levels.

7.1.2 Improving Service Levels

Second, we analyze the interesting case when the downstream manufacturers decide to improve their service levels through the implementation of BCT to achieve greater operations success or competitive advantages.

As the amount of growth in service level is becoming increasingly low when the buffer stock level increases, to be efficient, it is reasonable that the manufacturers would plan to improve their buffer stock service levels only up to a certain pre-determined upper limit ψ . Therefore, with the same budget as before, the improved buffer stock service level supported by the BCT is $\alpha_{New,BCT}^{CO} = \Phi\left(\frac{\Phi^{-1}(\alpha)}{\Omega\sqrt{\gamma}}\right)$ under the fixed lead time case (if $\Phi\left(\frac{\Phi^{-1}(\alpha)}{\Omega\sqrt{\gamma}}\right) \le \psi$), or $\alpha_{New,BCT}^{UCL,CO} = \Phi\left(\frac{\sigma_i^{UCL}\Phi^{-1}(\alpha)}{\sigma_{i,LTR}^{UCL}\Omega_{UCL}^{LTR}}\right)$ under the stochastic lead time case (if $\Phi\left(\frac{\sigma_i^{UCL}\Phi^{-1}(\alpha)}{\sigma_{i,LTR}^{UCL}\Omega_{UCL}^{LTR}}\right) \le \psi$). However, if the upper limit is reached, then we have $\alpha_{New,BCT}^{CO} = \psi$ (under the fixed lead time case), or $\alpha_{New,BCT}^{UCL,CO} = \psi$ (under the stochastic lead time case). We summarize the results in Proposition 9.

Proposition 9. *If* $\Phi\left(\frac{\Phi^{-1}(\alpha)}{\Omega\sqrt{\gamma}}\right) \le \psi$ *under the fixed lead time case, or* $\Phi\left(\frac{\sigma_i^{UCL}\Phi^{-1}(\alpha)}{\sigma_{i,LTR}^{UCL}\Omega_{UCL}^{LTR}}\right) \le \psi$ *under the stochastic lead time case, when the manufacturers decide to improve their buffer stock service levels through the application of blockchain facilitated buffer stock sharing scheme using the same budget as before, Pareto improvement*[10] *can always be achieved. On the other hand, if* $\Phi\left(\frac{\Phi^{-1}(\alpha)}{\Omega\sqrt{\gamma}}\right) > \psi$ *(under the fixed lead time case), or* $\Phi\left(\frac{\sigma_i^{UCL}\Phi^{-1}(\alpha)}{\sigma_{i,LTR}^{UCL}\Omega_{UCL}^{LTR}}\right) > \psi$ *(under the stochastic lead time case), the supplier would suffer from a profit loss, and Pareto improvement cannot be achieved.*

Proposition 9 is a crucial result, which shows that the supplier will not suffer or benefit from the downstream utilization of BCT when the manufacturers decide to raise their buffer stock service levels using the same budget as before if the improved service levels are no higher than the pre-determined upper limit ψ . This finding can be explained by noting that although the overall demand uncertainty is reduced through risk pooling and the lead time is shortened through the use of blockchain for the manufacturers, the total ordering quantity of Material A for the supplier remains invariable due to the growths in manufacturers' service levels. Therefore, it implies that when the manufacturers choose to enhance their business by improving service levels, Pareto improvement could be achieved for both the downstream and upstream supply chain members. Consequently, we have proven that the manufacturers could enjoy the benefits brought by BCT and

[10] Pareto improvement: For a supply chain system, after a strategy is adopted, members involved should be benefited (or at least not worse off).

simultaneously protect the profit of the upstream supplier by adopting the service level improvement strategy using the same budget, instead of the buffer stock reduction tactic. However, if the required "improved service levels" are higher than ψ, the manufacturers would only increase their service levels up to this upper limit to avoid trivial improvement. Thus, the ordering quantity of Material A declines, which leads to a profit loss for the supplier.

As a remark, although we apply the terminology "supplier" in this section to study the impact of downstream application of BCT on the upstream agents, the insights generated are not restricted to material suppliers. Instead, they are also applicable to manufacturers (when downstream retailers apply the BCT mediated buffer stock sharing scheme) or even service providers, as we suggested in Section 1.

7.2 Logistics Services Considerations

In the previous analyses, we consider the downstream utilization of BCT without the consideration of logistics services (i.e. the delivery of Material A from the supplier to the manufacturer). However, the quality and cost of logistics services are crucial for the decisions of manufacturers and the impact of BCT on supply chain members. For example, in the past, when BCT was not applied, adopting ordinary logistics services[11] would benefit manufacturers with low transportation costs, but would also result in long lead time and low reliability (i.e., high variance in lead time). Therefore, it is reasonable that manufacturers may pursue superior delivery services offered by premier logistics services providers[12] (e.g., DHL, Fedex), to enjoy the merits of short lead time and high reliability (i.e., low variance in lead time), which contributes to reduction of buffer stock. However, the cost of premier logistics services is much higher than that of the ordinary logistics services. Since the development of BCT facilitates faithful buffer stock sharing scheme and helps reduce lead time through smart contracting or virtual currency, therefore, the ordinary logistics services could be applied to achieve a satisfactory buffer stock level while the transportation cost is not very high. However, the average lead time and variance of lead time of the ordinary logistics services with the downstream implementation of blockchain might still be larger than those of the premier logistics services[13]. Therefore, with the consideration of logistics services, whether the downstream use of BCT could benefit the manufacturers and the supplier simultaneously is an interesting topic to investigate. Consequently, in this section, we analytically compare the performances of the supply chain members between two scenarios: (i) Premier logistics services are adopted but blockchain is absent (named as Scenario PNB); and (ii) ordinary logistics services are used and the blockchain facilitated buffer stock sharing scheme is applied (named as Scenario CB)[14]. In addition, the case of premier logistics services with the application of BCT is excluded from analysis. This is because the manufacturer will always benefit, but the supplier will always suffer from the reduced buffer stock level in this case.

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[11] "Ordinary logistics services" are characterized by low cost, long lead time, and high lead time variance.

[12] "Premier logistics services" are characterized by high cost, short lead time, and low lead time variance.

[13] Note that if the average lead time and variance of lead time of the ordinary logistics services with the downstream implementation of blockchain are smaller than those of the premier logistics services, the manufacturer could always benefit, while the supplier would always suffer, from the application of blockchain. We intentionally exclude this case to focus on the interesting cases where both members could get benefits from the use of BCT.

[14] The buffer stock service levels keep unchanged during scenario switch for the manufacturers.

First, in Scenario PNB, we denote the cost of premier logistics services per delivery as k_p. The lead time when premier logistics services are utilized follows a normal distribution with the mean of $\mu_{L,pre}$ and variance of ${\sigma_{L,pre}}^2$. Accordingly, the buffer stock level for Manufacturer *i* is denoted by $BS_{i,UCL}^{pre}$, the cost for Manufacturer *i* is $MC_{i,pre}^{UCL}$, and the overall profit for the supplier is $SP_{sum,pre}^{UCL}$. Therefore, we have the following expressions:

$$\sigma_{i,pre}^{UCL} = \sqrt{\mu_{L,pre}{\sigma_i}^2 + {\mu_i}^2{\sigma_{L,pre}}^2}, \tag{87}$$

$$BS_{i,UCL}^{pre} = \sigma_{i,pre}^{UCL}\Phi^{-1}(\alpha), \tag{88}$$

$$MC_{i,pre}^{UCL} = \sigma_{i,pre}^{UCL}\Phi^{-1}(\alpha)\theta + k_p, \tag{89}$$

$$SP_{sum,pre}^{UCL} = (\sigma_{1,pre}^{UCL} + \sigma_{2,pre}^{UCL})\Phi^{-1}(\alpha)p. \tag{90}$$

Then, in Scenario CB, we represent the cost of ordinary logistics services per delivery as k_c. Note that $k_p > k_c > 0$. After the manufacturers establish the buffer stock sharing scheme with the application of BCT, an ordinary logistics service is used where the lead time follows a normal distribution with the mean of $\mu_{L,ord}$ and variance of ${\sigma_{L,ord}}^2$. Accordingly, the buffer stock level for Manufacturer *i* is denoted by $BS_{i,UCL}^{ord}$, the cost for Manufacturer *i* is $MC_{i,ord}^{UCL}$, and the overall profit for the supplier is $SP_{sum,ord}^{UCL}$. Note that $\mu_{L,ord} > \mu_{L,pre} > 0$, and $\sigma_{L,ord} > \sigma_{L,pre} > 0$. Therefore, we have the following:

$$\sigma_{i,ord}^{UCL} = \sqrt{\mu_{L,ord}{\sigma_i}^2 + {\mu_i}^2{\sigma_{L,ord}}^2}, \quad \sigma_{\Sigma,ord}^{UCL} = \sqrt{\mu_{L,ord}\sigma_{1+2}^2 + (\mu_1+\mu_2)^2{\sigma_{L,ord}}^2}, \quad \Omega_{UCL}^{ord} = \left(\frac{\sigma_{\Sigma,ord}^{UCL}}{\sigma_{1,ord}^{UCL} + \sigma_{2,ord}^{UCL}}\right), \tag{91}$$

$$BS_{i,UCL}^{ord} = \sigma_{i,ord}^{UCL}\Omega_{UCL}^{ord}\Phi^{-1}(\alpha), \tag{92}$$

$$MC_{i,ord}^{UCL} = \sigma_{i,ord}^{UCL}\Omega_{UCL}^{ord}\Phi^{-1}(\alpha)\theta + k_c, \tag{93}$$

$$SP_{sum,ord}^{UCL} = (\sigma_{1,ord}^{UCL} + \sigma_{2,ord}^{UCL})\Omega_{UCL}^{ord}\Phi^{-1}(\alpha)p. \tag{94}$$

(Observe that we can easily find that $\sigma_{i,ord}^{UCL} > \sigma_{i,pre}^{UCL}$ and $0 < \Omega_{UCL}^{ord} \le 1$.)

Proposition 10. *When the cost difference between premier logistics services and ordinary logistics services is sufficiently large ($k_p - k_c > \Phi^{-1}(\alpha)\theta(\sigma_{i,ord}^{UCL}\Omega_{UCL}^{ord} - \sigma_{i,pre}^{UCL})$), and the integrated "level of uncertainty" of Scenario CB is larger than the sum of those for the two manufacturers in Scenario PNB ($\sigma_{\Sigma,ord}^{UCL} > \sigma_{1,pre}^{UCL} + \sigma_{2,pre}^{UCL}$), both the upstream supplier and downstream manufacturers could benefit from the downstream application of BCT. Thus, Pareto improvement is achieved.*

Proposition 10 shows the condition when both the two supply chain members could benefit from the application of BCT when the premier logistics services are switched to ordinary delivery services. Several important managerial implications are generated as follows.

From the aspects of manufacturers, they could take the advantage of the low cost of ordinary delivery services if the expenditure for the premier logistics services is sufficiently high (i.e., the cost difference between the two services is higher than a threshold). The reason behind is that the average lead time and variance of lead time of the ordinary logistics services with the downstream implementation of blockchain are larger than those of the premier logistics services, which may lead to an increase in the required buffer stock levels. Therefore, the reduction in transportation costs caused by logistics services switch should be large enough to compensate for the growth in the buffer stock budget. Besides, it is natural to note that the cost difference threshold is directly proportional to $(\sigma_{i,ord}^{UCL}\Omega_{UCL}^{ord} - \sigma_{i,pre}^{UCL})$, which represents the increase of the "level of uncertainty" from Scenario PNB to Scenario CB for Manufacturer i. It is implied that when the premier logistics services are becoming increasingly efficient and reliable, or BCT is becoming decreasingly efficient in reducing lead time, it is more difficult for the manufacturers to enjoy the low-cost advantages of ordinary logistics services with the assistance of BCT, as the cost difference threshold becomes larger. Besides, it is identified that when the rate of inventory reduction with the ordinary logistics service under uncertain lead time becomes larger (i.e., Ω_{UCL}^{ord} declines), the Pareto improvement is increasingly easy to be realized.

From the aspects of the supplier, it could enjoy a profit growth only if the manufacturers' total ordering quantity increases after the switch of logistics services and the application of blockchain-facilitated buffer stock sharing scheme. As mentioned, the rise in the average lead time and variance of lead time from Scenario PNB to Scenario CB may lead to an increase in buffer stock levels. However, it is not necessary as the buffer stock sharing scheme helps to reduce individual buffer stock levels through risk pooling. Therefore, from Proposition 10, we could see that only when the integrated "level of uncertainty" $\sigma_{\Sigma,ord}^{UCL}$ (under Scenario CB) is larger than the sum of the two manufacturers' levels of uncertainty (i.e., $\sigma_{1,pre}^{UCL} + \sigma_{2,pre}^{UCL}$) (under Scenario PNB), the overall ordering quantity of Material A would grow, which benefits the supplier. Accordingly, it is interesting to conclude that when the uncertain demands of the two manufactures become increasingly negative-correlated (ρ is more negative), it is more challenging for the supplier to enjoy a profit growth, while it is easier for the manufacturers to reduce their financial burdens, with the downstream application of BCT. Besides, it is important to observe that the use of BCT in shortening lead time also imposes inverse impacts on the influence of blockchain application on the two members. For instance, when the blockchain greatly helps to enhance lead time (e.g., $\mu_{L,ord}$ and ${\sigma_{L,ord}}^2$ are just slightly lower than $\mu_{L,pre}$ and ${\sigma_{L,pre}}^2$, respectively), the possibility for the manufacturer to enjoy the blockchain-facilitated cost reduction is higher, while that for the supplier to enjoy the blockchain-supported profit gains is lower.

Consequently, in this section, we analytically examine the conditions when both the supplier and manufacturers could benefit from the blockchain facilitated buffer stock sharing scheme, to achieve Pareto improvement with the consideration of logistics services. Specifically, the crucial inverse impacts of i) the ability of BCT in reducing lead time, and ii) the demand correlation coefficient between the two manufacturers on the supply chain members (i.e., the supplier and the manufacturers), are demonstrated.

In the above analyses, stochastic lead time is used, while all the qualitative results remain unchanged if the lead time is fixed without uncertainty (i.e., $\sigma_{L,ord} = \sigma_{L,pre} = 0$).

8. Conclusion

8.1. Concluding Remarks and Managerial Insights

Buffer stock sharing under is an important and well-known operations strategy. However, the real-world implementation of buffer stock sharing is hindered by probable occurrence of the cheating problem (e.g., a member tells lies to achieve a higher profit for himself). We propose that the BCT can serve as a practical solution to it.

Motivated by the importance of buffer stock sharing schemes and the emergence of BCT with features such as trustworthy data with permanent record, smart contracting, and virtual currency, we have analytically explored in this paper how two manufacturers can achieve efficient buffer stock sharing with the use of BCT. We have first constructed the basic model with a deterministic lead time and quantified the benefit of adopting a buffer stock sharing scheme. In the absence of BCT, we have shown the occurrence of a cheating problem. Comparing between the "faithful participation" and "cheating" cases of the buffer stock sharing scheme, we have proved that the dishonest manufacturer is always benefited whereas the honest manufacturer's benefit is smaller under the cheating case (even though it will still gain a benefit and never suffer a loss even if the cheating problem arises). However, the occurrence of cheating problem implies that a faithful buffer stock sharing scheme is uneasy to establish. We have then proposed the use of BCT to help. Moreover, since BCT can reduce lead time by its features such as smart contracting and virtual currency payment, we have further incorporated this feature into our model and derived the analytical expression for the overall value of BCT (OVBCT). We have revealed that a larger correlation coefficient of the market demands faced by the manufacturers would lead to a drop in the rate of inventory reduction and the OVBCT for both manufacturers. If the market demand uncertainty faced by one manufacturer increases, the corresponding effect on OVBCT depends highly on the relative size of demand uncertainties of the two markets. This finding implies that it is critical to note the relative size of demand uncertainties of the two manufacturers' markets before we can determine the impacts brought by a change of demand uncertainty towards the value of BCT. After that, we have explored the case when the manufacturers want to use the same original buffer stock's budget to achieve a higher inventory service level with the use of BCT. We have shown that the BCT mediated buffer stock sharing scheme is very useful in this regard because it can always yield a higher inventory service level. In addition, to show the robustness of our research findings derived from the basic model, we have constructed the extended models for the cases of stochastic lead times, different lead times, different service levels and proved that the qualitative results obtained in the basic model are still valid. Novel buffer stock division rules are then derived which help make the buffer stock sharing scheme beneficial. In addition, an *n*-manufacturers alliance is further analytically explored and we have uncovered the optimal condition for the alliance members to consider expanding the number of participating manufacturers under the BCT mediated sharing scheme. Moreover, we have analyzed the diverse impacts of the downstream use of BCT on the upstream agents like suppliers. Finally, we have analyzed the condition to achieve Pareto improvement for the supply chain members (i.e., the supplier and the manufacturers) with the considerations of premier and ordinary logistics services.

This paper has analytically examined the value of BCT for the buffer stock sharing scheme. All important findings and insights have been analytically derived in close-form. Since the buffer stock sharing scheme and BCT are important measures in the real world, we believe that this paper not only contributes to the literature, but it also uncovers for the potential values of BCT in the buffer stock sharing scheme, which will benefit the operations managers in practice.

8.2. Managerial Implications

From the findings, we propose the following managerial implications:

Occurrence of cheating: Our findings have shown that cheating should commonly arise as the manufacturer is enticed to tell lies. The dishonest manufacturer is always benefited whereas the honest manufacturer's benefit is smaller when the cheating case occurs. Having said that, despite the potential occurrence of cheating, the negative impact brought by the cheating is basically no larger than the benefit generated from the buffer stock sharing scheme. Thus, an important managerial implication is: No matter whether cheating arises or not, it is in fact wise for the manufacturers to cooperate in buffer stock sharing.

Values of blockchain: Under the cheating problem, the honest manufacturer fails to achieve what he deserves to get. To overcome the cheating problem and further shorten lead time, the use of BCT is proven to be helpful. We interestingly reveal that the rate of inventory reduction achieved by the buffer stock sharing scheme is negatively related to the demand correlation coefficient. Therefore, under the case with smaller demand correlation coefficients, manufacturers are encouraged to establish the buffer stock sharing scheme using BCT (actually, when BCT is not applied, a smaller demand correlation coefficient also leads to a higher inventory saving through the inventory sharing scheme). If the market demand uncertainty faced by a manufacturer increases, the effect on OVBCT depends on the relative size of demand uncertainties of the two markets. This result has a good implication to managers because they should pay attention to the relative size of demand uncertainties of the two manufacturers' markets before deciding to invest BCT or not (because the impacts of BCT are related to them).

Service perspective: For those manufacturers who want to use the same buffer stock's budget to achieve a higher inventory service level, we prove that the BCT mediated buffer stock sharing scheme can always lead to an improved inventory service level. As such, engineering managers who aspire to improve inventory service should actively consider the use of BCT.

Robustness of findings: If the engineering manager is doubtful of the above proposals because the basic models look simple, they should be relieved to know that from our extended analyses (covering the cases with stochastic lead times, different lead times, different service levels, and multiple (n > 2) manufacturers participation), the qualitative insights and managerial implications derived above are valid in the more generalized and complex cases.

Supply chain: In a supply chain context, we have uncovered the impacts of the downstream utilization of BCT on the upstream supplier. Specifically, if the manufacturers choose to reduce their buffer stock levels, we interestingly show that the upstream supplier always suffers due to a decrease in profit, while Pareto improvement could be achieved if the manufacturers decide to improve their service levels. This implies that buffer stock sharing by default will not benefit all supply chain members and there is an inherent conflict between the upstream supplier's benefit and downstream manufacturers'. Regarding the use of BCT, it is important to note that the blockchain's ability in reducing lead time and the demand correlation coefficient between the two manufacturers have opposite impacts on the supply chain members (i.e., the supplier and the manufacturers). To overcome this challenge, we suggest engineering managers consider the proper choice of logistics services (ordinary logistics versus express premium logistics), which interestingly can help achieve Pareto improvement in the supply chain.

8.3. Future Research

In this paper, we have focused on exploring the manufacturer's buffer stock sharing. Future research can be conducted to examine challenging issues such as the role of BCT in strategic placement of buffer stock in multi-echelon supply chain systems (Graves and Willems 2000; Klosterhalfen et al. 2014; Brulard et al. 2018). In our models, we consider the case when only one manufacturer decides to cheat but not both. The dishonest manufacturer could lie on the correlation coefficient, and the other manufacturer will believe it. In practice, the honest manufacturer may not easily believe the information signal from the other manufacturer. It will be interesting to examine the case when both manufacturers may cheat.

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Appendix:

Table 2. Major notation used.

Decision Variables	
BS_i	The buffer stock level of Manufacturer *i*.
BS_i^{co}	The buffer stock level of Manufacturer *i* under the buffer stock sharing scheme.
$BS_{i,X}^{Y}$	The buffer stock level of Manufacturer *i* under the situations with X and Y.
$\widetilde{BS}_{i,X}^{Y}$	The buffer stock level of Manufacturer *i* when the manufacturers have different replenishment lead times under the different situations X and Y.
$\widehat{BS}_{i,X}^{Y}$	The buffer stock level of Manufacturer *i* when the manufacturers have different service levels under the different situations X and Y.
Parameters	
σ	The standard deviation of demand.
μ	The mean of demand.
ρ	The coefficient of correlation of the demands for two manufacturers.
α	The buffer stock service level.
L	The replenishment lead time.
θ	The unit cost of buffer stock per a certain duration of time (e.g., a year).
$\Phi^{-1}(\cdot)$	The inverse cumulative distribution function of standard normal distribution.
VBCT	The value of blockchain technologies for avoiding cheating.
OVBCT	The overall value of blockchain technologies for the buffer stock sharing scheme.
Ω	An important parameter to define the buffer stock level under the buffer stock sharing scheme.
RP	The profit reduction of the supplier when the manufacturers reduce buffer stock levels.
MC	The overall cost for manufacturers when the logistics service is considered.
SP	The overall profit for the supplier when the logistics service is considered.
Superscripts and subscripts	
CO	Situations when the two manufacturers join a co-opetitive buffer stock sharing scheme.
MH	Situations when cheating exists.
BCT	Situations when blockchain helps reduce lead time.
LTR	Situations when lead time is reduced by blockchain.
New	Situations when the buffer stock service level is improved by blockchain.
UCL	Situations when lead time is stochastic.
$\sum_{i=1}^{n}$	Situations with multiple manufacturers.
pre	Situations when premier logistics services are adopted but blockchain is absent.
ord	Situations when ordinary logistics services are used and the blockchain facilitated co-opetitive buffer stock sharing scheme is applied.
Tilde/"head"	
$\hat{}$	Situations with cheating.
$\tilde{}$	Situations when the two manufacturers have different lead times.
$\frown$	Situations when the two manufacturers have different service levels.

Table 3. Sensitivity analysis towards $OVBCT_i$ $(i = (1, 2))$.
[$\uparrow$ = increase; $\downarrow$ = decrease; $-$ = "no-change"]

	$OVBCT_1$	$OVBCT_2$
$\gamma\uparrow$	$\downarrow$	$\downarrow$
$\rho\uparrow$	$\downarrow$	$\downarrow$
$\theta\uparrow$	$\uparrow$	$\uparrow$
$\alpha\uparrow$	$\uparrow$	$\uparrow$
$L\uparrow$	$\uparrow$	$\uparrow$
$\sigma_1\uparrow$	$\uparrow$	$\downarrow$ if $\sigma_1 > \sigma_2$ $-$ if $\sigma_1 = \sigma_2$ $\uparrow$ if $\sigma_1 < \sigma_2$
$\sigma_2\uparrow$	$\downarrow$ if $\sigma_2 > \sigma_1$ $-$ if $\sigma_2 = \sigma_1$ $\uparrow$ if $\sigma_2 < \sigma_1$	$\uparrow$

Table 4. Sensitivity analysis towards $\alpha^{CO}_{New,BCT}$.
[$\uparrow$ = increase; $\downarrow$ = decrease; $-$ = "no-change"]

	$\alpha^{CO}_{New,BCT}$			$\alpha^{CO}_{New,BCT}$
$\gamma\uparrow$	$\downarrow$		$\sigma_1\uparrow$	$\downarrow$ if $\sigma_1 > \sigma_2$ $-$ if $\sigma_1 = \sigma_2$ $\uparrow$ if $\sigma_1 < \sigma_2$
$\rho\uparrow$	$\downarrow$			
$\theta\uparrow$	$-$			
$\alpha\uparrow$	$\uparrow$		$\sigma_2\uparrow$	$\downarrow$ if $\sigma_2 > \sigma_1$ $-$ if $\sigma_2 = \sigma_1$ $\uparrow$ if $\sigma_2 < \sigma_1$
$L\uparrow$	$-$			

SHORT BIO

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