The following publication X. Chen, Y. Li, H. Ma, H. Tang and Y. Xie, "A Novel Variable Exponential Discrete Time Sliding Mode Reaching Law," in IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 68, no. 7, pp. 2518-2522, July 2021 is available at https://doi.org/10.1109/TCSII.2021.3051904.

A Novel Variable Exponential Discrete Time Sliding Mode Reaching Law

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***Abstract*—A new variable exponential discrete-time sliding mode control (DSMC) reaching law is proposed to suppress the chattering phenomenon and accelerate the reaching speed for the switching function. The variable exponential reaching law consists of two-phase different exponential term. The main effect of the first phase exponential reaching law is to reduce reaching steps. The second phase exponential reaching law can decrease the magnitude of quasi-sliding-mode domain (QSMD). Otherwise, the disturbance term is restrained by second order difference function which can also significantly diminish the range of QSMD. The reaching steps of the reaching law to converge to QSMD are derived from this new reaching law. Meanwhile, the dynamic analysis of the DSMC system based on new reaching law is presented. Finally, the mathematical simulations are conducted to preliminarily verify the results of theoretical analysis.**

***Index Terms*—Sliding mode control (SMC), uncertain systems, dynamic analysis, reaching law, quasi-sliding-mode domain.**

I. Introduction

As one kind of nonlinear control strategy, sliding mode control (SMC) strategy has pretty good effectiveness for uncertain or incompletely modelled system. There are some advantages about SMC strategy compared with other feedback control system such as good performance to stabilize the complex nonlinear systems, robustness for the perturbation or unmodeled components and fast response [1]–[4]. Hence, SMC strategy has been widely applied into biomedical robot, aerospace industry, power control system and high precision motion control, etc [5], [6]. The sliding mode control can be divided into two strategies: discrete-time SMC strategy (DSMC) and continuous-time SMC strategy (CSMC). Compared with CSMC, the most prominent feature of DSMC is that its switching frequency is limited. According to this characteristic, the DSMC which can achieve low sampling frequencies has been the popular research fields as the industrial computers' applications in the practical control engineering [7]–[9].

This work was supported in part by National Natural Science Foundation of China (51575544), the General Research Fund (GRF) of the Research Grants Council (RGC) of Hong Kong, China (Grant No. PolyU 152137/19E).(Corresponding author: Yangmin Li).

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The DSMC which reserves the switching term is usually called the DSMC based reaching law. Obviously, the dynamics of DSMC system based reaching law includes two stages: the first stage is approaching the switching surface; the second stage is to realize the sliding motion in the sliding manifold. Due to the DSMC based reaching law still contains the switching function, the chattering phenomenon cannot be allayed. As for the DSMC based equivalent control, the over large control effort is inevitable for lack of the approaching process. Both for the DSMC based reaching law and the DSMC based equivalent control, the sliding mode state cannot stay on the sliding surface and reach the origin. It will produce a quasi-sliding-mode domain (QSMD) and a chattering motion around the origin [10], [11]. These problems may lead to slow control response and system instability. In order to figure out how to solve these problems, different DSMC strategies have been proposed by previous researchers all over the world [12]–[14]. In literature [13], Qu, et al. proposed a modified reaching law which can achieve the convergence of the system state. Meanwhile, the time steps for switching function to reach the QSMD and the range of sliding manifold for the closed-loop control system were calculated and simulated.

However, the chattering problem of DSMC is the main factor that restricts its performance. In order to alleviate the chattering amplitude of the sliding manifold (the width of QSMD) and further improve the approach speed, a new DSMC reaching law is designed in this paper. In second section, the objective model is considered as the combination of linear dynamic system and bounded nonlinear disturbance. In third section, the design procedure of the related DSMC reaching law is given. Then, the accurate quasi sliding mode domain width and reaching steps of this method are obtained through theoretical analysis. In fourth section, numerical simulations are conducted to verify the effect of this new DSMC strategy compared with cutting-edge research results in [13]. Finally, conclusion is presented to summarize this paper.

II. Preliminaries

The state-space representations of a physical system can be derived as

$$\dot{\mathbf{d}} = \mathbf{A}\mathbf{d} + \mathbf{B}v + \mathbf{P}. \qquad (1)$$

We apply zero-order-holder and set T as sample time. For example, $v(t) = v(n)$ in the interval time $[nT, (n+1)T)$. Then, the state-space equation of the second-order discrete time system can be derived as

$$\mathbf{d}_{n+1} = \mathbf{f}\mathbf{d}_n + \mathbf{\Gamma}v_n + \mathbf{P}_n \qquad (2)$$

where

$$\begin{array}{l} \mathbf{f}=e^{\mathbf{A}T}, \mathbf{\Gamma}=\int_0^T e^{\mathbf{A}\tau} d\tau \mathbf{B} \\ \mathbf{P}_n=\int_0^T e^{\mathbf{A}\tau} \mathbf{P}\left((n+1) T-\tau\right) d\tau . \end{array} \tag{3}$$

It is clearly that $\mathbf{f}$ and $\mathbf{\Gamma}$ are constant systems matrices. And we assume the overall perturbation of the system is bounded and continuous, thus $\mathbf{P}_n=\int_0^T e^{\mathbf{A}\tau} \mathbf{P}\left((n+1) T-\tau\right) d\tau$. Then, the switching function is

$$s(n)=\mathbf{c}^T \mathbf{d}_n . \tag{4}$$

III. Novel Variable Exponential Reaching Law

In this part, a novel variable exponential reaching law is proposed and the control system is designed to achieve working objectives. The new variable exponential reaching law is composed of self-adaptive exponential term, variable coefficient switching term and disturbance compensator.

The new DSMC reaching law called variable exponential reaching law SMC (VSMC) is proposed as

$$s(n+1)=\varpi \Lambda(n) s(n)-\frac{chf(s(n)) \mu}{\Lambda(n)} sign(s(n))+\varepsilon(n), \tag{5}$$

$$\Lambda(n)=\kappa+(1-\kappa) e^{-\varphi|s(n)|^\gamma}, \tag{6}$$

$$\varepsilon(n)=\mathbf{c}^T\left(\mathbf{P}_n-2 * \mathbf{P}_{n-1}+\mathbf{P}_{n-2}\right), \tag{7}$$

$$chf(s(n))=\left\{\begin{array}{c} 1,|s(n)|>\sigma \\ \frac{|s(n)|}{\sigma},|s(n)| \leq \sigma, \end{array}\right. \tag{8}$$

where

$$\mu>0,0<\varpi<1,0<\kappa<1,0<\sigma<1, \gamma>0, \varphi>0. \tag{9}$$

As learned from [15], the bound of the gradient for disturbance $\mathbf{P}_n$ can be written as

$$|\varepsilon(n)| \leq \delta \leq \mu . \tag{10}$$

Solving(2)(4)(5), the control signal $v(n)$ can be generated as

$$v(n)=-\left(\mathbf{c}^T \mathbf{\Gamma}\right)^{-1}\left[\begin{array}{c} \mathbf{c}^T \mathbf{f} \mathbf{d}_n-\varpi \Lambda(n) s(n)+2 \mathbf{c}^T \mathbf{P}_{n-1} \\ -\mathbf{c}^T \mathbf{P}_{n-2}+\frac{chf(s(n)) \mu}{\Lambda(n)} sign(s(n)) \end{array}\right]. \tag{11}$$

The value of $\mathbf{P}_{n-1}$ and $\mathbf{P}_{n-2}$ have to be solved in order to solute (11). According to (2), the $\mathbf{P}_{n-1}$ and $\mathbf{P}_{n-2}$ can be derived as

$$\begin{array}{l} \mathbf{P}_{n-1}=\mathbf{d}_n-\mathbf{f} \mathbf{d}_{n-1}-\mathbf{\Gamma} v_{n-1} \\ \mathbf{P}_{n-2}=\mathbf{d}_{n-1}-\mathbf{f} \mathbf{d}_{n-2}-\mathbf{\Gamma} v_{n-2} \end{array}. \tag{12}$$

IV. Stability analysis of the proposed reaching law

There are two theorems proved in this section about the proposed reaching law. These theorems demonstrate the designed sliding mode control systems is stable.

Theorem 1: For the DSMC system (2) with condition (10).

a) If the coefficients of proposed reaching law ϖ, μ, σ satisfy condition $\sqrt{\frac{\mu}{\sigma \varpi}} \leq \kappa$, the system trajectories $s(n)$ from any initial state value will enter this region which defined by VSMC as

$$\Delta=\frac{\delta}{1-\varpi \Lambda(n)+\frac{\mu}{\sigma \Lambda(n)}} . \tag{13}$$

b) Once the $s(n)$ enter the region Δ , it cannot be escaped. In other words, the function $s(n)$ is convergent function.

Proof: a) Firstly, according to Lyapunov function, $[s(n+1)-s(n)] \times[s(n+1)+s(n)]<0$ can be the convergent condition. In order to prove *Theorem 1*, four cases about $s(n)$ must be considered. The detailed process of proof is expounded as follows:

Case 1: If $s(n)>\sigma$, applying (8) and because of $0<\varpi<1$, $\mu>0$, so

$$s(n+1)-s(n)=(\varpi-1) \Lambda(n) s(n)-\frac{\mu}{\Lambda(n)}+\varepsilon(n) . \tag{14}$$

Due to (6)(9)(10), it can be known $(\varpi-1) \Lambda(n) s(n)<0$. Also, $-\frac{\mu}{\Lambda(n)}+\varepsilon(n)<0$. Therefore, $s(n+1)-s(n)<0$. Obviously, $s(n+1)$ is monotone decreasing function in interval $(\sigma,+\infty)$.

Case 2: If $0<s(n) \leq \sigma$, applying (8) and because of $0<\varpi<1$, $\mu>0$, so

$$s(n+1)=\left[\varpi \Lambda(n)-\frac{\mu}{\sigma \Lambda(n)}\right] s(n)+\varepsilon(n) . \tag{15}$$

In order to guarantee $s(n)$ is monotone decreasing function in interval $(0, \sigma]$, the coefficient $\varpi \Lambda(n)-\frac{\mu}{\sigma \Lambda(n)}$ needs to satisfy

$$0<\left[\varpi \Lambda(n)-\frac{\mu}{\sigma \Lambda(n)}\right]<1 . \tag{16}$$

Assuming

$$\left[\varpi \Lambda(n)-\frac{\mu}{\sigma \Lambda(n)}\right]<1, \tag{17}$$

because

$$0<\varpi \Lambda(n)<1, \frac{\mu}{\sigma \Lambda(n)}>0, \tag{18}$$

so (17) is true.

Similarly, assuming

$$\left[\varpi \Lambda(n)-\frac{\mu}{\sigma \Lambda(n)}\right]>0, \tag{19}$$

thus

$$\varpi \Lambda(n)^2-\frac{\mu}{\sigma}>0 . \tag{20}$$

Solving (20) leads to

$$\Lambda(n)>\sqrt{\frac{\mu}{\sigma \varpi}} \quad or \quad \Lambda(n)<-\sqrt{\frac{\mu}{\sigma \varpi}} . \tag{21}$$

Due to $\kappa<\Lambda(n)<1$, it can be obtained

$$\sqrt{\frac{\mu}{\sigma \varpi}} \leq \kappa . \tag{22}$$

Hence, under the conditions (10)(22) and reaching law (15), the following deduction of system state can be derived

$$\begin{aligned} s(n+1)-s(n) & =\left[\varpi \Lambda(n)-\frac{\mu}{\sigma \Lambda(n)}-1\right] s(n)+\varepsilon(n) \\ & \leq\left[\varpi \Lambda(n)-\frac{\mu}{\sigma \Lambda(n)}-1\right] s(n)+\delta . \end{aligned} \tag{23}$$

When

$$\left[\varpi\Lambda\left(n\right)-\frac{\mu}{\sigma\Lambda\left(n\right)}-1\right]s\left(n\right)+\delta<0, \tag{24}$$

$s\left(n+1\right)-s\left(n\right)<0$ holds. Since (16) and $s\left(n\right)>0$, solving (24), the solution is

$$s\left(n\right)>\Delta_1=\frac{\delta}{1-\varpi\Lambda\left(n\right)+\frac{\mu}{\sigma\Lambda(n)}}. \tag{25}$$

Also, by deriving summation formula, we get

$$\begin{aligned}s\left(n+1\right)+s\left(n\right)&=\left[\varpi\Lambda\left(n\right)-\tfrac{\mu}{\sigma\Lambda(n)}+1\right]s\left(n\right)+\varepsilon\left(n\right)\\&\geq\left[\varpi\Lambda\left(n\right)-\tfrac{\mu}{\sigma\Lambda(n)}+1\right]s\left(n\right)-\delta.\end{aligned} \tag{26}$$

When

$$\left[\varpi\Lambda\left(n\right)-\frac{\mu}{\sigma\Lambda\left(n\right)}+1\right]s\left(n\right)-\delta>0, \tag{27}$$

$s\left(n+1\right)+s\left(n\right)>0$ holds. Since (16) and $s\left(n\right)>0$, solving (27), the solution is

$$s\left(n\right)>\Delta_2=\frac{\delta}{1+\varpi\Lambda\left(n\right)-\frac{\mu}{\sigma\Lambda(n)}}. \tag{28}$$

In order to guarantee $s\left(n+1\right)-s\left(n\right)<0$ and $s\left(n+1\right)+s\left(n\right)>0$, the switching state $s\left(n\right)$ must satisfies

$$\left|s\left(n\right)\right|>\Delta=\max\left\{\Delta_1,\Delta_2\right\}. \tag{29}$$

As $0<\varpi\Lambda\left(n\right)-\frac{\mu}{\sigma\Lambda(n)}<1$, it can be obtained

$$\Delta_1>\Delta_2,\Delta_1>\delta. \tag{30}$$

Therefore, the $s\left(n\right)$ is monotone decreasing function in interval $\left(\Delta_1,\sigma\right]$.

Case 3: If $s\left(n\right)<-\sigma$, applying (8) and because of $0<\varpi<1$, $\mu>0$, so

$$s\left(n+1\right)-s\left(n\right)=\left(\varpi-1\right)\Lambda\left(n\right)s\left(n\right)+\frac{\mu}{\Lambda\left(n\right)}+\varepsilon\left(n\right). \tag{31}$$

Similarly, in view of (6)(9)(10), we can get $\left(\varpi-1\right)\Lambda\left(n\right)s\left(n\right)>0$. At the same time, $\frac{\mu}{\Lambda(n)}+\varepsilon\left(n\right)>0$. Hence, $s\left(n+1\right)-s\left(n\right)>0$ is obtained. Thus, $s\left(n{+}1\right)$is monotone increasing function in interval $\left(-\infty,-\sigma\right)$.

Case 4: If $-\sigma\leq s\left(n\right)<0$, applying (8) and because of $0<\varpi<1$, $\mu>0$, so

$$s\left(n+1\right)=\left[\varpi\Lambda\left(n\right)-\frac{\mu}{\sigma\Lambda\left(n\right)}\right]s\left(n\right)+\varepsilon\left(n\right). \tag{32}$$

Similarly, the following results can be derived using the same analytical method as mentioned in case 2:

If the switching state $s\left(n\right)$ satisfies

$$\left|s\left(n\right)\right|>\Delta=\max\left\{\Delta_{11},\Delta_{22}\right\}, \tag{33}$$

where

$$\Delta_{11}=\frac{-\delta}{1-\varpi\Lambda\left(n\right)+\frac{\mu}{\sigma\Lambda(n)}}, \tag{34}$$

$$\Delta_{22}=\frac{-\delta}{1+\varpi\Lambda\left(n\right)-\frac{\mu}{\sigma\Lambda(n)}}. \tag{35}$$

both $s\left(n+1\right)-s\left(n\right)>0$ and $s\left(n+1\right)+s\left(n\right)<0$ can hold. Due to

$$\Delta_1=\Delta_{11},\Delta_2=\Delta_{22}, \tag{36}$$

and in view of (30), we can obtain $s\left(n\right)$ is monotone increasing function in interval $\left[-\sigma,-\Delta_1\right)$.

Combined with the above conclusions (29)(30)(33)(36), it can be summarized that the QSMD of the new reaching law is $\Delta=\Delta_1=\Delta_{11}$.

b) According to proof (a), the switching function $s\left(n\right)$ is convergent when $\left|s\left(n\right)\right|>\Delta$.Therefore, once $s\left(n{+}1\right)$ enters the QSMD, it cannot escape.

Theorem 2: The steps of the system trajectory reaching QSMD are no more than constant values k^* when the initial states are different. The values of convergent steps k^* can be represented as follows:

$$\begin{gathered}k^*=k_1^*+k_2^*=\log_{\varpi}\frac{{\psi_2}^*\left[\frac{\mu}{{\psi_3}^*}-\delta\right]+\sigma(1-\varpi)}{|s(0)|{\psi_1}^*(1-\varpi)+{\psi_2}^*\left[\frac{\mu}{{\psi_3}^*}-\delta\right]}\\+\log_{\varpi}\frac{\delta}{\delta-(1-\varpi)\sigma\theta(0)},|s(0)|>\sigma\\k^*=k_2^*=\log_{\varpi}\frac{\delta}{\delta-(1-\varpi)|s(0)|\theta(0)},|s(0)|\leq\sigma.\end{gathered} \tag{37}$$

Proof: Case 1: If $s(n)>\sigma,n=0,1,\cdots,k$, the value expression of $s(k)$ can be derived from (5). In order to save page layout, here's main derivation process as follows:

$$\begin{aligned}s(k)=&\varpi^k s(0)\prod_{n=0}^{k-1}\Lambda(n)\\&-\sum_{i=0}^{k-2}\varpi^{k-1-i}\underbrace{\left[\Lambda(k-1)\times\cdots\times\Lambda(i+1)\right]}_{k-1-i}\\&\times\left[\frac{\mu}{\Lambda(i)}-\varepsilon(i)\right]-\left[\frac{\mu}{\Lambda(k-1)}-\varepsilon(k-1)\right].\end{aligned} \tag{38}$$

When $s\left(n\right)$ reaches the boundary $s\left(k\right)=\sigma$, an algebra yields

$$\begin{aligned}&s(k)\leq\varpi^k s(0)\psi_1-\psi_2\left[\frac{\mu}{\psi_3}-\delta\right]\frac{1-\varpi^k}{(1-\varpi)}=\sigma\\&\Rightarrow\frac{\varpi^k s(0)\psi_1(1-\varpi)-\psi_2\left[\frac{\mu}{\psi_3}-\delta\right]\left[1-\varpi^k\right]-\sigma(1-\varpi)}{(1-\varpi)}=0\\&\Rightarrow\frac{\varpi^k\left\{s(0)\psi_1(1-\varpi)+\psi_2\left[\frac{\mu}{\psi_3}-\delta\right]\right\}-\psi_2\left[\frac{\mu}{\psi_3}-\delta\right]-\sigma(1-\varpi)}{(1-\varpi)}=0.\end{aligned} \tag{39}$$

In *Theorem 1*, we know $s\left(n\right)$ is convergent. Thus, a real number k_1^* can be yielded as

$$k_1^*=\log_{\varpi}\frac{\left[\frac{\mu}{{\psi_3}^*}-\delta\right]+\frac{\sigma(1-\varpi)}{{\psi_2}^*}}{s(0)\Lambda(0)\left(1-\varpi\right)+\left[\frac{\mu}{{\psi_3}^*}-\delta\right]}, \tag{40}$$

where

$${\psi_1}^*=\prod_{n=0}^{k^*-1}\Lambda(n),{\psi_2}^*=\prod_{n=1}^{k^*-1}\Lambda(n) \tag{41}$$

$${\psi_3}^*=\frac{\left[\varpi^{k^*-1}+\varpi^{k^*-2}+\cdots+1\right]}{\frac{\varpi^{k*-1}}{\Lambda(0)}+\frac{\varpi^{k*-2}}{\Lambda(1)}+\cdots+\frac{1}{\Lambda(k^*-1)}}. \tag{42}$$

Case 2: If $s(n) < -\sigma, n = 0, 1, \cdots, k$, in a similar way as in the derivation of (39), when $s(n)$ reaches the boundary $s(k) = -\sigma$, an algebra yields

$$\begin{array}{l} s(k) \geq \varpi^k s(0)\psi_1 + \psi_2 \left[\frac{\mu}{\psi_3} - \delta\right] \frac{1-\varpi^k}{(1-\varpi)} = -\sigma \\ \Rightarrow \frac{\varpi^k \left\{s(0)\psi_1(1-\varpi) - \psi_2\left[\frac{\mu}{\psi_3} - \delta\right]\right\} + \psi_2\left[\frac{\mu}{\psi_3} - \delta\right] + \sigma(1-\varpi)}{(1-\varpi)} = 0. \end{array} \tag{43}$$

Then, we can find the same real number k_1^*, it yields that

$$k_1^* = \log_\varpi \frac{\left[\frac{\mu}{\psi_3{}^*} - \delta\right] + \frac{\sigma(1-\varpi)}{\psi_2{}^*}}{|s(0)|\,\Lambda(0)\,(1-\varpi) + \left[\frac{\mu}{\psi_3{}^*} - \delta\right]}. \tag{44}$$

Case 3: If $0 \leq s(n) \leq \sigma, n = 0, 1, \cdots, k$, in view of (5)-(8), then

$$\begin{array}{l} s(n+1) = \varpi \Lambda(n)\, s(n) - \frac{|s(n)|\mu}{\sigma \Lambda(n)} + \varepsilon(n) \\ \quad = \varpi \left[\Lambda(n) - \frac{\mu}{\varpi\sigma\Lambda(n)}\right] s(n) + \varepsilon(n). \end{array} \tag{45}$$

Assuming

$$\Lambda(n) - \frac{\mu}{\varpi\sigma\Lambda(n)} = \theta(n), \tag{46}$$

thus, it follows from (45) that

$$\begin{array}{l} s(k) = \varpi\theta(k-1) + \varepsilon(k-1) \\ = \varpi^k s(0) \prod\limits_{n=0}^{k-1} \theta(n) \\ - \sum\limits_{i=0}^{k-2} \varpi^{k-1-i} \underbrace{[\theta(k-1) \times \cdots \times \theta(i+1)]}_{k-1-i} \\ \times \varepsilon(i) + \varepsilon(k-1), \end{array} \tag{47}$$

where $s(0)$ is the initial state of $s(n)$. Since $|\varepsilon(n)| \leq \delta$, so

$$\begin{array}{l} s(k) \leq \varpi^k s(0) \prod\limits_{n=0}^{k-1} \theta(n) \\ + \sum\limits_{i=0}^{k-2} \varpi^{k-1-i} \underbrace{[\theta(k-1) \times \cdots \times \theta(i+1)]}_{k-1-i} \times \delta + \delta. \end{array} \tag{48}$$

Setting

$$\varphi_1 = \prod_{n=0}^{k-1} \theta(n), \varphi_2 = \prod_{n=1}^{k-1} \theta(n), \tag{49}$$

then

$$\begin{array}{l} s(k) \leq \varpi^k s(0)\varphi_1 + \sum\limits_{i=0}^{k-2} \varpi^{k-1-i}\varphi_2 \times \delta + \delta \\ \quad \leq \varpi^k s(0)\varphi_1 + \delta\varphi_2 \sum\limits_{i=0}^{k-1} \varpi^{k-1-i} \\ \quad \leq \varpi^k s(0)\varphi_1 + \delta\varphi_2 \frac{1-\varpi^k}{1-\varpi}. \end{array} \tag{50}$$

Due to $s(n)$ is convergent which has been proven in *Theorem 1*, there must exist a real number k_2^* which denotes the system trajectory will firstly cross zero in the $k_2^* + 1$ steps. Solving

$$\varpi^{k_2^*} s(0)\varphi_1 + \delta\varphi_2 \frac{1-\varpi^{k_2^*}}{1-\varpi} = 0 \tag{51}$$

leads to

$$\begin{array}{l} -\delta\varphi_2 \left[1 - \varpi^{k_2^*}\right] = (1-\varpi)\, \varpi^{k_2^*} s(0)\varphi_1 \\ \Rightarrow \left[\delta\varphi_2 - (1-\varpi)\, s(0)\varphi_1\right] \varpi^{k_2^*} = \delta\varphi_2. \end{array} \tag{52}$$

Then, we can get

$$k_2^* = \log_\varpi \frac{\delta\varphi_2}{\delta\varphi_2 - (1-\varpi)\, s(0)\varphi_1}. \tag{53}$$

Case 4: If $-\sigma \leq s(n) < 0, n = 0, 1, \cdots, k$, with the condition (5)-(8), similar to the process of (45)-(50), the following derivation can be obtained

$$\varpi^{k_2^*} |s(0)|\, \varphi_1 + \delta\varphi_2 \frac{1-\varpi^{k_2^*}}{1-\varpi} = 0. \tag{54}$$

Solving (54) leads to

$$\begin{array}{l} k_2^* = \log_\varpi \frac{\delta\varphi_2}{\delta\varphi_2 - (1-\varpi)|s(0)|\varphi_1} \\ \quad = \log_\varpi \frac{\delta}{\delta - (1-\varpi)|s(0)|\theta(0)}. \end{array} \tag{55}$$

If $|s(0)| = \sigma$,thus

$$k_2^* = \log_\varpi \frac{\delta}{\delta - (1-\varpi)\,\sigma\theta(0)}. \tag{56}$$

According to the aforementioned four cases, the system state trajectory can firstly cross the sliding surface within $k_1^* + k_2^*$ steps when the initial state $|s(0)| > \sigma$; If the initial state $|s(0)| \leq \sigma$, the system state trajectory need at least k_2^* steps to firstly cross the sliding surface. Therefore, the *Theorem 2* holds.

V. Numerical Examples

Assuming the discrete time second-order system with time-varying uncertainties in (2) can be written as

$$\begin{array}{c} \mathbf{f} = \begin{bmatrix} 1.2 & 0.1 \\ 0 & 0.6 \end{bmatrix}, \mathbf{\Gamma} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ \mathrm{P}_n = a \times \begin{bmatrix} 0 \\ 2\sin(2nT\pi) + 0.5 \end{bmatrix}, \\ \mathbf{d}_0 = \begin{bmatrix} 2 \\ -6 \end{bmatrix}, \mathbf{c}^T = \begin{bmatrix} 5 & 1 \end{bmatrix} \end{array} \tag{57}$$

to implement the mathematical simulation. Two simulation cases are implemented for the analysis of the system dynamics. According to condition (7) and [13, eq.(7)(12)], the magnitude of disturbances can be calculated using MATLAB software. After calculation, we know the magnitude of disturbances of proposed algorithm $\delta = 0.0079$ is much smaller than that in the paper [13] which the upper bound of disturbance is $\delta_{13} = 0.1255$.

Case 1: In this example, the same parameters are adopted for the new reaching law and proposed reaching law in [13]. The parameters are settled as $a = 1, \varpi = 0.85, \mu = 0.07, \kappa = 0.7, \varphi = 20, \gamma = 10$, sampling frequency T=0.01s,$\sigma = 0.5$ which satisfy the condition (10)(22), and the switching states of the two reaching laws are shown in Fig. 1. According to (13), the calculational QSMD result of proposed method is $\Delta = 0.0275$. The simulation result can demonstrate the derivation of the proposed method. As compared with the QSMD $\Delta_{13} = 0.3641$ in [13], the magnitude of new method's QSMD is significantly diminished. Taking (37) into account, the convergent steps of the proposed reaching law are $k^* = 4$ that is accord with the result shown in Fig. 1. Obviously, the reaching performance of new method is better than that in [13].

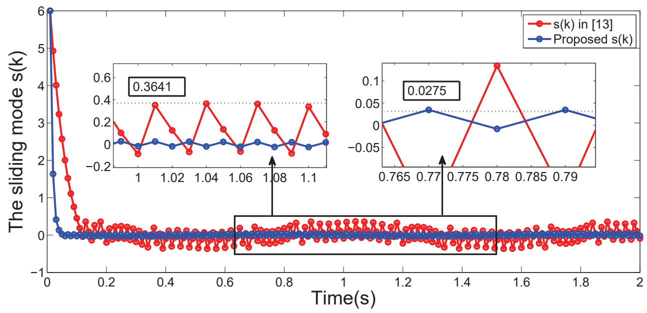

Fig. 1. Changes of switching state in Case 1.

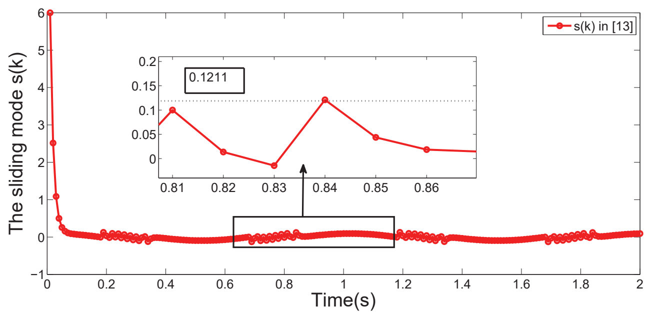

Fig. 2. Changes of switching state [13] in Case 2.

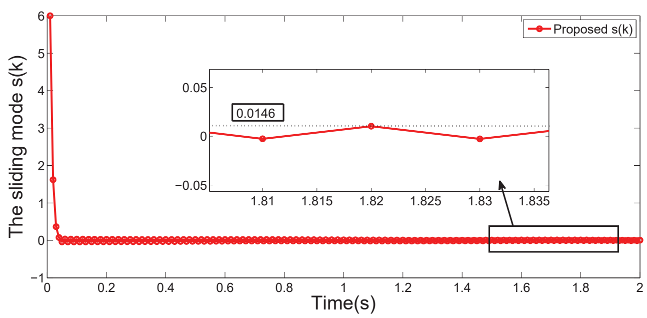

Fig. 3. Changes of switching state with the proposed VSMC in Case 2.

Case 2: The objective is to find the minimum QSMD in this example. The parameters need to be carefully tuned to get the best QSMD and convergence rate in the numerical example. After implementing different parameters, the optimal coefficients are obtained. Choosing $a = 1, \varpi = 0.9, \mu = 0.36, \kappa = 0.9, \varphi = 20, \gamma = 10$, sampling frequency T=0.01s,σ=0.8, the simulation is conducted under this condition. Fig. 2 shows the switching state of reaching law in paper [13, eq.(23)]. Fig. 3 shows the switching state of this proposed VSMC. According to (13), the width of QSMD in Fig. 3 is $\Delta = 0.0146$. By comparison, it is found that the control performance of the VSMC is obviously better than that of the literature [13].

VI. Conclusion

A novel DSMC reaching law is described in this article which can significantly improve the performance of the switching function for reaching speed and chattering suppression. The new reaching law contains a variable exponential term and second order difference error term. The convergence and effectiveness of this new reaching law were proved by mathematical analysis. Meanwhile, the width of QSMD and the reaching steps of the DSMC system were theoretically analysed. Finally, the numerical simulation was established to verify the theoretical analysis of this new reaching law. The advantages of this novel VSMC are reflected in these simulations by comparing with recently cutting-edge study results in literature [13]. In practical engineering, the proposed controller can significantly reduce control error and increase control speed. In the future, the proposed new sliding mode control will be applied in practical industrial fields such as high precision motion control, robot control and biomechanical control. Some practical applications of this novel VSMC will be further investigated.

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